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Offline-online retail collaboration via pickup partnership

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Abstract : We study an increasingly popular retail practice called *pickup partnership* that allows online retailers to offer an in-store pickup service by partnering with a physical store. In practice, online retailers use two policies for such partnerships: (i) paying a fixed fee for each pickup order to the offline partner (*fixed fee policy*), or (ii) offering a coupon with each pickup order to customers to be redeemed at the offline partner's store (*coupon policy*). Our goal is to examine each policy and identify the best approach for online retailers to adopt in a pickup partnership. We develop a stylized model that captures the key features of a pickup partnership. We find that although the coupon policy allows the online retailer to gain greater market coverage compared to the fixed fee policy, it does not always increase the online retailer's profit. We characterize when an online retailer should establish a pickup partnership with an offline partner using the fixed fee versus the coupon policies. We also find that these two policies may entail inefficiencies when the incentives of the two parties are not aligned. To alleviate such inefficiencies, we prescribe a new policy that aims to align both parties' incentives. This paper provides the first theoretical analysis of the emerging business model of in-store pickup partnerships and serves as a prescription for online retailers who want to establish such partnerships. By proposing a new policy, it also strives to make pickup partnerships more efficient than current practices.

Keywords : In-store pickup service, partnership, omnichannel retailing, retail operations

1 Introduction

Customer demand for increasingly convenient shopping (Boston Retail Partners 2021) has exacerbated the competition between pure online retailers and multichannel retailers. To stay ahead in this competition, taking advantage of their physical presence to enhance the customer experience (Chen et al. 2021), many multichannel retailers have started offering in-store pickup services that allow customers to pick up online orders in physical stores(Gao and Su 2017). Due to their convenience, in-store pickup services have rapidly become popular among customers. In 2020, in-store pickup sales in the United States doubled the previous year's total, and this trend is expected to continue to grow at an annual rate of at least 15% until 2024 (Chevalier 2021). To remain competitive and respond to the growing customer demand for in-store pickup services, pure online retailers have recently started forming strategic partnerships with brick-and-mortar retailers (referred to as offline partners in the rest of this paper), which is termed *pickup partnership*.

A pickup partnership enables an online retailer to use an offline partner's stores as pickup locations for customers who prefer to pick up their online orders from a nearby store at no additional shipping cost. Under this fulfillment option, the online retailer ships the customer order to the offline partner store chosen by the customer and notifies the customer when the order is ready for pickup. When the customer arrives for pickup, the offline partner handles the process using its own staff and informs the online retailer at the end of the process. Amazon Hub Counter (AHC) is a widely known example of such pickup partnerships. Through AHC, physical retailers collaborate with Amazon to make their stores assisted-pickup locations for Amazon orders. Examples include Rite Aid, GNC, Health Mart, and Stage Stores in the United States, NEXT in the U.K., Librerie Giunti al Punto, Fermopoint, and SisalPay in Italy, and ParcelPoint in Australia. Similarly, Uniqlo, a Japanese fashion retailer, has partnered with 5,700 7-Eleven convenience stores in Tokyo to offer in-store pickup services.[1]

Pickup partnerships promise several benefits to both online retailers and offline partners.For online retailers, offering a convenient in-store pickup option should translate into increased sales. In addition, compared to direct-delivery shipments, in-store pickup fulfillment is likely to reduce online retailers' freight costs due to the possibility of pooling several orders and hence decreasing (expensive) last-mile delivery costs (Morganti et al. 2014). For offline partners, in-store pickups induce additional foot traffic that can generate increased revenue through cross-selling. It has been reported that 45% of customers who used in-store pickup services made an in-store purchase during the pickup visit (UPS 2015). In addition, online retailers may compensate offline partners for handling each pickup order, creating an additional revenue stream for offline partners. These benefits suggest that the trend of forming pickup partnerships is likely to persist or even expand in the future.

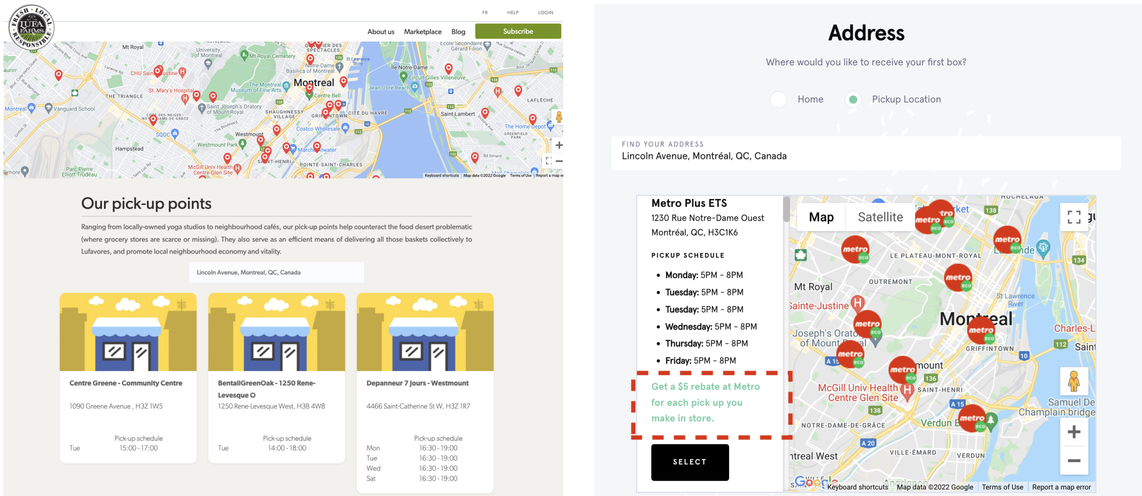
This trend raises the question of the best way to establish pickup partnerships between online retailers and offline partners. In practice, we observe two different policies (see Figure 1 for illustrative examples). The first is a *fixed fee policy* under which the online retailer pays a fixed commission to the offline partner for each in-store pickup order (Morganti et al. 2014, Fang et al. 2019).[2] For example, this type of pickup partnership has been established between Maturin and LOCO, between Lufa Farms (a Canadian online grocery retailer) and local stores as illustrated in Figure 1(a), and between Amazon and thousands of AHC retailers.[3] The second is a *coupon policy* under which the online retailer issues a coupon to the customers who select the in-store pickup delivery option. Customers can redeem this coupon to make a discounted purchase at the offline partner's store. If the coupon is redeemed, the online retailer will then reimburse the offline partner for the discounted amount. The pickup partnership between Cookit (a meal-kit retailer) and Metro (a Canadian grocery retailer) is based on the coupon policy, as illustrated in Figure 1(b). It is worth noting that even though the compensation payment for each in-store pickup order is similar between the two policies, the recipient of the payment

[1] https://ww.fashionnetwork.com/news/uniqlo-japan-launches-in-store-pickup-service-with-7-eleven,625015.html

[2] https://join.healthmart.com/pharmacy-marketing-and-promotions/becoming-an-amazon-hub-counter/

[3] https://www.pudoinc.com/member-benefits/pudopoint-counter/

depends on the policy. Under the fixed fee policy, the online retailer pays the offline partner, whereas under the coupon policy, the payment is made to the customer, making the two policies structurally different.



(a) Lufa farm and local stores (b) Cookit and Metro

Figure 1: Illustration of the fixed fee policy (Left) and the coupon policy (Right)

When establishing a pickup partnership, should online retailers pay a fixed fee directly to the offline partner or incentivize customers by offering a coupon? We are not aware of any academic study on pickup partnerships. From a practitioner perspective, the fixed fee policy may be more desirable as it generates a readily visible revenue stream for offline partners through the fixed commission paid for each in-store pickup order. At the same time, the coupon policy may also be desirable since incentivizing customers will likely drive more traffic to offline partners' stores, resulting in more cross-selling opportunities. Overall, it is not clear which of the two policies is more beneficial. This leads to our first two research questions: *How should an online retailer choose between fixed fee and coupon policies when establishing a pickup partnership? What type of online retailer is more suitable for the fixed fee policy versus the coupon policy?*

The presence of fixed fee and coupon policies in practice does not necessarily imply that they always help retailers establish an efficient pickup partnership. After all, a pickup partnership is tantamount to a contract between two parties. The supply chain contract literature (see Tsay et al. 1999) has established that inefficient contracts are common in practice (Loch and Wu 2008) and arise due to misaligned incentives between the two parties (Pavlov et al. 2022). From this perspective, just like inefficient contracts, pickup partnerships may also entail inefficiencies when the incentives between the online retailer and the offline partner are misaligned. This leads to our next two research questions: *Do fixed fee and coupon policies result in an inefficient pickup partnership? If so, can we propose an alternative policy that mitigates such inefficiencies?*

To answer our research questions, we develop a stylized model that captures the key features of a pickup partnership. Specifically, we consider an online retailer who sells a product only via an online channel. The online retailer contemplates the opportunity to offer an in-store pickup service to her customers through a partnership with an offline partner. The online retailer's objective is to maximize her profit while ensuring that the proposed partnership is also beneficial to the offline partner. When the pickup partnership is established, customers strategically decide between the direct-delivery and in-store pickup options to maximize their utility. We first analytically examine the effect of the partnership on the demand and profits of both partners under each policy. We then identify conditions under which each policy is beneficial; that is, both partners earn higher profits (in a non-strict sense) under the pickup partnership than they would without that. Armed with these results, we compare the two policies and characterize when each policy is optimal for the two parties. We then infer what type of online retailers are more suitable for the fixed fee policy versus the coupon policy when establishing a pickup partnership. Lastly, we examine the conditions under which the two policies generate inefficiencies and prescribe an alternative policy, termed the *hybrid policy*, to mitigate those inefficiencies.

Our study makes several contributions. First, we find that establishing a pickup partnership (regardless of policy) affects the demand for the online retailer's product in two ways: (i) the pickup partnership enables the online retailer to expand its market coverage due to the increased convenience of the direct-delivery option; (ii) the in-store pickup option incentivizes some existing customers to switch their delivery mode to the in-store pickup option. While the former effect (i.e., market expansion) on demand increases the online retailer's profit, the latter effect (i.e., demand shift) can actually hurt the online retailer's profit if the profit margin from in-store pickup orders is lower than the profit margin from direct-delivery orders. Hence, the pickup partnership will be beneficial as long as these two demand streams result in a net profit gain. Second, our results reveal that the two demand streams induced by the partnership are more substantial under the coupon policy. Indeed, beyond offering the convenience of the in-store pickup option (as in the fixed fee policy), the coupon policy also incentivizes additional customers to use the in-store pickup option to take advantage of the coupon. These additional customers do not necessarily increase the online retailer's profit, especially when the in-store pickup fulfillment is less profitable, on a per item basis, than the direct-delivery fulfillment. Thus, choosing between the fixed fee and coupon policies when establishing a pickup partnership requires a careful assessment of both partners' cost structures. Third, when examining the pickup partnership with respect to the offline partner's cost structure, we find that the online retailer should use the coupon policy if the offline partner can manage in-store pickups at a low handling cost. A low handling cost allows the online retailer to pay a low compensation per in-store pickup order, hence increasing the profit margin of in-store pickup orders. Consequently, the additional customers attracted by the coupon policy (relative to the fixed fee policy) will increase the online retailer's profit. When the offline retailer's handling cost is moderate, we show that the fixed fee policy is the best option for the partnership, but when the offline partner's handling cost is high, a partnership becomes plausible only with high compensation, so that the online retailer is better off not establishing a pickup partnership. When examining the pickup partnership with respect to the online retailer's cost structure, we find that a pickup partnership is not profitable for online retailers with low direct-delivery fulfillment costs or high in-store pickup fulfillment costs. Otherwise, the fixed fee policy is more suitable for online retailers with moderate direct-delivery fulfillment costs, moderate in-store pickup fulfillment costs, or low-priced products, whereas the coupon policy is more beneficial for online retailers with high direct-delivery fulfillment costs, low in-store pickup fulfillment costs, or high-priced products.

Finally, while our model suggests that the two policies used in practice can ensure a profitable partnership, it remains unclear whether these policies allow online retailers to fully unlock the potential benefits of such partnerships. In fact, we find that both policies can be inefficient in the sense that selecting the optimal policy (either fixed fee or coupon) for the partnership comes with an opportunity cost for the online retailer. In such cases, the online retailer has to establish the partnership under a suboptimal compensation value. To mitigate such inefficiencies, we propose a new hybrid policy that leverages the features of both the fixed fee and the coupon policies. In particular, the hybrid policy allows the online retailer to split the compensation such that a portion is paid to the offline partner as a fixed fee commission, with the remainder offered as a coupon to customers. We find numerically that inefficient pickup partnerships occur frequently and that the profit improvement generated from the hybrid policy can be substantial.

The rest of the paper is organized as follows. In Section 2, we review the related literature, and in Section 3, we formalize our model. We then analyze the model and derive several analytical results in Section 4. We consider various extensions in Section 5. In Section 6, we identify inefficiencies induced by the current policies and propose a new policy. Finally, we conclude and outline the managerial implications of our results in Section 7.

2 Literature review

This paper is related to three streams of literature: in-store pickup services, retail partnerships, and coupon promotions.

In-store pickup services

The recent growth of in-store pickup services (e.g., click-and-collect and ship-to-store) has led to an increase in research on that topic. The literature has examined two types of in-store pickup services: buy-online-pickup-in-store (BOPIS) and ship-to-store (STS). The major difference between the two is the order fulfillment point. BOPIS orders are fulfilled using store inventory and can thus only be placed for products available in a store (Gao and Su 2017), whereas STS orders are fulfilled using the distribution center inventory and can be placed for any product available online, regardless of whether it is stocked in any store (Ertekin et al. 2021).

Gallino and Moreno (2014) empirically show that even though using BOPIS can reduce online sales, the sales generated from the additional store traffic can make retailers better off when offering such services. Focusing on individual products, Gallino et al. (2017) find that STS services may shift sales from high-selling products to low-selling products. In the same vein, Ertekin et al. (2021) find that STS has a heterogenous effect on sales of online-only products versus products available both online and offline. The authors conclude that considering the STS effect when choosing channel(s) to sell a product can improve the performance of in-store pickup services. Akturk and Ketzenberg (2021) evaluate the competitive impact of BOPIS. They show that both online and store sales at a focal retailer are adversely affected after the competitors' launch of a BOPIS service. Focusing on customer behavior, Song et al. (2020) find that BOPIS has a positive effect on offline purchase frequency and on online purchase amounts. Glaeser et al. (2019) demonstrate that the location of the pickup stores can have a significant effect on BOPIS profitability.

Among analytical studies, Gao and Su (2017) examine the impact of BOPIS on store operations. The authors find that despite enabling retailers to increase demand from new customers, BOPIS may not be suitable for products that sell well in stores. Hu et al. (2022) demonstrate that retailers can leverage the additional demand induced by BOPIS to improve their store fill rates. Ertekin et al. (2021) illustrate that when implementing STS, retailers should offer easy-to-substitute products only online and difficult-to-substitute products both online and in stores. Similarly, Cao et al. (2016) find that in-store pickups may not be suitable for all products. Finally, Gao et al. (2022) show that it might be optimal for retailers to reduce their physical store presence under BOPIS.

The studies in this literature primarily examine in-store pickup services when the retailer owns both the online and offline channels. In contrast, our paper investigates in-store pickup services when offered by a pure online retailer that partners with an offline store. Thus, some of the highlighted benefits of BOPIS or STS for multichannel retailers (e.g., cross-selling, additional store traffic) will not be present for online retailers under the pickup partnership. More importantly, unlike a pickup partnership, which can be implemented using different policies, traditional in-store pickup services (whether BOPIS or STS) are quite standard across retailers. Therefore, existing studies on traditional in-store pickup services cannot help identify which policy retailers should use when establishing a pickup partnership. Overall, we contribute to this literature by developing a theoretical model (i) to demonstrate when and how a pickup partnership should be established, (ii) to identify how online retailers should choose between fixed fee and coupon policies, and (iii) to propose an alternative policy to improve the pickup partnership efficiency.

Retail partnerships

These kinds of partnerships have been studied in several settings, ranging from supply chain contracts and coordination (see Cachon and Lariviere 2005, for a comprehensive review) to coalition and coopetition contracts (Nagarajan and Sošić 2007, Cohen and Zhang 2022, Yuan et al. 2021). Our work is closely related to a growing stream in this literature that studies offline-online partnerships to enhance omnichannel retailing offerings, such as buy online, return in-store (Hwang et al. 2021, Nageswaran et al. 2021) and search offline, buy online (i.e., showrooming) (Dan et al. 2021). In this stream, Nageswaran et al. (2021) theoretically examine the potential of a return partnership between a pure online retailer and an offline partner that serves as the in-store return location for the online retailer. The authors find that such a return partnership can be formed either when there is only a small product assortment overlap between the two parties, or when the offline partner has a small number of physical locations. Hwang et al. (2021) empirically show that such return partnerships generate additional sales for the offline partner. Most studies on offline showrooming focus on a single company (e.g., Bell et al. 2018, Gao et al. 2022). Dan et al. (2021) analytically study how an online retailer should choose between competing and non-competing offline retailers to offer a physical showrooming service and how the type of offline retailer (competing or non-competing) can affect an online retailer's pricing strategy under an exogenous commission fee. In these studies, the focus is primarily on the types of offline partners that should be selected as partners. By contrast, our study focuses on how online retailers should select partnership policies according to offline partners' characteristics. Even when we extend our review to the broader retail partnership literature, we could not find any study with guidance on how online retailers and offline partners should establish a pickup partnership. We contribute to this stream by studying the impact of the two pickup partnership policies on the decisions and payoffs of the key stakeholders.

