A Branch-and-Price Method for an Inventory Routing Problem in the LNG Business

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18. June 2008

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Agenda

- The LNG Inventory Routing Problem (LNG-IRP)
- Column generation
  - Decomposition
  - The master problem
  - The subproblems
  - Branch-and-price
- Computational results
- Concluding remarks

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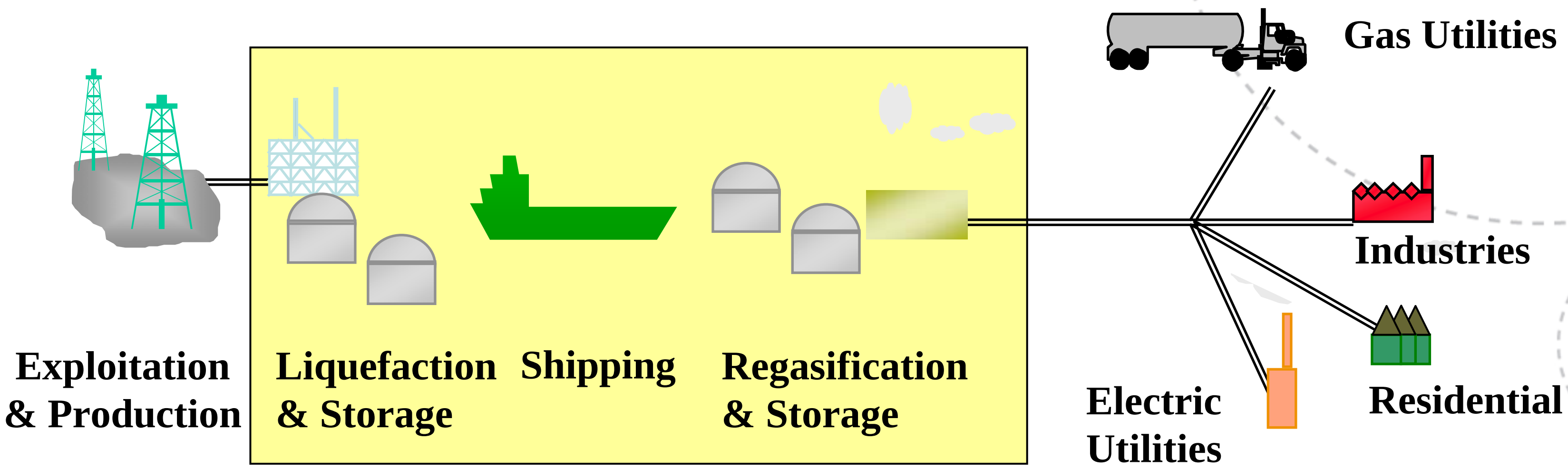
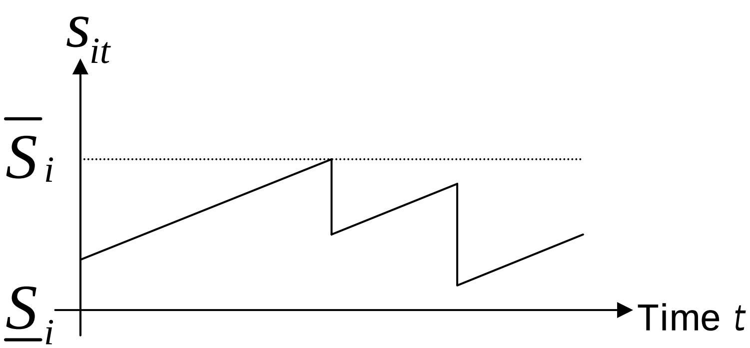
The LNG-IRP



- Maximize supply chain profit – 2-3 months planning horizon
- Decide LNG production and sales levels on day to day basis
- Optimal ship routes and schedules with corresponding optimal unloading quantities
  - The ship is fully loaded when it sails from a pick-up port
  - A ship can visit several consecutive delivery ports unloading a number of cargo tanks before returning to a pick-up port

