



# A Branch-and-Price Method for an Inventory Routing Problem in the LNG Business

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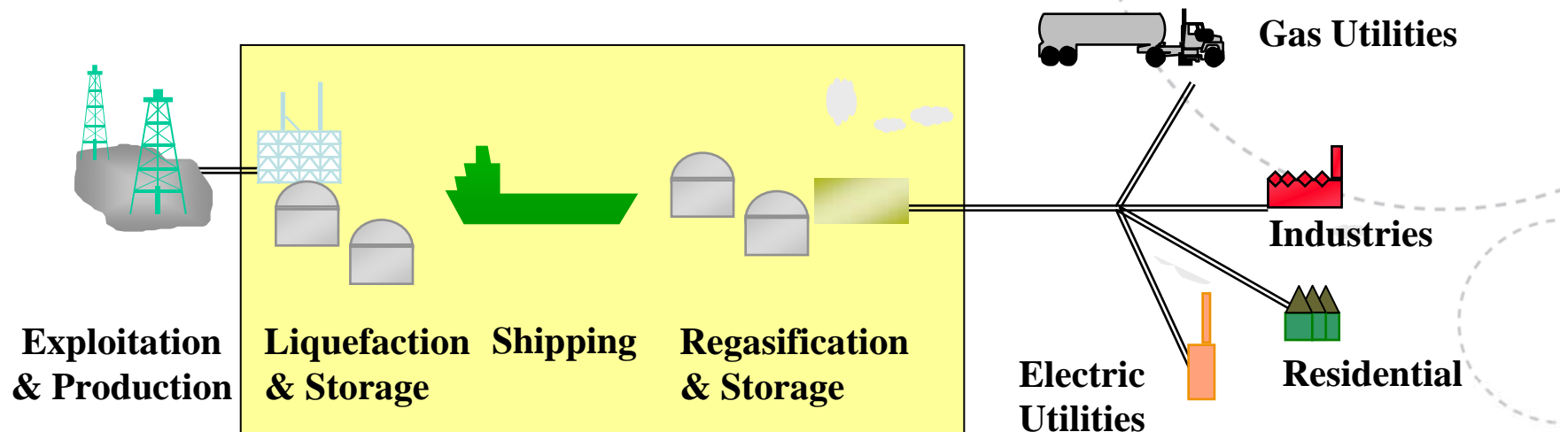
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# Agenda

- The LNG Inventory Routing Problem (LNG-IRP)
- Column generation
  - Decomposition
  - The master problem
  - The subproblems
  - Branch-and-price
- Computational results
- Concluding remarks

# The LNG-IRP

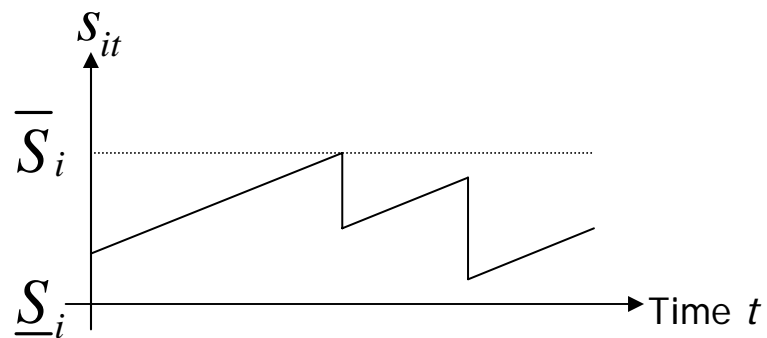


- Maximize supply chain profit – 2-3 months planning horizon
- Decide LNG production and sales levels on day to day basis
- Optimal ship routes and schedules with corresponding optimal unloading quantities
  - The ship is fully loaded when it sails from a pick-up port
  - A ship can visit several consecutive delivery ports unloading a number of cargo tanks before returning to a pick-up port

