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An option-based QML approach for estimating structural models

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Abstract : We develop a maximum-likelihood (ML) approach based on option markets for estimating structural models. We use dynamic programming and finite elements, derive the likelihood function, and, then, simultaneously solve and estimate the setting. We conduct a case study on Ford at the end of 2006, when the carmaker experienced financial difficulties. The stock- and the option-based ML output similar credit-risk parameters. Both align with rating agencies assessments.

Keywords : Structural model; historical estimation; maximum likelihood; option contracts

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1 Introduction

We derive and implement an option-based quasi-maximum likelihood (QML) approach for estimating structural models. We use dynamic programming and finite elements, explicit the likelihood function, and, then, simultaneously solve and estimate the setting. We conduct a case study on Ford Motor Company, reffered to as Ford in the following, at the end of 2006. Our findings show strong consistency between the stock- and the option-based QML approaches for liquid option contracts. This paper builds on the stock-based QML approach of Ben-Abdellatif and al. (2025).

Corporate bonds are typically issued on active primary markets and, then, exchanged on less liquid secondary markets, whence the tremendous need for theoretical benchmarks to use in conjunction with market signals and accounting values. Table 1 reports key statistics of US corporate bonds (issued principal amount, trading volume, and outstanding amounts), as published by the Securities Industry and Financial Markets Association (SIFMA). The row "Trading" refers to the average daily trading volume. The issuance of US corporate bonds is recovering since the low records in 2022–2023 resulting mainly from the Federal Reserve policy of raising interest rates to fight inflation.

Table 1: Principal amount of US corporate bonds (in \$ Billion)

Year	2018	2019	2020	2021	2022	2023	2024
Issued	1,388	1,462	2,376	2,053	1,397	1,504	1,966
Trading	33	36	41	39	40	43	52
Outstanding	8,504	8,861	9,810	10,348	10,430	10,727	11,221

The estimation of the structural model is challenging, as its underlying process, the firm asset value, is not directly observable. This delayed the derivation of statistical estimation methods by a quarter of a century after the seminal construction of Merton (1974). The first attempt went back to Duan (1994), who built a likelihood function in Merton (1974) model given the time series of the firm asset value. Duan (2000) then proposed a correction based on the time series of the underlying stock price, which gave rise to several analytical extensions in fundamental structural models with simple capital structures (Merton 1974, Black and Cox 1976, and Leland 1994). For more realistic structural models, the QML approach of Ben-Abdelattif et al. (2025) can be used. This construction combines dynamic programming and finite elements to simultaneously solve and estimate extended Geske-like (1977) settings. This dynamic program supports only a numerical but not a statistical error, and space but not time discretization. For high-dimensional structural models, on can build on the simulated-ML approach of Bruche (2005). The difference between quasi-ML and simulated-ML is similar to the difference between quasi-explicit calculus and Monte Carlo estimation. The former is more efficient whenever applicable, while the latter is more flexible.

ML methods discussed in the literature usually rely on stock prices (the observations) of the underlying company. Their associated firm asset values (the pseudo observations) are thus inferred from the match between theoretical equity values and their market counterparts. The option-based QML construction replaces market stock prices by market option premiums. We carry out a case study on Ford, and compare the stock- and the option-based QML approaches.

The rest of the paper is organized as follows. Section 2 presents the model and derives the likelihood function. Section 3 is a case study, and Section 4 is a conclusion.

2 Model resolution and estimation

We consider the structural model of Ben-Abdellatif et al. (2025) with the following balance sheet:

$$a + \mathrm{TB}_t(a) - \mathrm{BC}_t(a) = D_t^s(a) + D_t^j(a) + \mathcal{E}_t(a), \qquad \text{for all } t \text{ and } a = A_t, \tag{1}$$

where TB represents tax benefits, BC bankruptcy costs, D^s the senior debt, D^j the junior debt, $D^s + D^j = D$ the debt portfolio, $\mathcal{E}$ the firm's equity, and $a = A_t$ a given level of the firm asset value at time t. The left-hand side of eq. (1), indicated by $\mathrm{TV}_t(a)$, is referred to as the total value of the firm at (t, a). The debt portfolio promises interest and capital payments indicated by $d_n = d_n^s + d_n^j$ with $d_n^s = C_n^s + P_n^s$ and $d_n^j = C_n^j + P_n^j$, where d_n, d_n^s, and d_n^s, are the total, senior, and junior payment due at $t_n \in \mathcal{P} = \{t_0 = 0, \ldots, t_N = T, \ldots, t_{N^D} = T^D\}$. Time $t_N = T$ and time $t_{N^D} = T^D$ represent the present date and the debt maturity date, respectively, while $[0, T]$ represent the estimation time window. The default condition at $t_n \in \mathcal{P}$ is shown to take the form

