Lufthansa Systems

A Dynamic Reduce and Generate Approach for Airline Crew Scheduling

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Column Generation
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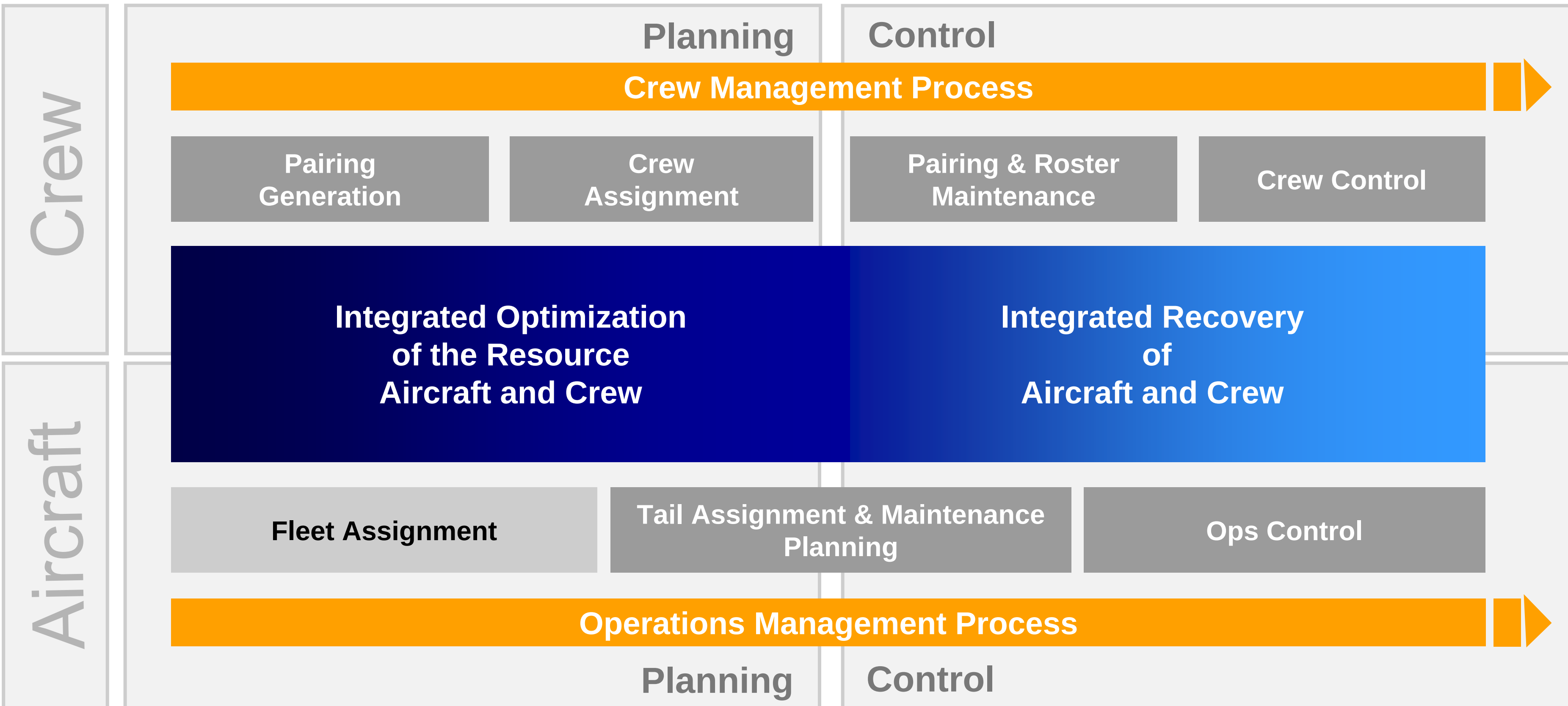
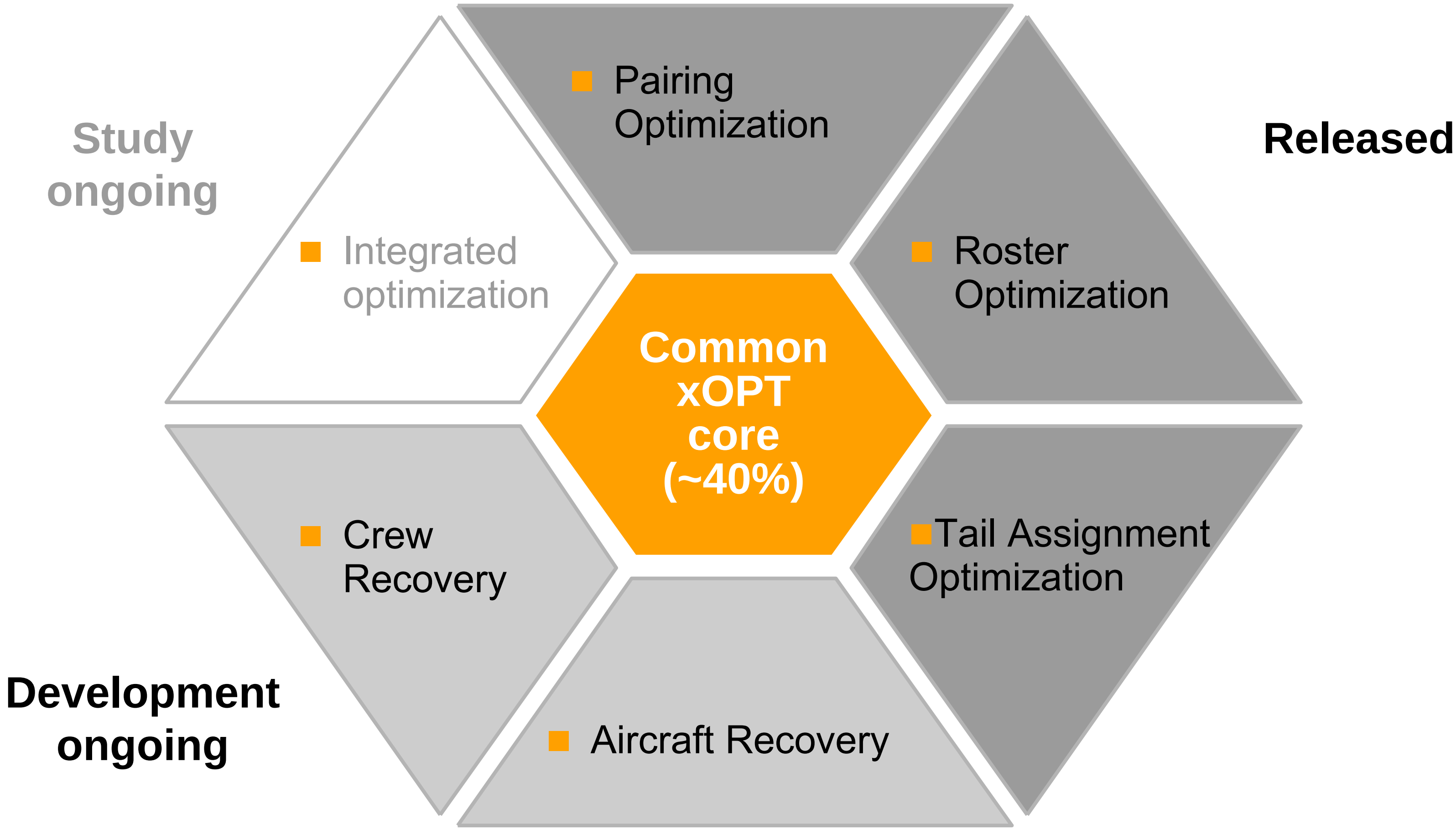
OASIS Program (2005-2010)
Optimization Algorithms for Seamless and Integrated Solvers