# Inventory management



Liquefaction plant  $i$



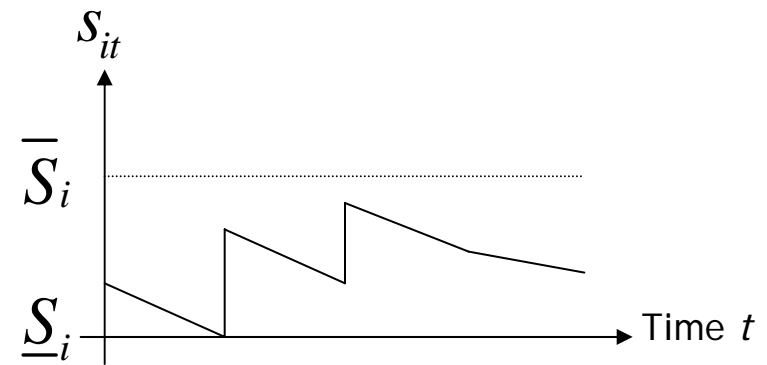
$$\underline{S}_i \leq \text{inventory } (s_{it}) \leq \bar{S}_i$$

$$\underline{Y}_{it} \leq \text{production } (y_{it}) \leq \bar{Y}_{it}$$

Inventory balance

Berth constraints

Regasification terminal  $i$



$$\underline{S}_i \leq \text{inventory } (s_{it}) \leq \bar{S}_i$$

$$\underline{Y}_{it} \leq \text{sales } (y_{it}) \leq \bar{Y}_{it}$$

Inventory balance

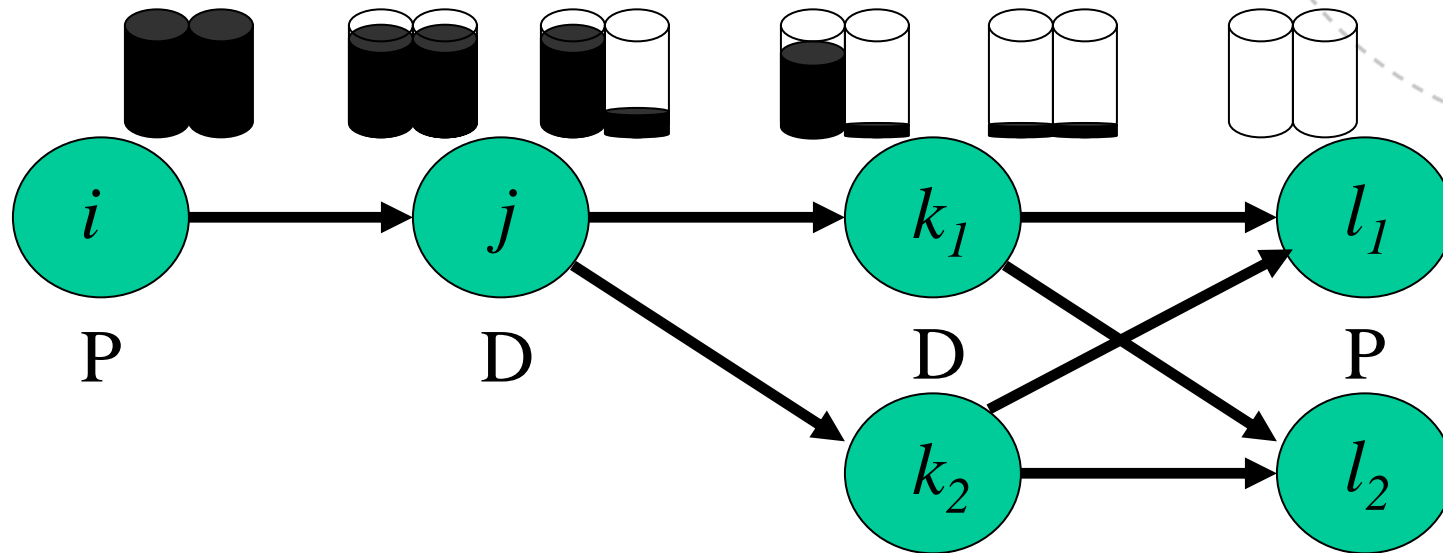
Berth constraints

# LNG Ships

- Heterogeneous fleet
- Each ship: 4-6 cargo tanks
- LNG transported at boiling state ( $-162^{\circ}\text{C}$ )
  - Boil-off from each cargo tank (Fixed % of tank capacity per day)
  - Used as fuel for the ship
  - Some LNG needed in tank to keep it cool
- Each tank should be unloaded once before refilling
  - Ships' cargo tanks should be as close as possible to full or empty to avoid sloshing
  - Need to leave just enough cargo in tanks to cover the boil-off for the rest of the trip to a pickup port



