

Real-time battery optimization for remote microgrids

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Real-time battery optimization for remote microgrids

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Abstract : In this paper, a new deterministic model is developed for the real-time optimization of microgrids with diesel gensets and battery energy storage systems. This new model ensures a better usage of batteries that prevents undesirable behavior, such as intermittent battery charge and discharge. Furthermore, the model enables implicitly controlling battery cycling, based on information provided by upper optimization layers. Also, since some constraints associated with batteries are bilinear (therefore nonlinear) and because performance is critical for real-time optimization, many approximations are proposed to linearize the problem. Among these, the constant voltage approximation and the relative state of charge calculation approximation yield an acceptable performance for real-time optimization, while keeping the approximation error small with respect to the relative state of charge.

Keywords : Real-time optimization; mixed-integer linear programming; deterministic unit commitment; microgrids; battery energy storage systems

1 Introduction

Remote microgrids in Quebec are electricity generation and distribution networks that are not connected to Hydro-Quebec's main grid. These microgrids serve isolated communities, mostly located in remote regions. They are powered by thermal power plants using diesel gensets, making them more expensive and more polluting than the main grid, which is primarily powered by hydroelectricity. Hydro-Quebec is working on decarbonizing remote microgrids to reduce their environmental impact. To achieve this, it is necessary to couple renewable energy sources, such as wind power, with the gensets to reduce fuel consumption and greenhouse gas (GHG) emissions [29]. This coupling brings many challenges to microgrids (i.e., weak grids), particularly due to the variability of wind generation [28]. Indeed, in these networks, the introduction of this variable generation quickly reaches a high proportion compared to that of the gensets. Consequently, it may be necessary to keep several gensets running to compensate for a sudden drop in wind generation and to maintain the grid's voltage and frequency [6, 27]. To avoid keeping gensets running (and thus consuming fuel), several other grid-forming resources can be used to provide these services to the grid, allowing time if needed to start the gensets. Among these resources, battery energy storage systems are the most commonly used. In this article, we will use the terms *batteries* and *storage system* to refer to these systems, in order to simplify the reading when there is no ambiguity.

Many heuristic approaches are proposed in the literature for coupling renewable energies with gensets [3, 11, 15, 18]. These methods offer suboptimal solutions, and it is often difficult to assess their quality. In a real-time optimization context, Yuchong et al. [14] developed an approach based on decision trees, demonstrating its effectiveness by comparing it to an exact mixed-integer optimization. However, this approach relies on assumptions that are not applicable in our context. For example, the assumption of starting and stopping all gensets simultaneously is a simplification of our problem, where each generator's configuration is considered individually. Furthermore, the time step considered in the work of Yuchong et al. [14] is 5 minutes. Such a time step does not allow us, in our context, to take into account several constraints related to the warm-up and cool-down phases of each generator. Other heuristics are based on reinforcement learning [9, 10, 30], which perform well in terms of speed in contexts with a moderate number of actions. In our context, the time step we consider makes the number of possible actions excessively high.

Before addressing the techno-economic modeling of wind turbines in the real-time optimization of remote microgrids, we will focus in this article on the modeling of these storage systems and their integration into the mathematical model presented in [16], which studied gensets only. This previous study was based on mixed-integer linear programming, which allowed for the optimal, real-time planning of diesel gensets for the next 48 hours. This horizon is discretized into one-minute periods (a total of 2880 minutes in the planning horizon). Moreover, the modeling included various constraints: each generator produces electrical power within a fixed range defined by a minimum and maximum load, and must respect warm-up and cool-down durations, as well as a minimum runtime after each startup. There were also constraints to meet the ISO 8528-1 standards [1] and for load sharing among the running gensets.

Integrating the battery modeling makes the mathematical model complex to solve exactly. Indeed, in addition to the constraints aforementioned, some constraints associated with batteries are non-linear, significantly influencing the problem's complexity. In this paper, we propose and compare several linear approximation methods for these non-linear formulations.

Very often, considering battery modeling in a linear program [4, 12–14] leads to neglecting the consistency between certain physical quantities (power, current, and voltage). Furthermore, for the consideration of longer time steps: Javier et al. [26] propose an efficient linear formulation to simultaneously manage energy generation and demand over a one-day rolling horizon with 15-minute time steps (96 periods in total). In our context, this represents an oversimplification of the problem. Sciolti et al. [23, 24] addressed the issue of physical consistency between current, voltage, and power

in a tactical/strategic context, where they solve over an entire year with hourly precision, effectively approximating non-linear battery management. To do this, they approximate non-linearities empirically or using McCormick envelopes [19]. To accelerate the resolution, valid inequalities were added to reduce symmetry, and CPLEX's *warm start* feature was used.

To our knowledge, no work has been published on battery modeling in the context of real-time optimization with a one-minute time step. It is important to note that such a time step requires taking into account different types of constraints applicable during the operation of the gensets.

2 Model

A battery energy storage system consists of an assembly of electrochemical cells connected in series to form a string (which increases the voltage), and several of these strings are connected in parallel (which increases the current) to form modules (batteries). Several modules are connected in series into module strings, and the module strings are connected in parallel to form the “battery” part of the battery storage system. Since batteries provide DC voltage and current, a power converter is needed to transform the DC voltage into the AC voltage of the power grid, and a transformer is used at the output to adjust the voltage level, provide galvanic isolation, and eliminate triplen harmonics (with the addition of a delta winding).

2.1 Objective function

As mentioned in [16], the costs associated with maintenance, as well as the costs related to the energy consumed by the auxiliary services, are not typically considered in the operating costs of a power plant, but the power of the auxiliaries contributes to the total load of the microgrid. Consequently, there is no operating cost associated with the battery energy storage system. However, to avoid excessive cycling that would lead to premature aging of the asset, a penalty is added to control cycling. The objective function is then given by:

$$\begin{aligned} \text{minimize} \quad & \text{genset costs} + \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} \epsilon_b^B (x_{b,t}^{BD} + x_{b,t}^{BC}) \\ & + \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} c_b^B (n_{b,t}^{BD} + n_{b,t}^{BC}) \end{aligned} \quad (1)$$

where $x_{b,t}^{BD}$ (resp., $x_{b,t}^{BC}$) is a binary variable representing the discharging (resp., charging) state of battery b at period t . A positive penalty given by ϵ_b^B is imposed upon the activation of each of these variables. This ensures that these variables are zero, in an optimal solution, when the battery in question is not being used (battery b is neither charging nor discharging at period t).

