Les Cahiers du GERAD ISSN: 0711–2440

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G–2026–10

March 2026

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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A decision support optimization model to reduce congestion in fitness centres with an account for users' preferences

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March 2026
Les Cahiers du GERAD
G–2026–10

Abstract : It is well known that physical activity plays a key role in maintaining good health. Over the past few decades, there has been a significant increase in both the number of fitness centres worldwide and the number of individuals who use them. In addition to contributing to public health, fitness centres also play an important role in the economy, generating revenue by attracting new members and retaining existing ones. During peak hours, however, the number of customers present in a facility often exceeds the available equipment capacity. As a result, some customers are forced to wait for equipment to become available or to deviate from their planned workout routines, which can lead to dissatisfaction and member attrition. In this paper, we propose a decision support optimization model designed to reduce peak-hour congestion in fitness centres while improving customer experience. The model is intended to support an online booking system in which customers submit provisional workout plans along with their preferences. The system then outputs a final, congestion-free workout plan for each customer that closely matches these preferences. The proposed model aims to maximize equipment utilization while ensuring that each customer can complete their workout with minimal idle time. The model is tested on realistic data sets, and the computational results show that, in addition to supporting the proposed booking system, it can reduce customer idle time by up to 19% compared to a first-come, first-served operational policy.

Keywords : Integer programming; network optimization; decision support models

Acknowledgements: This work was partly supported by the Canadian Natural Sciences and Engineering Research Council under grant 2021–03307 This support is greatly appreciated. The authors have no competing interests to declare that are relevant to the content of this article.

1 Introduction

It is well known that physical activity plays a key role in maintaining good health. The World Health Organization defines physical activity as any bodily movement produced by skeletal muscles that requires energy expenditure [3]. It also recognizes physical inactivity as the fourth leading risk factor for global mortality, accounting for approximately 6% of deaths worldwide. Moreover, physical inactivity is estimated to be responsible for approximately 21–25% of breast and colon cancers, 27% of diabetes cases, and nearly 30% of the burden of ischaemic heart disease [3]. On the other hand, regular and adequate levels of physical activity provide numerous benefits. For adults, physical activity contributes to reducing the risk of hypertension, coronary heart disease, stroke, diabetes, breast and colon cancer, depression, and falls. It also helps improve bone and functional health and constitutes a key determinant of energy expenditure, which is fundamental to energy balance and weight control.

People around the world are becoming increasingly health-conscious and are engaging more frequently in physical activities. For example, in Canada, more than 16.2 million individuals aged 12 and over reported participating in at least 150 minutes of moderate- to vigorous-intensity aerobic physical activity per week [4]. This trend has been accompanied by a rapid growth in both the number of fitness centres worldwide and the number of their users. Fitness centres are health, recreational, and social facilities dedicated to exercise, sports, and other physical activities. They offer a wide range of services, including organized group classes such as spinning, yoga, and martial arts; organized and informal team sports; and individual fitness opportunities such as cardiovascular training, weight training, and swimming.

In addition to contributing to public health, fitness centres play a significant role in the economies of most developed countries. They generate revenue by recruiting new members and encouraging existing members to renew their memberships [5]. As in any business, individuals are more likely to join or renew a membership when they perceive sufficient value for their money, particularly in terms of service quality. To this end, fitness centre owners already employ various incentives to retain existing customers and attract new ones [5]. These efforts are often complemented by incentives offered by employers to encourage employees to remain physically active [6, 7].

From the customers' perspective, Afthinos et al. [8] identify several variables that influence satisfaction with fitness centres. Among the most important is the *availability* of adequate space during exercise, implying that overcrowded facilities are likely to generate dissatisfaction. Indeed, peak hours at fitness centres are often characterized by customers waiting idly for equipment to become available or being forced to use equipment that was not part of their planned workout, thus deviating from their training routines. Although some fitness centres offer discounted off-peak memberships to mitigate this issue, overcrowding remains a persistent problem due to the growing popularity of fitness centres and the limited availability of many customers, who can often exercise only after work.

During the COVID-19 pandemic [9], the fitness industry was significantly affected, as many fitness centres were forced to close or operate under severe capacity restrictions. Nevertheless, this period prompted fitness associations in many countries to collaborate on the development of joint health and safety guidelines, some of which continue to benefit the industry beyond the pandemic. One such measure aimed at addressing overcrowding was the introduction of online booking systems (e.g., in Canada [10] and New Zealand [11]). When efficiently implemented, these systems can provide a long-term solution to the longstanding issue of peak-hour congestion. While online booking systems are already used in many fitness centres, their application is often limited to group classes with small capacities. Extending these systems to cover the entire training facility would require customers to reserve available time slots in advance, with fully booked slots becoming unavailable to additional users.

