Les Cahiers du GERAD ISSN: 0711–2440

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G–2023–08

March 2023

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2023
– Bibliothèque et Archives Canada, 2023

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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Scheduling maintenance with uncertain duration on power transmission systems

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March 2023
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G–2023–08

Abstract : For planning the operation of power transmission systems, which transport the energy produced by generation plants to customers centers, it is essential to establish when and which transmission lines are unavailable due to preventive maintenance. In a previous work, we proposed a mixed-integer formulation for the transmission maintenance scheduling problem that schedules preventive maintenance while keeping network connected and maximizing customers' supply in case of unexpected line failures. Starting from this previous deterministic model, we now propose a mixed-integer formulation that considers the uncertainty on preventive maintenance duration, over a one-year period. This stochastic formulation considers the existence of an unknown delay in maintenance tasks at the planning time. Task delays are represented by scenarios with associated probability of occurrence. This large-scale problem is solved using a proposed algorithm which decomposes the formulation into a master problem, solved with CPLEX through Benders decomposition, and sub problems that validate the solution found. We present computational results for the 24-bus IEEE network that demonstrate that the proposed algorithm reaches optimality more efficiently than the direct solution of the stochastic extensive formulation. When the algorithm runs larger instances the optimality level achieved is not as good as for smaller instances, but it presents a very good tolerance gap equivalent to the default CPLEX/AMPL value.

Keywords : Optimization under uncertainty, power transmission networks, scheduling, mixed-integer linear optimization

Acknowledgements: This research was supported by the NSERC-Hydro-Quebec-Schneider Electric Industrial Research Chair on Optimization for Smart Grids.

1 Introduction

Power transmission systems are composed of various equipment, such as transformers and transmission lines, these systems connect power generation plants to consumption centers. For planning the operation of power transmission systems and ensure their reliable operation, it is essential to know when each transmission line will be unavailable due to maintenance. As described in the literature, the transmission maintenance scheduling (TMS) problem seeks to determine the periods in which the transmission lines and their associated equipment will be removed for preventive maintenance.

The authors of the literature review [6] carry out a major and broad analysis of the available literature on scheduling maintenance, while in [5], the maintenance schedules discussed are those of deregulated power systems. Maintenance scheduling problems can be generation related, generation maintenance scheduling (GMS), or transmission related (TMS); ocasionally both problems are treated together. Literature on GMS is much more extensive than the one available on the TMS [6]. In this paper, we address the TMS problem.

Some maintenance scheduling problems described in the literature deal with uncertainties. The deterministic formulations eventually handle uncertainties by applying reservation constraints [6]. In the GMS model proposed by [22], the prices of energy and fuel are uncertain and scenarios are modeled by the author with the use of Monte Carlo method. In both GMS models presented by [9] and [11], the uncertainty treated is the power demand, its perfect value is not known and a group of scenarios with its associated probability is established in order to solve the stochastic problems. Besides that, sometimes unexpected breakdowns are represented by the forced outage rate that serves as the probability that the line or the generator is unavailable. This probabilistic approach is used in the GMS problems that also have constraints for the network [7, 12, 13, 19]. Commonly, authors use stochastic reliability indices, for example, the expected energy not served is used by [3, 7, 12, 13, 19], usually this index is minimized or constraints limit its value, and the loss of load probability is used in a levelized risk method in [15] and [21]. The TMS [16] considers the balance of potential equipment fault risk; it obtains the risk value through Monte Carlo simulation. In the GMS [3], a stochastic model simulates arbitrary forced outages with the Monte Carlo method. The stochastic TMS of [23] uses different wind speed scenarios and seeks to reduce wind power curtailment when considering network topology optimization. In this paper, the uncertainty about the duration of delays in preventive maintenance tasks for the TMS problem will be treated using stochastic methods. A similar work is unknown to the authors.

In our previous work [18], a deterministic TMS formulation is proposed. In the mixed integer linear problem (MILP), preventive maintenance does not isolate buses or divide the network, and the objective is to carry out the maintenance in preferred weeks, also maximizing customers supply in the event of an unexpected failure of a line.

Driven by the interests and requirements of a major Canadian electricity utility, the objective of this work is to design a stochastic TMS model, starting from the literature and from our previous work [18], that takes into account the uncertainty in the duration values for the maintenance tasks, for a one-year period. Duration values are not precisely known at the time maintenance is planned. The major objective is to obtain an effective yearly scheduling solution that considers all possible scenarios and is insensitive to uncertainties in task duration.

Our main contribution is to provide an optimization TMS model that expressly allow for possibility the occurrence of delays in maintenance tasks, making use of a group of scenarios that represent the scheme of delays and its associated probability of occurrence. The complete annual schedule is determined for transmission line maintenance whose duration are not known at the planning time. The proposed stochastic extensive TMS formulation is a demanding large-scale MILP to be solved by off-the-shelf solvers, therefore, we introduce a specially designed decomposition algorithm to solve the TMS model in a reduced computational time.

This paper is organized as follows. Section 2 explain and details the proposed stochastic MILP formulation for the TMS problem. Section 3 presents the new specialized algorithm, and Section 4 reports the results of the computational tests on the IEEE 24-bus Reliability Test System [20]. Section 5 completes the paper presenting the conclusion.

2 Mathematical formulation

In this section, we present our proposed formulation. We begin by introducing the required notation in subsection 2.1. Then we describe a previously proposed TMS in subsection 2.2. Lastly, we detail our stochastic model in subsection 2.3.

2.1 Basic notation

Parameters

A_l^t	Preference level (target time) for the maintenance of transmission line l at period t;
B_l	Susceptance of the transmission line l, in siemens;
D_k^t	Maximum demand at bus k, period t, in MW;
E_l	Minimum duration of maintenance task at line l, in weeks;
F_l^{Lim}	Maximum power flow capacity of line l, in MW;
G_{ki}^+	Upper limit of generation of unit i at bus k, in MW, when unit is on;
G_{ki}^-	Lower limit of generation of unit i at bus k, in MW, when unit is on;
L_t	Maximum maintenance tasks allowed at period t;
M_{ck}	Maintenance line connected to generator bus k, where the only other connected line is c;
O_l	Origin bus of line l;
Pr_s	Probability of occurrence of the scenario s;
R_l	Destination bus of line l;
S_{kl}	Incidence matrix bus-line, $+1$ if line l leaves bus k, -1 if line l enters bus k;
S_{lk}^T	Incidence matrix bus-line transposed;
T	Number of periods of time, in weeks;
U_{ij}	Matrix with elements $+1$ when there is a line between bus i and bus j, 0 otherwise;
W_l	Delay duration of maintenance task at line l, in weeks;

Sets

Δ	All lines of the grid, indexed by l or c;
Δ^{M1}	All lines scheduled for 1 week maintenance, indexed by l, $\Delta^{M1} \subseteq \Delta^M$;
Δ^{M2}	All lines scheduled for 2 weeks maintenance, indexed by l, $\Delta^{M2} \subseteq \Delta^M$;
Δ^{M3}	All lines scheduled for 3 weeks maintenance, indexed by l, $\Delta^{M3} \subseteq \Delta^M$;
Δ^M	All lines that will undergo maintenance, indexed by l, $\Delta^{M1} \cup \Delta^{M2} \cup \Delta^{M3}$;
Δ^{NM}	All lines not scheduled for maintenance, indexed by l, $\Delta \setminus \Delta^M$;
Γ	All buses of the network, index k;
Γ^0	Super-source bus for test of connectivity of the grid, indexed by i, j;
Γ^G	All buses directly connected with a generator, indexed by k;
Γ^{N0}	All buses of the network except the super-source bus, indexed by j, k, $\Gamma \setminus \Gamma^0$;
Γ^{NG}	All buses not directly connected with a generator, indexed by k, $\Gamma \setminus \Gamma^G$;
Γ^{NR}	All buses except the reference, index k, $\Gamma \setminus \Gamma^R$;
Γ^R	Reference bus of the network, indexed by k;
Λ	All periods of time, indexed by t;
Λ^C	Convenient periods, indexed by t, $\Lambda \setminus \Lambda^I$;
Λ^I	Inconvenient periods, indexed by t;
Ω	All scenarios of the delay in maintenance duration, indexed by s;
Ω^C	All pairs (c, k) where c $\in \Delta$ and k $\in \Gamma^G$;
Ω^{N1}	Pairs representing lines c and M_{ck}, connected to generator bus k, indexed by (c, k);
Ω^{Ok}	All pairs of line c and bus k with different design of Ω^{N1}, indexed by (c, k), $\Omega^C \setminus \Omega^{N1}$;
Υ^U	Generation units per power plant, indexed by i;
Ξ	Pairs of lines that will undergo maintenance simultaneously, indexed by (l, u);