The binary variables $x_{b,t}^{BD}$ and $x_{b,t}^{BC}$ are used to formally define (see section 2.6) the binary variables $n_{b,t}^{BD}$ and $n_{b,t}^{BC}$ present in the last term of the objective function, where:

- $n_{b,t}^{BD}$ equals 1 if battery b changes state [from discharging to charging/idle] or [from charging/idle to discharging] at period t ;
- $n_{b,t}^{BC}$ equals 1 if battery b changes state [from charging to discharging/idle] or [from discharging/idle to charging] at period t .

The parameter c_b^B corresponds to the penalty applied to the storage system b to penalize these state changes and thereby control its cycling. However, state changes should not be directly associated with cycles. To calculate cycles, a cycle counting method, such as the *rainflow* counting algorithm [2, 7, 8, 21], can be used *a posteriori* to adjust the penalty c_b^B over the planning horizon. This optimization of the use of the energy storage system to maximize its usage over its useful lifespan is handled by higher-level optimization layers (i.e., those looking at longer-term planning horizons).

2.2 Demand and reserve constraints

The global demand constraints in a microgrid system can be expressed by the fact that the sum of the generated active power equals the sum of the consumed active power (supply-demand balance). We then have:

$$P_t^G + \sum_{b \in \mathcal{B}} (p_{b,t}^{BD} - p_{b,t}^{BC}) = D_t, \quad \forall t \in \mathcal{T} \quad (2)$$

where, for each period t of the horizon $\mathcal{T}$, the sum of the active power generated by the generators P_t^G and the active powers discharged $p_{b,t}^{BD}$ by the storage systems $\mathcal{B}$ is equal to the sum of the net load D_t and the active powers charged $p_{b,t}^{BC}$ by the storage systems $\mathcal{B}$. It should be noted that D_t corresponds to the net load, i.e., the uncontrollable load (including losses) minus the uncontrollable generation (e.g., customer solar panels without remote control). Consequently, there can be high variability and uncertainty in the net load, and a certain reserve must be maintained in the controllable generation to ensure grid security. This gives rise to the second following global constraint on the reserve:

$$\bar{P}_t^G + \sum_{b \in \mathcal{B}} \bar{P}_{b,t}^B \geq D_t + R_t, \quad \forall t \in \mathcal{T} \quad (3)$$

where $\bar{P}_t^G$ represents the total active power available from the generators and $\bar{P}_{b,t}^B$ represents the active power available from the storage system b at period t . The sum of these must be greater than or equal to the net load D_t plus the required reserve R_t at period t .

2.3 Voltage, current, power and state of charge constraints

For each battery $b \in \mathcal{B}$ and each period $t \in \mathcal{T}$, we define the continuous variable $S_{b,t}^B$ representing the relative SOC (state of charge), where the term “relative” distinguishes it from the (absolute) SOC, which decreases with battery degradation. The value of this variable must always be between S_b^{MIN} and S_b^{MAX} with: $0 \leq S_b^{MIN} \leq S_{b,t}^B \leq S_b^{MAX} \leq 1$. Introducing lower and upper limits on the relative state of charge prevents a (voltage) limiter from being activated in the battery management system (BMS) when the current (in absolute value) equals the nominal current I_b^{NOM} . This phenomenon is caused by Peukert’s law [22], where the effective capacity of a battery decreases with the discharge current (resp., increases with the charge current), but the actual discharge capacity would be greater if the discharge current were lower (resp., lower if the charge current were lower). Physically, this can be explained by the battery’s equivalent resistance and its internal voltage drop, as the battery management system monitors the voltage level to establish lower and upper limits for discharging and charging the battery (the low battery cut-off voltage and high battery cut-off voltage).

To update the relative SOC of each battery at each period, we use a coulomb counting method [20, eq. 4], which is the algorithm generally used in practice in the BMS, by introducing the following constraints:

$$\begin{cases} S_{b,1}^B = S_b^0 - \frac{I_{b,1}^B}{n^H Q_b^{MAX}}, & \forall b \in \mathcal{B}, t = 1 \\ S_{b,t}^B = S_{b,t-1}^B - \frac{I_{b,t}^B}{n^H Q_b^{MAX}}, & \forall b \in \mathcal{B}, \forall t \in [2, |\mathcal{T}|] \end{cases} \quad (4)$$

where $I_{b,t}^B$ is a continuous variable representing the DC current value of the storage system b at period t . This value is divided by n^H , the number of periods in an hour, and multiplied by Q_b^{MAX} , the maximum capacity of the storage system b in ampere-hours (Ah). It is assumed that since the real-time planning horizon is relatively short, this maximum capacity is constant, i.e., the degradation of the storage system over this planning horizon is neglected. The calculation of degradation will be performed in a higher-level optimization layer to update this maximum capacity according to the battery’s state of health (SOH). When the storage system b is charging (resp., discharging), the current $I_{b,t}^B$ is negative (resp., positive). Thus, we increase (resp., decrease) the SOC value at period t compared to period $t - 1$ by the positive (resp., negative) value $-\frac{I_{b,t}^B}{n^H Q_b^{MAX}}$. Note that S_b^0 represents the SOC value of battery b at the beginning of the horizon.

On the one hand, the value of the current $I_{b,t}^B$ is defined using the following equations:

$$1000 \times P_{b,t}^B = I_{b,t}^B V_{b,t}^B, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (5)$$

where the constant 1000 appears for unit conversion reasons (kW to W) and where the current respects the following bounds:

$$-I_b^{NOM} \leq I_{b,t}^B \leq I_b^{NOM}, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (6)$$

where I_b^{NOM} is the nominal current of the storage system b , $P_{b,t}^B$ and $V_{b,t}^B$ respectively represent the battery power on the DC side and the DC voltage of the storage system b at period t . The power $P_{b,t}^B$ is related to the active power charged/discharged from the grid by the AC/DC conversion constraints:

$$P_{b,t}^B = \frac{1}{\eta_b} p_{b,t}^{BD} - \eta_b p_{b,t}^{BC}, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (7)$$

where η_b is the efficiency of the conversion system (including the converter, the filter, and the transformer) for the storage system b . It's assumed that the efficiency is symmetric, i.e., it is the same during discharge and charge, in contrast to some papers, such as [25, eq. 1].