## Example with P-D-D-P and Boil-off



The unloading quantity at node  $j$  cannot be decided before the ship returns to a pick-up port. Assume the pick-up port is  $l_2$ .

$$\text{Unloading quantity of a tank} = \text{Tank capacity} \cdot (1 - B) \cdot (T_{l_2} - T_i)$$

# Ship paths

- Geographical route P1→D2→D1→P2→D1→P2→D2
- Schedule T1 T2 T3 T4 T5 T6 T7
- Quantity **Q1** Q2 Q3 **Q4** Q5 **Q6** Q7

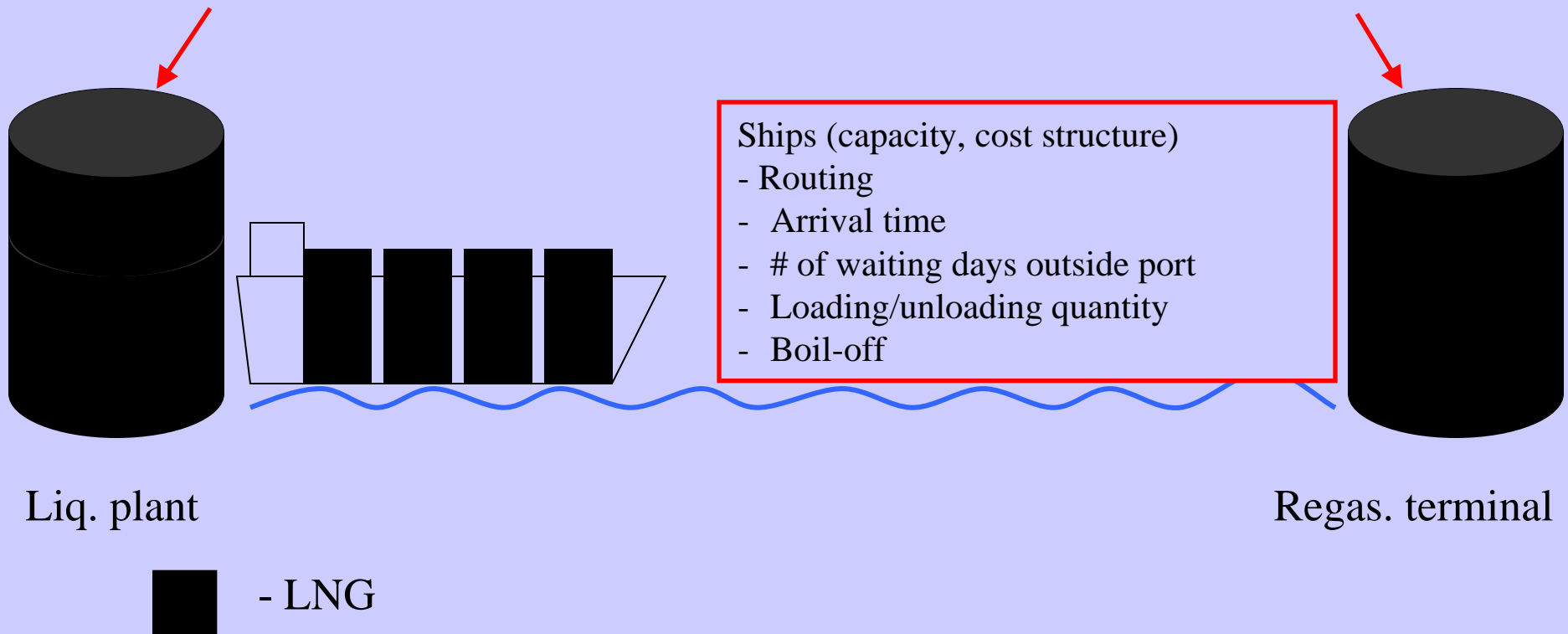
# Inventory management and routing

Constrained inventory and prod. capacity

- LNG production volume
- Ship arrival time
- Loading quantity
- Berth capacity (number of ships)

