A Branch-and-Price Method for an Inventory Routing Problem in the LNG Business

Marielle Christiansen\textsuperscript{a}
Guy Desaulniers\textsuperscript{b,c}, Jacques Desrosiers\textsuperscript{b,d}, Roar Grønhaug\textsuperscript{a}

18. June 2008

\textsuperscript{a}Norwegian University of Science and Technology, \textsuperscript{b}GERAD, \textsuperscript{c}Ecole Polytechnique de Montreal, \textsuperscript{d}HEC Montreal
Agenda

- The LNG Inventory Routing Problem (LNG-IRP)
- Column generation
  - Decomposition
  - The master problem
  - The subproblems
  - Branch-and-price
- Computational results
- Concluding remarks
The LNG-IRP

- Maximize supply chain profit – 2-3 months planning horizon
- Decide LNG production and sales levels on day to day basis
- Optimal ship routes and schedules with corresponding optimal unloading quantities
  - The ship is fully loaded when it sails from a pick-up port
  - A ship can visit several consecutive delivery ports unloading a number of cargo tanks before returning to a pick-up port
Inventory management

Liquefaction plant $i$

- Inventory balance
  - $S_i \leq \text{inventory } (s_{it}) \leq \bar{S}_i$
  - $Y_{it} \leq \text{production } (y_{it}) \leq \bar{Y}_{it}$

Berth constraints

Regasification terminal $i$

- Inventory balance
  - $S_i \leq \text{inventory } (s_{it}) \leq \bar{S}_i$
  - $Y_{it} \leq \text{sales } (y_{it}) \leq \bar{Y}_{it}$

Berth constraints
LNG Ships

- Heterogeneous fleet
- Each ship: 4-6 cargo tanks
- LNG transported at boiling state (-162°C)
  - Boil-off from each cargo tank (Fixed % of tank capacity per day)
  - Used as fuel for the ship
  - Some LNG needed in tank to keep it cool
- Each tank should be unloaded once before refilling
  - Ships’ cargo tanks should be as close as possible to full or empty to avoid sloshing
  - Need to leave just enough cargo in tanks to cover the boil-off for the rest of the trip to a pickup port
Example with P-D-D-P and Boil-off

The unloading quantity at node $j$ cannot be decided before the ship returns to a pick-up port. Assume the pick-up port is $l_2$.

Unloading quantity of a tank = $\text{Tank capacity} \cdot (1 - B) \cdot (T_{l_2} - T_i)$
Ship paths

- Geographical route: P1 → D2 → D1 → P2 → D1 → P2 → D2
- Schedule: T1, T2, T3, T4, T5, T6, T7
- Quantity: Q1, Q2, Q3, Q4, Q5, Q6, Q7
Inventory management and routing

Constrained inventory and prod. capacity
- LNG production volume
- Ship arrival time
- Loading quantity
- Berth capacity (number of ships)

Sufficient amount of LNG available
- LNG sale
- Ship arrival time
- Unloading quantity
- Berth capacity (number of ships)

Liq. plant
- LNG

Rengas. terminal
- LNG

Ships (capacity, cost structure)
- Routing
- Arrival time
- # of waiting days outside port
- Loading/unloading quantity
- Boil-off
Decomposition for Col. Gen.

• Master Problem
  – Sales and production at port $i$, $y_{it}$
  – Inventory management at port $i$, $s_{it}$
  – Port capacity, $N_i^{CAP}$