Variables

h_{ij}^{kt}	Binary, equals 1 if grid connectivity test flow goes through arc (i, j) when testing the non-isolation of bus k at period t, else 0;
w_{ij}^t	Unitary flow capacity of arc (i, j), if it exists and operates at period t, else 0;
$\hat{gen}_{ck}^t$	Generation at bus k, time t when line c fails, MW;
$\hat{d}_{ck}^t$	Demand at bus k, time t when line c fails, MW;

$\hat{f}^t_{cl}$ Power flow at line l, time t when line c fails, MW, $l \neq c$;
$\hat{g}^{ct}_{ik}$ Generation of unit i at bus k, time t when line c fails, MW;
$\hat{y}^t_{ij}$ Equals 1 if maintenance removal of arc with origin at bus i and destination at bus j and its associated equipment is scheduled for period t, else 0;
$\hat{\theta}^t_{ck}$ Voltage angle of bus k, time t when line c fails;
f^t_l Power flowing in the line l at period t, in MW;
g^t_{ik} Generation of unit i at bus k period t, in MW;
gen^t_k Total power generated at bus k period t, in MW;
n^t_{ck} Equals 0 if line M_{ck}, linked to generator bus k, as well as line c, is in maintenance at time t, else 1;
θ^t_k Voltage angle of bus k at period t;
s^t_l Binary, is 1 if maintenance of line l and its associated equipment starts at period t, else 0;
x^t_l Binary, is 1 if minimum duration E_l of maintenance on line l and its associated equipment is scheduled at time t, else 0;
e^t_l Binary, is 1 if minimum duration E_l of maintenance on line l and its associated equipment ends at period t, else 0;
z^{ts}_l Binary, is 1 if there is a maintenance delay W_l on line l and occurs at time t for scenario s, else 0;
y^{ts}_l Binary, is 1 if there is a maintenance of type x^t_l or z^{ts}_l on line l and its associated equipment scheduled at time t, else 0;

2.2 Features of the deterministic TMS model from [18]

Since the stochastic TMS formulation proposed in Section 2.3, which is the main subject of this paper, is derived and based on the deterministic TMS formulation described in [18], we have provided the full deterministic formulation in Appendix A. We now recall the main features of this model.

In [18], the deterministic MILP model allocates preventive maintenance tasks, which are the removal of transmission lines for their maintenance, over a total horizon of 52 weeks. The mathematical formulation uses a graph to represent the electricity transmission grid, with its generations units, power flows, and customer demands.

The objective represents the trade-off between maximizing the preferred targeted week for the maintenance of each line and maximizing the quantity of electricity supplied to customers when the power grid accounts for the original network, minus the lines in maintenance and minus a possible single line loss.

The basic constraints of a scheduling problem are represented by constraints (23)–(34). They avoid maintenance in inappropriate weeks, respect the maximum amount of weekly tasks allowed and guarantee the simultaneous maintenance of parallel lines, i.e., lines that have the same origin and destination bus. Furthermore, the maintenance duration for each transmission line is known, and varies between 1 and 3 weeks.

The direct current power flow (DCPF) model is the same used by the authors of [8, 10, 12, 14], rather than a transportation model. It is a linearization of the alternate current power flow (ACPF) and less computationally expensive. The constraints (35)–(47) represent the DCPF. The power flows of lines and voltages angles of buses are variables and respect their operating limits.

To represent the possibility of losing a network line due to an unexpected failure (excluding the lines undergoing preventive maintenance on that week), an idea known as N-1 network security, constraints (48)–(63) are enforced. This modeling is induced by the security-constrained unit commitment of [1].

In order to avoid bus isolation and grid splitting when preventive maintenance tasks are scheduled, a network connectivity test is performed. This is accomplished through constraints (64)–(75). These constraints reflect the sending of one artificial unit of flow from the bus called super-source to every other bus through the network, to check its connectivity. In this model, depending on network topology, it is possible that the removal of a specific line will always cause bus isolation.

2.3 The proposed stochastic TMS formulation

By solving the deterministic model of Section 2.2, appropriate solutions can be obtained; however, maintenance interventions may suffer delays in their duration and this uncertainty about the duration of the tasks must be accounted for.

Delays in maintenance duration can cause major problems for the utilities. The management of the teams responsible for these maintenance activities can be harshly affected. If a task is late, a possible consequence is having to reschedule or postpone the tasks that would begin next. From then on the possible cascade effects regarding successive postponements of activities can even lead to the non-realization of all desired tasks for the year.

Although the deterministic TMS can be solved several times for different duration values, the results obtained are different for each case and it is not possible to establish one perfect decision for all uncertain situations simultaneously. From the deterministic formulation proposed in Section 2.2, it is possible to build its stochastic version and, as intended, express the uncertainty of the duration in maintenance tasks.

The two-stage stochastic model proposed in Section 2.3 determines in the first-stage the starting week of each task and derives in the second-stage all the remaining variables. The annual scheduling problem is fully resolved at the end of the two-stage resolution.

2.3.1 A scenario-based two-stage formulation

We first make the assumption that there is a normal duration for each of the maintenance tasks. This normal duration corresponds to a minimum basic value that always occurs for each of the tasks. To represent the scheduling of this normal duration, we use the binary variables x_l^t, which assume value 1 when the normal maintenance of line l occurs in week t. To help determine the scheduling of the normal maintenance duration x_l^t, we use the variables $s_l^t,\ e_l^t \in \{0,1\}$ representing start and end times for each activity. These start, end and assignment decisions for the normal duration of tasks, (s_l^t, e_l^t, x_l^t), must be taken at the time the overall schedule is planned and are the first-stage decisions.

Second, we have to consider the possible delays that may occur for each of the maintenance tasks; these are stochastic delays. Considering that the parameters that determine the values of the delays come from historical data, various scenarios of delay occurrences can be established. Therefore, the stochastic delay is represented by a discrete and finite set of scenarios $\Omega = 1, 2, ..S$. Besides that, for each scenario there is an associated occurrence probability $pr_1, pr_2, ...pr_S$. The delay parameter, represented by W, has then its notation indexed to each scenario and the delay vector for a scenario s is represented by $W^s = [W_1^s, W_2^s, .., W_l^s]$, where W_l^s is the maintenance delay of line l in scenario s. Each scenario $s \in \Omega$ corresponds to one particular realization of the delay vector.

To represent the scheduling of the total duration of the maintenance of line l under scenario s, we introduce two groups of binary variables: z_l^{ts}, $t \in \Lambda$, which indicate whether or not a delay is incurred in week t for the maintenance of line l under scenario s, and y_l^{ts}, which indicate whether or not any maintenance is performed on line l during week t under scenario s.

From the model of Section 2.2, it is possible to create a corresponding stochastic formulation by associating a set of second-stage decisions to each possible behavior of the delay. Therefore, the proposed stochastic formulation is as follows:

$$\max \sum_{t\in\Lambda}\sum_{l\in\Delta} A_l^t x_l^t + \sum_{s\in\Omega}\sum_{t\in\Lambda}\sum_{l\in\Delta} A_l^t z_l^{ts} Pr_s + \sum_{s\in\Omega}\sum_{t\in\Lambda}\sum_{k\in\Gamma}\sum_{c\in\Delta} \hat{d}_{ck}^{ts} Pr_s \tag{1}$$

$$\text{s.t.} \sum_{t\in\Lambda^C} s_l^t = 1 \qquad \forall\ l \in \Delta^M \tag{2}$$