On the other hand, the voltage $V_{b,t}^B$ is related to $V_{b,t}^{BOC}$, the open-circuit DC voltage of the storage system b at period t , and to the DC current $I_{b,t}^B$ by the constraints:

$$V_{b,t}^B = V_{b,t}^{BOC} - R_b^B I_{b,t}^B, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (8)$$

where R_b^B is the equivalent internal resistance of the battery in storage system b . As can be seen in [5, Fig. 4], the open-circuit voltage of the battery (similar to the open-circuit voltage of a cell) varies non-linearly according to the relative SOC. However, considering that the limits S_b^{MIN} and S_b^{MAX} are introduced to avoid activating limiters in the battery management system and since the curve is relatively linear in the middle (neglecting the hysteresis phenomenon), a linear regression is used to approximate the relationship between the voltage $V_{b,t}^{BOC}$ and the SOC. Let A_b and B_b be respectively the slope and the y-intercept of the line resulting from this regression. We then have the following constraints:

$$V_{b,t}^{BOC} = A_b S_{b,t}^B + B_b, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (9)$$

2.4 Constraints to define the binary variables of charge/discharge states

For each battery b , we introduced the binary variable $x_{b,t}^{BD}$ representing the discharging state of battery b at period t . The variable $x_{b,t}^{BD}$ equals 1 if battery b discharges at period t . In order to include these variables in the formulation, the following constraints must be added:

$$p_{b,t}^{BD} \leq x_{b,t}^{BD} P_b^{NOM}, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (10)$$

where P_b^{NOM} is the nominal active power of the storage system b . When a storage system b discharges at period t , then $p_{b,t}^{BD} > 0$. In this case, constraints (10) force the activation of the variable $x_{b,t}^{BD}$ to set it to 1 (its only possible non-zero value). When the storage system is not in a discharging state, then $x_{b,t}^{BD}$ equals 0 thanks to the penalty ϵ_b^B in (1) applied to its default activation. Briefly, the role of this penalty is to consider that the value assigned by default to this variable in an optimal solution is 0. Following a similar reasoning, we defined the variable $x_{b,t}^{BC}$ for the charging state of the storage system b at period t and also the following constraints:

$$p_{b,t}^{BC} \leq x_{b,t}^{BC} P_b^{NOM}, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T}. \quad (11)$$

Finally, since a battery cannot be charged and discharged simultaneously, this behavior is enforced using the following constraints:

$$x_{b,t}^{BD} + x_{b,t}^{BC} \leq 1, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T}. \quad (12)$$

2.5 Limits on available power constraints

For each storage system b , the available power $\bar{P}_{b,t}^B$ at period t is used to bound the discharged active power from above during the same period. Conversely, during charging, the active power is added to the nominal power of the storage system b to determine the available power $\bar{P}_{b,t}^B$, because from the grid's perspective, it represents a load that can be shed. Thus, this translates into the following constraints:

$$p_{b,t}^{BD} \leq \bar{P}_{b,t}^B, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (13)$$

$$\bar{P}_{b,t}^B \leq P_b^{NOM} + p_{b,t}^{BC}, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T}. \quad (14)$$

2.6 Constraints to count the state changes

Let $n_{b,t}^{BD}$ be a binary variable that equals 1 if battery b changes state [from discharging to charging/idle] or [from charging/idle to discharging] at period t . This state change can be expressed by the equation $n_{b,t}^{BD} = |x_{b,t-1}^{BD} - x_{b,t}^{BD}|$. Indeed, $n_{b,t}^{BD}$ must take the value zero if the battery is discharging (or is not discharging) during the two consecutive periods $t-1$ and t . In this case, we have $|x_{b,t-1}^{BD} - x_{b,t}^{BD}| = |1 - 1| = 0$ (or $|x_{b,t-1}^{BD} - x_{b,t}^{BD}| = |0 - 0| = 0$). In the case where the battery is in a discharging state at period $t-1$ and not at period t (or discharging at period t and not at period $t-1$), we have $|x_{b,t-1}^{BD} - x_{b,t}^{BD}| = |0 - 1| = 1$ (or $|x_{b,t-1}^{BD} - x_{b,t}^{BD}| = |1 - 0| = 1$). Since the absolute value function is non-linear, it can be linearized by the following constraints, thanks to the penalty c_b^B present in the objective function (1):

$$\begin{cases} x_{b,0}^{BD} - x_{b,1}^{BD} \leq n_{b,1}^{BD}, & \forall b \in \mathcal{B}, t = 1 \\ x_{b,t}^{BD} - x_{b,t-1}^{BD} \leq n_{b,t}^{BD}, & \forall b \in \mathcal{B}, \forall t \in [2, |\mathcal{T}|] \\ x_{b,1}^{BD} - x_{b,0}^{BD} \leq n_{b,1}^{BD}, & \forall b \in \mathcal{B}, t = 1 \\ x_{b,t-1}^{BD} - x_{b,t}^{BD} \leq n_{b,t}^{BD}, & \forall b \in \mathcal{B}, \forall t \in [2, |\mathcal{T}|] \end{cases} \quad (15)$$

where $x_{b,0}^{BD}$ is an “initial” parameter that equals 1 if battery b is discharging at the beginning of the horizon, otherwise this parameter equals 0. Similarly, we define the parameter $x_{b,0}^{BC}$ which equals 1 if battery b is initially charging, and equals 0 otherwise.

Analogously, we define the binary variable $n_{b,t}^{BC}$ which equals 1 if battery b changes state [from charging to discharging/idle] or [from discharging/idle to charging] at period t (i.e., $n_{b,t}^{BC} = |x_{b,t-1}^{BC} - x_{b,t}^{BC}|$). This equation is linearized using the following constraints:

$$\begin{cases} x_{b,0}^{BC} - x_{b,1}^{BC} \leq n_{b,1}^{BC}, & \forall b \in \mathcal{B}, t = 1 \\ x_{b,t}^{BC} - x_{b,t-1}^{BC} \leq n_{b,t}^{BC}, & \forall b \in \mathcal{B}, \forall t \in [2, |\mathcal{T}|] \\ x_{b,1}^{BC} - x_{b,0}^{BC} \leq n_{b,1}^{BC}, & \forall b \in \mathcal{B}, t = 1 \\ x_{b,t-1}^{BC} - x_{b,t}^{BC} \leq n_{b,t}^{BC}, & \forall b \in \mathcal{B}, \forall t \in [2, |\mathcal{T}|]. \end{cases} \quad (16)$$

The total number of state changes over the planning horizon is then given by:

$$\sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} (n_{b,t}^{BD} + n_{b,t}^{BC}) \quad (17)$$

2.7 Generator constraints

All constraints presented in the article [16], for the set of generators $\mathcal{G}$, are considered in this study. However, since the plant controller must be able to regain control in case of problems and deactivate

the optimizer, it is necessary to add new constraints to be compatible with the current operating method in our remote microgrids. Indeed, as the priority order of the generators must be entered manually, it is currently not possible to dynamically change the priorities of the generators via the optimizer. Thus, the manually entered unit priorities must be respected, and they will be sent to the optimizer. The new constraints therefore consist of considering a priority order among the generators.

