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Optimisation and energy evaluation of batch pan scheduling in a white sugar refinery

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Abstract : The Batch Pan Scheduling Problem represents a major operational challenge in white sugar refineries, where production planning must balance throughput objectives against high and fluctuating energy demands driven by crystallisation. This paper examines batch pan scheduling from an integrated production and energy-aware perspective. A discrete-time mixed-integer linear programming model and a mixed-integer multi-objective formulation are developed to represent the initiation of batch crystallisation cycles across sequential pan stages while accounting for total energy consumption. The models capture material flow dependencies, stage capacity constraints, and the temporal structure of industrial sugarhouse operations. The single-objective model maximises the number of completed batch cycles at the final pan stage within a given planning horizon, directly representing final sugar production. Building on this solution, the multi-objective model incorporates the achieved production level as a constraint with controlled relaxation to minimise total energy consumption. Numerical experiments investigate the influence of planning horizon length on scheduling flexibility and energy performance. Results indicate that extending the planning horizon enhances temporal coordination, enabling a more even distribution of operations and yielding substantial reductions in energy consumption without compromising production output. Furthermore, the multi-objective formulation achieves consistently stable energy performance, with total energy requirements comparable to or lower than those obtained from the single-objective model under the same horizon length. These findings demonstrate the critical role of planning horizon selection in energy-aware batch scheduling and show that energy savings and variability reduction can be realised through improved temporal coordination. The proposed framework provides a transparent and effective decision-support tool for energy-efficient production planning in batch process industries.

Keywords : Batch scheduling; sugar refinery; mixed-integer linear programming; multi-objective optimisation

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1 Introduction

White sugar production is a large-scale industrial activity characterized by complex material flows, stringent product quality specifications, and exceptionally high energy consumption. Among all processing units in a sugar refinery, the crystallisation section, commonly referred to as the sugarhouse, plays a decisive role in determining both the production efficiency and the energy performance. Sucrose crystallisation is carried out in batch operated vacuum pans, where evaporation and crystal growth occur simultaneously under reduced pressure. Operating under vacuum enables crystallisation at relatively low temperatures. This limits sucrose degradation, reduces the formation of colored compounds, and improves the quality and uniformity of sugar crystals. Each vacuum pan batch produces a semi solid mixture known as massecuite, typically composed of approximately 50% crystalline sugar and 50% mother liquor, which is typically less concentrated than the liquor fed into the pan, depending on the crystallisation technique employed. After crystallisation, the massecuite is discharged into strike receivers and subsequently processed in centrifugals, where sugar crystals are separated from the mother liquor. Additional water and steam are frequently required during centrifugation for washing and dissolution of fine crystals, thereby further coupling production decisions with steam or energy demand [12].

Crystallisation in white sugar refineries is inherently a multi stage process. To satisfy the stringent purity requirements of white sugar, refineries operate cascaded crystallisation stages in which the liquor discharged from one stage serves as the feedstock for the subsequent stage. As a result, downstream operations are conditioned on the completion of multiple upstream batch cycles, giving rise to strong inter stage dependencies and increasingly scheduling constraints.

We define the problem that coordinates these batch operations as the Batch Pan Scheduling Problem (BPSP). The problem seeks to determine the initiation times of batch cycles over a finite planning horizon such that material availability (mother liquor), equipment capacity, and operational constraints are satisfied while optimizing key performance indicators. These indicators typically include production of the final sugar, energy efficiency and energy demand (steam) stability. In this study, throughput is measured as the number of completed batch cycles at the final pan stage within the planning horizon, corresponding to the production of final sugar.

From an operational perspective, the BPSP is particularly challenging due to the dominant role of steam. Steam is consumed intensively during evaporation and centrifugation, and poorly coordinated batch starts can generate large fluctuations in energy demand (steam). As a result, batch process scheduling increasingly emphasizes production planning that explicitly accounts for energy considerations, whereby production decisions are coordinated to smooth energy demand (steam) profiles over time.

In addition to energy considerations, scheduling decisions must ensure sufficient liquor availability prior to pan initiation, adequate strike receiver capacity before massecuite discharge, and proper coordination of centrifugals to achieve the required sugar color specifications. The target sugar color is attained upon completion of the batch cycle in the final crystallisation stage. These requirements show the need for optimisation scheduling frameworks capable of simultaneously capturing material availability constraints and the logical constraints that govern the feasibility of initiating each stage.

Batch process scheduling has been studied within the operations research and chemical engineering communities. Classical scheduling theory focuses on sequencing and timing decisions for jobs processed on one or more machines, with objectives such as makespan minimization, tardiness reduction, and resource utilization [10, 11]. While these models provide valuable theoretical insights, they often assume independent jobs and neglect material-flow and energy interactions, making them insufficient for complex process industries such as sugar manufacturing.

To overcome these limitations, specialized modeling frameworks have been developed for batch and semi continuous process industries. The State Task Network (STN) and Resource Task Network

(RTN) paradigms explicitly represent tasks, resources, and material states, enabling the modeling of batch operations, intermediate storage, and shared utilities [6, 7]. These frameworks have become the foundation for many short term scheduling models in the chemical and food processing industries. A critical modeling choice in batch scheduling concerns the representation of time. Discrete time formulations divide the planning horizon into uniform time intervals and associate binary variables with task initiation decisions. Although such models may result in large mixed integer linear programs (MILPs), they are particularly well suited for systems dominated by logical rules and event driven behavior. Continuous time formulations, by contrast, reduce the number of time points but introduce additional complexity related to task sequencing and overlap [1]. The sugar industry has motivated a number of specialized scheduling and control models due to its unique combination of batch crystallisation, strong inter stage dependencies, and high energy intensity. A seminal contribution is the work of [12], who developed a discrete event and Mixed Logical Dynamical (MLD) framework for modeling and optimisation of sugar plants. Their approach transformed logical pan program rules into mixed integer constraints and embedded them within a Model Predictive Control (MPC) framework, demonstrating significant improvements in energy demand (steam) stability.

Subsequent studies explored the integration of scheduling and control in sugar refineries. [9] proposed a set based formulation for scheduling mixed batch continuous sugar processes with variable cycle times. [2] and [3] demonstrated the effectiveness of MPC for managing energy consumption and product quality in industrial sugar factories. These contributions underline that energy-aware scheduling is not merely a cost-saving measure, but a fundamental requirement for sustainable and stable refinery operation [4].