$$\sum_{t\in\Lambda^C} e_l^t = 1 \qquad \forall\, l \in \Delta^M \tag{3}$$
$$x_l^t \geq s_l^t \qquad \forall\, l \in \Delta^M,\ t \in \Lambda^C \tag{4}$$
$$x_l^t \geq e_l^t \qquad \forall\, l \in \Delta^M,\ t \in \Lambda^C \tag{5}$$
$$s_l^t \geq x_l^t - x_l^{t-1} \qquad \forall\, l \in \Delta^M,\ t \in \Lambda^C,\ t > 1 \tag{6}$$
$$s_l^{first(\Lambda^C)} \geq x_l^{first(\Lambda^C)} \qquad \forall\, l \in \Delta^M \tag{7}$$
$$e_l^{t-1} \geq x_l^{t-1} - x_l^t \qquad \forall\, l \in \Delta^M,\ t \in \Lambda^C,\ t > 1 \tag{8}$$
$$e_l^{last(\Lambda^C)} \geq x_l^{last(\Lambda^C)} \qquad \forall\, l \in \Delta^M \tag{9}$$
$$\sum_{u\ \in\ Q_l..last^{\Lambda^C}} s_l^u \geq 1 \qquad l \in \Delta^M \tag{10}$$
$$\sum_{t\in\Lambda^C} x_l^t = E_l \qquad \forall\, l \in \Delta^M \tag{11}$$
$$\sum_{t\in\Lambda^C} z_l^{ts} = W_l^s \qquad \forall\, l \in \Delta^M,\ s \in \Omega \tag{12}$$
$$x_l^t + z_l^{ts} = y_l^{ts} \qquad \forall\, l \in \Delta^M,\ t \in \Lambda^C,\ s \in \Omega \tag{13}$$
$$\sum_{t\in\Lambda} \sum_{l\in\Delta^{NM}} y_l^{ts} = 0 \qquad \forall s \in \Omega \tag{14}$$
$$\sum_{t\in\Lambda^I} \sum_{l\in\Delta} y_l^{ts} = 0 \qquad \forall s \in \Omega \tag{15}$$
$$\sum_{l\in\Delta^M} y_l^{ts} \leq L^t \qquad \forall\, t \in \Lambda^C,\ s \in \Omega \tag{16}$$
$$y_l^{ts} - y_u^{ts} = 0 \qquad \forall t \in \Lambda, (l,u) \in \Xi, l \neq u, s \in \Omega \tag{17}$$

$$e_l^t \leq z_l^{t+v,s} \qquad \forall l \in \Delta^M, s \in \Omega, v \in 1..W_l^s, t \in first^{\Lambda^C}..(last^{\Lambda^C} - v), W_l^s \neq 0 \tag{18}$$
$$e_l^t \leq 0 \qquad \forall l \in \Delta^M, s \in \Omega, v \in 1..W_l^s, t \in (last^{\Lambda^C} - v + 1)..last^{\Lambda^C}, W_l^s \neq 0 \tag{19}$$

The proposed objective function (1) is the same as the one formulated in [18] and it has three parts. The first and second parts maximize the maintenance predilection of each task for certain weeks; the parameter A_l^t expresses the preference level, or target time, for the maintenance of line l at week t. In the first part of objective (1) the binary variables x_l^t assume value 1 if the nominal maintenance duration of line l is scheduled at week t and in the second part of objective (1) the binary variables for total duration y_l^{ts} assume value 1 if maintenance of line l is scheduled at week t in the scenario s. The third part of the objective maximizes all customer demand, $\hat{d}_{ck}^{ts} \in \mathbb{R}^+$, which indicates the energy in megawatts required by bus k, at period t, when eventually line c of the network was lost unexpectedly under scenario s. That is, this part of the objective maximizes the amount of energy supplied to customers when a line fails unexpectedly.

For all expected weeks of normal maintenance duration x_l^t, there is a variable $s_l^t,\ e_l^t \in \{0,1\}$ representing start and end of each activity. In addition, the weeks for scheduling the tasks must be sequential, so, as soon as the activity starts, it continues in the following weeks until its expected duration is completed. Besides, there is a minimum week from which each task can start; parameter Q_l indicates from which week the maintenance of line l can start. This is all guaranteed by constraints (2)–(10).

Since $x_l^t,\ z_l^{ts},\ y_l^{ts} \in \{0,1\}$ are the variables that allocate tasks in certain periods, constraints (11), (12) and (13) above, indicate that, for each maintenance task, the nominal normal duration E_l plus the stochastic delay duration W_l^s represent the total duration in weeks of the activity.

The weeks of delay for each activity z_l^{ts} must occur exactly in the sequence of the normal duration of activities x_l^t. The delay begins the week following the end of the normal duration and continues for sequential delay weeks until the activity is fully completed. In addition, so that the activities does not carry over to the following year, the end of the normal duration e_l^t must occur before the total value of the delay duration. All this is guaranteed by constraints (18)–(19).

The scheduling constraint (14) ensures no scheduling of lines $l \in \Delta^{NM}$ that do not need maintenance. This project is based on the needs of a major Canadian company, and therefore constraint (15) prevents tasks from being placed in weeks $t \in \Lambda^I$ with severe weather conditions. Besides, constraint (16) limits the amount of maintenance tasks that can take place in the same period of time, $\Lambda^C = \Lambda \setminus \Lambda^I$ and constraint (17) ensures the simultaneous maintenance of pairs of lines $\in \Xi$ as required for lines that have the same origin and destination bus, namely parallel lines.

In addition to the objective function (1) and constraints (2)–(19) described above, the stochastic model also includes constraints (35)–(75) of the Appendix A. However, since all variables are of the second-stage, these constraints from the Appendix A must be adapted in order to associate each scenario with a set of second-stage decisions.

Note that the first-stage of this stochastic TMS model is defined by constraints (2)–(11), while the remaining constraints are part of the second-stage. In the objective function (1), the first term is first-stage and the second and third terms are part of the second-stage.

The stochastic model above is robust in the sense that each stochastic scenario predicts exactly which lines are delayed and for how many weeks. Each scenario represents relevant delays. The amount of scenarios is limited and they are deterministic. The scheduling for the entire year can be determined by this single problem after the solution of the second-stage.

3 New specialized algorithm

The vast majority of TMS problems with mixed integer formulations have constraints that can be separated into various groups. Consequently, the treatment of these models often involve decomposition methods that seek to facilitate the computational effort when treating these large models.

Although the formulation proposed in Section 2.3 can be directly solved, this new algorithm proposed takes advantage of the separability element and uses decomposition to solve it, while maintaining the solution at a good quality level and improving the resolution time.

The stochastic problem of Section 2.3 is partitioned into three smaller problems, one called master problem and two sub problems. The master problem consists of the objective (1), the constraints (2)–(19) that represent the main scheduling problem and the constraints (48)–(63) from the MILP of Appendix A, which represent the attempt to provide maximum customer supply even if an unexpected failure occurs. The first sub problem contains the constraints of power flow, which are the constraints (35)–(47) that preserve the continuous electrical operation of the power grid. The second sub problem consists of the connectivity constraint, which are the constraints (64)–(75), they keep all buses connected to the network.

Variables and constraints from the sub problems do not directly impact the solution of the master problem, both sub problems are in reality a verification that there is a continuous operation of the grid and that all buses remain connected to the grid. Both sub problems have objective value zero.

Consequently, the problem is solved as indicated by Algorithm 1.

First, the master problem (1), (2)–(19) and (48)–(63), is solved by Benders decomposition, which is available via the MILP solver we use. Here the relative tolerance between the best bound and the best integer bound to stop the optimization process is set to virtually zero, leading to a better optimal integer solution.

