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Formation of international environmental agreements and payoff allocation

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Abstract : Dealing with climate change requires that all countries engage in costly efforts to reduce their emissions. Reaching this objective has so far been elusive because it is in the best interest of each country to let the others do the effort and benefit itself from a better environment. The presence of negative externalities and strategic behavior have made game theory a natural paradigm to design an international environmental agreement (IEA) that codifies what countries should do. Considering that countries are sovereign and no supranational entity can impose an agreement, a stream of literature adopted a noncooperative mode of play to the formation of an environmental coalition. On the other hand, as joint optimization of all countries' payoff leads to the best outcome, cooperative games approach has also been proposed to share the cost of climate change. Both approaches have their pros and cons.

In this paper, we propose a model of coalition formation that combines both cooperative and noncooperative modes of play. Starting from any given coalition, we implement a Markov process that shows sequentially which countries join or leave the coalition until reaching an absorbing state. All possible sequential scenarios are considered and an allocation to the player is made taking into account individual rationality. An illustration with vulnerable and invulnerable countries to pollution is given.

Keywords : Coalition formation; international environmental agreement; Markov process; Shapley value

Résumé : La lutte contre le changement climatique exige que tous les pays s'engagent dans des efforts coûteux pour réduire leurs émissions. Atteindre cet objectif s'est jusqu'à présent avéré difficile, car chaque pays a intérêt à laisser les autres faire l'effort et à bénéficier lui-même d'un environnement meilleur. La présence d'externalités négatives et de comportements stratégiques a fait de la théorie des jeux un paradigme naturel pour la conception d'un accord environnemental international (AIE) codifiant les actions des pays. Considérant que les pays sont souverains et qu'aucune entité supranationale ne peut imposer un accord, de nombreux travaux ont adopté un mode de jeu non coopératif pour la formation d'une coalition environnementale. D'autre part, l'optimisation conjointe des bénéfices de tous les pays conduisant au meilleur résultat, une approche de jeux coopératifs a également été proposée pour partager le coût du changement climatique. Chaque approche présente ses avantages et ses inconvénients.

Dans cet article, nous proposons un modèle de formation de coalition combinant des modes de jeu coopératifs et non coopératifs. À partir d'une coalition donnée, nous mettons en œuvre un processus markovien qui indique séquentiellement quels pays rejoignent ou quittent la coalition jusqu'à atteindre un état stable. Tous les scénarios séquentiels possibles sont envisagés et une allocation est effectuée au joueur en tenant compte de la rationalité individuelle. Une illustration est donnée avec des pays vulnérables et invulnérables à la pollution.

Mots clés : Formation de coalition; accord environnemental international; processus markovien; valeur de Shapley

1 Introduction

In this paper, we propose a Markov process approach to the formation of an international environmental agreement (IEA) that combines both cooperative and noncooperative games aspects. IEAs are concerned by transboundary (or global) pollution problems where the emissions generated by any country affect the well-being of all. The global environment is a public good that suffers from the lack of sufficient efforts by the agents (polluting countries) to keep it clean. For each country to internalize the cost of its emissions, it must either be incentivized through carrots, e.g., transfers or subsidies, or through sticks, e.g., taxes and quotas. The absence of a supranational environmental agency makes the implementation of such instruments nearly impossible in practice in the context of global pollution. The thinking is that an IEA could avoid, or at least alleviate the impact of this tragedy of the common type of situation.

From the above, it is clear that any design of an IEA must at least account for the facts that the countries' payoffs are interdependent (by the very public good nature of the environment), and that each country behaves strategically by choosing its pollution level considering the others' reactions. These features have made game theory a natural paradigm to model IEAs, which explains the large number of publications in this area during the last three decades or so. The literature can be classified along two main dimensions: (i) the mode of play, i.e., cooperative or noncooperative, and (ii) the temporal nature of the process of coalition formation, i.e., static or dynamic. We review below the part of the literature that is directly relevant to our approach, and next highlight our contribution.

Noncooperative game approach to IEAs. The premise in this literature stream is straightforward: In the absence of an international institution that can force sovereign countries to join an IEA, the participation must be voluntary based, and the agreement self-enforcing (stable in an equilibrium sense). The objective function of a country is typically made of two pieces: a concave increasing benefit function that depends only on own emissions, which are a by-product of production, and a convex increasing damage cost function that depends on total emissions (here lies the negative externality). The endogenous size of a stable agreement is determined by two rules referred to as internal and external stability. An IEA is internally stable if no member can improve its payoff by leaving it, and externally stable if no non-member can achieve a better outcome by joining the agreement. These rules were originally proposed in d'Aspremont et al. (1983) to study the stability of collusive pricing by a cartel. The payoffs at the basis of the comparisons involved in the no-entry and no-exit conditions are computed assuming that the members (signatories) act as a single player and choose the emissions levels that jointly maximize their welfare, while each non-member (non-signatory) optimizes its individual payoff.

The seminal papers in this stream are Barrett (1994) and Carraro and Siniscalco (1993), where the authors showed that, without additional incentives, only a few (typically 2 or 3) countries will join an IEA. Diamantoudi and Sartzetakis (2006) showed that, independently of the number of countries, the IEA will include at most 4 countries when the benefit and cost functions are quadratic, an assumption often made in the literature. The reason behind this grim result is free riding. Indeed, each country is better off letting the others invest in costly pollution abatement and enjoys a better environment without paying itself the cost. In these seminal papers, and in most of the literature that followed, the countries are symmetric. Consequently, it does not really matter which country joins and which does not, the main point being how many of the n countries will end up signing the agreement. Pavlova and de Zeeuw (2013) assumed asymmetries in both the benefit and cost functions and obtain that large stable coalitions will materialize only if they include countries that have relatively high marginal benefits and a relatively low marginal environmental damage, which is uninteresting as these countries do not contribute much to the common good. That is, we have a larger IEA but not much less emissions. Ansink et al. (2019) proposed a four-stage model of coalition formation with a noncooperative setting and use the concept of subgame perfect Nash equilibrium to find the IEA. The feature of the model is that the players can differently participate in the agreement by choosing of being a member, supporter, and a free-rider. Like in the standard IEA game, members coordinate their abatement levels and all other countries (supporters and free-riders) act as singletons. The authors show that under mild conditions on the costs and benefits of contributing to the public good (e.g., abatement of greenhouse gas emissions), there exist equilibria with support.

In an attempt to enlarge the size of the agreement, scholars extended the basic model in various directions, such as Stackelberg leadership (e.g., Diamantoudi and Sartzetakis (2006); Rubio and Ulph (2006)), transfer schemes (Hoel and Schneider (1997); Fuentes-Albero and Rubio (2010)), reputation effects (e.g., Hoel and Schneider (1997); Jeppesen and Andersen (1998); Cabon-Dhersin and Ramani (2006); Sacco and Zaccour (2019)), issue linkages (e.g., Barrett (1997); Botteon and Carraro (1998); Katsoulacos (1997); Carraro and Siniscalco (1997, 1998); Le Breton and Soubeyran (1997); Mohr and Thomas (1998)), punishments (Hoel and Schneider (1997); Bahn et al. (2009); Breton et al. (2010)), imposing a ratification threshold (see, e.g., Courtois and Haeringer (2005); Rubio and Casino (2005); Carraro et al. (2009); Weikard et al. (2015)), seeking a regional rather than global agreements (Asheim et al. (2006)), considering modest emission-reduction targets (Finus and Maus (2008)). Also, see Barrett (2003) for a general discussion.

Finally, we note that a series of contributions considered investments in both emissions abatement and adaptation to climate change (e.g., Lazkano et al. (2016); Masoudi and Zaccour (2018); Breton and Sbragia (2017, 2019a,b); Rohrer and Rubio (2024)).

For additional references on noncooperative IEA games, we refer the interested reader to the surveys in Finus (2001, 2008); Wagner (2001); Mitchell (2003); Jørgensen et al. (2010); Benchekroun and Long (2012); Carraro (2013); Marrouch and Chaudhuri (2016).

Process of coalition formation. The IEA game is played in two stages. In the first stage, a coalition of s countries forms, while the remaining $n-s$ countries stay out. In the second stage, a noncooperative emissions game is played to determine the equilibrium emissions and payoffs. Recall that the coalition acts as a single player and each of the $n-s$ non-member players optimizes individually its payoff. One checks if this coalition satisfies the internal and external stability conditions. If it is the case, then we have a stable IEA. Next, set $s = s+1$ and redo the two steps. In the basic model, there exists only one stable coalition, but one cannot rule out the existence of multiple stable coalitions in more sophisticated models.

The process just described assumes that the players are myopic. Diamantoudi and Sartzetakis (2006) endow countries with foresightedness. If a country contemplates leaving or joining a coalition, then it will determine its benefit considering its action could incentivize other countries to defect from the coalition or to join it. The implication is that the resulting coalition is not necessarily of size $s-1$ or $s+1$ but could be of any size. The authors derive the conditions for the existence of a unique set of farsighted stable IEAs.

In the above cited papers, the emissions game is static. A series of contributions retained a dynamic game model to capture the fact that the environmental damage is caused by the accumulation of pollution and by current one. The membership game could be played once or at each stage. Breton et al. (2010) proposed an evolutionary game model, with a state equation governing the evolution of the pollution stock, and a replicator dynamics equation describing the evolution of the membership. Interestingly, the authors obtain that, depending on the parameter values, a variety of stable coalitions can materialize, including the grand coalition.