Although such booking systems allow fitness centres to control capacity and promote customer safety, we argue that mathematical optimization can be used to further enhance their efficiency. In practice, facility capacity is often calculated without considering customers' individual workout plans or preferences. For instance, consider a one-hour time interval during which 40 customers arrive at a facility equipped with only 10 treadmills, all intending to use a treadmill for their workout. In this case, 30 customers would be unable to follow their planned routines and might either switch to alternative exercises or leave without exercising. If workout plans were shared within the booking system, however, it would be possible to reassign 30 of these customers to different time slots and allocate the available treadmill capacity to customers with different exercise plans. Consequently, incorporating customers' workout plans into the booking system is essential for improving efficiency through mathematical optimization. This problem bears similarities to the assignment of trajectories in air traffic flow management (ATFM). In ATFM, airlines independently plan optimal trajectories for their flights, but because these trajectories share network resources with limited capacities—such as airports and airspace sectors—they must be coordinated and optimized to produce feasible final trajectories [12–17].

In this paper, we propose a network optimization model designed to support online booking systems for fitness centres. More specifically, the model is meant to support a transition from a "First Come First Served" booking system to an "optimization-driven" booking system. The model incorporates customers' individual workout plans and seeks to (i) minimize total delay, defined as the difference between a customer's intended workout start time and the actual time at which they are admitted to the facility; (ii) minimize deviations from customers' workout plans; and (iii) maximize facility utilization, measured by the number of customers who can be accommodated.

Our main contribution is the development of a network-based mathematical optimization model that enhances the efficiency of fitness centre booking systems. In the proposed model, a fitness centre is represented as a fully connected network in which clusters of similar equipment form the nodes. Each customer's workout plan is modeled as a path through this network, with each node corresponding to a specific exercise type. The model is solved using a commercial solver, and computational results based on realistic data sets demonstrate that it can reduce total delay by up to 17.5% and customers' idle time by up to 19%.

The remainder of this paper is organized as follows. Section 2 provides a detailed description of the problem and the underlying assumptions. The proposed optimization model is presented in Section 3. Computational results are discussed in Section 4, and concluding remarks are given in Section 5.

2 Problem description

In general, the fitness industry consists of companies that own one or more fitness centres or operate franchise networks. The business model of each fitness centre typically revolves around a set of activities, such as physiotherapy, group classes, personal training, and free workouts. Among these, free workout activities usually occupy the largest portion of the available space. The free workout area of a fitness centre is generally organized into clusters of similar equipment, such as treadmills, elliptical machines, stationary bikes, rowing machines, free weights, and other types of exercise equipment. Each cluster provides a specific set of benefits to customers.

Customers subscribe to different types of memberships, which grant access to various services offered by the facility, such as saunas, hydro-massage beds, or showers, or provide flexibility to use multiple fitness centres operated by the same company. In this paper, we consider a highly capacity-constrained setting involving a fitness company that owns several fitness centres located within a given geographical region and within reasonable travel distance for its customers. A representative example is a network of fitness centres belonging to the same company and located within a large city, where customers may choose which facility to attend on a given day.

On the demand side, we consider a set of customers who regularly use the facilities to perform workouts. Depending on the type of membership held, a customer may be allowed to use the facility on any day of the week, at any time, or even at different branches of the same fitness company. The choice of a specific fitness centre on a given day may depend on several factors, including personal preferences, proximity, perceived attractiveness of the facility, availability of training partners, and the expected level of congestion.

We assume that fitness centres have implemented online booking systems to better control facility capacity. This assumption is related to the fact that our model is designed to support the digital transition from a "First Come First Served" booking system to an "Optimization-driven" booking system. Such systems are intended to determine which customers are allowed to enter a facility during each time interval. Unlike traditional booking systems that rely solely on aggregate facility capacity—defined, for example, by the total number of equipment items that can be used while respecting health and safety guidelines—we consider a booking system that also accounts for customers' workout plans. Specifically, customers are assumed to be willing to share their workout plans in advance, and the system admits customers whose plans are not in conflict, thereby reducing congestion. We argue that the effectiveness of such a system can be significantly enhanced through mathematical optimization. Accordingly, we propose a model that disaggregates facility capacity to the level of equipment clusters, allowing utilization to be maximized based on workout plans rather than on aggregate facility capacity.

We focus on a tactical-level planning problem, covering one or more days in advance, in which customers submit their workout plans for a specific day through the online booking system. For each customer, a workout plan consists of (i) an intended arrival time, (ii) the sequence of equipment to be used, (iii) the planned duration of use for each piece of equipment, and (iv) the preferred fitness centre location, provided that the customer's membership allows for such flexibility. Customers may also specify alternative equipment options to be used if their preferred equipment is unavailable, as well as a set of alternative arrival times. Supported by the optimization model, the booking system assigns a final workout plan to each customer while ensuring that the capacity of each equipment cluster is not exceeded at any time. The assigned workout plan may be identical to the originally submitted plan or may include deviations. We consider two types of deviations: time deviations, which affect arrival times or durations of equipment use, and exercise deviations, which occur when a customer is assigned to alternative equipment instead of the initially planned one. In addition, the final workout plan may be assigned to a fitness centre other than the customer's preferred location, provided that such reassignment is allowed by the membership and acceptable to the customer.