• Subproblem for each ship $v$
  – Ship routing and scheduling, $X_{jvtr} \lambda_{vr}$
  – Ship inventory management
    • Number of tanks unloaded at the delivery port, $L_{ivtr} \lambda_{vr}$
    • Volume loaded/unloaded at the ports including boil-off, $Q_{ivtr} \lambda_{vr}$
Master Problem

\[
\begin{align*}
\text{max } & \sum_{i \in N_D, t \in T} R_{EVi}, y_{it} - \sum_{i \in N, t \in T} C_{OSTi}, y_{it} - \sum_{v \in V, r \in R_v} C_{vr}, \lambda_{vr}, \\
& s_{it} - s_{i(t-1)} + I_i y_{it} - \sum_{v \in V, r \in R_v} I_i O_{i vtr}, \lambda_{vr} = 0, \quad \forall i \in N, t \in T, \\
& \sum_{v \in V, r \in R_v} Z_{ivtr}, \lambda_{vr} \leq N_i^{CAP}, \quad \forall i \in N, t \in T, \\
& \sum_{r \in R_v} \lambda_{vr} = 1, \quad \forall v \in V, \\
& S_i \leq s_{it} \leq S_i, \quad \forall i \in N, t \in T, \\
& Y_{it} \leq y_{it} \leq Y_{it}, \quad \forall i \in N, t \in T, \\
L_{ivtr}, \lambda_{vr} \in \{0,1,\ldots,W_v^{MX}\}, \quad \forall i \in N^D, v \in V, t \in T, \\
\sum_{r \in R_v} X_{ijvtr}, \lambda_{vr} \in \{0,1\}, \quad \forall i \in N, j \in N, v \in V, t \in T, \\
\lambda_{vr} \geq 0, \quad \forall v \in V, r \in R_v.
\end{align*}
\]
Valid ineq. - aggregated berth constr.

- By use of problem characteristics (inventory limits, production and sale limits, berth constraints, ship capacities, shortest round trip for a ship), we can calculate the upper and lower limits on the number of visits to a port for all time intervals.
The Subproblems (1:2)

- Heterogeneous fleet $\rightarrow$ One subproblem for each ship
- Reduced cost for a ship route variable

\[
\text{Max } \bar{C}_{vr} = -C_{vr} - \sum_{i \in N} \sum_{t \in T} (Z_{ivtr} \beta_{it} - I_i Q_{ivtr} \alpha_{it}) - \theta_v.
\]

- Longest path subproblems with side constraints caused by unloading restrictions in number of tanks and boil-off
The Subproblems (2:2)

- A node: Feasible combination of time and port
  - Unloading in number of cargo tanks at delivery ports

- The boil-off complicates the problem
  - Do not know the exact amount of cargo unloaded at the delivery ports before the ships return to a pick-up port
  - DP where partial paths can only be compared in pick-up nodes
Accelerating strategies in col. gen.

- Greedy Heuristic for solving the subproblems
  - Assume full unloading and does not consider boil-off
  - Post calculate boil-off
  - Topological sorted acyclic network without any complicating side constraints
  - When the greedy heuristic stops generating improving columns, switch to the exact DP algorithm

- Remove all berth constraints and add violated once during B&P

- Add several columns between each call to RMP
  - Several runs of the greedy heuristic
  - Manipulate the cost between each run to give incentive to find columns which traverse different arcs
Branch-and-Price

Depth-first B&B strategy with backtracking for the column generation

Four branching strategies
1. Branch on berth constraints in RMP (and aggregated berth constraints – valid inequalities)
2. Branch on the sum of all ships sailing from a specific port in a given time period (nodes in the subproblem)
3. Branch on the arcs in the subproblem, \( \sum_{r \in R_v} X_{ijvr} \lambda_{vr} \in \{0,1\} \)
4. Branch on deliveries (tanks), \( L_{ivtr} \lambda_{vr} \in \{0,1,\ldots,W_v^{MX}\} \)
# Computational Results – based on real world planning problems

