A Dynamic Reduce and Generate Approach for Airline Crew Scheduling

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Column Generation
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Agenda

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Crew Optimization Problems

Solution Methods

Final Remarks
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OASIS Program (2005-2010)
Optimization Algorithms for Seamless and Integrated Solvers

Planning | Control
---|---
Crew Management Process
- Pairing Generation
- Crew Assignment
- Pairing & Roster Maintenance
- Crew Control

Integrated Optimization of the Resource Aircraft and Crew
- Fleet Assignment
- Tail Assignment & Maintenance Planning
- Ops Control

Integrated Recovery of Aircraft and Crew

Operations Management Process
- Planning
- Control
Crew scheduling as part of an airline optimization suite

- Study ongoing
  - Integrated optimization
- Development ongoing
  - Pairing Optimization
  - Roster Optimization
  - Tail Assignment Optimization
  - Aircraft Recovery
- Released
  - Crew Recovery
  - Common xOPT core (~40%)
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### Pairing and Roster Construction as Part of an Airline Planning & Control Process

#### Processes for Aircraft and Crew Planning & Control

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**Aircraft**
- Network planning
- Fleet assignment + maintenance routing
- Fleet reassignment + adaptation of maintenance routing
- Tail assignment
- Aircraft recovery

**Crew**
- Pairing construction
- Roster construction and roster maintenance
- Crew recovery
Problem Description: Pairing Construction

- **Output:** Anonymous lines of work (from base to base)

- **Main constraints:**
  - Legality rules for individual pairings: e.g. max. duty time
  - Distribution between crew bases (+ other bounds on the number of pairings with certain features)

- **Objective:**
  - Minimize cost (transports, hotels, …)
  - Maximize robustness and suitability of pairings for roster construction (next planning step)
Problem Description: Roster Construction

**Output:** Assignment of pairings to crew members

**Main constraints:**
- Legality rules for individual rosters: e.g. max. duty time per certain period, distribution of off-days..
- Legality rules for combination of rosters: qualification constraints, don’t fly with, …

**Objective:**
- Minimize cost (by additional transports, hotels, overtime pay, …)
- Maximize crew satisfaction (fair distribution of unattractive duties, respecting preferences based on maximum fulfillments, …)
Seamless Optimization

- Seamless Pairing and Roster Optimization:
  - Changes in the input data
  - Stable solution which is similar to nominal solution, e.g. few changes of off time
  - Minor changes in the solution strategy
Recovery, Robust and Integrated Optimization

- **Crew and Aircraft Recovery (2008-2009):**
  - Objective: Minimize impact of disruptions
  - Crew recovery is an integration of pairing and rostering
  - Shorter run times (few minutes vs. several hours)

- **Robust Crew and Aircraft Planning (2008-2009):**
  - Objective: minimization of total planning and operational costs and increase of punctuality
  - Cooperation with Konrad Zuse Institute Berlin and University of Paderborn

- **Integrated Optimization (2009 - 2010):**
  - Integrated recovery
  - Integrated planning
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Scheduling Problem Formulation

- **General scheduling problem:** Assignment of resources to paths through a network

- **Constraints:**
  - **Network** structure
  - **Horizontal constraints** for single paths (e.g. max. duty time for a crew member)
  - **Vertical constraints** for multiple paths (e.g. min. number of crew members with a certain qualification)

- **Common solution approach: Column Generation**
Scheduling Problem Formulation - Pairing Event Network

Nodes:
- Leg (blue)
- Cin/Cout (yellow)

Edges:
- Transport (red)
IP and LP Formulation

Enumeration of all legal paths in the network gives the IP-formulation:

Min \( c(x,s) \)
\[ \begin{align*}
Ax &= b+s & (1) \\
x &\in \{0,1\}^n, \ s &\in S
\end{align*} \]

- \( c_i \) is the cost of a path and \((c_i, A_i)\) is called a column
- \((1)\) are partitioning and ‘vertical’ legality constraints
- Replacing \( \{0,1\} \) by \([0,1]\) gives the LP-formulation
- Column generation generates a restricted IP with fewer columns
Reduce-and-Generate Approach for solving large-scale Scheduling Problems

- Using traditional column generation with static problem reduction it is not possible to solve large-scale crew scheduling problems with many transport and flight connections in reasonable time.