Coupon promotions

There is a large literature stream on coupon redemption (e.g., Reibstein and Traver 1982, Danaher et al. 2015), coupon effects on customer behavior (e.g., Narasimhan 1984, Neslin et al. 1985, Heilman et al. 2002, Su et al. 2014), and marketing effects and optimal coupon scheduling (e.g., Sethuraman and Mittelstaedt 1992, Reimers and Xie 2019, Baardman et al. 2019). We position our work with respect to papers that consider the role of coupon promotions in channel coordination. Among those, Martin-Herran and Sigué (2015) find that manufacturers prefer coupon promotions over a cooperative pricing strategy. Li et al. (2020) evaluate how issuing coupons by either manufacturers or retailers can affect the supply chain profit. Pauwels et al. (2011) show that offering online promotions can also increase the demand for the offline channel, creating a channel synergy effect. Despite all these valuable contributions, there is no study that leverages coupons to facilitate a pickup partnership between an online retailer and an offline partner. Our study contributes to this literature by demonstrating how a coupon promotion can be used to design an effective mechanism for pickup partnerships.

3 Model description

In this section, we develop a stylized model to characterize the key features of a pickup partnership between an online retailer and an offline partner. In the following subsections, we describe our modeling framework, introduce a baseline policy in which the pickup partnership does not exist, and consider two different pickup partnership scenarios by building on the baseline policy.

3.1 Modeling framework

The model consists of an online retailer that sells a product through its online channel at price p. Consistent with the literature that models interactions between retailers and customers using the circular location model (e.g., Balasubramanian 1998, Shulman et al. 2009, Gao et al. 2022), we assume that the online retailer serves customers who are uniformly distributed on the circumference of a circular city with a circumference of one (Salop 1979). The online retailer's warehouse is located at the center of the circular city and is thus equidistant from all customers. Without loss of generality, we assume that the size of the market is normalized to one.

To provide customers with an in-store pickup service for online orders, the online retailer considers establishing a partnership with an offline partner. When such a partnership does not exist, the online retailer can only offer a direct-delivery option to its customers under which orders are shipped directly to customers. If a pickup partnership is established, in addition to the direct-delivery option, customers are now able to select a free in-store pickup option. With this option, the online retailer ships orders to the offline partner, and customers pick up their orders by visiting the offline partner at their convenience. We assume that the offline partner's store is randomly located on the circumference of the circle (In Section 5.2, we extend the model where the offline partner has more than one store). We also assume that the product sold by the online retailer is not offered by the offline partner.

Customers : We assume that customers make purchasing decisions based on their utility. The valuation of product is v for all customers. If customers opt for the direct-delivery option, they incur a "hassle" cost h_o that includes both the shipping cost and the inconvenience of waiting for the delivery. We assume that customers are heterogeneous with respect to h_o such that $h_o \sim U[0,1]$. If customers opt for the in-store pickup option, they incur a hassle cost of $h_p x$ for visiting the store to pick up their order, where h_p is the hassle cost per unit distance and x is the distance between a customer's location and the offline partner's location. Since customers are uniformly distributed on the circumference of the circular city, we have $x \sim U[0,1/2]$.

Online retailer : When the online retailer fulfills an order via direct delivery, she incurs a direct delivery fulfillment cost of $c_o < p$ due to the logistics required to ship the order from her warehouse to the customer's doorstep. When the online retailer fulfills an order via the in-store pickup option, she incurs an in-store pickup fulfillment cost of c_p due to the logistics required to ship the order from her warehouse to the offline partner. Following the literature (Morganti et al. 2014), we assume that, compared to the direct-delivery fulfillment option, the online retailer can save on logistics costs with the in-store pickup fulfillment option by pooling multiple orders into a single delivery; that is, $c_p \leq c_o$. Without loss of generality, we assume a zero procurement cost.

Offline partner : When the offline partner acts as an in-store pickup point, she incurs a handling cost c_s for each pickup order because she must temporarily store the order and assign staff to process in-store pickups. Customers who visit the offline partner to pick up their orders can generate cross-selling opportunities for the offline partner. To capture this effect, following the cross-selling literature (Gao and Su 2017, Ertekin et al. 2021), we assume that the offline partner earns a profit r from customers who visit the offline partner to pick up their order.

3.2 Baseline policy

Under the baseline policy, the online retailer does not form a partnership with the offline partner and consequently offers only the direct-delivery option to her customers. Under this benchmark scenario, the utility of purchasing with the direct-delivery option amounts to $u_o^B = v - p - h_o$, where the superscript B denotes the *baseline policy*. We let d_o^B denote the online retailer's endogenous demand from customers who opt for the direct-delivery option under the baseline policy. Then, the online retailer's profit under the baseline policy is given by (details of the demand derivation appear in Appendix B)

$$\pi_o^B = (p - c_o) d_o^B. \tag{1}$$

Since there is no partnership, the offline partner does not earn any profit from the online retailer's customers (i.e., $\pi_s^B = 0$) under the baseline policy.

3.3 Fixed fee policy

Under the fixed fee policy, the online retailer and the offline partner establish a pickup partnership under which the online retailer pays the offline partner a fixed fee α for each in-store pickup order. Subsequently, the online retailer offers both direct-delivery and in-store pickup options to her customers. Customers who opt for the direct-delivery option earn a utility $u_o^F = v - p - h_o$, where the superscript F denotes the *fixed fee policy*. Customers who opt for the in-store pickup option earn a utility $u_p^F = v - p - xh_p$.[4] We let d_o^F and d_p^F denote the online retailer's demand for the direct-delivery and in-store pickup options, respectively, under the fixed fee policy. Then, the online retailer's profit under this policy is

$$\pi_o^F(\alpha) = (p - c_o)d_o^F + (p - c_p - \alpha)d_s^F. \tag{2}$$

In turn, the offline partner's profit from the partnership under the fixed fee policy is equal to

$$\pi_s^F(\alpha) = (r + \alpha - c_s)d_s^F. \tag{3}$$

3.4 Coupon policy

The coupon policy establishes a pickup partnership between the online retailer and offline partner under which the online retailer provides a coupon with monetary value β to customers who opt for the in-store pickup option. Customers can then redeem the coupon to receive a discount on any purchase made at the offline partner. The online retailer reimburses the offline partner the amount β for any redeemed coupon.

We assume that customers who opt for the in-store pickup option will make a purchase from the offline partner to redeem the coupon with probability θ.[5] Therefore, under this policy, customers who opt for the direct-delivery option earn a utility $u_o^C = v - p - h_o$, where the superscript C denotes the *coupon policy*. Customers who opt for the in-store pickup option earn a utility $u_p^C = v + \theta\beta - p - xh_p$. Consistent with the literature showing that customers increase purchase amounts when they have a coupon (Gupta 1988, Krishna and Shoemaker 1992, Gopalakrishnan and Park 2021, Bawa and Shoemaker 2004), we assume that when the coupon is redeemed, the offline partner's profit from the cross-selling opportunity increases by β (i.e., the profit due to cross-selling becomes $r + \beta$ with probability θ). We let d_o^C and d_p^C denote the online retailer's demand for the direct-delivery and in-store pickup options, respectively, under the coupon policy. Consequently, the online retailer's profit is equal to

$$\pi_o^C(\beta) = (p - c_o)d_o^C + (p - c_p - \theta\beta)d_s^C, \tag{4}$$

whereas the offline partner's profit amounts to

$$\pi_s^C(\beta) = (r + \theta\beta - c_s)d_s^C. \tag{5}$$

An alternative way to model the offline partner's profit is to assume that the customer will use the coupon for additional purchase with probability ζ, and use the coupon to cover the cost of r with probability $1 - \zeta$. In this case, the offline partner's profit will change to $\pi_c^C(\beta) = (r + \zeta\theta\beta - c_s)d_s^C$. Our findings remain valid under this alternative modeling (see Appendix C).

[4] An alternative way to model the customer utility under the fixed-fee policy is to consider the cross-selling effect into the customer utility. In other words, the customer may consider the possible utility from the additional purchase from the store during her pickup trip when making decision between direct-delivery and in-store pickup service. Our findings still remain valid under this alternative modeling (see Appendix C for details).

[5] An alternative way to model the coupon redemption probability θ is to assume that θ increases with β, implying that customers are more likely to redeem coupons as their value increases. Our findings remain valid under this alternative modeling framework.

Under this modeling framework, the timeline of events shown in Figure 2 unfolds as follows:

1. The online retailer decides whether and with which policy to form a pickup partnership with the offline partner. Under the fixed fee policy, the online retailer determines the optimal parameter α to maximize her profit $\pi_o^F(\alpha)$, subject to the offline partner's rationality constraint; that is, $\pi_s^F(\alpha) \geq 0$. Under the coupon policy, the online retailer determines the optimal coupon value β that maximizes her profit $\pi_o^C(\beta)$, subject to the offline partner's rationality constraint $\pi_s^C(\beta) \geq 0$.
2. The offline partner accepts or rejects the partnership.
3. The customers decide whether to purchase the product from the online retailer and, if so, select one of the available delivery options.
4. The online retailer fulfills the order based on each customer's preferred delivery option.
5. The online retailer pays the offline partner according to the partnership policy.

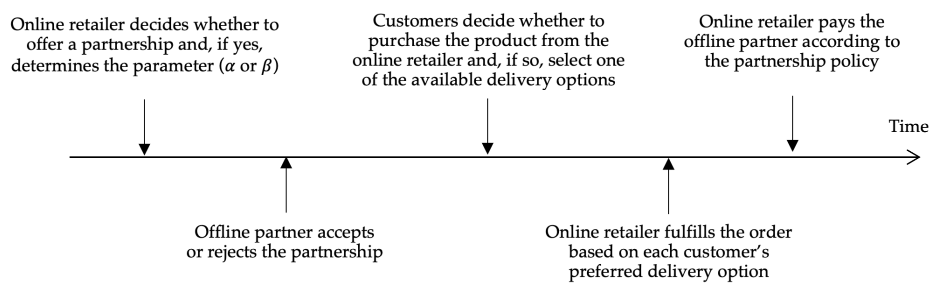


Figure 2: Timeline of events

Figure 3 reports the partnership policies considered by the online retailer and offline partner and the customer decision tree under each policy. Table 1 summarizes the customer utilities, online retailer's profit, and offline partner's profit under each policy. Appendix A.1 summarizes the notation used throughout the paper.

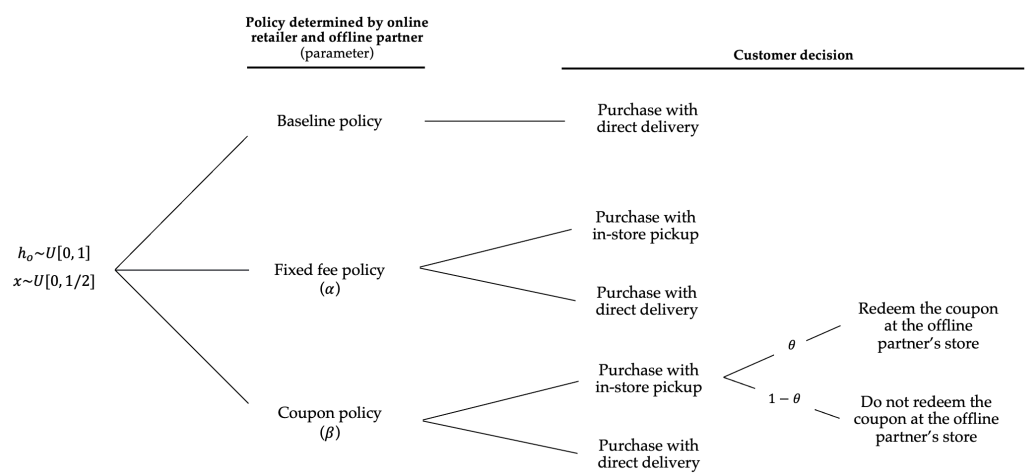


Figure 3: Partnership policies and customer decision trees

4 Results

In this section, we first assess the feasibility and potential benefits of the fixed fee and coupon policies by comparing each policy to the baseline policy. We then compare all three policies to identify the

Table 1: Summary of customer utilities and profit functions

Policy	Customer utility	Online retailer's profit	Offline partner's profit
Baseline	$u_o^B = v - p - h_o$	$(p - c_o)d_o^B$	0
Fixed fee	$u_o^F = v - p - h_o$ $u_p^F = v - p - xh_p$	$(p - c_o)d_o^F + (p - c_p - \alpha)d_s^F$	$(r + \alpha - c_s)d_s^F$
Coupon	$u_o^C = v - p - h_o$ $u_p^C = v - p - xh_p + \theta\beta$	$(p - c_o)d_o^C + (p - c_p - \theta\beta)d_s^C$	$(r + \theta\beta - c_s)d_s^C$

most preferred policy for both the online retailer and the offline partner. Finally, we examine whether a given policy is suitable for certain types of online retailers as characterized by the various model parameters.

4.1 Fixed fee policy

We start by comparing the fixed fee policy to the baseline policy. We let Δd_o^i, Δd_s^i, and Δd^i denote the differences in direct-delivery demand, in-store pickup demand, and total demand, respectively, between a pickup partnership policy, where $i \in \{F, C\}$, and the baseline policy. We thus have $\Delta d^i = \Delta d_o^i + \Delta d_s^i$. We derive analytical expressions for Δd_o^F, Δd_s^F, and Δd^F in the following proposition (details on the demand derivations appear in Appendix B).

Proposition 1. *Compared to the baseline policy, the fixed fee policy affects direct-delivery demand, in-store pickup demand, and total demand by*

$$\Delta d_o^F = - \underbrace{\frac{(v-p)^2}{h_p}}_{\substack{\text{Demand shift due to}\\ \text{convenience}}}$$

$$\Delta d_s^F = \underbrace{\frac{2(v-p)(1-v+p)}{h_p}}_{\substack{\text{Market expansion due to}\\ \text{convenience}}} + \underbrace{\frac{(v-p)^2}{h_p}}_{\substack{\text{Demand shift due to}\\ \text{convenience}}}$$

$$\Delta d^F = \underbrace{\frac{2(v-p)(1-v+p)}{h_p}}_{\substack{\text{Market expansion due to}\\ \text{convenience}}}$$

Proposition 1 reveals that the fixed fee policy induces two effects on the online retailer's demand relative to the baseline policy. First, some customers who were not purchasing under the baseline policy (due to the inconvenience caused by its high direct-delivery hassle cost) will now make a purchase via the in-store pickup option under the fixed fee policy. Specifically, these customers find visiting the offline partner's store to pick up their order more convenient due to the lower hassle cost. We call this effect the *market expansion effect* induced by the fixed fee policy. Second, due to the convenience of in-store pickups, some existing customers under the baseline policy will now choose this option. We call this effect the *demand shift effect* induced by the fixed fee policy. As a result, the fixed fee policy decreases direct-delivery demand due to its demand shift effect and creates a new demand stream through in-store pickups due to its market expansion and demand shift effects. Subsequently, the total demand increases only due to the market expansion effect since the demand shift effect simply transfers the existing demand from the direct-delivery option to the in-store pickup option.

A natural question that arises is how these demand changes affect the online retailer's and offline partner's profits, and whether it is beneficial to establish a pickup partnership under the fixed fee policy. We formally answer this question in Proposition 2. We let $\Delta\pi_o^i(\alpha)$ and $\Delta\pi_s^i(\alpha)$ denote the profit differences for the online retailer and the offline partner, respectively, between a pickup partnership policy, where $i \in \{F, C\}$, and the baseline policy.

Proposition 2. *(a) Compared to the baseline policy, the fixed fee policy with α affects the online retailer's and offline partner's profits by*

$$\Delta\pi_o^F = \underbrace{\frac{2(v-p)(1-v+p)}{h_p}(p-c_p-\alpha)}_{\textit{Increase due to market expansion}} + \underbrace{\frac{(v-p)^2}{h_p}(c_o-c_p-\alpha)}_{\textit{Change due to demand shift}}$$

$$\Delta\pi_s^F = \underbrace{\frac{(v-p)(2-v+p)}{h_p}(r+\alpha-c_s)}_{\textit{Change due to market expansion and demand shift}}$$

(b) It is beneficial for both parties to establish a pickup partnership under the fixed fee policy if and only if $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha} = \max\{0, c_s - r\}$ and $\bar{\alpha} = p - c_p - \frac{v-p}{2-v+p}(p-c_o)$.

Proposition 2 shows that the two demand streams induced by the fixed fee policy are key to its profitability for both parties. For the online retailer, the market expansion effect induced by the fixed fee policy increases profit due to the additional margins obtained from the new customers. However, the impact of the demand shift effect on the profit is more intricate. When α is relatively small, the cost of direct-delivery (c_o) is higher than the cost of in-store pickup ($c_p + \alpha$). Subsequently, the demand shift effect results in another profit increase for the online retailer due to the additional margins obtained from existing customers who alter their delivery option when the pickup partnership is available. By contrast, a sufficiently high α will make the in-store pickup option more costly for the online retailer, so customers generating the demand shift effect will lower the profit. In that case, the gain from new customers will be sufficient to compensate the loss from existing customers so long as $\alpha \leq \bar{\alpha}$. Otherwise (i.e., when $\alpha > \bar{\alpha}$), the fixed fee policy will decrease the online retailer's profit. For the offline partner, the margin from each new or existing customer who opts for the in-store pickup option is equal to $r + \alpha - c_s$. When $\alpha \geq \underline{\alpha}$, this margin is positive, so that the fixed fee policy will benefit the offline partner. Otherwise (i.e., when $\alpha < \underline{\alpha}$), the offline partner will be worse off under the fixed fee policy. We note that even if there is no fixed fee compensation (i.e., $\alpha = 0$), the pickup partnership can still be beneficial for the offline partner when the profit from cross-selling purchases is high enough (i.e., $r > c_s$).