<table>
<thead>
<tr>
<th>Id.</th>
<th>s/p/t</th>
<th>Arcs</th>
<th>Path flow</th>
<th>B-P</th>
<th>#MIPsol/BB-nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.MIP/Total (s)</td>
<td>Gap</td>
<td>1.MIP/Total (s)</td>
</tr>
<tr>
<td>1</td>
<td>2/5/30</td>
<td>257</td>
<td>0/0</td>
<td>0</td>
<td>0/0</td>
</tr>
<tr>
<td>2</td>
<td>2/5/45</td>
<td>647</td>
<td>4/973</td>
<td>0</td>
<td>0/9</td>
</tr>
<tr>
<td>3</td>
<td>2/5/60</td>
<td>1144</td>
<td>70/36000</td>
<td>27</td>
<td>2/338</td>
</tr>
<tr>
<td>4</td>
<td>3/4/30</td>
<td>429</td>
<td>0/14</td>
<td>0</td>
<td>0/10</td>
</tr>
<tr>
<td>5</td>
<td>3/4/45</td>
<td>1213</td>
<td>0/13625</td>
<td>0</td>
<td>65/1219</td>
</tr>
<tr>
<td>6</td>
<td>3/4/60</td>
<td>2110</td>
<td>223/36000</td>
<td>28</td>
<td>114/36000</td>
</tr>
<tr>
<td>7</td>
<td>5/6/30</td>
<td>859</td>
<td>0/39</td>
<td>0</td>
<td>1/14</td>
</tr>
<tr>
<td>8</td>
<td>5/6/45</td>
<td>2815</td>
<td>13/36000</td>
<td>16</td>
<td>2348/36000</td>
</tr>
<tr>
<td>9</td>
<td>5/6/60</td>
<td>5613</td>
<td>8724/36000</td>
<td>43</td>
<td>8454/36000</td>
</tr>
</tbody>
</table>
## More results solved by B&P

<table>
<thead>
<tr>
<th>Id.</th>
<th>s/p/t</th>
<th>Arcs</th>
<th>1.MIP Sec.</th>
<th>Total Sec.</th>
<th># MIP Sol.</th>
<th>BB Node (1000)</th>
<th>RMP Sec.</th>
<th>gSP / eSP Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3/4/45</td>
<td>1213</td>
<td>65</td>
<td>1219</td>
<td>32</td>
<td>43.4</td>
<td>1010</td>
<td>11 / 169</td>
</tr>
<tr>
<td>10</td>
<td>2/3/75</td>
<td>2744</td>
<td>4</td>
<td>26527</td>
<td>29</td>
<td>305.2</td>
<td>18333</td>
<td>81 / 7601</td>
</tr>
<tr>
<td>11</td>
<td>2/4/75</td>
<td>4834</td>
<td>1877</td>
<td>19707</td>
<td>13</td>
<td>76.5</td>
<td>10394</td>
<td>135 / 8980</td>
</tr>
<tr>
<td>12</td>
<td>2/5/75</td>
<td>1681</td>
<td>2</td>
<td>2889</td>
<td>12</td>
<td>30.2</td>
<td>2437</td>
<td>24 / 384</td>
</tr>
<tr>
<td>13</td>
<td>3/4/75</td>
<td>3010</td>
<td>10826</td>
<td>36000</td>
<td>14</td>
<td>255.6</td>
<td>29793</td>
<td>323 / 5273</td>
</tr>
</tbody>
</table>
Concluding Remarks

- New type of problem
  - Extension of the maritime inventory routing problem

- Both master problem and subproblems are complicated

- Real sized instances are solved to optimality by col. gen.

- Future research
  - Improve B&P by reducing the size of the search tree and the time spent in the master problem
  - Different decomposition
  - Developing more valid inequalities
  - Developing solution methods for extended LNG-IRP’s
A Branch-and-Price Method for an Inventory Routing Problem in the LNG Business

Marielle Christiansen⁹
Guy Desaulniers⁷,⁸, Jacques Desrosiers⁷,⁹, Roar Grønhaug⁹

18. June 2008

⁹Norwegian University of Science and Technology, ⁷GERAD,
⁸Ecole Polytechnique de Montreal, ⁹HEC Montreal