$$a = A_n < b_n,$$

where $A_n = A_{t_n}$ is the firm asset value and b_n the endogenous default barrier at t_n. It is worth noting that $b_t = 0$, for $t \notin \mathcal{P}$.

We consider an American-style equity option whose payoff function (exercise value) at $t_n \in \{t_0 = 0, \ldots, t_N = T\}$ is

$$\mathcal{V}_n^e(e) = \begin{cases} \max(e - K, 0), & \text{for the call option,} \\ \max(K - e, 0), & \text{for the put option,} \end{cases}$$

where $e = \mathcal{E}_n(a) \geq 0$ and $a = A_n$ are the firm equity and the firm asset value at t_n, respectively, $K > 0$ the option exercise price, $t_0 = 0$ the option inception date, and, for ease of notation, $t_N = T$ the option maturity date. No-arbitrage pricing gives

$$\mathcal{V}_n^h(e) = \begin{cases} \mathbb{E}^*\left[e^{-r(t_{n+1}-t_n)}\mathcal{V}_{n+1}(\mathcal{E}_{n+1}(A_{n+1})) \mid \mathcal{E}_n(a) = e\right], & \text{for } e > 0, \\ 0, & \text{for } e = 0, \end{cases}$$

where $\mathbb{E}^*[. \mid \mathcal{E}_n(a) = e]$ is the conditional expectation sign under a (the) risk-neutral probability measure, $\mathcal{V}_n^h(e)$ the option holding value at (t_n, e) with $\mathcal{E}_n(a) = e$, and r the risk-free rate with the convention that $\mathcal{V}_T^h(e) = 0$, for $e \geq 0$. The option overall value function at t_n is

$$\mathcal{V}_n(e) = \max\left(\mathcal{V}_n^e(e), \mathcal{V}_n^h(e)\right), \qquad \text{for } e \geq 0,$$

which characterizes the optimal exercise decision at (t_n, e) via the condition

$$\mathcal{V}_n^e(e) > \mathcal{V}_n^h(e).$$

For European equity options, set $\mathcal{V}_n^e = 0$, for $t_n < T$.

Since the option value process $\mathcal{V}$ is Markov, its joint density function at $\{t_0 = 0, \ldots, t_N = T\}$, given survival, evaluated at the market option prices $v_0^m, \ldots, v_T^m$, can be written as

$$f_{\mathcal{V},\theta}(v_0^m, \ldots, v_T^m) = \prod_{n=1}^{N} f_{\mathcal{V}_{n+1},\theta}\left(v_n^m \mid v_{n-1}^m\right), \tag{2}$$

where θ is the vector of unknown parameters of A with the convention that $f_{\mathcal{V}_0,\theta}(v_0) = 1$. Since $f_{\mathcal{V}_n,\theta}\left(v_n^m \mid v_{n-1}^m\right)$ is the first derivative with respect to v of the conditional cumulative density function $F_{\mathcal{V}_n,\theta}\left(v \mid v_{n-1}^m\right)$, evaluated at the option price v_n^m, one has

$$F_{\mathcal{V}_n,\theta}\left(v \mid v_{n-1}^m\right) = \mathbb{P}\left(\mathcal{V}_n(\mathcal{E}_n(A_n)) \leq v \mid v_{n-1}^m\right),$$

where $\mathbb{P}$ is the physical probability measure. The condition $\mathcal{V}_{n-1}(e_{n-1}) = v_{n-1}^m$ is equivalent to $e_{n-1} = \mathcal{E}_{n-1}(a_{n-1})$ and $\mathcal{E}_{n-1}^{-1}\left(\mathcal{V}_{n-1}^{-1}\left(v_{n-1}^m\right)\right) = a_{n-1} = A_{n-1}$. This results in

$$F_{\mathcal{V}_n,\theta}\left(v \mid v_{n-1}^m\right) = \begin{cases} F_{A_n,\theta}\left(\mathcal{E}_n^{-1}\left(\mathcal{V}_n^{-1}(v)\right) \mid a_{n-1}\right), & \text{for a call option.} \\ 1 - F_{A_n,\theta}\left(\mathcal{E}_n^{-1}\left(\mathcal{V}_n^{-1}(v)\right) \mid a_{n-1}\right), & \text{for a put option.} \end{cases}$$

