Les Cahiers du GERAD ISSN: 0711–2440

Pricing and unauthorized channel strategies for a global manufacturer considering import taxes

X. Yu, T. Xiao, G. Zaccour

G–2023–24

July 2023

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2023
– Bibliothèque et Archives Canada, 2023

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2023
– Library and Archives Canada, 2023

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Pricing and unauthorized channel strategies for a global manufacturer considering import taxes

Xiaohui Yu [a, b]

Tiaojun Xiao [a]

Georges Zaccour [b, c]

[a] *Center for Behavioral Decision and Control, School of Management and Engineering, Nanjing University, Nanjing, China, 210093*

[b] *Department of Decision Sciences, HEC Montréal & GERAD, Montréal (Qc), Canada, H3T 2A7*

[c] *Chair in Game Theory and Management, HEC Montréal, Montréal (Qc), Canada, H3T 2A7*

xiaohui.yu@hec.ca
xiaotj@nju.edu.cn
georges.zaccour@gerad.ca

July 2023
Les Cahiers du GERAD
G–2023–24

Abstract : To cover the import taxes, a manufacturer typically charges a higher price in a foreign market than in its domestic market. The price difference can lead to an unauthorized distribution channel, where an agent purchases products from the manufacturer's domestic market and resells them as gray market goods in the manufacturer's foreign market through cross-border e-commerce. Such practice is challenging to manufacturers in many sectors. We develop Stackelberg game models to investigate differentiated-pricing (setting a higher price in the foreign market) and equal-pricing (setting the same price in the domestic and foreign markets) strategies for a manufacturer who is facing an unauthorized channel. We find that the optimal pricing strategy of the manufacturer is influenced by the following critical parameters: consumers' acceptance degree of gray market product, the import tax on the manufacturer's authorized product, and the tax incentives for cross-border e-commerce. When the three parameters are high, an unauthorized channel can benefit the manufacturer, and a differentiated-pricing strategy should be adopted to encourage an unauthorized channel. When the three parameters are somewhat low, an unauthorized channel is harmful to the manufacturer, but a differentiated-pricing strategy should still be adopted to allow an unauthorized channel. However, when the three parameters are very low, it is better for the manufacturer to adopt an equal-pricing strategy to deter a harmful unauthorized channel.

Keywords: Unauthorized channel, pricing strategy, import tax, gray market, game theory

Résumé : Pour couvrir les taxes à l'importation, un fabricant facture généralement un prix plus élevé sur un marché étranger que sur son marché domestique. La différence de prix peut conduire à une importation parallèle par un agent qui achète des produits sur le marché intérieur du fabricant et les revend en tant que produits gris sur le marché étranger du fabricant via un commerce électronique transfrontalier. Une telle pratique est un défi pour les fabricants dans de nombreux secteurs.

Nous développons des modèles de jeu Stackelberg pour étudier les stratégies de tarification différenciée (fixation d'un prix plus élevé sur le marché étranger) et de tarification égale (fixation du même prix sur les marchés nationaux et étrangers) pour un fabricant confronté à l'importation parallèle. Nous obtenons que la stratégie de prix optimale du fabricant est influencée par les paramètres critiques suivants : le degré d'acceptation des produits du marché gris par les consommateurs, la taxe à l'importation sur le produit autorisé par le fabricant et les incitations fiscales pour le commerce électronique transfrontalier. Lorsque les trois paramètres sont élevés, l'importation parallèle peut profiter au fabricant, et une stratégie de prix différenciés devrait être adoptée pour encourager l'importation parallèle. Lorsque les trois paramètres sont assez bas, l'importation parallèle est préjudiciable au fabricant, mais une stratégie de prix différenciés devrait tout de même être adoptée pour permettre l'importation parallèle. Cependant, lorsque les trois paramètres sont très faibles, il est préférable que le fabricant adopte une stratégie d'égalisation des prix pour dissuader les importations parallèles préjudiciables.

Mots clés : Importation parallèle, stratégie de prix, taxe d'importation, marché gris, théorie des jeux

Acknowledgements: This research was supported in part by National Natural Science Foundation of China under Grant (No. 72171108); and China Scholarship Council under Grant (No. 202106190120).

1 Introduction

Parallel importation refers to a business activity in which a genuine product is imported into a country/region and sold as a gray market product without the permission of the intellectual property right-holder.[1] Parallel importation usually results in the encroachment of unauthorized channels on manufacturers' own distribution channels. Owing to efficient logistics networks, thriving cross-border e-commerce, and economic globalization, unauthorized channels have experienced rapid growth in the past few decades, involving various products such as cars, electrical appliances, food, cosmetics, and luxury goods. A.T. Kearney, a consulting company, estimated that global sales of gray products in unauthorized channels reached 1,500 billion dollars in 2016, and is still increasing (Wang et al. (2020)).

An unauthorized distribution channel often occurs when a manufacturer sells the same product in both its domestic market and a foreign market. Due to import tariffs, the manufacturer tends to set a higher selling price in the foreign market. The price difference can incentivize an importer to purchase products in the manufacturer's domestic market (low-price market) and resell them as gray market products in the manufacturer's foreign market (high-price market). For example, American versions of the iPhone have been frequently diverted to inland China. Since Omega needs to pay import taxes in America, the prices of Omega watches in America are higher than in Switzerland. Costco purchased the Swiss versions of Omega watches and resold them in America, leading to a 11-year court battle between Omega and Costco.[2] Similarly, Cettire privately purchases Prada, Burberry and Gucci products in Europe and resells them as gray market products in Australia.[3]

The development of cross-border e-commerce has facilitated the emergence of unauthorized distribution channels. To encourage such commerce, many countries impose lower taxes on products imported through cross-border e-commerce than those imported through other trade channels. For example, in inland China, if a product is imported through cross-border e-commerce and the value of the product is less than 2,000 CNY (approximately 285 dollars), it can enjoy zero tariffs and reduced value-added tax (70 percent of the normal VAT).[4] In South Korea, the government does not impose both tariffs and value-added tax on a product imported through cross-border e-commerce if its value is less than 150 dollars.[5] In New Zealand, the government does not impose tariffs on a product imported through cross-border e-commerce if its value is less than 1,000 NZD (approximately 640 dollars), with goods and services tax (GST) being the same as that in general trade case.[6] Motivated by tax incentives, importers can purchase products in low-price countries and import them to high-price countries through cross-border e-commerce. For example, Netease Koala and Tmall operate their own cross-border e-commerce platforms and parallel import various products from overseas markets into inland China.[7] LOTTE and Cettire also use their own cross-border e-commerce platforms to parallel import products into South Korea and Australia, respectively.[8]

Some manufacturers turn a blind eye to unauthorized channels, while others take actions to curb them. As many countries do not outlaw parallel importation, it is difficult for manufacturers to curb unauthorized channels through legal proceedings. Therefore, they typically use managerial tools to control them, and one practical effective method is equal pricing. British luxury manufacturer Mulberry has announced the launch of a global pricing alignment strategy that will ensure consumers

[1]WTO: https://www.wto.org/english/thewto_e/glossary_e/parallel_imports_e.htm

[2]Costco and Omega: https://www.greenbergglusker.com/publications/omega-drops-gray-market-high-court-case-against-costco/

[3]Cettire:https://www.businessoffashion.com/news/global-markets/australian-luxury-e-commerce-upstart-cettire-posts-strong-sales-growth/

[4]Inland China (tax):https://mag.wcoomd.org/magazine/wco-news-84/cross-border-e-commerce-in-china/

[5]South Korea (tax): https://docs.zonos.com/country-guides/asia/south-korea

[6]New Zealand (tax):https://www.customs.govt.nz/personal/duty-and-gst/faqs/

[7]Netease Koala (platform): https://www.kaola.com/, Tmall (platform):https://pages.tmall.com/wow/jinkou/act/zhiyingchaoshi?from=zebra:offline

[8]LOTTE (platform):https://www.lotteon.com/p/display/main/ellotte?mall_no=2, Cettire(platform):https://www.cettire.com/ca/?utm_source=google&utm_medium=cpc&utm_campaign=&gclid=EAIaIQobChMIy_uToKbn-wIVwOTjBxOfyAfaEAAYASAAEgKFR_D_BwE

pay the same price for its products no matter where they purchase them in the world.[9] Luggage manufacturer Briggs & Riley and car manufacturer NIO have also adopted a global unified pricing strategy.[10] To curb unauthorized channels, Danish Jewelry Pandora has lowered its prices on all products by 15 percent in inland China to align them with the brand's global prices.[11]

Given the above discussion, our objective is to address the following two questions:

1. Is an unauthorized channel beneficial or harmful to a global manufacturer?
2. When should a global manufacturer adopt equal (respectively, differentiated) pricing to block (respectively, allow) an unauthorized channel?

To address the two questions, we develop Stackelberg game models with a manufacturer and an importer as players. The manufacturer sells a product in both its domestic market and in a foreign market incurring import taxes. The manufacturer can adopt equal-pricing strategy (setting the same price in the two markets) or differentiated-pricing strategy (setting a higher price in the foreign market). If the manufacturer implements different prices in the two countries, then the importer can purchase the product in the manufacturer's domestic market and resell them as gray market products in the manufacturer's foreign market through cross-border e-commerce (i.e., the importer can create an unauthorized channel). If the manufacturer sells the product at the same price in both markets, then it leaves no room for the importer to create a profitable unauthorized channel.

Interestingly, we find that an unauthorized distribution channel can benefit the manufacturer when consumers' acceptance degree of gray market product, the import tax on the manufacturer's authorized product, and the import tax incentives for cross-border e-commerce are high. In such cases, differentiated-pricing strategy is the best choice for the manufacturer. Surprisingly, we also find that differentiated pricing could still be the best manufacturer's equilibrium strategy when an unauthorized channel is somewhat harmful to the manufacturer (i.e., the three parameters are somewhat low). However, if the three parameters are very low, then equal pricing must be adopted.

The above results are obtained assuming that (i) the market potential, and (ii) the consumer's willingness to pay (WTP), in both markets are the same. We extend our model by relaxing in turn these assumptions and obtain that our results are robust to having different market scales and vary little when we allow for different willingness to pay in the two markets.

2 Literature review

This research belongs to the literature on unauthorized channels, which is summarized in Table 1.

Empirical studies have examined various factors that affect unauthorized channels, such as consumers' perception of gray market products (Liu et al. (2012)), product availability and price differences (Zhao et al. (2016)), product features (Lee (2006)), and distribution control (Myers (1999)). Analytical models have also been developed to investigate unauthorized channels. In this stream of the literature, a manufacturer implements price discrimination between two markets due to different selling costs, e.g., import taxes, between the two markets (Ahmadi and Yang (2000)), or different market bases (scales or potential) and price sensitivities between the two markets (Ahmadi et al. (2015); Iravani et al. (2016)), or different consumers' willingness to pay between the two markets (Li et al. (2018); Huang et al. (2019); Ding et al. (2019); Zhang et al. (2021); Wang et al. (2023)). Such price discrimination often triggers the emergence of unauthorized channels.

[9] Mulberry: https://www.mulberry.com/plugins/investor_relations/pdf/Global_Pricing_Final.pdf

[10] NIO: https://inf.news/en/auto/b31eafee6e5205290f1d1dda2fab24f1.html, Briggs & Riley: https://www.prnewswire.com/news-releases/briggs--riley-aligns-global-pricing-300288844.html

[11] Jewelry Pandora: https://insideretail.asia/2018/07/24/pandora-china-cuts-prices-to-fight-grey-market-trading/

Table 1: A comparative analysis of research on unauthorized channels.

Articles	Differences between markets	Supply chain structure	Decisions	Impact on manufacturer	Strategy of manufacturer
Retailer unauthorized channel					
Xiao et al. (2011)	Consumers' WTP	Direct-indirect	Prices	Harmful	/
Zhang (2016)	Consumers' WTP	Direct-indirect Indirect-indirect	Prices; rebate	Harmful B or H	Offering rebates to consumers directly
Li et al. (2018)	Consumers' WTP	Direct-indirect	Prices; quantities	Harmful	/
Ding et al. (2019)	Consumers' WTP	Direct-indirect	Prices	Harmful	Punishing retailers who engaged in gray markets
Cao and Zhang (2019)	Consumers' WTP	Indirect-indirect	Prices; quantities; qualities	B or H	Quality differentiation between markets
Zhang et al. (2021)	Consumers' WTP	Direct-indirect	Prices	Harmful	Providing money-back guarantees for authorized products
Third-party unauthorized channel					
Yang et al. (1998)	Market scale; Consumers' WTP	Direct-direct	Prices	/	Pricing strategy
Ahmadi and Yang (2000)	Selling cost; market scale; price sensitivity	Direct-direct	Prices	B or H	/
Xiao et al. (2011)	Consumers' WTP	Direct-direct Direct-indirect	Prices	Harmful B or H	/
Ahmadi et al. (2015)	Market scale; price sensitivity	Direct-direct	Prices; quantities	Harmful	Pricing strategy
Iravani et al. (2016)	Market scale; price and service sensitivities	Direct-direct	Prices; services	Harmful	Providing service to authorized products
Li et al. (2018)	Consumers' WTP	Direct-indirect	Prices; quantities	B or H	/
Huang et al. (2019)	Consumers' WTP	Direct-direct	Prices; quality	Harmful	Staggering the time of product release
Zhang et al. (2021)	Consumers' WTP	Direct-direct	Prices	Harmful	Providing money-back guarantees for authorized products
Huang et al. (2020)	Consumers' WTP	Direct-indirect	Prices	B or H	Staggering the time of product release
This paper	Import tax; market scale; Consumers' WTP	Direct-direct	Prices	B or H	Pricing strategy

Note:
(i) "B or H" refers to "beneficial or harmful".
(ii) Ahmadi and Yang (2000) assume that the manufacturer is a coordinating representative who makes centralized decisions. This is essentially "direct-direct" structure.
(iii) Xiao et al. (2011), Li et al. (2018), and Zhang et al. (2021) study both third-party and retailer unauthorized channels.

Various forms of unauthorized channels have been well researched. In one form, a third-party independent importer purchases products in one market and resells them in another market (Ahmadi and Yang (2000); Ahmadi et al. (2015); Iravani et al. (2016); Huang et al. (2019); Wang et al. (2023)). This study also focuses on third-party unauthorized channel. In a second form, a retailer sells the products in other markets that are not allowed for by the manufacturer (Zhang (2016); Shao et al. (2016); Ding et al. (2019); Cao and Zhang (2020)). Some studies explore both third-party and retailer unauthorized channels (Xiao et al. (2011); Li et al. (2018); Zhang et al. (2021)). While most studies retain a framework with a single manufacturer, some consider a competitive setup (typically two manufacturers) (Shavandi et al. (2015); Li et al. (2016); Taleizadeh et al. (2017); Li et al. (2021)).

Is an unauthorized channel beneficial or harmful to the manufacturer? The answer varies with the model's setting. When a manufacturer sells products directly in two markets with different consumers' willingness to pay (or price sensitivities), a third-party unauthorized channel harms the manufacturer's profit (Xiao et al. (2011); Ahmadi et al. (2015); Iravani et al. (2016); Huang et al. (2019); Zhang et al. (2021)). However, when a manufacturer sells products directly in one market and indirectly in another market, a third-party unauthorized channel may benefit the manufacturer by reducing double marginalization (Xiao et al. (2011); Li et al. (2018); Huang et al. (2020a)). When it comes to retailer unauthorized channel, if a manufacturer sells products directly in one market and through a retailer in another market, the manufacturer will be harmed if the retailer invades the manufacturer's direct sales market (Xiao et al. (2011); Zhang (2016); Li et al. (2018); Ding et al. (2019); Zhang et al. (2021)). However, if a manufacturer sells indirectly in both markets, it may welcome a retailer unauthorized channel (Zhang (2016); Cao and Zhang (2020)). Finally, Ahmadi and Yang (2000) find that when a manufacturer sells products directly in two markets with different selling costs, a third-party unauthorized channel may benefit the manufacturer.

To curb unauthorized channels, researchers have proposed a series of strategies to manufacturers, including providing service to authorized products (Iravani et al. (2016); Taleizadeh et al. (2017)), offering rebates directly to consumers (Zhang (2016)), staggering the time of product release (Huang et al. (2019); Huang et al. (2020a)), offering differentiated quality products to different markets (Cao and Zhang (2020)), using RFID technology to trace products and punishing retailers who create unauthorized channels (Ding et al. (2019); Ding et al. (2021)), and providing money-back guarantees for authorized products (Zhang et al. (2021)).