Inventory management

Liquefaction plant ***i***



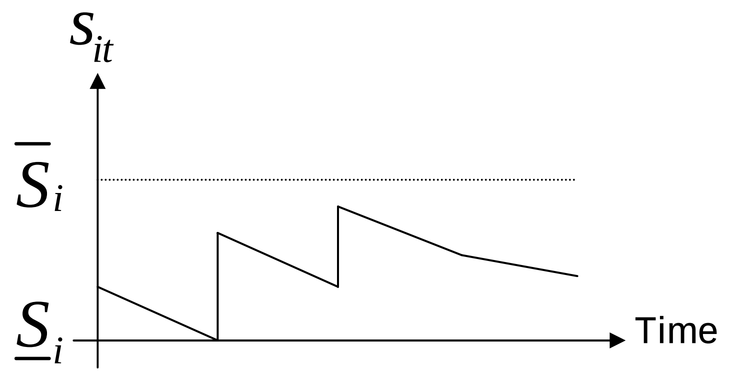
$\underline{S}_i \leq \text{inventory } (s_{it}) \leq \overline{S}_i$

$\underline{Y}_{it} \leq \text{production}(y_{it}) \leq \overline{Y}_{it}$

Inventory balance

Berth constraints

Regasification terminal ***i***



$\underline{S}_i \leq \text{inventory } (s_{it}) \leq \overline{S}_i$

$\underline{Y}_{it} \leq \text{sales}(y_{it}) \leq \overline{Y}_{it}$

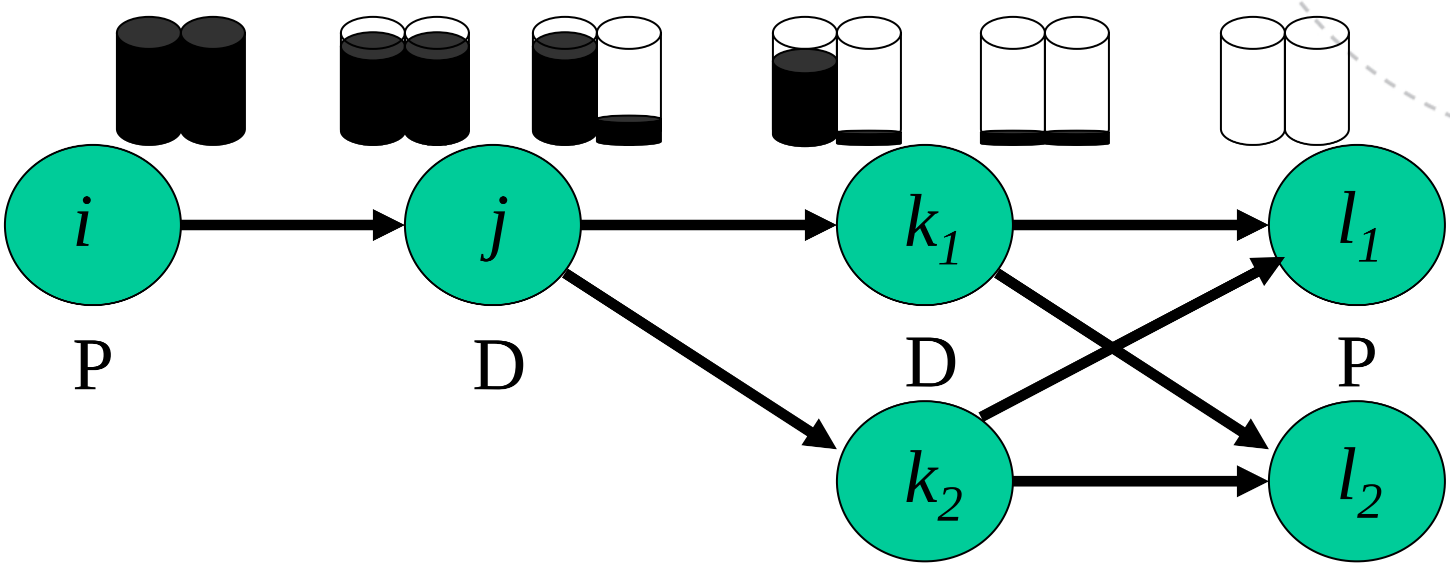
Inventory balance

Berth constraints

LNG Ships

- Heterogeneous fleet
- Each ship: 4-6 cargo tanks
- LNG transported at boiling state (-162^{o}C)
  - Boil-off from each cargo tank (Fixed % of tank capacity per day)
  - Used as fuel for the ship
  - Some LNG needed in tank to keep it cool
- Each tank should be unloaded once before refilling
  - Ships' cargo tanks should be as close as possible to full or empty to avoid sloshing
  - Need to leave just enough cargo in tanks to cover the boil-off for the rest of the trip to a pickup port

Example with P-D-D-P and Boil-off



The unloading quantity at node *j* cannot be decided before the ship returns to a pick-up port. Assume the pick-up port is l_2.

