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Computational investigation of residential users through load aggregator smart grid power optimization problem via mean field game theory

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Abstract : Incentive-based demand response aggregators are widely recognized as a powerful strategy to increase the flexibility of residential community microgrid (RCM) while allowing consumers' assets to participate in the power system operation in critical peak times. This paper presents the computational investigation for an incentive-based residential community [1], where an incentive-based pricing mechanism is adopted to encourage peak demand reduction and share the incentive demand curve with the residential community through the aggregator. Nash Equilibrium strategies, which minimize the total RCM energy cost functions, are found by applying mean field game theory. Computational investigations of the decentralized RCM electric energy problem via mean field game theory are presented for the MG with controllable load household and connected to a main grid with an incentive-based pricing strategy.

Keywords: Keywords: RCM, aggregator, demand response, Mean field game theory, dynamical game, pricing mechanism, dynamical pricing, stochastic control

1 Introduction

The smart grid is much automated and can be integrated into the main supply with distributed generation (DG) sources (primarily renewable), energy storage systems (ESSs), and advanced metering infrastructure. Due to using high technology devices in generation and distribution systems, the smart grid represents an advanced digital system [2]. In addition, the smart grid functionalities, such as real-time monitoring, load balancing, and accurate billing, require the granular collection of smart metering data at frequent time intervals [3].

In this paper, we propose a computational game-theoretical investigation for the incentive-based residential community microgrid with a load aggregator (RCMLG) via an application of Mean Field Game theory (MFG). Mean field game theory is a mathematical framework that studies the behavior of large populations of interacting agents. It combines concepts from game theory and statistical physics to analyze situations where each agent aims to optimize its own objective function while accounting for the aggregate behavior of the entire population. In mean-field games, the interactions between agents are approximated by a mean field, representing the average effect of all other agents on each individual's decision-making process. This approach allows for the analysis of complex systems with a large number of agents, making it applicable to various fields such as economics, biology, and engineering. Mean field game theory has been extensively studied and developed since its introduction by Lasry and Lions [4, 5, 6], and separately by Caines, Huang, and Malhame [7, 8] in 2006, leading to valuable insights into the dynamics and equilibria of large-scale interactive systems.

This paper represents a continuation of the work in [1]. The energy network considered in this framework consists of a main grid generation company (MA) that adopts an incentive-based pricing mechanism to reduce peak demand and it shares the incentive demand curve with the residential microgrid (MG) through the aggregator. The aggregator's objective is to maximize the welfare of the microgrid by finding the optimal equilibrium electricity price in the microgrid. In the constructed MG, household demand is divided into two types: (i) fixed, and (ii) controllable. Households communicate with each other and the main grid through the aggregator and they aim to minimize their individual electricity bill. The numerical outcomes of this research will lead to (i) A new general pricing methodology for the MG connected to an incentive-based pricing MA, (ii) the Nash-Equilibrium strategies for the households and community MG aggregator, and (iii) real-time optimization resulting in a reduction of both households' energy bill and MA peak demand cost.

1.1 MG architecture [1] and paper objectives

Considering the block diagram in [1] represented in Figure 1, the community MG is composed of an aggregator and N households, in the formulated game N is a sufficiently large number. By construction, the aggregator mediates the energy exchange between the microgrid and the main grid along with the energy exchange within the microgrid (i.e. between the households themselves and the households and the main grid).

1.2 Computational investigation for the MGO problem in literature

This subsection surveys mathematical techniques used in literature to model stochasticity in MGO problems. It discusses classical mathematical algorithms, machine learning algorithms, and the MFG computational approach.

- Classical Mathematical Algorithms: Various mathematical solvers improve Microgrid (MG) management capabilities. A general algebraic modeling system (GAMS) is used to evaluate linear equations and handle uncertainties in generation [9]. Multi-scenario mixed-integer linear programming MILP models were used in [10]. In addition, sparse nonlinear optimizer (SNOPT) which is based on GAMS was used in [11] to address scheduling and optimization problems. These algorithms demonstrate effectiveness in handling large-scale MILP problems.

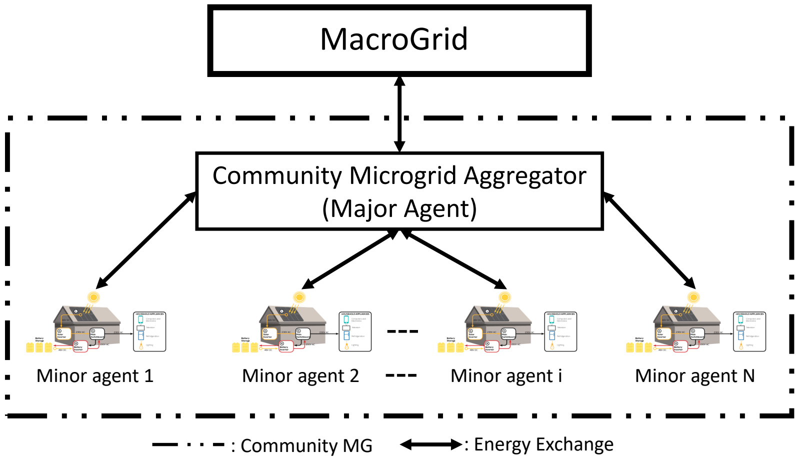

Figure 1: Major-minor agent community microgrid block diagram

- Machine Learning Algorithms: Noyan et al. introduced stochastic methods to approximate the probability distribution function (PDF) of random scenarios in [12]. Scenario-based generation, using discrete distribution sets and Monte Carlo Simulation (MCS), handles uncertainties in photovoltaic (PV) and wind turbines (WT) where introduced in [13]. Fuzzy theory and robust optimization are applied to renewable energy resources (RERs) and address parametric uncertainty was introduced in [14].
- MFG Computational Approach: Mean field games theory utilizes differential calculus to represent players as a continuum. It simplifies the handling of player entries/exits and enables seamless modeling of player generations. The introduction of a social dimension incorporates statistical data on other players. Mean field games offer advantages over N-player games and provide initial approximations to N-player solutions. MFG is a decentralized approach that requires minimal communication between agents. Its offline-solving capability makes it promising for real-time microgrid power network optimization.