Rubio and Ulph (2007) retained a dynamic game model in discrete time where in each stage, the players solve both an emissions game and a membership game so that the size of the coalition varies over time with the pollution stock. At the unique steady state, we have both the steady-state value of the pollution stock and the stable IEA. Note that the authors assume the existence of an upper bound on non-signatory emissions and that signatories are randomly selected in each stage. Rubio and Ulph (2007) extends Rubio and Casino (2005) where the membership game is played once.

de Zeeuw (2008) extends the static farsightedness behavior process in Diamantoudi and Sartzetakis (2006) to a dynamic game including a pollution stock. In Rubio and Ulph (2007) and de Zeeuw (2008), both the membership game and the emissions game are dynamic.

Cooperative game approach to IEAs. Van der Ploeg and de Zeeuw (1992) and Dockner and Long (1993) were among the firsts to characterize and contrast cooperative and noncooperative solutions to a dynamic (differential) game model of pollution control. They showed that cooperation leads to a higher collective welfare than the sum of individual welfares, and total cooperative emissions are lower than their noncooperative counterparts. These results are an invitation to design an IEA that includes all countries. This is the starting point of the cooperative game literature on IEAs. Chander and Tulkens (1992, 1995, 1997) introduced the concept of γ-core, which was after applied to IEAs in, e.g., Germain et al. (2003). In the γ-characteristic function concept, the payoff of a coalition is given by the Nash equilibrium outcome in the noncooperative game played by the coalition (acting as a single player) and the non-members acting individually. Interesting to note that this is also how the emissions game is played in the noncooperative approach. Eyckmans and Tulkens (2003) provide some simulations for assessing the coalitionally stable sharing of the cost of climate change. Breton et al. (2006) discuss different definitions of characteristic functions and their impact on deterring free riding.

A cooperative outcome is collectively optimal, but is not an equilibrium, and therefore not self-supported. A country can deviate from the grand coalition and achieve a higher payoff than the one it obtains by remaining in the coalition. To make a cooperative solution an equilibrium, one can use trigger strategies that punish harshly and credibly any deviation from the cooperative trajectory. This established result in repeated and state-space games of infinite duration might not be easy to apply in the international arena. Chander and Tulkens did not rely on this argument to sustain their γ-core IEA, but on the fact that any deviation would lead to the grand coalition to fall apart, which makes the deviation uninteresting, i.e., leads to a lower payoff. Whereas here we have either an IEA including all players or no coalition at all, partial coalitions are feasible in the noncooperative approach.

Petrosjan and Zaccour (2003) propose a time-consistent Shapley value to share the cost of an IEA among the n countries. As in the γ-core approach, the sustainability of the grand coalition is supported by the fact that a deviation along the cooperative state trajectory at any date before reaching the planning horizon will lead all players to switch to their noncooperative emissions in the subgame starting at that date. As the Shapley value is an imputation, it is not rational for any player to deviate.

To wrap up, the (typically) small coalition outcome of a noncooperative game approach to the formation of an IEA is entirely driven by the internal and external stability concepts. The IEA inclusive of all countries obtained in a cooperative game approach is driven by the view that any deviation will destroy completely the agreement. Diamantoudi and Sartzetakis (2006) argue that farsightedness constitutes a vision between the two. Indeed, when a country considers deviating, it should not assume optimistically (myopically) that the coalition will remain intact, nor pessimistically (they call it phobically) that the coalition will completely collapse.

Contributions of the present paper. A first originality of our approach is that it combines both cooperative and noncooperative games thinking to design an IEA, and not either of the two modes of play as it is done in the literature. Indeed, we use both the internal and external stability conditions and we define a sharing mechanism of coalition collective outcome among its members.

The second original feature is that our coalition formation process is sequential and visible, that is, at each step, we determine endogenously which country has an interest in joining or leaving the coalition formed at the preceding step. As we will see, this defines a Markov process of coalition formation, and any absorbing state of the Markov chain is a stable IEA. We show that absorbing classes must be absorbing states or periodic classes.

The third original feature concerns the sharing of benefits among participants in the IEA, once an absorbing state is reached. All classical cooperative game solution, e.g., the Shapley value, the core, and the nucleolus, assume that the grand coalition will form, which is far from being the case in the context of an IEA. Instead of using such solutions, we propose here to implement the Markovian process value introduced in Faigle and Grabisch (2012, 2013), which perfectly fits our situation and, interestingly, is a generalization of the Shapley value. Indeed, supposing that the process of formation of a coalition obeys a Markov chain, Faigle and Grabisch (2012) proposed the notion of scenario-value, which gives the marginal contribution of every player along the scenario. The process-value is the expected value of the scenario-values, taken over all possible scenarios with the probability of their occurrence. (A scenario is any sequence of coalitions generated by the Markov chain starting from some fixed coalition, e.g., the empty coalition, and reaching an absorbing state.) Therefore, it does not matter whether the final coalition (the IEA) is the grand coalition or not, and all possibilities of reaching the final coalition are taken into account in the computation. We derive some general properties of the Markovian process value applied to the formation of IEAs and show its flexibility through a detailed example. We note that the Markovian process value is axiomatized, which provides a normative foundation to the method of sharing benefits we are proposing.

Finally, our approach integrates the idea that the players are farsighted, which we implement by considering only those scenarios leading to a scenario-value that is an imputation, i.e., any country knows that entering this scenario will lead to a payoff at least equal to what it can get by acting alone.

The rest of the paper is organized as follows: In Section 2, we recall the Markovian process value and introduce in Section 3 the environmental game model. In Section 4, we discuss the IEA formation process and provide a detailed example in Section 5. We briefly conclude in Section 6.

2 The Markovian process value[1]

In this section, we introduce a value for TU-games suitable for Markovian coalition formation processes, which encompasses the classical Shapley value. We refer the reader to Faigle and Grabisch (2012, 2013) for a full exposition.

We consider a random process of coalition formation among players from set N, supposing that the process is Markovian and homogeneous, i.e., its transition matrix does not depend on time. Specifically, the set of states is the set of all possible coalitions 2^N, and for any $S, T \in 2^N$, the transition from S to T, denoted by $S \to T$, occurs with probability $p_{S,T}$ and does not depend on the history. We denote by $P = [p_{S,T}]$ the transition matrix, which is row-stochastic.

Fixing a finite horizon τ, a *scenario* $\mathcal{S}$ of length $t \leqslant \tau$ is a sequence of coalitions $S_0 S_1 \cdots S_t$ with $S_i \in 2^N$, $i = 0, \ldots, t$, which can be produced by the Markov chain, i.e., $p_{S_i,S_{i+1}} > 0$ for every $i = 0, \ldots, t-1$. The probability of its occurrence is simply

$$\mathbb{P}(\mathcal{S}) = \mathbb{P}(S_0) \prod_{i=0}^{t-1} p_{S_i,S_{i+1}}, \tag{1}$$

where $\mathbb{P}(S_0)$ is the probability of starting from coalition S_0. Note that there may be repetitions of coalitions in the scenario, S_t is not necessarily the grand coalition N, and S_0 is not necessarily the empty set. Also, we take the convention that if S is a terminal state of the Markov chain (i.e., $p_{S,S} = 1$), then any scenario containing S terminates at S, deleting the repetitions $SS \cdots$.

Denote by $S \Delta T := (S \cup T) \setminus (S \cap T)$ the symmetric difference of S and T. We call *simple scenario* any scenario $\mathcal{S} = S_0 S_1 \cdots S_t$ such that $|S_i \Delta S_{i+1}| \leqslant 1$ for $i = 0, \ldots, t$, i.e., in a transition at most one player is *active*, which means either entering the coalition S_{i+1} or leaving the coalition S_i. Accordingly, we call *simple* a coalition formation process producing only simple scenarios.

[1]This section draws heavily on Faigle and Grabisch (2012, 2013).

Let $\mathcal{G}(N)$ be the set of TU-games on N. Given a simple Markov coalition process with transition matrix P, denote by $\mathfrak{S}(P,\tau,S_0)$ the set of all (simple) scenarios produced by the Markov process of length at most τ and starting at S_0. For any simple scenario $\mathcal{S} = S_0S_1\cdots S_t \in \mathfrak{S}(P,\tau,S_0)$, we define the *scenario-value* $\phi^{\mathcal{S}} : \mathcal{G}(N) \to \mathbb{R}^N$ by

$$\phi_i^{\mathcal{S}}(v) = \sum_{k\,:\,S_k\Delta S_{k+1}\ni i} (v(S_{k+1}) - v(S_k)), \quad i \in N. \tag{2}$$

(For the general case with nonsimple scenarios, see Faigle and Grabisch (2012, 2013).) Then, the *process-value* $\phi : \mathcal{G}(N) \to \mathbb{R}^N$ for τ, S_0 is defined by

$$\phi_i(v) = \sum_{\mathcal{S}\in\mathfrak{S}(P,\tau,S_0)} \mathbb{P}(\mathcal{S})\phi_i^{\mathcal{S}}(v), \quad i \in N. \tag{3}$$

Note that since all scenarios are beginning with S_0, we assume $\mathbb{P}(S_0) = 1$. Also, S_0 must be chosen such that $p_{S_0,T} > 0$ at least for one coalition $T \neq \varnothing$ such that $|S_0\Delta T| = 1$. A natural choice is to choose S_0 such that $p_{T,S_0} = 0$ for all $T \neq \varnothing$, i.e., S_0 is a "source" in graph-theoretic terms. One may also take several starting points S_0 with some probability and consider the expected value.

