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Investment in emissions abatement capacity when consumers value the environmental performance of the supply chain

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Abstract : We examine the abatement investment and pricing decisions within a supply chain where consumers prioritize environmental performance. The product's green reputation is influenced by the gap between its unit pollution rate and an industry standard that declines over time. The abatement capacity investment is managed by the manufacturer, but the retailer has the option to share the associated cost. Our findings reveal that cost-sharing cooperation achieves an economically Pareto-optimal outcome compared to the no-cost-sharing scenario. From an environmental perspective, while it encourages greater abatement capacity investment and lowers the unit pollution rate, in most cases, the associated increase in demand may counteract these benefits, potentially leading to higher total emissions. Furthermore, as the industry standard declines more rapidly, firms tend to reduce their abatement investments. Lastly, firms with lower initial green reputation or abatement capacity – the "brown firms" – are less likely to catch up with the evolving standard, as higher initial states drive a virtuous cycle of increased investment, enhanced abatement capacity, reduced emissions, and further goodwill gains.

Keywords : Supply chain; pollution abatement; cost sharing; differential games

Résumé : Nous examinons les décisions d'investissement et de tarification en matière de réduction des émissions au sein d'une chaîne d'approvisionnement où les consommateurs privilégient la performance environnementale. La réputation écologique du produit est influencée par l'écart entre son taux de pollution unitaire et une norme industrielle qui diminue au fil du temps. L'investissement en capacité de réduction est géré par le fabricant, mais le distributeur a la possibilité de partager les coûts associés. Nos résultats révèlent que la coopération en matière de partage des coûts permet d'obtenir un résultat optimal au sens de Pareto par rapport au scénario sans partage des coûts. D'un point de vue environnemental, si elle encourage un investissement accru en capacité de réduction et réduit le taux de pollution unitaire, dans la plupart des cas, l'augmentation de la demande qui en découle peut contrebalancer ces avantages, entraînant potentiellement une augmentation des émissions totales. De plus, à mesure que la norme industrielle diminue plus rapidement, les entreprises ont tendance à réduire leurs investissements en matière de réduction des émissions. Enfin, les entreprises jouissant d'une réputation écologique ou une capacité de réduction initiale plus faible ont moins de chances de rattraper l'évolution de la norme, car des niveaux initiaux plus élevés favorisent un cercle vertueux d'investissement accru, de capacité de réduction renforcée, de réduction des émissions et de gains de clientèle supplémentaires.

Mots clés : Chaîne d'approvisionnement; réduction de la pollution; partage des coûts; jeux différentiels

1 Introduction

Consumers have been telling pollsters repeatedly that they are paying more attention to environment when making purchasing decisions. To illustrate, the Global Sustainability Study 2021 reported that a huge majority of respondents (85%) indicate that they have adopted a more sustainable buying behavior in the past five years (Simon-Kucher & Partners, 2021), and a study by Green Print (2021) found that 64% of Americans are willing to pay more for greener products. If a firm's environmental record matters, then it makes sense for firms to invest in costly abatement emissions actions and in new less-polluting production technologies. More than three decades ago, Porter (1996) conjectured that it pays to be green, meaning that pollution control could be a win-win strategy for the firm and the environment. For a firm to benefit from being environmentally friendly, consumers must be aware of its greening actions. The title of the above cited article, "*GreenPrint Survey Finds Consumers Want to Buy Eco-Friendly Products, but Don't Know How to Identify Them*", clearly highlights the importance for consumers of environmental information. Eco-labelling of products is by now a classical example of information provided by firms to help consumers recognize green products (see,e.g., Mason, 2006). Another (more general) channel is environmental information disclosure (EID), which is our focus here.

The popularity of EID has increased over time, at least among large corporations. Indeed, 82% of firms in the S&P 500 index disclosed their environmental record in 2018, up from 53% in 2012 (Yu et al., 2018). Now, this percentage is above 95%. When it is voluntary-based, EID is used by the firm to boost its green reputation among consumers (see, e.g., Kriström and Lundgren, 2003) and its stance as an innovative and socially responsible corporation among all its stakeholders. Given the ever-growing importance of environmental issues for citizens, one can assume that regulators will, sooner or later, render mandatory for firms to disclose their pollution levels. For instance, the California Air Resources Board has been asked by the government to put a system in place for reporting emissions by January 1, 2025 (Lambert, 2023).

In the 1990s, a series of studies focused on measuring the financial and economic effects of so-called Public Disclosure Program (PDP), which is a planned information strategy used by the regulator to reveal the environmental performance of firms. Hamilton (1995) reports that the stock value decreases on average by $4.1 million on the day that the list of polluters is released. Badrinath and Bolster (1996) find that, on average, there is a loss in value of about $14.3 million during the week of the settlement. Klassen and McLaughlin (1996) find that market valuation increases on average by $80.5 million following the announcement of an environmental award. Konar and Cohen (1997) obtain that firms with the largest decline in stock price when the information is made public reduce their emissions more than their industry peers.

More recent contributions that assessed the relationship between EID and the firm's financial performance reached mixed conclusions. Whereas Aragón-Correa et al. (2016) and Liu and Zhang (2017) find a negative relationship, Qiu et al. (2016) obtained no relationship, and Wang et al. (2020) reported a positive relationship. Wang et al. (2020) attributed these differences to the fact that the link between EID and financial performance is not necessarily direct, but depends on some moderator and mediator variables.

In this paper, we follow Wang et al. (2020) argument by assuming that the EID does not influence the supply chain performance (sales and profits) directly, but indirectly through its impact on the manufacturer's environmental reputation. This line of thinking has been adopted in the supply chain literature, to which our paper belongs.

The rest of the paper is organized as follows: In Section 2, we review the literature, state our research questions and highlight our contribution. In Section 3, we introduce the model, and in Section 4, we characterize the equilibrium solutions. Section 5 provides some numerical results and Section 6 discusses the managerial implications and briefly concludes.

2 Literature, research questions, and contribution

2.1 Brief literature review

We first note that a green reputation (goodwill) cannot be built overnight but over time through a series of investments in, e.g., emissions abatement, new production processes, recycling policies, advertising, etc. Second, a supply chain is made of independent institutions that interact strategically, i.e., the decisions of any agent affect the payoffs of all. To account for these two straightforward features, we adopt in this paper a dynamic game approach.

Some assumptions are common in the supply chain literature considering a low-carbon objective. First, the manufacturer invests in some pollution reduction activities. Denote by $I(t)$ the investment effort in pollution reduction at time t, and by $E(t)$ the state variable measuring the emissions level (intensity or propensity). Second, the supply chain's members, frequently only the manufacturer, conduct some advertising activities $A(t)$ to promote their green policies and procedures. Third, emissions, advertising, and possibly demand (which is typically proportional to emissions) affect the evolution of the green goodwill $G(t)$. Finally, the demand $D(\cdot)$ is assumed to depend negatively on the price, positively on the goodwill, and also on $E(t)$. The impact of $E(t)$ on demand can assume any sign depending on how $E(t)$ is measured. Importantly, the assumption is that consumers know the value of $E(t)$; otherwise, this variable cannot be a driver of demand.

Now, we discuss the relevant contributions. Zhou and Ye (2018) consider the following two-state and demand equations:

$$\begin{aligned}\dot{E}(t) &= mI(t) - \delta E(t),\\ \dot{G}(t) &= \beta A(t) + \theta E(t) - \tau G(t),\\ D(t) &= h(p)(\eta E(t) + \gamma G(t)),\end{aligned}$$

where all self-explanatory parameters are positive, and $h(p)$ is a decreasing function of price. Here, $E(t)$ represents the emissions reduction level at time t and has a positive impact on both goodwill and demand. (Note that the impact of $E(t)$ on demand is counted twice, directly, and indirectly through its impact on the goodwill.) In this framework, pollution level per se or pollution accumulation do not play a role.

Liu and De Giovanni (2019) retain one state variable, called environmental performance (EP) instead of emissions reduction, whose evolution is given by

$$\dot{E}(t) = mI(t) - g(\alpha - \beta p(t) + \eta E(t)) - \delta E(t),$$

where $D(t) = \alpha - \beta p(t) + \eta E(t)$, and $gD(t)$ measures total emissions. Comparatively to Zhou and Ye (2018), here pollution is accounted for explicitly, with EP depending positively on investment in emissions reduction and negatively on demand. Interestingly, a good environmental performance boosts sales and consequently emissions, which, in turn, deteriorates EP. This circular relationship is due to the fact that the investment in emissions reduction does not affect the intensity of emitting per unit produced.

In Liang and Futou (2020), the goodwill depends on low-carbon publicity A_r by the retailer and carbon reduction level b_m by the manufacturer, i.e.,

$$\dot{G}(t) = \phi A_r(t) + \theta b_m(t) - \tau G(t).$$

The authors also consider a state variable, called low-carbon reference $z(t)$, whose evolution is described as follows:

$$\dot{z}(t) = \kappa(G(t) - z(t)).$$

The demand depends linearly on $G(t)$ and $z(t)$, and is independent of price (a clear drawback). Note that the low-carbon reference depends on advertising, which could open the door to greenwashing.

Indeed, the supply chain can invest heavily in advertising and very little in emissions reduction and still claim a low-carbon policy. Further, it is not clear what would be the common measurement unit of $G(t)$ and $z(t)$ to be able to compute the difference. Zhang and Yu (2022) also consider the low-carbon reference and analyze the altruistic behaviors of supply chain members.

Huang (2023) adopts a model with two state variables, that is, emissions reduction level, which depends on a emissions reduction effort, and goodwill, which depends on manufacturer's brand advertising and emissions reduction level. Further, the author includes in the analysis two market segments (rural and urban markets), with demand in both markets being a function of price, goodwill, and retailer's promotional effort. Consumers in the urban market have better environmental awareness, hence the corresponding demand is also influenced by the emissions reduction level. Again, as emissions reduction and advertising influence additively the goodwill dynamics, the manufacturer can improve its green reputation without any significant reduction in emissions.

A series of studies investigate emission reduction under the carbon cap-and-trade policy. Under this framework, if a player (e.g., manufacturer, supplier, or retailer) emits less than his allocated carbon quota, he can sell surplus emission credits in the carbon trading market for profit. On the contrary, firms exceeding their quota must purchase additional carbon credits to remain compliant. Within this line of research, emission levels are explicitly modelled and compared to a threshold, $\bar{e}$, representing the carbon permit allocation. Emission reduction efforts or target reduction quantities lower unit/total pollution levels (Peng et al., 2023), and simultaneously enhance green goodwill (Wang et al., 2021; Cai and Jiang, 2023; Jiang et al., 2024; Zhu et al., 2024a). However, unlike these studies, Zhu et al. (2024b) models the manufacturer's low-carbon goodwill as a function of the gap between allocated carbon permits and instantaneous total emissions, rather than directly linking it to reduction efforts or quantities.

Cap-and-trade is also explored in De Giovanni (2021), although the focus is on identifying the conditions under which adopting smart applications and AI systems is economically beneficial for the whole supply chain. The AI solutions improve inventory management by eliminating white noise, with implications for emissions that depend on the efficiency and effectiveness of the technology.

The idea that what matters to consumers is relative emissions rather than absolute levels is relevant and is also retained in Yao et al. (2022), where the demand and green goodwill are given by

$$\begin{aligned} D(t) &= \alpha - \beta p(t) + \eta G(t), \\ \dot{G}(t) &= \phi A(t) + \sigma(\bar{e} - \xi D(t)) - \tau G(t). \end{aligned}$$

Here, pollution is proportional to demand, and the supply chain enjoys a higher green goodwill if it pollutes less than the acceptable standard $\bar{e}$, which may represent the industry average or the lowest emissions level within the industry. As noted above, the fact that the goodwill also involves advertising could lead to situations where the manufacturer's green goodwill increases despite polluting more than the standard.

A noteworthy limitation across these studies is the assumption of a constant benchmark $\bar{e}$ (emission cap or industry average), which becomes less realistic in games of infinite duration.