1.3 Paper objective

In this paper, the objective is to optimize the cost of the energy exchanged in community MG which is operating in a grid-connected mode where the MA adopts an incentive-based pricing mechanism. The focus is on numerically optimizing the MG optimization problem (MGO). In this context and as seen in Figure 1, the community MG is composed of a sufficiently large number of households and an aggregator. Households that are located within the geographical area of the MG communicate and exchange energy with each other and with the main grid through the aggregator of the community MG. The objective of the aggregator is to maximize the social welfare of the MG households. The aggregator sets the equilibrium price in the MG and sends the aggregated demand to the main grid provider. The aggregator collects the price of electricity from the main grid and then forwards the MG equilibrium price and the main grid electricity price to households. Households in the MG react to the message and the prices provided by the aggregator. Households' objective function is to minimize their energy cost and their demand is assumed to be divided into two types:(i) fixed demand, and (ii) controllable demand. Fixed demand represents all demand of all the appliances that cannot be controlled. Controllable demand represents all appliances that can be monitored and controlled as they are running.

The scope of this paper is to numerically solve the MGO via an application means of dynamical game theory. The objective of the connected households is to minimize their total cost and the aggregator to maximize the social welfare of the MG via means of major agent minor agents mean field game theory. This paper presents a high-level formulation of the load aggregated energy problem and a proof of concept that major-minor agent mean field game theory is a plausible way to find the optimal pricing mechanism deployed in the microgrid and the optimal load management strategies for the micro-grid households in an incentive main grid formulation.

1.4 Paper outline

The rest of the paper is structured as follows: Section II presents a summary of the MGO system dynamics presented in [1]. Section III focuses on the formulation MGO via an application of mean-field game theory. Section IV discusses the algorithmic implementation of the MGO. Section V presents the numerical simulation results for the MGO along with the numerical robust analysis for the proposed algorithms. Finally, in section VI we conclude the paper.

2 MGO system dynamics

This section presents a high-level summary of the MGO problem system dynamics from [1]. For the detailed formulation, we refer the readers to [1].

2.1 Main grid pricing mechanism

In this framework, the power network scheme studies a case MG in Montreal, Quebec where there is only one main energy company generator, Hydro-Quebec (HQ). The current electricity market price in this paper is denoted by p_{MA} dollars per kWh. HQ is motivated to adopt an incentive-based pricing mechanism. Currently, HQ is offering two portfolios for households to choose from:

(a) Winter Credit Option (WCO),
(b) Rate Flex Dynamic Pricing (RFD)

The aim of these portfolios is to reduce peak costs. For the scope of this paper, Portfolio (a) will be studied. Portfolio (a) works as follows: the day before a peak demand event, households will receive a notification about the peak event, and during an event, households are entitled to a 0.5$/kWh for every kilowatt-hour (kWh) curtailed (i.e. that is, not consumed compared with your usual energy use).

Figure 2 depicts the main grid pricing scheme for the WCO portfolios, the readers can infer that the peak threshold credit line is denoted by τ and here $\tau = 4Kwh$; and thus, during a peak event, if the household's demand is less than 4KWh, the household will be credited MA credit $= 0.5\$/Kwh$.

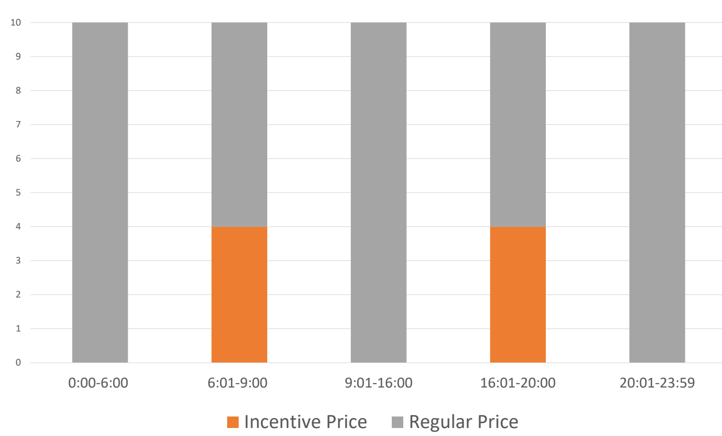

Figure 2: Winter Credit Option (WCO)–Pricing scheme

2.2 MG equilibrium price: P_{MG}

The microgrid price will be divided into two components: (i) the base price [15], the Walrasian equilibrium price that maximizes the social welfare, and (ii) the incentive/penalty during peak time. For more details about the Walrasian equilibrium, we refer the readers to [15] and for the incentive/penalty component, we refer the reader to [1]. Denote by $p^w_{MG}(t)$ the Walrasian equilibrium MG price, $p^w_{MG}(t)$ is proportional to $\bigtriangledown C_u(G_{MG}(t))$ where C_u is the price function which is concave with respect to $G_{MG}(t)$ where $G_{MG}(t)$ is the aggregated load in the MG i.e.

$$G_{MG}(t) = \sum_{i=1}^{i=N} y^i(t) = N \times E\{y(t)\} = N \int_{\Omega_y} y\mu_y(y,s)dy \tag{1}$$

where Ω_y is the range of $y(t)$ for all $0 \leq t \leq T$.

The AG will adopt the Walrasian Equilibrium during the off-peak-event time, and as its base price during the peak event. The AG will set an incentive and penalty credit during peak events to guarantee the grid functioning in the winter credit incentive threshold. During peak-event, the MG will be considered as one entity and the aggregated threshold is $\mathcal{T} = \tau \times N$. The aggregated credited saving of the MG at time t is $\lambda(t)$ where

$$\lambda(t) = \left(\mathcal{T} - \sum_{i=1}^{N} \left(\gamma^i(t) - r^i(t)\right)\right) \tag{2}$$

The AG aims to guarantee fairness and to set p_{MG} as Nash-Equilibrium during the peak-event.