Setting $\mathcal{V}_n(e) = v$ and $\mathcal{E}_n(a) = e$ gives

$$f_{\mathcal{V}_n,\theta}\left(v_n^m \mid v_{n-1}^m\right) = \begin{cases} +\frac{f_{A_n,\theta}(a_n|a_{n-1})}{\mathcal{E}_n(a_n)' \times \mathcal{V}_n(e_n)'}, & \text{for a call option.} \\ -\frac{f_{A_n,\theta}(a_n|a_{n-1})}{\mathcal{E}_n(a_n)' \times \mathcal{V}_n(e_n)'}, & \text{for a put option.} \end{cases} \tag{3}$$

where the conditional density function $f_{A_n,\theta}(. \mid A_{n-1} = a_{n-1})$ of A is known in closed form, as is usually assumed in the literature. This is an acceptable assumption for A to play its role of a state process. The lognormal dynamics is an example. The theoretical asset values $a_0, \ldots, a_T$ and equity values $e_0, \ldots, e_T$ can be referred to as the pseudo observations associated to the observations $v_0^m, \ldots, v_T^m$. The equity value e_n need not match the firm market cap e_n^m, for $n = 0, \ldots, N$, as implicitly assumed by the stock-based QML approach. This is the cost to pay to rely on option markets instead of stock markets for estimating structural models.

Likewise, the basic component of the stock-based likelihood function is

$$f_{\mathcal{E}_n,\theta}\left(e_n^m \mid e_{n-1}^m\right) = \frac{f_{A_n,\theta}(a_n \mid a_{n-1})}{\mathcal{E}_n(a_n)'}, \tag{4}$$

given that the firm equity can be seen as a trivial financial derivative on A with $\mathcal{V}_n(e) = e$ and $\mathcal{V}_n(e)' = 1$, for all $e > 0$. For the stock-based QML, $f_{\mathcal{E}_n,\theta}\left(e_n^m \mid e_{n-1}^m\right) \simeq f_{A_n,\theta}(a_n \mid a_{n-1})$ for a high–credit quality firm as $\mathcal{E}_n(a)' \to 1$, when $a \to \infty$. QML considers the pseudo observations as if they were directly observable. In this case, any increase in the firm asset value profits only to equityholders, while bondholders have already attained their full potential. A similar result holds true for call options on high–credit quality firms since $\mathcal{E}_n(a)' \to 1$ and $\mathcal{V}_n(e)' \to 1$, when $a \to \infty$.

We now use dynamic programming (DP) and simultaneously solve and estimate the model as follows:

1. Select a liquid vanilla equity option;
2. Set a value for the unknown vector parameters θ in a mesh of grid points $\mathcal{G}_\theta$;
3. Use DP and finite elements (here piecewise linear interpolations) to evaluate the in– and out–balance sheet securities;
   (a) Compute the endogenous default barriers;
   (b) Characterize the option exercise strategy;
4. Compute the pseudo-observations $e_0, \ldots, e_T$ then $a_0, \ldots, a_T$ consistently with the observations $v_0^m, \ldots, v_T^m$;
5. Exchange $\mathcal{E}_n(a_n)'$ and $\mathcal{V}_n(e_n)'$ for their DP counterparts $\widehat{\mathcal{E}}_n(a_n)'$ and $\widehat{\mathcal{V}}_n(e_n)'$, which is straightforward when DP is coupled with piecewise linear interpolations;
6. Compute the log-likelihood function;
7. Go to step 2 and repeat until the log-likelihood function reaches a maximum.

This dynamic program is flexible enough to accommodate arbitrary underlying Markov processes, debt payment schedules, seniority classes, options embedded in corporate bonds, and options on equity.

The code lines are written in C and compiled under CPP. We use the GSL software library to ease solving and estimating the model. The experiments are run under the lognormal assumption for which the unknown vector of parameters $\theta = (\mu, \sigma)$ is made of a drift parameter μ and a volatility parameter σ of the state process A.