Proposition 2 also establishes that when the fixed fee compensation lies in $[\underline{\alpha}, \bar{\alpha}]$, neither party will be worse off under the fixed fee policy, resulting in a beneficial partnership.[6] Otherwise, one of the parties will always be worse off; hence, a pickup partnership will not be established under the fixed fee policy. We note that as α increases, the profit will decrease (resp., increase) for the online retailer (resp., offline partner). This implies that the optimal value of α is $\underline{\alpha}$ for the online retailer and $\bar{\alpha}$ for the offline partner. However, the optimal α that maximizes one party's profit will result in the other party earning no profit from the partnership. To avoid this situation and ensure that both parties are strictly better off under the fixed fee policy, the pickup partnership can be established by choosing a value of α such that $\underline{\alpha} < \alpha < \bar{\alpha}$. Such a well-designed partnership will yield a win-win situation, in a similar spirit as a revenue-sharing contract between a supplier and a retailer in supply chain management (Cachon and Lariviere 2005).

4.2 Coupon policy

We compare the demand under the coupon and baseline policies in the following proposition.

[6] The word beneficial is used in a non-strict sense throughout the paper.

Proposition 3. *Compared to the baseline policy, the coupon policy affects direct-delivery demand, in-store pickup demand, and total demand by*

$$\Delta d_o^C = -\underbrace{(\underbrace{\frac{(v-p)^2}{h_p}}_{\textit{due to convenience}} + \underbrace{\frac{(v-p)}{h_p}\hat{\beta}}_{\textit{due to promotion}})}_{\textit{Demand shift}}$$

$$\Delta d_s^C = \underbrace{\underbrace{\frac{2(v-p)(1-v+p)}{h_p}}_{\textit{due to convenience}} + \underbrace{\frac{(1-v+p)}{h_p}\hat{\hat{\beta}}}_{\textit{due to promotion}}}_{\textit{Market expansion}} + \underbrace{\underbrace{\frac{(v-p)^2}{h_p}}_{\textit{due to convenience}} + \underbrace{\frac{(v-p)}{h_p}\hat{\beta}}_{\textit{due to promotion}}}_{\textit{Demand shift}}$$

$$\Delta d^C = \underbrace{\underbrace{\frac{2(v-p)(1-v+p)}{h_p}}_{\textit{due to convenience}} + \underbrace{\frac{(1-v+p)}{h_p}\hat{\hat{\beta}}}_{\textit{due to promotion}}}_{\textit{Market expansion}}$$

where $\hat{\beta}$ and $\hat{\hat{\beta}}$ are increasing functions of β, as shown in Appendix B. Proposition 3 conveys that, similar to the fixed fee policy, the coupon policy induces both market expansion and demand shift effects, albeit with higher magnitudes. This is due to the fact that the coupon policy provides two levers to influence demand. First, as in the fixed fee policy, it attracts new customers and shifts some existing customers from direct-delivery to in-store pickup due to the increased convenience of the in-store pickup option. Second, unlike the fixed fee policy, the coupon policy induces additional new and existing customers who are incentivized by the monetary value of the coupon. In particular, some customers who were not purchasing under the fixed fee policy (despite its convenience) will now make a purchase under the coupon policy to take advantage of the discount they receive at the offline partner store when picking up their online order. Consequently, the coupon policy induces a greater market expansion effect relative to the fixed fee policy. Similarly, some existing customers who use the direct-delivery option under both the baseline and the fixed fee policies will opt for the in-store pickup option under the coupon policy to take advantage of the coupon at the offline partner, resulting in a greater demand shift effect compared to that under the fixed fee policy. Thus, the pickup partnership's effects on direct-delivery demand, in-store pickup demand, and total demand are greater under the coupon policy than under the fixed fee policy.

Proposition 4 characterizes the corresponding change in profit for both partners and the condition when it is beneficial to establish a pickup partnership under the coupon policy.

Proposition 4. *(a) Compared to the baseline policy, the coupon policy affects the online retailer's and offline partner's profit by*

$$\Delta\pi_o^C = \underbrace{(\frac{2(v-p)(1-v+p)}{h_p} + \frac{(1-v+p)}{h_p}\hat{\hat{\beta}})(p-c_p-\theta\beta)}_{\textit{Increase due to market expansion}} + \underbrace{(\frac{(v-p)^2}{h_p} + \frac{(v-p)}{h_p}\hat{\beta})(c_o-c_p-\theta\beta)}_{\textit{Change due to demand shift}}$$

$$\Delta\pi_s^C = \underbrace{(\frac{(v-p)(2-v+p)}{h_p} + \frac{(1-v+p)}{h_p}\hat{\hat{\beta}} + \frac{(v-p)}{h_p}\hat{\beta})(r+\theta\beta-c_s)}_{\textit{Change due to market expansion and demand shift}}$$

(b) There exist two thresholds $\underline{\beta}$ and $\bar{\beta}$ such that it is beneficial for both parties to establish a pickup partnership under the coupon policy if and only if $\beta \in [\underline{\beta}, \bar{\beta}]$. Within this range, for the online retailer, the optimal coupon value is $\beta^* = \max\{\underline{\beta}, \check{\beta}\}$.

The closed-form expression for $\check{\beta}$ is reported in Appendix B. Proposition 4 shows that the profit implications of the demand change under the coupon policy are similar to those under the fixed fee policy. In short, the online retailer will be better off under the coupon policy so long as the additional profit margin earned from new customers induced by the market expansion effect offsets the loss from existing customers induced by the demand shift effect (i.e., when $\beta < \bar{\beta}$). Similarly, the offline partner will benefit from the partnership if she can collect a positive margin from customers who pick up their orders (i.e., when $\beta \geq \underline{\beta}$). Thus, when the monetary value of the coupon lies in $[\underline{\beta}, \bar{\beta}]$, no party is worse off under the coupon policy relative to the baseline policy.

As in the fixed fee policy, the offline partner prefers the highest possible value of the coupon (i.e., $\bar{\beta}$) to maximize her profit under the coupon policy, which comes at the expense of a zero gain for the online retailer. However, unlike the fixed fee policy, the optimal coupon value for the online retailer is not necessarily the minimum feasible value (i.e., $\underline{\beta}$) under the coupon policy. The rationale is that although an increase in β will decrease the profit margin from an in-store pickup order for the online retailer, it may also lead to more customers (both new and existing) opting for the in-store pickup delivery option to take advantage of the higher discount β. If the net profit from these customers offsets the decrease in profit margin per in-store pickup order, then the optimal coupon value for the online retailer would be $\beta^* > \underline{\beta}$. Such a coupon also provides a strictly positive gain from the partnership for the offline partner, resulting in a win-win situation for the two parties. Otherwise, the optimal coupon value for the online retailer is $\beta^* = \underline{\beta}$, which makes the offline partner indifferent between the baseline and coupon policies. In that case, as with the fixed fee policy, a win-win pickup partnership under the coupon policy can be established with $\underline{\beta} < \beta < \bar{\beta}$.

4.3 Optimal policy

Having characterized each pickup partnership in the previous subsections, we next compare all three policies to identify the optimal policy for both the online retailer and the offline partner. To do so, we first find the optimal solution for each policy and then compare the three optimal solutions to determine the best policy. To ensure that the two pickup partnership policies are compared objectively, we impose the constraint that the average compensation per in-store pickup order is the same under both policies (i.e., $\alpha = \theta\beta$). Our results still hold when we evaluate the model under optimal α and β for the online retailer (see Appendix C). Proposition 5 characterizes the results of this analysis conditional on c_s. We condition the analysis on c_s because it represents the offline partner's operational cost related to the partnership. Therefore, given that the process to establish a partnership starts with the online retailer selecting a partnership policy, the proposition conditioned on c_s can enable the online retailer to make that choice based on the offline partner's operational characteristics. We examine the sensitivity of the optimal policy with respect to other parameters in Section 4.4.

Proposition 5. *There exist two thresholds $\underline{c}_s$ and $\bar{c}_s$ such that it is optimal for the online retailer and the offline partner*

- *not to establish a pickup partnership when $c_s > \bar{c}_s$,*
- *to establish a pickup partnership under the fixed fee policy with parameter $\alpha \in [c_s - r, \bar{c}_s - r]$ when $\underline{c}_s < c_s \leq \bar{c}_s$, and*
- *to establish a pickup partnership under the coupon policy with parameter $\beta \in [\max\{0, \frac{c_s - r}{\theta}\}, \frac{\underline{c}_s - r}{\theta}]$ when $c_s \leq \underline{c}_s$.*

The rationale behind Proposition 5 is as follows. When the offline partner's in-store pickup handling cost is high (i.e., $c_s > \bar{c}_s$), she finds the partnership beneficial only if the compensation for each in-store pickup order (i.e., α or β) is sufficiently high. However, such a high compensation makes the online retailer worse off with any pickup partnership (as we show in Figure 4, when $c_s > \bar{c}_s$, the online retailer is better off under the baseline policy). Therefore, a beneficial partnership does not exist, making the baseline policy the best option.

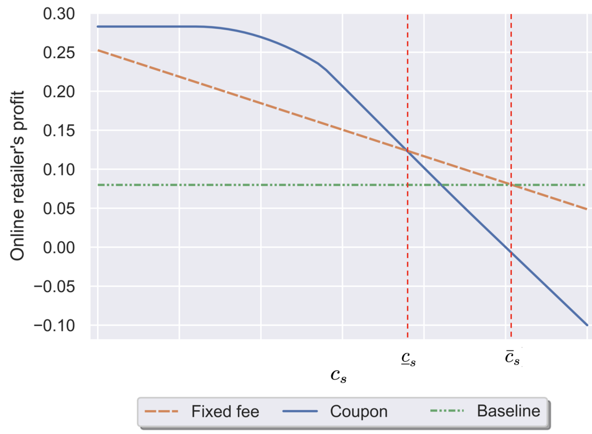


Figure 4: The online retailer's profit under the fixed fee, coupon, and baseline policies

When a beneficial partnership exists (i.e., $c_s \le \bar{c}_s$), the optimal policy is determined by the additional customers (both new and existing) who opt for the in-store pickup delivery option only under the coupon policy (i.e., customers forming the "market expansion due to promotion" and "demand shift due to promotion" segments shown in Proposition 3). When the in-store pickup handling cost is moderate (i.e., $\underline{c}_s < c_s \le \bar{c}_s$), a beneficial partnership (under both fixed fee and coupon policies) can be established only with a moderately high compensation (β or α). In this case, the coupon policy increases the profit by boosting the demand relative to the fixed fee policy. However, a moderately high compensation makes the in-store pickup fulfillment more costly (and thus less profitable) than direct-delivery fulfillment for the online retailer. Therefore, the coupon policy also decreases the profit by making more existing customers who would choose the direct-delivery option under the fixed fee policy switch to the more costly in-store pickup option. When the coupon value is moderately high, the increase in profit due to the demand boost effect cannot offset the loss due to the demand shift effect. As a result, the online retailer is worse off under the coupon policy with a moderately high β than under the fixed fee policy with an economically equivalent compensation (i.e., a moderately high α), making the fixed fee policy optimal. This can be seen in Figure 4, which shows that the online retailer's profit is higher under the fixed fee policy when $\underline{c}_s < c_s \le \bar{c}_s$.

Finally, when the in-store pickup handling cost is low (i.e., $c_s \le \underline{c}_s$), a beneficial partnership under both fixed fee and coupon policies can be established with low compensation. In this case, the net profit from the additional new and existing customers induced by the coupon policy with a low β makes the online retailer better off relative to the fixed fee policy. Thus, the coupon policy is optimal when $c_s \le \underline{c}_s$.

4.4 Comparative statics

In this section, we investigate whether a given optimal policy is more suitable for certain types of online retailers that can be characterized based on three key model parameters; namely c_o, c_p, and p. To do so, we examine the sensitivity of the optimal policy with respect to c_o, c_p, and p. All the technical details related to this analysis appear in Appendix D.

We make three main observations. First, we find that the benefit of the pickup partnership increases as the direct-delivery fulfillment cost (c_o) increases and the in-store pickup fulfillment cost (c_p) decreases. Under a high c_o and a low c_p, the direct-delivery becomes a more costly (and less profitable) fulfillment method for the online retailer relative to the in-store pickup delivery. Consequently, the online retailer will earn a higher profit with the pickup partnership (under either a fixed fee or a coupon policy) due to customers choosing the relatively less costly in-store pickup delivery option. Second, between the two partnership policies, the coupon policy outperforms the fixed fee policy as c_o increases and c_p decreases. Since the coupon policy generates more in-store pickup demand than the fixed fee policy (as characterized by Proposition 3), then the increasing benefit of the pickup partnership (as c_o increases and c_p decreases) becomes more pronounced under the coupon policy. Third, as the product price (p) increases, the coupon policy becomes more profitable than the fixed fee policy. An increase in p has two competing effects. While it increases the profit margin for both direct-delivery orders and in-store pickup orders, it also leads to lower demand for both types of orders. Since the coupon policy will generate a higher demand than the fixed fee policy (due to its promotional lever), the negative impact of the price increase on demand is more mitigated under the coupon policy.

Overall, as summarized in Table 2, these results suggest that a pickup partnership is not suitable for online retailers with low direct-delivery fulfillment cost (c_o) or high in-store pickup fulfillment cost (c_p) (e.g., online jewelry and luxury fashion retailers). The fixed fee policy is suitable for retailers with moderate direct-delivery cost, moderate in-store fulfillment cost, or low-priced products (e.g., online farmer marketplaces, online supermarkets), whereas the coupon policy is suitable for retailers with high direct-delivery cost, low in-store fulfillment cost, or high-priced products (e.g., meal kit companies, cosmetics retailers).

Table 2: Optimal policy based on online retailer characteristics

		$c_o(c_p)$		
		High (Low)	Moderate (Moderate)	Low (High)
p	High	Coupon	Coupon/Fixed fee	Baseline
	Low	Coupon/Fixed fee	Fixed fee	Baseline

5 Extensions

Having established the fundamental features of the pickup partnership under two different policies, we now consider several extensions of our theoretical model. In particular, we examine the pickup partnership with (i) a budget constraint, (ii) multiple pickup locations, and (iii) the consideration of consumer surplus. In this section, we summarize the results from these analyses; the technical details appear in Appendix E.

5.1 Budget constraint

In our main model, we assumed that the online retailer was willing to pay any compensation (α or β) for each in-store pickup so long as it is profitable. In practice, however, the online retailer may want to spend a specific budget to establish a pickup partnership. In this subsection, we evaluate how such a budget constraint affects the optimal pickup partnership policy.

We define the partnership budget K as the total amount of money allocated by the online retailer to compensate the offline partner for all in-store pickup orders. Note that the expected overall compensation to meet all potential in-store pickup orders equals αd_s^F under the fixed fee policy and $\theta\beta d_s^C$ under the coupon policy (we highlight that d_s^F and d_s^C are different). Therefore, the budget constraint imposes that the expected overall compensation for in-store pickup orders should not exceed the online retailer's budget (i.e., $\alpha d_s^F \leq K$ under the fixed fee policy and $\theta\beta d_s^C \leq K$ under the coupon policy).[7] Under a budget constraint, we make three observations.

[7]Alternatively, the budget can also be defined per in-store pickup order such that the compensation for each in-store pickup order cannot exceed a certain budget. Under this type of budget constraint, we find consistent results.

First, when the budget is small (i.e., $K < \underline{K}$ where the expression of $\underline{K}$ is provided in Appendix E), for each in-store pickup order, the online retailer can compensate the offline partner with a maximum fixed fee of $\alpha < \underline{\alpha}$ under the fixed fee policy and a maximum coupon value of $\beta < \underline{\beta}$ under the coupon policy. As Propositions 2 and 4 imply, such compensation levels are not high enough to incentivize the offline partner to accept the pickup partnership under any policy. As a result, the pickup partnership cannot be established under a small budget. Second, when the budget is moderate (i.e., $\underline{K} \geq K > \bar{K}$; the expression of $\bar{K}$ appers in Appendix E), the maximum compensation that the online retailer can pay is $\alpha > \underline{\alpha}$ under the fixed fee policy and $\beta < \underline{\beta}$ under the coupon policy, making the partnership beneficial only under a fixed fee policy. In this case, it is beneficial for both parties to establish a pickup partnership under the fixed fee policy so long as $\alpha \in [\underline{\alpha}, \frac{K}{d_s^F}]$. Third, when the budget is large (i.e., $K \geq \bar{K}$), both policies are beneficial. In this case, the online retailer can choose an optimal policy as characterized in Proposition 5, along with the budget consideration. As such, an optimal fixed fee policy can be implemented with parameter $\alpha \in [\underline{\alpha}, \min\{\bar{c}_s - r, \frac{K}{d_s^F}\}]$. Similarly, an optimal coupon policy can be implemented with parameter $\beta \in [\underline{\beta}, \min\{\frac{c_s - r}{\theta}, \frac{K}{\theta d_s^C}\}]$.