- Incentive credit:

$$\iota^i(t) = \begin{cases} (\tau + r^i(t)) \times \text{MA credit} & r^i(t) \geq 0 \& \gamma^i(t) = 0 \\ (\tau - \gamma^i(t)) \times \text{MA credit} & r^i(t) = 0 \& \gamma^i(t) \leq \tau \end{cases} \tag{3}$$

- Penalty credit for household i when $\gamma^i(t) \geq \tau$ & $r^i(t) = 0$

$$\iota^i(t) = (\tau - \gamma^i(t)) \times \text{MA credit}, \tag{4}$$

where $r^i(t)$ and $\gamma^i(t)$ represents the amount of energy sold or bought by household i at time t.

2.3 Household generation and operation cost dynamics

In this framework, we consider the scenario where households are homogeneous i.e. same technology for generation and storage is deployed at the household's site. The generated energy Θ^i by household i at time t is governed by the following stochastic differential equation [15]:

$$d\Theta^i = P^i_{PV}dt + \epsilon^i_\Theta dW^i_\Theta, \tag{5}$$

where W^i_Θ, $1 \leq i \leq N$ are N independent Wiener process (i.e. Brownian Motion). Denote the probability density function governing the energy generation by $\mu_\theta(\theta, t)$. For the scope of this paper, the propagation of the probability density function for the generation function will be found by analyzing the data collected over the span of 5 years for an MG consisting of 300 households.

The generation cost is mainly due to the operation and maintenance costs (O&M) of the PVs and the cost of storage. Assuming the cost of operation and maintenance O&M is $K_{O\&M} = 0.0535(\$/kWh)$ for generation and $C_{B,M} = 0.001(\$/kWh)$ for storage; the cost of generation for household i is given to be:

$$C^i(t) = K_{O\&M} \times \Theta^i(t) + C_{B,M} \times C_{max} \times dt, \tag{6}$$

where C_{max} is the battery maximum capacity which is assumed uniform for all households i, $1 \leq i \leq N$ and dt is the time increment.

2.4 Energy load management

For simplicity, we assume that household i, $1 \leq i \leq N$ has a time-varying energy demand profile denoted by $y^i(t)$. Households in this framework are assumed to possess a smart house technology with two types of demand: (i) fixed demand denoted by $y^i_f(t)$, and (ii) controllable demand denoted by $y^i_c(t)$. Household i demand $y^i(t)$ is given by:

$$y^i(t) = y^i_f(t) + y^i_c(t) \tag{7}$$

Denote by $\mu_{y_f}(y_f, t)$ the probability density function representing the fixed load appliances. Similarly to the generation density function, the probability density function governing the fixed demand will be calculated from collected data over the span of 4 years for 300 households.

For the controllable load, households monitor the electricity price and accordingly, they set the mode of operation for the controllable appliances.

Using such modes of transition between modes of operation in the controllable appliances, the nature of controllable appliances, and the fact that a load of each mode of operation depends on external conditions (such as external temperature, appliance insulation, etc.), the increment stochastic differential equation will be deployed to model the propagation of the controllable load at time t for each appliance. Thus the total controllable load $y_c(t)$ will be represented by the following equation:

$$dy_c = u_c(t)dt + \epsilon_c W_c(t); \tag{8}$$

where $u_c(t)$ represents the increment control vector action for each controllable appliance and $W_c(t)$ is a Brownian motion covering the stochastic nature of the controllable appliances, and $y^i_c(t)$ is bounded by $Y^c_{min} \leq y^i_c(t) \leq Y^c_{max}$ for all $1 \leq i \leq N$. In this framework, households are assumed to be identical, hence Y^c_{min} and Y^c_{max} are considered uniform for all households and represent the boundary conditions for the controllable load to be in the household's comfort mode of operation. In other words, Y^c_{min} and Y^c_{max} represent the minimum load and maximum load respectively for the controllable appliances to function within the minimum mode and maximum mode of their respective comfortable mode of operation.

2.5 Household energy profile[1]

Each household has an energy profile consisting of load, generation, storage, energy withdrawn, and energy sold. $\Theta^i(t)$ and $y^i(t)$ are the amount of energy generated by household i and the load of household i at time t respectively. The set of decision variables, $u^i_c(t)$, $u^i_r(t)$, and $u^i_\gamma(t)$, represent the controllable load, the discharging control action (i.e. the amount of energy sold by household i, and the charging control (i.e. the amount of energy withdrawn from the MG to charge the battery of household i) respectively. Define

$$\delta^i(t) := \Theta^i(t) + b^i(t) - y^i(t) \tag{9}$$

as the net energy after meeting the current demand. The dynamics of $b^i(t)$, $r^i(t)$ and $\gamma^i(t)$ are as follows:

$$b^i(t + dt) = b^i(t) + P^i_{pv}(t)dt - u^i_c(t)dt - dy_f(t)dt + \epsilon_\theta dW_\theta - \epsilon_c dW_c - r^i(t) + \gamma^i(t) \tag{10}$$

$$r^i(t) = \begin{cases} u^i_r(t)\delta^i(t) \\ \text{s.t. } u^i_r(t) = 0 \text{ when } \delta^i(t) \leq 0 \text{ \& } 0 \leq u^i_r(t) \leq 1 \end{cases} \tag{11}$$

$$\gamma^i(t) = \begin{cases} u^i_\gamma(t) C_{max} dt & \text{if } \delta^i(t) \leq 0 \\ u^i_\gamma(t)\left(C_{max}dt - \delta^i(t)\right) & \text{if } \delta^i(t) > 0 \\ s.t.\ 0 \leq u_\gamma(t) \leq 1 & \text{for all } 0 \leq t \leq T \end{cases} \tag{12}$$

where $0 \leq b^i(t) \leq C_{max}$ for all t and where C_{max} is the storage battery capacity for household i. Two scenarios to be considered; (i) scenario one (**SC1**) where $\delta^i(t) \leq 0$ i.e. there is a shortage at household i and household i is a pure consumer, and (ii) scenario two (**SC2**) where $\delta(t) \geq 0$ where household i is a prosumer i.e. household i has the potential to sell his overflow energy or charge his battery for future use.