Time is a critical component of the problem. The booking system operates on a day-by-day basis and considers the business hours of each fitness centre. Accordingly, we define a planning horizon that covers the operating hours of the facilities, which may span up to 24 hours. To capture variations in equipment usage and customer activity, the planning horizon is discretized into short time periods, typically representing intervals of 5 or 10 minutes. This level of granularity allows customers to specify workout start times that closely match their preferences and provides managers with improved visibility into equipment utilization at the cluster level. Disaggregating capacity in this manner also offers operational advantages, such as the ability to schedule equipment cleaning by cluster through temporary capacity adjustments, without requiring full facility shutdowns.

3 The capacity constrained network flow model

We model this problem as a capacity constrained network flow problem, where the flow is determined by the customers using the fitness centre equipment in a sequential manner. The model is aimed at a fitness company that has one or more fitness centres located within a given geographical region. We will denote the set of these fitness centres as $\mathcal{J}$ with each centre indexed by j. In our mathematical formulation, we represent this setting as a *disconnected graph* where each sub-graph element corresponds to the layout of a single fitness centre represented as a network. In fact, in our model consideration, each fitness centre of the company is represented as a fully connected network, where the nodes corresponds to the equipment in the centre and the arcs represent the possibility of moving from one equipment to another. We also model the workout plan of each customer as a path which starts at the entry point and ends at the exit point of the facility. The sequence of the nodes on the path then represents the sequence of execution of the customer's workout plan. The problem setting can thus be represented as a collection of disjoint directed graphs $G = \bigcup_{j \in \mathcal{J}} G_j$. Each sub-graph $G_j = (\mathcal{N}_j; \mathcal{E}_j)$, wherein $\mathcal{N}_j$ is the set of nodes (equipment) and $\mathcal{E}_j$ is the set of arcs connecting the nodes, will be the graph representing the network for the fitness centre j. This representation as a disconnected graph is due to the fact that a workout plan can only start and end in the same fitness centre, while any centre can be selected for a workout plan. The time horizon considered will be discretized into time periods. For example, a planning horizon of 24 hours can be discretized into time periods of 5 minutes lengths, yielding the set of discretized time periods to contain 288 periods. The details about the model and its parameters are given below.

3.1 Data notation

The data for our mathematical model is composed of the following:

$\mathcal{J}$: is the set of the fitness centres owned by the company.

$\mathcal{I}$: is the set of all the customers of the fitness company.

$\mathcal{T}$: is the set of (discretized) time periods.

i, j, k, n and t: are respectively the indexes of customers, fitness centre, cluster of equipment, node (equipment or entry/exit point) and time period.

$\mathcal{L}_i$: is the set of fitness centres feasible for the training of customer $i \in \mathcal{I}$.

$\mathcal{N}_j$: is the set of all the equipment in the fitness centre $j \in \mathcal{J}$.

$\mathcal{K}_j$: is the set of the clusters of equipment in the fitness centre $j \in \mathcal{J}$.

$\mathcal{S}_{k,j}$: is the set of the equipment belonging to cluster $k \in \mathcal{K}_j$ at the fitness centre $j \in \mathcal{J}$.

a_j and d_j: are respectively the check-in and the check-out points of the fitness centre $j \in \mathcal{J}$.

$\underline{t}^i_j$ and $\overline{t}^i_j$: are respectively the planned arrival time and departure time of customer $i \in \mathcal{I}$ at the fitness centre $j \in \mathcal{L}_i$.

$\mathcal{N}^i_j$: is the set of feasible equipment in the workout plan of customer $i \in \mathcal{I}$ at the fitness centre $j \in \mathcal{L}_i$. This includes the equipment in the preferred workout plan as well as alternative equipment.

$\Delta^+_{i,j}(n)$ and $\Delta^-_{i,j}(n)$: are the set of the nodes that are respectively subsequent and preceding to node $n \in \mathcal{N}^i_j$ in the workout plan of customer $i \in \mathcal{I}$ at the fitness centre $j \in \mathcal{L}_i$.

$T^i_{n,j} = \left[\underline{T}^i_{n,j}, \overline{T}^i_{n,j}\right]$: is the feasible time interval for customer $i \in \mathcal{I}$ to use equipment $n \in \mathcal{N}^i_j$ at the fitness centre $j \in \mathcal{L}_i$. For example, if the customer's plan is to start using equipment n from period 10 for 20 minutes in a 5-minutes discretization setting, then $T^i_{n,j} = \{10; 11; 12; 13\}$.

$\mathcal{T}^i_j = \left[\underline{T}^i_{a_j,j}, \overline{T}^i_{d_j,j}\right]$: is the feasible time interval for customer $i \in \mathcal{I}$ to workout at the fitness centre $j \in \mathcal{L}_i$.

$\alpha^i_{n,n',j}$: is the number of time period planned by customer $i \in \mathcal{I}$ to use equipment $n \in \mathcal{N}^i_j$ before moving onto equipment $n' \in \Delta^+_{ij}(n)$ at the fitness centre $j \in \mathcal{L}_i$.

$A^t_{k,j}$: is the capacity of cluster $k \in \mathcal{K}_j$ at the fitness centre $j \in \mathcal{J}$ during time period $t \in \mathcal{T}$.