In summary, the budget allocated by the online retailer to the pickup partnership will determine the policy to be chosen. Due to the additional demand it generates for in-store pickup orders, the coupon policy requires a higher budget than the fixed fee policy. Therefore, for online retailers with a limited budget, a pickup partnership is beneficial only under a fixed fee policy. A partnership with a coupon policy is beneficial only for online retailers with a higher budget.

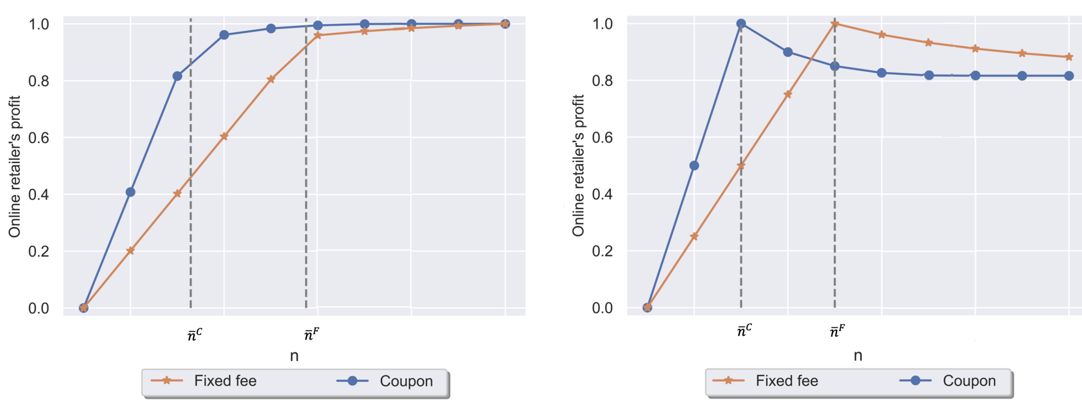
5.2 Multiple pickup locations

So far, we have assumed that the offline partner had only one pickup location. While the single-location assumption allows us to characterize the fundamental features of pickup partnership policies, in practice, offline partners can designate multiple stores as pickup locations. In such cases, it is managerially important to understand (i) whether the optimal pickup partnership policy depends on the number of in-store pickup locations, and (ii) how retailers should determine the optimal number of in-store pickup locations for a given pickup partnership policy. Next, we extend our model to examine the scenario with multiple pickup locations.

We assume that the offline partner has $n \geq 1$ stores that are spread uniformly along the circumference of a circular city. Thus, the distance between two nearby stores is $1/n$. Since customers are uniformly distributed on the circumference of the circular city (Salop 1979), $x \sim U[0, 1/2n]$. This implies that customers' hassle cost for visiting the nearest store to pick up an order (i.e., $h_p x$) decreases as the number of pickup locations increases (regardless of partnership policy). Consequently, more customers will find the in-store pickup option more convenient than the direct-delivery option, increasing the demand for in-store pickups.

Under this modeling framework, we find that the number of pickup locations can change the optimal policy structure derived in Proposition 5 only when the coupon policy is optimal (i.e., when $c_s \leq \underline{c}_s$). In Figure 5, we plot the relationship between the number of pickup locations and the optimal policy structure for that case. Since the demand for in-store pickup orders increases with n, once the number of pickup locations in the pickup partnership reaches a certain threshold, the online retailer will cover the entire market (i.e., each customer in the market will make a purchase via either the direct-delivery or in-store pickup option). The notations $\bar{n}^F$ and $\bar{n}^C$ in Figure 5 represent this threshold for the fixed fee and coupon policies, respectively, when $c_s \leq \underline{c}_s$. Since the total demand (i.e., demand for both direct-delivery and in-store pickup) is higher under the coupon policy relative to the fixed fee policy (Proposition 3), the online retailer can cover the entire market with fewer pickup locations under the coupon policy (i.e., $\bar{n}^C \leq \bar{n}^F$).

We next summarize our findings. First, the online retailer's profit increases with n under both partnership policies so long as $n < \bar{n}^C$ and a partnership is beneficial. Indeed, as n increases, the market expansion and demand shift effects under both policies become more substantial. Consequently, the coupon policy remains more profitable than the fixed fee policy. Second, when $\bar{n}^C \leq n < \bar{n}^F$, since the market is still not fully covered under the fixed fee policy, the market expansion and demand shift effects are still at play, and thus, profit continues to rise with n under the fixed fee policy. However, since the entire market is fully covered under the coupon policy, any increase in n will only amplify the demand shift effect of the coupon policy, whereas the market expansion effect remains the same. If the margin from an in-store pickup order is greater than the margin from a direct-delivery order (as illustrated in Figure 5(a)), then the online retailer's profit will continue to increase with n, albeit at a decreasing rate, under the coupon policy. In this case, the coupon policy still remains more profitable than the fixed fee policy. By contrast, if the in-store pickup fulfillment is more costly to the online retailer (as illustrated in Figure 5(b)), the profit under the coupon policy will decrease with n due to customers changing their preferences from direct delivery to in-store pickup. In this case, the two policies become equally profitable with a certain number of pickup locations (refer to the point in Figure 5(b) at which both lines intersect), and any increase in n beyond that point will make the fixed fee policy optimal. Third, when $n > \bar{n}^F$, the entire market is covered under both policies. Therefore, any increase in n will only amplify the demand shift effect under both policies while keeping the relative profitability the same. Hence, the coupon policy is optimal when the direct-delivery fulfillment is more costly (Figure 5(a)), whereas the fixed fee policy is optimal when the in-store pickup fulfillment is more costly (Figure 5(b)). Fourth, when n becomes large enough, the entire market makes a purchase only via the in-store pickup option under both policies, so that the online retailer becomes indifferent between the two policies (as illustrated in Figure 5(a)).



(a) Direct-delivery fulfillment is more costly

(b) In-store pickup fulfillment is more costly

Figure 5: The effect of n on the online retailer's profit under fixed fee and coupon policies

The above discussion suggests that the optimal number of pickup locations for a partnership depends on the comparison between the profit margin from an in-store pickup order and the profit margin from a direct-delivery order. When the margin is higher for the in-store pickup order, as illustrated in Figure 5(a), the minimum number of pickup locations, which enables the online retail to fully cover the market using only the in-store pickup option, is optimal under both partnership policies. When the margin is higher for the direct-delivery order, as illustrated in Figure 5(b), then the optimal number of pickup locations is equal to the smallest number that ensures full market coverage (i.e., $\bar{n}^F$ for the fixed fee policy and $\bar{n}^C$ for the coupon policy).

Overall, our analysis suggests that a larger number of pickup locations does not necessarily yield a higher profit for the partnership. Before negotiating the number of pickup locations, online retailers should carefully compare their profit margins between the direct-delivery and in-store pickup options. When the in-store pickup fulfillment is more costly than the direct-delivery fulfillment, online retailers can establish a partnership under the coupon policy using a smaller number of pickup locations. Otherwise, a larger number of locations will increase the partnership profitability.

5.3 Total welfare

In our main model, the optimal partnership policy was chosen to maximize the profit of the online retailer while ensuring that the offline partner is not worse off relative to the setting without a partnership. With the increasing awareness of social responsibility, retailers may alternatively seek partnership solutions that also consider the customers' interests (Goering 2012, Bénabou and Tirole 2010). Therefore, in this subsection, we evaluate how the optimal policy changes when the online retailer seeks to maximize the total welfare earned by all stakeholders (i.e., herself, the offline partner, and customers).

Our analysis reveals that when maximizing total welfare (i.e., the sum of the online retailer profit, offline partner profit, and consumer surplus), the threshold $\bar{c}_s$ increases. In other words, the range under which a beneficial pickup partnership exists will expand. This result is expected because the pickup partnership (under any policy) will always increase the consumer surplus relative to the baseline policy, since it offers an additional delivery option for customers. Therefore, the increase in consumer surplus is yet another benefit of establishing a pickup partnership. It is worth noting that even in this case, the pickup partnership may still not be beneficial for high values of c_s.

More importantly, our analysis shows that when setting the objective as the total welfare, the fixed fee is no longer the optimal policy for a pickup partnership. Recall that as shown in Proposition 5, when the online retailer maximizes her own profit, the fixed fee policy is optimal when the profit from the additional new customers induced by the coupon policy cannot offset the loss from the additional existing customers (switching from the direct-delivery to in-store pickup option), again induced by the coupon policy. When the online retailer maximizes total welfare, the consumer surplus is higher under the coupon policy than under the fixed fee policy, since the probability of redeeming the coupon provides an additional utility to customers. Consequently, compared to the fixed fee policy, the profit from additional new customers, coupled with the higher consumer surplus, will always offset the loss from additional existing customers under the coupon policy. Therefore, the online retailer is always better off by establishing the pickup partnership under the coupon policy relative to the fixed fee policy when considering the total welfare.

6 Pickup partnership with hybrid policy

An optimal partnership policy, as we identified in Proposition 5, aims to maximize the online retailer's profit subject to the offline partner's rationality constraint. Thus, to establish a pickup partnership, the online retailer has to offer either a fixed fee policy or a coupon policy while ensuring that the offline partner is not worse off relative to the baseline policy. In some situations, this may force the online retailer to select a partnership parameter that is not necessarily a profit maximizer for herself, implying that an optimal partnership policy can entail inefficiencies for the online retailer. Equivalently, we investigate when the offline partner's rationality constraint is tight. In this section, we first examine under which cases such inefficiencies exist for the two policies (fixed fee and coupon) used in current practices. We then prescribe a novel pickup partnership policy that alleviates such inefficiencies.

6.1 Inefficiency from fixed fee and coupon policies

We start by examining the optimal fixed fee policy. Recall from Proposition 5 that, when $\underline{c}_s < c_s \leq \bar{c}_s$, the online retailer prefers the fixed fee policy over the coupon policy with a moderately high β^*. Figure 6 illustrates the corresponding market segmentation under the optimal fixed fee policy. We observe that the optimal fixed fee policy with α generates a partial market coverage so that some customers (as depicted by the dotted region at the top-right corner in Figure 6) leave the market without making a purchase. Note that the online retailer can achieve the same market segmentation under the coupon policy by setting the coupon value to zero (i.e., $\beta = 0$). Consistent with Proposition 3, this implies that with any positive coupon value under a hypothetical coupon policy, the online retailer would generate additional sales from some of the customers in the dotted region, although a positive coupon value would also motivate some existing customers to change their delivery mode from direct-delivery to in-store pickup. As shown in Proposition 5, when the coupon value is low enough, the net profit change with any positive coupon value under that hypothetical coupon policy relative to the optimal fixed fee policy would be positive, representing an opportunity cost for the online retailer under the optimal fixed fee policy. When $\underline{c}_s < c_s \leq \bar{c}_s$, since the online retailer can encourage the offline partner to establish a partnership under the coupon policy using only a moderately high β^* (which makes the net profit change under the coupon policy compared to the optimal fixed fee policy negative), she prefers to establish the pickup partnership under a fixed fee policy despite its opportunity cost. Therefore, the aforementioned opportunity cost represents the inefficiency of the pickup partnership under the optimal fixed fee policy (relative to the hypothetical coupon policy with a low β).

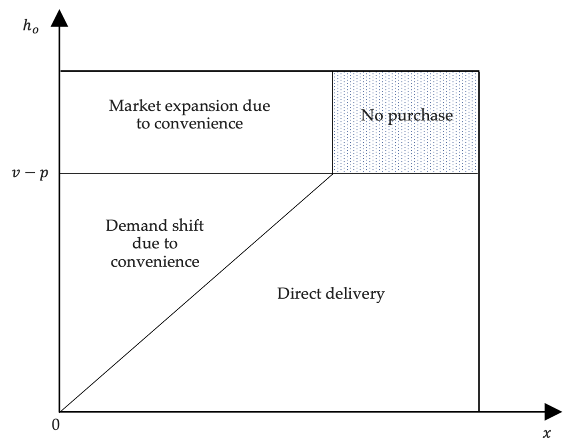


Figure 6: Market segmentation under the optimal fixed fee policy

We next examine the optimal coupon policy. As discussed in Proposition 5, when $c_s \leq \underline{c}_s$, the online retailer can induce the offline partner to establish a pickup partnership under the coupon policy with a relatively low β^*, making the coupon policy optimal. Figure 7 illustrates the corresponding market segmentation under the optimal coupon policy. We observe that the optimal coupon policy with β^* allows the online retailer to cover the entire market (i.e., all customers will make a purchase via direct-delivery or in-store pickup). However, as shown in Figure 7, β^* under the optimal coupon policy is greater than the minimum coupon value β_m under a hypothetical coupon policy that can allow the online retailer to just cover the entire market. This implies that the market expansion effect of the coupon policy is maximized when $\beta = \beta_m < \beta^*$. Thus, when the coupon value increases from β_m to β^*, the online retailer no longer generates new customers. Rather, as illustrated by the dotted region in Figure 7, an increase in the coupon value beyond β_m only induces more existing customers to change their delivery option from direct-delivery to in-store pickup. When the profit margin from an in-store pickup order is lower than the profit margin from a direct-delivery order, the customers in the dotted region will decrease the online retailer's profit relative to the profit under the hypothetical coupon policy with β_m. Despite this profit loss, when $c_s \leq \underline{c}_s$, the online retailer constructs the optimal coupon policy with $\beta^* > \beta_m$, because any β lower than β^* will make the offline partner worse off under the partnership. Thus, in order to fairly compensate the offline partner under the optimal coupon policy, the online retailer will absorb the loss from the customers in the dotted region. Hence, the absorbed loss from these customers represents the inefficiency of the pickup partnership under the optimal coupon policy (relative to the hypothetical coupon policy with β_m).

Overall, we find that, despite being optimal, both the fixed fee and coupon policies may entail inefficiencies for the online retailer. In the next subsection, we prescribe an alternative partnership policy to alleviate such inefficiencies.

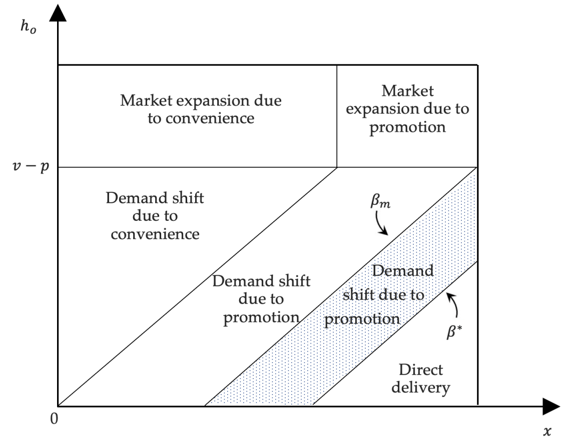


Figure 7: Market segmentation under the optimal coupon policy

6.2 Hybrid policy

We show that an alternative partnership can be established such that for each in-store pickup, the online retailer can compensate the offline partner with a total compensation γ, of which α_h is paid to the offline partner as a fixed fee, and β_h (where $\beta_h = \frac{\gamma-\alpha_h}{\theta}$) is offered to the customers as a coupon. We term this policy the *hybrid policy*. We note that in this setting, the average compensation per in-store pickup order becomes equivalent to those under the fixed fee and coupon policies (i.e., $\gamma = \alpha = \theta\beta$).

Customer utilities for direct-delivery and in-store pickup options under the hybrid policy remain the same as under the coupon policy. We let d_o^H and d_p^H denote the online retailer's demand for direct-delivery and in-store pickup options, respectively, where the superscript H denotes the hybrid policy. Then, the online retailer's profit is equal to

$$\pi_o^H(\gamma) = (p - c_o)d_o^H + (p - c_p - \gamma)d_s^H, \tag{6}$$

whereas the offline partner's profit is equal to

$$\pi_s^H(\gamma) = (r + \gamma - c_s)d_s^H. \tag{7}$$

Proposition 6 characterizes how the optimal policy structure presented in Proposition 5 changes in the presence of the hybrid policy.

Proposition 6. *There exist thresholds $\bar{c}_p$, $\underline{\underline{c}}_s$, and $\bar{\bar{c}}_s$ such that*

(a) if $c_p < \bar{c}_p$, it is optimal for the online retailer and the offline partner

- *not to establish a pickup partnership when $c_s > \bar{c}_s$,*
- *to establish a pickup partnership under the fixed fee policy with parameter $\alpha \in [c_s - r, \bar{c}_s - r]$ when $\bar{\bar{c}}_s < c_s \le \bar{c}_s$,*
- *to establish a pickup partnership under the hybrid policy with parameters $\beta_h = \beta_m$ and $\alpha_h \in [\max\{c_s - r - \theta\beta_m, 0\}, \bar{\bar{c}}_s - r - \theta\beta_m]$ when $\underline{\underline{c}}_s < c_s \le \bar{\bar{c}}_s$, and*
- *to establish a pickup partnership under the coupon policy with parameter $\beta \in [\max\{0, \frac{c_s-r}{\theta}\}, \frac{\underline{\underline{c}}_s-r}{\theta}]$ when $c_s \le \underline{\underline{c}}_s$.*

(b) otherwise (i.e., $c_p \ge \bar{c}_p$), the optimal policy structure from Proposition 5 remains the same.

The closed-form expressions for $\bar{c}_p$, $\underline{\underline{c}}_s$, and $\bar{\bar{c}}_s$ appear in Appendix C. Proposition 6 reveals that the hybrid policy can be a valuable lever for the online retailer to improve partnership efficacy only when the profit margin of the in-store pickup order is smaller than the profit margin of the direct-delivery order (i.e., $c_p < \bar{c}_p$ and $\underline{\underline{c}}_s < c_s \leq \bar{\bar{c}}_s$). In this case, the optimal policy structure characterized in Proposition 5 (labeled as "Without hybrid policy" in Figure 8) changes as shown by the "With hybrid policy" case in Figure 8. The figure shows that the retailer is better off under the hybrid policy relative to the coupon policy in Region I and relative to the fixed fee policy in Region II.