Crew scheduling as part of an airline optimization suite



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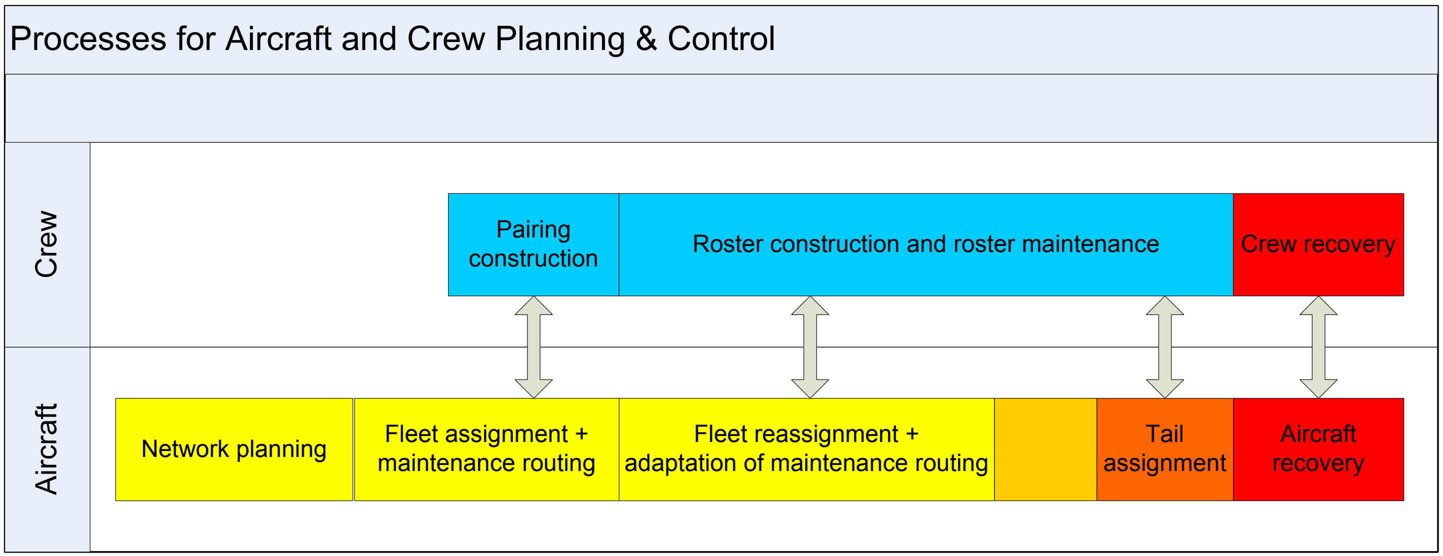
Pairing and Roster Construction as Part of an Airline Planning & Control Process



Problem Description: Pairing Construction

- **Output:** Anonymous lines of work (from base to base)

- **Main constraints:**
  - Legality rules for individual pairings: e.g. max. duty time
  - Distribution between crew bases (+ other bounds on the number of pairings with certain features)

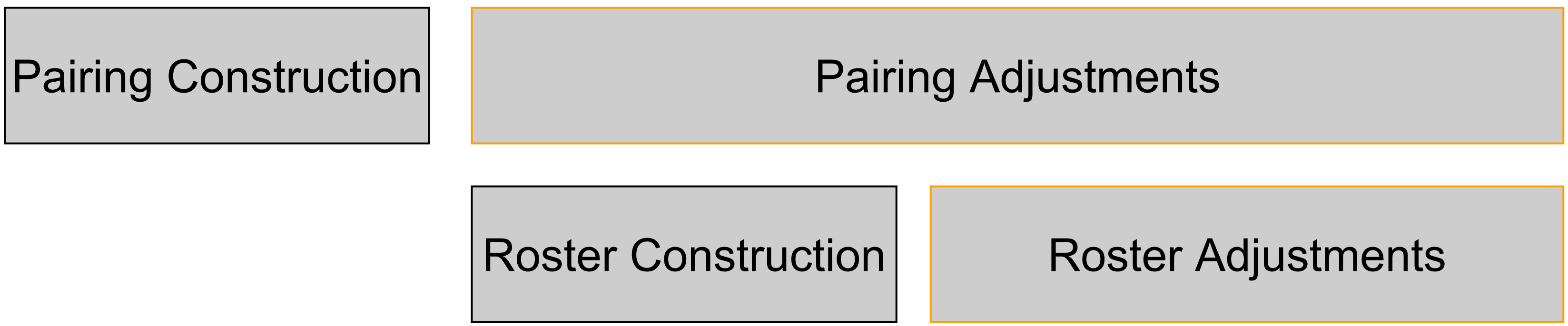
- **Objective:**
  - Minimize cost (transports, hotels, …)
  - Maximize robustness and suitability of pairings for roster construction (next planning step)