2.2 Research questions

As highlighted in the introduction, consumers are increasingly willing to shift their consumption towards greener products. To do it, they must be informed about each product's carbon footprint. Pollution being an inevitable by-product of production, a firm's environmental performance should be judged in its specific context. Indeed, emissions vary between sectors, and what should matter to consumers and all stakeholders is how the firm performs relatively to the industry (or similar companies), and not in absolute terms. Further, investing in emissions reduction is costly, which would clearly impact the pricing policies in the supply chain.

In this paper, we consider a two-player supply chain (a manufacturer and a retailer) facing consumers who are concerned by the environmental performance of firms. The manufacturer invests in emissions abatement and decides upon the wholesale price it charges to the retailer. The latter decides on the retail price. Two business models are considered, namely, a standard wholesale pricing model and a scenario in which the retailer participates in the investment cost of the manufacturer.

Our objective is to answer the following research questions:

1. What is the manufacturer's equilibrium investment in costly pollution reduction, with and without cost sharing?
2. What are the equilibrium wholesale and retail prices and how they compare in the two business models?
3. Under what conditions is it profitable to outperform the industry over time?
4. Is cost sharing of investment in pollution reduction Pareto optimal?

To deal with these research questions, we propose a finite-horizon dynamic game model, with two players, a manufacturer and a retailer, and two state variables, namely, the manufacturer's environmental reputation, and abatement capacity. The manufacturer chooses the investment in emissions abatement capacity (EAC) and the wholesale price, and the retailer decides on the participation rate in the investment cost in EAC and the price to consumers.

We determine and compare equilibrium strategies and outcomes in two scenarios. In one scenario, the retailer contributes to the manufacturer's investment cost in EAC. The game has three stages and is played sequentially. First, the retailer announces its participation rate in the investment cost. Next, the manufacturer chooses the wholesale price and investment in EAC. Finally, the retailer sets the price to consumers. In the other scenario, the manufacturer bears alone the investment cost and the first stage vanishes. In both scenarios, feedback (Markovian or state-dependent) strategies are considered. In the no-cost sharing scenario, we seek a Stackelberg equilibrium with the manufacturer acting as leader and the retailer as follower, whereas in the cost-sharing scenario, we determine a three-stage sequential equilibrium.

2.3 Contribution

Our contribution to the literature and our understanding of the impact of emissions reduction when it matters to consumers is fourfold:

1. From the discussion above, it seems clear that conceptually and intuitively, environmental performance of the manufacturer (and hence the supply chain) must be measured by the emissions propensity, that is, the pollution generated by one unit of production. This escaped the literature reviewed above (and most of the papers reviewed in De Giovanni and Zaccour, 2022). The variables used, e.g., emissions reduction, low-carbon publicity, etc., are not capturing, in a physical sense, the quantity of carbon saved, especially if one considers the firing back phenomenon obtained, or could be potentially realized.
2. Consumers know that we are yet far from a zero-carbon economy and what normally should influence their purchasing choices in the relative performance of one firm with respect to a standard. Some of the above-cited papers are capturing this aspect. However, they are assuming that this standard is constant over time, a hardly acceptable assumption especially that they are considering an infinite planning horizon. We contribute to this literature by letting the standard evolves over time to account for the fact that the industry will not keep its arms crossed for ever.
3. Information is crucial, which explains the popularity of environmental information disclosure programs. Such information is reliable because it is based on a physical measurement of pollution. We stick to this information in our work and do not, on purpose, include advertising to avoid greenwashing and deceiving consumers. Advertising can obviously play an important role in boosting demand, but should not be considered as a determinant of green reputation.

4. By determining and contrasting strategies and outcomes in two business models, namely, no-cost-sharing scenario, which is a standard wholesale pricing model in a supply chain, and a cost sharing scenario where the retailer participates in the environmental effort, we shed a light on how the contractual setup in a supply chain can affect the outcomes from the three standing points of supply chain's member, consumer's demand, and the environmental performance.

3 Model

Consider a dynamic supply chain formed of one manufacturer (player M) and a retailer (player R). Time is continuous and denoted by $t \in [0, T]$. The manufacturer produces a single good at marginal cost $c \geq 0$ and transfers it to the retailer that sells it to consumers. The demand $D(t)$ depends on the price $p(t)$ set by the retailer, and on the manufacturer's green reputation, $G(t)$. We adopt the following functional form:

$$D(t) = \alpha + \gamma G(t) - \beta p(t), \tag{1}$$

where α, β, and γ are positive parameters. In this specification, the demand is decreasing, as it should be, in the price, and increasing in the green reputation of the manufacturer. The size of the market is given by $\alpha + \gamma G(t)$, where α is interpreted as the intrinsic market potential that goes up, if the product's green reputation is positive, or down, otherwise. The linearity of the demand in its both arguments can be justified on the ground that it can be derived from consumer's maximization of a quadratic utility. Such functional form has been popular in the literature to which this paper contributes; see the survey in De Giovanni and Zaccour (2022).

Production, which is here equal to demand, pollutes the environment. The total emissions $E(t)$ are given by

$$E(t) = F(K(t))\, D(t),$$

where $K(t)$ is the installed abatement capacity (or emissions reduction) at time t, and $F(K(t))$ is the propensity to pollute per unit of production. The function $F : [0, \infty) \to [0, \theta]$ satisfies the following properties:

$$F(0) = \theta_0 > 0, \quad F'(K) \leq 0,$$

where θ is the emissions per unit of production when no abatement capacity is available. We assume that $F(K)$ is decreasing in K and is nonnegative at any time $t \in [0, T]$. As we consider a short-time (finite-horizon) planning period, we suppose that it is affine, i.e.,

$$F(K(t)) = \theta_0 - \theta_1 K(t),$$

where θ_0 and θ_1 are positive parameters, with θ_1 measuring the speed at which emissions could be reduced.

The manufacturer can increase its abatement capacity $K(t)$ by investing $I(t)$ at time t. The dynamics of $K(t)$ is governed by the following standard capital accumulation differential equation:

$$\dot{K}(t) = I(t) - \delta K(t), \qquad K(0) = K^0, \tag{2}$$

where K^0 is the initial capacity and $\delta \in (0, 1)$ is a decay parameter. The investment cost is convex increasing and given by $C(I) = \frac{1}{2} k I^2$.

The environmental reputation of the manufacturer depends on how much it pollutes, per unit produced, comparatively to the industry. We suppose that the propensity to pollute in the industry at time t, denoted by $\bar{e}(t)$, can be approximated by the following decreasing time function:

$$\bar{e}(t) = \epsilon + \sigma e^{-\nu t},$$

where ϵ, σ and ν are positive parameters, with $\epsilon + \sigma$ corresponding to the initial emissions propensity. Due to technological progress, $\bar{e}(t)$ goes down over time and converges to ϵ when t goes to infinity.

Put differently, ϵ is the lowest achievable emissions per unit of production. The difference in pollution propensity between the firm and the industry is given by

$$\Delta(t) = \epsilon + \sigma e^{-\nu t} - (\theta_0 - \theta_1 K(t)).$$

We shall refer to $\Delta(t)$ as the manufacturer's environmental performance. If $\Delta(t) > 0$, then the green reputation of the manufacturer goes up; otherwise, it goes down. Consequently, we model the evolution of the manufacturer's green reputation (GR) by the following differential equation:

$$\dot{G}(t) = \psi\Delta(t) - \mu G(t), \quad G(0) = G^0, \tag{3}$$

where G^0 is the initial reputation and $\mu \in (0,1)$ is the decay parameter.

Denote by $w(t)$ the wholesale price charged by the manufacturer to the retailer at time $t \in [0,T]$, and by $\rho \in (0,1)$ the common discount rate. We shall characterize and compare the results in two scenarios.

No cost sharing. In this scenario, the profit functions of the manufacturer and retailer are given, respectively, by

$$\pi_M = \int_0^T e^{-\rho t}\left((w(t) - c)D(t) - \frac{1}{2}kI^2(t)\right)dt + e^{-\rho T}S_M(G(T), K(T)), \tag{4}$$

$$\pi_R = \int_0^T e^{-\rho t}(p(t) - w(t))D(t)\,dt + e^{-\rho T}S_R(G(T), K(T)), \tag{5}$$

where $S_M(G(T), K(T))$ and $S_R(G(T), K(T))$ are the salvage values of the manufacturer and retailer's profits, respectively. The noncooperative game is played à la Stackelberg, with the manufacturer acting as a leader and the retailer as a follower. A feedback Stackelberg equilibrium is sought, that is, a subgame-perfect Markovian equilibrium where the strategies are function of the state variables. In this scenario, we have a finite-horizon two-player differential game, with two state variables G and K. The manufacturer chooses the wholesale price w and the investment in abatement capacity I, and the retailer decides the price p.

Cost sharing. In this scenario, the retailer shares with the manufacturer the investment cost in the abatement capacity. Denote by $B(t) \in [0,1]$ the retailer's share. The profit functions of the manufacturer and retailer are then given, respectively, by

$$\tilde{\pi}_M = \int_0^T e^{-\rho t}\left((w(t) - c)D(t) - \frac{1}{2}k(1 - B(t))I^2(t)\right)dt + e^{-\rho T}S_M(G(T), K(T)), \tag{6}$$

$$\tilde{\pi}_R = \int_0^T e^{-\rho t}\left((p(t) - w(t))D(t) - \frac{1}{2}kB(t)I^2(t)\right)dt + e^{-\rho T}S_R(G(T), K(T)). \tag{7}$$

Comparatively to the previous scenario, we have an additional control by the retailer $(B(t))$ and the game is played in three stages. First, the retailer announces its participation rate in the investment cost. Next, the manufacturer chooses the wholesale price and investment in abatement. Finally, the retailer sets the price to consumers. Again, we consider feedback strategies in this three-stage sequential game.

Remark 1. The no-cost-sharing scenario is embedded in the cost-sharing scenario. Indeed, it suffices to set $B(t) = 0$ in (6)–(7) to obtain (4)–(5). For this reason, we start by presenting the results of the cost-sharing scenario.

4 Equilibrium results

In this section, we state the two propositions characterizing the equilibria in the two scenarios.

4.1 Cost sharing

To determine subgame-perfect equilibrium in this sequential game, we proceed backward. That is, we first determine the retailer's price as function of the investment in abatement capacity by the manufacturer and the wholesale price. Second, we substitute for p into the objective function of the manufacturer and determine I and w as functions of the retailer's participation rate B in the cost. Finally, we substitute the reaction functions p, w, and I into the objective of the retailer and optimize for B.

The equilibrium strategies and state trajectories in the three-stage game are characterized in the following proposition.

Proposition 1. Assuming an interior solution, the feedback-sequential equilibrium strategies in the cost-sharing scenario are given by

$$\begin{aligned} p(t,G) =& \frac{1}{4\beta}\left[3\gamma G(t) + 3\alpha + \beta c\right], \\ w(t,G) =& \frac{1}{2\beta}\left[\gamma G(t) + \alpha + \beta c\right], \\ I(t,G,K) =& \frac{2r_1(t)+m_1(t)}{2k}K(t) + \frac{2r_3(t)+m_3(t)}{2k}G(t) + \frac{2r_4(t)+m_4(t)}{2k}, \\ B(t,G,K) =& \frac{(2r_1(t)-m_1(t))K(t) + (2r_3(t)-m_3(t))G(t) + (2r_4(t)-m_4(t))}{(2r_1(t)+m_1(t))K(t) + (2r_3(t)+m_3(t))G(t) + (2r_4(t)+m_4(t))}. \end{aligned}$$

The value functions of the manufacturer and the retailer are

$$\begin{aligned} V_M\left(t,G,K\right) &= \frac{1}{2}m_1(t)K^2 + \frac{1}{2}m_2(t)G^2 + m_3(t)KG + m_4(t)K + m_5(t)G + m_6(t), \\ V_R\left(t,G,K\right) &= \frac{1}{2}r_1(t)K^2 + \frac{1}{2}r_2(t)G^2 + r_3(t)KG + r_4(t)K + r_5(t)G + r_6(t), \end{aligned}$$

where $m_i(t)$ and $r_i(t)$, $i = 1,\ldots,6$, are determined by the system of differential equations (16) to (27) provided in the proof in Appendix A.