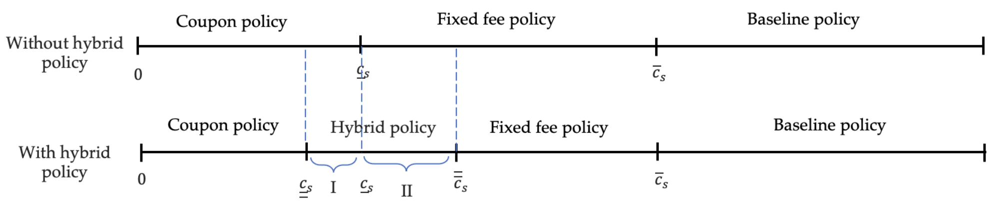


Figure 8: Optimal policy with and without the hybrid policy

In Region I of Figure 8 (i.e., when $\underline{\underline{c}}_s \leq c_s < \underline{c}_s$), in the absence of the hybrid policy, the online retailer establishes the partnership using the coupon policy with a relatively low β^* while absorbing the loss from the customers in the dotted region of Figure 7, as discussed in Section 6.1. The hybrid policy enables the online retailer to mitigate this loss. In particular, as illustrated in Figure 9, the online retailer will set the optimal hybrid policy coupon value β_h^* to β_m to cover the entire market, making the customers in Region A better off with the direct-delivery option and hence eliminating the loss from these customers. Since β_h^* is not high enough for the offline partner to accept the partnership, the online retailer will pay the remaining α_h of the hybrid policy compensation γ as a fixed fee to ensure that the offline partner is not worse off under the partnership.

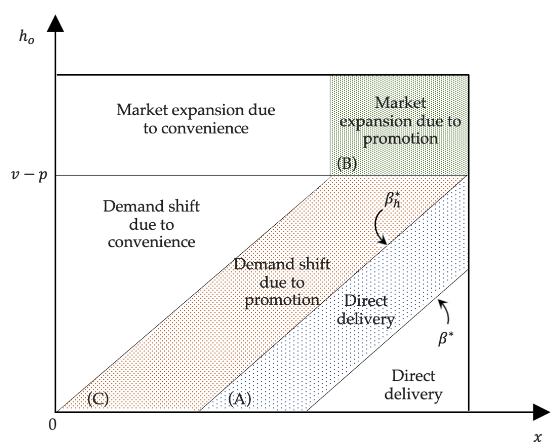


Figure 9: Market segmentation under the optimal hybrid policy

In Region II of Figure 8 (i.e., when $\underline{c}_s \leq c_s < \bar{\bar{c}}_s$), in the absence of the hybrid policy, the online retailer establishes the partnership using the fixed fee policy, while missing the additional profit opportunity from customers who leave the market without making any purchase (i.e., customers in the blue dotted region in Figure 6), as discussed in Section 6.1. As illustrated in Figure 9, the hybrid policy enables the online retailer to attract those customers. In particular, by setting the optimal hybrid policy coupon value β_h^* to β_m, the online retailer will cover the entire market, including the customers in Region B, thus generating an additional profit. However, this will also induce some of the existing customers (illustrated by Region C in Figure 9) to change their preference from direct-delivery under the fixed fee policy to in-store pickup under the hybrid policy, generating an additional loss. Since β_h^* represents a low β, as implied by Proposition 5, the former's additional profit becomes greater than the latter's additional loss, ultimately making the online retailer better off under the hybrid policy.

We next conduct a numerical study to quantify the extent to which the hybrid policy can benefit the online retailer. Using a wide range of model parameters, our study results in 5,540 instances for which the partnership is beneficial to both parties under either the fixed fee or coupon policies.[8] We find that the hybrid policy can improve the online retailer's profit for 24.06% (1,333 of 5,540) of those instances. The average improvement in the online retailer's profit amounts to 5.14% (with a standard deviation of 3.13%), with minimum and maximum values of 0.78% and 14.45%.

To summarize, our proposed hybrid policy allows the online retailer to minimize the potential inefficiencies of the pickup partnership established under either the fixed fee policy or the coupon policy while ensuring that the partnership remains beneficial for the offline partner. To our knowledge, such a hybrid policy has not yet been deployed in practice, potentially due to the fact that pickup partnerships in retail are still in a fairly nascent stage. Nevertheless, our results suggest that online retailers should consider implementing a hybrid policy when establishing a pickup partnership since such a policy is likely to yield a more efficient partnership, especially when in-store pickup delivery fulfillment is costly.

7 Conclusion

To survive in the omnichannel era, many pure online retailers (e.g., Amazon, Cookit, Maturin) have recently started to form pickup partnerships with offline stores to provide their customers with convenient in-store pickup services. Despite this evolving business model, the literature on how to design and assess the impact of a pickup partnership is non-existent. To our knowledge, this paper is the first to develop an analytical model to theoretically examine the benefits of implementing a pickup partnership. In particular, we first analyze the two policies, the fixed fee and the coupon policies, that practitioners use to form pickup partnerships. We then characterize whether and how online retailers should choose between these two policies. We also convey that despite being optimal, both policies entail inefficiencies leading to an opportunity cost for the online retailer. We then prescribe a new type of policy that mitigates such inefficiencies.

Our results indicate that the cost structures of the online retailer and offline partner determine whether the two parties should form a pickup partnership and if so, which policy they should use. Specifically, we find that the coupon policy is particularly suitable for offline partners that can manage the in-store pickup process efficiently (i.e., those with low in-store pickup handling cost) and online retailers with a high direct-delivery fulfillment cost, a low in-store pickup fulfillment cost, or high-priced products. In contrast, the fixed fee policy is suitable for offline partners with a moderate in-store pickup handling cost and online retailers with a moderate direct-delivery fulfillment cost, a moderate in-store fulfillment cost, or low-priced products. We also find that the partnership will not be beneficial for offline partners with a high in-store pickup handling cost or for online retailers with a low direct-delivery fulfillment cost or a high in-store fulfillment cost.

We later extend our model to examine the pickup partnership under three different scenarios. First, we find that the presence of a budget constraint may reduce the online retailer's willingness to adopt a coupon policy, making the fixed fee policy a more suitable option for online retailers with limited budgets. Second, we consider an offline partner with multiple pickup locations and find that a larger number of pickup locations does not necessarily yield a higher profit for the partnership, especially

[8]The parameter values are as follows: $p \in [0.7, 0.95]$ with 0.05 increments, $c_o \in [0.05p, 0.25p, 0.5p, 0.75p, 0.95p]$, $c_p \in [c_o, 0.75c_o, 0.5c_o, 0.25c_o, 0]$, $h_p \in [0.45, 0.75]$ with 0.1 increments, and $c_s \in [0, 1]$ with 0.05 increments. Other parameters are set to $v = 1$, $r = 0$, and $\theta = 1$ (The results are not sensitive to the values of v and θ. The increase of r intensifies the benefit of the partnership as it increases the offline partner's utility without harming the online retailer.). After dropping the instances that do not satisfy model assumptions, the numerical study results in 5,946 instances, of which, 5,540 (i.e., 93.17%) lead to a beneficial pickup partnership for both parties.

when in-store pickup fulfillment is more costly than direct-delivery fulfillment. Third, we consider an online retailer maximizing total welfare for all parties (including customers) and find that the coupon policy always outperforms the fixed fee policy due to the additional utility earned by customers from the discounted coupons.

Finally, we find that both policies entail inefficiencies due to the misaligned incentives of the online retailer. More precisely, to ensure that the offline partner is not worse off with the partnership (relative to no partnership), in some cases the online retailer has to propose a partnership that does not necessarily maximize her own profit. In such cases, the online retailer cannot fully leverage the potential of the partnership. To address this issue, we propose a hybrid policy that allows the online retailer to split the compensation amount per in-store pickup order between the offline partner (in the form of a fixed fee) and customers (in the form of a coupon). We then show that our proposed hybrid policy allows the online retailer to minimize these inefficiencies, while ensuring that the pickup partnership remains attractive for the offline partner. In a numerical study, we find that such inefficient partnerships can be common and that the profit improvement generated by the hybrid policy is substantial.

Our results provide several managerial implications for retailers seeking to establish a pickup partnership. First, a newly established partnership may require a fixed investment cost for the offline partner (e.g., setting up a pickup point in the store, assigning staff to process pickups) and the volume of in-store pickup orders is likely to increase over time as customers become more familiar with this service. Thus, the offline partner's handling cost is likely to be high in a newly established partnership. However, it will likely decline over time as the volume of in-store pickup orders increases and the offline partner improves the process through learning-by-doing. Therefore, a direct implication of our study is that a pickup partnership should initially be established using the fixed fee policy. As the offline partner becomes more efficient in processing in-store pickup orders, both parties can be better off by switching from the fixed fee policy to the coupon policy. In fact, our glance at the current pickup partnerships in the industry reveals an observation consistent with our implication. We observe that at this nascent stage of pickup partnerships, as implied by our study, firms mostly prefer the fixed fee policy relative to the coupon policy. Second, our results suggest that the parties are better off by customizing the partnership not only based on the stage of their relationship but also on the specific characteristics of the business setting. For example, for low-priced staple items, the fixed fee policy is more beneficial, whereas for high-priced niche items, the coupon policy is better. Since the hybrid policy encompasses both the fixed fee and coupon policies, it allows the online retailer to be more flexible in the partnership implementation. Third, although our results show that the optimal parameters for both the fixed fee and coupon policies depend on the characteristics of the business setting, we believe that the coupon policy may be an easier-to-implement option. For example, under the fixed fee policy, if the online retailer wants to set a fee that depends on the product price, both the online and offline partners need to keep track of actual transactions to determine the total transfer amount owing to the pickup partnership. Under the coupon policy, however, the online retailer can easily set a different coupon value based on the product price and make a transfer to the offline partner based on the redeemed amount.

Admittedly, more studies are needed to examine the growing trend of pickup partnerships. We can think of at least two potential avenues for future research. First, it would be interesting to investigate how the online retailer's and offline partner's operational decisions (e.g., assortment, price, inventory decisions) are affected by partnership policies. Second, it might be beneficial to empirically investigate the long- and short-term effects of each partnership policy for both the online retailer and the offline partner.

Appendix A Tables and figures

Table A.1: Summary of notation

Symbol	Definition
Notation related to the customer	
v	Customer's valuation for the product
h_o	Customer's hassle cost of using the direct-delivery option (e.g., shipping cost and delivery time)
x	Distance between customer's location and pickup location
h_p	Customer's hassle cost per unit of distance to visit the pickup location
θ	Probability that the customer redeems the coupon at the offline partner's store when picking up the order
r	Offline retailer's cross-selling profit per customer who picks up the order in-store
p	Price of the product
Notation related to the online retailer	
c_o	Online retailer's handling cost for each direct-delivery order (e.g., direct shipping cost)
c_p	Online retailer's handling cost for each in-store pickup order (e.g., cost of shipping to the pickup location)
α	Compensation value paid by the online retailer to the offline partner for each in-store pickup order
β	Monetary value of the coupon offered by the online retailer to be redeemed at the pickup location
d_o^i	Online retailer's expected demand for direct-delivery orders under policy $i \in \{F, C\}$
d_s^i	Online retailer's expected demand for in-store pickup orders under policy $i \in \{F, C\}$
π_o^i	Online retailer's expected profit under policy $i \in \{F, C\}$
Notation related to the offline partner	
c_s	Offline partner's handling cost per in-store pickup order (e.g., staff and storage)
π_s^i	Offline partner's expected profit from the partnership under policy $i \in \{F, C\}$

Appendix B Demand functions

To avoid trivial cases, in all appendices, we assume that under the baseline and fixed fee policies, there are some customers who leave the market, that is, $v - p < 1$ and $(v-p)/h_p < 1/2$. At the end of this section, we will explain, how would be the results if we relax these two assumption.

B.1 Baseline policy

Under the baseline policy, based on their utility, customers can either purchase the product via direct delivery or leave the market. Since the customer's utility from leaving the market is zero, a customer will purchase the product if her utility from buying is positive. Thus, since $h_o \sim U[0,1]$ and $x \sim U[0,1/2]$, the online retailer's demand under the baseline policy is simply $d_o^B = v - p$.

B.2 Fixed fee policy

Under the fixed fee policy, customer decisions are as follows:

a. When $h_o \le \min\{v-p, xh_p\}$, the customer purchases via direct delivery.
b. When $x \le \min\{\frac{h_o}{h_p}, \frac{v-p}{h_p}\}$, the customer purchases via in-store pickup.
c. When $v - p < \min\{h_o, xh_p\}$, the customer leaves the market.

Therefore, the demand for the direct-delivery and in-store pickup options are given by:

$$d_s^F = \frac{(v-p)(2-v+p)}{h_p} \quad \text{and} \quad d_o^F = \Big[1 - \frac{(v-p)}{h_p}\Big](v-p).$$

B.3 Coupon policy

Under the coupon policy, customer decisions are as follows:

a. When $h_o \leq \min\{v-p, xh_p - \theta\beta\}$, the customer purchases via direct delivery.
b. When $x \leq \min\{\frac{h_o+\theta\beta}{h_p}, \frac{v-p+\theta\beta}{h_p}\}$, the customer purchases via in-store pickup.
c. When $v-p < \min\{h_o, xh_p - \theta\beta\}$, the customer leaves the market.

Therefore, the demand for the direct-delivery and in-store pickup options are given by:

$$d_s^C = \begin{cases} \frac{2\theta\beta+(2-v+p)(v-p)}{h_p}, & 0 \leq \beta \leq \frac{h_p-2(v-p)}{2\theta} \\ (1-\frac{(h_p-2\theta\beta)^2}{4h_p}) & \frac{h_p-2(v-p)}{2\theta} \leq \beta \leq \frac{h_p}{2\theta} \\ 1 & \beta \geq \frac{h_p}{2\theta} \end{cases} \quad \text{and} \quad d_o^C = \begin{cases} (1-\frac{(v-p+2\theta\beta)}{h_p})(v-p), & 0 \leq \beta \leq \frac{h_p-2(v-p)}{2\theta} \\ \frac{(h_p-2\theta\beta)^2}{4h_p}, & \frac{h_p-2(v-p)}{2\theta} \leq \beta \leq \frac{h_p}{2\theta} \\ 0, & \beta \geq \frac{h_p}{2\theta} \end{cases}$$

Appendix C Proofs of statements

Proof of Proposition 1. The proof of Proposition 1 follows directly from the demand function derived in Appendix B. □

Proof of Proposition 2.

(a) For the online retailer, we have

$$\Delta\pi_o^F = (p-c_o)\Delta d_o^B + (p-c_p-\alpha)\Delta d_s^F \implies \Delta\pi_o^F = \frac{2(v-p)(1-v+p)}{h_p}(p-c_p-\alpha) + \frac{(v-p)^2}{h_p}(c_o-c_p-\alpha),$$

and for the offline partner, we have

$$\Delta\pi_s^F = (r+\alpha-c_s)\Delta d_s^F = \frac{(v-p)(2-v+p)}{h_p}(r+\alpha-c_s).$$

(b) The offline partner will accept the pickup partnership offer if and only if $\Delta\pi_s^F \geq 0$. Since $\Delta d_s^F > 0$ and $\alpha \geq 0$, $\Delta\pi_s^F$ is positive if

$$r+\alpha-c_s \geq 0 \implies \alpha \geq \max\{0, c_s - r\} = \underline{\alpha}.$$

Similarly, the online retailer will initiate the partnership under the fixed fee policy if and only if $\Delta\pi_o^F \geq 0$, and thus

$$\Delta\pi_o^F = \frac{2(v-p)(1-v+p)}{h_p}(p-c_p-\alpha) + \frac{(v-p)^2}{h_p}(c_o-c_p-\alpha) \geq 0 \implies \alpha \leq p-c_p-\frac{v-p}{2-v+p}(p-c_o) = \bar{\alpha}.$$

Therefore, the partnership under the fixed fee policy is beneficial only when $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. □

Proof of Proposition 3. The proof of Proposition 3 directly follows from the demand functions derived in Appendix B. We note that $\hat{\beta}$ and $\hat{\hat{\beta}}$ are given by:

$$\hat{\beta} = \begin{cases} 2\theta\beta & 0 \leq \beta \leq \frac{h_p-2(v-p)}{2\theta} \\ \frac{4\theta\beta(hp+\theta\beta)-h_p^2}{v-p} + h_p - v + p & \frac{h_p-2(v-p)}{2\theta} \leq \beta \leq \frac{h_p}{2\theta} \\ (h_p - v + p) & \beta \geq \frac{h_p}{2\theta} \end{cases} \quad \text{and} \quad \hat{\hat{\beta}} = \begin{cases} 2\theta\beta & 0 \leq \beta \leq \frac{h_p-2(v-p)}{2\theta} \\ \frac{(h_p-2v+2p)}{2} & \beta \geq \frac{h_p-2(v-p)}{2\theta} \end{cases}$$

□

Proof of Proposition 4.

(a) This result can be shown by substituting the demand function into the profit function reported in Section 3.