Algorithm 1 New specialized algorithm

Ensure: all: $count_1, count_2, a_1, a_2, end \leftarrow 0$
1: Solve Master Problem 1
2: **if** runs out of memory without solution **then**
3:     Solve Master Problem 2
4: **end if**
5: **if** optimal solution found **then**
6:     $Y_l^{ts} \leftarrow y_l^{ts}$
7: **else**
8:     Break: Master Problem has no solution
9: **end if**
10: Solve Sub Problem of Power Flow
11: **if** optimal solution found **then**
12:     $end \leftarrow end + 1$
13: **else**
14:     $count_1 \leftarrow count_1 + 1$
15:     $\hat{Y}_{l,count_1}^{ts} \leftarrow Y_l^{ts}$
16: **end if**
17: **for** $s \in$ Scenarios **do**
18:     $a_1 \leftarrow a_1 + 1$
19:     Solve Sub Problem of Grid Connectivity 1
20:     **if** optimal integer solution found **then**
21:         $end \leftarrow end + 1$
22:     **else**
23:         **for** $t \in$ Convenient Periods **do**
24:             $a_2 \leftarrow a_2 + 1$
25:             Solve Sub Problem of Grid Connectivity 2
26:             **if** optimal integer solution found **then**
27:             **else**
28:                 $count_2 \leftarrow count_2 + 1$
29:                 $\forall l \in \Delta : \tilde{Y}_{l,count_2}^{ts} \leftarrow Y_l^{ts}$
30:                 **for** $t_2 \in$ Convenient Periods, $t_2 \neq t$ **do**
31:                     $\forall l \in \Delta : \tilde{Y}_{l,count_2}^{t_2 s} \leftarrow \tilde{Y}_{l,count_2}^{ts}$
32:                 **end for**
33:                 **for** $s_3 \in$ Scenarios, $s_3 \neq s$ **do**
34:                     **for** $t_4 \in$ Convenient Periods **do**
35:                         $\forall l \in \Delta : \tilde{Y}_{l,count_2}^{t_4 s_3} \leftarrow \tilde{Y}_{l,count_2}^{t_4 s}$
36:                     **end for**
37:                 **end for**
38:             **end if**
39:         **end for**
40:         $a_2 \leftarrow 0$
41:     **end if**
42: **end for**
43: $a_1 \leftarrow 0$
44: **if** $end \geq s + 1$ **then**
45:     Break
46: **end if**
47: $end \leftarrow 0$
48: Solve Master Problem 1
49: **if** runs out of memory without solution **then**
50:     Solve Master Problem 2
51: **end if**
52: Go to step 5.

If the Benders decomposition in CPLEX fails to reach a result due to lack of machine memory, which might happen in larger instances especially in the case where a large number of lines are scheduled annually, the master problem is solved by Benders decomposition again, but now the gap tolerance between the solution bounds is the default from CPLEX/AMPL. This optimal integer solution reached is not as good as the previous one.

If the master problem reaches an optimal integer solution, regardless of the established gap, the binary solution y_l^{ts} from the master problem is sent as parameter Y_l^{ts} to the sub problems of power flow and grid connectivity. If the master problem has no solution the algorithm stops.

Then, the sub problem of power flow, with objective zero and constraints (35)–(47) is solved using the values of the variables obtained in the master problem. If there is a feasible solution then the parameter *end* increases one unit, if not a cut, formulated as (20), is added to the master problem.

After that, a sub problem of grid connectivity, with objective zero and constraints (64)–(75), is solved for each scenario of the stochastic problem. If there is a feasible integer solution the parameter *end* increases one unit for each scenario. If not, within that specific scenario, for each week in which maintenance tasks are allowed a sub problem of grid connectivity is solved. In case the tasks scheduled for that week disconnect buses from the grid, cuts formulated as (21) are added to the master problem.

If the power flow sub problem and the grid connectivity sub problems per scenario have a feasible solution, the complete problem is solved. If not, cuts of the types (20) and (21) are generated and added to the master problem. After that the master problem is solved again with Benders decomposition and gap tolerance virtually zero. And again, if there is not enough memory to reach a better solution the master problem is solved another time with a default value for the gap. Finally, the algorithm goes again to step 5 of the Algorithm 1.

Constraints (20) and (21) represent the set of cuts produced by the the sub problems and added to the master problem. Each cut eliminates the binary optimal solution previously found by the master problem for the variable y. The formulation of the cuts are inspired by the combinatorial Benders cuts, see, e.g. [4].

$$\sum_{\substack{l\in\Delta^M,\ t\in\Lambda^C,\ s\in\Omega:\\ \hat{Y}_{lc}^{ts}=0}} y_l^{ts} + \sum_{\substack{l\in\Delta^M,\ t\in\Lambda^C,\ s\in\Omega:\\ \tilde{Y}_{lc}^{ts}=1}} (1-y_l^{ts}) \geq 1, \qquad \forall c \in 1..count_1 \tag{20}$$

$$\sum_{\substack{l\in\Delta^M,\ v\in\Lambda^C:\\ \tilde{Y}_{lc}^{vs}=0, ord(v)=a_2}} y_l^{ts} + \sum_{\substack{l\in\Delta^M,\ v\in\Lambda^C:\\ \tilde{Y}_{lc}^{vs}=1, ord(v)=a_2}} (1-y_l^{ts}) \geq 1, \qquad \forall c \in 1..count_2, t\in\Lambda^C, s\in\Omega \tag{21}$$

In section 4 are reported the results obtained for the simulations with the power grid from [20], using both the extensive stochastic formulation and the new specialized algorithm.

4 Computational experiments

All computational experiments were accomplished employing CPLEX 20.1 on a Linux server with 16GB of RAM and 8 CPUs (Intel(R) Core(TM) i7-2600 CPU @ 3.40GHz) via AMPL.

4.1 Case study system and maintenance data

The transmission power grid employed in the tests is the IEEE-24-Reliability Test System (RTS), which is meticulously detailed in [20]. This network is also used for the tests in [8, 12, 17]. The grid, as shown in Figure 1, contains 10 generation plants, 24 buses, 38 lines and 17 buses with customers.

For the tests, all grid lines undergo maintenance throughout the year, except the line number 11 shown in Figure 1, as its removal always isolates bus number 7 from the network and makes the problem infeasible. A maximum of three tasks are allowed per week. In order to avoid severe winter conditions and meet the needs of a relevant Canadian company, maintenance takes place between May and October, as indicated by red vertical lines in Figure 2. Figure 2 shows the preferred weeks, or target time, A_l^t for disconnections of each transmission line. The weeks with the highest preference are indicated in a darker color.

The weekly demand for each bus of the grid is shown in the first and tenth Tables of [20]. And the minimum and maximum operating generation values are the ones indicated in the first Table of [18]. The power flow limits of the lines F_l are set up to 80% of the limits indicated in [20].

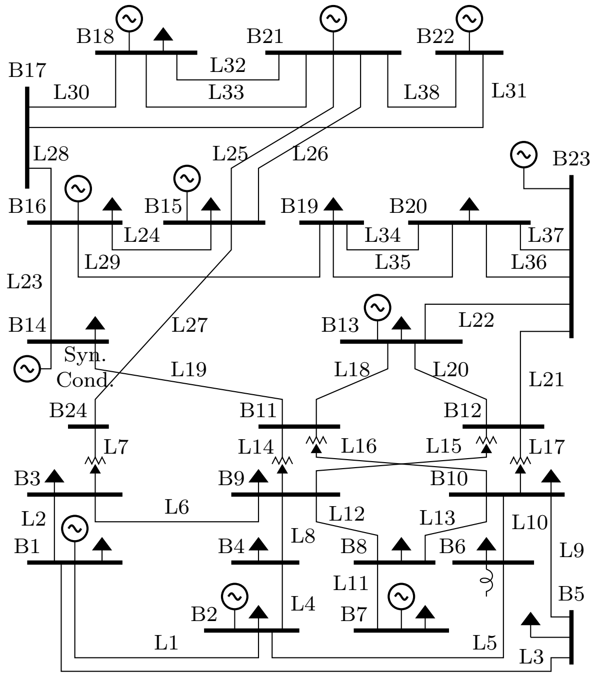

Figure 1: Power transmission grid IEEE RTS-24, from [20]

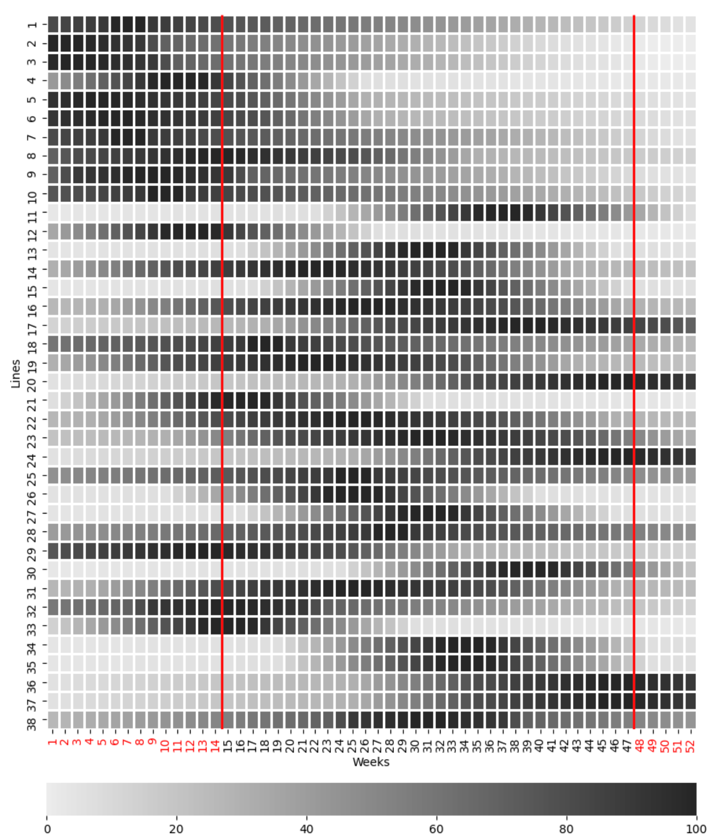

Figure 2: Weekly preference level (target time) A_l^t, for each line maintenance, [18]