Unloading quantity of a tank = $\text{Tank capacity} \cdot (1 - B) \cdot (T_{l_2} - T_i)$

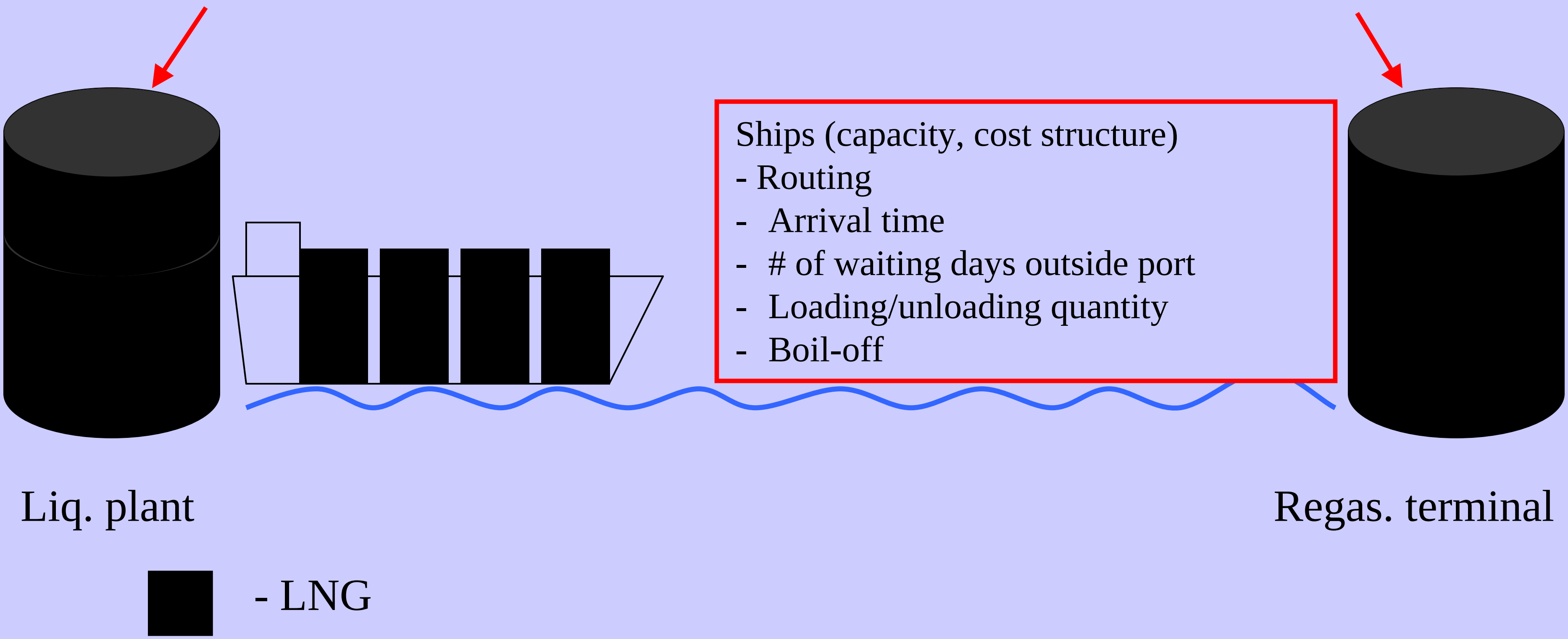
Ship paths

• Geographical route	P1→D2→D1→P2→D1→P2→D2						
• Schedule	T1	T2	T3	T4	T5	T6	T7
• Quantity	**Q1**	Q2	Q3	**Q4**	Q5	**Q6**	Q7

Inventory management and routing

Constrained inventory and prod. capacity
- LNG production volume
- Ship arrival time
- Loading quantity
- Berth capacity (number of ships)

Sufficient amount of LNG available
- LNG sale
- Ship arrival time
- Unloading quantity
- Berth capacity (number of ships)



Decomposition for Col. Gen.