(b) The existence proof of $\underline{\beta}$ follows a similar argument to that of $\underline{\alpha}$. Thus, we only show the existence of $\bar{\beta}$, and we can then find β^*. To show the existence of $\bar{\beta}$, it is enough to show that $\pi_o^C(\beta)$ is a continuous and unimodal function of β (i.e., there exists a $\check{\beta}$ such that $\pi_o^C(\beta)$ is increasing for $\beta < \check{\beta}$ and decreasing for $\beta \geq \check{\beta}$) and that $\pi_o^B \leq \pi_o^C(\beta = 0)$.

One can easily show that $\pi_o^B < \pi_o^C(0)$ and that $\pi_o^C(\beta)$ is a continuous function. Thus, we only need to show that $\pi_o^C(\beta)$ is a unimodal function of β. The online retailer's profit under the coupon policy can be written as the following piece-wise function of β:

$$\pi_o^C(\beta) = \begin{cases} (1 - \frac{2\theta\beta+v-p}{h_p})(v-p)(p-c_o) + \frac{2\theta\beta+(2-v+p)(v-p)}{h_p}(p-c_p-\theta\beta), & 0 \leq \beta \leq \frac{h_p-2(v-p)}{2\theta} \\ \frac{(h_p-2\theta\beta)^2}{4h_p}(p-c_o) + (1 - \frac{(h_p-2\beta)^2}{4h_p})(p-c_p-\theta\beta), & \frac{h_p-2(v-p)}{2\theta} \leq \beta \leq \frac{h_p}{2\theta} \\ (p-c_p-\theta\beta) & \beta \geq \frac{h_p}{2\theta} \end{cases} \tag{C.1}$$

The first term in $\pi_o^C(\beta)$ is a quadratic function of β, which is denoted by $A(\beta)$. We thus have

$$\frac{\partial A(\beta)}{\partial\beta} = \frac{2\theta}{h_p}\Big[p - c_p - (v-p)(p-c_o) - \frac{(2-v+p)(v-p)}{2} - 2\theta\beta\Big].$$

The root of $\frac{\partial A(\beta)}{\partial\beta}$ is given by:

$$\beta_A^* = \frac{p-c_p}{2\theta} - \frac{(v-p)(p-c_o)}{2\theta} - \frac{(2-v+p)(v-p)}{4\theta}.$$

This root maximizes $A(\beta)$ if $0 \leq \beta_A^* \leq \frac{h_p-2(v-p)}{2\theta}$. In other words, β_A^* maximizes $A(\beta)$ if

$$(v-p)c_o + (1-v+p)p + \frac{(2+v-p)(v-p))}{2} - h_p \leq c_p \leq (v-p)c_o + (1-v+p)p - \frac{(v-p)(2-v+p)}{2}.$$

If $c_p > (v-p)c_o + (1-v+p)p - \frac{(v-p)(2-v+p)}{2}$, then $\beta_A^* < 0$, meaning that $A(\beta)$ is decreasing in β for $0 \leq \beta \leq \frac{h_p-2(v-p)}{2}$. If $c_p < (v-p)c_o + (1-v+p)p + \frac{(2+v-p)(v-p))}{2} - h_p$, then $\beta_A^* > \frac{h_p-2(v-p)}{2\theta}$, meaning that $A(\beta)$ is increasing in β for $0 \leq \beta \leq \frac{h_p-2(v-p)}{2\theta}$.

The second term in $\pi_o^C(\beta)$ is a cubic function of β, which is denoted by $B(\beta)$ and can be written as follows:

$$\begin{aligned} B(\beta) &= (p - c_p - \theta\beta) + \frac{h_p^2 + 4(\theta\beta)^2 - 4h_p\theta\beta}{4h_p}(c_p + \theta\beta - c_o) \\ &= \frac{1}{h_p}(\theta\beta)^3 + \frac{c_p - c_o - h_p}{h_p}(\theta\beta)^2 + (c_o - c_p - 1 + \frac{h_p}{4})\theta\beta + (\frac{h_p}{4}(c_p - c_o) + p - c_p). \end{aligned}$$

Since $\frac{\theta^3}{h_p} > 0$, Figure C.1(a) depicts the only possible pattern for $B(\beta)$.

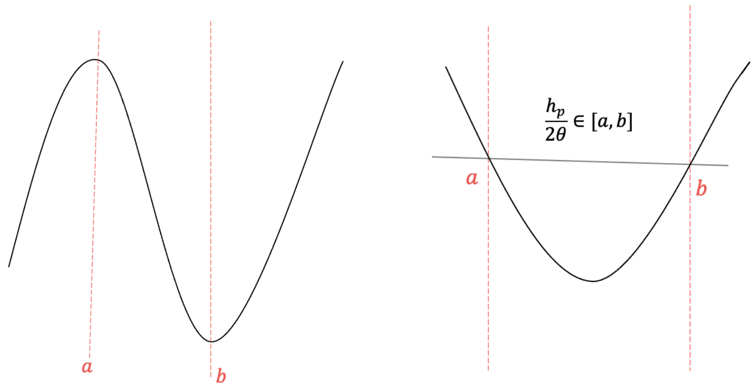
The first derivative of $B(\beta)$ is given by:

$$\frac{\partial B(\beta)}{\partial\beta} = -\theta + \frac{h_p - 2\theta\beta}{h_p}\theta(c_o - c_p) - \frac{h_p - 2\theta\beta}{h_p}\theta^2\beta + \frac{(h_p - 2\theta\beta)^2}{4h_p}\theta, \quad \frac{h_p - 2(v-p)}{2\theta} < \beta \leq \frac{h_p}{2\theta}.$$

When $\beta = \frac{h_p}{2\theta}$, we have $\frac{\partial B(\beta)}{\partial\beta} = -\theta < 0$. Therefore, $\frac{h_p}{2\theta} \in [a, b]$, where a and b are the roots of $\frac{\partial B(\beta)}{\partial\beta}$ (see Figure C.1).

When $\beta = \frac{h_p-2(v-p)}{2\theta}$, we have

$$\begin{aligned} \frac{\partial B(\beta)}{\partial\beta} &= \frac{2(v-p)}{h_p}\theta(c_o - c_p) - \frac{2(v-p)}{h_p}(\frac{h_p}{2} - v + p)\theta + \frac{(v-p)^2}{h_p}\theta - \theta \\ &= \Big[\frac{2(v-p)}{h_p}(c_o - c_p) - (v-p) + \frac{3(v-p)^2}{h_p} - 1\Big]\theta. \end{aligned}$$



(a) Possible Pattern for $B(\beta)$ (b) Plot of $\frac{\partial B(\beta)}{\partial \beta}$

Figure C.1

If $c_p \leq c_o + \frac{3(v-p)}{2} - \frac{h_p}{2} - \frac{h_p}{2(v-p)}$, then $\frac{\partial B(\beta)}{\partial \beta} \geq 0$ for $\beta = \frac{h_p - 2(v-p)}{2\theta}$. It means that $\frac{h_p-2(v-p)}{2\theta} < a$, where a is the first root of $\frac{\partial B(\beta)}{\partial \beta}$ (see Figure C.1). Therefore, when $c_p \leq c_o + \frac{3(v-p)}{2} - \frac{h_p}{2} - \frac{h_p}{2(v-p)}$, a maximizes $B(\beta)$. Since $\frac{\partial B(\beta)}{\partial \beta}$ is a quadratic function of β, a is given by:

$$a = \frac{1}{6\theta}\Big[2(h_p + c_o - c_p) - \sqrt{(2c_p + h_p)^2 + 4c_o(c_o - h_p - 2c_p) + 12h_p}\Big].$$

When $c_p > c_o + \frac{3(v-p)}{2} - \frac{h_p}{2} - \frac{h_p}{2(v-p)}$, $\frac{\partial B(\beta)}{\partial \beta} < 0$ for $\beta = \frac{h_p-2(v-p)}{2\theta}$. Therefore, $\frac{h_p-2(v-p)}{2\theta} \in [a, b]$, where a and b are the roots of $\frac{\partial B(\beta)}{\partial \beta}$. Thus, $B(\beta)$ is decreasing in β when $\beta \in [\frac{h_p-2(v-p)}{2\theta}, \frac{h_p}{2\theta}]$ and $\beta = \frac{h_p-2(v-p)}{2\theta}$ maximizes $B(\beta)$.

The third term of $\pi_o^C(\beta)$ is a linear function of β, which is denoted by $C(\beta)$. Since $\frac{\partial C(\beta)}{\partial \beta} = -\theta < 0$, $C(\beta)$ is decreasing in β.

We define $c_1 = c_o - \frac{h_p}{2} - \frac{h_p}{2(v-p)} + \frac{3(v-p)}{2}$, $c_2 = (v-p)c_o + (1-v+p)p + \frac{(2+v-p)(v-p)}{2} - h_p$, and $c_3 = (v-p)c_o + (1-v+p)p - \frac{(v-p)(2-v+p)}{2}$. Since we assume that $v - p < 1$ and $\frac{v-p}{h_p} \leq \frac{1}{2}$ (i.e., there are some customers who leave the market under the baseline and fixed fee policies), we can show that $c_3 \geq c_2 \geq c_1$. Since $\pi_o^C(\beta)$ is continuous, $\pi_o^C(\beta)$ can have only one of four possible patterns as shown in Figure C.2. We thus conclude that $\pi_o^C(\beta)$ is a continuous and unimodal function.

Since $\pi_o^C(\beta)$ is a continuous and unimodal function, there exists a unique $\check{\beta}$ (see closed-form expression below) that maximizes $\pi_o^C(\beta)$.

$$\check{\beta} = \begin{cases} \frac{1}{6\theta}\Big[2(h_p + c_o - c_p) - \sqrt{(2c_p + h_p)^2 + 4c_o(c_o - h_p - 2c_p) + 12h_p}\Big], & c_p \leq c_1 \\ \frac{1}{2\theta}\Big[h_p - 2(v-p)\Big], & c_1 < c_p \leq c_2 \\ \frac{1}{4\theta}\Big[2(p - c_p) - 2(v-p)(p-c_o) - 2(v-p) + (v-p)^2\Big], & c_2 < c_p \leq c_3 \\ 0, & c_p > c_3 \end{cases}$$

where $c_1 = c_o - \frac{h_p}{2} - \frac{h_p}{2(v-p)} + \frac{3(v-p)}{2}$, $c_2 = (v-p)c_o + (1-v+p)p + \frac{(2+v-p)(v-p)}{2} - h_p$, and $c_3 = (v-p)c_o + (1-v+p)p - \frac{(v-p)(2-v+p)}{2}$.

The variable $\check{\beta}$ maximizes the online retailer's profit under the coupon policy without considering the offline partner's rationality constraint. If $\beta = \check{\beta}$ does not satisfy the rationality constraint, then the online retailer should increase the value of β. Since $\pi_o^C(\beta)$ is decreasing for $\beta > \check{\beta}$, in that case, the optimal value is the smallest value that satisfies the offline partner's rationality constraint, that is, $\frac{c_s - r}{\theta}$. As a result, the optimal value of β is $\max\{\frac{c_s - r}{\theta}, \check{\beta}\}$. □

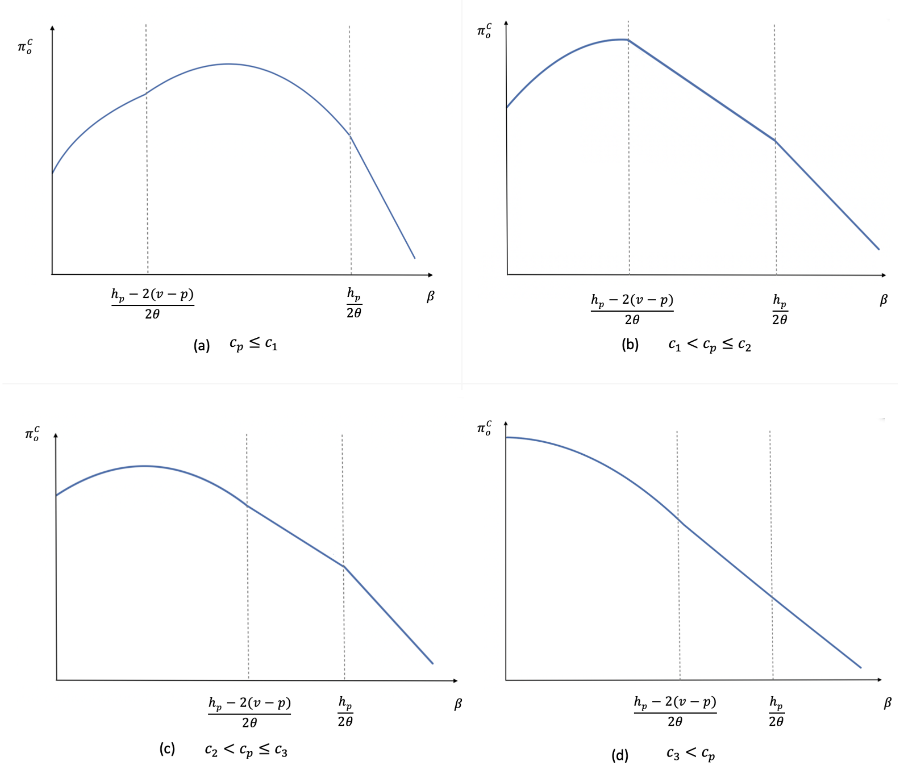


Figure C.2: The online retailer's profit under the coupon policy

Proof of Proposition 5. First, we show that there exists a unique $\bar{\bar{\beta}}$ such that under condition $\theta\beta = \alpha$, the online retailer will prefer the coupon policy over the fixed fee policy when $\beta \leq \bar{\bar{\beta}}$ (i.e., for $\alpha = \theta\beta \leq \theta\bar{\bar{\beta}}$, $\pi_o^F(\alpha) \leq \pi_o^C(\beta)$). We also show that $\theta\bar{\bar{\beta}} \leq \bar{\alpha}$ and $\bar{\bar{\beta}} < \bar{\beta}$. By combining these findings with Propositions 2 and 4, it is straightforward to show that the optimal policy is (i) the coupon policy when $c_s \leq r + \theta\bar{\bar{\beta}} = \underline{c}_s$, (ii) the fixed fee policy when $r + \theta\bar{\bar{\beta}} = \underline{c}_s < c_s \leq r + \bar{\alpha} = \bar{c}_s$, and (iii) the baseline policy when $c_s > \bar{c}_s$.

The existence of $\bar{\bar{\beta}}$ can be shown using the characteristics of $\pi_o^C(\beta)$ as discussed in the proof of Proposition 4. For $\frac{\alpha}{\theta} = \beta > \check{\beta}$, the online retailer's profit is decreasing in the partnership parameter under both the fixed fee and the coupon policies, and $|\frac{\partial \pi_o^C(\beta)}{\partial \beta}| > \theta|\frac{\partial \pi_o^F(\alpha)}{\partial \alpha}|$ for $\beta > \frac{h_p - 2(v-p)}{2\theta}$. Therefore, $\pi_o^C(\beta) = \pi_o^F(\alpha)$ has a unique solution.

Since $\pi_o^F(\alpha = 0) = \pi_o^C(\beta = 0)$, $\pi_o^C(\beta)$ is a unimodal function, and $\pi_o^F(\alpha)$ is a decreasing function of α, if we show that $\theta\bar{\bar{\beta}} \leq \bar{\alpha}$, then we can conclude that $\theta\bar{\bar{\beta}} < \bar{\alpha}$ (see Figure C.3). To show this, it is enough to show that $\pi_o^C(\frac{\bar{\alpha}}{\theta}) < \pi_o^F(\bar{\alpha}) = \pi_o^B$ where $\bar{\alpha} = p - c_p - \frac{v-p}{2-v+p}(p - c_o)$. The following tree situations are then possible:

1. $\bar{\alpha} > \frac{h_p}{2}$. In this case, we have

$$\pi_o^C(\frac{\bar{\alpha}}{\theta}) = (p - c_p - \bar{\alpha}) = \frac{v-p}{2-v+p}(p - c_o) < (v-p)(p - c_o) = \pi_o^B,$$

where the last inequality follows from $v - p < 1$.