Sufficient amount of LNG available

- LNG sale
- Ship arrival time
- Unloading quantity
- Berth capacity (number of ships)



Liq. plant

Regas. terminal

■ - LNG

# Decomposition for Col. Gen.

- Master Problem
  - Sales and production at port  $i$ ,  $y_{it}$
  - Inventory management at port  $i$ ,  $s_{it}$
  - Port capacity,  $N_i^{CAP}$
- Subproblem for each ship  $v$ 
  - Ship routing and scheduling,  $X_{ijvr} \lambda_{vr}$
  - Ship inventory management
    - Number of tanks unloaded at the delivery port,  $L_{ivtr} \lambda_{vr}$
    - Volume loaded/unloaded at the ports including boil-off,  $Q_{ivtr} \lambda_{vr}$

# Master Problem

$$\max \sum_{i \in N_D} \sum_{t \in T} R_{EVit} y_{it} - \sum_{i \in N_P} \sum_{t \in T} C_{OSTit} y_{it} - \sum_{v \in V} \sum_{r \in R_v} C_{vr} \lambda_{vr}, \quad (1)$$

$$s_{it} - s_{i(t-1)} + I_i y_{it} - \sum_{v \in V} \sum_{r \in R_v} I_i Q_{ivtr} \lambda_{vr} = 0, \quad \forall i \in N, t \in T, \quad (2) \quad \alpha_{it}$$

$$\sum_{v \in V} \sum_{r \in R_v} Z_{ivtr} \lambda_{vr} \leq N_i^{CAP}, \quad \forall i \in N, t \in T, \quad (3) \quad \beta_{it}$$

$$\sum_{r \in R_v} \lambda_{vr} = 1, \quad \forall v \in V, \quad (4) \quad \theta_v$$

$$\underline{s}_i \leq s_{it} \leq \bar{s}_i, \quad \forall i \in N, t \in T, \quad (5)$$

$$\underline{Y}_{it} \leq y_{it} \leq \bar{Y}_{it}, \quad \forall i \in N, t \in T, \quad (6)$$

$$L_{ivtr} \lambda_{vr} \in \{0, 1, \dots, W_v^{MX}\}, \quad \forall i \in N^D, v \in V, t \in T, \quad (7)$$

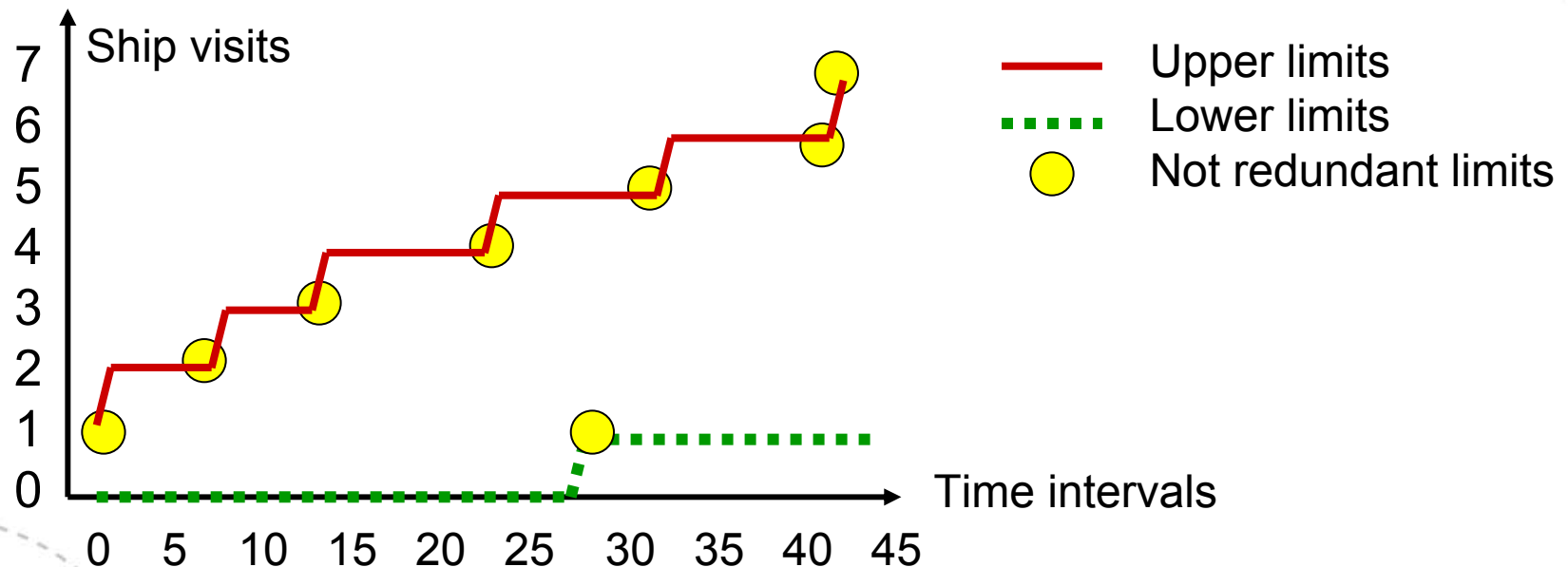
$$\sum_{r \in R_v} X_{ijvtr} \lambda_{vr} \in \{0, 1\}, \quad \forall i \in N, j \in N, v \in V, t \in T, \quad (8)$$