The role of manufacturer's pricing strategy in deterring unauthorized channels has also been studied. Some researchers argue that a manufacturer will be inevitably forced to adopt the same price across all markets as long as parallel importation is permitted (Richardson (2002); Valletti (2006); Gnecco et al. (2022)). However, other contributions suggest that a manufacturer can still adopt different prices even in the presence of unauthorized channels. The rationale is that gray market products may not be perceived to be of the same quality as authorized products due to differences in packaging or guarantees (Jelovac and Bordoy (2005); Autrey et al. (2014)). Yang et al. (1998) propose three pricing strategies for a manufacturer who is facing an unauthorized channel: differentiated-pricing to allow an unauthorized channel, equal-pricing to eliminate an unauthorized channel, and strategic-pricing, i.e., setting the price difference between markets to be equal to the cost of an unauthorized channel, which renders it unattractive. Ahmadi et al. (2015) further compare these three pricing strategies and find that a manufacturer should use prices to block an unauthorized channel when consumers' perception of gray market product is very high. Strategic-pricing is more valuable than equal-pricing, but equal-pricing is easier to implement and is sometimes a good substitute for strategic-pricing.

In Yang et al. (1998) and Ahmadi et al. (2015), an unauthorized channel is driven by the price gap between markets, which is caused by the difference in consumers' willingness to pay between markets (Yang et al. (1998)), or the differences in price sensitivity and market scale (Ahmadi et al. (2015)). We must note that the difference in import tax between markets is also a crucial driving force behind unauthorized channels. To illustrate, the prices of iPhones in China and India are higher than in America, not because Chinese and Indian consumers have a higher willingness to pay (or lower price sensitivity) than American consumers, but because China and India impose high import taxes on iPhones. Similarly, the prices of Omega watches in America are higher than in Europe because

Omega needs to pay import taxes in America. Before adopting equal-pricing strategy, Mulberry set higher prices in China than in its domestic British market because China imposes high import taxes on luxuries. Although Ahmadi and Yang (2000) do consider the difference in import tax between markets, they do not provide a comparative analysis of equal-pricing and differentiated-pricing. In particular, this paper fills the gap.

This paper makes several contributions to the existing literature. Firstly, to the best of our knowledge, it is the first study that compares equal-pricing and differentiated-pricing strategies of a manufacturer who is facing an unauthorized channel driven by the difference in import tax between markets. Since the driving force of an unauthorized channel in this paper differs from that of Ahmadi et al. (2015), the results are significantly different. For example, Ahmadi et al. (2015) suggest that a manufacturer should adopt equal-pricing or strategic-pricing to block an unauthorized channel when consumers have a high acceptance degree of gray market product. In contrast, our study shows that in such cases, a manufacturer should adopt differentiated-pricing strategy to allow an unauthorized channel. The rationale is that an unauthorized channel driven by the difference in import tax between markets has its unique characteristic: it can expand the total demand of the foreign market when consumers have a high acceptance degree of gray market product. Secondly, our paper reveals various characteristics of an unauthorized channel driven by the difference in import tax between markets, including both its positive and negative effects. Thirdly, our paper provides insights into the optimal pricing strategy of a global manufacturer who is facing this type of unauthorized channel, as well as the effects of critical market parameters, including the import tax on the manufacturer's authorized product, the import tax incentives for cross-border e-commerce, and consumers' acceptance degree of gray market product.

3 The model

Consider a manufacturer (denoted by M) who sells a single product in its own domestic market and a foreign market. Following Ahmadi and Yang (2000), Chen et al. (2022), and Li et al. (2023), we assume that the manufacturer sells the product to the foreign market through general trade, and it needs to pay an import tax (or tariff) $t > 0$ per unit. In the domestic market, the manufacturer does not need to pay import taxes. This is consistent with reality that many manufacturers establish wholly owned subsidiaries in foreign countries to import their products (including clear customs) into these countries through general trade. Following the literature, e.g., Zhang (2016), Shao et al. (2016), Ding et al. (2019), Zhang et al. (2021), we normalize, without loss of generality, the manufacturer's unit production cost to zero.

The manufacturer's selling prices of an authorized product in the domestic market and the foreign market are p_d and p_f, respectively. Typically, the manufacturer sets a higher selling price in the foreign market to cover the import taxes, i.e., $p_f > p_d$. Owing to the price gap, an importer (denoted by I) can purchase products in the manufacturer's domestic market and resell them as gray market products in the manufacturer's foreign market through cross-border e-commerce, paying an import tax $t_g > 0$ per unit. Since many countries levy lower import taxes on the products imported through cross-border e-commerce than those through general trade, we assume $t_g < t$. Let the importer's selling price of a gray market product in the foreign market be p_g, with $p_d < p_g < p_f$. Figure 1 describes the model with an unauthorized channel.

The market potentials (scales or bases) in both markets are equal and normalized to one. We will relax this assumption in Subsection 8.1. The consumers' willingness to pay (WTP) in the foreign and domestic markets are V_f and V_d, respectively. We suppose that consumers are heterogenous and let V_f and V_d be both uniformly distributed over the interval [0,1]. We will consider the situation that consumers in the two markets have different WTP in Subsection 8.2. We assume that consumers' WTP for gray market product is lower because it may not benefit from the high-quality service offered by the manufacturer, or consumers may need to deal with a foreign language on the package or in the system of gray market product. Let δ be consumers' acceptance degree of gray market product, $0 < \delta < 1$.

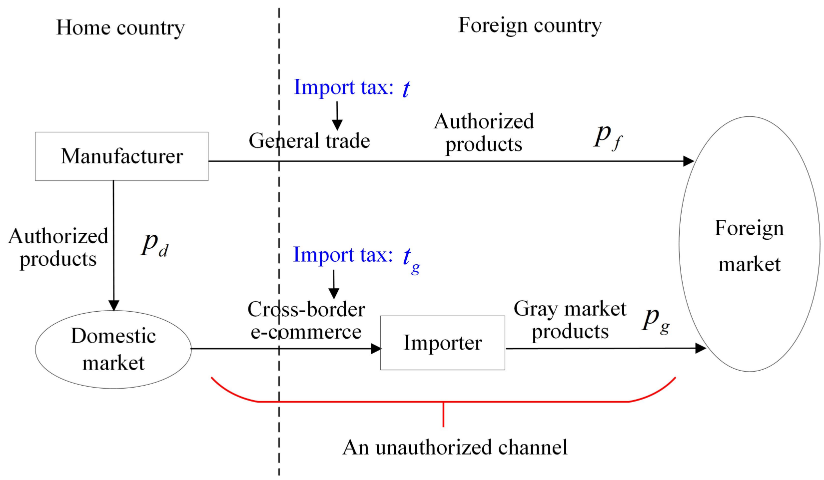


Figure 1: An unauthorized cross-border e-commerce channel.

To simplify the notation, we introduce some threshold parameters. Let $\hat{t}_1 = \frac{(1+4\delta-3\delta^2)t_g+2\delta(1-\delta)^2}{\delta(1+2\delta-\delta^2)}$ and $\hat{t}_2 = \frac{2-2\delta+t_g}{2-\delta}$. If $t \le \hat{t}_1$, then the gap between t and t_g is too small, and the importer will never create an unauthorized channel. If $t \ge \hat{t}_2$, then the manufacturer will not sell authorized products in the foreign market owing to the high import tax. To exclude trivial situations, we assume $\hat{t}_1 < t < \hat{t}_2$. Furthermore, to ensure $\hat{t}_2 > \hat{t}_1$, $\delta > \frac{5-\sqrt{17}}{4}$ and $t_g < \frac{\delta(5\delta-2\delta^2-1)}{1+4\delta-2\delta^2}$ should be satisfied. Similar assumptions are also made in other unauthorized channel models (Zhang (2016); Ding et al. (2019)). Table 2 lists the notations used throughout the paper.

Table 2: Notations.

Notations	Descriptions
Decision variables	
p_d	Manufacturer's selling price in the domestic market (authorized product).
p_f	Manufacturer's selling price in the foreign market (authorized product).
p_g	Importer's selling price in the foreign market (gray market product).
Native parameters	
V_d	Consumers' willingness to pay (WTP) in the domestic market, $V_d \sim U[0,1]$.
V_f	Consumers' willingness to pay (WTP) in the foreign market, $V_f \sim U[0,1]$.
δ	Consumers' acceptance degree of gray market product, $0 < \delta < 1$.
t	Import tax on unit product in general trade (paid by the manufacturer).
t_g	Import tax on unit product in cross-border e-commerce (paid by the importer).
The other parameter	
Δt	Tax incentives for cross-border e-commerce in the foreign market, $\Delta t = t - t_g$.
Functions	
Q_d	Manufacturer's demand in the domestic market (authorized product).
Q_f	Manufacturer's demand in the foreign market (authorized product).
Q_g	Importer's demand in the foreign market (gray market product).
π_M	Manufacturer's profit.
π_I	Importer's profit.

The manufacturer has two pricing strategies: differentiated-pricing (DP) and equal-pricing (EP). Under DP, the manufacturer sets different selling prices in the two markets, and the parallel importer may create an unauthorized channel. Under EP, the manufacturer sets a uniform selling price in the two markets, which implies that the creation of an unauthorized channel is unprofitable.

The time sequence of the game is as follows. The manufacturer first chooses its pricing strategy (EP or DP), and next sets simultaneously p_d and p_f. If the manufacturer implements DP, then the importer can choose to create an unauthorized channel and decide p_g if it does so. We analyze the following three cases:

Benchmark: There is no parallel importer, and the manufacturer adopts DP.

Case D: There is a parallel importer, and the manufacturer adopts DP.

Case E: There is a parallel importer, and the manufacturer adopts EP.

Note that the benchmark and E cases involve an optimization problem (the manufacturer is the only agent involved), whereas case D is a game, and a Stackelberg equilibrium will be sought. Comparing case D with the benchmark, we can investigate whether an unauthorized channel is beneficial or harmful to the manufacturer. Comparing cases D and E, we can explore when the manufacturer should adopt differentiated-pricing to allow an unauthorized channel, and when the manufacturer should adopt equal-pricing to block an unauthorized channel.

4 Equilibrium and optimal outcomes

We use the superscripts B, D, and E to characterize the benchmark, case D and case E, respectively. All proofs are provided in the Appendix.

4.1 Benchmark

In this scenario, there is no importer, and consequently the manufacturer chooses DP. (Note that if EP is optimal, then this solution will emerge as the optimal solution in this case.) In the domestic market, a consumer will purchase an authorized product when $V_d - p_d > 0$. In the foreign market, a consumer will purchase an authorized product when $V_f - p_f > 0$. We can derive the following demand functions: $Q_f^B = 1 - p_f$ and $Q_d^B = 1 - p_d$. The profit of the manufacturer is:

$$\pi_M^B(p_d, p_f) = \underbrace{(1-p_d)p_d}_{\text{Domestic market}} + \underbrace{(1-p_f)(p_f - t)}_{\text{Foreign market}} \tag{1}$$

The following proposition characterizes the manufacturer's optimal solution.

Proposition 1. In the benchmark, the optimal solution is given by:

$$p_f^{B*} = \frac{1+t}{2}, \qquad p_d^{B*} = \frac{1}{2}, \qquad Q_f^{B*} = \frac{1-t}{2}, \qquad Q_d^{B*} = \frac{1}{2}, \qquad \pi_M^{B*} = \frac{t^2 - 2t + 2}{4}.$$

It is easy to see that $p_f^{B*} > p_d^{B*}$ and $Q_f^{B*} < Q_d^{B*}$, that is, EP can never be optimal in this setup. Moreover, p_f^{B*} increases with t, which is a cost for the manufacturer.

4.2 Case D

Here, the manufacturer chooses DP, and there exists an importer who has the opportunity of purchasing products in the domestic market and reselling them in the foreign market. In the foreign market, a consumer will purchase an authorized product from the manufacturer when $V_f - p_f > \delta V_f - p_g$ and will buy a gray market product from the importer when $\delta V_f - p_g > 0$ and $\delta V_f - p_g > V_f - p_f$. In the domestic market, a consumer will buy an authorized product from the manufacturer when $V_d - p_d > 0$. Consequently, we obtain the following demands: $Q_f^D = 1 - \frac{p_f - p_g}{1-\delta}$, $Q_g^D = \frac{p_f - p_g}{1-\delta} - \frac{p_g}{\delta}$, and $Q_d^D = 1 - p_d$. Figure 2 represents the different thresholds.

The profits of the manufacturer and the importer are:

$$\pi_M^D(p_d, p_f, p_g) = \underbrace{(1-p_d)p_d}_{\text{Domestic}} + \underbrace{\left(1 - \frac{p_f - p_g}{1-\delta}\right)(p_f - t)}_{\text{Foreign (authorized product)}} + \underbrace{\left(\frac{p_f - p_g}{1-\delta} - \frac{p_g}{\delta}\right)p_d}_{\text{Foreign (gray product)}} \tag{2}$$

$$\pi_I^D(p_d, p_f, p_g) = \underbrace{\left(\frac{p_f - p_g}{1-\delta} - \frac{p_g}{\delta}\right)(p_g - p_d - t_g)}_{\text{Foreign (gray product)}} \tag{3}$$

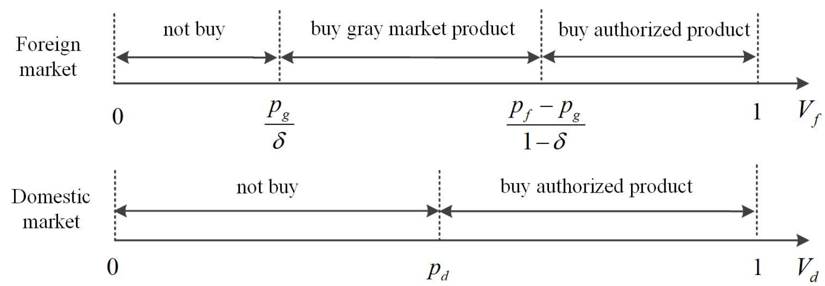


Figure 2: Market segments in case D.

To determine a Stackelberg equilibrium, we first consider the parallel importer's response to the manufacturer's prices decisions. Since $\frac{\partial^2 \pi_I^D(p_d,p_f,p_g)}{\partial p_g^2} = -\frac{2}{(1-\delta)\delta} < 0$, $\pi_I^D(p_d, p_f, p_g)$ is concave in p_g. Solving $\frac{\partial \pi_I^D(p_d,p_f,p_g)}{\partial p_g} = 0$ for p_g, we get the importer's reaction function $p_g^D(p_f, p_d) = \frac{p_d + t_g + p_f\delta}{2}$. As p_g^D is increasing in both manufacturer's prices, we conclude that there is strategic complementarity between the two players' strategies, i.e., when one increases the price, the other also increases the price. Inserting $p_g^D(p_f, p_d)$ into Equation (2), we obtain:

$$\pi_M^D(p_f, p_d) = \frac{2\delta[p_f(1 + p_d + t - \delta) + (p_d - p_d^2 - t)(1-\delta) - p_f^2] - (p_d + t_g + p_f\delta)[p_d - (p_f - t)\delta]}{2(1-\delta)\delta} \tag{4}$$

Expecting the importer's reaction, the manufacturer decides p_f and p_d to maximize $\pi_M^D(p_f, p_d)$. The equilibrium outcome of case D is summarized in Proposition 2.

Proposition 2. Equilibrium in case D.