$C^i_{n,j}$: is the penalty cost incurred when customer $i \in \mathcal{I}$ utilizes equipment $n \in \mathcal{N}^i_j$ at the fitness centre $j \in \mathcal{L}_i$. This cost is 0 if the customer had planned to used the equipment $n \in \mathcal{N}^i_j$.

γ^i: is the maximum total amount of time acceptable by customer $i \in \mathcal{I}$ to be idle while waiting for equipment to be free.

P^i_j: is the penalty of cost (dissatisfaction) if the workout plan of customer $i \in \mathcal{I}$ cannot be fulfilled at the fitness centre $j \in \mathcal{L}_i$.

3.2 The decision variables

We define the decision variables for our model following an idea that is well known in the network flow literature [12–14, 17]. Namely, these variables will define, for each customer, the equipment that is being used at a specific fitness centre for each time period.

$$x^i_{n,j}(t) = \begin{cases} 1 & \text{if customer } i \in \mathcal{I} \text{ is planned to use equipment } n \in \mathcal{N}^i_j \text{ at the fitness centre } j \in \mathcal{L}_i \\ & \text{by time period } t \in \mathcal{T}^i_{n,j} \\ 0 & \text{otherwise,} \end{cases}$$

It is assumed that a customer will use each equipment type for at least one time period. This is representative of a duration of about 5 to 10 minutes, which is very compatible with the reality.

3.3 The objective functions

In this section, we present two objective functions that can be used either individually or simultaneously as key performance indicators to guide the optimization model. These objectives are the total delay and the total deviation from customers' workout plans.

3.4 Delay

The aim of this objective function is to minimize the total delay in the start time of customers workouts. In other word, this is the difference between the time that they intended to start their workout and the time at which they are actually allowed into the fitness centre. For example, if a customer would ideally like to start her training at 6pm, the model will aim at assigning her a starting time that is as close as possible to 6pm.

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{L}_i} \sum_{t\in T^i_{a_j,j}} \left(t - \underline{t}^i_j\right) \left(x^i_{a_j,j}(t) - x^i_{a_j,j}(t-1)\right). \tag{1}$$

3.5 Deviation from workout plan

One of the main motivation of this paper is that the customers share their workout plans in advance to allow the fitness company to make a better planning of their fitness facilities. In return, these customers would expect minimal changes to their workout plan. This means that they would want to be allowed to used the equipment that they intend to use and spend enough time on these equipment, within a certain allowed limit. Thus any change of equipment to be used will result in the dissatisfaction for the customers. Usually deviation from workout plan will occur when the number of customers wanting to use equipment in a cluster is larger than the number of equipment available in that cluster.

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{L}_i} \sum_{n\in\mathcal{N}^i_j} C^i_{n,j} x^i_{n,j} \left(\overline{T}^i_{n,j}\right). \tag{2}$$

Note that the cost parameter $C^i_{n,j}$ is 0 if customer $i \in \mathcal{I}$ had planned to used the equipment $n \in \mathcal{N}^i_j$ at the fitness centre $j \in \mathcal{L}_i$. The cost is also modulated to account for the preferences of the customer in terms of utilizing alternative equipment in the event that their preferred equipment are not available.

3.6 The constraints

In this section we present the constraints of our model, whose role are to ensure that each customer is assigned at most a single workout plan at one fitness centre and that the number of people present in each cluster of equipment do not exceed the capacity of the cluster.

1. *Time connectivity constraints*:
   The first set of constraints is concerned with the time connectivity of the workout plans. These constraints follow naturally from the definition of the variables. Indeed, if a customer is using an equipment n *by* the time period t^*, then $x^i_{n,j}(t)$ must have a value of 1 for all later time periods $(t \geq t^*)$. They are stated as

$$x^i_{n,j}(t-1) - x^i_{n,j}(t) \leq 0, \qquad \forall i \in \mathcal{I}, j \in \mathcal{L}_i, n \in \mathcal{N}^i_j, t \in T^i_{n,j}. \tag{3}$$

2. *Clusters capacity constraints:*
   The second group of constraints are the clusters capacity constraints. These constraints ensure that the number of people using the equipment in each cluster do not exceed the cluster's capacity at any given time period. These constraints allow the fitness centre to enforce some health and safety measures by limiting the number of customers present in the centre. Note that by making the capacity of the clusters time dependent provides an opportunity to efficiently schedule the cleaning of the equipment by cluster, without having to shut down the entire facility.

$$\sum_{i \in \mathcal{I}} \sum_{n \in \mathcal{S}_{k,j} \cap \mathcal{N}^i_j} \max \left\{ 0, x^i_{n,j}(t) - \sum_{n' \in \Delta^+_{i,j}(n)} x^i_{n',j}(t) \right\} \leq A^t_{k,j}, \qquad \forall j \in \mathcal{J}, k \in \mathcal{K}_j, t \in \mathcal{T}. \tag{4}$$