In SC1, we have $\delta(t) \leq 0$ then $u_r^* = 0$, $0 \leq u_\gamma^*$ and:

$$db = C_{max} u_\gamma dt + P_{PV} dt - u_c dt - dy_f dt + \epsilon_b dW_b \tag{13}$$

denote by function $f := P_{Pv} - u_c - dy_f$ then db is can be written as follows:

$$db = (C_{max} u_\gamma + f)dt + \epsilon_b dW_b \tag{14}$$

On the other hand scenario SC2 where $\delta(t) > 0$ will be divided into two sub-cases:

Case 1: It is optimal to sell (i.e. $u_\gamma^* = 0$):

$$db = \frac{1 - u_r^*}{1 + u_r^*} f dt + \epsilon_b dW_b \tag{15}$$

Case 2: It is optimal to recharge (i.e. $u_r^* = 0$):

$$db = \frac{1 - u_\gamma^*}{1 + u_\gamma^*} f dt + \frac{u_\gamma^*}{1 + u_\gamma^*} C_{max} dt + \epsilon_b dW_b \tag{16}$$

Assuming P_{PV} and y are piece-wise continuous and differentiable and using (13), (15) and (16) the readers can prove that db is continuous and differentiable and that u_γ and u_r are continuous.

3 MGO: Dynamical game formulation

The objective of this framework is to develop a decentralized control synthesis under large population conditions such that each player's strategy uses only limited information. In particular, we assume that the aggregator's decisions are communicated to the households, while the state of each household is always known to itself but can be estimated by the aggregator using the mean field framework. The key feature of the MFG approach is finding the closed-loop solutions formed by the Mean Field Hamilton Jacobean Bellman equation (MF-HJB) and the Mean Field Fokker Planck Kolomogrov Equation (MF-FPK).

3.1 Mean field Fokker Planck equation

The aggregator in the constructed framework will be solving the Fokker Planck Kolomogrov equation affiliated with the controllable load and the generated energy and will find the propagation of $\mu(y_c, \theta, t)$ with respect to t. The aggregator at any time t is able to find the probability density function $\mu(y_c, y_f, \theta, t) := \mu(y_c, \theta, t) \times \mu_{y_f}(y_f, t)$. Recall Equations (5) and (8) representing the partial differential equation for the generation θ and the controllable load y_c the mean field Fokker Plank Kolomogrov (MF-FPK) equation will be presented in the following equation:

$$\begin{aligned}\frac{\partial \mu(y_c, \theta, t)}{\partial t} =& -\frac{\partial}{\partial \theta} P_{PV} \mu(y_c, \theta, t) + \frac{1}{2}\frac{\partial^2}{\partial \theta^2}(\epsilon_\theta^2 \mu(y_c, \theta, t)) \\ & - \frac{\partial}{\partial y_c}[u_c \mu(y_c, \theta, t)] + \frac{1}{2}\frac{\partial^2}{\partial y_c^2}(\epsilon_c^2 \mu(y_c, \theta, t))\end{aligned} \tag{17}$$

3.2 Mean field Hamilton Jacobean Bellman equation

The microgrid connection to the main grid has two modes of operation: (i) peak event, and (ii) off-peak demand. For the off-peak scenario, we will use the MFG formulation in [15] with one modification where we include the controllable load and the controllable appliances. The equilibrium price in this scenario is the Walrasian equilibrium, we need to note that the incentive and penalty control actions are inactive during the off-peak event. Regarding the second scenario, the peak-event, the cost function $L(b, y, \theta, \gamma, r, t)$ for a generic house i in the microgrid is represented as follows:

$$\begin{aligned} L(\bullet) :=& E \int_0^{T_f} (\gamma(t) (w_1(t) P_{MG}(t) + w_2(t) P_{MA}(t)) \\ & - r(t) P_{MG}(t) + \iota^i(t) + C_{O\&M}(t)) dt \end{aligned} \tag{18}$$

where w_1 and w_2 denote the fraction of demand consumed from the MG and the main grid respectively. From [1], w_1 and w_2 in Equation (18) are as follows:

$$\begin{aligned} w_1(t) =& \min \left\{ 1, \frac{\sum_{i=1}^N r^i(t)}{\sum_{i=1}^N \gamma^i(t)} \right\} \\ w_2(t) =& 1 - w_1(t) \end{aligned} \tag{19}$$

The cost function in (18) becomes:

$$\begin{aligned} L(\bullet) = \int_0^{T_f} & (C_{O\&M} + N \nabla C_u \left(\int_{\Omega_y} y \mu_y dy \right) (\gamma w_1 - r) \\ & + \gamma(t) w_2 P_{MA} + \iota) dt \end{aligned} \tag{20}$$

Hence, the cost to go $J(b, y, \theta, \gamma, r, s)$ is given by:

$$\begin{aligned} J(\bullet) =& E \left[\int_s^{T_f} L(b_t, y_t, \theta_t, \gamma_t, r_t) dt \right] \quad s.t. \\ & b_s = b, y_s = y, \theta_s = \theta, \gamma_s = \gamma, r_s = r \end{aligned} \tag{21}$$

and accordingly the value function $v(\bullet)$ is given by:

$$v(b, y, \theta, \gamma, r, s) = \inf_{u_\gamma, u_r} J(b, y, \theta, \gamma, r, s) \tag{22}$$

The generic household i cost function will be subject to two cases: (i) energy shortage case (i.e. $\delta^i \leq 0$) and (ii) energy surplus case (i.e. $\delta^i(t) \geq 0$). Assuming that all functions are sufficiently smooth then the mean field Hamilton-Jacobi-Bellman equations (MF-HJBs) for each scenario are:

- SC1: $\delta(t) \leq 0$, $u_r^* = 0$ and the MF-HJB-SC1 is:

$$\begin{aligned} -\frac{\partial v}{\partial t} =& - C_{O\&M} + P_{PV} \frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2} \frac{\partial^2 v}{\partial \theta^2} \\ & + \inf_{u_\gamma} \left\{ \gamma (w_1 P_{MG} + w_2 P_{MA}) + C_{max} u_\gamma \frac{\partial v}{\partial b} - \iota \right\} \\ & + \inf_{u_c} \left\{ u_c \frac{\partial v}{\partial y_c} + \frac{\epsilon_c^2}{2} \frac{\partial^2 v}{\partial y_c^2} \right\} \end{aligned} \tag{23}$$

  where $\iota(t) = (\tau - \gamma(t)) \times$ MA credit and thus u_γ^* and u_c^* are given by the following equations:

$$u_\gamma^* = \inf_{u_\gamma} \left\{ C_{max} u_\gamma \left(\frac{\partial v}{\partial b} + \text{MA credit} \right) \right\}$$

$$\Rightarrow u_\gamma^* = \begin{cases} 0 & \text{if } \frac{\partial v}{\partial b} + \text{MA credit} \geq 0 \\ 1 & \text{if } \frac{\partial v}{\partial b} + \text{MA credit} < 0 \end{cases} \tag{24}$$

$$u_c^* = -sgn\left(\frac{\partial v}{\partial y_c}\right) \tag{25}$$

- SC2: $\delta(t) > 0$ i.e. household can either sell the surplus, fill the battery or do nothing. The MF-HJB-SC2 is:

$$\begin{aligned}
-\frac{\partial v}{\partial t} =& C_{O\&M} + P_{PV}\frac{\partial v}{\partial \theta} + \frac{\epsilon_\theta^2}{2}\frac{\partial^2 v}{\partial \theta^2} \\
&+ \inf_{u_\gamma, u_r, u_c} \left\{\gamma\left(w_1 P_{MG} + w_2 P_{MA}\right) - rP_{MG} - \iota\right\} \\
&+ \inf_{u_\gamma, u_r, u_c} \left\{\left(\frac{1-u_\gamma}{1+u_\gamma} + \frac{1-u_r}{1+u_r}\right)(P_{PV} - dy)\frac{\partial v}{\partial b}\right\} \\
&+ \inf_{u_\gamma, u_c} \left\{\frac{u_\gamma}{1+u_\gamma} C_{max}\frac{\partial v}{\partial b}\right\} + \inf_{u_c}\left\{u_c\frac{\partial v}{\partial y_c} + \frac{\epsilon_c^2}{2}\frac{\partial^2 v}{\partial y_c^2}\right\}
\end{aligned} \tag{26}$$

thus:

$$\begin{aligned}
\{u_\gamma^*, u_r^*\} = &\inf_{u_\gamma, u_r} \left(\frac{1-u_\gamma}{1+u_\gamma} + \frac{1-u_r}{1+u_r}\right)(P_{PV} - dy)\frac{\partial v}{\partial b} \\
&+ \frac{u_\gamma}{1+u_\gamma} C_{max}\frac{\partial v}{\partial b}
\end{aligned} \tag{27}$$

$$u_c^* = -\, sgn\left(\frac{\partial v}{\partial y_c}\right) \tag{28}$$

The optimal solutions for the MF-HJB-SC2 equation in (26) are derived. The optimal solutions are presented in Table 1.

Table 1: Optimal solution considering SC2

Scenario	$P_{PV} - dy$	$\frac{\partial v}{\partial b}$+MA credit
$u^*r = 1$ $u^\gamma = 0$	≥ 0	≥ 0
$u^*r = 0$ $u^\gamma = 0$	> 0	< 0
$u^*r = 0$ $u^\gamma = 0$	< 0	≥ 0
$u^*r = 0$ $u^\gamma = 1$	≤ 0	< 0

3.3 Mean field loop

The mean field loop aims to find the fixed point convergence i.e. the pair of the probability density function and its corresponding set of optimal control actions. Recall the equations representing the MF-FPK and MF-HJB in (17), (23), and (26), the MFG loop can be summarized by the following diagram:

$$\begin{array}{ccc}
\mu(y_c, \theta, t) \xrightarrow{(1,3,4)} & & p_{MG}(t) \xrightarrow{(23,26)} v(b, y, \theta, \gamma, r, t) \\
\nwarrow & & \swarrow \\
\nwarrow & & \swarrow \\
& & u_c^*(t), u_\gamma^*(t), u_r^*(t)
\end{array} \tag{29}$$

The loop in Equation (29) summarizes the MFG approach, the coupling in the MF-HJB and MF-FPK, and the coupling in the cost functions and system dynamics of the generic agents and aggregators. The uniqueness of the MFG approach is represented by the converging solution of MF-FPK and control actions represented by $\mu(y_c, \theta, t)$ and $[u_c^*, u_\gamma^*, u_r^*]$ of the loop (29).

4 Algorithmic implementation for the MGO via MFG

In this section, we will present the computational procedures that address the MF-HJB equation in both scenarios: (i) a day with no peak event, and (ii) a day with peak event.

4.1 Boundary specifications and discretization techniques

To solve the MFG equations (HJB and FPK) using numerical methods, two aspects need to be determined: 1) the boundary conditions and 2) the approximation techniques employed to discretize the partial differential equations.

4.1.1 Boundary specifications

The following are the prescribed boundary specifications for implementing the MFG algorithm on the contructed MGO problem:

- $0 \leq t \leq T$, where T is to be quantified.
- $0 \leq u_r, u_\gamma \leq 1$ and the load control is $u_c := [-1, 1]$.
- Power is assumed to be positive and bounded i.e. $0 \leq P_{PV} \leq P_{PV}^{max}$, where P_{PV}^{max} is to be quantified.
- Controllable load $y_c^i(t)$ is bounded for all $1 \leq i \leq N$, i.e. $Y_{min}^c \leq y_c^i(t) \leq Y_{max}^c$, where Y_{min}^c and Y_{max}^c are defined.
- $\mu_\theta(\theta, t)$ and $\mu_y(y, t)$ are specified and known to the households.
- Final cost is zero i.e. $v(\theta, b, y, \gamma, r, T) = 0$.
- Final household i account balance $b^i(T) = 0$ for all $1 \leq i \leq N$.
- C_{max} is uniform for all households and is to be quantified.
- During peak events, MA credit, and the demand curve threshold τ are to be quantified and communicated to households through the microgrid aggregator.