The equilibrium GR and abatement capacity trajectories are obtained as a solution of the system of the first-order linear differential equations given in the proof.

Proof. See Appendix A. □

The expressions of $m_i(t)$ and $r_i(t)$, $i = 1,\ldots,6$, are very long and their signs cannot be determined analytically. Consequently, only few comments can be made on the results in the above proposition.

1. The pricing decisions are independent on the investment in abatement capacity, which is due to the fact that K does not appear explicitly in the demand function.
2. The wholesale and retail prices and the demand, given by

$$D(t) = \frac{1}{4}\left(\alpha + \gamma G(t) - \beta c\right),$$

are increasing in the green reputation of the manufacturer. A higher reputation enlarges the market potential and allows the retailer to increase its price. Further, the retailer's reaction function to the wholesale price is given by (see Appendix A)

$$p(t,G) = \frac{1}{2\beta}\left[\alpha + \gamma G(t) + w\left(t,G\right)\right],$$

which shows that the two prices are strategic complements, i.e., $p(t,G)$ is increasing in $w\left(t,G\right)$.
3. The investment strategy and the participation rate depend on both state variables K and G. Here, we cannot say much analytically on the relationships between the control and state variables. We shall postpone this discussion to the numerical illustrations section.

4.2 No cost sharing

In this section, we characterize the equilibrium in the scenario where the retailer does not contribute to the manufacturer's investment cost, that is, the first stage in the game does not exist.

Proposition 2. Assuming an interior solution, the feedback-Stackelberg equilibrium strategies in the no-cost-sharing scenario are as follows:

$$\tilde{p}(t,\tilde{G}) = \frac{1}{4\beta}\left[3\gamma\tilde{G}(t) + 3\alpha + \beta c\right],$$
$$\tilde{w}(t,\tilde{G}) = \frac{1}{2\beta}\left[\gamma\tilde{G}(t) + \alpha + \beta c\right],$$
$$\tilde{I}(t,\tilde{G},\tilde{K}) = \frac{1}{k}\left[\ell_1(t)\tilde{K}(t) + \ell_3(t)\tilde{G}(t) + \ell_4(t)\right].$$

The value function of the manufacturer is:

$$\tilde{V}_M\left(t,\tilde{G},\tilde{K}\right) = \frac{1}{2}\ell_1(t)\tilde{K}^2 + \frac{1}{2}\ell_2(t)\tilde{G}^2 + \ell_3(t)\tilde{K}\tilde{G} + \ell_4(t)\tilde{K} + \ell_5(t)\tilde{G} + \ell_6(t),$$

where $\ell_i(t)$, $i = 1, 2, \ldots, 6$, are determined by the system of differential equations (33) to (38) provided in the proof in Appendix A.

The value function of the retailer is:

$$\tilde{V}_R\left(t,\tilde{G},\tilde{K}\right) = \frac{1}{2}z_1(t)\tilde{K}^2 + \frac{1}{2}z_2(t)\tilde{G}^2 + z_3(t)\tilde{K}\tilde{G} + z_4(t)\tilde{K} + z_5(t)\tilde{G} + z_6(t),$$

where $z_i(t)$ $(i = 1, 2, \ldots, 6)$ are determined as a solution of the system of differential equations (39) to (44) provided in the proof.

The equilibrium GR and abatement capacity trajectories are obtained as a solution of the system of the first-order linear differential equations given in the proof.

Proof. See Appendix A. □

As in the previous scenario, the pricing strategies are independent of the abatement stock, and increasing the goodwill level leads to higher retail and wholesale prices. We note that the retail price is more responsive to changes in the manufacturer's green reputation compared to the wholesale price $\left(\frac{\partial p}{\partial G} = \frac{3\gamma}{4\beta} > \frac{\partial w}{\partial G} = \frac{\gamma}{2\beta} > 0\right)$, while manufacturer's profit margin is more sensitive than retailer's margin $\left(\frac{\partial(w-c)}{\partial G} = \frac{\gamma}{2\beta} > \frac{\partial(p-w)}{\partial G} = \frac{\gamma}{4\beta} > 0\right)$.

5 Numerical simulations

In this section, we first compare the equilibrium results in the two scenarios, and next conduct some sensitivity analysis.

The parameters used for numerical simulations are summarized in Table 1. Note that we have fixed once for all the values of 13 parameters to focus on the impact of the eight remaining parameters that are the most related to our research questions.[1] In the benchmark case, we have set $\epsilon + \sigma e = \theta_0$ and $K^0 = 0$, which means that the manufacturer and the industry have the same pollution propensity at the initial instant of time.

[1] The results for any constellation of parameter values can be obtained from the authors upon request.

Table 1: Benchmark case

Planning horizon and discount rate:	$T = 15$, $\rho = 0.1$;
Salvage functions:	$S_i(K(T), G(T)) = S_{iK}K(T) + S_{iG}G(T)$ with $S_{iK} = S_{iG} = 5$ $(i = M, R)$;
Demand parameters:	$\alpha = 20$, $\gamma = 4$, $\beta = 6$;
Cost parameters:	$k = 2$, $c = 1.5$;
Industry pollution propensity:	$\epsilon = 7$, $\sigma = 3$, $\nu = 0.05$;
M's pollution propensity:	$\theta_0 = 10$, $\theta_1 = 0.1$;
Goodwill dynamics:	$\psi = 0.5$, $\mu = 0.1$, $G^0 = 10$;
Abatement capacity dynamics:	$\delta = 0.1$, $K^0 = 0$.
Values considered in the sensitivity analysis (benchmark value is bold)	
$\nu \in \{0, 0.02, 0.04, \mathbf{0.05}, 0.07, 0.09, 0.1, 0.12, 0.15\}$	$\mu \in \{0.05, 0.07, 0.09, \mathbf{0.1}, 0.12, 0.14, 0.15\}$
$\theta_0 \in \{9.7, 9.8, 9.9, \mathbf{10}, 10.1, 10.2, 10.3\}$	$\delta \in \{0.05, 0.07, 0.09, \mathbf{0.1}, 0.12, 0.14, 0.15\}$
$\theta_1 \in \{0.05, 0.07, 0.09, \mathbf{0.1}, 0.12, 0.14, 0.15\}$	$G^0 \in \{4, 6, 8, \mathbf{10}, 12, 14, 16\}$
$\psi \in \{0.05, 0.09, 0.1, 0.2, 0.3, 0.4, \mathbf{0.5}, 0.6, 0.7, 0.8\}$	$K^0 \in \{\mathbf{0}, 2, 4, 5, 7, 9, 10\}$

5.1 Comparison of equilibrium results

We compare the results obtained in the two scenarios from the three standpoints of players' profits, consumer's demand, and the environment.

First, we observe that the pricing strategies are the same in both scenarios, that is, for any given reputation level, we have

$$\tilde{p}(t, G) = p(t, G) \quad \text{and} \quad \tilde{w}(t, G) = w(t, G).$$

As both retail and wholesale prices are increasing in G, the time paths (trajectories) in the no-cost-sharing scenario will be above their counterparts if $\tilde{G}(t) > G(t)$, and below if $\tilde{G}(t) < G(t)$. The ranking of the reputation trajectories will depend on the abatement capacity installed in the two scenarios, which in turn depends on the investments made.

Given the complex expressions of the investment strategies, we cannot compare them analytically and must proceed numerically. For all considered parameter values, we obtained that a cost-sharing program leads to a higher total instantaneous investment in emissions reduction ($I(t) > \tilde{I}(t)$). As a sharing mechanism is an incentive to both parties to engage more in investing to reduce emissions, this result is by no means surprising. (A similar result is systematically obtained in the literature dealing with cooperative advertising, which is also a cost sharing mechanism (see the surveys in Aust and Buscher, 2014; Jørgensen and Zaccour, 2014)). A larger investment leads to a larger abatement capacity $K(t)$, resulting in a lower emission rate $F(K(t))$ and a higher level of goodwill, that is, $G(t) > \tilde{G}(t)$. Consequently, we have the following

Claim 1. At any instant of time, the retail and wholesale prices are higher in the cost-sharing scenario than in the no-cost-sharing scenario.

The cost of emissions reduction is passed to consumers. However, this does not necessarily lead to lower demands because consumer's willigness-to-pay is shifted up by the rise of the product's environmental reputation. Indeed, computing the difference in the demands of the two scenarios, we obtain:

$$D(t) - \tilde{D}(t) = \alpha + \gamma G(t) - \beta p(t) - \left(\alpha + \gamma \tilde{G}(t) - \beta \tilde{p}(t)\right) = \frac{\gamma}{4}\left(G(t) - \tilde{G}(t)\right) > 0.$$

Consequently, we make the following

Claim 2. At any instant of time, the demand is higher in the cost-sharing scenario than in the no-cost-sharing scenario.

Further, comparing the profits over the planning horizon leads to the following result observed in all our simulations:

Claim 3. A cost-sharing program leads to a Pareto-optimal outcome, that is, $\pi_M > \tilde{\pi}_M$ and $\pi_R > \tilde{\pi}_R$.

This result means that the additional revenues in the cost-sharing scenario, generated by the higher investments in abatement capacity and the consequent rise of the goodwill, exceed the additional investment cost.

What about the environment that is the third party involved in the comparison? Here, the cost-sharing program has two effects. First, the enhanced emissions abatement capacity achieved through cost sharing leads to a further reduction in the emissions rate ($\theta_0 - \theta_1 K(t) < \theta_0 - \theta_1 \tilde{K}(t)$), resulting in cleaner production processes. Second, the further improved green reputation of the product attracts more environmentally conscious consumers who may have previously opted for substitutes or refrained from purchasing, that is, $D(t) > \tilde{D}(t)$ for all t. The cost-sharing mechanism is environmentally friendly if

$$\int_0^T e^{-\rho t}\left(\theta_0 - \theta_1 K(t)\right) D(t)dt \leq \int_0^T e^{-\rho t}\left(\theta_0 - \theta_1 \tilde{K}(t)\right)\tilde{D}(t)dt \Leftrightarrow$$

$$\int_0^T e^{-\rho t}\left[\theta_0\left(D(t) - \tilde{D}(t)\right) - \theta_1\left(K(t)D(t) - \tilde{K}(t)\tilde{D}(t)\right)\right] dt \leq 0.$$

Table 2 compares total emissions in the two scenarios. In most cases, they are higher in the cost-sharing scenario, which means that the increase in demand outweighs the reduction in per-unit pollution. However, under some conditions, the cost-sharing scheme can lead to lower total emissions over the entire planning horizon (see numbers in red). Specifically, this overall reduction in total emissions can be attained when the initial abatement capacity (K^0) and/or initial goodwill level (G^0) is high. In industries that are already relatively clean, the potential for market expansion due to improved environmental performance is limited, allowing the benefits of improved pollution efficiency to dominate. Similarly, if the depreciation rates of abatement capacity (δ) and/or goodwill (μ) are low, it becomes easier to maintain these levels. Consequently, the market transitions more rapidly to a cleaner state, where the limited potential for market expansion once again allows pollution efficiency to dominate, leading to a reduction in total emissions.

Both low and high levels of consumer environmental sensitivity (ψ) can lead to a reduction in total emissions, but through different mechanisms. When ψ is low, consumers place little importance on environmental performance, so the goodwill gains from abatement are minimal, resulting in only a limited increase in demand. On the contrary, when ψ is high, consumers are highly sensitive to pollution, forcing firms to maintain very low pollution rates. In this case, the reduction in emission rate becomes the dominant effect.