   The left-hand side of the inequality (4) represents the number of customers that are using an equipment in the cluster k at each time period. In fact, when a customer enters a cluster, $x^i_{n,j}(t)$ becomes 1 with $\sum_{n' \in \Delta^+_{i,j}(n)} x^i_{n',j}(t) = 0$ until the customer moves on to use a subsequent equipment, in which case, this customer can no longer be counted as present in the cluster (now $\sum_{n' \in \Delta^+_{i,j}(n)} x^i_{n',j}(t) = 1$). On the other hand, one should note that the term $x^i_{n,j}(t) - \sum_{n' \in \Delta^+_{i,j}(n)} x^i_{n',j}(t)$ will take the value -1 if the customer starts using an equipment that is adjacent to n without using equipment n. It will also be equal to -1 for any time period t that is beyond the feasible time interval of equipment n, i.e., $t > \overline{T}^i_{n,j}$ and $t \leq \overline{T}^i_{n',j}$. Thus, the maximum function is therefore introduced to handle these special cases in the constraints as in [13]. A linearized form of the constraint is given in Appendix 2.

3. *Network flow constraints:*
   The third group of constraints are the network flow constraints, which ensure that each assigned workout plan is unique and that the flow of customers within the facilities are as smooth as possible.

$$\sum_{j \in \mathcal{L}_i} x^i_{d_j,j}(\overline{T}^i_{d_j,j}) = 1, \qquad \forall i \in \mathcal{I}, \tag{5}$$

$$x^i_{n,j}(t) \leq \sum_{n' \in \Delta^-_{i,j}(n)} x^i_{n',j}(t - \alpha^i_{n',j}), \quad \forall i \in \mathcal{I}, j \in \mathcal{L}_i, n \in \mathcal{N}^i_j \setminus \{a_j\}, t \in T^i_{n,j}, \tag{6}$$

$$x^i_{n,j}(\overline{T}^i_{n,j}) \leq \sum_{n' \in \Delta^+_{i,j}(n)} x^i_{n',j}(\overline{T}^i_{n',j}), \qquad \forall i \in \mathcal{I}, j \in \mathcal{L}_i, n \in \mathcal{N}^i_j \setminus \{d_j\}, \tag{7}$$

$$\sum_{n' \in \Delta^+_{i,j}(n)} x^i_{n',j}(\overline{T}^i_{n',j}) \leq 1, \qquad \forall i \in \mathcal{I}, j \in \mathcal{L}_i, n \in \mathcal{N}^i_j \setminus \{d_j\}. \tag{8}$$

   The constraints (5) ensure that the workout plan of each customer should start and end in one single fitness centre. Constraints (6) state that a customer cannot start exercising with an equipment n at time t if she had not spend at least $\alpha^i_{n',j}$ time periods on the preceding equipment according the current workout plan. The constraints (7) state that each customer must have moved on to one of the subsequent equipment by the latest time period at which she is allowed to use equipment n. Finally, constraints (8) ensure that the customers can only use one equipment after another.

4. *Maximum workout time*:

$$x^i_{a_j,j}(t) - x^i_{d_j,j}(t + \bar{t}^i_j - \underline{t}^i_j + \gamma^i) \leq 0, \qquad \forall i \in \mathcal{I}, j \in \mathcal{L}_i, t \in T^i_{a_j,j}, t + \bar{t}^i_j - \underline{t}^i_j + \gamma^i \in T^i_{d_j,j}. \quad (9)$$

The fourth group of constraints ensure that the total amount of time spent in a fitness centre by each customer do not exceed the maximum time allowed by either the customer or the fitness centre. Indeed, a fitness may centre may set a time limit, of 1 hour for example, for each customer, while some customer may not want to spend more than a time limit which is less than that of the centre (45 minutes for example).

4 Computational experiments

The above mathematical model is a binary integer programming model in which, the order of magnitude of the decision variables is approximately $\mathcal{O}(|\mathcal{I}| \times |\mathcal{N}| \times |\mathcal{J}| \times |\mathcal{T}|)$. Solving such models for a complete scale with a large number of variables may necessitates the development of sophisticated solution approaches or the use of heuristics or metaheuristic methods. However, in order to test the ability of our mathematical model in generating optimized and feasible schedules, we consider a small and tractable example, but realistic. Indeed, our proposed optimization is very useful in highly capacity constrained situations. Thus, we have considered an example of peak-hour in a gym represented by a three-hours time interval that has been discretized in time periods of 15 minutes, for a total 12 time periods. We consider a set of 60 customers planning their workouts in a single gym facility. The workout plans of each customer is randomly assigned to contain the planned use of about 3 to 6 equipments. The layout of the fitness centre facility under consideration is shown in Figure 3, which is a replication of a real gym based in Montreal (Canada). The type of equipment and their numbers are shown directly in this figure. It is assumed the workout plan of each of the 60 customer should start and end within the 3-hour planning horizon.

The optimization model was coded using the R Package version 4.3.2 [1] environment and solved with the open source GLPK solver version 5.0 [2] on an Intel Core i7-13620 @ 2.40GHz machine with 64 GB of memory and running on Windows 11. In the implementation we mainly focus on analyzing three main results, namely, the total delay the total deviation from the customers' workout plans and the "*first in first out*" (FIFO) policy which represents a case where optimization is not used by the management. The latter policy allows us to better assess the contribution of the optimization model in supporting the online booking system.