We show now that the classical Shapley value is a particular case of the process-value. Consider the Markov process defined by the transition matrix

$$p_{S,T} = \begin{cases} \dfrac{1}{n-|S|}, & \text{if } S \subseteq T, |T \setminus S| = 1, \\ 1, & \text{if } S = T = N, \\ 0, & \text{otherwise.} \end{cases}$$

It is easy to see that there are $n!$ scenarios of length n produced by this process starting from $\varnothing$ and ending at N, with equal probability, which corresponds to the maximal chains in the Boolean lattice $(2^N, \subseteq)$. Therefore, the process-value coincides with the Shapley value (Faigle and Grabisch, 2012).

3 The environmental game model

Consider a set $N = \{1, \ldots, n\}$ of countries (players) contemplating the formation of an international environmental agreement (IEA). Each country i produces a quantity of goods. Assuming a monotone relationship between production and the level of carbon emission e_i, we can express the revenue of country i as a function of e_i. Let $e = (e_1, \ldots, e_n)$. Following Van der Ploeg and de Zeeuw (1992), we assume that the payoff function of country i has the form

$$\pi_i(e) = B_i(e_i) - D_i\Big(\sum_{j\in N} e_j\Big), \tag{4}$$

where B_i is the benefit function from economic activity induced by e_i, and D_i is the damage function. We assume that $B_i(e_i)$ is concave increasing and $D_i(\cdot)$ is convex increasing. Each country i aims at maximizing its payoff π_i by choosing the level of emissions e_i. As the damage function depends on the total emissions, each player's payoff depends on other players' decisions.

Suppose that the formation of an IEA can be modeled as a cooperative game. Consequently, we need first to determine the optimal collective outcome that is achieved by maximizing the sum of all countries' payoffs, and second to allocate this outcome to the players by using some solution concept of cooperative games. Instead of using one of the classical solution concepts, e.g., the Shapley value, the core, or the nucleolus, we propose to use a solution concept taking into account the process itself of coalition formation, which is the Markovian process value, described in Section 2.

For this second step, we must first compute the characteristic function (CF) values of all possible coalitions. Recall that the CF $v(\cdot)$ is defined by

$$v : 2^N \to \mathbb{R}, \qquad v(\varnothing) = 0.$$

We adopt here the γ-CF due to Chander and Tulkens (1997). In this setup, the γ-CF, denoted $v^\gamma(S)$, where $S \subseteq N$ is a coalition, is defined as the partial (Nash) equilibrium outcome of the noncooperative game between the members of coalition S, acting as a single player, and non-members acting individually. Formally, for any nonempty coalition $S \subseteq N$, we have

$$\begin{aligned} v^\gamma(S) &:= \sum_{i \in S} \pi_i(e_S^\gamma, \{e_j^\gamma\}_{j \in N \setminus S}) \qquad (5)\\ e_S^\gamma &:= \arg\max_{e_S} \sum_{i \in S} \pi_i(e_S, \{e_j^\gamma\}_{j \in N \setminus S})\\ e_j^\gamma &:= \arg\max_{e_j} \pi_j(e_S^\gamma, e_j, \{e_i^\gamma\}_{i \in N \setminus \{S \cup j\}}), \text{ for all } j \in N \setminus S, \end{aligned}$$

where $e_S = \{e_i\}_{i \in S}$, that is, the vector of emissions of the players belonging to coalition S. For $S = \varnothing$, as this means that the coalition formation process has not yet started, we set by convention $\pi_i(e_\varnothing^\gamma, \{e_j^\gamma\}_{j \in N}) = 0$ for all $i \in N$. This implies $v(\varnothing) = 0$, as expected.

From now on, we drop the superscript γ from v, and for simplicity write $e^S := (e_S^\gamma, \{e_j^\gamma\}_{j \in N \setminus S})$.

4 The IEA formation process

Consider that a coalition S of countries has formed. Any country i then faces either of the two questions: If $i \in S$, then is it payoff-improving to leave it? If $i \notin S$, then is it in its best interest to join S? To answer these questions, we consider the following quantities:

$$\delta_i^-(S) = \pi_i(e^S) - \pi_i(e^{S \setminus \{i\}}), \quad i \in S, \qquad (6)$$

$$\delta_i^+(S) = \pi_i(e^{S \cup \{i\}}) - \pi_i(e^S), \quad i \notin S. \qquad (7)$$

Player $i \in S$ has an incentive to leave S if $\delta_i^-(S) < 0$, and player $i \in N \setminus S$ is better off joining S if $\delta_i^+(S) > 0$. Note that due to our convention,

$$\delta_i^-(\{i\}) = \pi_i(e^{\{i\}}), \quad \delta_i^+(\varnothing) = \pi_i(e^{\{i\}}), \quad i \in N. \qquad (8)$$

We call *state* the current coalition. The quantities in (6)-(7) govern the process of coalition formation, starting from the empty coalition, as follows: for every state S,

- If for some $i \in S$, $\delta_i^-(S) < 0$, then i leaves S with probability proportional to $|\delta_i^-(S)|$:

$$p_{S, S \setminus \{i\}} = \frac{|\delta_i^-(S)|}{\sum_{\substack{j \in N \setminus S \\ \delta_j^+(S) > 0}} \delta_j^+(S) + \sum_{\substack{j \in S \\ \delta_j^-(S) < 0}} |\delta_j^-(S)|}. \qquad (9)$$

- If for some $i \notin S$, $\delta_i^+(S) > 0$, then i enters S with probability proportional to $\delta_i^+(S)$:

$$p_{S, S \cup \{i\}} = \frac{\delta_i^+(S)}{\sum_{\substack{j \in N \setminus S \\ \delta_j^+(S) > 0}} \delta_j^+(S) + \sum_{\substack{j \in S \\ \delta_j^-(S) < 0}} |\delta_j^-(S)|}. \qquad (10)$$

- Otherwise, i.e., for all $i \in S$, $\delta_i^-(S) \geqslant 0$, and for all $i \notin S$, $\delta_i^+(S) \leqslant 0$, the process remains in state S:

$$p_{S,S} = 1. \qquad (11)$$

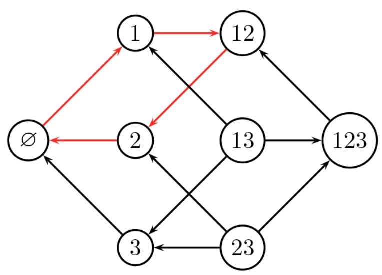
Equations (9)–(11) define a Markov chain on the set of states 2^N, with transition probabilities $p_{S,T}$ from a state S to a state T, where T is either $S \cup \{i\}$ or $S \setminus \{i\}$ for some country i. In order to avoid a disconnected transition graph, we suppose throughout this section that the payoff function is *decisive*, meaning that for every state S, $\delta_i^+(S) > 0$ or $\delta_i^-(S) < 0$ for at least one i.

Knowing the transition matrix $P = [p_{S,T}]$ permits to deduce all possible scenarios of coalition formation that can happen, and to compute the probability of their occurrence. Observe that by construction all scenarios are simple, and therefore, the scenario-values and the process-value can be computed according to (2) and (3).

We list hereunder some properties of the transition graph G obtained from the transition matrix P.

Lemma 1.

1. Except absorbing states, no state can have a loop, i.e., a transition to itself.
2. Given two states S and T, there is a transition from S to T or from T to S iff $|S\Delta T| = 1$. In addition, if there is a transition from S to T, there cannot be a transition from T to S. In other words, there is no cycle of size 2 in G.
3. There is no cycle of odd length, except loops (length $= 1$).
4. Cycles of length 4 which are absorbing classes exist, for any number of countries (see example below with 3 countries).



Proof.

1. This is by construction of the transition matrix (formulas (9) to (11)).
2. The first assertion comes from the fact that the payoff function is supposed to be decisive. The second assertion is true because $S\Delta T$ is a single country i, and a transition from S to T means that for country i, its payoff is better under the state S than in the state T.
3. Suppose a cycle of odd length exists, say $S_1 \to S_2 \to \cdots \to S_{2k} = S_1$, with $k > 0$. Denote by s the cardinality of S_1 and suppose it is an odd number. As all transitions involve exactly one country, the cardinality of S_2 is even, and therefore the cardinality of S_3 is odd again. It follows that eventually the cardinality of S_{2k} is even, which is impossible. The other case works the same.
4. Consider the cycle $\varnothing \to 1 \to 12 \to 2 \to \varnothing$. It obeys the properties given in 1), and hence is a possible transition graph. Now, to make it an absorbing class, it suffices that for 1, all transitions with $1i$, $i \neq 2$, are in the direction $1i \to 1$, for 12, all transitions with $12i$, $i \neq 1, 2$, are in the direction $12i \to 12$, and similarly for 2 and $\varnothing$. □

By the second property above, one sees that the transition graph of the Markov process is obtained from the Hasse diagram of the Boolean lattice $(2^N, \subseteq)$, where each link is replaced by an arrow.

We recall that an absorbing class is *aperiodic* if the greatest common divisor of the length of all cycles in the class is 1; otherwise, the class is said to be *periodic*, with the period being the greatest common divisor. The following corollary is a direct consequence of the above Lemma, and describes which types of absorbing class we can have.