$$\lambda_{vr} \geq 0, \quad \forall v \in V, r \in R_v. \quad (9)$$

Dual  
variables

# Valid ineq. - aggregated berth constr.

- By use of problem characteristics (inventory limits, production and sale limits, berth constraints, ship capacities, shortest round trip for a ship), we can calculate the upper and lower limits on the number of visits to a port for all time intervals



# The Subproblems (1:2)

- Heterogeneous fleet → One subproblem for each ship
- Reduced cost for a ship route variable

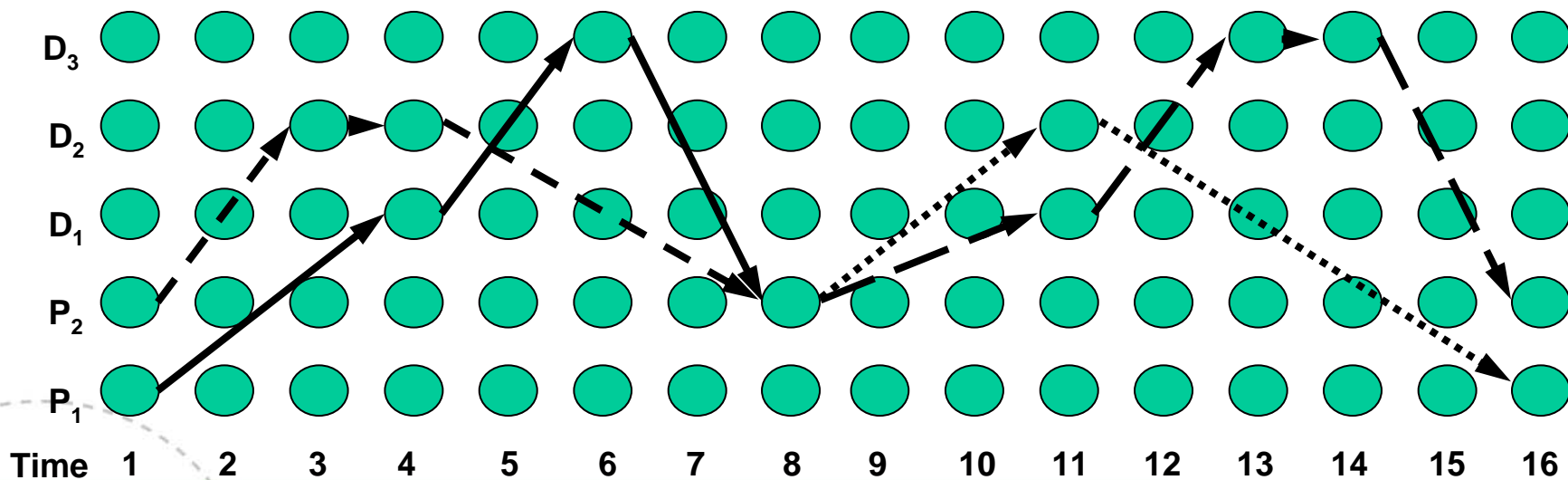
$$\text{Max } \bar{C}_{vr} = -C_{vr} - \sum_{i \in N} \sum_{t \in T} (Z_{ivtr} \beta_{it} - I_i Q_{ivtr} \alpha_{it}) - \theta_v.$$