Let σ be the permutation that gives the priority order of each generator:

$$\begin{array}{ccc} \sigma & : & \{1, 2, \dots, |\mathcal{G}|\} \rightarrow \{1, 2, \dots, |\mathcal{G}|\} \\ & & g \mapsto \sigma(g) \end{array}$$

When we have $\sigma(g) < \sigma(g')$, then generator g has higher priority than generator g' . In other words, generator g must be used first before using generator g' (i.e., starting generator g before generator g' , and conversely, stopping generator g' before stopping generator g). Also, since $\sigma(g) < \sigma(g')$, it goes without saying that $g \neq g'$. We then have the following new constraints:

$$v_{\sigma(g),t}^G \leq v_{\sigma(g'),t}^G, \quad \forall t \in \mathcal{T}, \forall g, g' \in \mathcal{G} \mid \sigma(g) < \sigma(g') \quad (18)$$

where $v_{g,t}^G$ represents the binary variable that equals 1 if generator $g \in \mathcal{G}$ is running during period $t \in \mathcal{T}$.

Furthermore, the introduction of these priority constraints enables the explicit integration of moving average power constraints through a higher optimization layer. More specifically, this upper layer determines the generators' commitment priority order based, among other factors, on the moving average power limits associated with each unit. The resulting priority order is then communicated to the real-time optimization layer through the permutation σ , thereby maintaining high computational efficiency while promoting compliance with the corresponding limits.

3 Linear approximations

The model described in the previous section is a mathematical program that contains bilinear constraints (5). Solving such a (non-convex) problem exactly will be very costly in terms of time. Our context, which falls within the framework of real-time optimization, requires us to propose other methods in order to reduce the computation time. For this purpose, three approximation methods are proposed in the following sections.

3.1 Constant voltage approximation

The first approximation is based on the assumption that the battery voltage $V_{b,t}^B$ is constant and equal to its nominal voltage. By doing so, the bilinear relationship (5) becomes linear, and the battery power becomes directly proportional to the current. Consequently, if the numerator and the denominator in constraints (4) are multiplied by the nominal voltage of the battery, constraints (4) become:

$$\begin{cases} S_{b,1}^B = S_b^0 - \frac{P_{b,1}^B}{n^H E_b^{MAX}}, & \forall b \in \mathcal{B}, t = 1 \\ S_{b,t}^B = S_{b,t-1}^B - \frac{P_{b,t}^B}{n^H E_b^{MAX}}, & \forall b \in \mathcal{B}, \forall t \in [2, |\mathcal{T}|] \end{cases} \quad (19)$$

where E_b^{MAX} is the nominal energy of battery b in kWh (if the Q_b^{MAX} given in the problem is the nominal capacity of the battery in Ah).

This approximation is frequently used in the literature, for example in [17, 22], but it is important to mention that by assuming the battery voltage is constant, the internal voltage drop constraints (8) are neglected. Therefore, constraints (9) are no longer useful. Furthermore, since the nominal battery voltage is usually equal to its open-circuit voltage when fully discharged, this generally results in an

underestimation of the voltage $V_{b,t}^B$. However, due to the bilinear relationship (5), this approximation implies an overestimation (in absolute value) of the battery current. Thus, during discharge, the relative SOC will be underestimated, while during charge, it will be overestimated. In addition, the upper or lower current bounds (6) could be reached before the upper or lower bounds of $P_{b,t}^B$ are reached. Thus, neglecting constraints (6) and (5) in the classical approximation could lead to an infeasible solution.

3.2 McCormick approximation

This method is based on the convex relaxation of the bilinear constraints (5), which consists of adding linear constraints, called *McCormick* envelopes (see [19]). These constraints form the convex envelope of the function $(I_{b,t}^B, V_{b,t}^B) \mapsto \frac{I_{b,t}^B \times V_{b,t}^B}{1000}$. To do this, we first define V_b^{MIN} and V_b^{MAX} , the best bounds for each battery b that we can obtain for $V_{b,t}^B$. Indeed, since $S_b^{MIN} \leq S_{b,t}^B \leq S_b^{MAX}$, then according to constraints (6), (8) and (9) we can deduce that:

$$\begin{cases} V_b^{MIN} = A_b S^{MIN} + B_b - R_b I_b^{NOM}, & \forall b \in \mathcal{B} \\ V_b^{MAX} = A_b S^{MAX} + B_b + R_b I_b^{NOM}, & \forall b \in \mathcal{B}. \end{cases} \quad (20)$$

Thus, the *McCormick* envelopes in our context are the following:

$$\begin{cases} P_{b,t}^B * 1000 \leq V_b^{MIN} I_{b,t}^B + I_b^{NOM} V_{b,t}^B - V_b^{MIN} I_b^{NOM}, \\ \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \\ P_{b,t}^B * 1000 \leq -I_b^{NOM} V_{b,t}^B + V_b^{MAX} I_{b,t}^B + I_b^{NOM} V_b^{MAX}, \\ \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \\ P_{b,t}^B * 1000 \geq V_b^{MIN} I_{b,t}^B - I_b^{NOM} V_{b,t}^B + V_b^{MIN} I_b^{NOM}, \\ \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \\ P_{b,t}^B * 1000 \geq V_b^{MAX} I_{b,t}^B + I_b^{NOM} V_{b,t}^B - I_b^{NOM} V_b^{MAX}, \\ \forall b \in \mathcal{B}, \forall t \in \mathcal{T}. \end{cases} \quad (21)$$

However, this new formulation contains the following flaw: if $P_{b,t}^B = 0$, there is no guarantee that $I_{b,t}^B = 0$. Indeed, the solver will push $I_{b,t}^B$ to take the smallest negative value, because this increases the SOC without any penalty, which is highly cost-effective, but very negatively affects the behavior and consistency between the physical quantities $S_{b,t}^B$, $I_{b,t}^B$, $P_{b,t}^B$ and $V_{b,t}^B$. To fill this gap, we must strengthen the formulation by adding lower and upper bounds for the value of the current $I_{b,t}^B$ of battery b at period t .