4.1.2 Discretization techniques

To achieve numerical solutions for the partial equations, it is imperative to meet these requirements and employ the designated approximation techniques. The time, energy, household balance, and demand are represented by Δt, $\Delta\theta$, Δb, and Δy, respectively, as step sizes. For the numerical algorithms to converge effectively, these step sizes must fulfill the essential conditions outlined in [16], which ensure the convergence of the Fixed Point Argument Method. Specifically, in this scenario, the conditions are as follows:

$$\begin{gathered}\frac{|P_{PV}^{max}|\Delta t}{(\Delta\theta)^2} \leq \frac{1}{2};\ \frac{\Delta t}{(\Delta y)^2} \leq \frac{1}{2};\\ \frac{|\Upsilon|\Delta t}{(\Delta b)^2} \leq \frac{1}{2};\ \frac{|Y_{max}^c|\Delta t}{(\Delta y_c)^2} \leq \frac{1}{2}\end{gathered} \tag{30}$$

where $\Upsilon := \max\ [C_{max}, \max\ (P_{PV} - \delta y)]$. The methods employed to discretize the first and second-order derivatives of a function g with respect to the variable ρ are outlined below:

1st order derivative:

$$\text{Forward in } \rho : \frac{\partial g(., \rho)}{\partial \rho} = \frac{g(., \rho + \Delta\rho) - g(., \rho)}{\Delta\rho} \tag{31}$$
$$\text{Backward in } \rho : \frac{\partial g(., \rho)}{\partial \rho} = \frac{g(., \rho) - g(., \rho - \Delta\rho)}{\Delta\rho}$$

2nd order derivative:

$$\frac{\partial^2 g}{\partial \rho^2} = \frac{g(., \rho + \Delta\rho) - 2g(., \rho) - g(., \rho - \Delta\rho)}{(\Delta\rho)^2} \tag{32}$$

4.2 Algorithmic procedure for the uniform households

The presented numerical algorithm focuses on the constructed residential community microgrid where statistical information is derived from historically collected data. In this framework, we are assuming that the households possess knowledge of the statistical information, denoted by $\mu_\theta(t, \theta)$ and $\mu_y(t, y)$. The MFG algorithmic implementation provides a numerical solution for the converging solution for mean-field equations presented in the MFG-loop in Equation (29). Algorithm 1 constitutes an iterative method for finding a a fixed point solutionfor the loop in Equation (29) with a converging error ϵ_f to be defined:

Algorithm 1 MFG-FPK and MFG-HJB numerical solution

Input: $\mu_0(\theta, y)$ C_{max} θ_{min} θ_{max} Y^c_{min} Y^c_{max} T ϵ_f

Initialization:
Solve (17) by assuming $u^0_r = 0$, $u^0_\gamma = 0$, $u^0_c = -1$ and denote the solution by $\mu^0_t(\theta, y)$ and assign $k = 1$

Loop: At iteration k, execute the following:
Substitute $\mu^{k-1}_t(\theta, y)$ in (23) and (26) and solve the MF-HJB in both scenarios in the backward direction and denote the solution by $v^k(\theta, y, t)$
Find the optimal controls $u^k_{r,\gamma,c}$ using the following equations (24), (25), (27), and (28) and Table 1
Substitute $u^k_{r,\gamma,c}$ in (23) and (26) and find $\mu^k_t(\theta, y)$ by solving the MF-FPK in (17) in the forward direction
If $|\mu^k_t(\theta, y) - \mu^{k-1}_t(\theta, y)| < \epsilon_f$, let $\mu^k_t(\theta, y) = \mu_t(\theta, y)$ and exit the loop.

Output: $\mu_t(\theta, y)$

Algorithm 2 is employed to generate an optimal strategy for an individual household within the MG, here optimality is defined as the best response strategy. It is important to note that each household, denoted as i (where $1 \leq i \leq N$), possesses complete information about their respective state variables and system dynamics, as indicated in the input section of Algorithm 2. Furthermore, household i also observes the values of $\gamma^i(t)$ and $r^i(t)$, and has control over $u^i_\gamma(t)$, $u^i_r(t)$, and $u^i_c(t)$. The generic household best response algorithm is as follows:

5 Numerical result and robustness analysis: Montreal MG case study

In this section, we examine the computational results of the proposed algorithms for solving the constructed MGO problem, using the Montreal Microgrid case study as the basis. The residential microgrid considered in this study assumes that all households have uniform generations and load potentials. In other words, each household deploys similar solar panel systems and utilizes similar household appliances.

Algorithm 2 Computation of a generic agents' best response control along a sample path during both off-peak event day and peak-event days

For a generic household of the community MG $1 \leq i \leq N$:
Input: Convergent probability density functions (i.e. $\mu_\theta(t,\theta)$, $\mu_y(t,y)$ from Algorithm 1), $P_{MA}(t)$ $\forall t$, C_{max}, $\Omega_\theta := [\theta_{min}, \theta_{max}]$, $\Omega_{y_f} := [y^f_{min}, y^f_{max}]$, and $\Omega_{y_c} := [Y^c_{min}, Y^c_{max}]$

Off-Peak Event Input: MA credit is set to zero, threshold $\tau(t) = 0$, and incentive and penalty $\iota^i(t)$ is set to zero. Here P_{MG} is found via (1)

Peak Event Input: MA credit, and threshold $\tau(t)$ are defined and communicated to the MG households via the aggregator. $\iota^i(t)$ is found using (3) and (4) in case of incentive and in case of penalty respectively. Subsequently $P^i_{MG}(t)$ is found.

Initialization:
Generate two Brownian Process W^i_θ & W^i_c for both $\theta^i(t)$ and $y^i_c(t)$ respectively.
Set $k = 0$, $t = T$, $v(T) = b(T) = 0$, $u^i_\gamma(T) = 0$,
$u^i_r(T) = 0$, and $u^i_c(T) = 0$.