(i) The manufacturer's selling prices in the two markets (p_d^{D*} and p_f^{D*}), the corresponding demands (Q_d^{D*} and Q_f^{D*}), and the profit (π_M^{D*}) are as follows:

$$p_d^{D*} = \frac{\delta(3-\delta) - t_g}{2(1 + 2\delta - \delta^2)}$$

$$p_f^{D*} = \frac{1 + t + \delta(3 + 2t + t_g) - \delta^2(2 + t)}{2(1 + 2\delta - \delta^2)}$$

$$Q_d^{D*} = \frac{2 + t_g + \delta(1-\delta)}{2(1 + 2\delta - \delta^2)}$$

$$Q_f^{D*} = \frac{2 + t_g + t(\delta - 2) - 2\delta}{4(1-\delta)}$$

$$\pi_M^{D*} = \frac{t_g^2(1+\delta^2) + 2t_g\delta[t(\delta^2 - 2\delta - 1) - 2(\delta-1)^2]}{8\delta(1 + \delta - 3\delta^2 + \delta^3)} + \frac{t^2(2 + 3\delta - 4\delta^2 + \delta^3) - 4t(1 + \delta - 3\delta^2 + \delta^3) + 2(1 + 5\delta - 9\delta^2 + 3\delta^3)}{8(1 + \delta - 3\delta^2 + \delta^3)}$$

(ii) The importer's selling price in the foreign market (p_g^{D*}), the corresponding demand (Q_g^{D*}), and the profit (π_I^{D*}) are as follows:

$$p_g^{D*} = \frac{t\delta(1+2\delta-\delta^2)+2\delta(2+\delta-\delta^2)+t_g(1+4\delta-\delta^2)}{4(1+2\delta-\delta^2)}$$

$$Q_g^{D*} = \frac{t\delta(1+2\delta-\delta^2)-2\delta(1-\delta)^2-t_g(1+4\delta-3\delta^2)}{4\delta(1+\delta-3\delta^2+\delta^3)}$$

$$\pi_I^{D*} = \frac{[t_g(1+4\delta-3\delta^2)+2\delta(1-\delta)^2-t\delta(1+2\delta-\delta^2)]^2}{16\delta(1-\delta)(1+2\delta-\delta^2)^2}$$

By the assumption that $\hat{t}_1 < t < \hat{t}_2$, we clearly see that $t > \hat{t}_1$ ensures $Q_g^{D*} > 0$ and $t < \hat{t}_2$ implies $Q_f^{D*} > 0$. We find that $Q_d^{D*} > 0$ holds because the manufacturer needs not to pay import taxes in the domestic market.

The equilibrium outcomes lead to the following results.

Proposition 3. We have:

(i) $p_f^{D*} > p_f^{B*}$, $p_d^{D*} < p_d^{B*}$.

(ii) $\frac{\partial p_d^{D*}}{\partial t_g} < 0$, $\frac{\partial p_d^{D*}}{\partial \delta} > 0$, $\frac{\partial p_f^{D*}}{\partial t} > 0$, $\frac{\partial p_f^{D*}}{\partial t_g} > 0$, $\frac{\partial p_g^{D*}}{\partial t} > 0$, $\frac{\partial p_g^{D*}}{\partial t_g} > 0$, $\frac{\partial p_g^{D*}}{\partial \delta} > 0$.

(iii) When $t > \frac{2\delta(1-\delta)+t_g(1+4\delta-\delta^2)}{\delta(1+2\delta-\delta^2)}$, then an unauthorized channel increases the total demand in the foreign market ($Q_f^{D*} + Q_g^{D*} > Q_f^{B*}$); otherwise, an unauthorized channel lowers (or not affect) the total demand in the foreign market ($Q_f^{D^*} + Q_g^{D^*} \leq Q_f^{B^*}$).[12] Moreover, we have: $\frac{\partial(Q_f^{D*}+Q_g^{D*}-Q_f^{B*})}{\partial t} > 0$, $\frac{\partial(Q_f^{D*}+Q_g^{D*}-Q_f^{B*})}{\partial \delta} > 0$, and $\frac{\partial(Q_f^{D*}+Q_g^{D*}-Q_f^{B*})}{\partial t_g} < 0$.

(iv) The equilibrium profit of the manufacturer increases with δ ($\frac{\partial \pi_M^{D*}}{\partial \delta} > 0$), and decreases with t and t_g ($\frac{\partial \pi_M^{D*}}{\partial t} < 0$, $\frac{\partial \pi_M^{D*}}{\partial t_g} < 0$); the equilibrium profit of the importer increases with t and δ ($\frac{\partial \pi_I^{D*}}{\partial t} > 0$, $\frac{\partial \pi_I^{D*}}{\partial \delta} > 0$), and decreases with t_g ($\frac{\partial \pi_I^{D*}}{\partial t_g} < 0$).

Comparing case D with the benchmark, we find that when an unauthorized channel occurs, the manufacturer will increase the selling price in the foreign market ($p_f^{D*} > p_f^{B*}$) and lower the selling price in the domestic market ($p_d^{D*} < p_d^{B*}$). Therefore, an unauthorized channel can widen the price difference of the manufacturer's authorized product between the domestic and foreign markets. We also find that an unauthorized channel can increase the total demand in the foreign market when δ and t are high, or t_g is low. The reason for this is that when consumers have a high acceptance of gray market product (δ is high), they can derive a high level of utility from purchasing them. As a result, the sales of gray product are high, which contributes to a high total demand in the foreign market. When the import tax on gray product (t_g) is low, the importer will set a low price for gray product ($\frac{\partial p_g^{D*}}{\partial t_g} > 0$), which also allows consumers to obtain a high utility when purchasing them and therefore expands the sales of gray product. Finally, when the import tax on the manufacturer's authorized product (t) is high, the manufacturer will set a high price in the foreign market to compensate for the import tax burden. This would make the product unaffordable for most consumers. If an unauthorized channel occurs, more consumers will have the opportunity to purchase the product because they can afford gray product sold at a lower price, which leads to an increase in the total demand.

4.3 Case E

In this case, the manufacturer adopts equal pricing to avoid the threat of an unauthorized channel. The demands are given by $Q_d^E = 1 - p_d$ and $Q_f^E = 1 - p_f$. The manufacturer maximizes the following

[12] The condition $t > \frac{2\delta(1-\delta)+t_g(1+4\delta-\delta^2)}{\delta(1+2\delta-\delta^2)}$ in Proposition 3 (iii) can be written into $t_g < \frac{t\delta(1+2\delta-\delta^2)-2\delta(1-\delta)}{1+4\delta-\delta^2}$; or can be written into $\delta > \hat{\delta}$, where $\hat{\delta}$ is the second real root of $f(\delta) = t\delta^3 - (2+2t+t_g)\delta^2 + (2-t+4t_g)\delta + t_g = 0$.

profit:

$$\pi_M^E(p_d, p_f) = \underbrace{(1-p_d)p_d}_{\text{Domestic market}} + \underbrace{(1-p_f)(p_f-t)}_{\text{Foreign market}}$$
$$\text{subject to } p_d = p_f$$

Solving this optimization problem yields the results in the following proposition.

Proposition 4. When the manufacturer implements equal prices in both markets, the optimal solution is as follows:

$$p_d^{E^*} = p_f^{E^*} = \frac{2+t}{4}; \qquad Q_d^{E^*} = Q_f^{E^*} = \frac{2-t}{4}; \qquad \pi_M^{E^*} = \frac{(t-2)^2}{8}$$

Comparing case E with the benchmark, we obtain $p_d^{B*} < p_d^{E*} = p_f^{E*} < p_f^{B*}$, which shows that under EP, the manufacturer chooses a price between p_d^{B*} and p_f^{B*}.

5 The impact of unauthorized channel

Is an unauthorized channel beneficial or harmful to the manufacturer? The answer is in the following proposition.

Proposition 5. Comparing π_M^{B*} and π_M^{D*}, we get:

(i) For $t > \hat{t}_3$, an unauthorized channel benefits the manufacturer ($\pi_M^{B*} < \pi_M^{D*}$, $\frac{\partial(\pi_M^{D^*}-\pi_M^{B^*})}{\partial t} > 0$).

(ii) For $t = \hat{t}_3$, an unauthorized channel has no impact on the manufacturer's profit ($\pi_M^{B*} = \pi_M^{D*}$).

(iii) For $t < \hat{t}_3$, an unauthorized channel harms the manufacturer ($\pi_M^{B*} > \pi_M^{D*}$, $\frac{\partial(\pi_M^{D^*}-\pi_M^{B^*})}{\partial t} < 0$), where $\hat{t}_3 = \frac{t_g}{\delta} + \sqrt{2}(1-\delta+t_g)\sqrt{\frac{1-\delta}{\delta(1+2\delta-\delta^2)}}$.

The interesting result that an unauthorized channel benefits the manufacturer when $t > \hat{t}_3$ can be explained as follows: In the benchmark, the manufacturer's profits in the domestic and foreign markets are $p_d^{B*}Q_d^{B*}$ and $(p_f^{B*}-t)Q_f^{B*}$, respectively. When an unauthorized channel occurs, they become $p_d^{D*}Q_d^{D*}$ and $(p_f^{D*}-t)Q_f^{D*}+p_d^{D*}Q_g^{D*}$. Clearly, $p_d^{D*}Q_d^{D*} < p_d^{B*}Q_d^{B*}$, which indicates a decrease in the manufacturer's domestic-market profit. The cause is that an unauthorized channel forces the manufacturer to reduce p_d ($p_d^{D*} < p_d^{B*}$), which affects the manufacturer's optimal pricing in the domestic market. However, the manufacturer's foreign-market profit may become higher than in the benchmark (i.e., $(p_f^{D*}-t)Q_f^{D*}+p_d^{D*}Q_g^{D*}$ may be larger than $(p_f^{B*}-t)Q_f^{B*}$. This is because an unauthorized channel has both negative and positive effects. Specifically, in the benchmark, consumers in the foreign market purchases authorized products from the manufacturer at price p_f^{B*}, and the manufacturer pays an import tax t when it sells a unit authorized product in the foreign market. When an unauthorized channel occurs, some foreign consumers turn to buying gray products from the importer who acquires these products in the domestic market by paying a relatively lower unit price p_d^{D*} to the manufacturer ($p_d^{D*} < p_f^{B*}$). This is one of the negative effects of an unauthorized channel: it encroaches the manufacturer's income. However, the import tax on unit gray product sold in the foreign market is paid by the importer. The manufacturer sells these products to the importer in the domestic market without paying any import tax. This is an important positive effect of an unauthorized channel: it helps the manufacturer to avoid import taxes. From $p_f^{D*} > p_f^{B*}$, we can see that the manufacturer obtains a higher unit income from consumers who remain loyal to the manufacturer's authorized channel, which is another positive effect of an unauthorized channel. In terms of market demand, Proposition 3 shows that when t is low, an unauthorized channel decreases the total demand in the foreign market ($Q_f^{D*}+Q_g^{D*} < Q_f^{B*}$), which is another negative effect of an unauthorized channel. However, as the value of t increases, this negative effect gradually diminishes and can eventually turn into a positive effect. Once t surpasses a certain threshold, an unauthorized channel can increase the total demand in the foreign market ($Q_f^{D*} + Q_g^{D*} > Q_f^{B*}$).

When $t < \hat{t}_3$, the import tax levied on the manufacturer's authorized product is low, and thus the positive effect of an unauthorized channel (i.e., helping the manufacturer to avoid import taxes) is not obvious and cannot compensate for the manufacturer's losses caused by an unauthorized channel (including the loss of demand in the foreign market, the incomes that has been encroached, and so on). Therefore, an unauthorized channel harms the manufacturer. Conversely, when $t > \hat{t}_3$, the import tax levied on the manufacturer's authorized product is high, an unauthorized channel can help the manufacturer avoid a significant amount of import taxes. Also, since t is high, an unauthorized channel can increase the total demand in the foreign market (or the decrease in demand caused by an unauthorized channel is not significant). Therefore, the positive effects of an unauthorized channel outweigh the negative effects of an unauthorized channel, making it beneficial to the manufacturer.

This result explains why some manufacturers turn a blind eye to unauthorized channels. This result is consistent with Ahmadi and Yang (2000) who consider the difference in selling cost between two markets but do not further compare differentiated-pricing with equal-pricing strategies. This result is different from Ahmadi et al. (2015), Iravani et al. (2016), Huang et al. (2019), and Zhang et al. (2021) who investigate the same supply chain structure shown in Figure 1 but find that an unauthorized channel is always harmful to the manufacturer. The cause is that their models do not consider the difference in import tax between the two markets. In their models, the price differential between the two markets is driven by the different price sensitivity or consumers' willingness to pay in the two markets. Therefore, unauthorized channels in their models cannot help the manufacturer to avoid import taxes.

6 Manufacturer's optimal pricing strategy

Comparing case E with the benchmark confirms the obvious result that the profit under (the unconstrained) DP, which is a price discrimination strategy, is superior to the constrained EP ($\pi_M^{B*} > \pi_M^{E*}$). The interesting question is what is the manufacturer's optimal pricing strategy when the parallel importer is around?

Proposition 6. Comparing π_M^{D*} and π_M^{E*}, we obtain that:

(i) When $t < \hat{t}_4$, EP yields higher profit than DP ($\pi_M^{E*} > \pi_M^{D*}$).
(ii) When $t = \hat{t}_4$, EP and DP lead to the same profit ($\pi_M^{E*} = \pi_M^{D*}$).
(iii) When $t > \hat{t}_4$, DP gives higher profit than EP ($\pi_M^{E*} < \pi_M^{D*}$),

where $\hat{t}_4 = t_g + (1-\delta)\sqrt{\frac{2\delta(1-\delta)+4\delta t_g - t_g^2(1+\delta)}{\delta(1+2\delta-\delta^2)}}$

Proposition 6 suggests that equal pricing may be better than differentiated pricing when the manufacturer faces the threat of an unauthorized channel. To find out the underlying reason for this interesting result, we combine the results in Propositions 5 and 6 in Corollary 1.

Corollary 1. When the manufacturer faces the threat of an unauthorized channel, we have:

(i) If $t \geq \hat{t}_3$, an unauthorized channel benefits the manufacturer (or has no effect on the manufacturer's profit), and the manufacturer chooses DP to encourage it ($\pi_M^{D*} \geq \pi_M^{B*} > \pi_M^{E*}$).
(ii) If $\hat{t}_4 < t < \hat{t}_3$, an unauthorized channel harms the manufacturer's profit, but still allows for it by choosing DP ($\pi_M^{B*} > \pi_M^{D*} > \pi_M^{E*}$).
(iii) If $t \leq \hat{t}_4$, then we have $\pi_M^{B*} > \pi_M^{E*} \geq \pi_M^{D*}$. The manufacturer blocks an unauthorized channel by implementing EP.

Although the manufacturer adopts DP when $t \geq \hat{t}_3$ or when $\hat{t}_4 < t < \hat{t}_3$, the reasons behind this pricing strategy are totally different. When $t \geq \hat{t}_3$, an unauthorized channel benefits (or has no effect on) the manufacturer. Consequently, the manufacturer is better off adopting DP as it implements price discrimination and encourages an unauthorized channel. When $t < \hat{t}_3$, an unauthorized channel is harmful to the manufacturer, and both DP (in case D) and EP (in case E) have disadvantages. Under DP, a harmful unauthorized channel will occur. Adopting EP can block a harmful unauthorized channel, but at the expense of giving up price discrimination. From $\pi_M^{B*} > \pi_M^{E*}$ and $\frac{\partial(\pi_M^{B*} - \pi_M^{E*})}{\partial t} > 0$, we know that as the import tax in the foreign market t increases, the difference between the domestic and foreign markets will increase, and as a result the manufacturer's losses resulting from giving up price discrimination will also increase (i.e., the disadvantage of EP will increase). From $\pi_M^{B*} > \pi_M^{D*}$ and $\frac{\partial(\pi_M^{B*} - \pi_M^{D*})}{\partial t} < 0$ (Proposition 5, when $t < \hat{t}_3$), we see that as the import tax t increases, an unauthorized channel can help the manufacturer save more import taxes, and thus the manufacturer's losses caused by an unauthorized channel will decrease (i.e., the disadvantage of DP will decrease). This explains the existence of the threshold value $\hat{t}_4$. When $t < \hat{t}_4$, the losses from giving up price discrimination are lower than the losses caused by an unauthorized channel ($\pi_M^{E*} > \pi_M^{D*}$), and thus the manufacturer will adopt EP. When $\hat{t}_4 < t < \hat{t}_3$, the losses from giving up price discrimination are higher than the losses caused by an unauthorized channel ($\pi_M^{E*} < \pi_M^{D*}$), and thus the manufacturer will adopt DP.