Beyond operational efficiency and economic considerations, the sugarcane processing industry also faces pressure to reduce its environmental footprint in response to climate change mitigation objectives. Sugar manufacturing is a highly energy intensive industrial activity, relying predominantly on steam for evaporation, crystallisation, and downstream separation, which contributes to greenhouse gas emissions when fossil fuels are used [13]. As decarbonization policies and energy efficiency targets tighten, reducing energy consumption in sugar refineries has become a critical lever for mitigating climate impacts. In this context, optimisation based batch scheduling offers a practical and cost effective approach to energy reduction at the operational level. By coordinating batch initiation decisions across multiple crystallisation stages and smoothing steam demand profiles over time, energy-aware scheduling can lower energy consumption and associated emissions without requiring major capital investments for green energy technology. [4] addressed this challenge by proposing strategies for monitoring energy consumption in sugarcane processing factories, highlighting the role of continuous measurement in supporting energy efficient operations

More recently, energy-aware scheduling has emerged as a major research direction across process industries. Reviews such as [5] highlight the growing importance of explicitly incorporating energy considerations into scheduling models, particularly in the context of decarbonization and industrial sustainability. Nevertheless, many existing approaches rely on complex hybrid models or detailed process simulations that may limit scalability and applicability at the planning level.

This work contributes to the literature by proposing a discrete-time mixed integer linear programming (MILP) formulation designed to reflect the structural characteristics of white sugar refinery operations. The proposed model captures essential inter-stage material dependencies and batch initiation logic. In addition, the study introduces a multi-objective modelling framework that explicitly incorporates energy-related objective, enabling the simultaneous consideration of production performance and energy efficiency within the scheduling decision process.

This paper is organized as follows: Section 1 introduces the problem context, Section 2 presents mathematical formulations, including single objective and multi-objective model, Section 3 discusses the numerical results, and Section 4 concludes the work.

2 Problem presentation and mathematical model

We consider a white sugar refinery operating four crystallisation stages, each associated with a dedicated vacuum pan. A stage is defined as the completion of a full batch cycle resulting in the production of a specific sugar color. The stages are organized sequentially with dependencies. Stage 1 corresponds to the completion of a batch cycle in pan 1, producing the first sugar grade. Stage 2 completes a batch cycle in pan 2 and requires fine liquor from two completed stage 1 cycles. Stage 3 similarly requires liquor from two stage 2 cycles, while stage 4 produces the final sugar color using liquor obtained from two stage 3 cycles. Each pan cycle consists of five main processing steps with fixed duration: filling, evaporation, discharge, centrifugation, and sugar extraction. Steam consumption is concentrated in the evaporation and centrifugation steps, making the timing of pan initiations a critical driver of energy demand. The strong coupling between material flows and steam usage motivates the explicit inclusion of energy considerations in the scheduling process.

The batch pan scheduling problem addressed in this work is to determine the initiation times of batch cycles for each stage over a finite planning horizon such that material availability constraints are satisfied, pan capacities are respected, production is maximized, and operational stability at the end. The following schemes present a typical multi stage crystallisation process in a refinery.

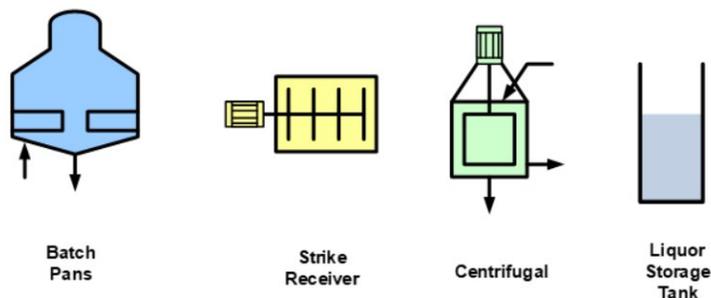


Figure 1: Equipment required per stage.

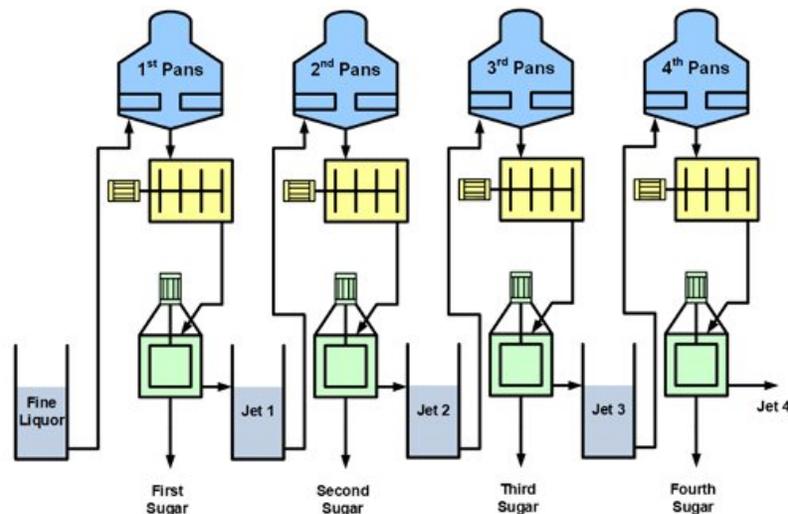


Figure 2: Multiple stage crystallisation process.

We now present a discrete time mixed integer linear programming formulation of the Batch Pan Scheduling Problem.

2.1 Sets and parameters

- T : planning horizon length.
- $K = \{1, 2, 3, 4\}$: set of crystallisation stages.
- C : capacity of each vacuum pan.
- D^k : energy demand (steam) associated with initiating a batch at stage k .

2.2 Decision variables

- X_t^k : takes 1 if a batch at stage $k \in K$ is initiated at time $t = 0, \dots, T$, and 0 otherwise.
- V_t^k : volume of liquor available at stage $k \in K$ at time $t = 0, \dots, T$.