Problem Description: Roster Construction

- **Output:** Assignment of pairings to crew members
- **Main constraints:**
  - Legality rules for individual rosters: e.g. max. duty time per certain period, distribution of off-days..
  - Legality rules for combination of rosters: qualification constraints, don‘t fly with, …
- **Objective:**
  - Minimize cost (by additional transports, hotels, overtime pay, …)
  - Maximize crew satisfaction (fair distribution of unattractive duties, respecting preferences based on maximum fulfillments, …)

Seamless Optimization

- Seamless Pairing and Roster Optimization:
  - Changes in the input data
  - Stable solution which is similar to nominal solution, e.g. few changes of off time
  - Minor changes in the solution strategy



Recovery, Robust and Integrated Optimization

- Crew and Aircraft Recovery (2008-2009):
  - Objective: Minimize impact of disruptions
  - Crew recovery is an integration of pairing and rostering
  - Shorter run times (few minutes vs. several hours)

- Robust Crew and Aircraft Planning (2008-2009):
  - Objective: minimization of total planning and operational costs and increase of punctuality
  - Cooperation with Konrad Zuse Institute Berlin and University of Paderborn

- Integrated Optimization (2009 - 2010):
  - Integrated recovery
  - Integrated planning

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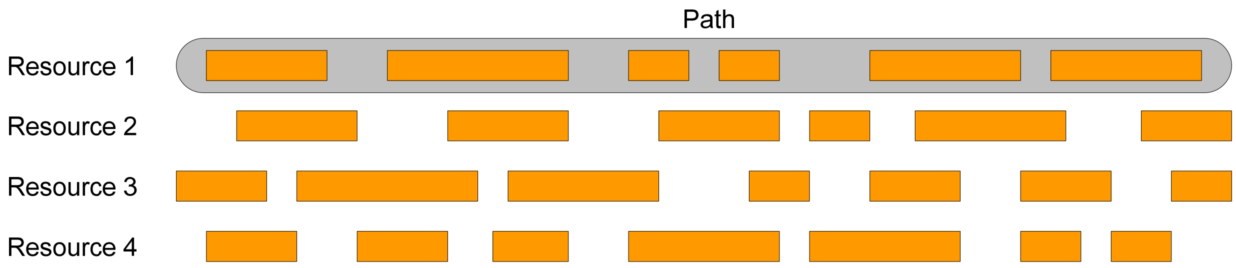
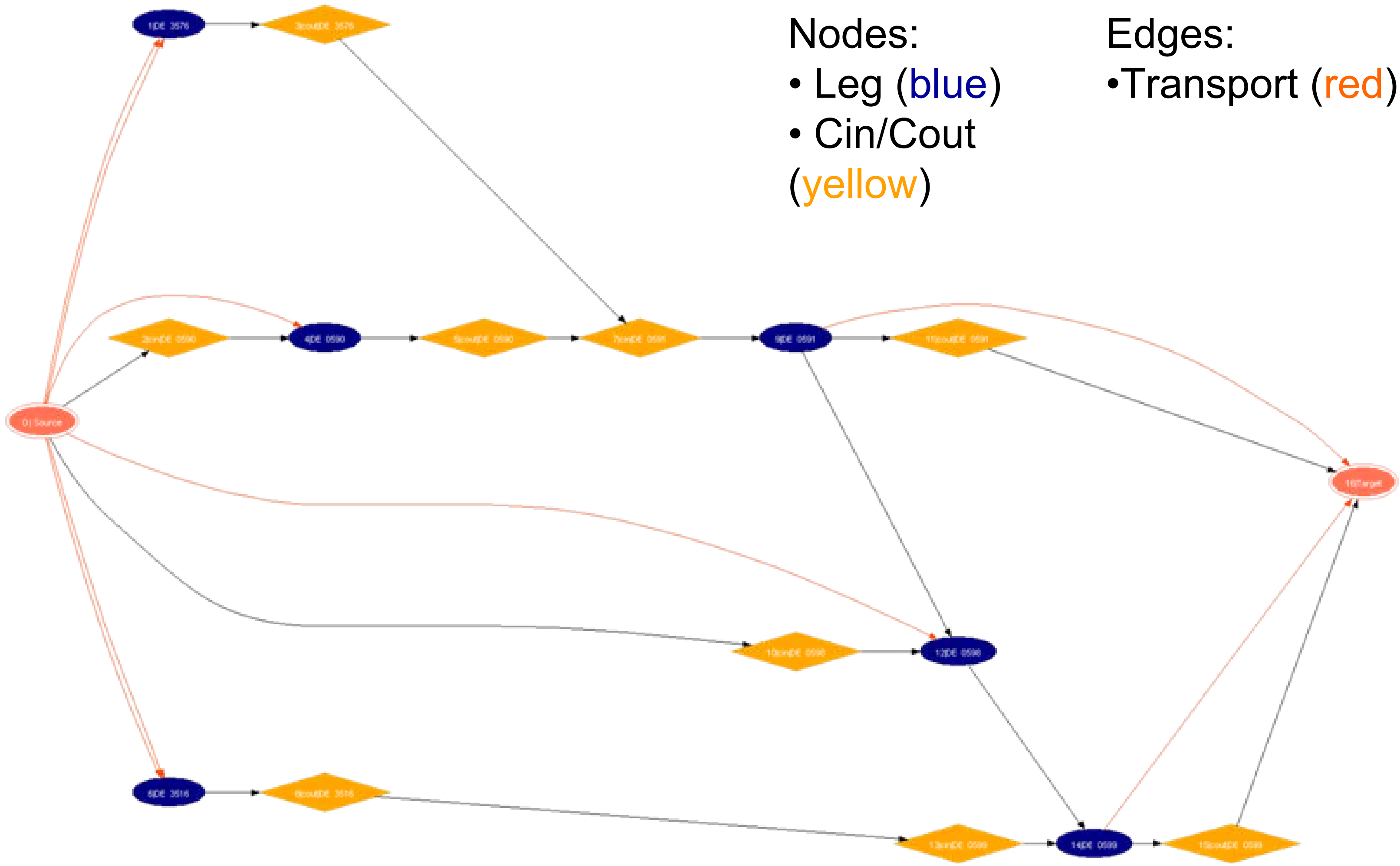
Scheduling Problem Formulation



- **General scheduling problem:** Assignment of resources to paths through a network
- **Constraints:**
  - Network structure
  - Horizontal constraints for single paths (e.g. max. duty time for a crew member)
  - Vertical constraints for multiple paths
    (e.g. min. number of crew members with a certain qualification)

- **Common solution approach: Column Generation**

Scheduling Problem Formulation - Pairing Event Network



Nodes:
- Leg (blue)
- Cin/Cout (yellow)

Edges:
- Transport (red)

IP and LP Formulation

Enumeration of all legal paths in the network gives the IP-formulation:

$$
\begin{aligned}
\text{Min}\quad & c(x,s) \\
& Ax = b+s \qquad\qquad (1) \\
& x \in \{0,1\}^n,\ s \in S
\end{aligned}
$$

- c_i is the cost of a path and (c_i, A_i) is called a column
- (1) are partitioning and ‘vertical’ legality constraints
- Replacing {0,1} by [0,1] gives the LP-formulation
- Column generation generates a restricted IP with fewer columns

Reduce-and-Generate Approach for solving large-scale Scheduling Problems

- Using traditional column generation with static problem reduction it is not possible to solve large-scale crew scheduling problems with many transport and flight connections in reasonable time