Corollary 1. Consider the Markov process induced by some decisive payoff functions $\pi_1, \ldots, \pi_n$ as described. Then, the absorbing classes are either singletons (absorbing states) or periodic classes.

Proof. Consider an absorbing class $\mathcal{C}$. If $|\mathcal{C}| = 1$, then it reduces to an absorbing state (examples below will show that this situation is possible). Suppose now that $|\mathcal{C}| > 1$. As shown in Lemma 1 (Item 3), there is no cycle of odd length, except loops. Therefore, the greatest common divisor of the length of all cycles cannot be 1 and equals at least 2. Consequently, all classes not reduced to a single state are periodic. □

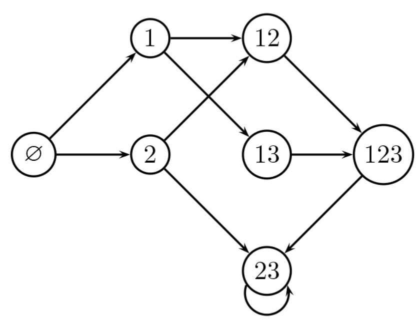
4.1 Myopic and farsighted behavior

A country i is *myopic* if it decides to enter or leave a coalition only considering (6) and (7). If all countries are myopic, an assumption that has been largely adopted in the literature, the process of coalition formation will exactly follow the Markov process described above.

A country i is said to be *farsighted till horizon* τ if it engages in a scenario $\mathcal{S} \in \mathfrak{S}(P, \tau, \varnothing)$ only if after τ steps, the scenario-value for country i satisfies $\phi_i^{\mathcal{S}}(v) \geqslant v(\{i\})$. Such a scenario is called *feasible for country* i. A scenario is *feasible* if it is feasible for every country. Observe that to be farsighted at horizon $\tau = 1$ is equivalent to be myopic, because in our context the first transition is always of the form $\varnothing \to \{i\}$. Then, limiting the scenario to the first step yields $\phi_i^{\mathcal{S}}(v) = v(\{i\}) \geqslant v(\{i\})$, which is always satisfied.

Remark 1. Feasibility is similar to individual rationality in cooperative game theory with a nuance. In a classical cooperative game, it is the sharing of the grand coalition's payoff that must be individually rational, i.e., each player gets at least $v(\{i\})$. Here, it is the sharing of the (not necessarily grand) coalition's payoff that must be individually rational.

Example 1. Let us give an illustrative example, considering $n = 3$, and the transition graph given below.



There are four scenarios, which are:

$$
\begin{aligned}
\mathcal{S}_1 :&\quad \varnothing \to 1 \to 12 \to 123 \to 23\\
\mathcal{S}_2 :&\quad \varnothing \to 1 \to 13 \to 123 \to 23\\
\mathcal{S}_3 :&\quad \varnothing \to 2 \to 12 \to 123 \to 23\\
\mathcal{S}_4 :&\quad \varnothing \to 2 \to 23
\end{aligned}
$$

Assuming the horizon is $\tau = 4$ and using formula (2) to calculate the scenario-values, the conditions for feasibility of the scenarios are:

- Scenario $\mathcal{S}_1$:

$$
\begin{aligned}
\phi_1^{\mathcal{S}_1} &= v(1) + v(23) - v(123) \geqslant v(1) \Leftrightarrow v(23) \geqslant v(123),\\
\phi_2^{\mathcal{S}_1} &= v(12) - v(1) \geqslant v(2) \Leftrightarrow v(12) \geqslant v(1) + v(2),\\
\phi_3^{\mathcal{S}_1} &= v(123) - v(12) \geqslant v(3) \Leftrightarrow v(123) \geqslant v(12) + v(3).
\end{aligned}
$$

- Scenario $\mathcal{S}_2$:

$$
\phi_1^{\mathcal{S}_2} = v(1) + v(23) - v(123) \geqslant v(1) \Leftrightarrow v(23) \geqslant v(123),
$$

$$\phi_2^{\mathcal{S}_2} = v(123) - v(13) \geqslant v(2) \Leftrightarrow v(123) \geqslant v(13) + v(2),$$
$$\phi_3^{\mathcal{S}_2} = v(13) - v(1) \geqslant v(3) \Leftrightarrow v(13) \geqslant v(1) + v(3).$$

- Scenario $\mathcal{S}_3$:

$$\phi_1^{\mathcal{S}_3} = v(12) - v(2) + v(23) - v(123) \geqslant v(1),$$
$$\phi_2^{\mathcal{S}_3} = v(2) \geqslant v(2) \text{ (always true)},$$
$$\phi_3^{\mathcal{S}_3} = v(123) - v(12) \geqslant v(3) \Leftrightarrow v(123) \geqslant v(12) + v(3).$$

- Scenario $\mathcal{S}_4$:

$$\phi_1^{\mathcal{S}_4} = 0 \text{ (country 1 is inactive)},$$
$$\phi_2^{\mathcal{S}_4} = v(2) \geqslant v(2) \text{ (always true)},$$
$$\phi_3^{\mathcal{S}_4} = v(23) - v(2) \geqslant v(3) \Leftrightarrow v(23) \geqslant v(2) + v(3).$$

Observe that if v is superadditive, all scenarios are feasible for countries 2 and 3. Feasibility for country 1 involves more constraints, not incompatible with superadditivity, unless v is nonnegative. Indeed, $v(123) \geqslant v(1) + v(23) \geqslant v(23)$, making scenario $\mathcal{S}_1$ infeasible for country 1.

Now, if the horizon is limited to $\tau = 2$, all conditions are either trivially satisfied or of the type $v(ij) \geqslant v(i) + v(j)$.

The next proposition gives some conditions about feasibility.

Proposition 1. Suppose that v is superadditive. The following holds:

1. Suppose in addition that the Markov process has all absorbing classes which are singletons, say $S_1, \ldots, S_k$. Take a country $i \in N \setminus (S_1 \cup \cdots \cup S_k)$ (i.e., not in an absorbing class) and suppose that $v(\{i\}) > 0$. Then any scenario beginning with $\varnothing \to \{i\}$ is infeasible for i for some horizon τ.
2. Consider a scenario terminating at S and $i \in S$. If i has never left the scenario, then the scenario is feasible for i.

Proof.

1. Consider such an $i \in N$. As i belongs to none of the absorbing classes, it must at some time τ go out the current coalition, say S:

$$\mathcal{S} = \varnothing \to \underbrace{\{i\} \to \cdots \to S}_{i \text{ present}} \to S \setminus \{i\} \to \cdots$$

The scenario-value on horizon τ is then

$$\phi_i^{\mathcal{S}}(v) = v(\{i\}) - 0 + v(S \setminus \{i\}) - v(S) \leqslant 0 < v(\{i\})$$

by superadditivity of v.

2. Let us consider a scenario $\mathcal{S}$ terminating at S, and consider $i \in S$ such that i enters at some point, say T, and never leaves. We obtain

$$\phi_i^{\mathcal{S}} = v(T \cup \{i\}) - v(T) \geqslant v(\{i\})$$

by superadditivity of v. □

Remark 2. 1. The first result must be taken with care, as it is possible that i enters again the scenario later: $T \to T \cup \{i\}$. Even if it has to leave again, say from set T', the sign of the process-value is not necessarily negative:

$$\phi_i^{\mathcal{S}}(v) = v(\{i\}) + v(S \setminus \{i\}) - v(S) + v(T \cup \{i\}) - v(T) + v(T' \setminus \{i\}) - v(T').$$

Hence, everything depends on the horizon.

2. A similar remark holds for the second result. Consider $i \in S$, entering and leaving at some points, for example:

$$\cdots \to T \to T \cup \{i\} \to \cdots \to T' \to T' \setminus \{i\} \to \cdots \to S \setminus \{i\} \to S.$$

   Then

$$\phi_i^{\mathcal{S}} = \underbrace{v(T \cup \{i\}) - v(T)}_{\geqslant v(\{i\})} + \underbrace{v(T' \setminus \{i\}) - v(T')}_{\leqslant -v(\{i\})} + \underbrace{v(S) - v(S \setminus \{i\})}_{\geqslant v(\{i\})}.$$

   Nothing can be said in general.
3. The foregoing results give some insight in the following question: When does the scenario-value yield an imputation? The question concerns only the countries who are active in the given scenario, as one may consider that the other countries i should simply receive their individual payoff $\pi_i(e^S)$, where S is the final state of the scenario. While there is no general answer to this question, we can say that every country entering the scenario and not leaving it will receive a payoff at least equal to its individual payoff.

5 An example of formation of an IEA

We consider the set of n countries $N = \{1, \ldots, n\}$, where k countries are vulnerable to pollution (these countries form the set K), whereas the rest $n - k$ countries are not (set $\bar{K}$), where $K \cup \bar{K} = N$. One interpretation is that vulnerable players are developed countries whose citizens are highly sensitive to environmental issues, whereas invulnerable players are the developing countries having to deal with economic and social pressing issues while the environment is not a priority. Another interpretation is that invulnerable countries face a very small environmental damage, which we set equal to zero to simplify the analysis. The idea of dividing the set of countries into vulnerable and non-vulnerable players has been initiated in Smala Fanokoa et al. (2011), and later developed in Masoudi and Zaccour (2013), and Su and Parilina (2023, 2024).