2.3 Single-objective model

The original operational objective is to minimize the variability of energy demand (steam) over the planning horizon, which can be expressed as the variance of total steam consumption given by the following:

$$\mu = \frac{1}{|T|} \sum_{t=0}^{|T|} \left(\sum_{k=1}^4 D^k X_t^k - \frac{1}{|T|} \sum_{t=0}^{|T|} \sum_{k=1}^4 D^k X_t^k \right)^2 \quad (1)$$

However, this objective is nonlinear. To preserve linearity and maintain computational tractability, we adopt an objective related to maximizing sugar production, as effective scheduling of operations in process industries has the potential to generate substantial economic returns [9]. Since the final sugar product is obtained only when stage 4 is initiated, maximizing the number of stage 4 batch starts serves as an indicator for maximizing the sugar output rate:

$$\max Z = \sum_{t=0}^T X_t^4 \quad (2)$$

After each batch initiation, a distinct sugar grade is produced whose colour reflects the colour of the remaining mother liquor; as nearly 50% of sucrose is crystallised at each stage, the liquor colour changes and the sugar quantity decreases progressively from first to fourth sugar, after which all sugars are blended to produce a final sugar of average quality while satisfying liquor balance constraints.

Material balance constraints.

$$V_{t+1}^{k+1} = V_t^{k+1} - C X_t^{k+1} + \frac{C}{2} X_t^k, \quad k = 1, 2, 3, t = 0, \dots, T-1. \quad (3)$$

Availability constraints.

$$X_t^k \leq \frac{V_t^k}{C}, \quad k = 1, 2, 3, 4, t = 0, \dots, T. \quad (4)$$

Variable bounds.

$$0 \leq V_t^k \leq C, \quad k = 1, 2, 3, 4, t = 0, \dots, T. \quad (5)$$

These constraints ensure material feasibility, enforce inter stage dependencies, and prevent infeasible batch initiations. For the previous model, we introduce additional constraints when a lower bound \underline{D} and an upper bound \bar{D} on energy consumption are specified. These constraints are formulated as follows.

Total energy constraint.

$$\underline{D} \leq \sum_{t=0}^T \sum_{k \in K} D^k X_t^k \leq \overline{D} \quad (6)$$

The single objective model is used to maximise the product. Using that model, the energy consumption will be evaluate by time horizon size. This evaluation helps to determine a flexible time horizon with less energy consumption.

2.4 Assumptions and modeling remarks

This section clarifies the underlying assumptions adopted in the proposed formulation and provides additional modeling remarks that justify the structural choices made in the mathematical model. Explicitly stating these assumptions improves model transparency and facilitates interpretation of the numerical results, as recommended in the batch scheduling and process optimisation literature [5, 7].

2.4.1 Modeling assumptions

The following assumptions are made throughout this study:

1. **Discrete time representation.** The planning horizon is discretized into uniform time periods. This choice is motivated by the event driven nature of batch pan operations, where decisions are naturally associated with batch start times. Discrete time formulations are well suited for capturing logical constraints and inter stage dependencies in batch processes [7].
2. **Deterministic processing times.** The durations of the five pan cycle steps are assumed to be fixed and known in advance. While processing times may vary in practice, deterministic approximations are commonly used at the planning level [9].
3. **Single batch per pan at a time.** Each vacuum pan can process at most one batch at any given time, consistent with standard industrial operation.
4. **Fixed pan capacity.** All pans are assumed to have identical and constant capacity C .
5. **Aggregated material representation.** Liquor volumes are tracked in aggregate form without explicit compositional detail. This level of abstraction is appropriate for scheduling decisions [12].
6. **Deterministic inter stage conversion.** Two upstream batches are required to initiate one downstream batch, represented through fixed conversion ratios.
7. **No equipment failures.** Equipment is assumed to be continuously available; maintenance and breakdowns are not considered.

2.4.2 Modeling remarks

The discrete time MILP formulation reflects a deliberate trade off between model fidelity and computational tractability. While hybrid and continuous time models offer higher resolution, they often lead to large-scale and difficult to solve optimisation problems. The proposed formulation captures the dominant structural features of the batch pan scheduling problem while remaining suitable for medium term to long term planning.

The surrogate objective function that maximizes the number of stage 4 batch starts. Although it does not explicitly minimize steam variability, it aligns with industrial production goals and preserves linearity. Extensions to multi objective formulations incorporating explicit energy smoothing terms are straightforward. Finally, the model is intended for planning and scheduling rather than real time control. Detailed dynamic behavior and quality control are better addressed by lower level control strategies such as Model Predictive Control, which complement the planning level decisions generated by the proposed framework [2, 12].

2.5 Multi-objective model

In this section, we present a linear multi-objective model. Instead of minimizing the energy variation, we optimize the total energy, defined as

$$\min Z_2 = \sum_{t=0}^T \sum_{k=1}^4 D^k X_t^k. \quad (7)$$

Simultaneously, we maximize the total number of stage 4 batch starts:

$$\max Z_1 = \sum_{t=0}^T X_t^4. \quad (8)$$

These two objectives are conflicting: improving one generally leads to the deterioration of the other. The new model considers the same constraints as the single-objective model, together with the following additional constraint, which keeps the total number of stage 4 batches initiation near optimal:

$$\sum_{t=0}^T X_{4,t} \geq (1 - \epsilon) Z_1^*. \quad (9)$$

Here, Z_1^* denotes the optimal value obtained from the single-objective model. This formulation allows an $\epsilon\%$ degradation of the primary objective. This method is known as the ϵ -constraint method for multi-objective linear programming [8]. More formally, let us define the feasible region of the model as

$$\mathcal{F} = \{(X, V) : (3), (4), (5)\}. \quad (10)$$

First, we optimize the total number of stage 4 batches while ignoring energy consumption:

$$Z_1^* = \max_{(X,V) \in \mathcal{F}} \sum_{t=0}^T X_{4,t}. \quad (11)$$

After obtaining Z_1^* , we redefine the admissible solution set as

$$\mathcal{F}_\epsilon = \{(X, V) \in \mathcal{F} : Z_1(X, V) \geq (1 - \epsilon)Z_1^*\}. \quad (12)$$

The feasible set \mathcal{F}_ϵ contains solutions that maintain near-maximal production while allowing at most an $\epsilon\%$ loss in output of the first objective. Finally, we minimize the total energy consumption:

$$Z_2^* = \min_{(X,V) \in \mathcal{F}_\epsilon} \sum_{t=0}^T \sum_{k=1}^4 D^k X_t^k. \quad (13)$$

This procedure first implicitly determines the Pareto frontier by maximizing production and then explores this frontier to identify the minimum-energy solution.

In the next section, we provide the numerical investigation to evaluate our model output.