We provide numerical examples to illustrate Corollary 1 and gain more insights. We define $\Delta t = t - t_g$, which measures the import tax incentives for cross-border e-commerce in the foreign market. Let $t = 0.25$, $\delta = 0.7$, and $\Delta t = 0.2$. Figures 3–5 show how the critical parameters (t, δ, and Δt) affect the manufacturer's profits in the three cases. In Figures 3–5, the condition $\hat{t}_1 < t < \hat{t}_2$ is satisfied.

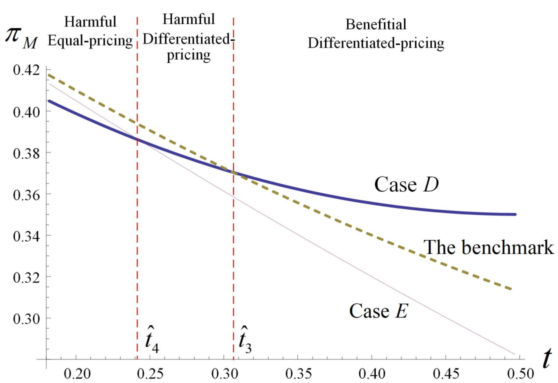


Figure 3: Manufacturer's profit versus t.

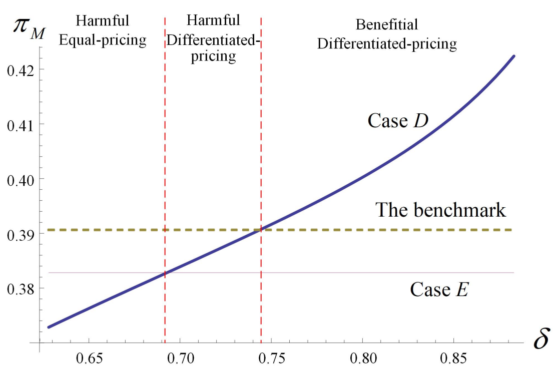


Figure 4: Manufacturer's profit versus δ.

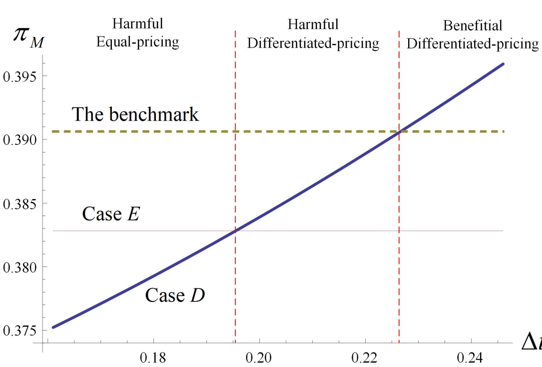


Figure 5: Manufacturer's profit versus Δt.

The line of case E is always below the line of the benchmark, which is consistent with the result that DP is superior to EP when there is no importer. In Figure 3, as the import tax t decreases, the difference between the two markets decreases (the disadvantage of EP decreases), and thus the line of case E gradually approaches the benchmark line. On the other hand, as the import tax t decreases (when $t < \hat{t}_3$), the positive effect of an unauthorized channel that helps the manufacturer save import taxes becomes less significant, and therefore the line of case D gradually moves away from the benchmark line. The line of case E and the line of case D intersects at point $\hat{t}_4$.

Figures 4 and 5 present new results: (i) when δ and Δt are high, an unauthorized channel is beneficial to the manufacturer, and therefore the manufacturer should adopt DP. (ii) when δ and Δt are somewhat low, an unauthorized channel harms the manufacturer, but the manufacturer should still allow it by adopting DP. (iii) when δ and Δt are very low, it is better for the manufacturer to adopt EP to deter a harmful unauthorized channel. These results are counterintuitive. Indeed, when consumers' acceptance degree of gray product (δ) and the tax incentives for cross-border e-commerce (Δt) are high, an unauthorized channel becomes a serious competitor of the manufacturer. However, when one looks at the impact of an unauthorized channel on market demand (typically ignored in the literature), then the results make much sense.

In the three cases, the profits that the manufacturer obtains from consumers in the foreign and domestic markets are: $\underbrace{(p_f^{B*}-t)Q_f^{B*}}_{\text{foreign}}+\underbrace{p_d^{B*}Q_d^{B*}}_{\text{domestic}}$ (in the benchmark), $\underbrace{(p_f^{E*}-t)Q_f^{E*}}_{\text{foreign}}+\underbrace{p_d^{E*}Q_d^{E*}}_{\text{domestic}}$ (in case E), and $\underbrace{(p_f^{D^*}-t)Q_f^{D^*}+p_d^{D*}Q_g^{D*}}_{\text{foreign}}+\underbrace{p_d^{D*}Q_d^{D*}}_{\text{domestic}}$ (in case D). In case E, the rationale for EP is to block an unauthorized channel by sacrificing some profits (i.e., giving up price discrimination), and therefore the profits that the manufacturer obtains from both markets are lower than those in the benchmark ($p_d^{E*}Q_d^{E*} < p_d^{B*}Q_d^{B*}$ and $(p_f^{E*}-t)Q_f^{E*} < (p_f^{B*}-t)Q_f^{B*}$). By adopting DP in case D, the manufacturer allows an unauthorized channel, and thus some consumers in the foreign market turns to buying gray product from the importer. The manufacturer obtains a lower price from consumers who buy gray products ($p_d^{D*} < p_f^{E*} < p_f^{B*}$) but does not need to pay the import tax t for these consumers. In addition, the manufacturer obtains lower profit in the domestic market ($p_d^{D*}Q_d^{D*} < p_d^{B*}Q_d^{B*}$) because an unauthorized channel affects its optimal pricing in the domestic market ($p_d^{D*} < p_d^{B*}$). Moreover, Proposition 3 shows that when δ or Δt are low, an unauthorized channel will decrease the total demand in the foreign market ($Q_f^{D*}+Q_g^{D*} < Q_f^{B*} < Q_f^{E*}$). However, as either δ or Δt increase, $Q_f^{D*}+Q_g^{D*}$ in case D will gradually rise. Note that Q_f^{B*} in the benchmark and Q_f^{E*} in case E are only dependent on t and will not be affected by changes in δ or Δt. Eventually, when δ or Δt exceeds certain thresholds, an unauthorized channel can increase the demand in the foreign market.

When market conditions are very unfavorable for an unauthorized channel (i.e., δ and Δt are very low), an unauthorized channel would result in a very significant decrease in the total demand in the foreign market ($Q_f^{D*}+Q_g^{D*}$ will be very low). Whether we compare case D with the benchmark or with case E, the positive effects of an unauthorized channel (such as helping the manufacturer avoid import taxes and obtain more profits from loyal consumers) cannot outweigh the negative effects of an unauthorized channel, particularly the considerable loss of demand in the foreign market (i.e., $\pi_M^{B*} > \pi_M^{E*} > \pi_M^{D*}$). Therefore, the manufacturer should adopt EP to block an unauthorized channel.

When market conditions are somewhat unfavorable for an unauthorized channel (i.e., δ and Δt are somewhat low), the loss of demand in the foreign market caused by an unauthorized channel is somewhat significant. Therefore, when comparing case D with the benchmark, the positive effects of an unauthorized channel are still not enough to outweigh its negative effects (including the loss of demand and reduced unit income in the foreign market, and the loss of profits in the domestic market). However, when we compare case D with case E, the outcome will be different because the manufacturer has already sacrificed some profits in case E ($\pi_M^{E*} < \pi_M^{B*}$). Consequently, the negative effects of an unauthorized channel are not so high. For example, the difference between $p_d^{D*}$and p_f^{E*} is lower than the difference between p_d^{D*} and p_f^{B*}, which implies the reduced unit income is not that high. Also, the loss of profits in the domestic market is not that high because the manufacturer's profit in the domestic market in case E is lower than that in the benchmark. Therefore, when comparing case D with case E, the negative effects of an unauthorized channel could be outweighed by its positive effects ($\pi_M^{D*} > \pi_M^{E*}$). The manufacturer should still adopt DP to allow an unauthorized channel.

When market conditions are favorable for an unauthorized channel (i.e., δ and Δt are high), an unauthorized channel may increase the total demand in the foreign market (or the decrease in demand caused by an unauthorized channel is not significant). Regardless of whether we compare case D with case E or with the benchmark, the positive effects of an unauthorized channel can outweigh the negative effects (i.e., $\pi_M^{D*} > \pi_M^{B*} > \pi_M^{E*}$). Therefore, the manufacturer will adopt DP to encourage an unauthorized channel.

7 Consumer surplus and social welfare

One common way to measure consumer surplus is by summing up the differences between the maximum price that each consumer is willing to pay and the actual selling price (Han et al. (2017); Zhou et al. (2022)). Social welfare is a weighted sum of consumer surplus and firms' profits (Huang et al. (2020b)). In reality, profits earned by firms from sales in foreign markets often flow back to their home country through remittances or other means, as firms' headquarters and financial management are typically situated in their country of origin. Therefore, we include the manufacturer's profit in the social welfare of the domestic market. Additionally, importers usually bring gray products to their home countries for sale. For instance, Taobao, Costco, Cettire, and Lotte import gray products to China, America, Australia, and South Korea, respectively. Therefore, we include the importer's profit in the social welfare of the foreign market. Following Ross (1984) and Corchón and Torregrosa (2022), we use α to denote the weight of consumer surplus in the social welfare of the foreign market, and $1-\alpha$ to denote the weight of the importer's profit in the social welfare of the foreign market, which are determined by the foreign market's regulator. Similarly, we use β to represent the weight of consumer surplus in the social welfare of the domestic market, and $1-\beta$ to represent the weight of the manufacturer's profit in the social welfare of the domestic market. In Table 3, we list the expressions of consumer surplus and social welfare of the two markets under the three cases. Proposition 7 shows the relationship between consumer surplus across these three cases.

Table 3: Consumer surplus and social welfare.

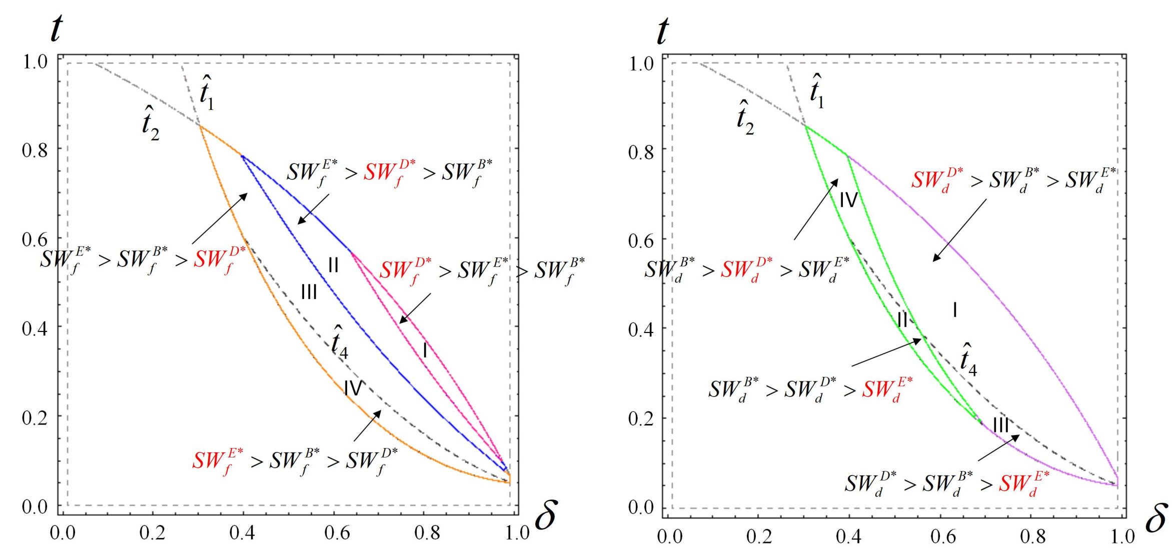
Case	Market	Consumer surplus	Social welfare
Benchmark	Foreign	$CS_f^B = \int_{p_f^{B*}}^1 (v_f - p_f^{B*})\, dv_f$	$SW_f^B = \alpha CS_f^B$
Benchmark	Domestic	$CS_d^B = \int_{p_d^{B*}}^1 (v_d - p_d^{B*})\, dv_d$	$SW_d^B = \beta CS_d^B + (1-\beta)\pi_M^{B*}$
Case E	Foreign	$CS_f^E = \int_{p_f^{E*}}^1 (v_f - p_f^{E*})\, dv_f$	$SW_f^E = \alpha CS_f^E$
Case E	Domestic	$CS_d^E = \int_{p_d^{E*}}^1 (v_d - p_d^{E*})\, dv_d$	$SW_d^E = \beta CS_d^E + (1-\beta)\pi_M^{E*}$
Case D	Foreign	$CS_f^D = \int_{\frac{p_f^{D*}-p_g^{D*}}{1-\delta}}^1 (v_f - p_f^{D*})\, dv_f + \int_{\frac{p_g^{D*}}{\delta}}^{\frac{p_f^{D*}-p_g^{D*}}{1-\delta}} (\delta v_f - p_g^{D*})\, dv_f$	$SW_f^D = \alpha CS_f^D + (1-\alpha)\pi_I^{D*}$
Case D	Domestic	$CS_d^D = \int_{p_d^{D*}}^1 (v_d - p_d^{D*})\, dv_d$	$SW_d^D = \beta CS_d^D + (1-\beta)\pi_M^{D*}$

Proposition 7. The consumer surplus of the domestic market in the three cases satisfies the inequalities: $CS_d^D > CS_d^B > CS_d^E$. The consumer surplus of the foreign market in the three cases satisfies the inequalities: $CS_f^E > CS_f^B > CS_f^D$.

Propositions 1–4 present the relationship between the manufacturer's selling prices in the three cases ($p_d^{D*} < p_d^{B*} < p_d^{E*} = p_f^{E*} < p_f^{B*} < p_f^{D*}$). Specifically, when the parallel importer appears, the manufacturer will increase its selling price in the domestic market (the lower price market) if it adopts EP to block an unauthorized channel or decrease its selling price in the domestic market if it adopts

DP to allow an unauthorized channel ($p_d^{D*} < p_d^{B*} < p_d^{E*}$). It is intuitive that the relationship between consumer surplus is opposite to that of the selling prices ($CS_d^D > CS_d^B > CS_d^E$). In the foreign market (the higher price market), the manufacturer will decrease its selling price if it adopts EP ($p_f^{E*} < p_f^{B*}$). Therefore, we have $CS_f^E > CS_f^B$. If the manufacturer chooses DP to allow an unauthorized channel, although lower-priced gray products appear, consumer surplus of the foreign market is lower than that under the benchmark ($CS_f^D < CS_f^B$) due to the manufacturer raising its price of authorized products ($p_f^{D*} > p_f^{B*}$) and consumers discounting the valuation of gray market products. In fact, providing multiple products to segment the market is essentially a way to capture more consumer surplus.

As the expressions of social welfare are very complex, we conduct a numerical study to illustrate them. Let $t_g = 0.05$ and $\alpha = \beta = 0.2$. Figure 6 presents the comparisons of the social welfare of the foreign and domestic markets in the three cases. In Figure 6(a), the pink region represents the area where $SW_f^D > SW_f^E > SW_f^B$, the blue region represents the area where $SW_f^E > SW_f^D > SW_f^B$, the orange region represents the area where $SW_f^E > SW_f^B > SW_f^D$. In Figure 6(b), the purple region represents the area where $SW_d^D > SW_d^B > SW_d^E$, the light green region represents the area where $SW_d^B > SW_d^D > SW_d^E$. Proposition 6 implies that the manufacturer chooses DP (case D) when $t > \hat{t}_4$; otherwise, it chooses EP (case E). $\hat{t}_4$ is the gray dashed lines in Figure 6. It is important to note that if the manufacturer chooses DP (respectively, EP), then as long as the social welfare in case D (respectively, case E) is higher than the social welfare in the benchmark, a (potential) unauthorized channel can have a positive effect on social welfare. Therefore, we have Result 1.



(a) Foreign market. (b) Domestic market.

Figure 6: Social welfare comparison.

Result 1. For the foreign market (Figure 6(a)), we obtain that:

(i) In regions I, II and IV, a (potential) unauthorized channel increases social welfare.

(ii) In region III, an unauthorized channel decreases social welfare.

For the domestic market (Figure 6(b)), we obtain that:

(i) In region I, an unauthorized channel increases social welfare.