The resulting scheduling for the 60 customers are shown in Figures 1, 2a and 2b. In these figures, each horizontal line represent the scheduled workout plan of each customer. The different colours show the clusters in which they are working out at each time period, the "•" represents the estimated start time of the workout, while the "◦" represents the exit. The blank space between two workout represents idle time for the customer while waiting for a machine to become available so that they can use. Finally, the dashed line represent when a customer is using an alternative equipment from their planned schedule. In these graphs, Figure 1 show the scheduling under the FIFO booking policy, i.e., the scheduling if optimization is not used to support the booking system. In this graph, it can be seen that all the customers are arriving on time, but because of the capacity restriction, a significant number of customers have to idle in between two workouts (blank spaces), which either extend their time in the facility or negatively affect their satisfaction with regarding the gym.

The graph in Figure 2a shows the scheduling when optimization is supporting the booking schedule with a focus on minimizing the total delay. It can be seen in this graph that customers' workout sessions are more compact, with less idleness in between. It can also be observed that the total duration of the makespan is reduced compared to FIFO. Furthermore, it appears that for a majority of customers, the model suggests a slightly early start of the session than the initially planned arrival time: these customers are therefore informed of a new arrival time, earlier but still within the defined tolerance. Conversely, only a small number of customers see their schedule slightly delayed, which helps to smooth