3 Numerical experiments

In this section, numerical experiments are conducted to evaluate the performance of the proposed scheduling models, namely the single-objective and multi-objective formulations, under different planning horizons. Detailed scheduling patterns are analyzed for two representative horizon lengths, $T = 40$

and $T = 42$, across the four crystallisation stages indexed by $k \in \{1, 2, 3, 4\}$. These two horizon lengths are selected to illustrate their impact on the scheduling structure and its visual representation. Each stage is associated with a fixed steam energy demand coefficient D^k , representing the energy consumption incurred when initiating a batch cycle at stage k . Specifically, the energy demand parameters are set to $D^1 = 8.0$, $D^2 = 6.5$, $D^3 = 9.0$, and $D^4 = 7.5$. The capacity of each vacuum pan is assumed to be constant and equal to $C = 100$.

In the first set of experiments, we present the optimal solution of the single-objective model without energy optimisation. We then analyze the multi-objective model, which determines the energy-optimal solution for a given total number of scheduled batches with the parameter $\epsilon = 0.01$. Using the solutions obtained from both models, we evaluate the energy variation according to formula (1) for different planning horizons.

In addition to the detailed scheduling analysis, the impact of the planning horizon length on energy performance is systematically examined by computing the total energy consumption for horizons ranging from $T = 10$ to $T = 170$ for both models. This horizon sensitivity analysis allows us to assess how extending the planning horizon affects the optimal scheduling policy and the resulting cumulative energy consumption, thereby providing insights into the trade-off between temporal flexibility and energy efficiency in multi-stage batch production systems.

Finally, the model is solved under explicit bounds on total energy consumption to investigate the effect of energy constraints on scheduling decisions. In particular, lower and upper bounds on total energy consumption, denoted by $\underline{D} = 0$ and $\overline{D} = 500$ or $\overline{D} = 1000$, are imposed for planning horizons ranging from $T = 10$ to $T = 170$. These constrained experiments illustrate how restricting the admissible energy range influences the timing and coordination of batch initiations across stages, further clarifying the interaction between energy limits and production scheduling decisions. All models are solved using the Gurobi Optimizer, version 12.0.1.

3.1 Scheduling and total energy by time horizon

Figures 3 and 4 present the optimal scheduling patterns obtained for two different planning horizons, namely $T = 40$ and $T = 42$. Each figure illustrates the initiation times of batch cycles across the four crystallisation stages and highlights how scheduling decisions evolve as the available planning horizon changes.

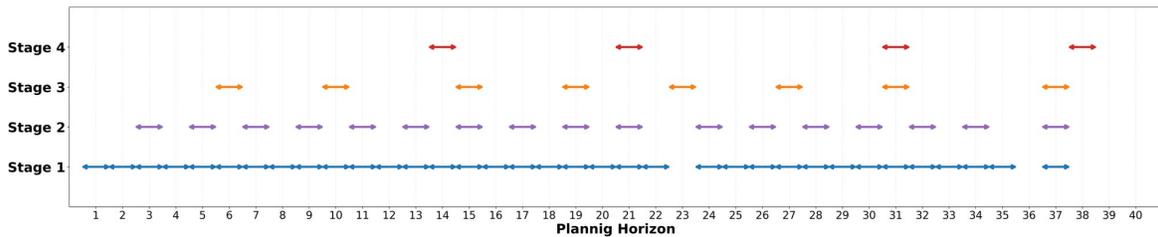


Figure 3: Scheduling with time horizon $T=40$.

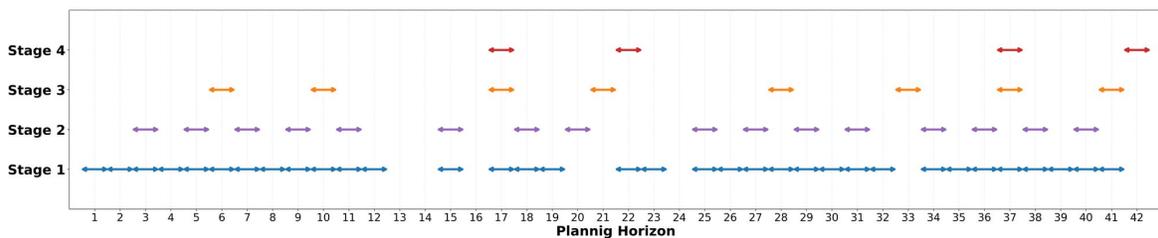


Figure 4: Scheduling with time horizon $T=42$.

For $T = 40$, batch initiations particularly in the downstream stages are relatively concentrated in time. This clustering reflects the limited temporal flexibility available to the optimizer, which forces downstream batches to be initiated as soon as material availability constraints are satisfied. Consequently, steam intensive operations overlap more frequently, resulting in a total energy consumption of 492.5.

When the planning horizon is extended to $T = 42$, the optimizer exploits the additional temporal flexibility to redistribute batch initiations more evenly over time. In particular, the initiation of stage 3 and stage 4 batches is slightly delayed and better spaced, reducing the simultaneous execution of steam demanding operations. This improved temporal coordination leads to a lower total energy consumption of 478.0, while maintaining the same number of completed final stage batches.

These results highlight that total energy consumption is not determined solely by the number of completed final stage batches, but is strongly influenced by the temporal distribution of batch initiations across the planning horizon. Even modest extensions of the horizon can enable smoother scheduling patterns that reduce energy usage without compromising production output. The energy variation in this case is 37.17 for $T = 40$ and 31.87 for $T = 42$. Without energy optimisation, the flexible schedule provides lower total energy consumption and lower variation.

Figures 5 and 6 present the schedules obtained under the energy-optimised model while enforcing the same number of stage 4 starts. For the two planning horizons, the resulting total energy is 462.0, while the total variation equals 34.41 for $T = 40$ and 39.28 for $T = 42$. Compared with the schedule produced by the single-objective model, the inclusion of the energy criterion reshapes the timing of operations and yields lower total energy consumption. In addition, the variability of energy demand is slightly reduced for $T = 40$, whereas it increases for $T = 42$, suggesting that the smoothing effect of the optimisation depends on the horizon length. By comparing the total variation between the single-objective and multi-objective models, we observe that when $T = 42$, the smallest variation is achieved. Figures 7 and 8 illustrate the energy variation profile.