(ii) In regions II, III, and IV, a (potential) unauthorized channel decreases social welfare.

In regions I and II of Figure 6(a), the manufacturer adopts DP to allow an unauthorized channel (case D), which can increase the social welfare of the foreign market ($SW_f^D > SW_f^B$). The reason for this is that in the absence of parallel importer (the benchmark), the manufacturer sells products to the foreign market and repatriates the profits earned back to its home country. This leads to a significant outflow of wealth from foreign consumers. An unauthorized channel enables the importer in foreign market to participate and gain a share of the profits, contributing to increased economic activity and wealth accumulation within the foreign market. This is one of the reasons why many countries incentivize their importers to engage in parallel import trade, as it can get a share of the profits of foreign manufacturers and mitigate the outflow of their country's wealth to other countries.

In region IV of Figure 6(a), the manufacturer adopts EP to block an unauthorized channel (case E), which also increases the social welfare of the foreign market ($SW_f^E > SW_f^B$). The reason for this is that under EP, the manufacturer reduces its selling price in the foreign market, resulting in an increase in social welfare. This is another reason why many countries encourage parallel importation, as the potential threat of unauthorized channels can compel foreign manufacturers to charge lower prices in those countries even if they are subject to import taxes on their products. This ultimately benefits consumers.

8 Extensions

To test the robustness of our results and obtain new findings, we extend the basic model in two directions, denoted by subscripts E_1 and E_2.

8.1 Different market scales

In the basic model, the market potential is normalized to one in both countries. In this subsection, we let the foreign and domestic market potential be $N_f > 0$ and $N_d > 0$, respectively. We introduce the following thresholds: $\hat{t}_{1,E_1} = \frac{(4N_d\delta - 3N_d\delta^2 + N_f)t_g + 2N_d\delta + 2N_d\delta^3 - 4N_d\delta^2}{N_f\delta + 2N_d\delta^2 - N_d\delta^3}$, and $\hat{t}_{2,E_1} = \frac{2+t_g-2\delta}{2-\delta}$. To avoid trivial cases, we assume that $\hat{t}_{1,E_1} < t < \hat{t}_{2,E_1}$. If $t \leq \hat{t}_{1,E_1}$, the importer will never create an unauthorized channel, and if $t \geq \hat{t}_{2,E_1}$, the manufacturer will not sell authorized products in the foreign market. Everything else remains as that in the basic model.

In the benchmark (no importer), the profit of the manufacturer is:

$$\pi_{M,E_1}^B(p_d, p_f) = \underbrace{N_d(1-p_d)p_d}_{\text{Domestic market}} + \underbrace{N_f(1-p_f)(p_f-t)}_{\text{Foreign market}}$$

In case D, the profits of the manufacturer and the importer are:

$$\pi_{M,E_1}^D(p_d, p_f, p_g) = \underbrace{N_d(1-p_d)p_d}_{\text{Domestic}} + \underbrace{N_f\left(1-\frac{p_f-p_g}{1-\delta}\right)(p_f-t)}_{\text{Foreign (authorized product)}} + \underbrace{N_f\left(\frac{p_f-p_g}{1-\delta}-\frac{p_g}{\delta}\right)p_d}_{\text{Foreign (gray product)}} \quad (5)$$

$$\pi_{I,E_1}^D(p_d, p_f, p_g) = \underbrace{N_f\left(\frac{p_f-p_g}{1-\delta}-\frac{p_g}{\delta}\right)(p_g-p_d-t_g)}_{\text{Foreign (gray product)}}$$

To determine the equilibrium, we first derive the importer's reaction function $p_{g,E_1}^D(p_f, p_d) = (p_d + t_g + p_f\delta)/2$, which is the same as that in the basic model. Again, the two players' prices are strategic complements. Inserting $p_{g,E_1}^D(p_f, p_d)$ into Equation (5), we obtain the manufacturer's profit function:

$$\pi_{M,E_1}^D(p_f, p_d) = \frac{2p_d\delta N_d(1-p_d-\delta+p_d\delta) + N_f\{\delta(p_f-t)[t_g+2(1-\delta)-p_f(2-\delta)] + p_d\left[(2p_f-t)\delta-t_g\right]-p_d^2\}}{2(1-\delta)\delta}$$

In case E, the manufacturer maximizes the following profit function:

$$\pi^{E}_{M,E_1}(p_d,p_f) = \underbrace{N_d(1-p_d)p_d}_{\text{Domestic market}} + \underbrace{N_f(1-p_f)(p_f-t)}_{\text{Foreign market}}$$
$$\text{subject to } p_d = p_f$$

The equilibrium and optimal outcomes of the three cases are summarized in Table 4.

Comparing the manufacturer's equilibrium/optimal profits in the three cases, we can derive Proposition 8.

Table 4: Equilibrium and optimal outcomes for asymmetric market scales.

	The benchmark	Case E	Case D
p^*_{f,E_1}	$\frac{1+t}{2}$	$\frac{N_d+N_f+N_f t}{2(N_d+N_f)}$	$\frac{N_f(1+t)+N_d\delta[3+t_g+t(2-\delta)-2\delta]}{2A_1}$
p^*_{d,E_1}	$\frac{1}{2}$	$\frac{N_d+N_f+N_f t}{2(N_d+N_f)}$	$\frac{N_d(2-\delta)\delta+N_f(\delta-t_g)}{2A_1}$
p^*_{g,E_1}	/	/	$\frac{N_f[t_g+\delta(2+t)]+\delta N_d A_2}{4A_1}$
Q^*_{f,E_1}	$\frac{N_f(1-t)}{2}$	$\frac{N_f(N_d+N_f-N_f t)}{2(N_d+N_f)}$	$\frac{N_f[2+t_g-2\delta-t(2-\delta)]}{4(1-\delta)}$
Q^*_{d,E_1}	$\frac{N_d}{2}$	$\frac{N_d(N_d+N_f-N_f t)}{2(N_d+N_f)}$	$\frac{N_d[N_d(2-\delta)\delta+N_f(2+t_g-\delta)]}{2A_1}$
Q^*_{g,E_1}	/	/	$\frac{N_f[N_f(t\delta-t_g)-N_d\delta A_3]}{4\delta(1-\delta)A_1}$
π^*_{M,E_1}	$\frac{N_d+N_f(t-1)^2}{4}$	$\frac{(N_d+N_f-N_f t)^2}{4(N_d+N_f)}$	$\frac{2N_d^2\delta^2(2-3\delta+\delta^2)+N_f^2A_4+N_dN_f\delta A_5}{8\delta(1-\delta)A_1}$
π^*_{I,E_1}	/	/	$\frac{N_f\left[N_f(t_g-t\delta)+N_d\delta A_3\right]^2}{16(1-\delta)\delta A_1^2}$

Note: The full expressions of $A_1,\ldots,A_5$ are provided in Table A1 in the Appendix.

Proposition 8. When the foreign and domestic market scales are $N_f > 0$ and $N_d > 0$, there exist $\hat{t}_{3,E_1}$ and $\hat{t}_{4,E_1}$ such that the results in Corollary 1 still hold, where $\hat{t}_{3,E_1} = \frac{t_g}{\delta} + \sqrt{2}(1+t_g-\delta)\sqrt{\frac{N_d(1-\delta)}{\delta[N_f+N_d(2-\delta)\delta]}}$, and $\hat{t}_{4,E_1} = \frac{(N_f+N_d)t_g+(1-\delta)\sqrt{\frac{2N_d(N_f+N_d)A_6}{\delta A_1}}}{N_d(2-\delta)+N_f\delta}$.

Moreover, we have:

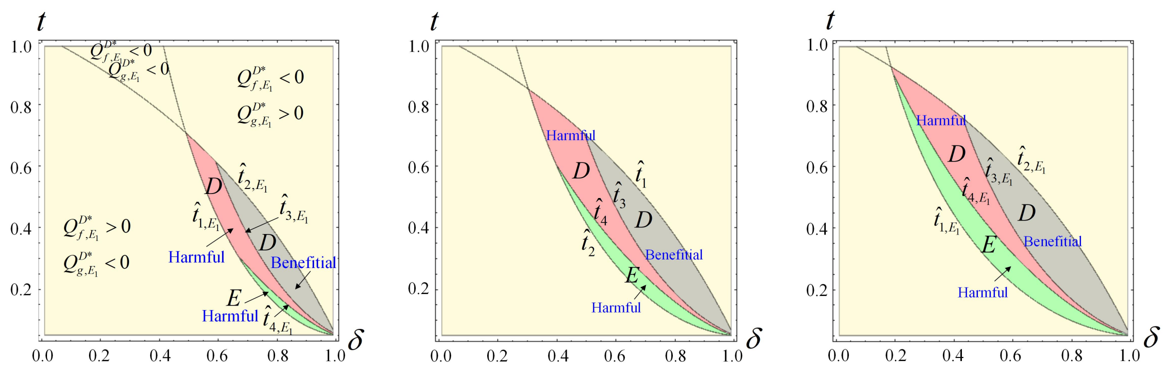
(i) The threshold $\hat{t}_{1,E_1}$ for the parallel importer to create unauthorized channel decreases with N_f ($\frac{\partial \hat{t}_{1,E_1}}{\partial N_f} < 0$) and increases with N_d ($\frac{\partial \hat{t}_{1,E_1}}{\partial N_d} > 0$).

(ii) The threshold $\hat{t}_{3,E_1}$ that an unauthorized channel benefits the manufacturer decreases with N_f ($\frac{\partial \hat{t}_{3,E_1}}{\partial N_f} < 0$) and increases with N_d ($\frac{\partial \hat{t}_{3,E_1}}{\partial N_d} > 0$).

(iii) The threshold $\hat{t}_{4,E_1}$ for the manufacturer to adopt DP and allow an unauthorized channel decreases with N_f ($\frac{\partial \hat{t}_{4,E_1}}{\partial N_f} < 0$) and increases with N_d ($\frac{\partial \hat{t}_{4,E_1}}{\partial N_d} > 0$).

(iv) When $N_f > \frac{N_d\delta[2+t_g(4-\delta)-2\delta(1+t)+t\delta^2]}{t\delta-t_g}$, an unauthorized channel increases the total demand in the foreign market ($Q^{D*}_{f,E_1} + Q^{D*}_{g,E_1} > Q^{B*}_{f,E_1}$); otherwise, it lowers (or does not affect) the total demand in foreign market ($Q^{D*}_{f,E_1} + Q^{D*}_{g,E_1} \leq Q^{B*}_{f,E_1}$).

Proposition 8 demonstrates that Corollary 1 remains robust even when the scales of the domestic and foreign markets are different. However, as one could expect, the threshold values change. To illustrate the results, we provide numerical examples. We set $t_g = 0.05$ and $N_d = 1$. In Figure 7, we depict how the critical parameters (t, δ and N_f) affect the manufacturer's optimal pricing strategy. We keep $N_d = 1$ unchanged and vary the value of N_f to observe the changes in the manufacturer's optimal pricing strategy. Note that if $N_f = 1$, then we recover the basic model.



(a) $N_f = 0.2$ (b) $N_f = 1$ (c) $N_f = 1.8$

Figure 7: The manufacturer's optimal pricing strategy when δ, t and N_f vary.

In Figure 7, the yellow areas show the situations of $Q^{D*}_{g,E_1} < 0$ or $Q^{D*}_{f,E_1} < 0$, which is of no economic interest. In the grey area, an unauthorized channel is beneficial to the manufacturer, and the manufacturer chooses DP ($\pi^{D*}_{M,E_1} > \pi^{B*}_{M,E_1} > \pi^{E*}_{M,E_1}$). In the red area, an unauthorized channel is harmful to the manufacturer, but the manufacturer still chooses DP ($\pi^{B*}_{M,E_1} > \pi^{D*}_{M,E_1} > \pi^{E*}_{M,E_1}$) to allow an unauthorized channel. In the green area, an unauthorized channel is harmful to the manufacturer, and the manufacturer adopts EP to deter an unauthorized channel ($\pi^{B*}_{M,E_1} > \pi^{E*}_{M,E_1} > \pi^{D*}_{M,E_1}$). As the value of N_f increases, the valid area ($Q^{D*}_{g,E_1} > 0$ and $Q^{D*}_{f,E_1} > 0$) becomes larger, and the grey, red, and green areas become larger.

Now, we explain the changes in the manufacturer's pricing strategy shown in Figure 7. In case D, as the scale of the foreign market (N_f) increases, the manufacturer not only reduces its selling price in the foreign market ($\frac{\partial p^{D*}_{f,E_1}}{\partial N_f} < 0$), but also lowers its price in the domestic market ($\frac{\partial p^{D*}_{d,E_1}}{\partial N_f} < 0$) to induce the importer to set a lower selling price of gray product ($\frac{\partial p^{D*}_{g,E_1}}{\partial N_f} < 0$). The reason for this is that, due to the increase in the scale of the foreign market, the manufacturer and the importer can obtain higher increases in their market demands by slightly lowering their selling prices. Since the importer will pay a lower p^{D*}_{d,E_1} when buying products in the domestic market, its motivation to create an unauthorized channel will increase ($\hat{t}_{1,E_1}$ will decrease). Since $\hat{t}_{2,E_1}$ does not vary with N_f and $\hat{t}_{1,E_1}$ decreases with N_f, the valid area ($\hat{t}_{1,E_1} < t < \hat{t}_{2,E_1}$) expands as N_f increases.

Item 4 in Proposition 8 indicates that as N_f increases, the total demand in the foreign market in case D will increase and ultimately exceeds the total demand in the benchmark. This implies that as N_f increases, one of the negative effects of an unauthorized channel (i.e., lowering the total demand in the foreign market) is gradually reduced and ultimately becomes a positive effect (i.e., expanding the total demand). Therefore, the threshold that an unauthorized channel benefits the manufacturer ($\hat{t}_{3,E_1}$) decreases with N_f, and the threshold for the manufacturer to adopt DP and allow an unauthorized channel ($\hat{t}_{4,E_1}$) also decreases with N_f, as shown in Figure 7. Since $\hat{t}_{2,E_1}$ does not vary with N_f and $\hat{t}_{3,E_1}$ decreases with N_f, the grey area expands with N_f. Although the threshold $\hat{t}_{4,E_1}$ decreases with N_f, the green area still expands with N_f because $\hat{t}_{1,E_1}$ decreases with N_f more quickly.

8.2 Different consumers' willingness to pay

In the basic model, V_f and V_d are both uniformly distributed over the interval $[0, 1]$. To assess the impact of having different willingness to pay in the two markets, we assume that $V_f \sim U[0, \theta_f]$ and $V_d \sim U[0, \theta_d]$ ($\theta_f > 0$, $\theta_d > 0$). To avoid trivial cases, we assume that $\hat{t}_{1,E_2} < t < \hat{t}_{2,E_2}$, where $\hat{t}_{1,E_2} = \frac{[\theta_d + \theta_f \delta t_g(4-3\delta)] - 2\delta\theta_f(1-\delta)(\theta_f\delta - \theta_d)}{\delta A_7}$ and $\hat{t}_{2,E_2} = \frac{t_g + 2\theta_f(1-\delta)}{2-\delta}$. If $t \leq \hat{t}_{1,E_2}$, the importer will never create an unauthorized channel, and if $t \geq \hat{t}_{2,E_2}$, the manufacturer will give up selling the authorized product in the foreign market.