Loop: At iteration k, execute the following:
Substitute $p_{pv}(t)$ in (5) and calculate $\theta^i(t)$
Using (9) find $\delta^i(t)$ and using (11), (12), and (8) find $\gamma^i(t)$, $r^i(t)$, and $y_c(t)$ respectively.
Calculate: $\Lambda_\theta = \int_{\Omega_\theta} \theta\mu_\theta(t,\theta)d\theta$, $\Lambda_y = \int_{\Omega_y} y\mu_y(t,y)dy$,
Using (19) find $w_1(t)$ and following that $w_2(t) = 1 - w_1(t)$ and using (1) find $P_{MG}(t)$
If $\delta^i(t) \leq 0$ then consider Mf-HJB-SC1 in (23) and thus $u^{*,i}_r(t-\Delta t) = 0$ and $u^{*,i}_\gamma(t-\Delta t)$ is found using (24) and $u^{*,i}_c(t-\Delta t)$ is found using (25)
Else, consider MF-HJB-SC2 in (26) and calculate $dy_f = \frac{y_f(t+\Delta t) - y_f(t)}{\Delta t}$ and using (27), and (28), and the results in Table 1 find $u^{*,i}_r(t-\Delta t)$, $u^{*,i}_\gamma(t-\Delta t)$, and $u^{*,i}_c(t-\Delta t)$
Increment k
If $T - \Delta t \times k \leq 0$ exit the loop.

Output: $v^i(t)$, $\theta^i(t)$, $P_{MG}(t)$, $u^i_r(t)$, $u^i_\gamma(t)$, & $u^i_c(t)$

5.1 Numerical results

The initial probability density function $\mu_0(\theta, y)$, represented in Figure 3, is assumed to be known to all agents. Applying Algorithm 1, the convergent solution $\mu_t(\theta, y_c)$ to the MF-FPK equation (17) is shown in Figure 4.

Figure 5 depicts the solution to the MF-HJB for the overall population in the normal day event. Here, we focused on presenting the probability density function during the time of the day where $\theta(t) > 0$ i.e. from 8h:00 to 20h:00.

Figure 6 depicts the results of Algorithm 2 for the generic agent i where $b^i(0) = 2.8$ and $\theta^i(0) = 0$ and for this experiment $C_{max} = 5$, $\theta_{min} = 0$, $\theta_{max} = 3kWh$ $y_c = [0.5, 2.5]$ and $T = 24$. The generic agent i is fed the converging results of Algorithm 1 (i.e. $\mu(\theta, y_c, t)$) which allows the generic agent i to find the equilibrium solutions of $p_m g(t)$, $w_1(t)$ and $w_2(t)$ for all t where $0 \leq t \leq T$.

Figure 7 depicts the generic agent i energy profile during peak-event. In this figure, we can see that during peak-event the controllable load optimal decision is to stay within the minimum mode of operation. In addition, the numerical results prove that the energy cost over the peak event day to a normal day has a reduction of 20.27%. The energy cost for the same generic agent during a normal day was 1.31$ compared to 1.09$ during the peak event day.

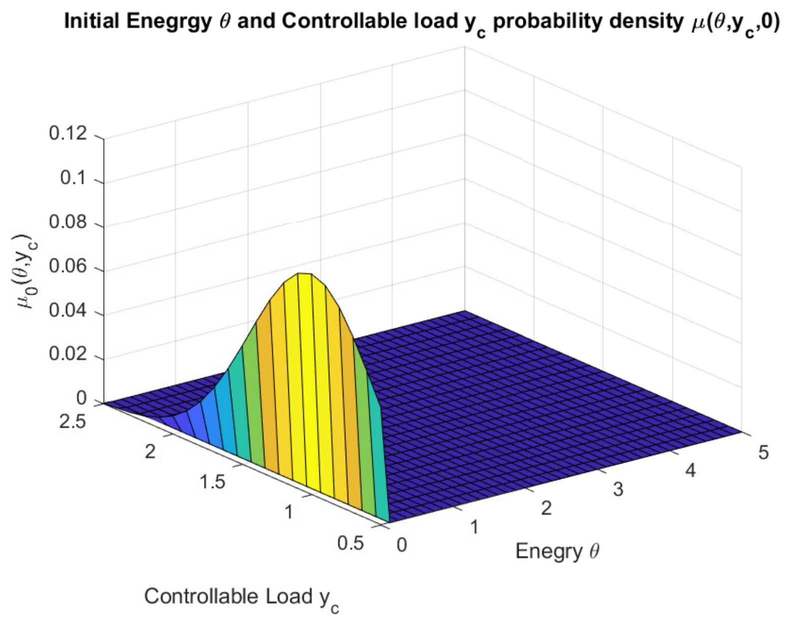

Figure 3: Controllable load and generation probability density function at $t = 0$

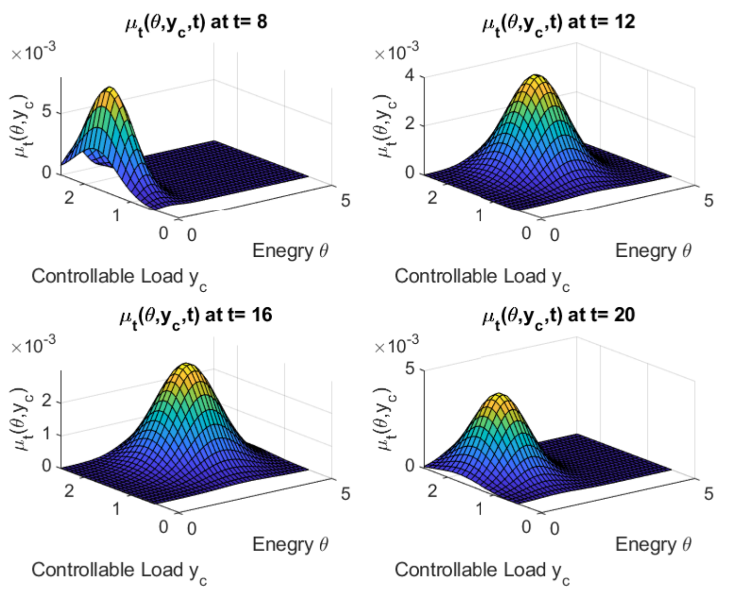

Figure 4: Propagation of the $\mu(\theta, y_c, t)$ versus time

5.2 Algorithms computational performances

Table 2 presents the algorithms' computational performance for the uniform cases, where $\epsilon_f = 10^{-4}$, where ϵ_f signifies the stopping condition for the numerical implementation of the finite difference method. The number of iterations represents the number of iterations required for executing Algorithm 1.