When abatement capacity translates to emission rate reduction more efficiently – indicated by a lower value of θ_0 or a higher value of θ_1 – the benefits of improved pollution efficiency surpass the effects of market expansion, leading to a net reduction in total emissions. Finally, when the industry standard declines slowly (represented by a small ν), total emissions under the cost-sharing arrangement are lower than in the non-cooperative scenario. As shown in the numerical simulations detailed in the following section, a smaller ν leads to higher sustainability investment, which, in turn, enhances the effect of emission rate reduction.

5.2 Sensitivity analysis

When testing the impact of a particular parameter, we keep all other parameters as specified in the benchmark case. Specifically, we have assessed the impact on strategies, state variables, and payoffs of parameters related to industry pollution propensity evolution (ν), abatement capacity (θ_1 and ψ), states depreciation rates (μ and δ), and initial state values (θ_0, G^0, and K^0). A summary of their effects on decision variables, state dynamics, and profits is provided in Table 3.

Table 2: Difference of Total Emissions $\Delta = Emissions^{CS} - Emissions^{WP}$

$\boldsymbol{K^0}$	0	2	4	5	7	9	10	15	20	
Δ	12.5991	11.2937	9.9203	9.2082	7.7331	6.1900	5.3931	1.1536	-3.5102	
$\boldsymbol{G^0}$	4	6	8	10	12	14	16	18	20	
Δ	19.3921	17.4901	15.2257	12.5991	9.6101	6.2589	2.5453	-1.5305	-5.9687	
$\boldsymbol{\delta}$	0.05	0.07	0.09	0.1	0.12	0.14	0.15	0.5	0.99	
Δ	0.6568	12.9691	13.4230	12.5991	10.5685	8.6663	7.8347	0.6706	0.1386	$K^0 = 0$
Δ	-114.662	-33.4494	-8.7987	-3.5102	1.4799	3.1407	3.4282	0.5972	0.1302	$K^0 = 20$
$\boldsymbol{\mu}$	0.05	0.07	0.09	0.1	0.12	0.14	0.15	0.5	0.99	
Δ	8.2301	14.1628	13.3908	12.5991	11.0462	9.7422	9.1879	2.7421	0.8424	$K^0 = 0$
Δ	-73.8979	-22.6051	-7.0137	-3.5102	0.1155	1.6808	2.1045	1.5983	0.3333	$K^0 = 20$
$\boldsymbol{\psi}$	0.05	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
Δ	-3.4220	-2.8460	-2.7003	-1.1107	1.1002	4.9209	12.5991	28.8904	-10.0659	
$\boldsymbol{\theta_0}$	7	8	9.7	9.8	9.9	10	10.1	10.2	10.3	
Δ	-46.6218	-21.5562	8.8338	10.1421	11.3972	12.5991	13.7477	14.8430	15.8852	
$\boldsymbol{\theta_1}$	0.05	0.07	0.09	0.1	0.12	0.13	0.14			
Δ	3.6845	6.2489	10.1889	12.5991	9.7892	-28.0399	-323.17			
$\boldsymbol{\nu}$	0	0.02	0.04	0.05	0.07	0.09	0.1	0.12	0.15	
Δ	6.1161	9.2518	11.6295	12.5991	14.1965	15.4360	15.9503	16.8118	17.7841	$K^0 = 0$
Δ	-13.7282	-8.9199	-5.1147	-3.5102	-0.7784	1.4394	2.3924	4.0438	6.0157	$K^0 = 20$

Note: Unless specified otherwise, we use the benchmark values from Table 1.

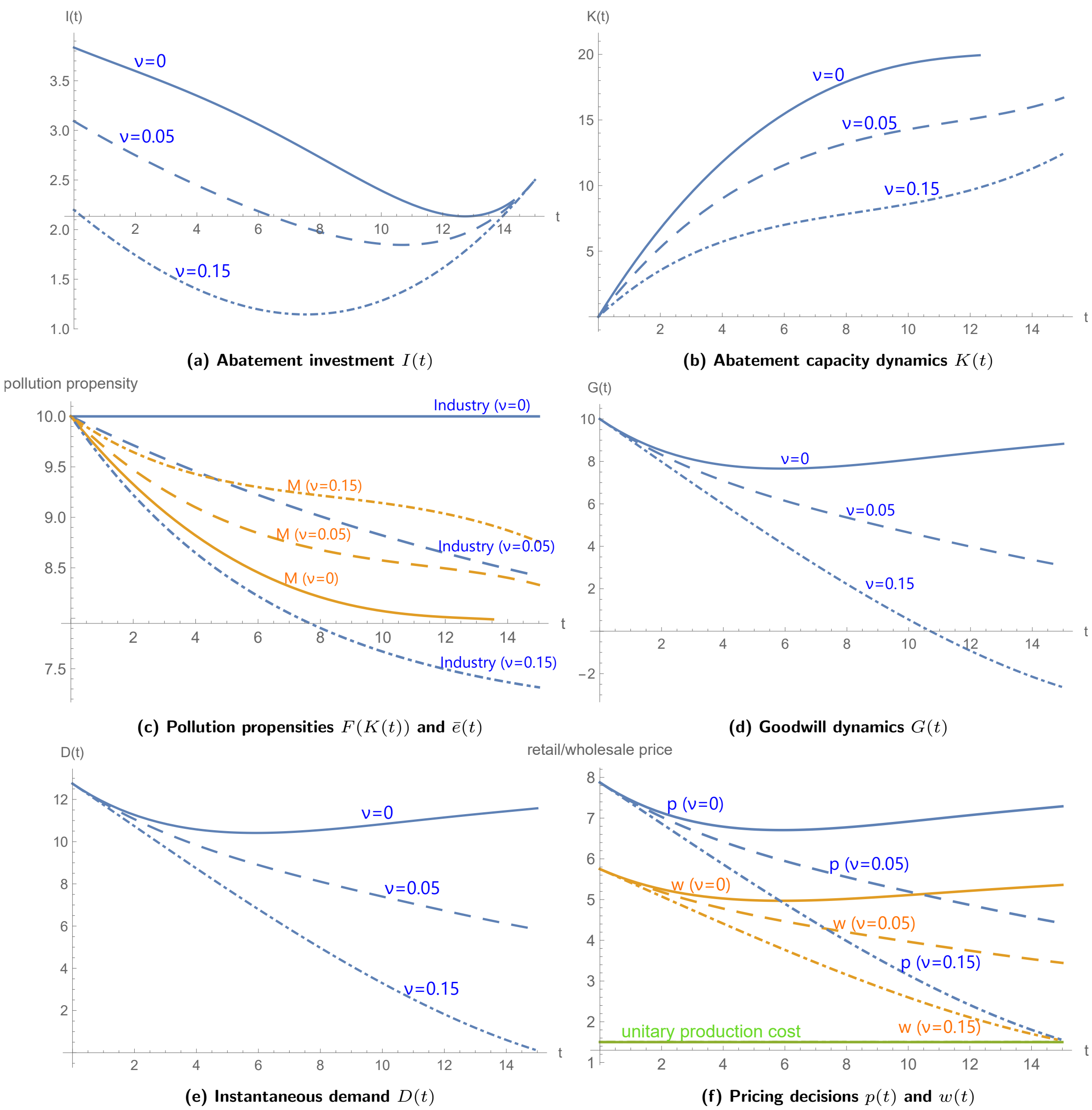
Table 3: Sensitivity analysis

Parameter	ν	θ_0	θ_1	ψ	μ	δ	G^0	K^0
				No cost sharing				
$\tilde{I}, \tilde{K}$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$
$\tilde{G}$, $\tilde{p}$, $\tilde{w}$	$-$	$-$	$+$	$-$, for $\psi < \bar{\psi}$, $+$, for $\psi \geq \bar{\psi}$	$-$	$-$	$+$	$+$
$\tilde{V}_M$, $\tilde{V}_R$, $\tilde{V}_{SC}$	$-$	$-$	$+$	$-$, for $\psi < \tilde{\psi}_i$, $i = M, R, SC$. $+$, for $\psi \geq \tilde{\psi}_i$, $i = M, R, SC$.	$-$	$-$	$+$	$+$
				Cost sharing				
I, K	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$
G, p, w	$-$	$-$	$+$	$-$, for $\psi < \bar{\bar{\psi}}$, $+$, for $\psi \geq \bar{\bar{\psi}}$,	$-$	$-$	$+$	$+$
V_M, V_R, V_{SC}	$-$	$-$	$+$	$-$, for $\psi < \hat{\psi}_i$, $i = M, R, SC$. $+$, for $\psi \geq \hat{\psi}_i$, $i = M, R, SC$.	$-$	$-$	$+$	$+$
B	$+$	$+$	?	?	?	?	$-$	$-$

The results in Table 3 allow for three general comments. First, the impact of each parameter is qualitatively the same in both scenarios. To save on space and avoid repetitions, we only show the graphs and results for the no-cost sharing scenario. Second, the direction of variation (positive or negative) in all variables of interest is the same when varying a parameter, except for the case of ψ. Specifically, if an increase in any parameter leads to an increase/decrease in abatement capacity investment I, the same effect is also observed for pricing decisions (retail price p and wholesale price w), state variables (abatement capacity K and goodwill G), and for individual and supply chain profits. This consistency arises from the rewarding domino effect of abatement capacity investment. An increase in this investment leads to enhanced capacity and a reduction in the manufacturer's pollution propensity. In turn, this positively impacts the evolution of goodwill, resulting in either its increase or a more gradual decline. When the firm has a stronger green reputation, both demand and price are higher, and so do the profit margins, as mentioned previously. These combined effects lead to higher profits.

5.2.1 Effects of industry standard evolution: ν

Figure 1 displays the investment in abatement capacity, pollution propensities, goodwill dynamics, and pricing under different values of ν. As depicted in Figure 1c, when $\nu = 0$, the industry emission propensity remains constant, a scenario considered in, e.g., Yao et al. (2022). Furthermore, we have considered two additional cases where the industry emission rate decreases almost linearly ($\nu = 0.05$) and in convex manner ($\nu = 0.15$), respectively.



(a) Abatement investment $I(t)$

(b) Abatement capacity dynamics $K(t)$

(c) Pollution propensities $F(K(t))$ and $\bar{e}(t)$

(d) Goodwill dynamics $G(t)$

(e) Instantaneous demand $D(t)$

(f) Pricing decisions $p(t)$ and $w(t)$

Figure 1: The effects of industry pollution propensity evolution (ν)

Remark 2. The investment increases in the last few periods in all three cases, which is driven by the salvage value. When discussing the effects of the parameters, we primarily focus on the earlier stages. We provide some figures in Appendix B where we set the salvage value equal to zero. Without any surprise, we recover the standard case of decreasing investments during the last periods. All other sensitivity analysis hold in this setup with zero-salvage value.

Figure 1a shows that as technology evolves faster, the manufacturer invests less in green innovation. Consequently, its abatement capacity builds up slowly (Figure 1b), leading to a higher emission rate (Figure 1c). Specifically, for a constant and quasi-linearly decreasing industry emission propensity (which could be the result of technological stagnation, industry-specific challenges, or limited resource availability), the manufacturer manages to keep its emission rate below the industry threshold throughout the entire planning horizon. This results in the goodwill remaining at a relatively high level in the constant case and decreasing relatively slowly in the quasi-linear case.

In contrast, in some industries, an intense technological race may lead to rapid technological progress, causing the industry average pollution rate to drop quickly, as illustrated by the case of $\nu = 0.15$. Since catching up with this new standard is highly costly, the manufacturer may prefer to endure a steep decrease in goodwill and pollute more than the average, even to the extent of acquiring a negative green reputation.

Regarding the pricing strategies in Figure 1f, as explained previously, both the retail and wholesale prices are contingent on goodwill, therefore exhibiting a similar pattern to the goodwill dynamics. The manufacturer consistently enjoys a higher profit margin than the retailer due to his leadership. Moreover, the retail price is more sensitive to changes in goodwill, as evidenced by a sharper decreasing trend compared with the wholesale price, whereas the manufacturer's profit margin displays greater sensitivity.