- Master Problem
  - Sales and production at port ***i*** , y_{it}
  - Inventory management at port ***i***, s_{it}
  - Port capacity, N_i^{CAP}

- Subproblem for each ship ***v***
  - Ship routing and scheduling, $X_{ijvtr}\lambda_{vr}$
  - Ship inventory management
    - Number of tanks unloaded at the delivery port, $L_{ivtr}\lambda_{vr}$
    - Volume loaded/unloaded at the ports including boil-off, $Q_{ivtr}\lambda_{vr}$

Master Problem

Dual variables

$$\max \sum_{i\in N_D}\sum_{t\in T} R_{EVit}\, y_{it} - \sum_{i\in N_P}\sum_{t\in T} C_{OSTit}\, y_{it} - \sum_{v\in V}\sum_{r\in R_v} C_{vr}\lambda_{vr}, \qquad (1)$$

$$s_{it} - s_{i(t-1)} + I_i y_{it} - \sum_{v\in V}\sum_{r\in R_v} I_i Q_{ivtr}\lambda_{vr} = 0, \qquad \forall i\in N, t\in T, \qquad (2) \qquad \alpha_{it}$$

$$\sum_{v\in V}\sum_{r\in R_v} Z_{ivtr}\lambda_{vr} \le N_i^{CAP}, \qquad \forall i\in N, t\in T, \qquad (3) \qquad \beta_{it}$$

$$\sum_{r\in R_v}\lambda_{vr} = 1, \qquad \forall v\in V, \qquad (4) \qquad \theta_v$$

$$\underline{S}_i \le s_{it} \le \overline{S}_i, \qquad \forall i\in N, t\in T, \qquad (5)$$

$$\underline{Y}_{it} \le y_{it} \le \overline{Y}_{it}, \qquad \forall i\in N, t\in T, \qquad (6)$$

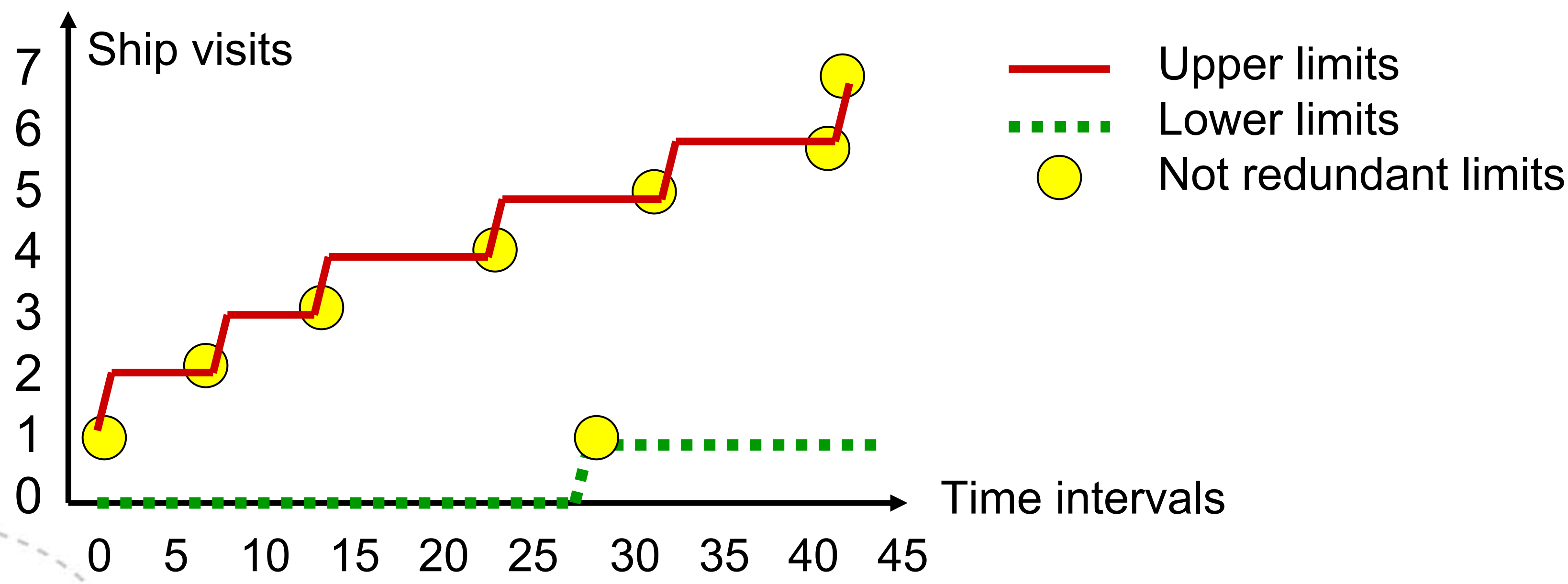
$$L_{ivtr}\lambda_{vr} \in \left\{0,1,\ldots,W_v^{MX}\right\}, \qquad \forall i\in N^D, v\in V, t\in T, \qquad (7)$$