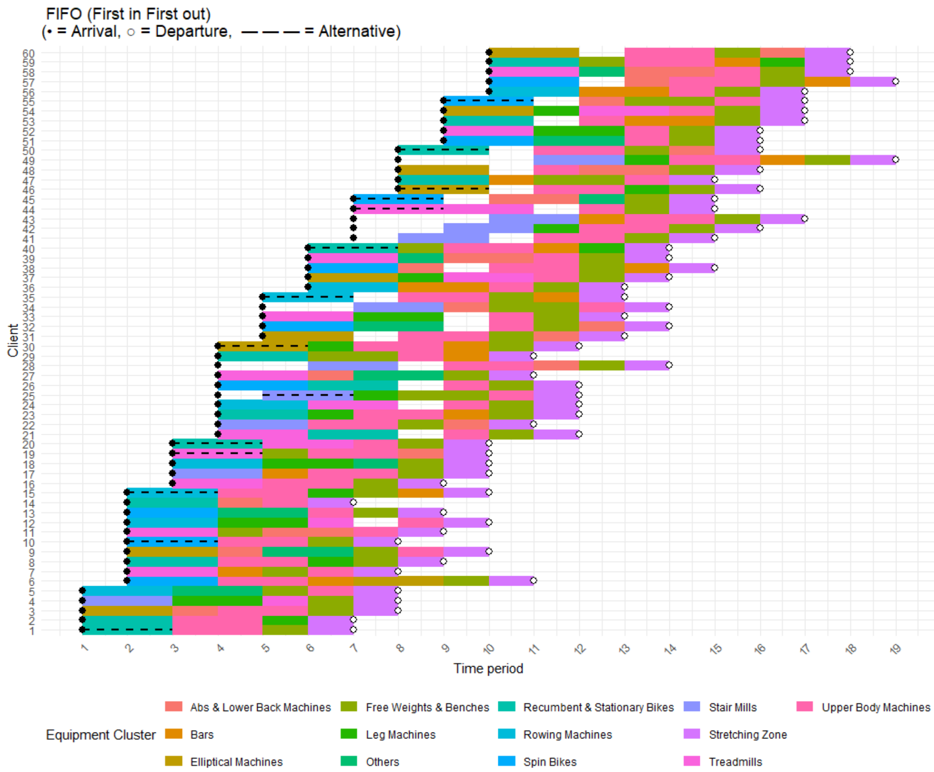


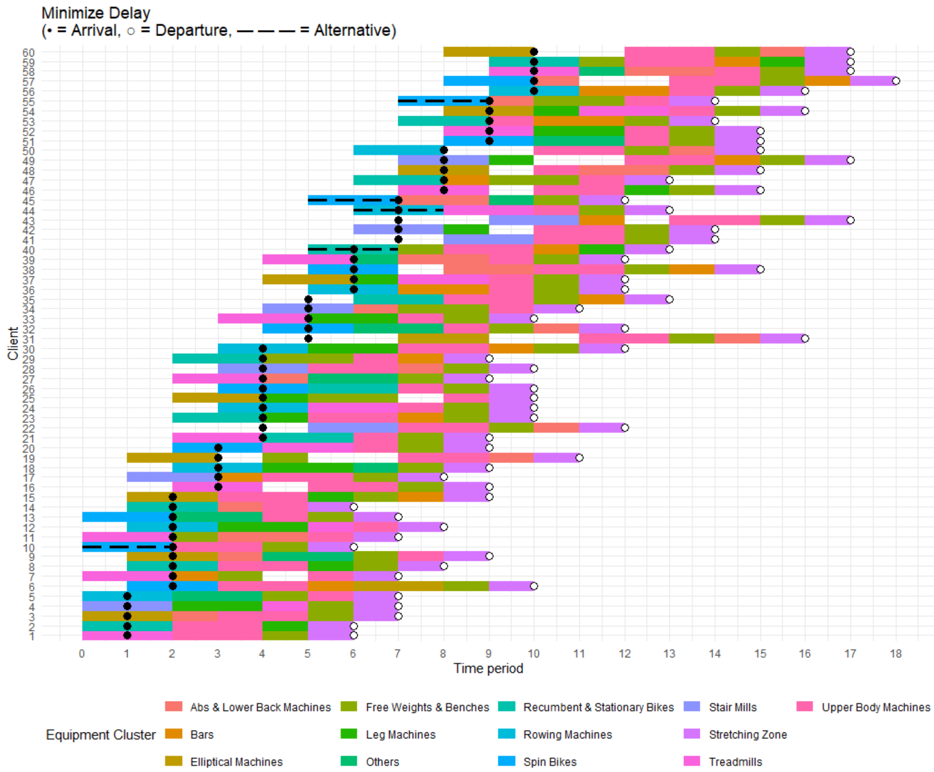
Figure 1: The scheduling under the FIFO planning

the congestion across clusters while avoiding unacceptable idleness. Finally, The graph in Figure 2b shows the scheduling when optimization is supporting the booking schedule with a focus on minimizing the total deviation from customers' workout plans. This graphs presents a structure very similar to that obtained when minimizing the delay. As before, the scheduling is generally compact and adheres to the same principles of anticipating or slightly shifting sessions within the clients' tolerance limits. The main difference lies in the use of alternatives: a slightly lower number of dashed segments are observed, reflecting a more sparing use of alternative equipment, consistent with the objective of minimizing the total deviation from customers' workout plans.

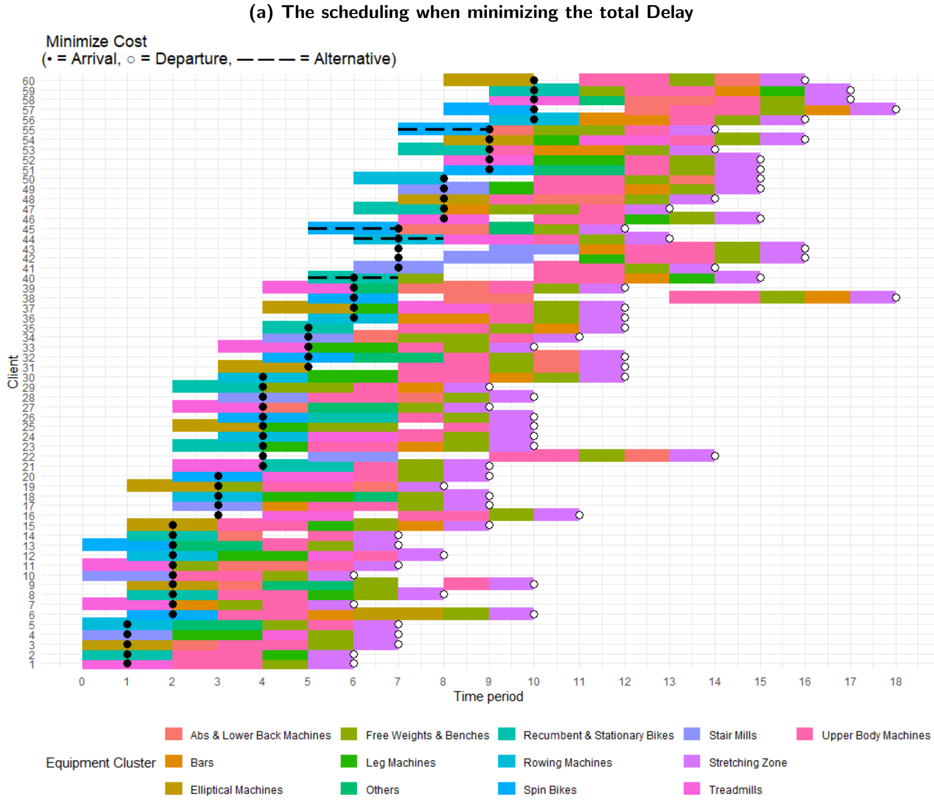
Overall, one of the most obvious conclusion is the clear superiority of the two optimization models compared to the FIFO approach used as a benchmark. This superiority is reflected in most performance indicators. In terms of customer experience, the optimized models reduce the average idle from approximately 39 to up to 12 minutes per customer.

5 Conclusion

In this paper, we addressed the problem of overcrowding in fitness centres during peak hours by proposing a mathematical optimization framework to support the transition from a "First Come First Served" booking system to an "optimization-driven" booking system. Motivated by the increasing demand for fitness facilities and the negative impact of congestion on customer satisfaction and workout quality, we modeled the interaction between customers' individual training plans and the limited capacity of fitness centre equipment. By representing a fitness centre as a network of equipment clusters and interpreting workout plans as paths within this network, the proposed model enables a more informed and efficient allocation of customers to time slots and resources.