It is worth mentioning that when goodwill becomes negative and threatens demand (periods 11 to 15 in the case of $\nu = 0.15$), both players' profit margins approach zero, as does the demand. In this case, the firm effectively exits the market.

5.2.2 Effects of own abatement capacity benefit: θ_1 and ψ

Next, we discuss the effect of θ_1, which measures how effectively the abatement capacity can reduce the manufacturer's emission rate, and ψ, which captures the impact of the difference in pollution propensity on green reputation.

The effects of θ_1 are straightforward: a higher value indicates that the installed capacity is more effective in abatement reduction. Therefore, the manufacturer would be more willing to invest in it. Higher investment increases the abatement capacity, reduces the manufacturer's emission rate, and enhances its goodwill, resulting in a larger demand, from which the retailer also benefits.

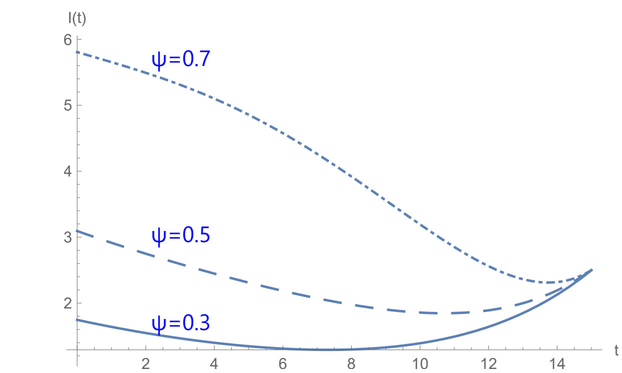
The effect of ψ is more complex. On one hand, it has a marketing impact: if the manufacturer manages to keep its pollution propensity lower than the industry threshold, its green reputation grows, and so does the demand. On the other hand, it also serves as a punitive factor. If the manufacturer pollutes more than the industry threshold, it will experience a decrease in goodwill that jeopardizes the demand.

Figures 2 and 3 demonstrate (all or part of) the decisions and states evolutions over time under different values of ψ. It can be observed from Figures 2a and 3a that regardless of whether the marketing or punishing effect is active, a larger value of ψ always induces more investment in green innovation. When the manufacturer outperforms the industry average ($\Delta(t) > 0$), it aims to increase or maintain the gap to maximize the advantage of its marketing effect. Conversely, when it functions as a punishing tool ($\Delta(t) < 0$), the manufacturer seeks to control the gap to minimize its negative impact on green reputation.

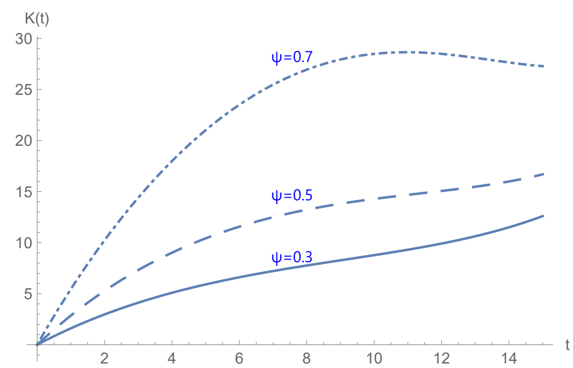
Moreover, when ψ takes moderate and high values, this enhancing effect is not linear - the increase in investment from $\psi = 0.5$ to 0.7 is much higher than that from 0.3 to 0.5. However, for small values of ψ (e.g., 0.05 to 0.2), such non-linear positive impact is not as pronounced.

Naturally, heavier investment is followed by a higher abatement capacity level and lower pollution propensity, as shown in Figures 2b and 3b, and Figures 2c and 3c, respectively. Nonetheless, the impact translated to goodwill evolution could be of two signs, depending on the range in which ψ resides.

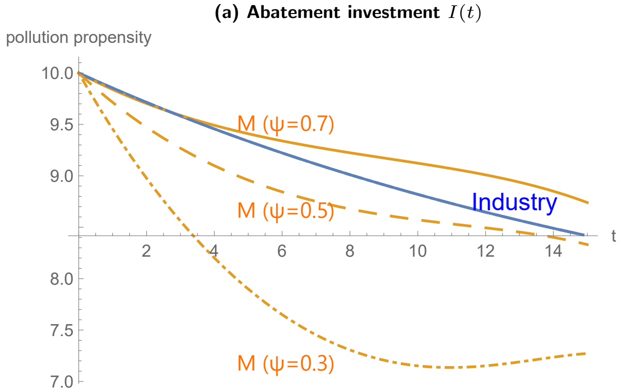
A more intuitive inference would be that a higher pollution rate is coupled with a lower level of goodwill, as plotted by Figures 2c and 2d, in line with the expectation that environmental friendliness ought to be rewarded, while poor performance in this regard should incur penalties. This holds true when ψ is in the range of moderate-high level.



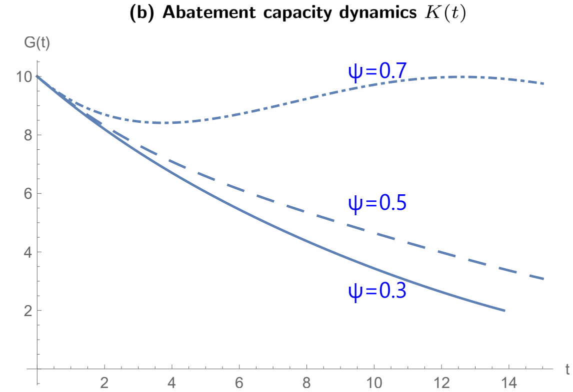
(a) Abatement investment $I(t)$



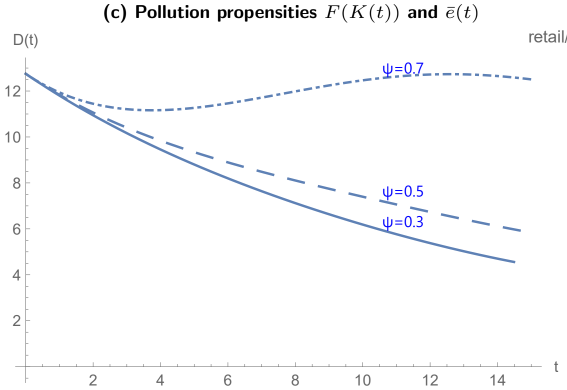
(b) Abatement capacity dynamics $K(t)$



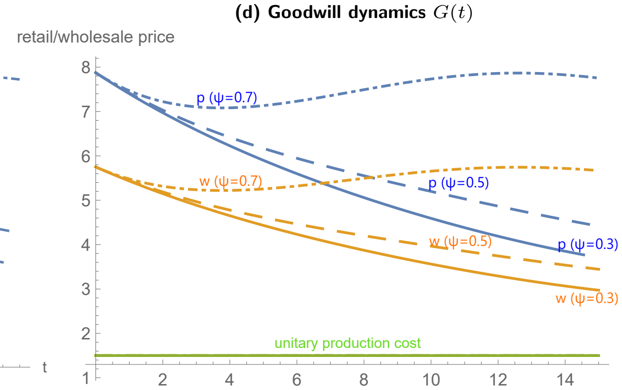
(c) Pollution propensities $F(K(t))$ **and** $\bar{e}(t)$



(d) Goodwill dynamics $G(t)$



(e) Instantaneous demand $D(t)$



(f) Pricing decisions $p(t)$ **and** $w(t)$

Figure 2: The effects of pollution impact on green reputation (ψ**)**

However, if the impact is trivial (e.g., close to the goodwill decay rate $\mu = 0.1$), the opposite is observed. In these cases, higher pollution propensity corresponds to a higher level of goodwill. Figure 3c shows that when ψ is very small, the limited investment in green innovation leads to more severe environmental pollution than the industry standard, negatively impacting the goodwill dynamics. A smaller ψ implies a larger gap, i.e., larger $|\Delta(t)|$. Surprisingly, as illustrated in Figure 3d, the goodwill corresponding to a smaller ψ (and a larger gap) turns out to be higher. This is due to the fact that

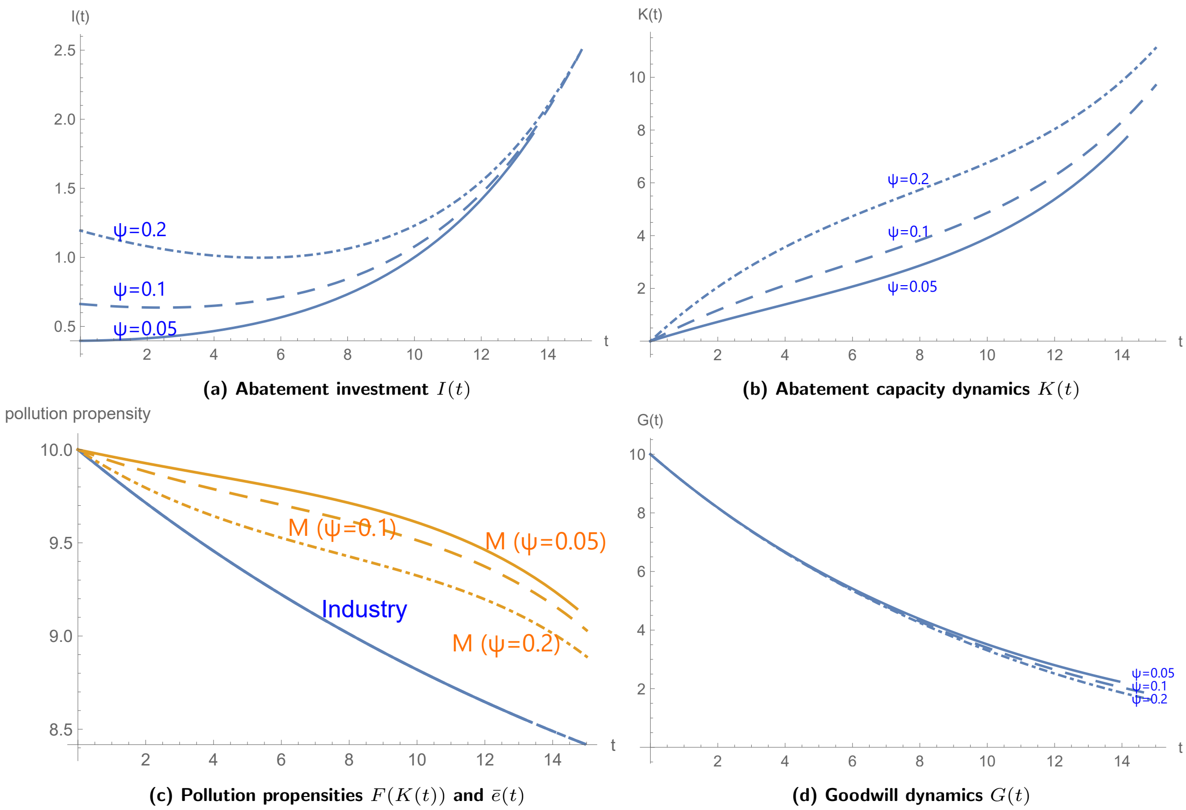


Figure 3: The negative effect of ψ on G (and D)

although a lower ψ implies larger $|\Delta(t)|$, if ψ is sufficiently small, the product $|\psi\Delta(t)|$ is smaller, resulting in a smaller impact on goodwill evolution.

Therefore, if consumers place little emphasis on environmental performance, or if the information related to emission rates is not transparent – causing ψ to have a small value – then a further decrease in consumers' environmental concern and/or information transparency (ψ) would lead the manufacturer to invest less, pollute more, yet enjoy better green reputation, resulting in a larger demand and a higher profit margin. Our numerical simulations show that for the benchmark case, this "punishment-free" phenomenon exists for $\psi < \bar{\psi} = 0.25$.