Table 2 details the maintenance duration parameters. The first row shows the values for the parameter Q_l, indicating from which week the maintenance of line l can start. The second row of Table 2 shows the maintenance minimum duration E_l, in weeks, for each line. Considering the deterministic and the stochastic TMS formulation presented in Sections 2.2 and 2.3, Table 2 describes the three sets of possible scenarios for the delays on maintenance duration w_l, where $0 \leq w_l \leq 2$. For the stochastic formulation in Section 2.3 the probability of each scenario $s = \{1, 2, 3\}$ occurring is 0.5, 0.3 and 0.2 respectively.

Table 2: Maintenance duration parameters

Line	1	2	3	4	5	6	7	8	9	10	11	12	13
Q_l	20	15	19	15	16	17	27	19	15	22		16	31
E_l	1	2	1	1	2	1	1	1	1	1		1	1
W_l^1	0	0	0	0	0	0	0	0	0	0		0	0
W_l^2	0	1	1	1	1	1	0	1	1	0		1	0
W_l^3	1	2	1	1	1	1	0	1	1	1		1	0
Line	14	15	16	17	18	19	20	21	22	23	24	25	26
Q_l	23	32	25	38	20	27	45	17	24	31	45	26	26
E_l	1	1	1	1	1	1	1	3	2	1	1	1	1
W_l^1	0	0	0	0	0	0	0	0	0	0	0	0	0
W_l^2	0	0	0	0	1	0	1	1	0	0	1	0	0
W_l^3	1	0	0	0	1	1	2	2	1	1	2	0	0
Line	27	28	29	30	31	32	33	34	35	36	37	38	
Q_l	32	29	20	39	24	18	18	35	35	42	42	30	
E_l	1	1	1	1	3	1	1	1	1	1	1	1	
W_l^1	0	0	0	0	0	0	0	0	0	0	0	0	
W_l^2	0	0	1	0	0	1	1	0	0	0	0	0	
W_l^3	0	0	1	0	0	1	1	0	0	1	1	0	

4.2 Case study results for deterministic TMS

We started by solving the deterministic TMS for the situation W^1, where there are no delays, $W_l^s = 0$ for all lines l. The optimal value of the objective function obtained and the required CPU time are indicated in Table 3 and the optimal solution for the scheduling is shown in Figure 3.

Table 3 details the optimal values achieved for the objective function as well as the required CPU times for the experiments. The results of three tests with the deterministic model and a test with the extensive stochastic model are presented.

The result of the first deterministic simulation, where W_l^1 values are observed, is easy to understand since the model will always try to allocate the preferential weeks for each maintenance, as illustrated by Figure 3, while respecting scheduling constraints, and when these do not cause isolation of buses, division of the grid or loss of attending maximum demand in case of line failure.

Note that the result found is not the same as that found in [18]. This occurs because in the model proposed in [18], a maximum of two activities can occur in the same week, while in this project we use the limit of three activities per week. Therefore, the objective function result found in this model is improved since more activities can be assigned to their preferred weeks.

When analyzing the solution, it is clear that, motivated by various factors, there is an uncertainty about the duration of maintenance. And from there, it is evaluated that in addition to the result found so far, there are two other possible delay situations, as stated in Table 2 in rows W_l^2 and W_l^3 occur. The question that arises then is whether the optimal solution is vulnerable to changes in maintenance duration delays.

Table 3: Optimal solution of deterministic and stochastic computational experiments

	Deterministic*			Stochastic**
	W^1	W^2	W^3	$W^{s=1,2,3}$
Objective	4.613.629,6	4.614.232,9	4.614.498,1	4.613.890,5
CPU time (seconds)	346	1402	3116	32513

*Mathematical formulation described in Section 2.2.
**Mathematical formulation described in Section 2.3.

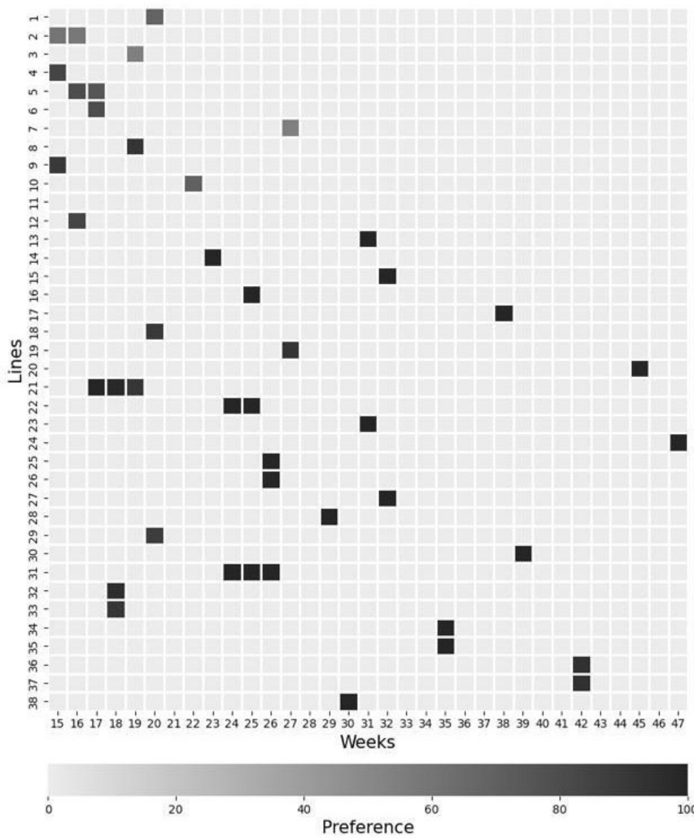

Figure 3: Optimal scheduling for the deterministic formulation with W_l^1

Thus, in the sequence, the deterministic TMS is solved again, but this time for situations where W^2 and W^3. The optimal values of the objective function obtained and the required CPU times are indicated in Table 3 and the optimal solutions for the scheduling are shown in Figures 4 and 5.

The results of these two deterministic simulations, with delays W^2 and W^3, allocate once again the tasks to their preferred weeks, as illustrated by Figures 4 and 5, as long as these allocations respect scheduling constraints, allow meeting the highest demand in case of a line failure and reject isolating the buses and dividing the grid. Furthermore, the objective function values indicated in Table 3 increase with the increase in the total amount of maintenance activities.

Comparing the scheduling results in Figures 3, 4 and 5, the start of maintenance in line 2 ranges from week 15 to 33, in line 3 ranges from week 19 to 27 and in line 22 ranges from 24 to 28. Besides, from Table 3, the optimal solution that represents the weekly preference of tasks and meets customers demands in a failure situation varies between 4.613.629, 6 and 4.614.498, 1. Finally, the optimal solution is significantly affected by the variations in maintenance duration.