Referring to the general framework introduced in Section 3, the benefit and damage functions (see (4)) are as follows:

$$B_i(e_i) = \alpha_i e_i - \frac{1}{2}e_i^2,\ i \in N,$$
$$D\,(E) = \begin{cases} \frac{1}{2}\beta E^2, & \text{for } i \in K, \\ 0, & \text{for } i \in \bar{K}, \end{cases}$$

where α_i and β are positive parameters, and $E = \sum_{i \in N} e_i$ is the total pollution.

Consider the scenario where coalition $S \subseteq N$, $|S| = s$, is formed in the game, with ℓ vulnerable players, $|S \cap K| = \ell$, and $s - \ell$ invulnerable players, $|S \cap \bar{K}| = s - \ell$. The players in coalition S maximize their joint payoff, while each non-member maximizes its own payoff.

Remark 3. The (usual) assumption in the literature is that all countries are vulnerable to pollution. This model is nested in ours. Indeed, it suffices to set $k = n$ and $l = s$ to recover the case with only vulnerable countries. Further, if symmetry is required, then one sets $\alpha_i = \alpha$ for all $i \in N$.

Assuming that the game is played noncooperatively, the following proposition gives the Nash equilibrium strategies. We use the notation $\alpha_S = \sum_{i \in S} \alpha_i$.

Proposition 2. When coalition $S \neq \varnothing$ is formed, the players' Nash equilibrium strategies are given by

$$e_i^S = \begin{cases} \alpha_i - \frac{\ell\beta}{1+\beta(s\ell+k-\ell)}\alpha_N, & \text{if } i \in S \text{ (any player in coalition } S\text{)}, \\ \alpha_i - \frac{\beta}{1+\beta(s\ell+k-\ell)}\alpha_N, & \text{if } i \in (N \setminus S) \cap K \text{ (vulnerable player outside } S\text{)}, \\ \alpha_i, & \text{if } i \in (N \setminus S) \cap \bar{K} \text{ (invulnerable player outside } S\text{)}. \end{cases} \quad (12)$$

If S is formed, the total equilibrium emissions are given by

$$E^S = \frac{1}{1+\beta(s\ell+k-\ell)}\alpha_N, \tag{13}$$

and the players' payoffs by

$$\pi_i^S = \begin{cases} \frac{\alpha_i^2}{2} - \frac{(\ell^2\beta^2+\beta)\alpha_N^2}{2(1+\beta(s\ell+k-\ell))^2}, & \text{if } i \in S\cap K \text{ (vulnerable player in } S),\\ \frac{\alpha_i^2}{2} - \frac{\ell^2\beta^2\alpha_N^2}{2(1+\beta(s\ell+k-\ell))^2}, & \text{if } i \in S\cap \bar{K} \text{ (invulnerable player in } S),\\ \frac{\alpha_i^2}{2} - \frac{(\beta^2+\beta)\alpha_N^2}{2(1+\beta(s\ell+k-\ell))^2}, & \text{if } i \in (N\setminus S)\cap K \text{ (vulnerable player outside } S),\\ \frac{\alpha_i^2}{2}, & \text{if } i \in (N\setminus S)\cap \bar{K} \text{ (invulnerable player outside } S). \end{cases} \tag{14}$$

Proof. The payoff of coalition S is

$$\pi_S(e_S, (e_j : j \in N\setminus S)) = \sum_{i\in S}\left(\alpha_i e_i - \frac{e_i^2}{2}\right) - \frac{1}{2}\ell\beta E^2,$$

while the payoff of any individual player $j \in N\setminus S$ is

$$\pi_j(e_S, \{e_i\}_{i\in N\setminus S}) = \begin{cases} \alpha_j e_j - \frac{e_j^2}{2} - \frac{1}{2}\beta E^2, & \text{if } i \in (N\setminus S)\cap K \text{ (vulnerable player outside } S),\\ \alpha_j e_j - \frac{e_j^2}{2}, & \text{if } i \in (N\setminus S)\cap \bar{K} \text{ (invulnerable player outside } S). \end{cases}$$

The first-order conditions of the optimization problems of the coalition S and all individual players when identifying the Nash equilibrium result in the system of equations:

$$\begin{cases} \alpha_i - e_i - \ell\beta E = 0, & \text{if } i \in S \text{ (any player player from } S),\\ \alpha_i - e_i - \beta E = 0, & \text{if } i \in (N\setminus S)\cap K \text{ (vulnerable player outside } S),\\ \alpha_i - e_i = 0, & \text{if } i \in (N\setminus S)\cap \bar{K} \text{ (invulnerable player outside } S). \end{cases}$$

Solving this system of linear equations, we obtain the strategies given by (12). After verifying the second-order conditions, we obtain that this profile of strategies admits the Nash equilibrium in the scenario when coalition S is formed and all other players behave as singletons. □

We can make several observations from Proposition 2:

1. The total emission E^S is decreasing in the number of vulnerable countries ℓ in coalition S, the total number of vulnerable countries k, and in the size s of coalition S.
2. The largest emissions e_i^S are made by invulnerable players outside S, and the smallest emissions are by any player from coalition S, independently if this player is invulnerable or not. The emissions of the vulnerable player outside S are in between.
3. Comparing the payoffs, we conclude that for both types of players, being a singleton (outside coalition S) is always beneficial compared to being a member of coalition S when S is fixed. However, this is not true if we consider whether a particular player joins or leaves coalition S, because in the latter case the size of the coalition changes after such transitions.

Corollary 2. In the cooperative scenario where coalition N forms, the players' emissions are given by

$$e_i^c = \alpha_i - \frac{\beta k}{1+\beta kn}\alpha_N, \text{ for any } i \in N, \tag{15}$$

and the total emissions by

$$E^c = \frac{1}{1+\beta kn}\alpha_N. \tag{16}$$

The total payoff of the grand coalition N is given by

$$\pi_N^c = \sum_{i\in N}\frac{\alpha_i^2}{2} - \frac{\beta k}{1+\beta kn}\alpha_N^2. \tag{17}$$

Proof. This immediately follows from Proposition 2 by substituting $s = n$, $\ell = k$ into Equations (12), (13), and (14). ☐

Corollary 3. In the fully noncooperative scenario, i.e., when all players from N behave as singletons, the players' Nash equilibrium strategies are as follows:

$$e_i^{nc} = \begin{cases} \alpha_i - \frac{\beta}{1+\beta k}\alpha_N, & \text{if } i \in K \text{ (vulnerable player)}, \\ \alpha_i, & \text{if } i \in \bar{K} \text{ (invulnerable player)}. \end{cases} \tag{18}$$

The total emissions are given by

$$E^{nc} = \frac{1}{1+\beta k}\alpha_N, \tag{19}$$

and player i's payoff by

$$\pi_i^{nc} = \begin{cases} \frac{\alpha_i^2}{2} - \frac{(\beta^2+\beta)\alpha_N^2}{2(1+\beta k)^2}, & \text{if } i \in K \text{ (vulnerable player)}, \\ \frac{\alpha_i^2}{2}, & \text{if } i \in \bar{K} \text{ (invulnerable player)}. \end{cases} \tag{20}$$

Proof. This follows directly from Proposition 2 by substituting $s = 1$, $\ell = 1$ for a vulnerable player and $s = 1$, $\ell = 0$ for an invulnerable player into the Equations (12) and (14). Then we can find the total emissions by summarising the equilibrium individual emissions over the set of players N. ☐

As it has been repeatedly shown in the literature (see Jørgensen et al. (2010) for a survey), total emissions are lower under (full) cooperation than under a noncooperative mode of play.

We examine when players (vulnerable and invulnerable) have incentives to join or leave coalition S. To do this, we compute $\delta_i^-(S)$ for any player $i \in S$ and $\delta_i^+(S)$ for any player $i \notin S$, depending on whether player i is vulnerable, $i \in K$, or not, $i \in \bar{K}$:

1. *Invulnerable player i in coalition S, $i \in S \cap \bar{K}$, $S \neq \{i\}$.* Taking into account the expression of player i's payoff given in Proposition 2, by formula (6) we obtain:

$$\delta_i^-(S) = -\frac{\ell^2\beta^2\alpha_N^2}{2(1+\beta(s\ell+k-\ell))^2} < 0. \tag{21}$$

2. *Invulnerable player outside coalition S, $i \in (N \setminus S) \cap \bar{K}$, $S \neq \varnothing, S \subseteq N \setminus \{i\}$.* By formula (7) we find that

$$\delta_i^+(S) = -\frac{\ell^2\beta^2\alpha_N^2}{2(1+\beta((s+1)\ell+k-\ell))^2} < 0. \tag{22}$$

3. *Vulnerable player i in coalition S, $i \in S \cap K$, $S \neq \{i\}$.* Taking into account the vulnerable player i's payoff defined in Proposition 2, by formula (6) we obtain:

$$\delta_i^-(S) = \left(\frac{(\beta^2+\beta)}{2(1+\beta((s-1)(\ell-1)+k-\ell+1))^2} - \frac{(\ell^2\beta^2+\beta)}{2(1+\beta(s\ell+k-\ell))^2}\right)\alpha_N^2, \tag{23}$$

which can be of any sign depending on parameters of the game.