Accordingly, this non-monotone impact is also present on profits. In Figure 4 we present the profits of the manufacturer and the entire supply chain under various ψ values. Similar to the case of goodwill dynamics, all profits initially decrease with ψ when it is small. This is primarily due to the absence of penalties for the manufacturer's non-adoption of green technology. However, at a certain threshold, an increase in ψ starts to have a positive effect instead, though the specific transition points differ.

This diminishing impact is particularly pronounced for the manufacturer's profit, as it is the party responsible for the investment in green technology. Consequently, the threshold for this transition is higher for the manufacturer than for the entire supply chain ($\tilde{\psi}_M = 0.37 > \tilde{\psi}_{SC}$). In contrast, the retailer's switching point is lower ($\tilde{\psi}_R = 0.12 < \tilde{\psi}_{SC}$). It is worth noting that, despite the non-monotone impact nature, both players ultimately benefit from serving highly environmentally conscious consumers, as profits tend to be higher when ψ values are large.

When the impact of pollution difference on green reputation is very high (e.g., $\psi = 0.7$), the manufacturer would allocate a large budget on abatement capacity from the beginning of planning horizon (0-5 periods), and gradually reduce it over time, until the final adjustment in the last few periods. Consequently, the manufacturer's emission rate is well below the industry average during the planning horizon, implying the growth of goodwill.

For a moderate impact like $\psi = 0.5$ (which is the benchmark case), the abatement investment remains relatively stable throughout the time, and the manufacturer's pollution propensity is slightly below the standard. However, due to its limited effect on goodwill, the natural decay dominates and the goodwill ends up decreasing slowly over time.

In the case of coordination, the impacts of θ_1 and ψ on the cost-sharing rate remain unclear. Figure 5 illustrates the effects of θ_1 on the support rate for abatement capacity investment offered by the retailer, revealing no consistent pattern in how the support rate is adjusted based on the effectiveness of emission reduction. However, the support rate is chosen so that the total investment I increases as θ_1 rises.

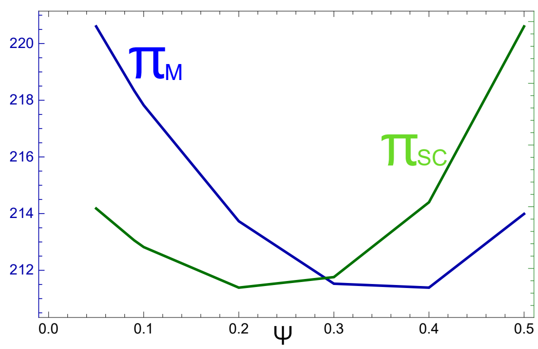


Figure 4: The non-monotone impact of ψ on π_M and π_{SC}

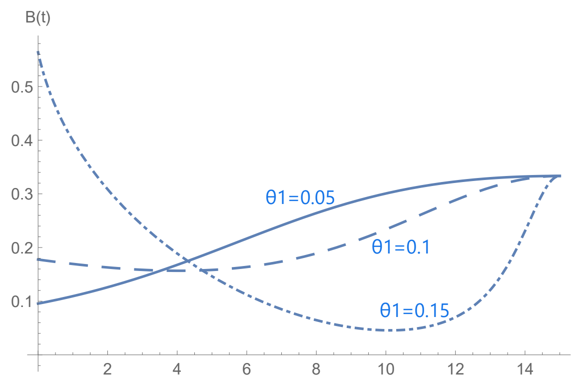


Figure 5: The effect of θ_1 on B

5.2.3 Effects of decay rates: μ and δ

Goodwill decay rate μ and abatement capacity depreciation rate δ both have negative impacts on investment, abatement capacity, goodwill, and pricing. This is intuitive: if it is more difficult to maintain the assets, the manufacturer is less likely to allocate a significant budget towards their build-up.

Similar to the cases of θ_1 and ψ, their effects on the cost-sharing rate in the cooperation scenario $B(t)$ are ambiguous.

5.2.4 Effects of initial status: G^0, K^0, and θ_0

Furthermore, our numerical simulations demonstrate that the initial states matter, which is expected in a finite-horizon model. Recall that the investment in green technology increases with the instantaneous goodwill level ($m_3(t) > 0$). Thus, a higher initial goodwill G^0 implies a higher initial investment, which then leads to higher abatement capacity, lower emission rate, and ultimately, an enhanced goodwill - creating a virtuous cycle.

Similarly, a higher initial abatement capacity K_0 induces more green investment ($m_1(t) > 0$), which in turn reinforces the capacity – another virtuous cycle.

Since $K(t)$ will ultimately translate to pollution propensity, we also test different values for θ_0, which, along with $K(0)$, determines the initial emission rate $F(K(0))$. In particular, for $K_0 = 0$, $\theta_0 = \epsilon + \sigma$ means that the manufacturer has exactly the same pollution propensity as the industry ($F(K(0)) = \bar{e}(0)$). If $\theta_0 > \epsilon + \sigma$, the manufacturer pollutes more than average, and vice versa for $\theta_0 < \epsilon + \sigma$. Therefore, we can use $K_0 = 0$ as the benchmark case and simply vary the values of θ_0 to observe how the starting pollution level affects the firms' decisions and the system dynamics.

Figure 6 shows that a better starting position, whether in the form of higher goodwill (Figure 6b) or a pollution rate below the industry average (Figure 6a), provides the manufacturer with greater momentum to invest more in green innovation and reduce his own pollution propensity.

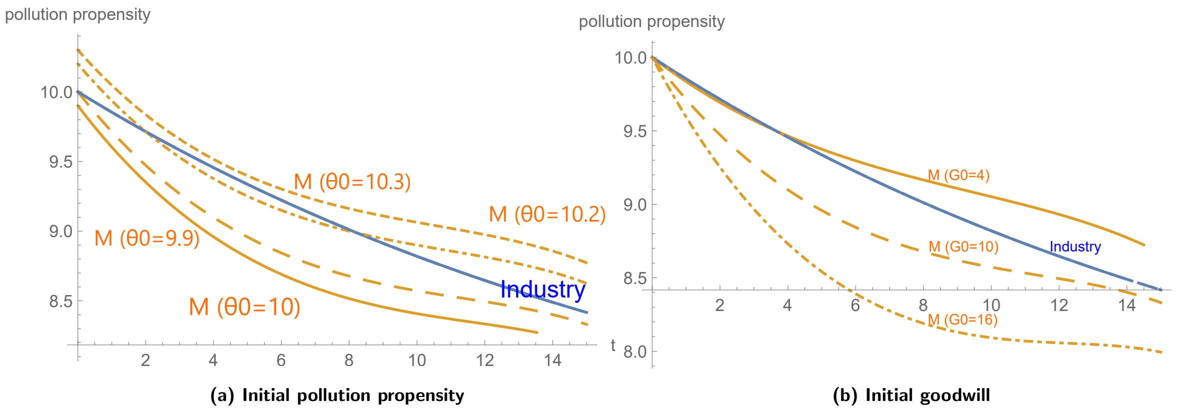


Figure 6: The effects of initial states

6 Managerial insights and concluding remarks

Starting from the premise that the environmental performance of a firm must be judged in comparison of others in the same industry, we looked at investments by a supply chain in pollution abatement in naturally a dynamic setup. We considered two scenarios, with and without participation of the retailer in the investment cost of the manufacturer, which is the prime concerned player by the environmental performance as it directly affects its reputation and consequently its demand. We wrap up our main managerial takeaways.

1. A cost-sharing scheme enhances investment in abatement capacity and lowers unit pollution rates. However, this does not always lead to a cleaner society, as the resulting increase in demand can offset these benefits and lead to higher total emissions. Cost-sharing is environmentally beneficial in markets that are already relatively clean, particularly when consumer environmental sensitivity is either very low or very high, investment efficiency is greater, and the industry standard decreases at a slower pace.
2. In an industry with flourishing technology revolution, which is accompanied by a rapid decrease in the average pollution rate, manufacturers tend to invest less in green innovation. However, companies with severe environmental unfriendliness will eventually face market exclusion.
3. When consumers exhibit low environmental concern or emission information is not transparent (resulting in a small ψ), manufacturers can potentially avoid the consequences of not engaging in green innovation.
4. To encourage abatement capacity investment, the government could enhance information disclosure and consumer education. Raising ψ to enter the positive-effect regime would require manufacturers to "pay" (in the form of goodwill) for their pollution.
5. If a company significantly lags behind the industry emission performance standard or has a notorious green reputation, catching up becomes less likely. In this scenario, the government can promote initiatives that facilitate the adoption of green technology or use regulatory tools to help/force these companies improve their initial status.

We made two simplifying assumptions in this paper that are worth relaxing. First, we supposed that the propensity to pollute decreases linearly in the abatement capacity. While this assumption is reasonable when the planning horizon is short, as it is here, it is still conceptually more attractive to retain a convex decreasing function to capture the marginal decreasing return to scale in abatement capacity. Second, we assumed that the competitive pressure on the manufacturer to invest is coming from consumers who base their purchasing decision based on the environmental performance of the industry whose emissions propensity is modeled by an exogenously given function of time. It would be interesting to introduce strategic competition at the manufacturing level to assess the robustness of our results.

Appendices

A Proofs of Propositions

Proof of Proposition 1. We write the retailer's optimization problem, and assuming the feedback-information structure, write its HJB equation:

$$\begin{aligned}\rho V_R\left(t,G,K\right)-\frac{\partial V_R}{\partial t} &= \max_{p}\left\{(p-w)\left(\alpha+\gamma G-\beta p\right)-\frac{1}{2}kBI^2\right. \\ &\left.+\frac{\partial V_R}{\partial G}\left(\psi\left(\epsilon+\sigma e^{-\nu t}-(\theta_0-\theta_1 K)\right)-\mu G\right)+\frac{\partial V_R}{\partial K}\left(I-\delta K\right)\right\},\end{aligned} \tag{8}$$

with boundary condition $V_R\left(T,G,K\right)=S_R(G(T),K(T))$.

The retailer $p(t,G,K)$ maximizing the right-hand-side term in brackets in (8), and we obtain its reaction function:

$$p(G,w)=\frac{\alpha+\gamma G+\beta w}{2\beta}. \tag{9}$$

Second, we solve the manufacturer's optimization problem and write the HJB equation:

$$\begin{aligned}\rho V_M\left(t,G,K\right)-\frac{\partial V_M}{\partial t} &= \max_{w,I}\left\{(w-c)\left(\alpha+\gamma G-\beta p\right)-\frac{1}{2}k(1-B)I^2\right. \\ &\left.+\frac{\partial V_M}{\partial G}\left(\psi\left(\epsilon+\sigma e^{-\nu t}-(\theta_0-\theta_1 K)\right)-\mu G\right)+\frac{\partial V_M}{\partial K}\left(I-\delta K\right)\right\},\end{aligned} \tag{10}$$

with boundary condition $V_M\left(T,G,K\right)=S_M(G(T),K(T))$.

Substituting the reaction function p given by (9) and maximizing the right-hand side of HJB Equation (10), we find the manufacturer's strategies:

$$w(G)=\frac{\alpha+\gamma G+\beta c}{2\beta}, \tag{11}$$

$$I(t,B,G,K)=\frac{1}{k(1-B)}\frac{\partial V_M}{\partial K}. \tag{12}$$

Substituting for w in the expression of p, we get

$$p(t,G)=\frac{3\alpha+3\gamma G+\beta c}{4\beta}. \tag{13}$$

Substituting for p in the demand function, we obtain

$$D(G)=\frac{\alpha+\gamma G-\beta c}{4}.$$

Next, substituting w, p, and I into HJB Equation (8) and then solve if for B:

$$\begin{aligned}\rho V_R\left(t,G,K\right)-\frac{\partial V_R}{\partial t} &= \max_{B\in[0,1]}\left\{\frac{(\alpha+\gamma G-\beta c)^2}{16\beta}-\frac{B}{2k(1-B)^2}\left(\frac{\partial V_M}{\partial K}\right)^2+\frac{1}{k(1-B)}\frac{\partial V_R}{\partial K}\frac{\partial V_M}{\partial K}\right. \\ &\left.+\frac{\partial V_R}{\partial G}\left(\psi\left(\epsilon+\sigma e^{-\nu t}-(\theta_0-\theta_1 K)\right)-\mu G\right)-\delta K\frac{\partial V_R}{\partial K}\right\},\end{aligned} \tag{14}$$

and maximizing the right-hand side term in brackets w.r.t. B, we find the derivative over B and get the equation:

$$-\frac{1+B}{2(1-B)}\frac{\partial V_M}{\partial K}+\frac{\partial V_R}{\partial K}=0,$$

and we obtain the strategy:

$$B(t,G,K)=\frac{2\frac{\partial V_R}{\partial K}-\frac{\partial V_M}{\partial K}}{2\frac{\partial V_R}{\partial K}+\frac{\partial V_M}{\partial K}}, \tag{15}$$

and $B(t,G,K)\in[0,1)$, that should be verified.