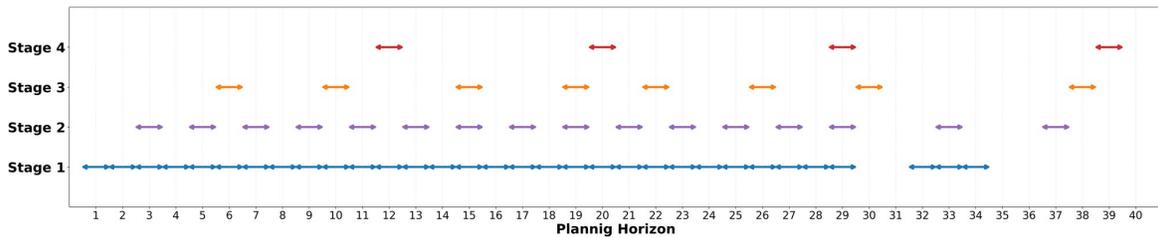


Figure 5: Scheduling with time horizon $T = 40$ and energy optimisation.

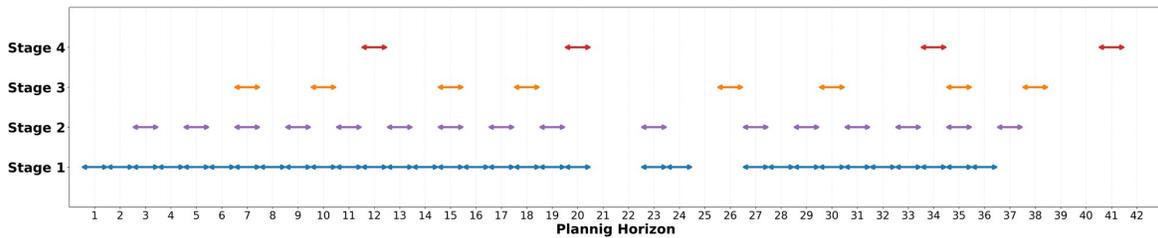


Figure 6: Scheduling with time horizon $T = 42$ and energy optimisation.

These results confirm that energy performance in batch pan scheduling is primarily driven by temporal flexibility and multi-objective coordination, where time horizon extensions combined with energy optimisation yield significant reductions in both total energy consumption and demand variability while preserving production levels.

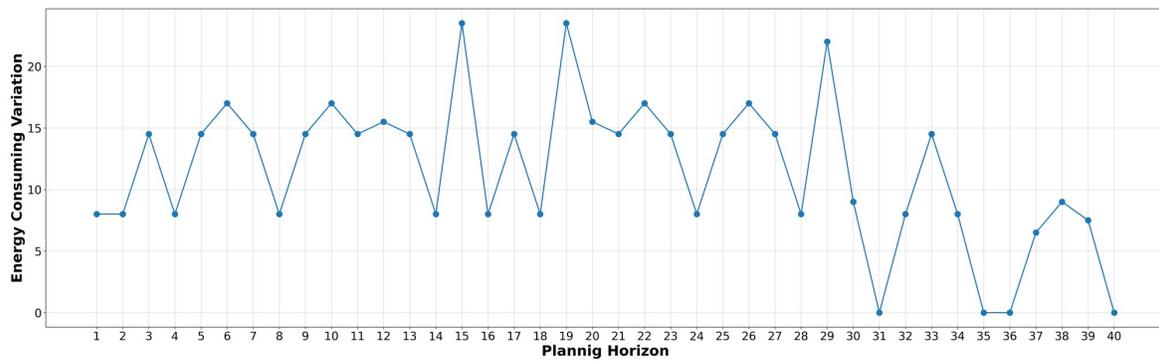


Figure 7: Energy variation with time horizon $T = 40$ by using the multi-objective model.

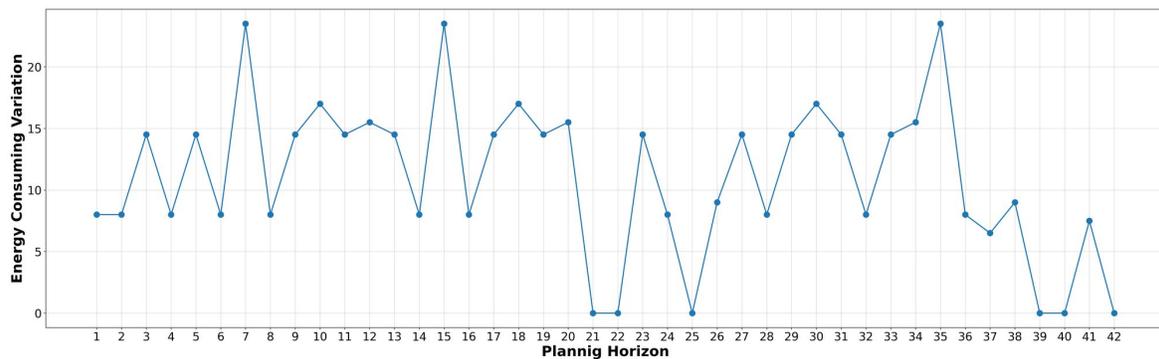


Figure 8: Energy variation with time horizon $T = 42$ by using the multi-objective model.

3.2 Total energy consumption by time horizon

This section examines the evolution of total energy consumption as a function of the planning horizon by considering single objective and multi-objective model. Figure 9 shows the total energy consumption together with the optimal number of stage 4 batch initiations for planning horizons ranging from $T = 10$ to $T = 170$ for our first model.

As expected, total energy consumption increases monotonically with the length of the planning horizon, reflecting the cumulative nature of steam usage over time. The number of stage 4 initiations exhibits a stepwise pattern, consistent with the discrete nature of batch production, where each increment corresponds to the completion of an additional batch cycle in the final crystallisation stage.

An important observation is that identical numbers of stage 4 initiations can be associated with different levels of total energy consumption across distinct planning horizons. This confirms that energy usage depends not only on production volume but also on how batch initiations are temporally coordinated. Longer planning horizons provide greater scheduling flexibility, allowing batch starts to be distributed more evenly over time, which mitigates periods of concentrated steam demand and improves overall energy efficiency.

These findings reinforce the importance of considering time-horizon effects when evaluating energy performance in multi-stage batch scheduling with the model that do not include energy optimisation, as production metrics alone do not fully capture energy-related outcomes.