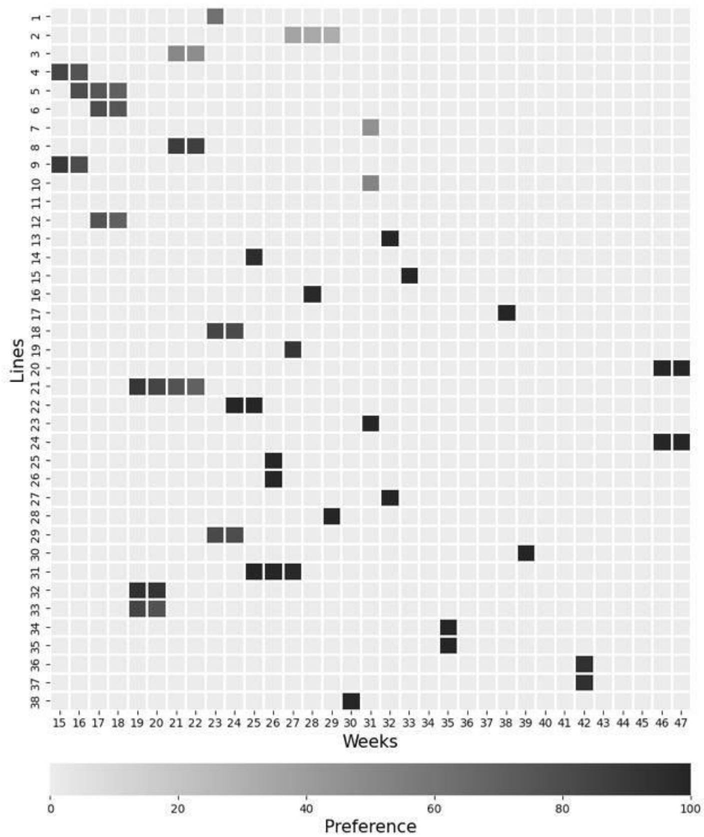

Figure 4: Optimal scheduling for the deterministic formulation with W_l^2

Unfortunately, it is not possible to predict maintenance delays in advance, and therefore it is impossible to determine precisely which of the delay scenarios will actually occur. And so, there is no such thing as perfect information for this problem. In this case, since there is a limit of tasks for the same week and several lines have maintenance preference for nearby weeks, if the maintenance is short, they tend to occur in weeks of their greatest preference and are arranged in closer weeks. As tasks becomes longer, more of them occur in less preferred weeks, however the total preference level value becomes greater due to more weeks of service.

4.3 Case study results for stochastic TMS

As we do not have enough information for an exact decision that would be optimal for all delay variations, we use the extensive stochastic model proposed in Section 2.3 to obtain a comprehensive decision.

The optimal value for the stochastic experiment is shown in Table 3 and the optimal solution for scheduling each task are indicated in Figure 6. As explained in Section 2.3, the start, end and minimum duration of the tasks are first-stage variables, indicated in blue in Figure 6 and the delayed weeks are second-stage variables, indicated in orange if connected to the scenario 2 and green if connected to scenario 3.

Figure 6 presents the stochastic solution, thus, it is clear that the start time for each of the tasks is common regardless of the scenario, that is, for any possibility of delay foreseen in the scenarios, the starting week is not changed. And then, considering all the possible scenarios handled, certain tasks may not experience delays and certain may experience long delays. The interval between the starting

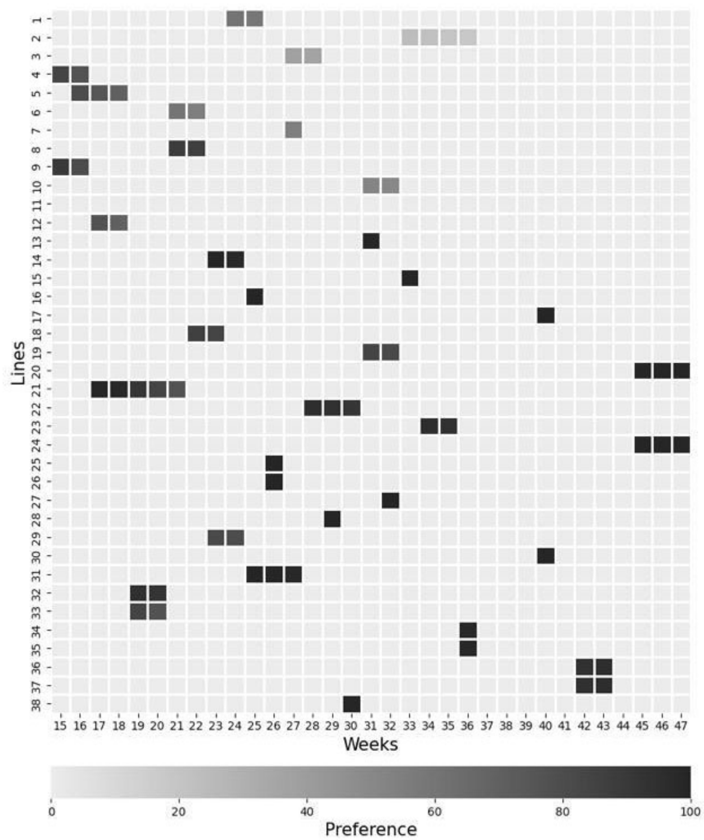

Figure 5: Optimal scheduling for the deterministic formulation with W_l^3

weeks of tasks that may experience delay is increased, as a result some tasks occur in less preferred weeks. Any interval between tasks would be better projected if the uncertain information was known.

In this result, it is evident that the planned delay is not in fact the actual delay, and that it is never possible to be sure which delay will materialize. This occurs for all stochastic models, planned uncertainties do not necessarily materialize as projected. In conclusion, it is impossible to find a perfect solution for all situations.

This TMS model cannot be treated by some typically stochastic approaches. For example, considering that in our model the delay of the activity is only known after the start of the initiated activity, or even more precisely at the moment when the task does not finish as planned, the approach known as wait and see cannot be used. Because for this method it would be necessary for the delay of each of the tasks to be known just before the start of the activity itself, and it would be enough if it were known in the very beginning week.

Another approach that is not applicable for this TMS would be to forget the delay uncertainties and treat them as a delay with an expected value. Thus, for example, all delays would be treated by their mean values. But, since the delay information is only known when the task is not completed and still, it is not known how long it lasts. If a delay lasts less than or equal to the mean value, tasks continue as planned. However, if the delay is greater than planned, a failure occurs and it is not possible to proceed with the rest of the planned schedule.

If all delay information were known at the time of designing, it would be possible to determine and choose the perfect scenario for the planning of each year. From the difference between having the correct information and using stochastic scenarios, the Expected Value of Perfect Information (EVPI)

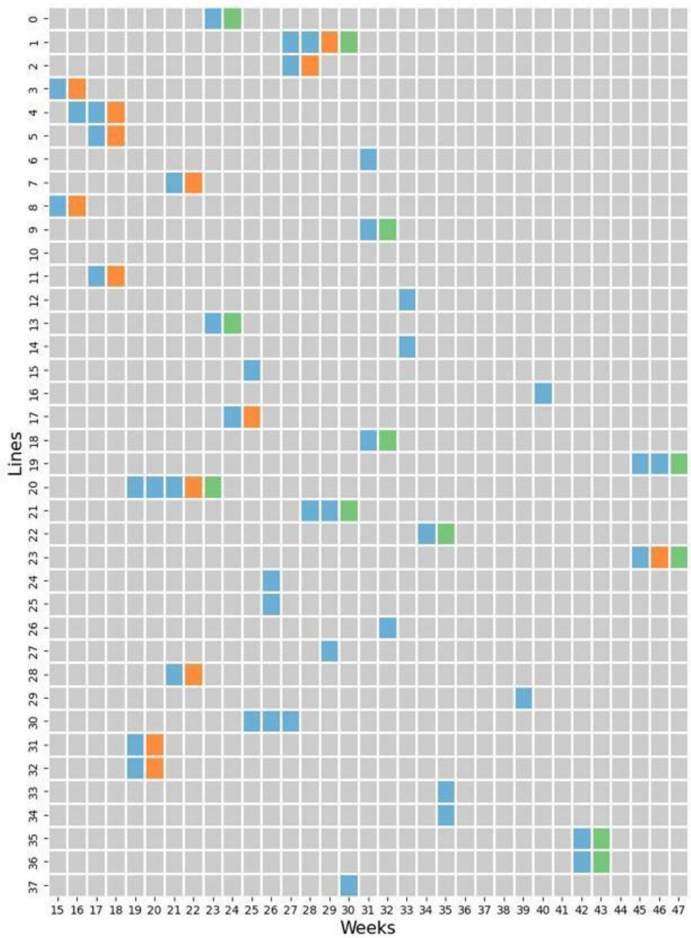

Figure 6: Optimal scheduling for the stochastic formulation with $W_l^{s=1,2,3}$

is obtained. According to [2], the value of EVPI indicates the maximum amount any decision maker would spend on obtaining accurate information instead of working with the uncertain data. If the occurrence of scenarios 1, 2 and 3 alternates, for example, over ten years in the proportion indicated in Section 4.1 and this information is known in advance, in this case the average objective value per year would be:

$$0,5 \cdot 4.613.629,6 + 0,3 \cdot 4.614.232,9 + 0,2 \cdot 4.614.498,1 = 4.613.984,3$$

which is better than the stochastic objective value shown in Table 3, and the EVPI: $4.613.984,3 - 4.613.890,5 = 93.8$. Note that the EVPI considers the values of the objective functions. But, in this scheduling problem with delays, the values of the optimal solutions for the allocation variables are equally relevant.