Insert (11)–(13), and (15) into the maximand on the right-hand-side of (8) and (10) and get the HJB equations:

$$\begin{aligned}\rho V_R(t,G,K)-\frac{\partial V_R}{\partial t}=&\frac{(\alpha+\gamma G-\beta c)^2}{16\beta}+\frac{1}{8k}\left(2\frac{\partial V_R}{\partial K}+\frac{\partial V_M}{\partial K}\right)^2\\&+\frac{\partial V_R}{\partial G}\left(\psi\left(\epsilon+\sigma e^{-\nu t}-(\theta_0-\theta_1 K)\right)-\mu G\right)-\delta K\frac{\partial V_R}{\partial K},\end{aligned}$$

$$\begin{aligned}\rho V_M(t,G,K)-\frac{\partial V_M}{\partial t}=&\frac{(\alpha+\gamma G-\beta c)^2}{8\beta}+\frac{1}{4k}\frac{\partial V_M}{\partial K}\left(2\frac{\partial V_R}{\partial K}+\frac{\partial V_M}{\partial K}\right)\\&+\frac{\partial V_M}{\partial G}\left(\psi\left(\epsilon+\sigma e^{-\nu t}-(\theta_0-\theta_1 K)\right)-\mu G\right)-\delta K\frac{\partial V_M}{\partial K}.\end{aligned}$$

Suppose that the manufacturer's and retailer's value functions are quadratic in K and G, and given by

$$\begin{aligned}V_M(t,K,G)=&\frac{1}{2}m_1(t)K^2+\frac{1}{2}m_2(t)G^2+m_3(t)KG+m_4(t)K+m_5(t)G+m_6(t),\\V_R(t,K,G)=&\frac{1}{2}r_1(t)K^2+\frac{1}{2}r_2(t)G^2+r_3(t)KG+r_4(t)K+r_5(t)G+r_6(t).\end{aligned}$$

Then, we write the right-hand and the left-hand side expressions in the HJB equations:

$$\begin{aligned}&\rho\left[\frac{1}{2}m_1(t)K^2+\frac{1}{2}m_2(t)G^2+m_3(t)KG+m_4(t)K+m_5(t)G+m_6(t)\right]\\&-\left[\frac{1}{2}\dot m_1(t)K^2+\frac{1}{2}\dot m_2(t)G^2+\dot m_3(t)KG+\dot m_4(t)K+\dot m_5(t)G+\dot m_6(t)\right]\\&=\frac{1}{8\beta}(\alpha+\gamma G-\beta c)^2+\frac{1}{4k}(m_1(t)K+m_3(t)G+m_4(t))\\&\quad\times(2r_1(t)K+2r_3(t)G+2r_4(t)+m_1(t)K+m_3(t)G+m_4(t))\\&\quad+[m_2(t)G+m_3(t)K+m_5(t)]\left[\psi\left(\epsilon+\sigma e^{-\nu t}-(\theta_0-\theta_1 K)\right)-\mu G\right]\\&\quad-\delta K\left[m_1(t)K+m_3(t)G+m_4(t)\right],\end{aligned}$$

$$\begin{aligned}&\rho\left[\frac{1}{2}r_1(t)K^2+\frac{1}{2}r_2(t)G^2+r_3(t)KG+r_4(t)K+r_5(t)G+r_6(t)\right]\\&-\left[\frac{1}{2}\dot r_1(t)K^2+\frac{1}{2}\dot r_2(t)G^2+\dot r_3(t)KG+\dot r_4(t)K+\dot r_5(t)G+\dot r_6(t)\right]\\&=\frac{1}{16\beta}(\alpha+\gamma G-\beta c)^2+\frac{1}{8k}(2r_1(t)K+2r_3(t)G+2r_4(t)+m_1(t)K+m_3(t)G+m_4(t))^2\\&\quad+[r_2(t)G+r_3(t)K+r_5(t)]\left(\psi\left(\epsilon+\sigma e^{-\nu t}-(\theta_0-\theta_1 K)\right)-\mu G\right)\\&\quad-\delta K\left[r_1(t)K+r_3(t)G+r_4(t)\right].\end{aligned}$$

Collecting terms corresponding to $(GK)^0, G^1, G^2, K^1, K^2, G^1K^1$, we obtain the system of equations:

$$\dot{m}_1(t) = (2\delta + \rho)m_1(t) - \frac{1}{2k}(m_1(t))^2 - \frac{1}{k}m_1(t)r_1(t) - 2\theta_1\psi m_3(t), \tag{16}$$

$$\dot{m}_2(t) = (2\mu + \rho)m_2(t) - \frac{\gamma^2}{4\beta} - \frac{1}{2k}(m_3(t))^2 - \frac{1}{k}m_3(t)r_3(t), \tag{17}$$

$$\dot{m}_3(t) = (\delta + \mu + \rho)m_3(t) - \frac{1}{2k}(m_1(t)m_3(t) + m_1(t)r_3(t) + m_3(t)r_1(t)) - \theta_1\psi m_2(t), \tag{18}$$

$$\begin{aligned}\dot{m}_4(t) = {} & (\delta + \rho)m_4(t) - \frac{1}{2k}(m_1(t)m_4(t) + m_1(t)r_4(t) + m_4(t)r_1(t)) \\ & - \psi(\epsilon - \theta_0 + \sigma e^{-\nu t})m_3(t) - \theta_1\psi m_5(t),\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{m}_5(t) = {} & (\mu + \rho)m_5(t) - (\epsilon - \theta_0 + \sigma e^{-\nu t})\psi m_2(t) - \frac{1}{2k}(m_3(t)m_4(t) + m_3(t)r_4(t) \\ & + m_4(t)r_3(t)) - \frac{(\alpha - \beta c)\gamma}{4\beta}\end{aligned} \tag{20}$$

$$\dot{m}_6(t) = \rho m_6(t) - (\epsilon - \theta_0 + \sigma e^{-\nu t})\psi m_5(t) - \frac{1}{4k}(m_4(t))^2 - \frac{1}{2k}m_4(t)r_4(t) - \frac{(\alpha - \beta c)^2}{8\beta}, \tag{21}$$

$$\dot{r}_1(t) = (\rho + 2\delta)r_1(t) - \frac{1}{4k}(2r_1(t) + m_1(t))^2 - 2\theta_1\psi r_3(t), \tag{22}$$

$$\dot{r}_2(t) = (2\mu + \rho)r_2(t) - \frac{\gamma^2}{8\beta} - \frac{1}{4k}(2r_3(t) + m_3(t))^2, \tag{23}$$

$$\dot{r}_3(t) = (\delta + \mu + \rho)r_3(t) - \frac{1}{4k}(2r_1(t) + m_1(t))(2r_3(t) + m_3(t)) - \theta_1\psi r_2(t), \tag{24}$$

$$\dot{r}_4(t) = (\delta + \rho)r_4(t) - \frac{1}{4k}(2r_1(t) + m_1(t))(2r_4(t) + m_4(t)) - \psi(\epsilon - \theta_0 + \sigma e^{-\nu t})r_3(t) - \theta_1\psi r_5(t), \tag{25}$$

$$\dot{r}_5(t) = (\mu + \rho)m_5(t) - (\epsilon - \theta_0 + \sigma e^{-\nu t})\psi r_2(t) - \frac{1}{4k}(2r_3(t) + m_3(t))(2r_4(t) + m_4(t)) - \frac{(\alpha - \beta c)\gamma}{8\beta} \tag{26}$$

$$\dot{r}_6(t) = \rho r_6(t) - (\epsilon - \theta_0 + \sigma e^{-\nu t})\psi r_5(t) - \frac{1}{8k}(2r_4(t) + m_4(t))^2 - \frac{(\alpha - \beta c)^2}{16\beta}, \tag{27}$$

with the boundary conditions $V_M(T, G, K) = S_M(G(T), K(T))$, $V_R(T, G, K) = S_R(G(T), K(T))$.

Once coefficients $m_i(t)$, $r_j(t)$, $i, j = 1, \ldots, 6$, are found, we solve the system of differential equations:

$$\begin{pmatrix}\dot{G}(t)\\ \dot{K}(t)\end{pmatrix} = \begin{pmatrix}-\mu & \psi\theta_1\\ \frac{2r_3(t)+m_3(t)}{2k} & \frac{2r_1(t)+m_1(t)}{2k} - \delta\end{pmatrix}\begin{pmatrix}G(t)\\ K(t)\end{pmatrix} + \begin{pmatrix}\psi(\epsilon + \sigma e^{-\nu t} - \theta_0)\\ \frac{2r_4(t)+m_4(t)}{2k}\end{pmatrix} \tag{28}$$

with initial conditions

$$G(0) = G^0,$$
$$K(0) = K^0,$$

to determine equilibrium GR and abatement capacity trajectories. □

Proof of Proposition 2. The proof is repeating the proof of Proposition 1 substituting $B(t) = 0$. The reaction function of the retailer at the second stage of the game is

$$\tilde{p} = \frac{\alpha + \gamma\tilde{G} + \beta\tilde{w}}{2\beta}. \tag{29}$$

Second, we solve the manufacturer's optimization problem by writing the HJB equation by substituting p given by (29) into manufacturer's HJB equation, and maximizing the right-hand side of HJB equation, we find the manufacturer's strategies:

$$\tilde{w}(t,\tilde{G}) = \frac{\alpha+\gamma\tilde{G}+\beta c}{2\beta}, \tag{30}$$

$$\tilde{I}(t,\tilde{G},\tilde{K}) = \frac{1}{k}\frac{\partial \tilde{V}_M}{\partial \tilde{K}}. \tag{31}$$

Substituting for $\tilde{w}$ in the expression of $\tilde{p}$, we get

$$\tilde{p}(t,\tilde{G}) = \frac{3\alpha+3\gamma\tilde{G}+\beta c}{4\beta}. \tag{32}$$

Substituting for $\tilde{p}$ in the demand function, we obtain

$$\tilde{D}(t,\tilde{G}) = \frac{\alpha+\gamma\tilde{G}-\beta c}{4}.$$

Substituting for $\tilde{w}$ and $\tilde{I}$ in the HJB equation, we obtain

$$\rho\tilde{V}_M\left(t,\tilde{G},\tilde{K}\right) - \frac{\partial\tilde{V}_M}{\partial t} = \frac{1}{8\beta}\left(\alpha+\gamma\tilde{G}-\beta c\right)^2 + \frac{1}{2k}\left(\frac{\partial\tilde{V}_M}{\partial\tilde{K}}\right)^2 + \frac{\partial\tilde{V}_M}{\partial\tilde{G}}\left(\psi\left(\epsilon+\sigma e^{-\nu t} - \left(\theta_0-\theta_1\tilde{K}\right)\right)-\mu\tilde{G}\right) - \delta\tilde{K}\frac{\partial\tilde{V}_M}{\partial\tilde{K}}.$$