Figure 10 compares the total energy consumption and the optimal number of Pan 4 initiations obtained from the single-objective and multi-objective models as the planning horizon increases. The multi-objective model consistently achieves slightly lower and more stable total energy consumption than the single-objective model across all horizons, indicating improved energy efficiency while main-

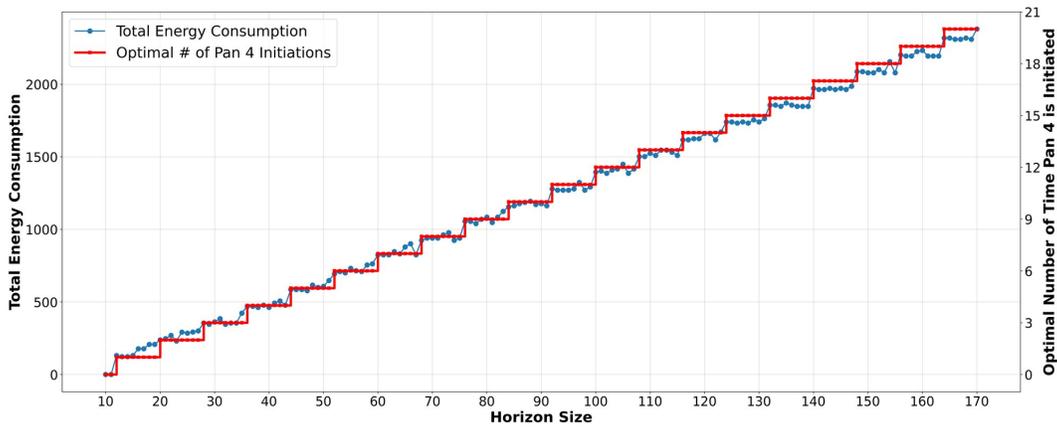


Figure 9: Total energy consumption and the corresponding optimal number of Pan 4 initiations.

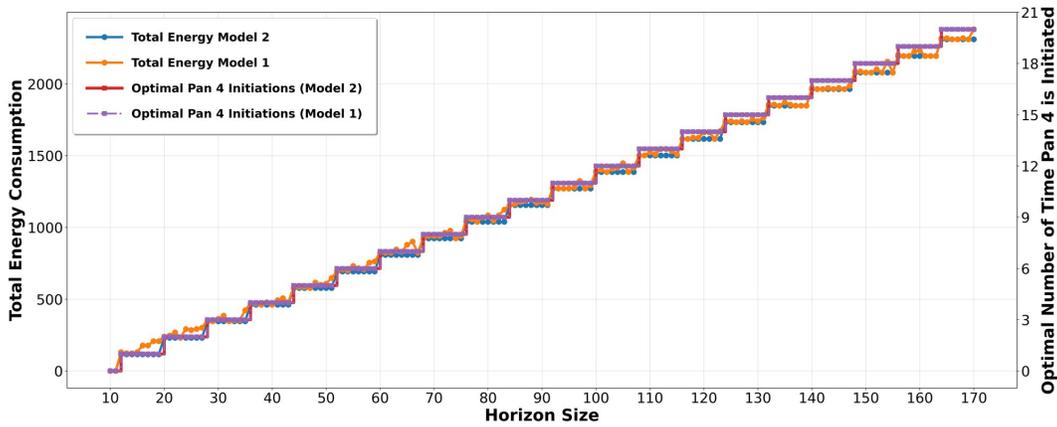


Figure 10: Total energy consumption the corresponding optimal number of Pan 4 initiations: Single Objective vs Multi-Objective Model.

taining equivalent production performance. The stepwise evolution of the curves corresponds to discrete increases in the optimal number of Pan 4 initiations, shown on the secondary axis, which occur at identical horizon thresholds for both models. This alignment confirms that both formulations preserve the same production structure. Overall, the results demonstrate that the multi-objective model dominates by achieving comparable throughput decisions with systematically lower energy consumption, highlighting the benefit of the energy-aware optimisation strategy.

Regarding total variation, Figure 11 illustrates the relationship between energy variation and the optimal number of Pan 4 initiations as the planning horizon increases for both models. For small horizons, energy variation rises sharply, reflecting limited scheduling flexibility and strong sensitivity to production horizon decisions. Beyond approximately 25–30 periods, both models enter a stable regime in which energy variation fluctuates around a nearly constant level despite the continuous increase in optimal initiations. The multi-objective model consistently exhibits slightly lower and smoother energy variation than the single-objective model, demonstrating improved energy balancing while maintaining identical production decisions, since both models generate the same initiation thresholds. Overall, the results show that increasing the number of optimal stage 4 initiations primarily drives total production expansion, whereas energy variation becomes largely independent of horizon length, with the multi-objective model achieving superior stability and energy efficiency along the optimal production trajectory.

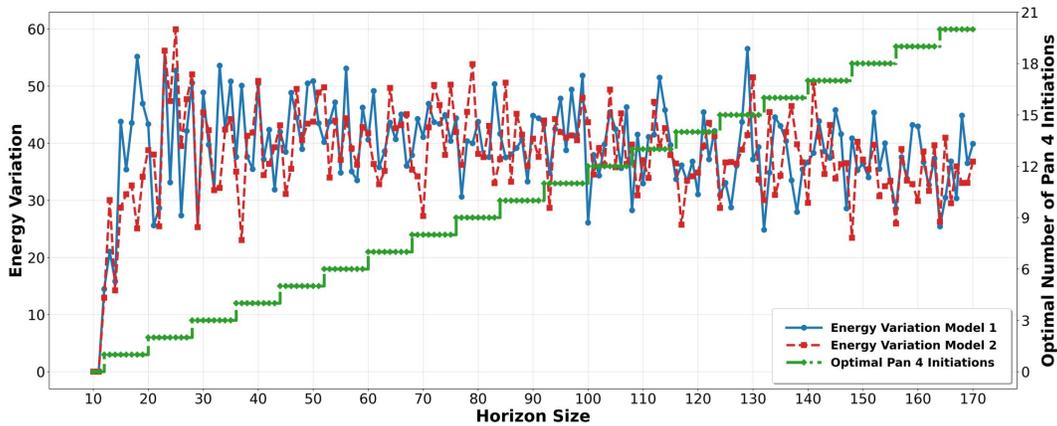


Figure 11: Total energy variation and the corresponding optimal number of Pan 4 initiations: Single Objective vs Multi-Objective Model.

3.3 Total Energy consumption with bounded total energy

We now investigate the effect of imposing explicit bounds on total energy consumption in both models. Figures 12 and 13 illustrate the total energy consumption and corresponding optimal number of stage 4 batch initiations as functions of the planning horizon size, subject to an upper energy bound of 500 and 1000, respectively.