4. *Vulnerable player outside coalition S, $i \in (N \setminus S) \cap K$, $S \neq \varnothing, S \subseteq N \setminus \{i\}$.* By formula (7) we find that

$$\delta_i^+(S) = \left(\frac{(\beta^2+\beta)}{2(1+\beta(s\ell+k-\ell))^2} - \frac{((\ell+1)^2\beta^2+\beta)}{2(1+\beta((s+1)(\ell+1)+k-\ell-1))^2}\right)\alpha_N^2, \tag{24}$$

which again can be of any sign depending on parameters of the game.

5. *Invulnerable player i joining $\varnothing$ or leaving $\{i\}$.* According to our convention and formulas (6), (7),

$$\delta_i^+(\varnothing) = \delta_i^-(\{i\}) = \frac{\alpha_i^2}{2} > 0.$$

6. *Vulnerable player i joining $\varnothing$ or leaving $\{i\}$.* According to our convention and formulas (6), (7),

$$\delta_i^+(\varnothing) = \delta_i^-(\{i\}) = \frac{\alpha_i^2}{2} - \frac{(\beta^2+\beta)\alpha_N^2}{2(1+\beta k)^2},$$

which can be of any sign depending on parameters of the game.

Based on the above cases of player's possible transitions, we can draw the following conclusions:

1. If an invulnerable country belongs to S, $|S| > 1$, it always has an incentive to leave S.
2. If an invulnerable country does not belong to S, $S \neq \varnothing$, it never has an incentive to join S.
3. The signs of $\delta_i^-(S)$ and $\delta_i^+(S)$ are independent of any values α_j, $j \in N$, except for $\delta_i^+(\varnothing)$ and for $\delta_i^-(\{i\})$.
4. If a vulnerable country i belongs to $S \neq \{i\}$, it has an incentive to leave S if

$$\begin{aligned}
&\beta^2(k^2 - 2k\ell - 3\ell^2 - 4k\ell^2 - k^2\ell^2 + 8\ell^3 + 4k\ell^3 - 4\ell^4 + 2k\ell s + 2\ell^2 s + 2k\ell^2 s \\
&- 8\ell^3 s - 2k\ell^3 s + 4\ell^4 s + 2\ell^3 s^2 - \ell^4 s^2) \\
&+ \beta(-4 - 2k + 6\ell + 2k\ell - 7\ell^2 - 2k\ell^2 + 4\ell^3 + 4s + 2ks - 6\ell s + 4\ell^2 s - 2\ell^3 s - s^2 + 2\ell s^2) \\
&+ (-3 + 2\ell - \ell^2 + 2s) < 0. \qquad (25)
\end{aligned}$$

As this expression does not depend on α_j, $j \neq i$, the vulnerable players are symmetric when deciding on leaving S.

5. If a vulnerable country does not belong to $S \neq \varnothing$, it has an incentive to join S if

$$\begin{aligned}
&\beta^2(2k\ell - 2k^2\ell - \ell^2 + 4k\ell^2 - k^2\ell^2 - 2\ell^3 + 2k\ell^3 - \ell^4 + 2ks + 2\ell^2 s - 4k\ell^2 s \\
&+ 4\ell^3 s - 2k\ell^3 s + 2\ell^4 s + s^2 + 2\ell s^2 - 2\ell^3 s^2 - \ell^4 s^2) \\
&+ \beta(2\ell - 2k\ell + 3\ell^2 - 2k\ell^2 + 2\ell^3 + 2s + 2ks - 2\ell^2 s - 2\ell^3 s + s^2 + 2\ell s^2) \\
&+ (2s - \ell^2) > 0. \qquad (26)
\end{aligned}$$

As this expression does not depend on α_j, $j \neq i$, the vulnerable players are symmetric when deciding on joining S.

6. At the origin of the scenario ($S_0 = \varnothing$), any invulnerable player can join $\varnothing$ (i.e., can start the coalition formation process). A vulnerable player i starts the coalition formation process iff

$$\alpha_i^2(1+\beta k) > \beta(1+\beta)\alpha_N^2.$$

7. After the first transition ($|S| = 1$), only a vulnerable player will join. A vulnerable player has an incentive to join coalition S consisting of another vulnerable player iff condition (26) holds true for $s = \ell = 1$, which is equivalent to the inequality

$$\beta^2(4 + 4k - 3k^2) + \beta(8 - 2k) + 1 > 0.$$

Any vulnerable player belonging to coalition S containing two vulnerable players has no incentive to leave the coalition iff

$$\beta^2(4 + 4k - 3k^2) + \beta(8 - 2k) + 1 < 0.$$

For $k = 2$ this inequality is equivalent to $\beta > -\frac{1}{4}$, and it is always satisfied.
The third vulnerable player has an incentive to join iff

$$(16 - 16k) + \beta(-48k - 16k^2) > 0 \Leftrightarrow \beta < -\frac{k-1}{k(k+3)},$$

which is never satisfied.
The fourth vulnerable player has an incentive to join iff

$$-6 + \beta(-120 - 36k) + \beta^2(-864 - 336k - 30k^2) > 0,$$

which is never satisfied.

8. As a result, we obtain that the absorbing state may be only a coalition consisting of two vulnerable players but never more, and it also depends on β and k ($\beta > 0$ for $k = 2$; $\beta \in (0, (1 + 2\sqrt{3})/11)$ for $k = 3$; $\beta \in (0, 1/2\sqrt{7})$ for $k = 4$; $\beta \in (0, (-1 + 2\sqrt{13})/51)$ for $k = 5$, etc.)
9. If the game v is superadditive, by combining the above results with Proposition 1 (Item 1), we find that any scenario starting with $\varnothing \to \{i\}$, where i is invulnerable, is infeasible.

5.1 Example

We apply the above results to an example with four countries $N = \{1, 2, 3, 4\}$, where countries 1, 2 and 3 are vulnerable to pollution, whereas country 4 is not. To go further in the analysis, we choose the following parameter values:

$$\alpha_1 = 6, \quad \alpha_2 = 5, \quad \alpha_3 = 4, \quad \alpha_4 = 3, \quad \beta = 0.05.$$

Table 1 reports the results of emissions, for all possible coalitions given in Proposition 2.

Table 1: Strategies (level of emissions)

Coalition	e_1	e_2	e_3	e_4	E
$\{1\}$	5.217	4.217	3.217	3	15.652
$\{2\}$	5.217	4.217	3.217	3	15.652
$\{3\}$	5.217	4.217	3.217	3	15.652
$\{4\}$	5.217	4.217	3.217	3	15.652
$\{1,2\}$	4.560	3.560	3.280	3	14.400
$\{1,3\}$	4.560	4.280	2.560	3	14.400
$\{1,4\}$	5.250	4.250	3.250	2.250	15.000
$\{2,3\}$	5.280	3.560	2.560	3	14.400
$\{2,4\}$	5.250	4.250	3.250	2.250	15.000
$\{3,4\}$	5.250	4.250	3.250	2.250	15.000
$\{1,2,3\}$	4.138	3.138	2.138	3	12.414
$\{1,2,4\}$	4.667	3.667	3.333	1.667	13.333
$\{1,3,4\}$	4.667	4.333	2.667	1.667	13.333
$\{2,3,4\}$	5.333	3.667	2.667	1.667	13.333
$\{1,2,3,4\}$	4.3125	3.3125	2.3125	1.3125	11.250

In this example, we have two asymmetries, one related to the environmental damage cost and the other in the different values of α. A higher value of α means that player has a more efficient economy. Table 2 gives the corresponding payoffs and the cooperative game v.

Table 2: Payoffs

Coalition S	$v(S)$	Players' Payoffs			
		$\pi_1(e^S)$	$\pi_2(e^S)$	$\pi_3(e^S)$	$\pi_4(e^S)$
$\{1\}$	11.569	11.569	6.069	1.569	4.500
$\{2\}$	6.069	11.569	6.069	1.569	4.500
$\{3\}$	1.569	11.569	6.069	1.569	4.500
$\{4\}$	4.500	11.569	6.069	1.569	4.500
$\{1,2\}$	18.058	11.779	6.279	2.557	4.500
$\{1,3\}$	13.558	11.779	7.057	1.779	4.500
$\{1,4\}$	16.313	12.094	6.594	2.094	4.219
$\{2,3\}$	8.058	12.557	6.279	1.779	4.500
$\{2,4\}$	10.813	12.094	6.594	2.094	4.219
$\{3,4\}$	6.313	12.094	6.594	2.094	4.219
$\{1,2,3\}$	21.741	12.414	6.914	2.414	4.500
$\{1,2,4\}$	23.444	12.667	7.167	3.333	3.611
$\{1,3,4\}$	18.944	12.667	7.833	2.667	3.611
$\{2,3,4\}$	13.444	13.333	7.167	2.667	3.611
$\{1,2,3,4\}$	27.813	13.412	7.912	3.412	3.076

Remark 4. We can easily notice that the characteristic function with values given in Table 2 is superadditive. However, this is not a general result. For example, if $\beta = 0.44$ and we keep all other parameters the same, then the game is not superadditive. We observe that $v(\{1\}) = -1.07$, $v(\{2\}) = -6.57$, and $v(\{1,2\}) = -7.92$. Since $v(\{1\}) + v(\{2\}) > v(\{1,2\})$, for vulnerable countries 1 and 2 it is not profitable to form a coalition.[2]

We proceed state by state, starting with the empty coalition. We omit the details of computation, solely based on (6) and (7), and in Figure 1 represent the transition graph (absorbing states are in boldface), together with all probabilities of transition in Table 3. We recall that these probabilities are proportional to $\delta_i^+(S)$ or $|\delta_i^-(S)|$.