In the benchmark, the demand functions for the manufacturer's authorized product in the foreign and domestic markets are $Q^B_{f,E_2} = \frac{\theta_f - p_f}{\theta_f}$ and $Q^B_{d,E_2} = \frac{\theta_d - p_d}{\theta_d}$, respectively. The profit of the manufacturer is:

$$\pi^B_{M,E_2}(p_d, p_f) = \underbrace{p_d(\frac{\theta_d - p_d}{\theta_d})}_{\text{Domestic market}} + \underbrace{(p_f - t)(\frac{\theta_f - p_f}{\theta_f})}_{\text{Foreign market}}$$

In case D, the demand for the manufacturer's authorized product in the foreign and domestic markets are $Q^D_{f,E_2} = \frac{1}{\theta_f}\left(\theta_f - \frac{p_f - p_g}{1-\delta}\right)$ and $Q^D_{d,E_2} = \frac{1}{\theta_d}(\theta_d - p_d)$, respectively. The demand function for the importer's gray market product in the foreign market is $Q^D_{g,E_2} = \frac{1}{\theta_f}\left(\frac{p_f - p_g}{1-\delta} - \frac{p_g}{\delta}\right)$. The profits of the manufacturer and the importer are:

$$\pi^D_{M,E_2}(p_d, p_f, p_g) = \underbrace{\frac{1}{\theta_d}(\theta_d - p_d)p_d}_{\text{Domestic}} + \underbrace{\frac{1}{\theta_f}(\theta_f - \frac{p_f - p_g}{1-\delta})(p_f - t)}_{\text{Foreign (authorized product)}} + \underbrace{\frac{1}{\theta_f}(\frac{p_f - p_g}{1-\delta} - \frac{p_g}{\delta})p_d}_{\text{Foreign (gray product)}} \tag{6}$$

$$\pi^D_{I,E_2}(p_d, p_f, p_g) = \underbrace{\frac{1}{\theta_f}\left(\frac{p_f - p_g}{1-\delta} - \frac{p_g}{\delta}\right)(p_g - p_d - t_g)}_{\text{Foreign (gray product)}}$$

To determine a Stackelberg equilibrium, we first derive the parallel importer's reaction function $p^D_{g,E_2}(p_f, p_d) = (p_d + t_g + p_f\delta)/2$, which is the same as that in the basic model and in the previous extension. Inserting $p^D_{g,E_2}(p_f, p_d)$ into Equation (6), we obtain the following expression for the manufacturer's profit to be maximized with respect to p_f and p_d:

$$\pi^D_{M,E_2}(p_f, p_d) = \frac{\theta_d\delta(p_f - t)[t_g - p_f(2-\delta) + 2\theta_f(1-\delta)] - p_d\theta_d\{t_g + [t - 2p_f - 2\theta_f(1-\delta)]\delta\} - p_d^2[\theta_d + 2\theta_f(1-\delta)\delta]}{2\theta_f\theta_d\delta(1-\delta)}$$

In case E, since the manufacturer blocks an unauthorized channel, the demand functions for the authorized product in the two markets remain the same as those in the benchmark. The profit of the manufacturer is given by:

$$\pi^E_{M,E_2}(p_d, p_f) = \underbrace{p_d(\frac{\theta_d - p_d}{\theta_d})}_{\text{Domestic market}} + \underbrace{(p_f - t)(\frac{\theta_f - p_f}{\theta_f})}_{\text{Foreign market}}$$
$$\text{subject to } p_d = p_f$$

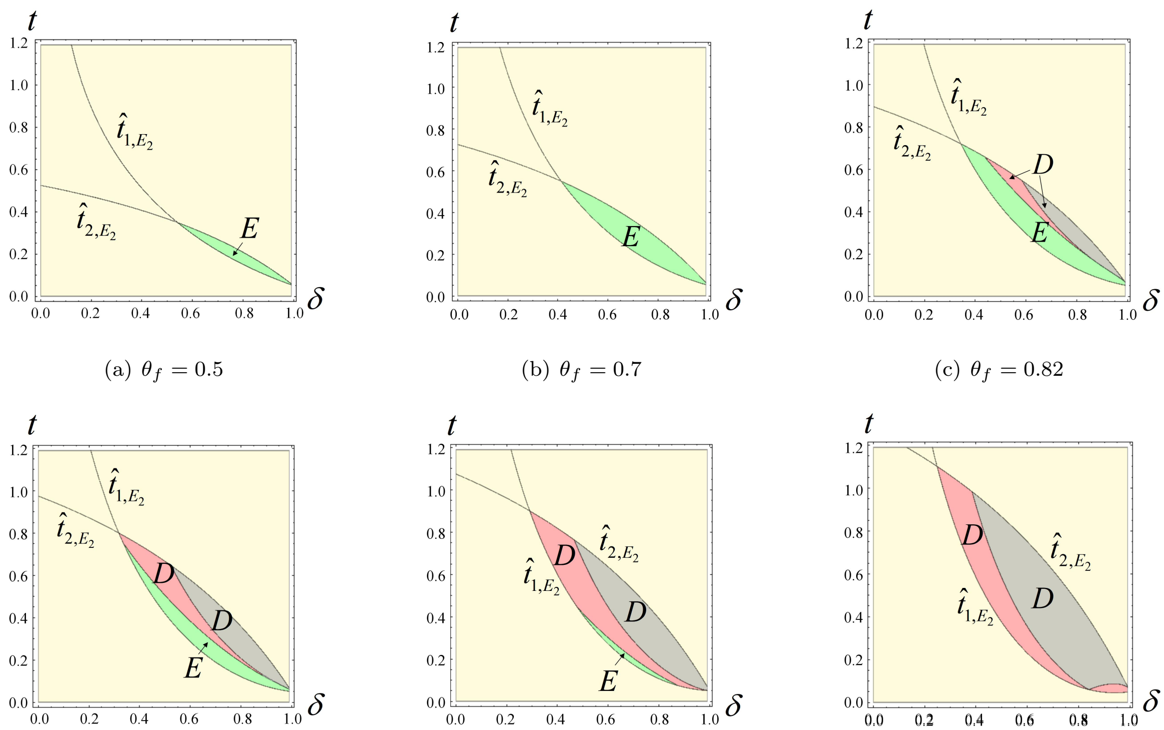
The equilibrium and optimal outcomes of the three cases are summarized in Table 5.

As the equilibrium profit of the manufacturer in case D is very complex, we conduct a numerical study to compare the equilibrium/optimal profits of the manufacturer in the three cases. Let $t_g = 0.05$ and $\theta_d = 1$. Figure 8 illustrates how the critical parameters (t, δ, and θ_f) affect the manufacturer's optimal pricing strategy. We keep $\theta_d = 1$ unchanged and test multiple values for θ_f to observe the changes in the manufacturer's optimal pricing strategy. Note that $\theta_f = 1$ would correspond to the basic model. In Figure 8, the gray, red, green, and yellow areas have the same meanings as in Figure 7. Result 2 is derived based on Figure 8.

Table 5: Equilibrium and optimal outcomes with different consumers' WTP.

	The benchmark	Case E	Case D
p^*_{f,E_2}	$\frac{\theta_f+t}{2}$	$\frac{\theta_d(t+2\theta_f)}{2(\theta_f+\theta_d)}$	$\frac{tA_7+\theta_f\left[\theta_d(1+\delta)+\delta t_g+2\theta_f\delta(1-\delta)\right]}{2A_7}$
p^*_{d,E_2}	$\frac{\theta_d}{2}$	$\frac{\theta_d(t+2\theta_f)}{2(\theta_f+\theta_d)}$	$\frac{\theta_d[\theta_f(3-\delta)\delta-t_g]}{2A_7}$
p^*_{g,E_2}	/	/	$\frac{t_g[\theta_d+\theta_f\delta(4-\delta)]+\delta\{tA_7+2\theta_f[2\theta_d+\theta_f\delta(1-\delta)]\}}{4A_7}$
Q^*_{f,E_2}	$\frac{\theta_f-t}{2\theta_f}$	$\frac{2\theta_f^2-t\theta_d}{2\theta_f^2+2\theta_f\theta_d}$	$\frac{t_g-t(2-\delta)+2\theta_f(1-\delta)}{4\theta_f(1-\delta)}$
Q^*_{d,E_2}	$\frac{1}{2}$	$\frac{2\theta_d-t}{2(\theta_f+\theta_d)}$	$\frac{t_g+2\theta_d+\theta_f\delta(1-\delta)}{2A_7}$
Q^*_{g,E_2}	/	/	$\frac{\delta(A_8+tA_7)-t_g[\theta_d+\theta_f\delta(4-3\delta)]}{4\theta_f\delta(1-\delta)A_7}$
π^*_{M,E_2}	$\frac{t^2-2t\theta_f+\theta_f(\theta_f+\theta_d)}{4\theta_f}$	$\frac{(t^2\theta_d+4\theta_f^2\theta_d-4t\theta_f^2)}{4\theta_f(\theta_f+\theta_d)}$	$\frac{t_g^2(\theta_d+\theta_f\delta^2)-2t_g\delta(tA_7-A_8)+\delta A_9}{8\theta_f\delta(1-\delta)A_7}$
π^*_{I,E_2}	/	/	$\frac{t_g[\theta_d+\theta_f\delta(4-3\delta)]-\delta(A_8+tA_7)^2}{16\theta_f\delta(1-\delta)A_7^2}$

Note: The full expressions of $A_7,\ldots,A_9$ are provided in Table A1 in the Appendix.



(a) $\theta_f = 0.5$ (b) $\theta_f = 0.7$ (c) $\theta_f = 0.82$

(d) $\theta_f = 0.95$ (e) $\theta_f = 1.05$ (f) $\theta_f = 1.25$

Figure 8: The manufacturer's optimal pricing strategy when δ, t and θ_f vary.

Result 2. The valid area ($\hat{t}_{1,E_2} < t < \hat{t}_{2,E_2}$) expands with θ_f. Moreover, we have:

(i) When $0.75 \leq \theta_f \leq 1.085$, there exist $\hat{t}_{3,E_2}$ and $\hat{t}_{4,E_2}$ for which Corollary 1 still holds.

(ii) When $\theta_f < 0.75$, the valid area is small, and the manufacturer will only adopt EP to block a harmful unauthorized channel.

(iii) When $\theta_f > 1.085$, the manufacturer will only adopt DP to allow an unauthorized channel, regardless of whether it is harmful or beneficial.

Result 2 shows that when consumers' willingness to pay in the two markets are different, the results in Corollary 1 are slightly changed. As consumers' WTP in the foreign market (θ_f) increases, the threshold $\hat{t}_{2,E_2}$ will also increase ($\frac{\partial \hat{t}_{2,E_2}}{\partial \theta_f} > 0$), which implies that the manufacturer has greater incentive to sell authorized products in the foreign market. Therefore, the valid area ($\hat{t}_{1,E_2} < t < \hat{t}_{2,E_2}$) expands with θ_f. Then, we explain why the manufacturer will only adopt EP when θ_f is low, and only adopt DP when θ_f is high. In the benchmark, the manufacturer's optimal selling prices in the foreign and domestic markets are $p^{B*}_{f,E_2} = (\theta_f + t)/2$ and $p^{B*}_{d,E_2} = \theta_d/2$, respectively. We can see that p^{B*}_{f,E_2} increases with θ_f, and p^{B*}_{d,E_2} increases with θ_d. This is intuitive: a manufacturer sets a higher selling price in a market with higher WTP to capture more consumer surplus. Thus, the difference between p^{B*}_{f,E_2} and p^{B*}_{d,E_2} is no longer solely determined by the import tax t but is also influenced by consumes' WTP in each market. When θ_f is low (i.e., $\theta_f < 0.75$), although the import tax t may lead the manufacturer to prefer a high price in the foreign market, the low WTP of foreign consumers may lead the manufacturer to prefer a lower price. These two effects offset each other, resulting in a small difference in the manufacturer's optimal selling price between the two markets (i.e., p^{B*}_{f,E_2} in the foreign market is slightly higher than p^{B*}_{d,E_2} in domestic market). In this case, if the parallel importer emerges, the manufacturer will suffer a very small loss if it adopts EP (i.e., the loss of giving up price discrimination is very small). Conversely, when θ_f is high (i.e., $\theta_f > 1.085$), the manufacturer tends to set a very high price in the foreign market not only because of the import tax but also because of the high WTP of foreign consumers. Thus, the difference in the manufacturer's price between the two markets is significant. If the importer emerges, the manufacturer will suffer a significant loss if it adopts EP (i.e., the disadvantage of giving up price discrimination is very high). Therefore, the manufacturer will only choose DP.

9 Conclusions

A manufacturer often sets a higher selling price in the foreign market than in its domestic market due to the requirement to pay import taxes. The price difference can lead to the emergence of an unauthorized channel: an importer purchases products in the manufacturers' domestic market and resells them as gray market products in the manufacturer's foreign market through cross-border e-commerce. We develop Stackelberg game models to explore differentiated-pricing and equal-pricing strategies of a manufacturer who is facing an unauthorized channel. We find that:

1. An unauthorized channel has both positive and negative effects. On the negative side, it erodes the manufacturer's income as some foreign consumers turn to buying gray products from the importer who acquires these products in the domestic market by paying a relatively lower unit price to the manufacturer. Additionally, the manufacturer obtains lower profits from domestic consumers because an unauthorized channel affects its optimal pricing in the domestic market. On the positive side, an unauthorized channel helps the manufacturer to avoid import taxes as the importer pays the import taxes for gray market products sold in the foreign market. Moreover, an unauthorized channel enables the manufacturer to earn higher unit income from loyal consumers who prefer the manufacturer's authorized channel. The impact of an unauthorized channel on the foreign market demand depends on various factors, such as consumers' acceptance degree of gray product, the import tax on the manufacturer's authorized product, and the tax incentives for cross-border e-commerce. When these parameters are low, an unauthorized channel can decrease the total demand in the foreign market, which is another negative effect. However, as the three parameters increase, the total demand in the foreign market gradually rises. When the three parameters exceed certain thresholds, an unauthorized channel can increase the demand in the foreign market, becoming a positive effect. This suggests that in practice, manufacturers need to carefully evaluate the positive and negative effects of an unauthorized channel and determine whether it is beneficial or harmful. Based on this analysis, appropriate actions can be taken to either prevent or welcome an unauthorized channel.
2. Pricing strategy can be used by the manufacturer to manage an unauthorized channel, and the optimal pricing strategy depends on critical market parameters. (i) When the market con-

ditions are favorable for an unauthorized channel (i.e., consumers' acceptance degree of gray market product, the import tax on the manufacturer's authorized product, and the tax incentives for cross-border e-commerce are high), an unauthorized channel benefits the manufacturer. Therefore, the manufacturer should choose differentiated pricing to encourage an unauthorized channel. In practice, manufacturers should start by gathering data on critical market parameters. After obtaining the necessary information, manufacturers should anticipate the parallel importers' pricing decisions for gray market products and decide their selling prices for each market. (ii) When the market conditions are somewhat unfavorable for an unauthorized channel (i.e., consumers' acceptance degree of gray market product, the import tax on the manufacturer's authorized product, and the tax incentives for cross-border e-commerce are somewhat low), an unauthorized channel is harmful to the manufacturer. However, the manufacturer should still choose differentiated pricing to allow an unauthorized channel. In practice, manufacturers should tolerate the existence of harmful unauthorized channels and take the same actions as they would under favorable market conditions. (iii) When the market conditions are very unfavorable for an unauthorized channel (i.e., consumers' acceptance degree of gray market product, the import tax on the manufacturer's authorized product, and import the tax incentives for cross-border e-commerce are very low), the manufacturer should adopt equal-pricing to block a harmful unauthorized channel. In practice, manufacturers can implement a global pricing alignment strategy, similar to Mulberry, Briggs & Riley and NIO, to set a uniform price in both domestic and foreign markets.

3. If the scales of the domestic and foreign markets are different, the manufacturer's optimal pricing strategy in item 2 remains robust. If domestic and foreign consumers have different willingness to pay, the manufacturer's optimal pricing strategy slightly changes. Specifically, when the willingness to pay of foreign consumers is similar to or slightly lower than that of domestic consumers, the manufacturer's optimal pricing strategy in item 2 remains robust. However, when the willingness to pay of foreign consumers is significantly lower than that of domestic consumers, the manufacturer should only adopt equal-pricing strategy. Conversely, the manufacturer should only adopt differentiated-pricing strategy as long as the willingness to pay of foreign consumers is slightly higher than that of domestic consumers. Therefore, in practice, manufacturers should carefully consider the differences in consumers' willingness to pay between domestic and foreign markets when choosing pricing strategies.

This paper focuses on an unauthorized channel created by an independent third-party importer. It is worth noting that unauthorized channels created by retailers are also prevalent. Further research is needed to explore how a manufacturer can use pricing strategy to manage its retailers' unauthorized channels. Additionally, this paper assumes that a single manufacturer sells the same product in both domestic and foreign markets. It would be interesting to explore how a manufacturer's optimal pricing strategy changes when it faces competition from other manufacturers in the foreign market, or when it offers horizontally or vertically differentiated products in domestic and foreign markets.

Appendix

A. Table A1

Table A1: Threshold values and simplified equations.