Assuming that the approach to this problem is one of risk aversion, a worst-case planning can be adopted as procedure. In this case, the worst scenario would be the one with the highest number of weekly task delays, scenario number three. Leading to a counter intuitive situation where the greatest result of maximizing the preference of allocation for each task is precisely in the worst scenario condition. So, for the case of risk aversion the deterministic scenario three solution of Figure 5 is optimal and would be adopted every year for ten years. Thus, by fixing the first-stage variables of the stochastic model according to its values from Figure 5, it is obtained 4.613.875,2 as stochastic objective solution. This result represents the annual achievement of preferences when a period of ten years is considered and imagining that the maintenance preference is repeated every year in the same weeks.

It can be seen that this solution is worse than the stochastic solution. It is also worth mentioning that the weekly preference level may vary over the years and also that not necessarily all lines undergo maintenance every year.

4.4 Computational costs

Finally, we present a discussion about the computational cost for solving our proposed extensive stochastic model and also the advantages of applying our proposed algorithm for solving the same model.

In Table 3 we can see that the CPU times increase as the difficulty of the model increases.

We also present, as we did in our previous work [18], a computational experiment independent from the ones presented in the previous sections, to evaluate the efficiency of the algorithm proposed in Section 3. Figure 7 displays the results, the columns in brown and blue indicate the standard CPLEX method of resolution and the algorithm proposed in Section 3, respectively. Each pair of columns with different colors represents a set of one line or a group of lines that undergo maintenance, the larger set AL includes all grid lines except the one that isolates the bus 7 and leads to an infeasible problem, line 11.

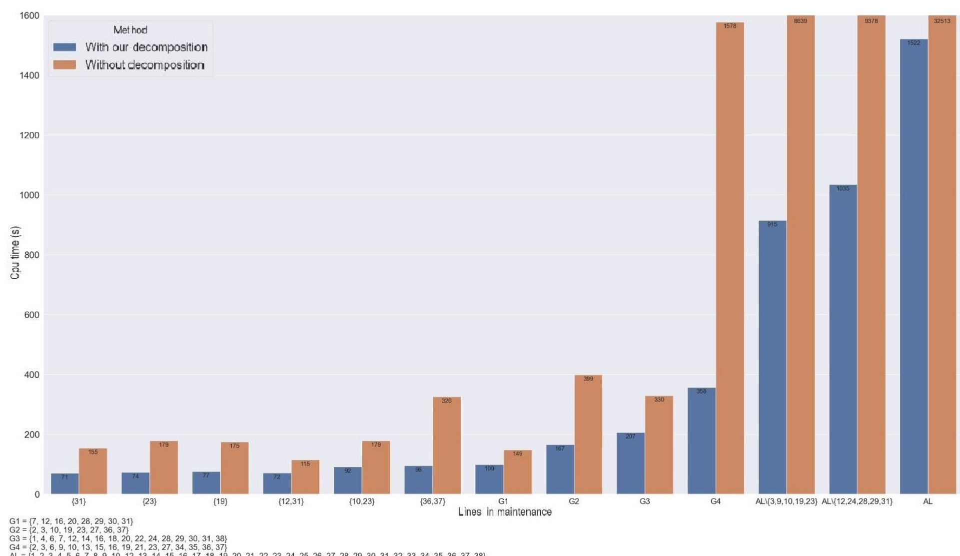

Figure 7: CPU times (seconds) for groups of lines that undergo maintenance

The CPU times have normally lower values when a small number of lines are disconnected, and the results are very similar for both methods. The CPU times grow with the increase in the quantity of lines that are disconnected, this growth is different when the two methods are compared, the new algorithm from Section 3 grows slower and thus indicating better performance.

However, it is important to note that for larger instances of Figure 7, where a great number of outages take place, the optimal value reached for the objective solution is better when solving the extensive model using the standard CPLEX method as indicated in Table 4. In Table 4 are displayed the three larger instances of Figure 7, for each of them we can see the two different optimal objective values reached when solving the stochastic extensive formulation using standard CPLEX resolution, without decomposition, or, with our decomposition proposed in Section 3.

Table 4: Optimal objective solution of stochastic computational experiments

	Instances		
	$AL \setminus \{3, 9, 10, 19, 23\}$	$AL \setminus \{12, 24, 28, 29, 31\}$	AL
Standard CPLEX	4.614.098,5	4.613.173,1	4.613.890,5
Algorithm Section 3	4.613.817,0	4.613.105,9	4.613.704,0

It is worth discussing what happens when solving these larger instances using CPLEX in these problems with integer variables. First, there is a standard relative tolerance that compares, for each branch-and-bound node, the values between the current best bound and the best integer solution, the process ends in this optimal solution found when the relative gap is within the previously stipulated value. For this paper, it was determined that this gap should be zero, and thus, the best possible objective is achieved. When the problem is solved without the algorithm proposed in Section 3, it takes a long time, as shown in Figure 7, but the best solution is always found. When the problem is solved with the algorithm, which uses the built-in benders decomposition from CPLEX, great progress can be seen in the resolution of the first nodes of the tree, but after a while the optimal values of the nodes stop improving significantly. Thus, the solver runs out of memory because it cannot achieve an optimal result within the stipulated zero relative gap and within the computer's available memory. In these cases, as the result achieved is already very good, we adopted the solver's standard gap in place of the zero gap, the results achieved are exactly those shown in Table 4.

Finally, it takes around 25 minutes to schedule the tasks on the 37 transmission lines with a satisfactory optimality, so the CPU times of our proposed algorithm is suitable and interesting even when solving the largest instances of the problem. We conclude that, compared with the off-the-shelf CPLEX resolution, the specific new algorithm we propose is more efficient, even if the optimal solution achieved for the largest cases is smaller, since it respects an acceptable tolerance.

5 Conclusion

This paper deals with the scheduling of preventive maintenance on power transmission lines for a one-year period. We propose an original stochastic MILP formulation for the TMS problem that, apart from the essential constraints from the literature and from [18], also incorporates the uncertainty on the duration of preventive maintenance tasks. This new extensive stochastic formulation is of large-scale and very demanding for the CPLEX solver, therefore, in addition, we propose a specially designed algorithm to solve more efficiently the model to optimality or optimality with a default gap tolerance for the case of larger instances. We executed computational experiments using the IEEE-24-RTS transmission grid and the results illustrate that the model reaches its intention of scheduling effective optimal annual maintenance that is insensitive to the uncertainty on the duration of the delays. Furthermore, we confirm that using our specialized algorithm the model can be resolved, in an acceptable time, to optimality or, for the larger and challenging instances, to optimality with a standard gap tolerance.

Appendix A TMS model proposed in [18]

The proposed model is formulated as follows:

$$\max \sum_{t \in \Lambda} \Big(\sum_{k \in \Gamma} \sum_{c \in \Delta} \hat{d}^t_{ck} + \sum_{l \in \Delta} A^t_l y^t_l \Big) \tag{22}$$

$$\text{s.t.} \quad y^t_l = 0 \qquad \forall\, l \in \Delta,\ t \in \Lambda^I \tag{23}$$

$$\sum_{t \in \Lambda} y^t_l = 0 \qquad \forall\, l \in \Delta^{NM} \tag{24}$$

$$\sum_{l\in\Delta^M} y_l^t \leq L^t \qquad \forall\ t \in \Lambda \tag{25}$$

$$\sum_{t\in\Lambda} y_l^t = 1 \qquad \forall\ l \in \Delta^{M1} \tag{26}$$

$$\sum_{t\in\Lambda} y_l^t = 2 \qquad \forall\ l \in \Delta^{M2} \tag{27}$$

$$y_l^{t=2} \geq y_l^{t=1} \qquad \forall\ l \in \Delta^{M2} \tag{28}$$

$$y_l^t \geq y_l^{t-1} - y_l^{t-2} \qquad \forall l \in \Delta^{M2}, t \in 3..T \tag{29}$$