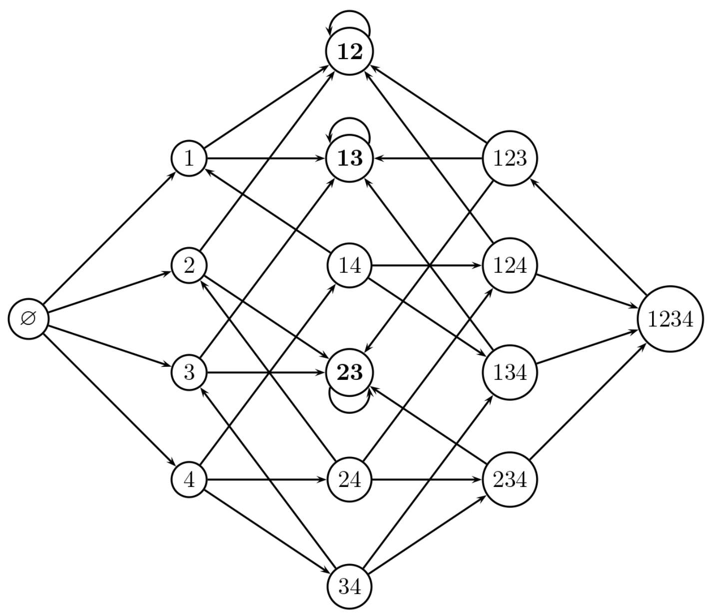


Figure 1: Transition graph

Table 3: Transition probabilities

$p_{\varnothing,1}$	0.488	$p_{24,2}$	0.296
$p_{\varnothing,2}$	0.256	$p_{24,124}$	0.352
$p_{\varnothing,3}$	0.066	$p_{24,234}$	0.352
$p_{\varnothing,4}$	0.190	$p_{34,3}$	0.296
$p_{1,12}$	0.500	$p_{34,134}$	0.352
$p_{1,13}$	0.500	$p_{34,234}$	0.352
$p_{2,12}$	0.500	$p_{123,12}$	0.333
$p_{2,23}$	0.500	$p_{123,13}$	0.333
$p_{3,13}$	0.500	$p_{123,23}$	0.333
$p_{3,23}$	0.500	$p_{124,12}$	0.918
$p_{4,14}$	0.333	$p_{124,1234}$	0.082
$p_{4,24}$	0.333	$p_{134,13}$	0.918
$p_{4,34}$	0.333	$p_{134,1234}$	0.082
$p_{14,1}$	0.296	$p_{234,23}$	0.918
$p_{14,124}$	0.352	$p_{234,1234}$	0.082
$p_{14,134}$	0.352	$p_{1234,123}$	1

[2] In this example, the equilibrium emissions are positive for all the players, so we have an interior solution with negative resulting payoffs.

We list now all possible scenarios starting from $\varnothing$ and compute probability of their occurrence by Equation (1) (product of all transition probabilities along the scenario, assuming the probability to start from $\varnothing$ is 1).

- $\mathcal{S}_1$: $\varnothing \to 1 \to 12$, $\mathbb{P}(\mathcal{S}_1) = 0.244$
- $\mathcal{S}_2$: $\varnothing \to 1 \to 13$, $\mathbb{P}(\mathcal{S}_2) = 0.244$
- $\mathcal{S}_3$: $\varnothing \to 2 \to 12$, $\mathbb{P}(\mathcal{S}_3) = 0.128$
- $\mathcal{S}_4$: $\varnothing \to 2 \to 23$, $\mathbb{P}(\mathcal{S}_4) = 0.128$
- $\mathcal{S}_5$: $\varnothing \to 3 \to 13$, $\mathbb{P}(\mathcal{S}_5) = 0.033$
- $\mathcal{S}_6$: $\varnothing \to 3 \to 23$, $\mathbb{P}(\mathcal{S}_6) = 0.033$
- $\mathcal{S}_7$: $\varnothing \to 4 \to 14 \to 124 \to 1234 \to 123 \to 12$, $\mathbb{P}(\mathcal{S}_7) = 0.0006$
- $\mathcal{S}_8$: $\varnothing \to 4 \to 14 \to 124 \to 1234 \to 123 \to 13$, $\mathbb{P}(\mathcal{S}_8) = 0.0006$
- $\mathcal{S}_9$: $\varnothing \to 4 \to 14 \to 124 \to 1234 \to 123 \to 23$, $\mathbb{P}(\mathcal{S}_9) = 0.0006$
- $\mathcal{S}_{10}$: $\varnothing \to 4 \to 14 \to 124 \to 12$, $\mathbb{P}(\mathcal{S}_{10}) = 0.0205$
- $\mathcal{S}_{11}$: $\varnothing \to 4 \to 14 \to 134 \to 13$, $\mathbb{P}(\mathcal{S}_{11}) = 0.0205$
- $\mathcal{S}_{12}$: $\varnothing \to 4 \to 14 \to 134 \to 1234 \to 123 \to 12$, $\mathbb{P}(\mathcal{S}_{12}) = 0.0006$
- $\mathcal{S}_{13}$: $\varnothing \to 4 \to 14 \to 134 \to 1234 \to 123 \to 13$, $\mathbb{P}(\mathcal{S}_{13}) = 0.0006$
- $\mathcal{S}_{14}$: $\varnothing \to 4 \to 14 \to 134 \to 1234 \to 123 \to 23$, $\mathbb{P}(\mathcal{S}_{14}) = 0.0006$
- $\mathcal{S}_{15}$: $\varnothing \to 4 \to 14 \to 1 \to 12$, $\mathbb{P}(\mathcal{S}_{15}) = 0.0094$
- $\mathcal{S}_{16}$: $\varnothing \to 4 \to 14 \to 1 \to 13$, $\mathbb{P}(\mathcal{S}_{16}) = 0.0094$
- $\mathcal{S}_{17}$: $\varnothing \to 4 \to 24 \to 2 \to 12$, $\mathbb{P}(\mathcal{S}_{17}) = 0.0094$
- $\mathcal{S}_{18}$: $\varnothing \to 4 \to 24 \to 2 \to 23$, $\mathbb{P}(\mathcal{S}_{18}) = 0.0094$
- $\mathcal{S}_{19}$: $\varnothing \to 4 \to 24 \to 124 \to 12$, $\mathbb{P}(\mathcal{S}_{19}) = 0.0205$
- $\mathcal{S}_{20}$: $\varnothing \to 4 \to 24 \to 124 \to 1234 \to 12$, $\mathbb{P}(\mathcal{S}_{20}) = 0.0006$
- $\mathcal{S}_{21}$: $\varnothing \to 4 \to 24 \to 124 \to 1234 \to 13$, $\mathbb{P}(\mathcal{S}_{21}) = 0.0006$
- $\mathcal{S}_{22}$: $\varnothing \to 4 \to 24 \to 124 \to 1234 \to 23$, $\mathbb{P}(\mathcal{S}_{22}) = 0.0006$
- $\mathcal{S}_{23}$: $\varnothing \to 4 \to 24 \to 234 \to 23$, $\mathbb{P}(\mathcal{S}_{23}) = 0.0205$
- $\mathcal{S}_{24}$: $\varnothing \to 4 \to 24 \to 234 \to 1234 \to 12$, $\mathbb{P}(\mathcal{S}_{24}) = 0.0006$
- $\mathcal{S}_{25}$: $\varnothing \to 4 \to 24 \to 234 \to 1234 \to 13$, $\mathbb{P}(\mathcal{S}_{25}) = 0.0006$
- $\mathcal{S}_{26}$: $\varnothing \to 4 \to 24 \to 234 \to 1234 \to 23$, $\mathbb{P}(\mathcal{S}_{26}) = 0.0006$
- $\mathcal{S}_{27}$: $\varnothing \to 4 \to 34 \to 3 \to 13$, $\mathbb{P}(\mathcal{S}_{27}) = 0.0094$
- $\mathcal{S}_{28}$: $\varnothing \to 4 \to 34 \to 3 \to 23$, $\mathbb{P}(\mathcal{S}_{28}) = 0.0094$
- $\mathcal{S}_{29}$: $\varnothing \to 4 \to 34 \to 134 \to 13$, $\mathbb{P}(\mathcal{S}_{29}) = 0.0205$
- $\mathcal{S}_{30}$: $\varnothing \to 4 \to 34 \to 134 \to 1234 \to 12$, $\mathbb{P}(\mathcal{S}_{30}) = 0.0006$
- $\mathcal{S}_{31}$: $\varnothing \to 4 \to 34 \to 134 \to 1234 \to 13$, $\mathbb{P}(\mathcal{S}_{31}) = 0.0006$
- $\mathcal{S}_{32}$: $\varnothing \to 4 \to 34 \to 134 \to 1234 \to 23$, $\mathbb{P}(\mathcal{S}_{32}) = 0.0006$
- $\mathcal{S}_{33}$: $\varnothing \to 4 \to 34 \to 234 \to 23$, $\mathbb{P}(\mathcal{S}_{33}) = 0.0205$
- $\mathcal{S}_{34}$: $\varnothing \to 4 \to 34 \to 234 \to 1234 \to 12$, $\mathbb{P}(\mathcal{S}_{34}) = 0.0006$
- $\mathcal{S}_{35}$: $\varnothing \to 4 \to 34 \to 234 \to 1234 \to 13$, $\mathbb{P}(\mathcal{S}_{35}) = 0.0006$
- $\mathcal{S}_{36}$: $\varnothing \to 4 \to 34 \to 234 \to 1234 \to 23$, $\mathbb{P}(\mathcal{S}_{36}) = 0.0006$

Let us assume that countries are farsighted. Observe that the game v is superadditive and that the absorbing classes are singletons (states 12, 13 and 23), as expected from our analysis in Section 5. Then, Proposition 1 (Item 1) can be applied, and we find that all scenarios where country 4 is entering first (scenarios $\mathcal{S}_7$ to $\mathcal{S}_{36}$) are infeasible. Also, from Proposition 1 (Item 2), the 6 remaining scenarios are feasible.