Index	Expression	Location
$\hat{t}_1$	$\frac{(1+4\delta-3\delta^2)t_g+2\delta(1-\delta)^2}{\delta(1+2\delta-\delta^2)}$	Section 3
$\hat{t}_2$	$\frac{2-2\delta+t_g}{2-\delta}$	Section 3
$\hat{t}_3$	$\frac{t_g}{\delta}+\sqrt{2}(1-\delta+t_g)\sqrt{\frac{1-\delta}{\delta(1+2\delta-\delta^2)}}$	Proposition 5
$\hat{t}_4$	$t_g+(1-\delta)\sqrt{\frac{2\delta(1-\delta)+4\delta t_g-t_g^2(1+\delta)}{\delta(1+2\delta-\delta^2)}}$	Proposition 6
$\hat{t}_{1,E_1}$	$\frac{(4N_d\delta-3N_d\delta^2+N_f)t_g+2N_d\delta+2N_d\delta^3-4N_d\delta^2}{N_f\delta+2N_d\delta^2-N_d\delta^3}$	Subsection 8.1
$\hat{t}_{2,E_1}$	$\frac{2+t_g-2\delta}{2-\delta}$	Subsection 8.1
$\hat{t}_{3,E_1}$	$\frac{t_g}{\delta}+\sqrt{2}(1+t_g-\delta)\sqrt{\frac{N_d(1-\delta)}{\delta[N_f+N_d(2-\delta)\delta]}}$	Proposition 8
$\hat{t}_{4,E_1}$	$\frac{(N_f+N_d)t_g+(1-\delta)\sqrt{\frac{2N_d(N_f+N_d)A_6}{\delta A_1}}}{N_d(2-\delta)+N_f\delta}$	Proposition 8
$\hat{t}_{1,E_2}$	$\frac{[\theta_d+\theta_f\delta t_g(4-3\delta)]-2\delta\theta_f(1-\delta)(\theta_f\delta-\theta_d)}{\delta A_7}$	Subsection 8.2
$\hat{t}_{2,E_2}$	$\frac{t_g+2\theta_f(1-\delta)}{2-\delta}$	Subsection 8.2
A_1	$N_f+N_d(2-\delta)\delta$	Table 4 and $\hat{t}_{4,E_1}$
A_2	$2+t_g(4-\delta)+2(1+t)\delta-(2+t)\delta^2$	Table 4
A_3	$2+t_g(4-3\delta)-2(2+t)\delta+(2+t)\delta^2$	Table 4
A_4	$t_g^2-2t_gt\delta+\delta[2+t^2(2-\delta)-4t(1-\delta)-2\delta]$	Table 4
A_5	$\delta t_g^2+\delta(2-\delta)[t^2(2-\delta)+4(1-\delta)-4t(1-\delta)]+2t_g[2\delta(2-t)-\delta^2(2-t)-2]$	Table 4
A_6	$N_d(1+2t_g-\delta)(2-\delta)\delta+N_f[2t_g\delta^2+(1-\delta)\delta^2-t_g^2(1+\delta)]$	$\hat{t}_{4,E_1}$
A_7	$\theta_d+\theta_f(2-\delta)\delta$	Table 5, $\hat{t}_{1,E_2}$, A_9
A_8	$2\theta_f(1-\delta)(\theta_f\delta-\theta_d)$	Table 5
A_9	$t^2(2-\delta)A_7-4t\theta_f(1-\delta)A_7+2\theta_f^2(1-\delta)[2\theta_f(1-\delta)\delta+\theta_d(1+4\delta-\delta^2)]$	Table 5

B. Proofs

Proof of Proposition 1. The Hessian matrix of $\pi_M^B(p_f,p_d)$ with respect to (p_f,p_d) is $\mathbf{H}^B=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}$. Since $|\mathbf{H}_1^B|=-2<0$ and $|\mathbf{H}_2^B|=4>0$, $\mathbf{H}^B$ is negatively definite. Therefore, $\pi_M^B(p_f,p_d)$ is jointly concave in (p_f,p_d). Solving the first-order conditions $\frac{\partial\pi_M^B(p_f,p_d)}{\partial p_f}=0$ and $\frac{\partial\pi_M^B(p_f,p_d)}{\partial p_d}=0$, we can derive p_f^{B*} and p_d^{B*}. Inserting p_f^{B*} and p_d^{B*} into $Q_f^B(p_f)$, $Q_d^B(p_d)$, and Equation (1), we can obtain Q_f^{B*}, Q_d^{B*}, and π_M^{B*}. □

Proof of Proposition 2. Since $0<\delta<1$, we can derive $\frac{\partial^2\pi_I^D(p_d,p_f,p_g)}{\partial p_g^2}=-\frac{2}{(1-\delta)\delta}<0$. Therefore, $\pi_I^D(p_d,p_f,p_g)$ is a concave function of p_g. Solving the first-order condition $\frac{\partial\pi_I^D(p_d,p_f,p_g)}{\partial p_g}=0$ for p_g, we can obtain the importer's reaction function $p_g^D(p_f,p_d)=\frac{p_d+t_g+p_f\delta}{2}$. Inserting $p_g^D(p_f,p_d)$ into Equation (2), we can obtain $\pi_M^D(p_f,p_d)$ in Equation (4). Expecting the importer's reaction, the manufacturer decides p_f and p_d to maximize $\pi_M^D(p_f,p_d)$. The Hessian matrix of $\pi_M^D(p_f,p_d)$ with respect to (p_f,p_d) is $\mathbf{H}^D=\begin{bmatrix}-\frac{2-\delta}{1-\delta} & \frac{1}{1-\delta}\\ \frac{1}{1-\delta} & -\frac{1+2\delta(1-\delta)}{(1-\delta)\delta}\end{bmatrix}$. Since $|\mathbf{H}_1^D|=-\frac{2-\delta}{1-\delta}<0$ and $|\mathbf{H}_2^D|=\frac{2[1+\delta(2-\delta)]}{\delta(1-\delta)}>0$, $\mathbf{H}^D$ is negatively definite. Therefore, $\pi_M^D(p_f,p_d)$ is jointly concave in (p_f,p_d). Solving the first-order conditions $\frac{\partial\pi_M^D(p_f,p_d)}{\partial p_f}=0$ and $\frac{\partial\pi_M^D(p_f,p_d)}{\partial p_d}=0$, we can derive p_f^{D*} and p_d^{D*}.Inserting p_f^{D*} and p_d^{D*} into the importer's reaction function $p_g^D(p_f,p_d)$, we can obtain p_g^{D*}. Inserting p_d^{D*}, p_f^{D*}, and p_g^{D*} into $Q_f^D(p_f,p_g)$, $Q_g^D(p_f,p_g)$, $Q_d^D(p_d)$, Equation (2), and Equation (3), we can obtain Q_f^{D*}, Q_g^{D*}, Q_d^{D*}, π_M^{D*}, and π_I^{D*}. □

Proof of Proposition 3.

(i) Since $0 < \delta < 1$ and $t_g > 0$, we can derive $p_f^{D*} - p_f^{B*} = \frac{(1-\delta+t_g)\delta}{2[1+\delta(2-\delta)]} > 0$. Similarly, we can derive $p_d^{D*} - p_d^{B*} < 0$.

(ii) Since $0 < \delta < 1$ and $t_g > 0$, we can obtain $\frac{\partial p_d^{D*}}{\partial \delta} = \frac{3-2\delta+2t_g(1-\delta)+\delta^2}{2(1+2\delta-\delta^2)^2} > 0$ and $\frac{\partial p_d^{D*}}{\partial t_g} = -\frac{1}{2[1+\delta(2-\delta)]} < 0$. In a similar way, we can derive all the other results.

(iii) Comparing the total demand in the foreign market between the benchmark and case D, we can derive $Q_f^{D*} + Q_g^{D*} - Q_f^{B*} = \frac{t\delta(1+2\delta-\delta^2)-2\delta(1-\delta)-t_g(1+4\delta-\delta^2)}{4\delta(1+2\delta-\delta^2)}$. Since $0 < \delta < 1$, we can obtain that the denominator $4\delta(1+2\delta-\delta^2) > 0$. Therefore, when $t\delta(1+2\delta-\delta^2) > 2\delta(1-\delta)+t_g(1+4\delta-\delta^2)$, we have $Q_f^{D*} + Q_g^{D*} - Q_f^{B*} > 0$; otherwise, we have $Q_f^{D*} + Q_g^{D*} - Q_f^{B*} \leq 0$. Since $0 < \delta < 1$ and $t_g > 0$, we can derive $2\delta(1-\delta) + t_g(1+4\delta-\delta^2) > 0$ and $\delta(1+2\delta-\delta^2) > 0$. Therefore, we can rewrite $t\delta(1+2\delta-\delta^2) > 2\delta(1-\delta) + t_g(1+4\delta-\delta^2)$ as $t > \frac{2\delta(1-\delta)+t_g(1+4\delta-\delta^2)}{\delta(1+2\delta-\delta^2)}$. In a similar way, we can derive all the other results.

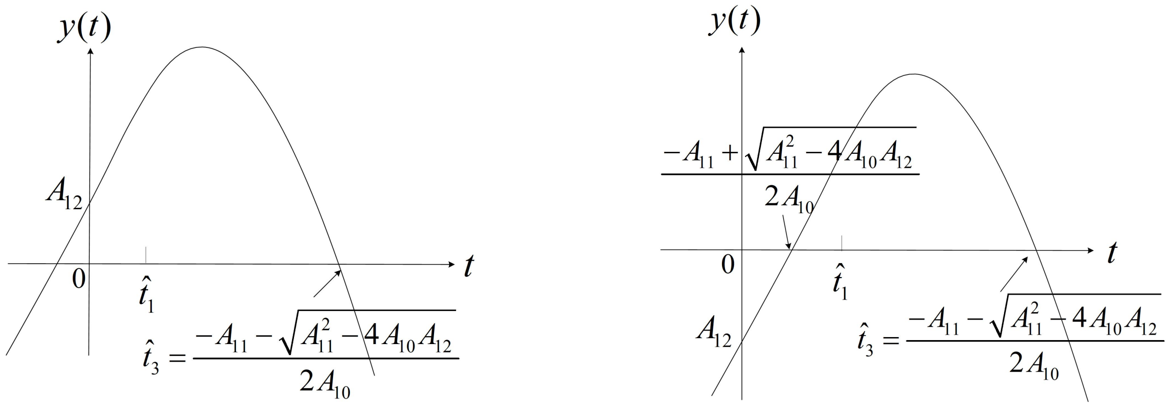
(iv) We can derive $\frac{\partial \pi_M^{D*}}{\partial t} = \frac{2\delta(2+3\delta-4\delta^2+\delta^3)t-2t_g\delta(1+2\delta-\delta^2)-4\delta(1+\delta-3\delta^2+\delta^3)}{8\delta(1+\delta-3\delta^2+\delta^3)}$. Since $8\delta(1+\delta-3\delta^2+\delta^3) > 0$, whether $\frac{\partial \pi_M^{D*}}{\partial t}$ is positive or negative depends on the numerator. Therefore, when $2\delta(2+3\delta-4\delta^2+\delta^3)t < 2t_g\delta(1+2\delta-\delta^2)+4\delta(1+\delta-3\delta^2+\delta^3)$, we have $\frac{\partial \pi_M^{D*}}{\partial t} < 0$. Since $0 < \delta < 1$ and $t_g > 0$, we have $2t_g\delta(1+2\delta-\delta^2)+4\delta(1+\delta-3\delta^2+\delta^3) > 0$ and $2\delta(2+3\delta-4\delta^2+\delta^3) > 0$. Therefore, when $t < \frac{2t_g\delta(1+2\delta-\delta^2)+4\delta(1+\delta-3\delta^2+\delta^3)}{2\delta(2+3\delta-4\delta^2+\delta^3)}$, we have $\frac{\partial \pi_M^{D*}}{\partial t} < 0$. Simplified $\frac{2t_g\delta(1+2\delta-\delta^2)+4\delta(1+\delta-3\delta^2+\delta^3)}{2\delta(2+3\delta-4\delta^2+\delta^3)}$ is exactly equal to $\hat{t}_2$. Since $t < \hat{t}_2$ always holds, we have $\frac{\partial \pi_M^{D*}}{\partial t} < 0$. In a similar way, we can derive all the other results. □

Proof of Proposition 4. The proof is similar to that of Proposition 1. □

Proof of Proposition 5. Comparing the manufacturer's optimal profits under the benchmark and case D, we have $\pi_M^{B*} - \pi_M^{D*} = \frac{A_{10}t^2+A_{11}t+A_{12}}{8\delta(1+\delta-3\delta^2+\delta^3)}$, where $A_{10} = -\delta^2(1+2\delta-\delta^2)$, $A_{11} = 2t_g\delta(1+2\delta-\delta^2)$, and $A_{12} = 2\delta(1-\delta)^3 + 4t_g\delta(1-\delta)^2 - t_g^2(1+\delta^2)$. Since $0 < \delta < 1$, we can derive $8\delta(1+\delta-3\delta^2+\delta^3) > 0$. Therefore, we only need to focus on $A_{10}t^2 + A_{11}t + A_{12}$. Define a quadratic function $y(t) = A_{10}t^2 + A_{11}t + A_{12}$. Since $0 < \delta < 1$, we can derive $A_{10} < 0$. Therefore, the graph of the quadratic function $y(t)$ (i.e., parabola) is concave downwards. Since $0 < \delta < 1$ and $t_g > 0$, we have $A_{11}^2 - 4A_{10}A_{12} > 0$. Therefore, the parabola and the t-axis have two intersections. Since $0 < \delta < 1$ and $t_g > 0$, we have $-\frac{A_{11}}{2A_{10}} > 0$. Therefore, the axis of symmetry of the parabola lies to the right of the y-axis . There are three cases:

1. If $A_{12} > 0$, the graph of the quadratic function $y(t)$ is illustrated in Figure A1(a). We can derive $\hat{t}_3 = \frac{-A_{11}-\sqrt{A_{11}^2-4A_{10}A_{12}}}{2A_{10}} = \frac{t_g}{\delta} + \sqrt{2}(1+t_g-\delta)\sqrt{\frac{1-\delta}{\delta(2\delta-\delta^2+1)}}$, which is the positive root of $y(t) = 0$. In Section 3, we know that t should satisfy $\hat{t}_1 < t < \hat{t}_2$. Since $0 < \delta < 1$ and $t_g > 0$, we can derive $\hat{t}_3 > \hat{t}_1$. However, the relationship between $\hat{t}_3$ and $\hat{t}_2$ is uncertain. We can derive the following results:
   (i) When $t < \hat{t}_3$, we have $y(t) = A_{10}t^2 + A_{11}t + A_{12} > 0$ and $\pi_M^{B*} - \pi_M^{D*} > 0$.
   (ii) When $t = \hat{t}_3$, we have $y(t) = A_{10}t^2 + A_{11}t + A_{12} = 0$ and $\pi_M^{B*} - \pi_M^{D*} = 0$.
   (iii) When $t > \hat{t}_3$, we have $y(t) = A_{10}t^2 + A_{11}t + A_{12} < 0$ and $\pi_M^{B*} - \pi_M^{D*} < 0$.
2. If $A_{12} = 0$, the results are the same as that in 1.
3. If $A_{12} < 0$, the graph of the quadratic function $y(t)$ is illustrated in Figure A1(b). Since $0 < \delta < 1$, and $t_g > 0$, we can derive that $\frac{-A_{11}+\sqrt{A_{11}^2-4A_{10}A_{12}}}{2A_{10}}$ (i.e., the left positive root of $y(t) = 0$) is lower than $\hat{t}_1$. Since t should satisfy $\hat{t}_1 < t < \hat{t}_2$, the left positive root is not within the scope of this research. We only focus on the right positive root $\hat{t}_3 = \frac{-A_{11}-\sqrt{A_{11}^2-4A_{10}A_{12}}}{2A_{10}}$. Since $0 < \delta < 1$

and $t_g > 0$, we can derive $\hat{t}_3 > \hat{t}_1$. However, the relationship between $\hat{t}_3$ and $\hat{t}_2$ is uncertain. Therefore, the results are the same as that in 1. Proposition 5 is proven.



(a) $A_{12} > 0$ (b) $A_{12} < 0$

Figure A1: The graph of $y(t)$.