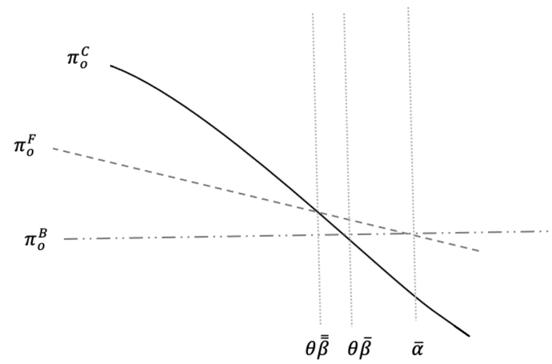


Figure C.3: Possible relationship between $\bar{\bar{\beta}}$, $\bar{\beta}$, and $\bar{\alpha}$

2. $\frac{h_p-2(v-p)}{2} < \bar{\alpha} \le \frac{h_p}{2}$. In this case, we have

$$\pi_o^C(\frac{\bar{\alpha}}{\theta}) = \Big[p-c_p-\bar{\alpha}+(c_p+\bar{\alpha}-c_o)\frac{(h_p-2\bar{\alpha})^2}{4h_p}\Big] = \Big[1-\frac{(h_p-2\bar{\alpha})^2}{4h_p}\Big]\frac{v-p}{2-v+p}(p-c_o)+\frac{(h_p-2\bar{\alpha})^2}{4h_p}(p-c_o).$$

We next show that if $\pi_o^C(\frac{\bar{\alpha}}{\theta}) \le \pi_o^B$, we then reach to a strict inequality. We have

$$\Big[1-\frac{(h_p-2\bar{\alpha})^2}{4h_p}\Big](\frac{v-p}{2-v+p})(p-c_o)+\frac{(h_p-2\bar{\alpha})^2}{4h_p}(p-c_o) \le (v-p)(p-c_o) \implies (h_p-2\bar{\alpha})^2 \le 2h_p(v-p)$$

$$\implies (h_p-2(p-c_p)+\frac{2(v-p)}{2-v+p}(p-c_o))^2 \le 2h_p(v-p).$$

Since $4(v-p)^2 \le 2h_p(v-p)$, then

$$\Big[h_p-2(p-c_p)+\frac{2(v-p)}{2-v+p}(p-c_o)\Big] < 2(v-p) \implies \frac{h_p-2(v-p)}{2} \le (p-c_p)-\frac{v-p}{2-v+p}(p-c_o) = \bar{\alpha}.$$

3. $\bar{\alpha} < \frac{h_p-2(v-p)}{2}$. As in the second case, if $\pi_o^C(\frac{\bar{\alpha}}{\theta}) \le \pi_o^B$, then we reach a strict inequality.

$$\pi_o^C(\frac{\bar{\alpha}}{\theta}) = \frac{2\bar{\alpha}+(2-v+p)(v-p)}{h_p}(p-c_p-\bar{\alpha})+(1-\frac{v-p+2\bar{\alpha}}{h_p})(v-p)(p-c_o) \le (p-c_o)(v-p)$$

$$\implies 2\bar{\alpha}(p-c_p-\bar{\alpha})+(2-v+p)(v-p)(p-c_p-\bar{\alpha}) < (v-p+2\bar{\alpha})(v-p)(p-c_o).$$

Since $\bar{\alpha} = (p-c_p)-\frac{v-p}{2-v+p}(p-c_o)$, we have

$$2\bar{\alpha}\frac{v-p}{2-v+p}(p-c_o)+(2-v+p)(v-p)\frac{v-p}{2-v+p}(p-c_o) \le (v-p+2\bar{\alpha})(v-p)(p-c_o)$$

$$\implies \frac{2\bar{\alpha}}{2-v+p}+v-p \le v-p+2\bar{\alpha} \implies 1 \le 2-v+p,$$

where last inequity follows from $v-p<1$. We then have $\theta\bar{\bar{\beta}} \le \bar{\alpha}$.

Therefore, we conclude that when $\alpha = \theta\beta < \theta\bar{\bar{\beta}}$, we have $\pi_o^C(\beta) > \pi_o^F(\alpha) > \pi_o^B$, when $\theta\bar{\bar{\beta}} < \alpha = \theta\beta < \bar{\alpha}$, we have $\pi_o^F(\alpha) > \max\{\pi_o^C(\beta), \pi_o^B\}$, and when $\alpha = \theta\beta > \bar{\alpha}$, we have $\pi_o^B > \pi_o^F(\alpha) > \pi_o^C(\alpha)$. Thus, based on Propositions 2 and 4, when $c_s \le r+\theta\bar{\bar{\beta}} = \underline{c}_s$, the online retailer's profit under a beneficial coupon policy is higher relative to the fixed fee policy, and the coupon policy is beneficial for any $\beta \in [\max\{0, \frac{c_s-r}{\theta}\}, \bar{\bar{\beta}}]$. When $\underline{c}_s < c_s \le r+\bar{\alpha} = \bar{c}_s$, the online retailer's profit under a beneficial fixed fee policy is greater than a beneficial coupon policy, and the fixed policy is beneficial for any $\alpha \in [c_s-r, \bar{c}_s-r]$. Lastly, when $c_s > \bar{c}_s$, neither a fixed fee policy nor a coupon policy can improve the online retailer's profit when compared to the baseline policy, and hence the baseline policy is optimal. □

We can show that the results hold when we consider the optimal α and β for the online retailer. Based on *Proposition 2*, we know that the fixed fee policy is beneficial for both partners if $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, and the online retailer's profit is decreasing in α. Therefore, the optimal α for the online retailer is $\alpha^* = \underline{\alpha} = \max\{0, c_s - r\}$. Based on *Proposition 4*, the optimal coupon value (i.e., β) is $\beta^* = \max\{\frac{c_s - r}{\theta}, \breve{\beta}\}$. Therefore, under the optimal condition, we still have $\alpha^* = \theta\beta^*$ unless when $c_s \leq \theta\breve{\beta} + r < \underline{c}_s$. The second part of the inequality results from our discussion above that $c_s = r + \theta\bar{\bar{\beta}}$. In this case, $\beta^* = \breve{\beta} \geq \frac{c_s - r}{\theta}$ and $\alpha^* = c_s - r$. Because of $\pi_o^F(\alpha = 0) = \pi_o^C(\beta = 0)$ and what we showed in the proof of *Proposition 4*, in this case, the online retailer will prefer the coupon policy over the fixed fee policy. Therefore, our results still hold under the optimal value of α and β.

Proof of Proposition 6. We first show that when $c_p > v - \frac{h_p}{2} - \frac{(v-p)(p-c_o)}{h_p} = \bar{c}_p$, the hybrid policy cannot be optimal, and so the optimal policy remains the same as in Proposition 5.

When $c_p > \bar{c}_p$, we have $\bar{\bar{\beta}} \leq \frac{h_p - 2(v-p)}{2\theta}$, so that $\pi_o^C(\beta = \frac{h_p - 2(v-p)}{2\theta}) < \pi_o^F(\alpha = \frac{h_p - 2(v-p)}{2})$. We next show that when $c_p > \bar{c}_p$, the optimal policy is either the coupon policy or the fixed fee policy.

Although the online retailer's profit is a piece-wise function of β, in this case, it is enough to focus on the first part of the function (i.e., when $\beta \leq \frac{h_p - 2(v-p)}{2\theta}$) because the coupon policy can be optimal only when $\beta \leq \frac{h_p - 2(v-p)}{2\theta}$. We assume that there exists an optimal hybrid policy with parameters β_h^* and α_h^* (where $\beta_h^*, \alpha_h^* > 0$). Namely, β_h^* and α_h^* maximize the online retailer's profit, while satisfying the offline partner's rationality constraint $(r + \alpha_h^* + \theta\beta_h^* - c_s)d_s^H \geq 0$. Thus, the online retailer's profit under the hybrid policy must be higher than the profit under the coupon policy with the coupon value $\beta^* = \frac{\theta\beta_h^* + \alpha_h^*}{\theta}$ and the fixed fee policy with $\alpha^* = \theta\beta_h^* + \alpha_h^*$.

We let $\pi_o^H(\gamma = \theta\beta_h + \alpha_h)$ denote the online retailer's profit under the hybrid policy. For the online retailer's profit under the hybrid policy to be higher relative to the coupon policy, we must have the following (without loss of generality, we assume $\theta = 1$):

$$\begin{aligned}
&\pi_o^H(\gamma^* = \beta_h^* + \alpha_h^*) > \pi_o^C(\beta^*) \\
&\implies \frac{2\beta_h^* + (2 - v + p)(v - p)}{h_p}(p - c_p - \beta_h^* - \alpha_h^*) + (1 - \frac{v - p + 2\beta_h^*}{h_p})(v - p)(p - c_o) > \\
&\quad \frac{2\beta^* + (2 - v + p)(v - p)}{h_p}(p - c_p - \beta^*) + (1 - \frac{v - p + 2\beta^*}{h_p})(v - p)(p - c_o) \\
&\implies \beta_h^* + \alpha_h^* > p - c_p - (v - p)(p - c_o).
\end{aligned} \tag{C.2}$$

In addition, the online retailer's profit under the hybrid policy must be higher relative to fixed fee policy, that is,

$$\begin{aligned}
&\pi_o^H(\gamma^* = \beta_h^* + \alpha_h^*) > \pi_o^F(\alpha^*) \\
&\implies \frac{2\beta_h^* + (2 - v + p)(v - p)}{h_p}(p - c_p - \beta_h^* - \alpha_h^*) + (1 - \frac{v - p + 2\beta_h^*}{h_p})D(v - p)(p - c_o) > \\
&\quad \frac{(2 - v + p)(v - p)}{h_p}(p - c_p - \alpha^*) + (1 - \frac{v - p}{h_p})(v - p)(p - c_o) \\
&\implies \beta_h * + \alpha_h^* < p - c_p - (v - p)(p - c_o).
\end{aligned} \tag{C.3}$$

Inequalities (C.2) and (C.3) cannot hold simultaneously, and so when $c_p > \bar{c}_p$, the hybrid policy cannot be optimal. The online retailer will opt either for the fixed fee policy or for coupon policy based on Proposition 5.

We next show that when $c_p < \bar{c}_p$ (i.e., $\bar{\bar{\beta}} > \frac{h_p - 2(v-p)}{2\theta}$), the hybrid policy can be optimal only when $\underline{\underline{c}}_s = r + \frac{h_p - 2(v-p)}{2} \leq c_s < r + p - c_p - (p - c_o)(v - p) = \bar{\bar{c}}_s$. When $c_p < \bar{c}_p$, there are three possible cases:

1. $c_s \leq \theta\breve{\beta} + r$. Based on Propositions 4 and 5, the online retailer's profit is maximized when $\beta = \breve{\beta}$, and since $c_s \leq \theta\breve{\beta} + r$, $\beta = \breve{\beta}$ satisfies the offline partner's rationality constraint, so that the coupon policy is optimal.

2. $\theta\check{\beta} + r < c_s \le r + \bar{\alpha}$. In this case, the online retailer can maximize her profit under the fixed fee and coupon policies with parameters $c_s - r$ and $\frac{c_s - r}{\theta}$, respectively (i.e., to maximize her profit, the online retailer will pay the minimum compensation value under either policy, and so the offline partner's rationality constraint is binding). Therefore, the online retailer determines the optimal decision by using the following optimization formulation:

$$\begin{aligned} \max_{\beta_h, \alpha_h} \quad & \pi_o(\gamma) = (p - c_o)d_o^H + (p - c_p - \alpha_h - \theta\beta_h)d_s^H \\ & (r + \alpha_h + \theta\beta_h - c_s)d_s^H \ge 0 \\ & \alpha_h + \theta\beta_h = c_s - r \\ & \alpha_h, \beta_h \ge 0. \end{aligned} \tag{C.4}$$

As a result, it is enough to solve

$$\max_{\beta_h} \pi_o(\beta_h) = (p - c_o)d_o^H + (p - c_p - c_s + r)d_s^H. \tag{C.5}$$

If $\beta_h^\dagger$ is the solution of Equation (C.5), then the solution of Equation (C.4) is $(\beta_h^*, \alpha_h^*) = (\beta_h^\dagger, c_s - r - \beta_h^\dagger)$. We will find $\beta_h^\dagger$ based on the first-order condition. The profit function $\pi_o(\beta_h)$ in Equation (C.5) can be written as

$$\pi_o(\beta_h) = \begin{cases} (p - c_o)(1 - \frac{v - p + 2\theta\beta_h}{h_p})(v - p) + (p - c_p - c_s + r)\frac{2\theta\beta_h + (2 - v + p)(v - p)}{h_p}, & 0 \le \beta_h \le \frac{h_p - 2(v-p)}{2\theta} \\ (p - c_p - c_s + r) - \frac{(h_p - 2\theta\beta_h)^2}{4h_p}(r + c_o - c_p - c_s), & \frac{h_p - 2(v-p)}{2\theta} < \beta_h \le \frac{h_p}{2\theta} \\ (p - c_p - c_s + r) & \beta_h \ge \frac{h_p}{2\theta} \end{cases}$$

Thus, the first derivative of $\pi_o(\beta_h)$ is given by:

$$\frac{\partial \pi_o}{\partial \beta_h} = \begin{cases} \frac{2\theta}{h_p}\Big[p - c_p - c_s + r - (p - c_o)(v - p)\Big], & 0 \le \beta_h < \frac{h_p - 2(v-p)}{2\theta} \\ \frac{\theta}{h_p}(h_p - 2\theta\beta_h)(r + c_o - c_p - c_s), & \frac{h_p - 2(v-p)}{2\theta} < \beta_h \le \frac{h_p}{2\theta} \\ 0 & \beta_h \ge \frac{h_p}{2\theta} \end{cases}$$

Therefore, when $c_s > r + p - c_p - (p - c_o)(v - p)$, we have $\beta_h^\dagger = 0$. In this case, the optimal policy is the fixed fee policy. When $c_s < r + p - c_p - (p - c_o)(v - p)$, we have $\beta_h^\dagger = \frac{h_p - 2(v-p)}{2\theta}$. In this case, the optimal policy can be either the coupon policy or the hybrid policy. More specifically, if $c_s \ge r + \frac{h_p - 2(v-p)}{2}$, then the hybrid policy with $\beta_h^* = \frac{h_p - 2(v-p)}{2\theta}$ and $\alpha_h^* = c_s - r - \frac{h_p - 2(v-p)}{2\theta}$ is optimal. Otherwise, the coupon policy with $\beta^* = \frac{c_s - r}{\theta}$ is optimal.

3. $c_s > r + \bar{\alpha}$. In this case, based on Proposition 5, none of the policies are beneficial. □

As mentioned above, to avoid trivial cases, we assume that $v - p < 1$ and $(v - p)/h_p < 1/2$. Here, we will explain what the results would be if we relax these assumptions. If we relax the assumption of $v - p < 1$, under the baseline model, all customers in the market will purchase from the online retailer. This happens because $h_o \le 1$. Therefore, in this case, the pickup partnership will be beneficial for the online retailer if and only if the profit margin of in-store pickup orders is higher than the profit margin of direct-delivery orders (i.e., $\alpha \le c_o - c_p$ or $\theta\beta \le c_o - c_p$). In this case, because the coupon policy encourages more customers to use in-store pickup service, the online retailer always prefers the coupon policy over the fixed fee policy. Therefore, when $v - p \ge 1$, the pickup partnership is beneficial for both parties if and only if $c_s \le r + c_o - c_p$, and the coupon-policy is preferred over the fixed-fee policy. If we relax assumption $(v - p)/h_p < 1/2$, under the fixed fee policy, all customers in the market will purchase from the online retailer. In this case, comparing the fixed fee policy and coupon policy is less interesting because the online retailer prefers the coupon-policy over the fixed-fee policy if and only if when the profit margin of the in-store pickup order is higher than the direct-delivery order. This happens because the additional market expansion of the coupon disappears when $(v - p)/h_p \ge 1/2$.

Our results still hold if we consider the cross-selling effect into the customer utility. Let's assume that γ is the average customer's additional utility from the cross-selling effect. In this case, the customer utility for in-store pickup orders under the fixed fee policy and coupon policy changes to $u_p^F = v + \gamma - p - xh_p$ and $u_p^C = v + \gamma - p - xh_p + \theta\beta$, respectively. In this case, the demand for the direct-delivery and in-store pickup options will change. Again, to avoid trivial cases, we assume that some of the customer will leave the market under the fixed-fee policy. In other words, we assume $0 \leq \gamma \leq (hp - 2(v-p))/2$.

The demand for the direct-delivery and in-store pickup options under the fixed-fee policy are given by:

$$d_s^F = \frac{2\gamma + (v-p)(2-v+p)}{h_p} \quad \text{and} \quad d_o^F = \Big[1 - \frac{(v-p+2\gamma)}{h_p}\Big](v-p).$$

The demand for the direct-delivery and in-store pickup options under the coupon policy are given by:

$$d_s^C = \begin{cases} \frac{2\theta\beta+2\gamma+(2-v+p)(v-p)}{h_p}, & 0 \leq \beta \leq \frac{h_p-2(v-p-\gamma)}{2\theta} \\ (1 - \frac{(h_p-2\theta\beta-2\gamma)^2}{4h_p}) & \frac{h_p-2(v-p-\gamma)}{2\theta} \leq \beta \leq \frac{h_p-2\gamma}{2\theta} \\ 1 & \beta \geq \frac{h_p-2\gamma}{2\theta} \end{cases}$$

and

$$d_o^C = \begin{cases} (1 - \frac{(v-p+2\theta\beta+2\gamma)}{h_p})(v-p), & 0 \leq \beta \leq \frac{h_p-2(v-p-\gamma)}{2\theta} \\ \frac{(h_p-2\theta\beta-2\gamma)^2}{4h_p}, & \frac{h_p-2(v-p-\gamma)}{2\theta} \leq \beta \leq \frac{h_p-2\gamma}{2\theta} \\ 0, & \beta \geq \frac{h_p-2\gamma}{2\theta} \end{cases}$$

Following the same proof procedure, we can show that the findings continue to hold under this alternative model.

Our findings hold even when we assume that the coupon will increases cross-selling effect for the offline partner only by probability ζ (i.e., when customers use the coupon for additional purchase with probability ζ, and use the coupon to cover the cost of r with probability $1-\zeta$). Note that this assumption does not impact the demand function because the customer utility from the coupon remains the same no matter that the customer uses it to purchase more or to cover the cost of r. Besides, the online partner should still pay β for each redeem coupon. For this reason, the profit margin of in-store pickup orders remain the same for the online retailer(i.e., $p - c_p - \theta\beta$). Therefore, our analyses remain as above except the lower bound for the coupon value (i.e., $\underline{\beta}$). Under this scenario, the coupon policy is beneficial for the offline partner if and only if $\beta \geq \underline{\beta} = \min\{0, \frac{c_s-r}{\zeta\theta}\}$.