For these 6 scenarios, the scenario-values are as follows:

Scenario $\mathcal{S}_1$: $\phi_1^{\mathcal{S}_1} = v(1) - v(\varnothing) = 11.569$, $\phi_2^{\mathcal{S}_1} = v(12) - v(1) = 6.489$. The two countries 3 and 4 are inactive, hence $\phi_3^{\mathcal{S}_1} = \phi_4^{\mathcal{S}_1} = 0$. However, they receive their individual payoff under the formation of coalition 12, i.e., $\pi_3(e^{12}) = 2.557$ and $\pi_4(e^{12}) = 4.500$, respectively.

Scenario $\mathcal{S}_2$: $\phi_1^{\mathcal{S}_2} = v(1) - v(\varnothing) = 11.569$, $\phi_3^{\mathcal{S}_2} = v(13) - v(1) = 1.989$. Countries 2 and 4 are inactive, and receive $\pi_2(e^{13}) = 7.057$ and $\pi_4(e^{13}) = 4.500$, respectively.

Scenario $\mathcal{S}_3$: $\phi_1^{\mathcal{S}_3} = v(12) - v(2) = 11.989$, $\phi_2^{\mathcal{S}_3} = v(2) - v(\varnothing) = 6.069$. Countries 3 and 4 are inactive, and receive $\pi_3(e^{12}) = 2.557$ and $\pi_4(e^{12}) = 4.500$, respectively.

Scenario $\mathcal{S}_4$: $\phi_2^{\mathcal{S}_4} = v(2) - v(\varnothing) = 6.069$, $\phi_3^{\mathcal{S}_4} = v(23) - v(2) = 1.989$. Countries 1 and 4 are inactive, and receive $\pi_1(e^{23}) = 12.557$ and $\pi_4(e^{23}) = 4.500$, respectively.

Scenario $\mathcal{S}_5$: $\phi_1^{\mathcal{S}_5} = v(13) - v(3) = 11.989$, $\phi_3^{\mathcal{S}_5} = v(3) - v(\varnothing) = 1.569$. Countries 2 and 4 are inactive, and receive $\pi_2(e^{13}) = 7.057$ and $\pi_4(e^{13}) = 4.500$, respectively.

Scenario $\mathcal{S}_6$: $\phi_2^{\mathcal{S}_6} = v(23) - v(3) = 6.489$, $\phi_3^{\mathcal{S}_6} = v(3) - v(\varnothing) = 1.569$. Countries 1 and 4 are inactive, and receive $\pi_1(e^{23}) = 12.557$ and $\pi_4(e^{23}) = 4.500$, respectively.

To compute the process-value, we must consider that scenarios 7 to 36 will not realize, therefore, we have to renormalize the probabilities of scenarios 1 to 6 to obtain

$$\mathbb{P}(\mathcal{S}_1) = \mathbb{P}(\mathcal{S}_2) = 0.301,\ \mathbb{P}(\mathcal{S}_3) = \mathbb{P}(\mathcal{S}_4) = 0.158,\ \mathbb{P}(\mathcal{S}_5) = \mathbb{P}(\mathcal{S}_6) = 0.041,$$

which yields the following values:

$$\phi_1 = 9.3468,\ \phi_2 = 4.1351,\ \phi_3 = 1.0407,\ \phi_4 = 0. \tag{27}$$

Interpretation of the results and discussion: If a scenario is observed, say $\mathcal{S}_1$, then $\phi^{\mathcal{S}_1}$ should be used for the payment. Recall that if a scenario $\mathcal{S}$ terminates at coalition S, the sum of the values $\sum_{i\in N} \phi_i^{\mathcal{S}} = v(S)$. Hence, the payoff is a sharing of $v(S)$, not among players in S, but among *all players that have been active in the scenario*. In our case, for all scenarios $\mathcal{S}_1$ to $\mathcal{S}_6$, this amounts to the same. Recall that the inactive players receive their individual payoff under the formation of coalition S.

Now suppose that no scenario has been observed, but one sees only the final result, i.e., the coalitions that are absorbing states, in our case, $12, 13$, and 23. Then, it makes sense to take the expected value over all scenarios leading to that absorbing state. In our example, taking $S = 12$ as absorbing state, only scenarios $\mathcal{S}_1$ and $\mathcal{S}_3$ lead to 12. The probabilities of their occurrence are 0.244 and 0.128, respectively, so after normalization, this yields 0.656 and 0.344. Taking the expected value yields the following payoff:

$$\phi_1 = 11.713,\ \phi_2 = 6.344,\ \phi_3 = \phi_4 = 0.$$

Countries 3 and 4 do not participate to the scenarios and receive $\pi_3(e^{12})$ and $\pi_4(e^{12})$, respectively.

Now taking $S = 13$, scenarios $\mathcal{S}_2$ and $\mathcal{S}_5$ are involved. After normalization, the probabilities of their occurrence are 0.881 and 0.119, respectively. This yields:

$$\phi_1 = 11.619,\ \phi_3 = 1.939,\ \phi_2 = \phi_4 = 0.$$

Finally, for $S = 23$, scenarios $\mathcal{S}_4$ and $\mathcal{S}_6$ are involved, with normalized probabilities 0.795 and 0.205, respectively. This yields:

$$\phi_2 = 6.155,\ \phi_3 = 1.903,\ \phi_1 = \phi_4 = 0.$$

Taking the process-value does not make so much sense, because this would mean that no final coalition (absorbing state) will be observed, in which case the value does not represent a sharing of something (no property of efficiency for the process-value). Moreover, there are countries active in some scenarios and inactive in other ones, and the obtained payoff (27) is not an imputation.

An important question is indeed whether the obtained payoff vector is an imputation, i.e., $\phi_i \geqslant v(\{i\})$ for every country i. Let us recall the values for $v(\{i\})$:

$$v(\{1\}) = 11.569,\ v(\{2\}) = 6.069,\ v(\{3\}) = 1.569.$$

Comparing these values with the above payoffs for *active* players, we see that in all cases, the obtained payoffs are imputations when restricted to active players. This is because of the definition of a feasible scenario, and as we have taken an average over feasible scenarios, the obtained payoff is an imputation. For the inactive country 4, its individual payoff is always equal to 4.500, which is exactly $v(\{4\})$.

Finally, let us remark that the payoffs obtained by our method radically differ from the Shapley value, and other similar concepts like the egalitarian Shapley value, even if we restrict to the subgame $v_{|S}$ where S is the final coalition. This is because the Shapley value would consider any ordering (scenario to reach S) as possible and equiprobable. Our analysis shows that some scenarios may reveal infeasible, and scenarios have probability of occurrence, reflecting externalities, that is, all the possible transitions that can occur along the coalition formation process.

6 Concluding remarks

In this paper, we apply a Markov process approach to the formation of IEAs, where any absorbing state of the Markov chain is a stable IEA. Our study contributes to several streams of research focusing on IEAs: coalition formation, and both cooperative and noncooperative game approaches. Despite the vast related literature, the study of IEAs remains of high importance.

There exists great potential for applying network theory to IEAs. Currarini et al. (2016) discuss a wide range of application of network economics to environmental problems, since they are typically related to local interactions and network structures. However, contributions applying network theory to IEAs seem still limited. Günther and Hellmann (2017) study the stability of IEAs in a repeated game framework, and characterize necessary and sufficient conditions for stability of an IEA when pollution has global and local effects. Local pollution spillovers are represented by a network, where a link between two countries (e.g., sharing the same border), indicates whether pollution of the countries affects each other. Clearly, incorporating the network structure enriches considerably the analysis of IEAs. In particular, Günther and Hellmann (2017) show that, while stable IEAs exist if the network structure is balanced, they might fail to exist for networks being highly asymmetric with respect to neighbors in the network.

Applying network theory to IEAs is a natural follow-up of the present paper. In particular, it would be interesting to apply a dynamic network formation process introduced in Caulier et al. (2015) to the analysis of IEAs. Caulier et al. (2015) used a similar approach to that of Faigle and Grabisch (2012, 2013), but applied to network instead of coalition processes. The authors investigated the dynamic random network formation processes, where links may appear and disappear at any time, i.e., allowing that the network and the set of active players may change over time. A scenario of the process, i.e., a sequence of networks, is the result of a stochastic process, typically a Markov chain. Following this dynamic network based approach, we could apply the link-based scenario allocation rule introduced and axiomatically characterized in Caulier et al. (2015), to allocate value generated by the IEA dynamic network formation process, i.e., to share benefits among all participants involved in the IEA.

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