According to constraints (5), we have $\frac{I_{b,t}^B}{1000} = \frac{P_{b,t}^B}{V_{b,t}^B}$, and since $V_b^{MIN} \leq V_{b,t}^B \leq V_b^{MAX}$, we obtain:

$$\begin{cases} \text{if } P_{b,t}^B > 0, & \text{then } \frac{P_{b,t}^B}{V_{b,t}^{MAX}} \leq \frac{I_{b,t}^B}{1000} \leq \frac{P_{b,t}^B}{V_{b,t}^{MIN}} \\ \text{if } P_{b,t}^B < 0, & \text{then } \frac{P_{b,t}^B}{V_{b,t}^{MIN}} \leq \frac{I_{b,t}^B}{1000} \leq \frac{P_{b,t}^B}{V_{b,t}^{MAX}} \\ \text{if } P_{b,t}^B = 0, & \text{then } I_{b,t}^B = 0 \end{cases}$$

Then, according to constraints (7) and (12) we can deduce that:

$$\begin{cases} \text{if } P_{b,t}^B > 0, & \text{then } P_{b,t}^B = \frac{1}{\eta_b} p_{b,t}^{BD} & \text{and } p_{b,t}^{BC} = 0 \\ \text{if } P_{b,t}^B < 0, & \text{then } P_{b,t}^B = -\eta_b p_{b,t}^{BC} & \text{and } p_{b,t}^{BD} = 0 \\ \text{if } P_{b,t}^B = 0, & \text{then } p_{b,t}^{BD} = 0 & \text{and } p_{b,t}^{BC} = 0 \end{cases}$$

As can be seen from the previous equations, the powers $p_{b,t}^{BD}$ and $p_{b,t}^{BC}$ cannot be simultaneously non-zero. This allows us to avoid introducing binary variables to represent the previous cases for the

bounds of $\frac{I_{b,t}^B}{1000}$, hence the following new constraints:

$$\begin{cases} \frac{I_{b,t}^B}{1000} \leq \frac{p_{b,t}^{BD}}{\eta_b V_b^{MIN}} - \frac{\eta_b p_{b,t}^{BC}}{V_b^{MAX}}, & \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \\ \frac{I_{b,t}^B}{1000} \geq \frac{p_{b,t}^{BD}}{\eta_b V_b^{MAX}} - \frac{\eta_b p_{b,t}^{BC}}{V_b^{MIN}}, & \forall b \in \mathcal{B}, \forall t \in \mathcal{T}. \end{cases} \quad (22)$$

3.3 Relative SOC Approximation

To improve the constant voltage approximation, a hybrid of the two previous methods can be used, where the bilinear constraints (5) are relaxed and the relative SOC is considered to be calculated from the power with (19), but while keeping the constraints associated with $I_{b,t}^B$ and $V_{b,t}^B$. Furthermore, since constraints (5) are relaxed, the previous constraints (22) are used to link the power to the current and voltage. Moreover, this allows tightening the bounds on the battery current, given by (6).

4 Results

To evaluate the performance of the proposed model, we conducted tests on 4 instances (I_1 , I_2 , I_3 and I_4), in which three generators and a battery energy storage system are modeled. The optimization is performed over a 48-hour planning horizon with a one-minute time step.

The results obtained for the 4 instances are summarized in Table 1. For each instance, the problem is solved with Gurobi, and the following is given for the exact method:

- The optimality gap (**Gap(%)**) of the solution found: this value reflects the quality of the planning associated with this solution. When this value is zero, it implies that the solver has proven optimality.
- The computation time (**T(s)**) required to obtain the planning. This time includes the time for building the mathematical program as well as its solving time in seconds. The computation time is limited to 10 hours for the resolution with the exact method.

For the approximation methods, the computation times required to obtain a solution with an optimality gap of less than 1% and with an optimality gap of 10^{-4} are given.

Table 1: Results

Inst.	Exact method		Computation time in sec.					
	Gap(%)	T(s)	McCor. appr.		Volt. appr.		SOC appr.	
			Gap ≤ 1%	Gap ≈ 0%	Gap ≤ 1%	Gap ≈ 0%	Gap ≤ 1%	Gap ≈ 0%
I_1	1.52	36000	84	158	48	68	48	64
I_2	4.70	36000	113	236	47	83	51	80
I_3	1.57	36000	169	679	55	188	52	174
I_4	2.84	36000	37	468	65	65	52	55
Avg.		36000	100.75	385.25	53.75	101	50.75	93.25

Regarding the exact method, for instance I_1 , Gurobi finds, after 10 hours of computation, a feasible solution with a 1.5% gap. Therefore, it is possible that the solution found is non-optimal. In this case, in addition to the actual costs that can be reduced if using another planning that is optimal or near-optimal, there is also the risk that the non-optimal planning could reveal undesirable behavior. For example, in Figure 1, it can be observed that there is “highly intermittent” battery charging during the periods [0, 650]. It should be noted that a non-zero gap (a solution for which optimality is not proven) does not necessarily imply the observation of such behavior (see Figure 2), but there is the possibility of observing it. The tests at this level demonstrate that the problem we are dealing with is