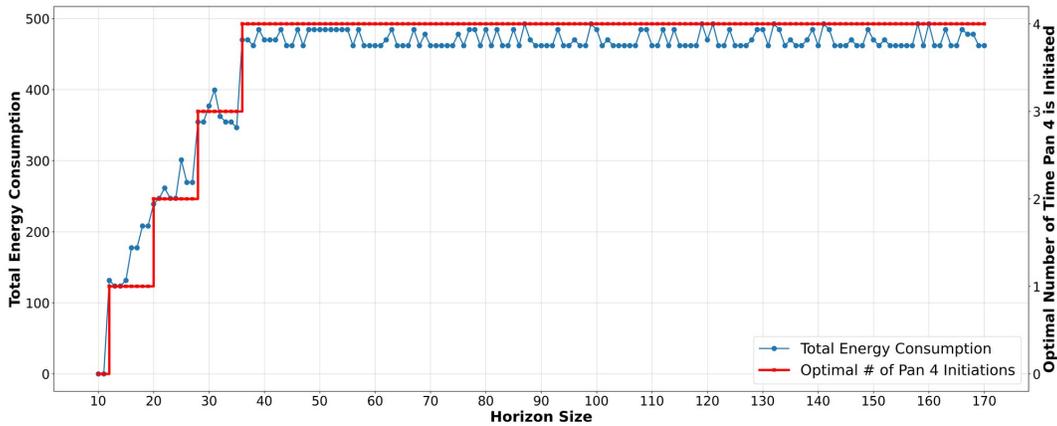


Figure 12: Total energy consumption as a function of the planning horizon and the corresponding optimal number of pan 4 initiations with a bounded total energy 500.

As the planning horizon increases, the number of stage 4 initiations again exhibits a stepwise behavior. Beyond a certain horizon length, production reaches its maximum feasible level under the imposed energy constraint, and the number of final-stage batches remains constant. Despite this fixed production level, total energy consumption varies across different planning horizons. Larger planning horizons provide additional flexibility that allows steam-intensive operations to be scheduled more evenly over time, thereby reducing overlap in energy demand and improving overall energy efficiency, even under binding energy constraints.

Regarding the multi-objective model, Figures 14 and 15 show that both models generate identical optimal Pan 4 initiation decisions as the planning horizon increases, indicating that production throughput is preserved under the imposed energy constraints. However, the multi-objective model consistently achieves lower and more stable total energy consumption than the single-objective model, demonstrating improved energy stability without sacrificing production performance.

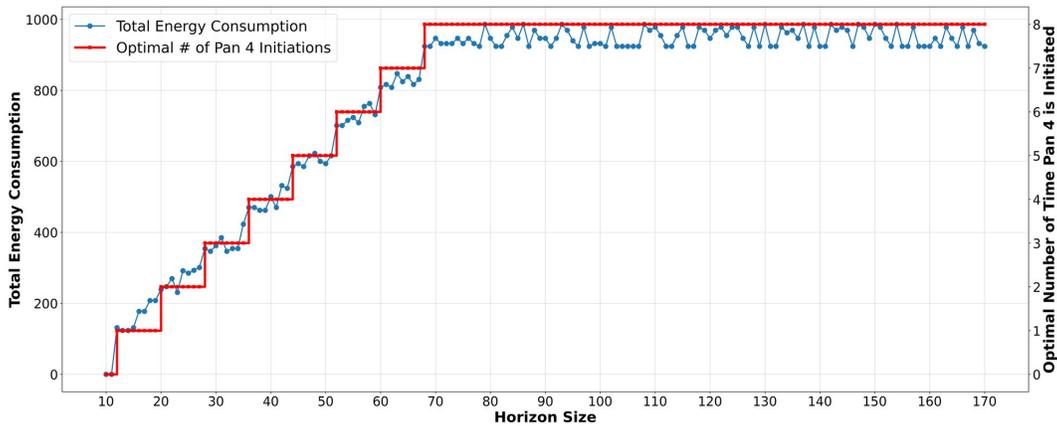


Figure 13: Total energy consumption as a function of the planning horizon and the corresponding optimal number of pan 4 initiations with a bounded total energy 1000.

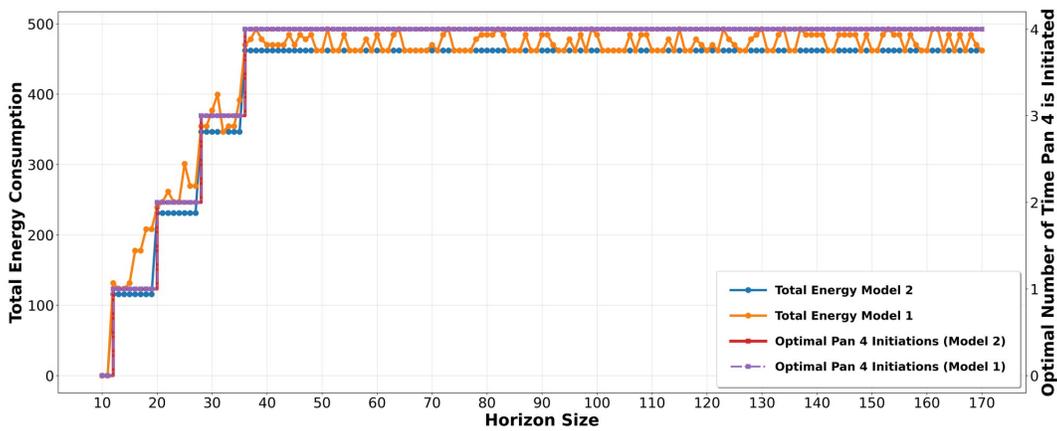


Figure 14: Total energy consumption as a function of the planning horizon and the corresponding optimal number of Pan 4 initiations under a bounded total energy of 500: Single-Objective vs Multi-Objective Model.