- Dynamic reduce-and-generate approach:
  - Successively solving reduced scheduling problems, called sub-problems, with given size
  - Determination of sub-problems using dynamic reduction of the network and the path pool
  - Sub-problems can be solved in reasonable time. Hence, good performance if number of main iterations is not too large.
  - Good solution quality if reduced networks contain legal paths with negative reduced costs. Then the final LP/IP objective value is near globally optimal.
  - Convergence if systematic change of restrictions

Static Problem Reduction (Fixing)

1. Fixations generated from the IP solver based on fractional LP solutions
   - master problem reduction (column and constraint fixations)
   - network reduction (node/arc fixations)

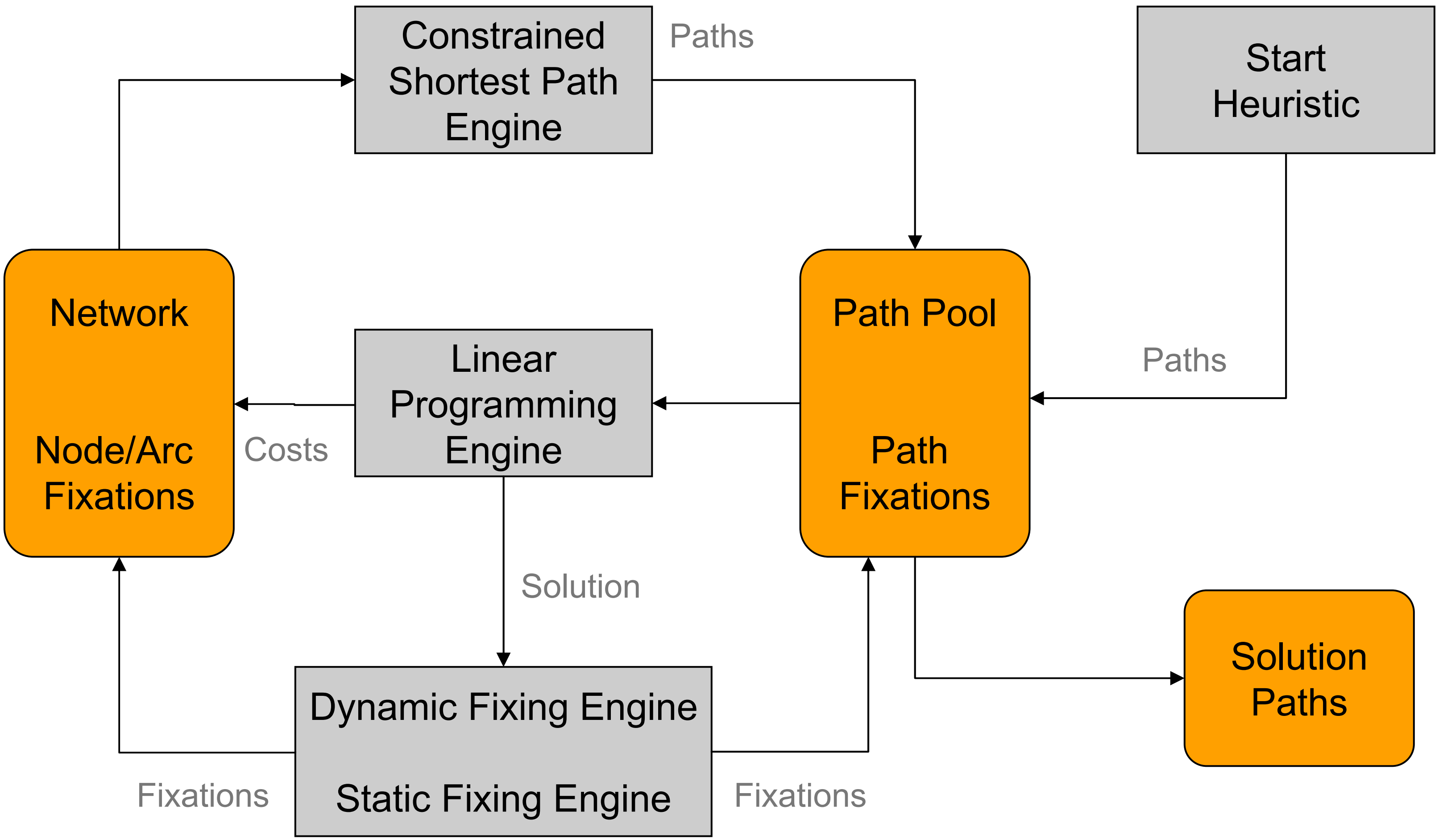
2. Restrictions generated from main algorithm
   - horizontal pruning constraints and limited path search
   - network reduction based on an approximated scheduling solution
   - window restrictions (block-coordinate search)