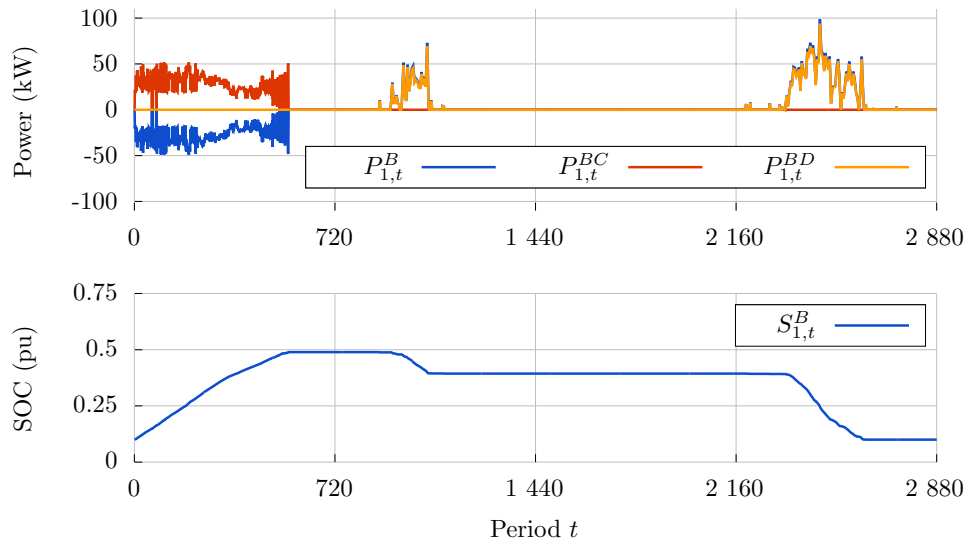


Figure 1: Instance I_1 : solution found by the exact model

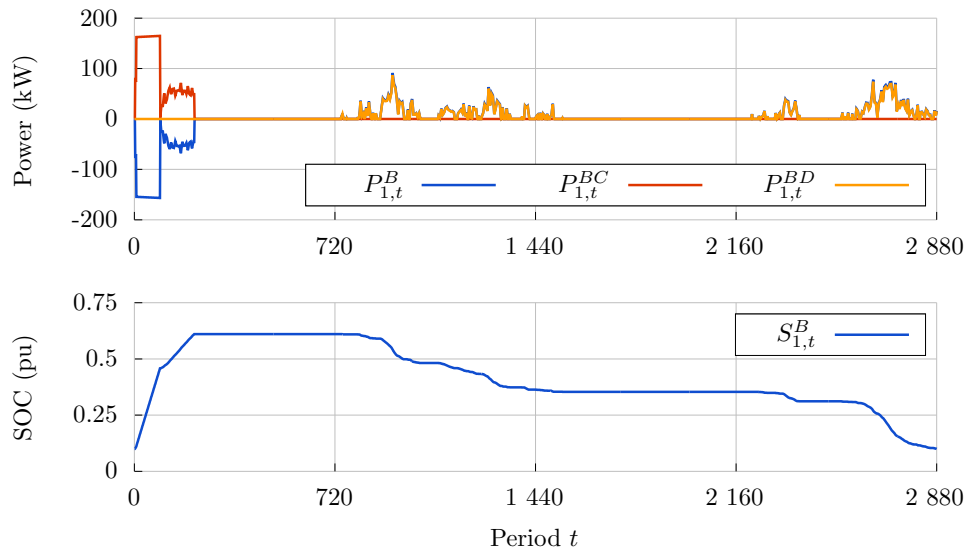


Figure 2: Instance I_2 : solution found by the exact model

very difficult in terms of complexity. Moreover, there is a lot of symmetry in the problem, as several solutions can be equivalent (within the tolerance gap) for charging and discharging the battery.

For the approximation methods, we successfully find an optimal solution for the 4 instances. The computation time is longer for the McCormick approximation method, taking an average of 6 minutes to obtain the optimal solution, and over a minute and a half to obtain a solution with an optimality gap of less than 1%. The other approximation methods are much faster. Indeed, the relative SOC calculation approximation method (resp., constant voltage approximation) finds the optimal solution, on average, in less than 94 (resp., 101) seconds. Moreover, a solution with an optimality gap of less than 1% can be obtained in less than 54 seconds on average. The relative SOC calculation approximation method is faster than the constant voltage approximation method. For the 4 instances, the speed of this method can be justified by the quality of the bounds on the current obtained thanks to constraints (22), as can be seen in Figures 3-4. Indeed, constraints (22) represent cuts that significantly reduce the

search space for the possible values of the current I^B . This will also influence the error in the SOC estimation (see Figures 9–10).

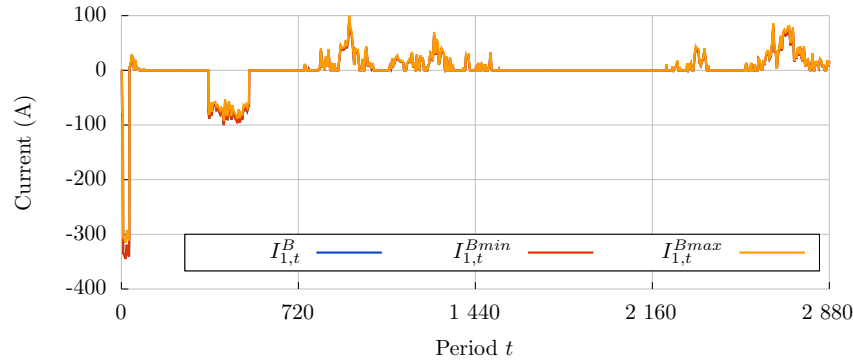


Figure 3: Instance I_2 : current bounds found by constraints (22)