3 Case study: Ford

We use the QML approach and conduct a case study on Ford, which experienced financial difficulties from 2005 to 2007. Option contracts have been widely used to implicitly calibrate structural models. Maglione (2020) and Carr and Wu (2011) recommend at– and out–of–the–money put options.

Following Geske et al. (2016), we use COMPUSTAT to resume the debt payment schedule of a corporate debt portfolio according to the key payment dates (in years)

$$\{1, 2, 3, 4, 5, 7\}.$$

The authors build on Merton (1974) setting and, thus, ignore coupon payments. The databases Capital IQ and EDGAR are used for validation purposes. On this register, we report the lack of information to resume the interest/capital debt payment schedule of a public company by credit class, payment date, seniority class, or nature of debt (public or private), which together represent the core input data for credit-risk analysis. We are confident that providers of accounting and financial databases can disclose more detailed information under confidentiality constraints. The time series of daily stock prices is taken from CRSP, while the time series of daily option prices and their characteristics are taken from OptionMetrics. We use the US Treasury yield curve, as published by the US Department of the Treasury, to compute the risk-free rate of a governmental debt portfolio with the same payment schedule as the corporate debt portfolio to be valued.

Ford faced financial difficulties from 2005 to 2007 mainly due to growing competition in the automotive industry amid the Great Recession. The company was downgraded by rating agencies following a series of significant losses that culminated in a $12.7 billion loss in 2006. We set $t_N = T$ at December 31, 2006, and the estimation window to one year with $N = 250$ observed stock and option prices.

The abbreviations ITM, ATM, and OTM stand for in-the-money, at-the-money, and out-of-the-money options, while YTM, CS, and CC stand for the yield to maturity, the credit spread, and the credit class of the corporate debt portfolio. It is worth noting that YTM and CS can be output by group of debt contracts, e.g. by seniority class. Table 2 reports the stock- and option-based QML estimation of $\theta = (\mu, \sigma)$. We investigate with multiple present dates and options characteristics. We report an OTM put related scenario.

Table 2: Stock- and option-based QML estimation of $\theta = (\mu, \sigma)$

Present date	Stock-based	Option-based		
T		ITM put	ATM put	OTM put
2006 year end	$(5\%, 2\%)$	$(5\%, 4\%)$	$(3\%, 4\%)$	$(-2\%, 5\%)$

As expected, the estimated volatility of Ford log-returns is low over the estimation window, as obtained by the stock- or the option-based QML. The OTM put estimation captures a realistic negative drift parameter, albeit irrelevant to compute YTM and CS.

Table 3: Credit-risk analysis of Ford at 2006 year end (in $ Billion)

Present date T	Accounting	Market	Theoretical values	
2006 year end	values	values	stock-based	option-based
A	278.6	–	408.6	408.8
$+$TB	–	–	2.6	2.5
$-$BC	–	–	0.0	0.0
Total	278.6	–	411.2	411.2
$+D$	280.9	–	397.5	397.3
$+\mathcal{E}$	−2.3	13.6	13.6	13.9
OTM put		0.8	0.0	0.8
YTM			5.8%	5.8%
CS		Bloomberg 5%		
CC		S&P BB−		

It is worth noting that the stock-based QML supposes a match between the theoretical equity value and the firm market capitalization at each observation date but not the option-based QML. Rather, the latter supposes a match between the theoretical option value and the market option price. The stock- and the option-based QML provide mainly similar results. This partial finding contradicts Maglione (2020) and Carr and Wu (2011), but remains consistent with the general no-arbitrage principle. Whether or not option contracts give rise to complementary information with respect to their underlying asset is a challenging issue that merits a further investigation.

4 Conclusion

We develop a quasi-maximum likelhood (QML) approach based on option prices for estimating structural models. We use dynamic programming and finite elements to derive and compute the likelihood function and, then, simultaneously solve and estimate the setting. We conduct a case study on Ford under the lognormal assumption at the end of 2006, when the company faced financial difficulties. The findings are aligned with the signals disclosed by rating agencies at that time.

More interesting, the stock- and the option-based QML approaches output similar estimations. Although this finding is coherent with the no-arbitrage principle, it contradicts the idea that option markets drive valuable information that complement the stock market signals. This issue deserves to be empirically investigated further on large samples of public companies.

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