$$\sum_{r\in R_v} X_{ijvtr}\lambda_{vr} \in \{0,1\}, \qquad \forall i\in N, j\in N, v\in V, t\in T, \qquad (8)$$

$$\lambda_{vr} \ge 0, \qquad \forall v\in V, r\in R_v. \qquad (9)$$

Valid ineq. - aggregated berth constr.

- By use of problem characteristics (inventory limits, production and sale limits, berth constraints, ship capacities, shortest round trip for a ship), we can calculate the upper and lower limits on the number of visits to a port for all time intervals



The Subproblems (1:2)

- Heterogeneous fleet → One subproblem for each ship

- Reduced cost for a ship route variable

$$\text{Max} \quad \overline{C}_{vr} = -C_{vr} - \sum_{i \in N} \sum_{t \in T} \left(Z_{ivtr} \beta_{it} - I_i Q_{ivtr} \alpha_{it} \right) - \theta_v .$$

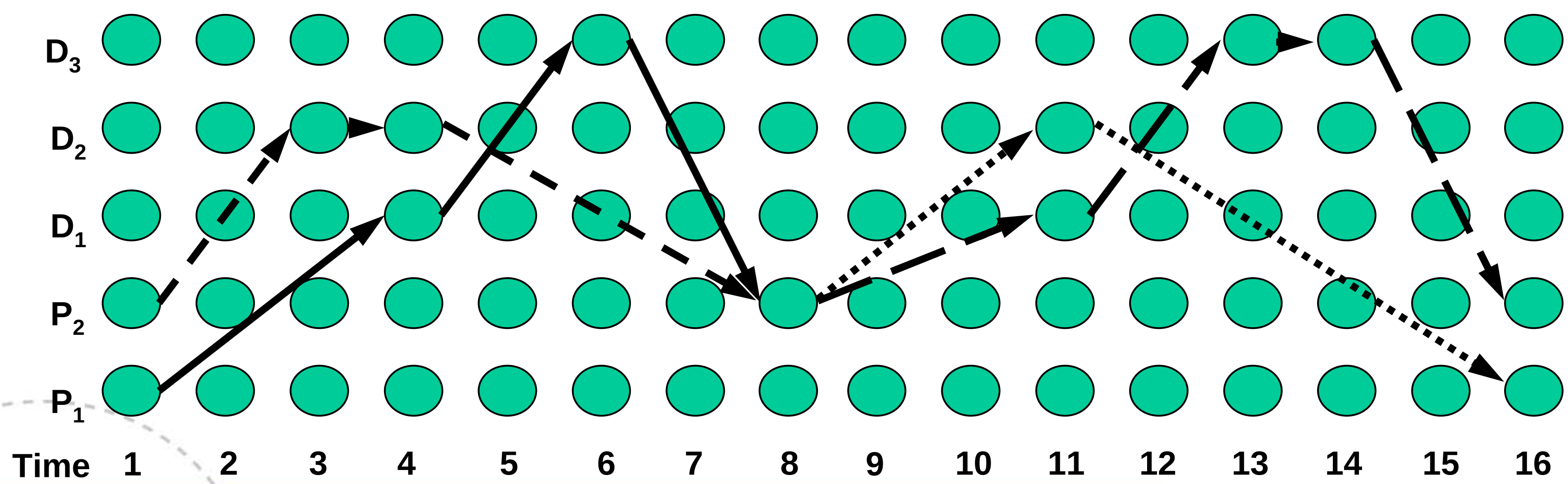
Ship route sailing cost | Port capacities | Loading/unloading quantities | Convexity constraint