□

Proof of Proposition 6. Comparing the manufacturer's equilibrium profits under case E and case D, we have $\pi_M^{E*} - \pi_M^{D*} = \frac{(\frac{A_{10}}{\delta})t^2 + A_{11}t + A_{12}}{8\delta(1+\delta-3\delta^2+\delta^3)}$. We have defined A_{10}, A_{11} and A_{12} in Proof of Proposition 5. Since $8\delta(1+\delta-3\delta^2+\delta^3) > 0$, we only focus on the numerator. Define a quadratic function $z(t) = \left(\frac{A_{10}}{\delta}\right)t^2 + A_{11}t + A_{12}$. We can derive $\hat{t}_4 = \frac{-A_{11} - \sqrt{A_{11}^2 - 4\left(\frac{A_{10}}{\delta}\right)A_{12}}}{2\left(\frac{A_{10}}{\delta}\right)} = t_g + (1-\delta)\sqrt{\frac{2\delta(1-\delta)+4t_g\delta - t_g^2(1+\delta)}{\delta(1+2\delta-\delta^2)}}$, which is the larger root of $z(t) = 0$. The proof is similar to that of Proposition 5. □

Proof of Corollary 1. Comparing $\hat{t}_3$ and $\hat{t}_4$, we have:

$$\begin{aligned}\hat{t}_3 - \hat{t}_4 =& \frac{t_g(1+\delta-3\delta^2+\delta^3) + \sqrt{2}(1-\delta+t_g)\sqrt{\delta(1+\delta-3\delta^2+\delta^3)}}{\delta(1+2\delta-\delta^2)} \\ &- \frac{(1-\delta)\sqrt{\delta(1+2\delta-\delta^2)\left[4t_g\delta + 2(1-\delta)\delta - t_g^2(1+\delta)\right]}}{\delta(1+2\delta-\delta^2)}\end{aligned}$$

Since $0 < \delta < 1$, $t_g > 0$, and $\hat{t}_1 < \hat{t}_2$, we can derive $\hat{t}_3 - \hat{t}_4 > 0$. Combining Propositions 5 and 6, we can derive Corollary 1. □

Proof of Proposition 7. We can calculate the consumer surplus of the domestic market in the three cases:

$$CS_d^B = \int_{p_d^{B*}}^1 (v_d - p_d^{B*})\, dv_d = \int_{\frac{1}{2}}^1 (v_d - \frac{1}{2})\, dv_d = \frac{1}{8}$$

$$CS_d^E = \int_{p_d^{E*}}^1 (v_d - p_d^{E*})\, dv_d = \int_{\frac{2+t}{4}}^1 (v_d - \frac{2+t}{4})\, dv_d = \frac{1}{32}(2-t)^2$$

$$CS_d^D = \int_{p_d^{D*}}^1 (v_d - p_d^{D*})\, dv_d = \int_{\frac{\delta(3-\delta)-t_g}{2(1+2\delta-\delta^2)}}^1 \left[v_d - \frac{\delta(3-\delta)-t_g}{2(1+2\delta-\delta^2)}\right] dv_d = \frac{(2+t_g+\delta-\delta^2)^2}{8(1+2\delta-\delta^2)^2}$$

Since $0<\delta<1$, $t>0$, $t_g>0$, and $\hat{t}_1<t<\hat{t}_2$, we can derive $CS_d^D>CS_d^B>CS_d^E$. Similarly, we can calculate the consumer surplus of the foreign market in the three cases:

$$CS_f^B=\int_{p_f^{B*}}^{1}(v_f-p_f^{B*})\,dv_f=\int_{\frac{1+t}{2}}^{1}(v_f-\frac{1+t}{2})\,dv_f=\frac{1}{8}(1-t)^2$$
$$CS_f^E=\int_{p_f^{E*}}^{1}(v_f-p_f^{E*})\,dv_f=\int_{\frac{2+t}{4}}^{1}(v_f-\frac{2+t}{4})\,dv_f=\frac{1}{32}(2-t)^2$$
$$CS_f^D=\int_{\frac{p_g^{D*}}{\delta}}^{\frac{p_f^{D*}-p_g^{D*}}{1-\delta}}(\delta v_f-p_g^{D*})\,dv_f+\int_{\frac{p_f^{D*}-p_g^{D*}}{1-\delta}}^{1}(v_f-p_f^{D*})\,dv_f$$
$$=\frac{1}{2}-\frac{[2-t_g+t(2-\delta)-2\delta]^2}{32(1-\delta)^2}+\frac{\{(2-t)\delta-(2+t)(2-\delta)\delta^2+t_g[1+(4-3\delta)\delta]\}^2}{32\delta(1+\delta-3\delta^2+\delta^3)^2}$$
$$-\frac{[2+t_g-t(2-\delta)-2\delta]\left[1+t+(3+2t+t_g)\delta-(2+t)\delta^2\right]}{8(1+\delta-3\delta^2+\delta^3)}$$

Since $0<\delta<1$, $t>0$, $t_g>0$, and $\hat{t}_1<t<\hat{t}_2$, we can derive $CS_f^E>CS_f^B>CS_f^D$. Proposition 7 is proven. □

Proof of outcomes in Tables 4 and 5. The proofs are similar to those of Propositions 1 and 2. □

Proof of Proposition 8. Comparing the manufacturer's optimal profits under the benchmark and case D (in Table 4), we have

$$\pi_{M,E_1}^{B*}-\pi_{M,E_1}^{D*}=\frac{A_{13}t^2+A_{14}t+A_{15}}{8\delta(1-\delta)[N_f+N_d(2-\delta)\delta]},$$

where

$$A_{13}=-N_f\delta^2(N_f+2N_d\delta-N_d\delta^2),$$
$$A_{14}=2N_ft_g\delta(N_f+2N_d\delta-N_d\delta^2),$$
$$\text{and } A_{15}=N_fN_d[2(1+2t_g)\delta-(6+8t_g+t_g^2)\delta^2+2(3+2t_g)\delta^3-2\delta^4]-N_f^2t_g^2.$$

Similar to the proof process of Proposition 5, we can obtain $\hat{t}_{3,E_1}=\frac{-A_{14}-\sqrt{A_{14}^2-4A_{13}A_{15}}}{2A_{13}}$.

Comparing the manufacturer's optimal profits under case E and case D, we have

$$\pi_{M,E_1}^{E*}-\pi_{M,E_1}^{D*}=\frac{A_{16}t^2+A_{17}t+A_{18}}{8\delta(N_f+N_d)(1-\delta)[N_f+N_d(2-\delta)\delta]},$$

where

$$A_{16}=-N_f\delta[N_f^2\delta+N_d^2(2-\delta)^2\delta+N_fN_d(2-\delta+2\delta^2-\delta^3)],$$
$$A_{17}=2N_ft_g\delta[N_f^2+N_d^2(2-\delta)\delta+N_fN_d(1+2\delta-\delta^2)],$$
$$A_{18}=N_fN_d\left[4t_g\delta(1-\delta)^2(N_d+N_f)+2\delta(1-\delta)^3(N_d+N_f)-t_g^2\delta^2(N_d+N_f)-t_g^2N_f\right]-N_f^3t_g^2.$$

Similar to the proof process of Proposition 5, we can obtain $\hat{t}_{4,E_1}=\frac{-A_{17}-\sqrt{A_{17}^2-4A_{16}A_{18}}}{2A_{16}}$.

Taking the derivative of $\hat{t}_{1,E_1}$ and $\hat{t}_{3,E_1}$ with respect to N_f, we can derive $\frac{\partial\hat{t}_{1,E_1}}{\partial N_f}=-\frac{2N_d(1-\delta+t_g)(1-\delta)}{[N_f+N_d(2-\delta)\delta]^2}$ and $\frac{\partial\hat{t}_{3,E_1}}{\partial N_f}=-\frac{N_f^3N_d^2(1-\delta+t_g)^4(1-\delta)^2\delta^4}{\sqrt{2}\{\delta^3N_f^2N_d(1-\delta)(1-\delta+t_g)^2[N_f+N_d(2-\delta)\delta]\}^{3/2}}$. Since $0<\delta<1$, $t_g>0$, $N_f>0$, and $N_d>0$, we can derive $\frac{\partial\hat{t}_{1,E_1}}{\partial N_f}<0$ and $\frac{\partial\hat{t}_{3,E_1}}{\partial N_f}<0$. Similarly, we can obtain all the other results.

Comparing the total demand in the foreign market between the benchmark and case D, we can derive $Q^{D*}_{f,E_1} + Q^{D*}_{g,E_1} - Q^{B*}_{f,E_1} = \frac{N_f\{N_f(t\delta - t_g) - N_d\delta[2+t_g(4-\delta)-2(1+t)\delta+t\delta^2]\}}{4\delta[N_f+N_d(2-\delta)\delta]}$. Since $0 < \delta < 1$, $N_f > 0$, and $N_d > 0$, we have $4\delta[N_f + N_d(2-\delta)\delta] > 0$. Therefore, we only need to focus on the numerator. When $N_f(t\delta - t_g) > N_d\delta[2 + t_g(4-\delta) - 2(1+t)\delta + t\delta^2]$, we have $Q^{D*}_{f,E_1} + Q^{D*}_{g,E_1} - Q^{B*}_{f,E_1} > 0$; otherwise, we have $Q^{D*}_{f,E_1} + Q^{D*}_{g,E_1} - Q^{B*}_{f,E_1} \leq 0$. Since $\hat{t}_{1,E_1} < t < \hat{t}_{2,E_1}$, $0 < \delta < 1$, $t_g > 0$, $N_f > 0$, and $N_d > 0$, we can derive $N_d\delta[2 + t_g(4-\delta) - 2(1+t)\delta + t\delta^2] > 0$ and $t\delta - t_g > 0$. Therefore, we can rewrite $N_f(t\delta - t_g) > N_d\delta[2 + t_g(4-\delta) - 2(1+t)\delta + t\delta^2]$ as $N_f > \frac{N_d\delta[2+t_g(4-\delta)-2(1+t)\delta+t\delta^2]}{t\delta - t_g}$. Proposition 8(iv) is proven. □

References

Ahmadi, R., Iravani, F., Mamani, H., 2015. Coping with gray markets: The impact of market conditions and product characteristics. Production and Operations Management 24, 762–777.

Ahmadi, R., Yang, B.R., 2000. Parallel imports: Challenges from unauthorized distribution channels. Marketing Science 19, 279–294.

Autrey, R.L., Bova, F., Soberman, D.A., 2014. Organizational structure and gray markets. Marketing Science 33, 849–870.

Cao, Q., Zhang, J., 2020. Gray market's product quality in the circular economy era. International Journal of Production Research 58, 308–331.

Chen, K., Wang, X., Niu, B., Chen, Y.J., 2022. The impact of tariffs and price premiums of locally manufactured products on global manufacturers' sourcing strategies. Production and Operations Management 31, 3474–3490.

Corchón, L.C., Torregrosa, R.J., 2022. Correction: Two extensions of consumer surplus. Series-Journal of the Spanish Economic Association 13, 557–579.

Ding, L., Hu, B., Ke, C., Wang, T., Chang, S., 2019. Effects of IoT technology on gray market: An analysis based on traceability system design. Computers & Industrial Engineering 136, 80–94.

Ding, L., Yuan, H., Hu, B., 2021. Adopt or not: Manufacturers' RFID decisions for gray marketing in a competitive environment. Computers & Industrial Engineering 151, 106957.

Gnecco, G., Pammolli, F., Tuncay, B., 2022. Welfare and research and development incentive effects of uniform and differential pricing schemes. Computational Management Science 19, 229–268.

Han, X., Yang, Q., Shang, J., Pu, X., 2017. Optimal strategies for trade-old-for-remanufactured programs: Receptivity, durability, and subsidy. International Journal of Production Economics 193, 602–616.

Huang, H., He, Y., Chen, J., 2019. Competitive strategies and quality to counter parallel importation in global market. Omega 86, 173–197.

Huang, H., He, Y., Chen, J., 2020a. Cross-market selling channel strategies in an international luxury brand's supply chain with gray markets. Transportation Research Part E: Logistics and Transportation Review 144, 102157.

Huang, H., Xing, X., He, Y., Gu, X., 2020b. Combating greenwashers in emerging markets: A game-theoretical exploration of firms, customers and government regulations. Transportation Research Part E: Logistics and Transportation Review 140, 101976.

Iravani, F., Dasu, S., Ahmadi, R., 2016. Beyond price mechanisms: How much can service help manage the competition from gray markets? European Journal of Operational Research 252, 789–800.

Jelovac, I., Bordoy, C., 2005. Pricing and welfare implications of parallel imports in the pharmaceutical industry. International Journal of Health Care Finance and Economics 5, 5–21.

Lee, B.C., 2006. Consumer perceived importance of channel authorization: A post hoc segmentation approach to dealing with gray markets. Australasian Marketing Journal 14, 10–22.

Li, H., Qing, Q., Wang, J., Hong, X., 2021. An analysis of technology licensing and parallel importation under different market structures. European Journal of Operational Research 289, 132–143.

Li, H., Shao, J., Zhu, S.X., 2018. Parallel importation in a supply chain: The impact of gray market structure. Transportation Research Part E: Logistics and Transportation Review 114, 220–241.

Li, H., Zhu, S.X., Cui, N., Li, J., 2016. Analysis of gray markets in differentiated duopoly. International Journal of Production Research 54, 4008–4027.

Li, W., Sun, H., Dong, H., Gan, Y., Koh, L., 2023. Outsourcing decision-making in global remanufacturing supply chains: The impact of tax and tariff regulations. European Journal of Operational Research 304, 997–1010.

Liu, W., Chang, L., Lin, J., 2012. Consumer lifestyle matters: Evidence from gray markets in China. Journal of Service Science and Management 5, 196–205.

Myers, M.B., 1999. Incidents of gray market activity among U.S. exporters: Occurrences, characteristics, and consequences. Journal of International Business Studies 30, 105–126.

Richardson, M., 2002. An elementary proposition concerning parallel imports. Journal of International Economics 56, 233–245.

Ross, T.W., 1984. Uncovering regulators' social welfare weights. The RAND Journal of Economics 15, 152–155.

Shao, J., Krishnan, H., McCormick, S.T., 2016. Gray markets and supply chain incentives. Production and Operations Management 25, 1807–1819.

Shavandi, H., Valizadeh Khaki, S., Khedmati, M., 2015. Parallel importation and price competition in a duopoly supply chain. International Journal of Production Research 53, 3104–3119.

Taleizadeh, A.A., Hadadpour, S., Cárdenas-Barrón, L.E., Shaikh, A.A., 2017. Warranty and price optimization in a competitive duopoly supply chain with parallel importation. International Journal of Production Economics 185, 76–88.

Valletti, T.M., 2006. Differential pricing, parallel trade, and the incentive to invest. Journal of International Economics 70, 314–324.

Wang, M., Huang, H., Liu, F., 2023. To adapt or to standardize? Cross-market green product design under parallel importation impact. Transportation Research Part E: Logistics and Transportation Review 174, 103133.

Wang, Y., Lin, J., Choi, T.M., 2020. Gray market and counterfeiting in supply chains: A review of the operations literature and implications to luxury industries. Transportation Research Part E: Logistics and Transportation Review 133, 101823.

Xiao, Y., Palekar, U., Liu, Y., 2011. Shades of gray—the impact of gray markets on authorized distribution channels. Quantitative Marketing and Economics 9, 155–178.

Yang, B.R., Ahmadi, R.H., Monroe, K.B., 1998. Pricing in separable channels: The case of parallel imports. Journal of Product & Brand Management 7, 433–440.

Zhang, J., 2016. The benefits of consumer rebates: A strategy for gray market deterrence. European Journal of Operational Research 251, 509–521.

Zhang, Y., Zhang, S.H., Feng, H., 2021. The impacts of money-back guarantees in the presence of parallel importation. European Journal of Operational Research 295, 170–182.

Zhao, K., Zhao, X., Deng, J., 2016. An empirical investigation of online gray markets. Journal of Retailing 92, 397–410.

Zhou, Y., Gao, X., Luo, S., Xiong, Y., Ye, N., 2022. Anti-counterfeiting in a retail platform: A game-theoretic approach. Transportation Research Part E: Logistics and Transportation Review 165, 102839.