Suppose that the manufacturer's value function is quadratic in K and G, and given by

$$\tilde{V}_M\left(t,\tilde{G},\tilde{K}\right) = \frac{1}{2}\ell_1(t)\tilde{K}^2 + \frac{1}{2}\ell_2(t)\tilde{G}^2 + \ell_3(t)\tilde{K}\tilde{G} + \ell_4(t)\tilde{K} + \ell_5(t)\tilde{G} + \ell_6(t).$$

We thus obtain a system of differential equations:

$$C_{M1}(t) = \frac{1}{2}\left[\dot{\ell}_1(t) - (2\delta+\rho)\ell_1(t) + \frac{1}{k}\left(\ell_1(t)\right)^2 + 2\theta_1\psi\ell_3(t)\right] = 0 \tag{33}$$

$$C_{M2}(t) = \frac{1}{2}\dot{\ell}_2(t) + \frac{\gamma^2}{8\beta} - \frac{2\mu+\rho}{2}\ell_2(t) + \frac{1}{2k}\left(\ell_3(t)\right)^2 = 0 \tag{34}$$

$$C_{M3}(t) = \dot{\ell}_3(t) - (\delta+\mu+\rho)\ell_3(t) + \frac{1}{k}\ell_1(t)\ell_3(t) + \theta_1\psi\ell_2(t) = 0 \tag{35}$$

$$C_{M4}(t) = \dot{\ell}_4(t) + \frac{1}{k}\ell_1(t)\ell_4(t) - (\delta+\rho)\ell_4(t) + \psi(\epsilon-\theta_0+\sigma e^{-\nu t})\ell_3(t) + \theta_1\psi\ell_5(t) = 0 \tag{36}$$

$$C_{M5}(t) = \dot{\ell}_5(t) - (\mu+\rho)\ell_5(t) + (\epsilon-\theta_0+\sigma e^{-\nu t})\psi\ell_2(t) + \frac{1}{k}\ell_3(t)\ell_4(t) + \frac{(\alpha-\beta c)\gamma}{4\beta} = 0 \tag{37}$$

$$C_{M6}(t) = \dot{\ell}_6(t) - \rho\ell_6(t) + (\epsilon-\theta_0+\sigma e^{-\nu t})\psi\ell_5(t) + \frac{1}{2k}\left(\ell_4(t)\right)^2 + \frac{(\alpha-\beta c)^2}{8\beta} = 0, \tag{38}$$

with the boundary conditions $\tilde{V}_M\left(T,\tilde{G},\tilde{K}\right) = S_M(\tilde{G}(T),\tilde{K}(T))$.

We note that

1. $C_{M1}(t), C_{M2}(t)$, and $C_{M3}(t)$ are independent of ℓ_4, ℓ_5, and ℓ_6, which implies that the first three differential equations can be solved separately, i.e.,

$$\dot{\ell}_1(t) = -\frac{1}{k}\left(\ell_1(t)\right)^2 + (2\delta+\rho)\ell_1(t) - 2\theta_1\psi\ell_3(t),$$

$$\dot{\ell}_2(t) = (2\mu+\rho)\,\ell_2(t) - \frac{1}{k}\left(\ell_3(t)\right)^2 - \frac{\gamma^2}{4\beta},$$

$$\dot{\ell}_3(t) = (\delta+\mu+\rho)\ell_3(t) - \frac{1}{k}\ell_1(t)\ell_3(t) - \theta_1\psi\ell_2(t).$$

2. Once we find ℓ_1, ℓ_2, and ℓ_3, we can solve the system of Equations (36) and (37) for ℓ_4 and ℓ_5, which are independent of ℓ_6.
3. Next, we solve (38) for ℓ_6.

For the retailer we obtain the HJB equations:

$$\begin{aligned}\rho\tilde{V}_R\left(t,\tilde{G},\tilde{K}\right)-\frac{\partial\tilde{V}_R}{\partial t}=&\frac{1}{16\beta}\left(\alpha+\gamma\tilde{G}-\beta c\right)^2\\&+\frac{\partial\tilde{V}_R}{\partial\tilde{G}}\left(\psi\left(\epsilon+\sigma e^{-\nu t}-\left(\theta_0-\theta_1\tilde{K}\right)\right)-\mu\tilde{G}\right)\\&+\frac{\partial\tilde{V}_R}{\partial\tilde{K}}\left(\frac{1}{k}\frac{\partial\tilde{V}_M}{\partial\tilde{K}}-\delta\tilde{K}\right),\end{aligned}$$

and assuming that $\tilde{V}_R\left(t,\tilde{G},\tilde{K}\right)=\frac{1}{2}z_1(t)\tilde{K}^2+\frac{1}{2}z_2(t)\tilde{G}^2+z_3(t)\tilde{K}\tilde{G}+z_4(t)\tilde{K}+z_5(t)\tilde{G}+z_6(t)$, we obtain the system of differential equations with respect to $z_1,\ldots,z_6$:

$$\dot{z}_1(t)=(\rho+2\delta)z_1(t)-\frac{2}{k}z_1(t)\ell_1(t)-2\theta_1\psi z_3(t),\tag{39}$$

$$\dot{z}_2(t)=(2\mu+\rho)z_2(t)-\frac{\gamma^2}{8\beta}-\frac{2}{k}z_3(t)\ell_3(t),\tag{40}$$

$$\dot{z}_3(t)=(\delta+\mu+\rho)z_3(t)-\frac{1}{k}z_1(t)\ell_3(t)-\frac{1}{k}z_3(t)\ell_1(t)-\theta_1\psi z_2(t),\tag{41}$$

$$\dot{z}_4(t)=(\delta+\rho)z_4(t)-\frac{1}{k}z_1(t)\ell_4(t)-\frac{1}{k}z_4(t)\ell_1(t)-\psi(\epsilon-\theta_0+\sigma e^{-\nu t})z_3(t)-\theta_1\psi z_5(t),\tag{42}$$

$$\dot{z}_5(t)=(\mu+\rho)\ell_5(t)-(\epsilon-\theta_0+\sigma e^{-\nu t})\psi z_2(t)-\frac{1}{k}z_3(t)\ell_4(t)-\frac{1}{k}z_4(t)\ell_3(t)-\frac{(\alpha-\beta c)\gamma}{8\beta}\tag{43}$$

$$\dot{z}_6(t)=\rho z_6(t)-(\epsilon-\theta_0+\sigma e^{-\nu t})\psi z_5(t)-\frac{1}{k}z_4(t)\ell_4(t)-\frac{(\alpha-\beta c)^2}{16\beta},\tag{44}$$

with the boundary condition $\tilde{V}_R\left(T,\tilde{G},\tilde{K}\right)=S_R(\tilde{G}(T),\tilde{K}(T))$.

We note that

1. Substituting ℓ_1 and ℓ_3 to Equations (39)–(41), we solve these equations for z_1, z_2, and z_3 separately from the rest of the equations.
2. We substitute ℓ_1, ℓ_4, z_1, and z_3 into Equations (42) and (43) and solve them for z_4 and z_5.
3. Finally, we solve Equation (44) for z_6.

Once coefficients $\ell_i(t)$, $i=1,\ldots,6$ are found, we solve the system of differential equations

$$\begin{pmatrix}\dot{\tilde{G}}(t)\\\dot{\tilde{K}}(t)\end{pmatrix}=\begin{pmatrix}-\mu&\psi\theta_1\\\frac{\ell_3(t)}{k}&\frac{\ell_1(t)}{k}-\delta\end{pmatrix}\begin{pmatrix}\tilde{G}(t)\\\tilde{K}(t)\end{pmatrix}+\begin{pmatrix}\psi(\epsilon+\sigma e^{-\nu t}-\theta_0)\\\frac{\ell_4(t)}{k}\end{pmatrix}\tag{45}$$

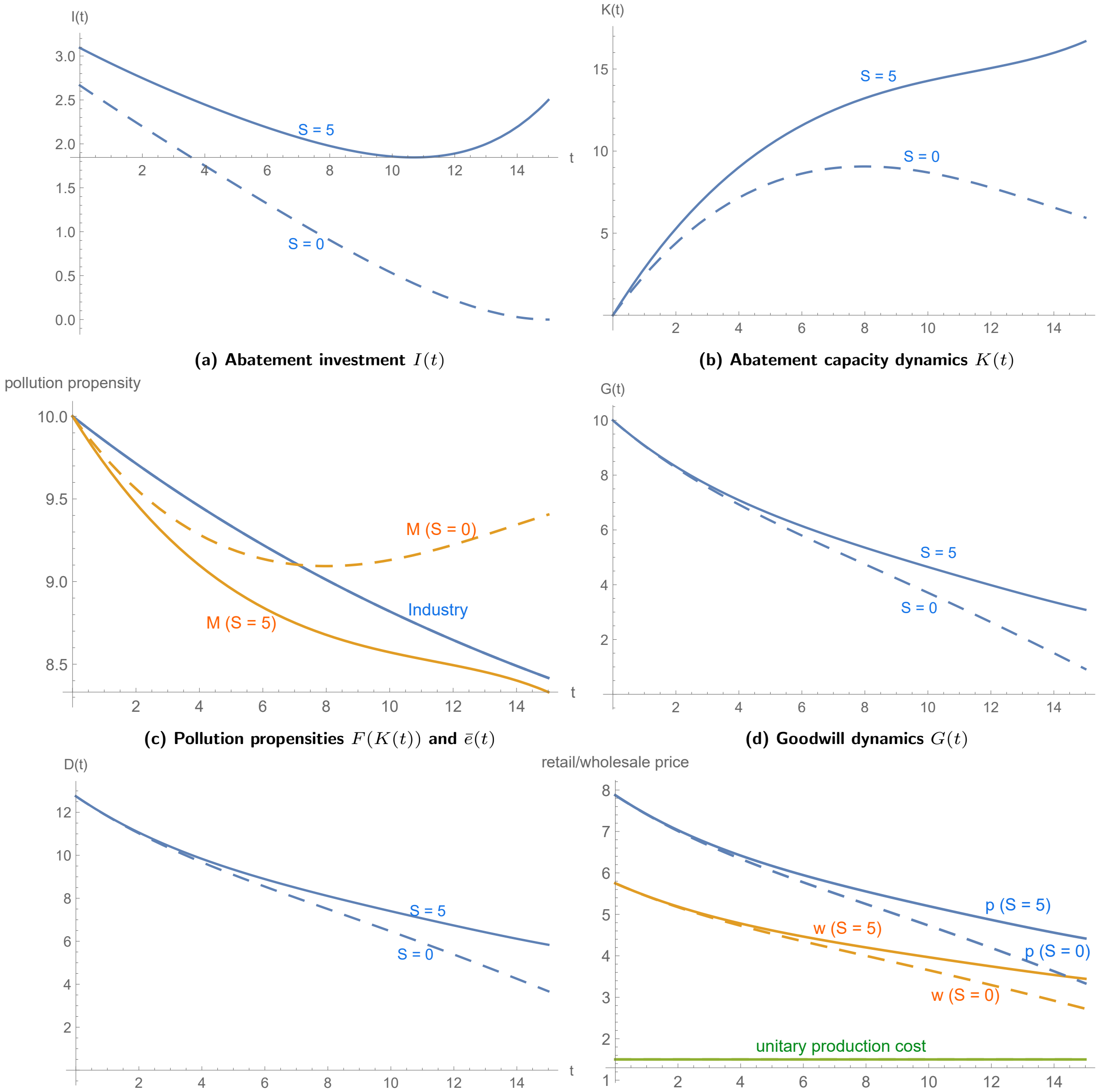
with initial conditions

$$\tilde{G}(0)=G^0,\tag{46}$$

$$\tilde{K}(0)=K^0,\tag{47}$$

to determine equilibrium GR and abatement capacity trajectories. □

B Additional figures



(a) Abatement investment $I(t)$

(b) Abatement capacity dynamics $K(t)$

(c) Pollution propensities $F(K(t))$ **and** $\bar{e}(t)$

(d) Goodwill dynamics $G(t)$

(e) Instantaneous demand $D(t)$

(f) Pricing decisions $p(t)$ **and** $w(t)$

Figure 7: The effects of salvage values

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