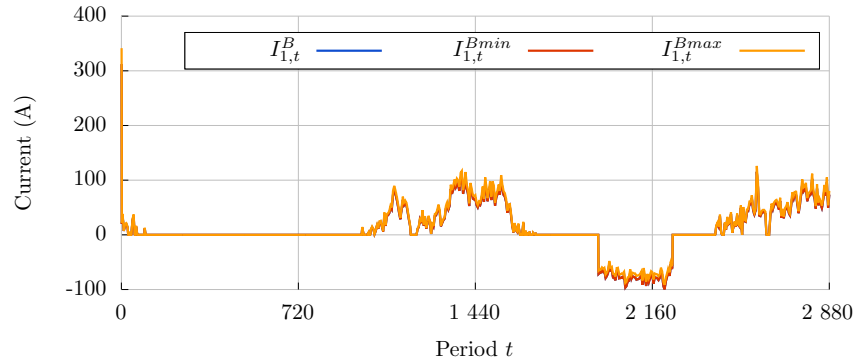
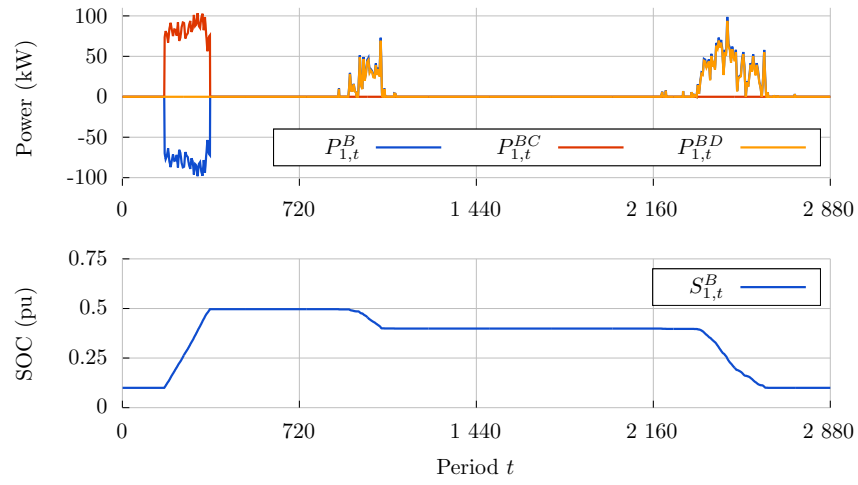
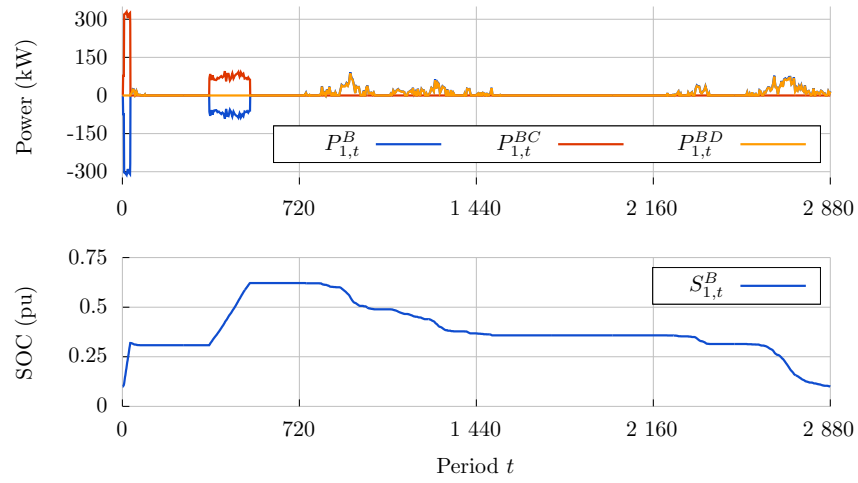
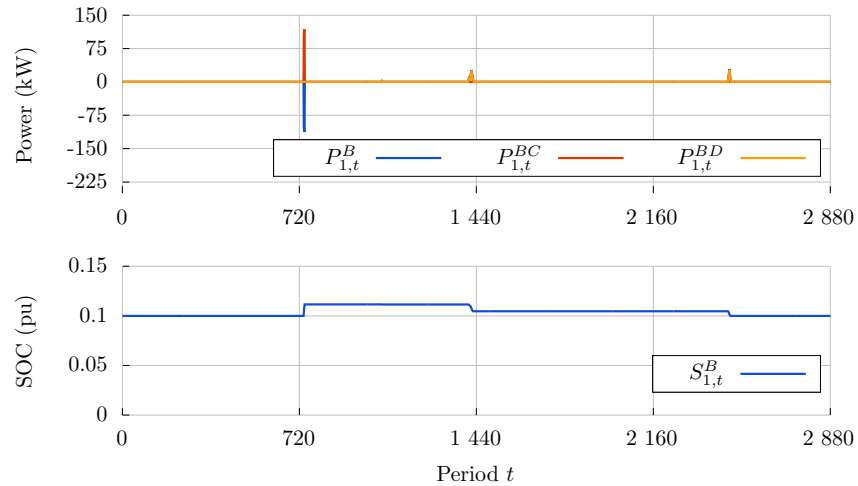


Figure 4: Instance I_4 : current bounds found by constraints (22)

It is important to emphasize that, across the four case studies considered, none of the approximation methods evaluated produced a solution exhibiting undesirable battery behavior, such as intermittent switching between charging and discharging modes, as illustrated by the examples presented in Figures 5–8.

Finally, to compare the quality of the proposed approximations, we look for the exact values associated with the power values estimated/used by the approximation method in question. To do this, the part of the solution associated with the generators is fixed and the problem is solved using the exact method. Indeed, this allows obtaining the same solution, but with the correct values for current, voltage, and power. Thanks to these values, the real value of the relative SOC can be calculated for each period of the horizon. According to Figures 9 and 10, it can be observed that the analysis proposed in subsection 3.1 is confirmed: the relative SOC will be underestimated during discharge and it will be overestimated during charge. It is important to emphasize that the remarkable overestimation of the SOC at the end of the horizon by the McCormick approximation is due to the overestimation of the voltage value during the same periods, as can be seen in Figures 11 and 12.

Figure 5: Instance I_1 : solution found by the approximation methodsFigure 6: Instance I_2 : solution found by the approximation methodsFigure 7: Instance I_3 : solution obtained with the Relative SOC Approximation method