Ship route sailing cost      Port capacities      Loading/unloading quantities      Convexity constraint

- Longest path subproblems with side constraints caused by unloading restrictions in number of tanks and boil-off

# The Subproblems (2:2)

- A node: Feasible combination of time and port
  - Unloading in number of cargo tanks at delivery ports
- The boil-off complicates the problem
  - Do not know the exact amount of cargo unloaded at the delivery ports before the ships return to a pick-up port
  - DP where partial paths can only be compared in pick-up nodes



# Accelerating strategies in col. gen.

- Greedy Heuristic for solving the subproblems
  - Assume full unloading and does not consider boil-off
  - Post calculate boil-off
  - Topological sorted acyclic network without any complicating side constraints
  - When the greedy heuristic stops generating improving columns, switch to the exact DP algorithm
- Remove all berth constraints and add violated once during B&P
- Add several columns between each call to RMP
  - Several runs of the greedy heuristic
  - Manipulate the cost between each run to give incentive to find columns which traverse different arcs

# Branch-and-Price

Depth-first B&B strategy with backtracking for the column generation

## Four branching strategies

1. Branch on berth constraints in RMP (and aggregated berth constraints – valid inequalities)
2. Branch on the sum of all ships sailing from a specific port in a given time period (nodes in the subproblem)
3. Branch on the arcs in the subproblem,  $\sum_{r \in R_v} X_{ijvtr} \lambda_{vr} \in \{0, 1\}$
4. Branch on deliveries (tanks),  $L_{ivtr} \lambda_{vr} \in \{0, 1, \dots, W_v^{MX}\}$

# Computational Results –

based on real world planning problems

Id.	s/p/t	Arcs	Path flow		B-P			#MIPsol/ BB-nodes	
			1.MIP/Total (s)	Gap	1.MIP/Total (s)	Gap			
1	2 /5/30	257	0/ 0	0	0/ 0	0	7/ 117		
2	2 /5/45	647	4/ 973	0	0/ 9	0	8/ 516		
3	2 /5/60	1144	70/ 36000	27	2/ 338	0	22/ 8 283		
4	3 /4/30	429	0/ 14	0	0/ 10	0	9/ 1 116		
5	3 /4/45	1213	0/ 13625	0	65/ 1219	0	32/ 43 391		
6	3 /4/60	2110	223/ 36000	28	114/ 36000	34	45/ 435 875		
7	5 /6/30	859	0/ 39	0	1/ 14	0	14/ 1 089		
8	5 /6/45	2815	13/ 36000	16	2348/ 36000	44	30/ 709 518		
9	5 /6/60	5613	8724/ 36000	43	8454/ 36000	69	15/ 364 576		

# More results solved by B&P

Id.	s/p/t	Arcs	1.MIP Sec.	Total Sec.	# MIP Sol.	BB Node (1000)	RMP Sec.	gSP / eSP Sec.
5	3/4/45	1213	65	1219	32	43.4	1010	11 / 169
10	2/3/75	2744	4	26527	29	305.2	18 333	81 / 7601
11	2/4/75	4834	1877	19707	13	76.5	10394	135 / 8980
12	2/5/75	1681	2	2889	12	30.2	2437	24 / 384
13	3/4/75	3010	10826	36000	14	255.6	29793	323 / 5273

# Concluding Remarks

- New type of problem
  - Extension of the maritime inventory routing problem
- Both master problem and subproblems are complicated
- Real sized instances are solved to optimality by col. gen.
- Future research
  - Improve B&P by reducing the size of the search tree and the time spent in the master problem
  - Different decomposition
  - Developing more valid inequalities
  - Developing solution methods for extended LNG-IRP's



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