- Dynamic reduce-and-generate approach:
  - Successively solving reduced scheduling problems, called sub-problems, with given size.
  - Determination of sub-problems using dynamic reduction of the network and the path pool.
  - Sub-problems can be solved in reasonable time. Hence, good performance if number of main iterations is not too large.
  - Good solution quality if reduced networks contain legal paths with negative reduced costs. Then the final LP/IP objective value is near globally optimal.
  - Convergence if systematic change of restrictions.
Static Problem Reduction (Fixing)

1. Fixations generated from the IP solver based on fractional LP solutions
   - master problem reduction (column and constraint fixations)
   - network reduction (node/arc fixations)

2. Restrictions generated from main algorithm
   - horizontal pruning constraints and limited path search
   - network reduction based on an approximated scheduling solution
   - window restrictions (block-coordinate search)
Dynamic Problem Reduction (Fixing)

**Dynamic network reduction** (node/arc fixations) in each path generation iteration generated from:

- network preprocessing
- labels computed by solving a relaxed pricing problem
- vertical information from partially rounded LP solutions
- ‘disjunctive’ path generation
- ‘block-coordinate search’
- dynamic transport generation
Reduce-And-Generate Approach

- Network
- Node/Arc Fixations
- Linear Programming Engine
- Dynamic Fixing Engine
- Static Fixing Engine
- Path Pool
- Path Fixations
- Constrained Shortest Path Engine
- Start Heuristic
- Solution Paths
Optimization Engines

- **Constrained Shortest Path Engine**
  - Generates legal paths with negative „reduced costs“, which improve the LP
  - Depth-first search using labels and dynamic arc sorting

- **Linear Programming Engine**
  - Solves the LP for
    a) computing reduced costs (dual solution)
    b) computing a fractional solution (primal solution)
  - Inexact proximal bundle algorithm

- **Fixing Engine**
  - Determines path or network fixations such that the difference of IP and LP objective values is very small
  - Perturbation heuristic based on fractional LP solutions
Solution Strategy

- **Pairing solution strategy:**
  - generating many paths per iteration
  - use of pruning constraints, i.e. in the beginning search for pairings with restrictions, e.g. length, number of transports

- **Roster solution strategy:**
  - generating few paths per iteration, because of long paths and ‘symmetry’
  - ‘simultaneous path generation’
  - ‘sub-problem refinement’ → linear performance

- Dynamic transport generation for all crew optimizers
Results

■ **Pairing Optimization**
  ■ Production quality solutions for various scenarios, covering 4 different airlines
  ■ Largest planning group:
    ■ Fleet of up to >40 aircraft
    ■ ~7500 legs for a planning period of 1 month
  ■ Base balancing in scenarios with multiple homebases (2-6).
  ■ Solution times: 0.5h-4.5h (Intel 3.0GHz)

■ **Roster Optimization**
  ■ Production quality solutions for various scenarios, covering 3 different airlines
  ■ Largest planning group:
    ■ >1000 crew members
    ■ ~5500 positions to be covered in a planning period of 1 month
  ■ Fair assignment + preferential bidding
  ■ Solution times: 0.5h-9.5h (Intel 3.0GHz)
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- Dynamic Reduce-and-Generate approach for solving all airline planning and control problems

- Results for large-scale scheduling problems show:
  - linear/quadratic performance regarding problem size
  - quality is often near globally optimal

- The problem structure can differ strongly depending on business requirements

- Potential for algorithmic improvements, e.g. further developments of dynamic reduction, parallelization, …

- Development of optimizers for integrated airline planning and control problems within OASIS program, which will improve the cost and robustness of the solutions

- See R&D web page for references and other information:
  [optimization]
  http://www.lhsystems.com