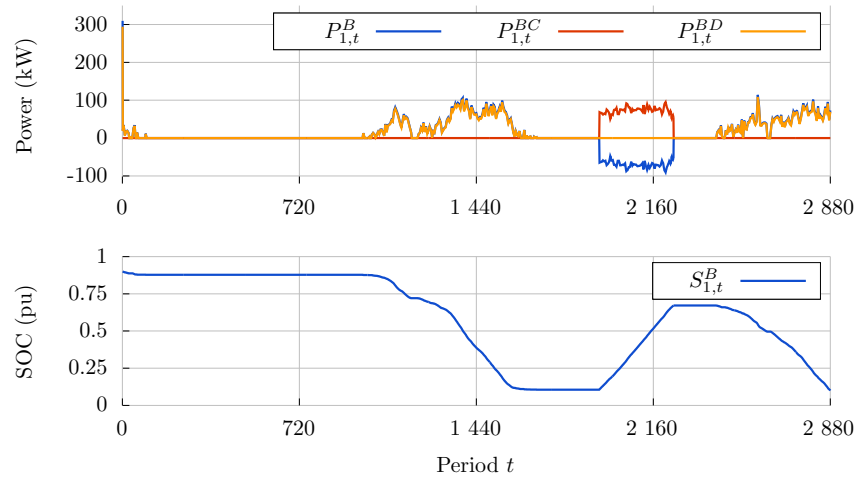


Figure 8: Instance I_4 : solution found by the approximation methods

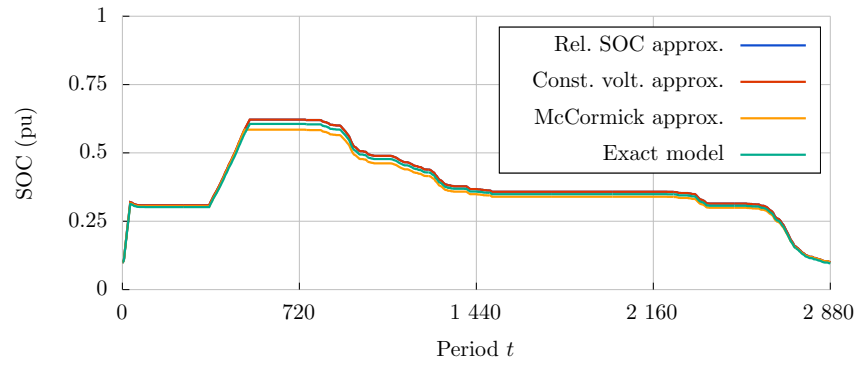


Figure 9: Instance I_2 : SOC comparison

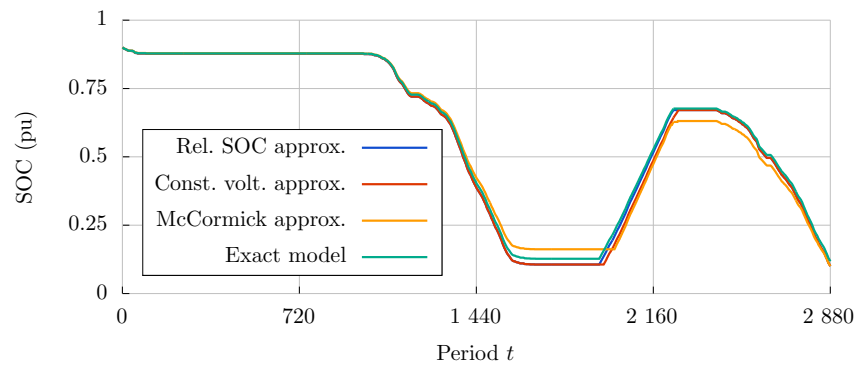
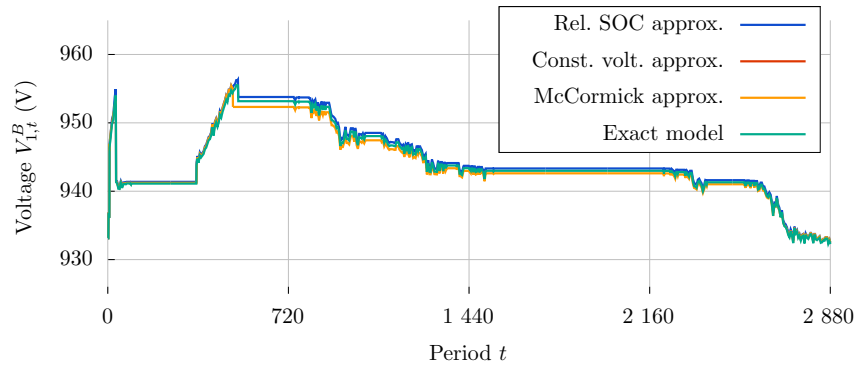
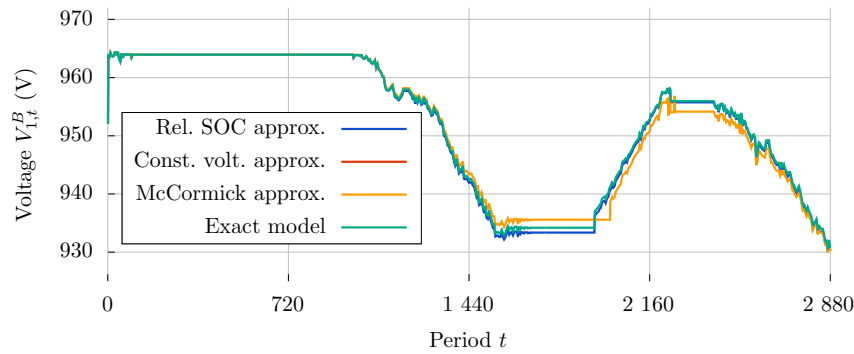


Figure 10: Instance I_4 : SOC comparison

Figure 11: Instance I_2 : voltage comparisonFigure 12: Instance I_4 : voltage comparison

5 Conclusion

In this article, a formulation for integrating battery energy storage systems into the modeling presented in article [16], which concerns diesel gensets, was proposed. In this new formulation, in addition to the constraints that ensure consistency between physical quantities (voltage, current, and power), constraints are added to avoid undesirable battery usage situations. Moreover, we also implicitly control battery cycling based on information sent by higher optimization layers. For the gensets, constraints were added to impose a prioritization order for their use, set by the user. The global mathematical program of this problem is classified as difficult: in 10 hours of computation for the four instances, Gurobi failed to find the optimal solution or to prove the optimality of the best solution obtained.

Three approximation methods are proposed to solve this mathematical program within an acceptable time frame according to our context, which falls within the framework of real-time optimization.

The first is based on the assumption that the battery voltage is constant and equals the nominal battery voltage. The second method is based on generating the convex envelope of the bilinear function used to define the relationship between power, voltage, and current. For this, we added the *McCormick* constraints, as well as other constraints to strengthen the formulation by ensuring good consistency between the physical quantities. The third approximation method, which is faster, consists of taking advantage of this consistency guaranteed by these latter constraints, where we update the relative SOC according to the power.

According to our tests, we observe that the constant voltage approximation and the relative SOC calculation approximation methods globally provide a very good approximation of the relative SOC.

The slight overestimation and underestimation observed do not influence the availability of the storage system in the context where the SOC bounds would be correctly adjusted to account for the approximation error.

Finally, it should be emphasized that the explicit integration of constraints related to the moving average power of the generators can have a negative impact on the computation time. However, these constraints can be taken into account implicitly in our study, notably through communication with a higher optimization layer. Indeed, thanks to the new constraints introduced in the formulation, concerning the prioritization order in the commitment of the generators, it is possible to set this order to respect, as much as possible, the moving average power limit specific to each generator.

These results pave the way for a deeper reflection on the use of heuristic methods to improve the resolution of this type of complex problem. Indeed, faced with the limits of solvers in a real-time optimization context, the development of customized heuristics could allow obtaining high-quality solutions with reduced computation times. A systematic exploration of these approaches constitutes a promising perspective for future work.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the authors used Gemini in order to translate and improve the readability of this work. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

CRedit authorship contribution statement

Rachid Hassani: Conceptualization, Methodology, Software, Writing – review & editing.

Mathieu Lambert: Conceptualization, Methodology, Software, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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