$$\sum_{t\in\Lambda} y_l^t = 3 \qquad \forall\ l \in \Delta^{M3} \tag{30}$$

$$y_l^{t=2} \geq y_l^{t=1} \qquad \forall\ l \in \Delta^{M3} \tag{31}$$

$$y_l^{t=3} \geq y_l^{t=2} \qquad \forall\ l \in \Delta^{M3} \tag{32}$$

$$y_l^t \geq y_l^{t-1} - \frac{1}{2}(y_l^{t-2} + y_l^{t-3}) \qquad \forall l \in \Delta^{M3}, t \in 4..T \tag{33}$$

$$y_l^t - y_u^t = 0 \qquad \forall t \in \Lambda, (l,u) \in \Xi, l \neq u \tag{34}$$

$$gen_k^t - D_k^t = \sum_{l\in\Delta} S_{kl}\ f_l^t \qquad \forall k \in \Gamma, t \in \Lambda \tag{35}$$

$$f_l^t = B_l \sum_{k\in\Gamma} S_{lk}^T\ \theta_k^t \qquad \forall l \in \Delta^{NM}, t \in \Lambda \tag{36}$$

$$\frac{f_l^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T\ \theta_k^t \geq -F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda \tag{37}$$

$$\frac{f_l^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T\ \theta_k^t \leq F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda \tag{38}$$

$$f_l^t \geq -F_l^{Lim} + F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda \tag{39}$$

$$f_l^t \leq F_l^{Lim} - F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda \tag{40}$$

$$\theta_k^t \geq -\pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda \tag{41}$$

$$\theta_k^t \leq \pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda \tag{42}$$

$$\theta_k^t = 0 \qquad \forall k \in \Gamma^{R}, t \in \Lambda \tag{43}$$

$$gen_k^t = 0 \qquad \forall\ k \in \Gamma^{NG}, t \in \Lambda \tag{44}$$

$$gen_k^t = \sum_{i\in\Upsilon^U} g_{ik}^t \qquad \forall k \in \Gamma^{G}, t \in \Lambda \tag{45}$$

$$g_{ik}^t \leq G_{ki}^+ \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{46}$$

$$g_{ik}^t \geq G_{ki}^- \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{47}$$

$$g\hat{e}n_{ck}^t - \hat{d}_{ck}^t = \sum_{\substack{l\in\Delta \\ l\neq c}} S_{kl}\ \hat{f}_{cl}^t \qquad \forall k \in \Gamma, t \in \Lambda, c \in \Delta \tag{48}$$

$$\hat{f}_{cl}^t = B_l \sum_{k\in\Gamma} S_{lk}^T\ \hat{\theta}_{ck}^t \qquad \forall l \in \Delta^{NM}, t \in \Lambda, c \in \Delta,\ c \neq l \tag{49}$$

$$\frac{\hat{f}_{cl}^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T \hat{\theta}_{ck}^t \geq -F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda, c \in \Delta, c \neq l \tag{50}$$

$$\frac{\hat{f}_{cl}^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T\ \hat{\theta}_{ck}^t \leq F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda, c \in \Delta, c \neq l \tag{51}$$

$$\hat{f}_{cl}^t \geq -F_l^{Lim} + F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda, c \in \Delta, l \neq c \tag{52}$$

$$\hat{f}_{cl}^{t} \leq F_l^{Lim} - F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda, c \in \Delta, l \neq c \tag{53}$$

$$\hat{\theta}_{ck}^{t} \geq -\pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda, c \in \Delta \tag{54}$$

$$\hat{\theta}_{ck}^{t} \leq \pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda, c \in \Delta \tag{55}$$

$$\hat{\theta}_{ck}^{t} = 0 \qquad \forall k \in \Gamma^{R}, t \in \Lambda, c \in \Delta \tag{56}$$

$$g\hat{e}n_{ck}^{t} = 0 \qquad \forall k \in \Gamma^{NG}, t \in \Lambda, c \in \Delta \tag{57}$$

$$g\hat{e}n_{ck}^{t} = \sum_{i \in \Upsilon^U} \hat{g}_{ik}^{ct} \qquad \forall k \in \Gamma^{G}, t \in \Lambda, c \in \Delta \tag{58}$$

$$\hat{g}_{ik}^{ct} \leq G_{ki}^{+}\ n_{ck}^{t} \qquad \forall k \in \Gamma^{G}, i \in \Upsilon^U, t \in \Lambda, c \in \Delta \tag{59}$$

$$\hat{g}_{ik}^{ct} \geq G_{ki}^{-}\ n_{ck}^{t} \qquad \forall k \in \Gamma^{G}, i \in \Upsilon^U, t \in \Lambda, c \in \Delta \tag{60}$$

$$\hat{d}_{ck}^{t} \leq D_k^t \qquad \forall k \in \Gamma, t \in \Lambda, c \in \Delta \tag{61}$$

$$n_{ck}^{t} = 1 \qquad \forall (c,k) \in \Omega^{Ok}, t \in \Lambda \tag{62}$$

$$n_{ck}^{t} = 1 - y_{(M_{ck})}^{t} \qquad \forall (c,k) \in \Omega^{N1}, t \in \Lambda \tag{63}$$

$$\hat{y}_{(O_l,R_l)}^{t} = y_l^t \qquad \forall l \in \Delta, t \in \Lambda^C \tag{64}$$

$$\hat{y}_{(R_l,O_l)}^{t} = y_l^t \qquad \forall l \in \Delta, t \in \Lambda^C \tag{65}$$

$$(1 - \hat{y}_{ij}^{t}) U_{ij} \geq w_{ij}^t \qquad \forall i \in \Gamma, j \in \Gamma, t \in \Lambda^C, j \neq i \tag{66}$$

$$w_{ij}^t \geq h_{ij}^{kt} \qquad \forall i,j \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq i \tag{67}$$

$$\sum_{\substack{j \in \Gamma \\ j \neq i}} h_{ij}^{kt} = 1 \qquad \forall i \in \Gamma^0, k \in \Gamma, t \in \Lambda^C, k \neq i \tag{68}$$

$$\sum_{\substack{i \in \Gamma \\ i \neq j}} h_{ij}^{kt} = 0 \qquad \forall j \in \Gamma^0, k \in \Gamma, t \in \Lambda^C, k \neq j \tag{69}$$

$$\sum_{\substack{i \in \Gamma \\ i \neq j}} h_{ij}^{kt} = \sum_{\substack{i \in \Gamma \\ i \neq j}} h_{ji}^{kt} \qquad \forall j,k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{70}$$

$$\sum_{\substack{i \in \Gamma \\ i \neq j}} h_{ij}^{kt} \leq 1 \qquad \forall j,k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{71}$$

$$\sum_{\substack{i \in \Gamma \\ i \neq j}} h_{ji}^{kt} \leq 1 \qquad \forall j,k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{72}$$

$$\sum_{\substack{i \in \Gamma \\ i \neq j}} h_{ij}^{kt} = 1 \qquad \forall j \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{73}$$

$$\sum_{\substack{j \in \Gamma \\ j \neq i}} h_{ij}^{kt} = 0 \qquad \forall i \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, i \neq k \tag{74}$$

$$h_{ij}^{kt} + h_{ji}^{kt} \leq 1 \qquad \forall i,j \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq i \tag{75}$$

$$y_l^t \in \{0,1\} \qquad \forall l \in \Delta, t \in \Lambda \tag{76}$$

$$g_{ik}^t \in \mathbb{R}^+ \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{77}$$

$$gen_k^t \in \mathbb{R}^+ \qquad \forall k \in \Gamma, t \in \Lambda \tag{78}$$

$$\hat{g}_{ik}^{ct} \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{79}$$

$$g\hat{e}n_{ck}^t \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma, t \in \Lambda \tag{80}$$

$$\hat{d}_{ck}^t \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma, t \in \Lambda \tag{81}$$

$$n_{ck}^t \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma^G, t \in \Lambda \tag{82}$$

$$\hat{y}_{ij}^t \in \mathbb{R}^+ \qquad \forall i, j \in \Gamma, t \in \Lambda^C, j \neq i \qquad (83)$$

$$w_{ij}^t \in \mathbb{R}^+ \qquad \forall i, j \in \Gamma, t \in \Lambda^C \qquad (84)$$

$$h_{ij}^{kt} \in \{0, 1\} \qquad \forall i, j, k \in \Gamma, t \in \Lambda^C \qquad (85)$$

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