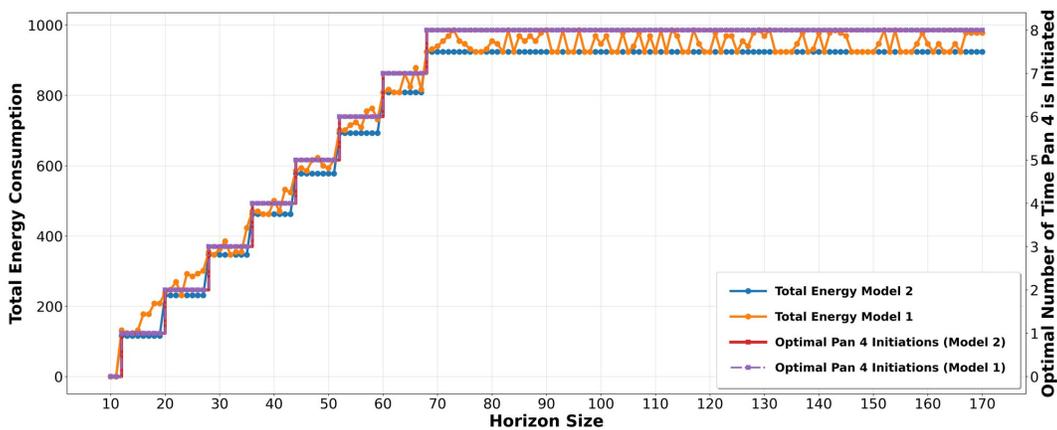


Figure 15: Total energy consumption as a function of the planning horizon and the corresponding optimal number of Pan 4 initiations under a bounded total energy of 1000: Single-Objective vs Multi-Objective Model.

Figures 16 and 17 illustrate the evolution of energy variation with respect to the planning horizon under different energy bounds. The multi-objective model consistently achieves lower and more stable

energy variation than the single-objective model while preserving the same optimal number of Pan 4 initiations, thereby demonstrating improved energy smoothing.

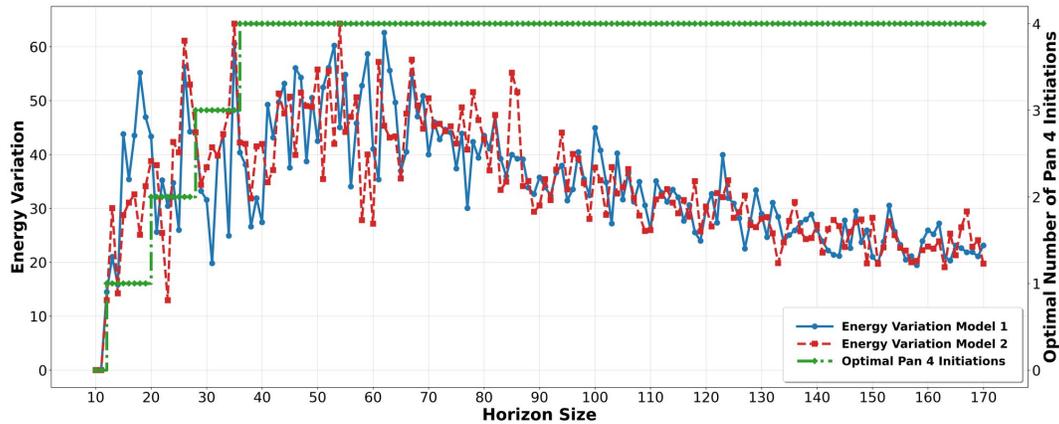


Figure 16: Total energy variation as a function of the planning horizon and the corresponding optimal number of Pan 4 initiations under a bounded total energy of 500: Single-Objective vs Multi-Objective Model.

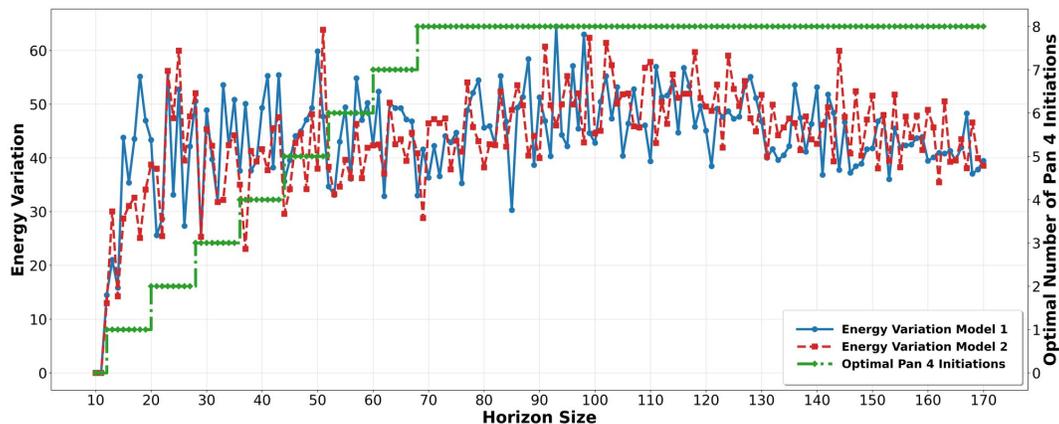


Figure 17: Total energy variation as a function of the planning horizon and the corresponding optimal number of Pan 4 initiations under a bounded total energy of 1000: Single-Objective vs Multi-Objective Model.

Overall, the multi-objective model outperforms the single-objective formulation by maintaining identical production decisions while systematically reducing total energy consumption and energy variation across all planning horizons. These results confirm that incorporating energy optimisation into the optimisation framework enhances operational stability and energy efficiency without compromising production output.

4 Conclusion

This paper addressed the Batch Pan Scheduling Problem in white sugar refineries using an energy-aware planning perspective. A discrete time mixed integer linear programming formulation was developed to model batch initiation decisions across four sequential crystallisation stages, explicitly capturing inter stage material dependencies and capacity constraints. Numerical experiments showed that the planning horizon length has a significant impact on energy performance. Although production remained constant across different horizons, total energy consumption varied due to differences in the temporal distribution of batch initiations. Longer horizons allowed batch starts to be more evenly spaced, reducing the overlap of steam intensive operations and lowering cumulative energy consumption. These

results demonstrate that energy efficiency in batch sugar production depends not only on the number of completed batches but also on the coordination of their initiation times when energy optimisation is not explicitly incorporated. By including total energy optimisation, in addition to maximising the number of stage 4 initiations, more stable results are obtained, providing greater flexibility in the selection of the planning horizon without increasing total energy consumption.

The proposed framework demonstrates the effectiveness of discrete-time scheduling models for analysing energy production trade-offs in multi-stage batch processes under both single-objective and multi-objective formulations. Future work will focus on extending the model to multi-objective formulations that explicitly incorporate energy smoothing objectives in addition to total energy minimisation. Furthermore, the inclusion of intermediate storage buffers will be considered to enhance the realism and flexibility of the scheduling problem, as well as to account for uncertainty in model parameters.

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