Dynamic Problem Reduction (Fixing)

Dynamic network reduction (node/arc fixations) in each path generation iteration generated from:

- network preprocessing
- labels computed by solving a relaxed pricing problem
- vertical information from partially rounded LP solutions
- ‘disjunctive’ path generation
- ‘block-coordinate search’
- dynamic transport generation

Reduce-And-Generate Approach



Optimization Engines

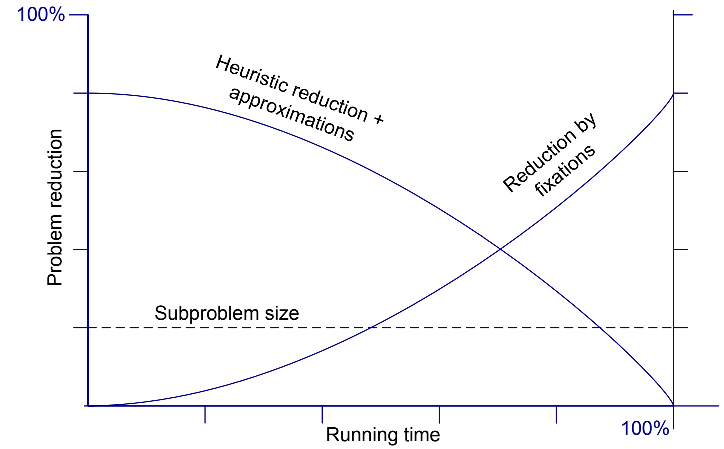
- Constrained Shortest Path Engine
  - Generates legal paths with negative „reduced costs“, which improve the LP
  - Depth-first search using labels and dynamic arc sorting

- Linear Programming Engine
  - Solves the LP for
    a) computing reduced costs (dual solution)
    b) computing a fractional solution (primal solution)
  - Inexact proximal bundle algorithm

- Fixing Engine
  - Determines path or network fixations such that the difference of IP and LP objective values is very small
  - Perturbation heuristic based on fractional LP solutions

Solution Strategy

- Pairing solution strategy:
  - generating many paths per iteration
  - use of pruning constraints, i.e. in the beginning search for pairings with restrictions, e.g. length, number of transports
- Roster solution strategy:
  - generating few paths per iteration, because of long paths and 'symmetry'
  - 'simultaneous path generation'
  - 'sub-problem refinement' → linear performance
- Dynamic transport generation for all crew optimizers



Results

- **Pairing Optimization**
  - Production quality solutions for various scenarios, covering 4 different airlines
  - Largest planning group:
    - Fleet of up to >40 aircraft
    - ~7500 legs for a planning period of 1 month
  - Base balancing in scenarios with multiple homebases (2-6).
  - Solution times: 0.5h-4.5h
    (Intel 3.0GHz)

- **Roster Optimization**
  - Production quality solutions for various scenarios, covering 3 different airlines
  - Largest planning group:
    - >1000 crew members
    - ~5500 positions to be covered in a planning period of 1 month
  - Fair assignment + preferential bidding
  - Solution times: 0.5h-9.5h
    (Intel 3.0GHz)

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- Dynamic Reduce-and-Generate approach for solving all airline planning and control problems
- Results for large-scale scheduling problems show:
  - linear/quadratic performance regarding problem size
  - quality is often near globally optimal
- The problem structure can differ strongly depending on business requirements
- Potential for algorithmic improvements, e.g. further developments of dynamic reduction, parallelization, …
- Development of optimizers for integrated airline planning and control problems within OASIS program, which will improve the cost and robustness of the solutions
- See R&D web page for references and other information:

  http://www.lhsystems.com [optimization]