The specifications of the computer on which the simulation was held are 1) Processor Intel(R) Core(TM) i5-6300U CPU @ 2.40GHz, 2496 Mhz, 2 Core(s), 4 Logical Processor(s), 2) RAM: 8GB and 3) system type: 64-operating system, x64-based processor.

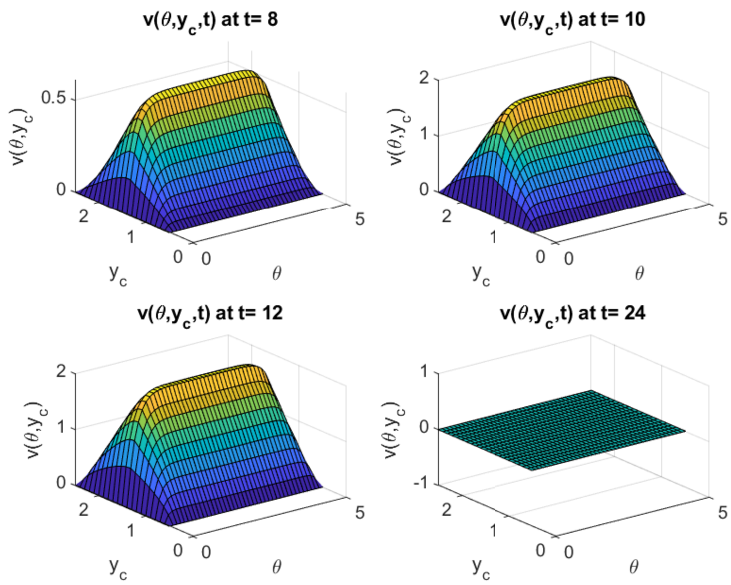

Figure 5: Propagation of the $v(\theta, y_c, t)$ versus time

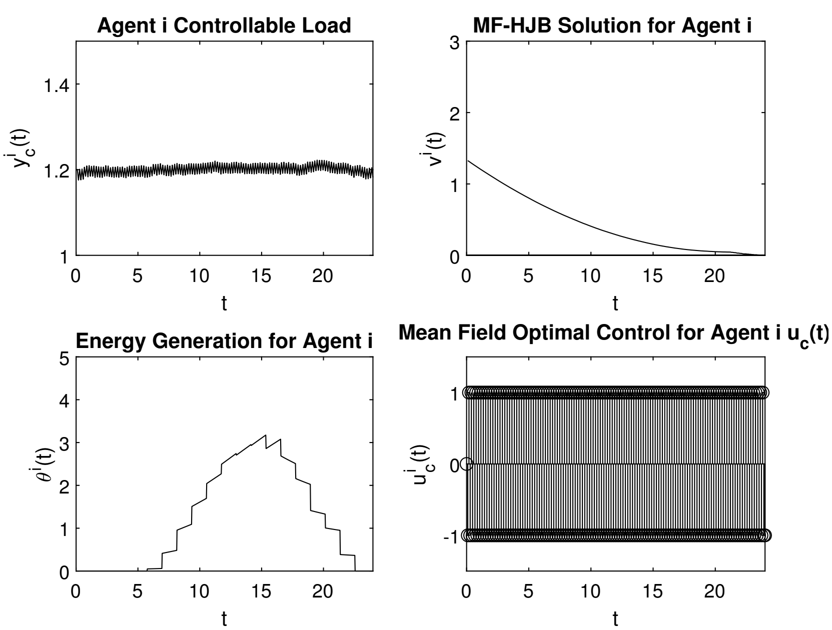

Figure 6: Generic agent energy profile

Table 2: Algorithms performance for the uniform household case

Scenario	# Iter.	Run.-Time sec	Memory KB	Dim.
Normal Day	7	263.271	32,422	4
Peak-Event Day	15	452.56	42,044	4

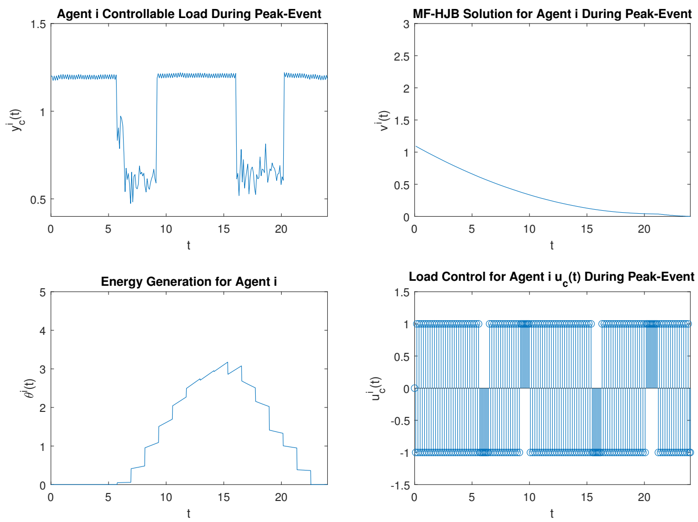

Figure 7: Generic agent energy profile during peak event

6 Conclusion

The paper introduces multiple algorithms that address the optimization problem in residential microgrid power network problems. These algorithms aim to find Nash Equilibrium strategies for all the households in the MG using the decentralized MFG methodology. The study analyzes various scenarios, including uniform households in both peak event and off-peak event scenario where the MG is in grid-connected mode and where the main grid deploys an incentive base pricing mechanism. In additionally, numerically we have demonstrated the following conclusions:

- The proposed MFG algorithm yields a convergent fixed point probability density function for the MFG-loop in Equation (29).
- Numerically, the algorithm presented in this paper produced a Nash equilibrium strategy for the constructed MGO problem resulting in reducing the energy bill for the households in the MG and simultaneously reducing the peak-event for the main grid.
- In terms of running time and complexity, the costs associated with implementing the decentralized mean field approach are lower compared to the centralized stochastic control case. Furthermore, these costs do not increase with the population size.

For future research, the algorithms introduced in this study can be expanded to address a broader problem formulation in which the residential microgrid will be consisting of non-uniform households that possess both controllable and shiftable loads, and the MG will be in a grid-connection mode where the main grid deploy a more general dynamic pricing (for example; time of use pricing mechanism).

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