(a) The scheduling when minimizing the total Delay



(b) The scheduling when minimizing the total deviation

Computational experiments conducted on realistic data sets demonstrate the effectiveness of the proposed approach. The results show that incorporating customers' training plans into the booking and allocation process can significantly reduce total delays and idle times, while simultaneously improving overall facility utilization. In particular, the model achieved reductions of up to 17.5% in total delay and up to 19% in customers' idle time, highlighting the potential of optimization-based decision support tools to improve both customer experience and operational efficiency in fitness centres.

Several directions for future research naturally emerge from this work. These include extending the model to handle stochastic arrivals and no-shows, incorporating fairness considerations among customers, and studying dynamic or real-time re-optimization as booking requests evolve. Additionally, integrating behavioral aspects, such as customers' willingness to accept alternative exercises or time slots, could further enhance the practical applicability of the model. Overall, this work illustrates how mathematical optimization can play a meaningful role in addressing real-world challenges in the fitness industry and beyond.

Appendix 1: Layout the the test facility

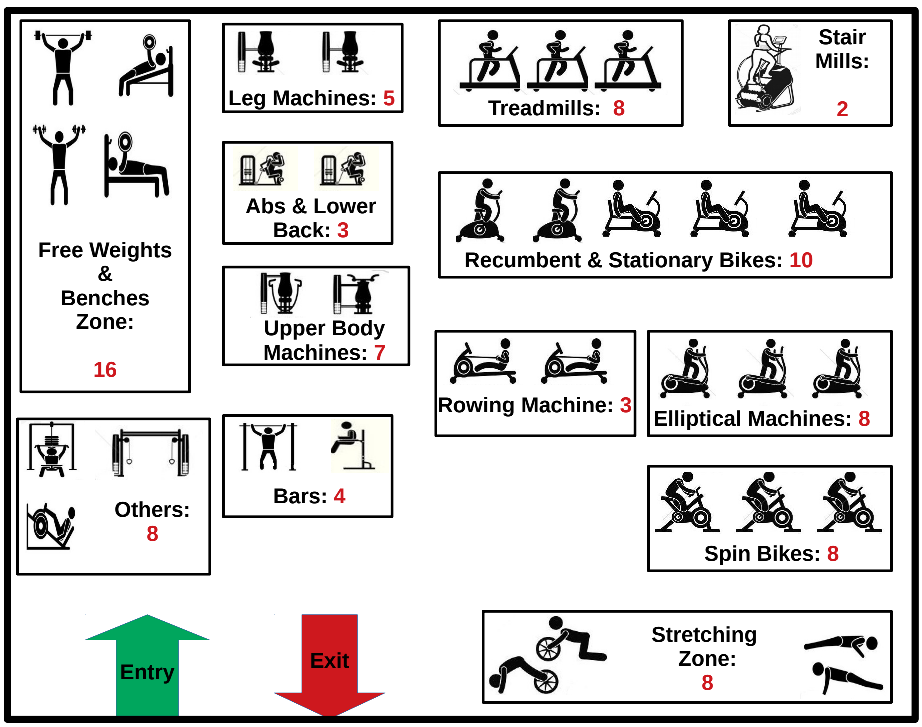


Figure 3: A realistic fitness centre layout: the numbers are the capacity of the clusters

Appendix 2: Linearization of the clusters capacity constraint (4)

It should be noted that the left hand-side of the clusters capacity constraints (4) is nonlinear. It can be linearized using the standard procedure of introducing an auxiliary variable $z_{i,n}^{k,j,t}$ for all $i \in \mathcal{I}, n \in \mathcal{S}_{k,j} \cap \mathcal{N}_j^i, j \in \mathcal{J}, k \in \mathcal{K}_j, t \in \mathcal{T}$, which will replace the max term in the constraint to become the following linear constraint

$$\sum_{i \in \mathcal{I}} \sum_{n \in \mathcal{S}_{k,j} \cap \mathcal{N}_j^i} z_{i,n}^{k,j,t} \leq A_{k,j}^t, \qquad \forall j \in \mathcal{J}, k \in \mathcal{K}_j, t \in \mathcal{T}.$$

The following linking constraints have to be added to make the linearization complete:

$$z_{i,n}^{k,j,t} \geq x_{n,j}^{i}(t) - \sum_{n' \in \Delta_{i,j}^{+}(n)} x_{n',j}^{i}(t) \quad \forall i \in \mathcal{I},\ n \in \mathcal{S}_{k,j} \cap \mathcal{N}_j^i, j \in \mathcal{J}, k \in \mathcal{K}_j, t \in \mathcal{T}$$

and

$$z_{i,n}^{k,j,t} \geq 0 \quad \forall i \in \mathcal{I},\ n \in \mathcal{S}_{k,j} \cap \mathcal{N}_j^i, j \in \mathcal{J}, k \in \mathcal{K}_j, t \in \mathcal{T}.$$

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