- Longest path subproblems with side constraints caused by unloading restrictions in number of tanks and boil-off

The Subproblems (2:2)

- A node: Feasible combination of time and port
  - Unloading in number of cargo tanks at delivery ports

- The boil-off complicates the problem
  - Do not know the exact amount of cargo unloaded at the delivery ports before the ships return to a pick-up port
  - DP where partial paths can only be compared in pick-up nodes



Accelerating strategies in col. gen.

- Greedy Heuristic for solving the subproblems
  - Assume full unloading and does not consider boil-off
  - Post calculate boil-off
  - Topological sorted acyclic network without any complicating side constraints
  - When the greedy heuristic stops generating improving columns, switch to the exact DP algorithm

- Remove all berth constraints and add violated once during B&P

- Add several columns between each call to RMP
  - Several runs of the greedy heuristic
  - Manipulate the cost between each run to give incentive to find columns which traverse different arcs

Branch-and-Price

Depth-first B&B strategy with backtracking for the column generation

Four branching strategies

1. Branch on berth constraints in RMP (and aggregated berth constraints – valid inequalities)

2. Branch on the sum of all ships sailing from a specific port in a given time period (nodes in the subproblem)

3. Branch on the arcs in the subproblem, $\sum_{r \in R_v} X_{ijvtr} \lambda_{vr} \in \{0,1\}$

4. Branch on deliveries (tanks), $L_{ivtr} \lambda_{vr} \in \{0,1,....,W_v^{MX}\}$

Computational Results –
based on real world planning problems

			Path flow		B-P		#MIPsol/ BB-nodes
Id.	s/p/t	Arcs	1.MIP/Total (s)	Gap	1.MIP/Total (s)	Gap	
1	2 /5/30	257	0/ 0	0	0/ 0	0	7/ 117
2	2 /5/45	647	4/ 973	0	0/ 9	0	8/ 516
3	2 /5/60	1144	70/ 36000	27	2/ 338	0	22/ 8 283
4	3 /4/30	429	0/ 14	0	0/ 10	0	9/ 1 116
5	3 /4/45	1213	0/ 13625	0	65/ 1219	0	32/ 43 391
6	3 /4/60	2110	223/ 36000	28	114/ 36000	34	45/ 435 875
7	5 /6/30	859	0/ 39	0	1/ 14	0	14/ 1 089
8	5 /6/45	2815	13/ 36000	16	2348/ 36000	44	30/ 709 518
9	5 /6/60	5613	8724/ 36000	43	8454/ 36000	69	15/ 364 576

More results solved by B&P

Id.	s/p/t	Arcs	1.MIP Sec.	Total Sec.	# MIP Sol.	BB Node (1000)	RMP Sec.	gSP / eSP Sec.
5	3/4/45	1213	65	1219	32	43.4	1010	11 / 169
10	2/3/75	2744	4	26527	29	305.2	18 333	81 / 7601
11	2/4/75	4834	1877	19707	13	76.5	10394	135 / 8980
12	2/5/75	1681	2	2889	12	30.2	2437	24 / 384
13	3/4/75	3010	10826	36000	14	255.6	29793	323 / 5273

Concluding Remarks

- New type of problem
  - Extension of the maritime inventory routing problem
- Both master problem and subproblems are complicated
- Real sized instances are solved to optimality by col. gen.
- Future research
  - Improve B&P by reducing the size of the search tree and the time spent in the master problem
  - Different decomposition
  - Developing more valid inequalities
  - Developing solution methods for extended LNG-IRP's

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