Appendix D Details for comparative statics

We let $\Delta\pi_o^{C-F}$ denote the difference in the online retailer's profit between the coupon policy and the fixed fee policy with the same average compensation value per pickup order (i.e., $\alpha = \theta\beta$). We first chracterize how $\Delta\pi_o^F$, $\Delta\pi_o^C$, and $\Delta\pi_o^{C-F}$ are changing with respect to c_p and c_o. We have

$$\frac{\partial\Delta\pi_o^F}{\partial c_p} = -\frac{(v-p)(2-v+p)}{h_p} < 0 \quad \text{and} \quad \frac{\partial\Delta\pi_o^C}{\partial c_p} = -\Big[\frac{(v-p)(2-v+p)}{h_p} + \frac{(1-v+p)}{h_p}\hat{\hat{\beta}} + \frac{(v-p)}{h_p}\hat{\beta}\Big] < 0.$$

Thus, we conclude that the profitability of the fixed fee and coupon policies decreases with c_p. We also have

$$\frac{\partial\Delta\pi_o^{C-F}}{\partial c_p} = -\Big[\frac{(1-v+p)}{h_p} + \frac{(v-p)}{h_p}\hat{\beta}\Big] < 0.$$

Since $\frac{\partial\Delta\pi_o^{C-F}}{\partial c_p} < 0$, we conclude that the profitability of the coupon policy decreases with c_p faster than that of the fixed fee policy.

Similarly, for c_o, we have

$$\frac{\partial \Delta\pi_o^F}{\partial c_o} = \frac{(v-p)^2}{h_p} > 0 \quad \text{and} \quad \frac{\partial \Delta\pi_o^C}{\partial c_o} = \frac{(v-p)^2}{h_p} + \frac{(v-p)}{h_p}\hat{\beta} > 0.$$

Thus, we conclude that the profitability of the fixed fee and coupon policies increases with c_o. We also have

$$\frac{\partial \Delta\pi_o^{C-F}}{\partial c_o} = \frac{(v-p)}{h_p}\hat{\beta} > 0.$$

Since $\frac{\partial \Delta\pi_o^{C-F}}{\partial c_o} > 0$, we conclude that the profitability of the coupon policy increases with c_o faster than the fixed fee policy. Next, we show how $\Delta\pi_o^{C-F}$ changes with respect to p.

$$\Delta\pi_o^{C-F} = \begin{cases} \frac{2\theta\beta(1-v+p)}{h_p}(p-c_p-\theta\beta+\frac{2\theta\beta(v-p)}{h_p}(c_o-c_p-\theta\beta)) & p \geq \theta\beta+v-\frac{h_p}{2} \\ \frac{(h_p-2v+2p)(1-v+p)}{2h_p}(p-c_p-\theta\beta)+(\theta\beta-\frac{(\theta\beta)^2}{h_p}-\frac{(h_p-2(v-p))^2}{4h_p})(c_o-c_p-\theta\beta) & p \leq \theta\beta+v-\frac{h_p}{2}, \beta \leq \frac{h_p}{2\theta} \\ \frac{(h_p-2(v-p))(1-v+p)}{2h_p}(p-c_p-\theta\beta)+(1-\frac{v-p}{h_p})(v-p)(c_o-c_p-\theta\beta) & p \leq \theta\beta+v-\frac{h_p}{2}, \beta > \frac{h_p}{2\theta} \end{cases}$$

More specifically, $\frac{\partial \Delta\pi_o^{C-F}}{\partial p}$ can be characterized as follows:

- When $p \geq \theta\beta+v-\frac{h_p}{2}$, we have

$$\frac{\partial \Delta\pi_o^{C-F}}{\partial p} = \frac{2\theta\beta}{h_p}(1-v+2p-c_o) \geq 0.$$

- When $p \leq \theta\beta+v-\frac{h_p}{2}$, $\theta\beta \leq \frac{h_p}{2}$, we have

$$\frac{\partial \Delta\pi_o^{C-F}}{\partial p} = \frac{1}{2h_p}\Big[(p-c_p-\theta\beta)(2(1-v+p)+(h_p-2(v-p))+(h_p-2(v-p))(1-v+p-2(c_o-c_p-\theta\beta))\Big].$$

  When $c_o \leq c_p+\theta\beta$, we can show that $\frac{\partial \Delta\pi_o^{C-F}}{\partial p} > 0$. When $c_o > c_p+\theta\beta$, it is enough to show that

$$2(p-c_p-\theta\beta)(1-v+p)+(p-c_p-\theta\beta)(h_p-2(v-p))+(h_p-2(v-p))(1-v+p) \geq 2(c_o-c_p-\theta\beta)(h_p-2(v-p))$$

  so that

$$\frac{2(p-c_p-\theta\beta)(1-v+p)}{(h_p-2(v-p))} + (p-c_p-\theta\beta) + (1-v+p) \geq 2(c_o-c_p-\theta\beta),$$

  where the above inequality follows from $p+c_p+\theta\beta+1-v+p \geq 2c_o$.

- When $p \leq \theta\beta+v-\frac{h_p}{2}$, $\theta\beta > \frac{h_p}{2}$, we have

$$\frac{\partial \Delta\pi_o^{C-F}}{\partial p} = \frac{1}{h_p}((1-v+p)(p-c_p-\theta\beta) + \frac{(h_p-2(v-p))(p-c_p-\theta\beta)}{2} + \frac{(h_p-2(v-p))(1-v+p)}{2} + (c_o-c_p-\theta\beta)(2(v-p)-h_p)).$$

  Since $p+c_p+\theta\beta+1-v+p \geq c_o$, we can show that $\frac{\partial \Delta\pi_o^{C-F}}{\partial p} > 0$ when $p \leq \theta\beta+v-\frac{h_p}{2}$ and $\theta\beta > \frac{h_p}{2}$.

  As a result, by increasing p, the coupon policy becomes more profitable for the online retailer.

Appendix E Proofs of statements from Section 5

D.1 Budget constraint

In this subsection, we provide the details of our findings related to the model in the presence of the budget constraint. We let K denote the total budget. There exists $\underline{K} = \max\{0, c_s - r\}d_s^F$ so that when $K < \underline{K}$, neither the fixed fee policy nor the coupon policy are feasible. This follows from the fact that the minimum compensation value under which the offline partner is not worse off under the fixed fee policy is $\max\{0, c_s - r\}$, and the demand for in-store pickup orders is d_s^F. As a result, the minimum budget under which the fixed fee policy is beneficial is $\max\{0, c_s - r\}d_s^F$. Note that the minimum budget under the coupon policy is higher than $\max\{0, c_s - r\}d_s^F$. Indeed, based on Proposition 3, the demand for in-store pickup orders is always higher under the coupon policy than under the fixed fee policy (i.e., $d_s^F < d_s^C$). Thus, when $K > \underline{K}$, the fixed fee policy is beneficial. The maximum fixed fee value that the online retailer can pay to the offline partner under the budget constraint is $\frac{K}{d_s^F}$. However, based on Proposition 5, the fixed fee policy will be beneficial only when $\alpha \in [\underline{\alpha}, \bar{c}_s - r]$. Therefore, when $K > \underline{K}$ the fixed fee policy with parameter $\alpha \in [\underline{\alpha}, \max\{\bar{c}_s - r, \frac{K}{d_s^F}\}]$ is beneficial.

We next identify conditions under which the coupon policy is beneficial (for the model with a budget constraint). Recall that the offline partner is not worse off under the coupon policy only when $\beta > \underline{\beta} = \max\{0, \frac{c_s - r}{\theta}\}$. Therefore, the minimum budget under which the coupon policy is beneficial is $\bar{K} = \max\{0, \frac{c_s - r}{\theta}\}d_s^C$, which is characterized as

$$\bar{K} = \begin{cases} 0, & c_s - r \le 0 \\ \frac{2(c_s - r) + (2 - v + p)(v - p)}{h_p}(c_s - r), & 0 < c_s - r \le \frac{h_p - 2(v - p)}{2} \\ (1 - \frac{(h_p - 2(c_s - r))^2}{4h_p})(c_s - r) & \frac{h_p - 2(v - p)}{2} \le c_s - r \le \frac{h_p}{2} \\ c_s - r & c_s - r \ge \frac{h_p}{2} \end{cases}$$

As a result, the coupon policy is beneficial only when $K \ge \bar{K}$. In this case, the maximum coupon value that the online retailer can pay is $\frac{K}{\theta d_s^C}$. Based on Proposition 5, the coupon policy is beneficial when $\beta \in [\underline{\beta}, \frac{c_s - r}{\theta}]$. Therefore, when $K > \bar{K}$, the coupon policy is beneficial for any $\beta \in [\underline{\beta}, \min\{\frac{c_s - r}{\theta}, \frac{K}{\theta d_s^C}\}]$. Since $\bar{K} > \underline{K}$, when $K > \bar{K}$, both the fixed fee and the coupon policies are beneficial and they become optimal strategies based on the results of Proposition 5.

D.2 Multiple pickup locations

First, we examine how the optimal policy changes when there is more than one pickup location. To do so, we evaluate how the demand function varies when there are n pickup locations under the fixed fee and coupon policies. We will then infer the optimal policy.

The demand for the direct-delivery (d_o^F) and in-store pickup (d_s^F) options, when there are n pickup locations under the fixed fee policy are given by (recall that when n pickup locations are available, the maximum distance between a customer and a pickup location is $\frac{1}{2n}$ based on the circle model from Salop (1979)):

$$d_o^F = \begin{cases} (1 - \frac{n(v-p)}{h_p})(v - p), & n < \lceil \frac{h_p}{2(v-p)} \rceil \\ \frac{h_p}{4n}, & n \ge \lceil \frac{h_p}{2(v-p)} \rceil \end{cases} \quad \text{and} \quad d_s^F = \begin{cases} \frac{n(2-v+p)(v-p)}{h_p}, & n < \lceil \frac{h_p}{2(v-p)} \rceil \\ (1 - \frac{h_p}{4n}), & n \ge \lceil \frac{h_p}{2(v-p)} \rceil \end{cases}$$

The demand under the baseline policy is independent of n and remains to be $(v - p)$. We can now substitute the demand function into the online retailer's and offline partner's profit functions under the fixed fee and baseline policies, and then, by comparing them, we can find the conditions under which the fixed fee policy is beneficial.

As in the main model, the offline partner is better-off under the fixed fee policy if and only if $\alpha \geq \underline{\alpha} = \max\{c_s - r, 0\}$ (using the same argument as in the main model). The online retailer is better-off under the fixed fee policy if and only if $\alpha \leq \bar{\alpha}$, where $\bar{\alpha}$ is given by:

$$\bar{\alpha} = \begin{cases} (p - c_p) - \frac{v-p}{2-v+p}(p - c_o), & 1 \leq n < \lceil \frac{h_p}{2(v-p)} \rceil \\ (p - c_p) - \frac{4n(v-p)-h_p}{4n-h_p}(p - c_o), & n \geq \lceil \frac{h_p}{2(v-p)} \rceil \end{cases}$$

Thus, similar to Proposition 2, the fixed fee policy is beneficial $\forall \alpha \in [\underline{\alpha}, \bar{\alpha}]$.

Under the coupon policy, the demand functions for the direct-delivery and in-store pickup options are given by:

$$d_o^C = \begin{cases} \left[1 - \frac{n(v-p+2\theta\beta)}{h_p}\right](v - p), & 0 \leq \beta < \max\{0, \frac{h_p - 2n(v-p)}{2n\theta}\} \\ \frac{(h_p - 2n\theta\beta)^2}{4nh_p}, & \max\{0, \frac{h_p-2n(v-p)}{2n\theta}\} \leq \beta < \frac{h_p}{2n\theta} \\ 0 & \beta \geq \frac{h_p}{2n\theta} \end{cases}$$

$$d_s^C = \begin{cases} \frac{n}{h_p}(2\theta\beta + (2 - v + p)(v - p)), & 0 \leq \beta < \max\{0, \frac{h_p-2n(v-p)}{2n\theta}\} \\ \left[1 - \frac{(h_p-2n\theta\beta)^2}{4nh_p}\right], & \max\{0, \frac{h_p-2n(v-p)}{2n\theta}\} \leq \beta < \frac{h_p}{2n\theta} \\ 1 & \beta \geq \frac{h_p}{2n\theta} \end{cases}$$

Thus, the online retailer's profit under the coupon policy with n pickup locations is given by:

$$\pi_o^C(\beta) = \begin{cases} \left[1 - \frac{n(v-p+2\theta\beta)}{h_p}\right](v - p)(p - c_o)+ & 0 \leq \beta < \max\{0, \frac{h_p-2n(v-p)}{2n\theta}\} \\ \frac{n}{h_p}\left[2\theta\beta + (2 - v + p)(v - p)\right](p - c_p - \theta\beta) & \\ \frac{(h_p-2n\theta\beta)^2}{4nh_p}(p - c_o) + (1 - \frac{(h_p-2n\theta\beta)^2}{4nh_p})(p - c_p - \theta\beta), & \max\{\frac{h_p-2n(v-p)}{2n\theta}, 0\} \leq \beta < \frac{h_p}{2n\theta} \\ (p - c_p - \theta\beta) & \beta \geq \frac{h_p}{2n\theta} \end{cases}$$

As in Proposition 4, we can show that $\pi_o^C(\beta)$ is unimodal so that there exist unique $\bar{\beta}$ and $\underline{\beta}$ such that the coupon policy is beneficial when $\beta \leq [\underline{\beta}, \bar{\beta}]$. Consequently, the optimal coupon value from the online retailer's perspective is $\beta^* = \max\{c_s - r, \check{\beta}\}$. The expression of $\check{\beta}$ depends on the vale of n as follows:
(i) If $n < \lceil \frac{h_p}{2(v-p)} \rceil$, then we have

$$\check{\beta} = \begin{cases} \frac{1}{6n\theta}\left[2(n(c_o - c_p) + h_p) - \sqrt{4n(c_o - c_p)(n(c_o - c_p) - h_p) + h_p(h_p + 12n)}\right], & c_p \leq c_1 \\ \frac{h_p-2n(v-p)}{2n\theta}, & c_1 < c_p \leq c_2 \\ \frac{(p-c_p)}{2\theta} - \frac{(v-p)(p-c_o)}{2\theta} - \frac{(2-v+p)(v-p)}{4\theta}, & c_2 < c_p \leq c_3 \\ 0, & c_p > c_3 \end{cases}$$

Here $c_1 = c_o - \frac{h_p}{2n} + \frac{3(v-p)}{2} - \frac{h_p}{2n(v-p)}$, $c_2 = p - (v - p)(p - c_o - 2) + \frac{(v-p)^2}{2} - \frac{h_p}{n}$, and $c_3 = p - (v - p)(p - c_o + 1) + \frac{(v-p)^2}{2}$.
(ii) If $n \geq \lceil \frac{h_p}{2(v-p)} \rceil$, then we have

$$\check{\beta} = \begin{cases} \frac{1}{6n\theta}\left[2(n(c_o - c_p) + h_p) - \sqrt{4n(c_o - c_p)(n(c_o - c_p) - h_p) + h_p(h_p + 12n)}\right], & c_p \leq c_o + \frac{h_p}{4n} - 1 \\ 0 & c_p > c_o + \frac{h_p}{4n} - 1 \end{cases}$$

The derivation of the above expressions is similar to the derivation of the expressions in the proof of Proposition 4.

Therefore, when $n < \lceil \frac{h_p}{2(v-p)} \rceil$, all of our previous findings still hold. When $n > \lceil \frac{h_p}{2(v-p)} \rceil$ (i.e., the case of market saturation under the pickup partnership), the coupon policy can be optimal only when the profit margin of in-store pickup orders is higher than the profit margin of direct-delivery orders (i.e., $c_p + \theta\beta < c_o$).

Based on the above findings, we can now obtain the optimal number of pickup locations under each policy. Suppose that $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, the online retailer's profit as a function of n is given by:

$$\pi_o^F = \begin{cases} (p-c_o)(1-\frac{n(v-p)}{h_p})(v-p) + (p-c_p-\alpha)(2-v+p)\frac{(v-p)n}{h_p}, & 1 \le n \le \lceil \frac{h_p}{2(v-p)} \rceil \\ \frac{h_p}{4n}(p-c_o) + (p-c_p-\alpha)(1-\frac{h_p}{4n}), & n \ge \lceil \frac{h_p}{2(v-p)} \rceil \end{cases}$$

We assume that n is a continuous variable, so that the derivative of π_o^F can be written as

$$\frac{\partial \pi_o^F}{\partial n} = \begin{cases} -\frac{(p-c_o)(v-p)^2}{h_p} + (p-c_p-\alpha)\frac{(2-v+p)(v-p)}{h_p}, & 1 \le n < \frac{h_p}{2(v-p)} \\ \frac{h_p}{4n^2}(c_o-c_p-\alpha), & n \ge \frac{h_p}{2(v-p)} \end{cases}$$

Since we assume that $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, we have $\frac{\partial \pi_o^F}{\partial n} > 0$ when $n < \frac{h_p}{2(v-p)}$. Thus, the optimal n under the fixed fee policy is given by:

$$n^* = \begin{cases} +\infty & \alpha \le c_o - c_p \\ \lfloor \frac{h_p}{2(v-p)} \rfloor & \alpha > c_o - c_p \end{cases}$$

Under the coupon policy, we have:

$$\frac{\partial \pi_o^C}{\partial n} = \begin{cases} \frac{(v-p)}{h_p}\Big[(2-v+p)(p-c_p-\theta\beta) - (p-c_o)(v-p)\Big] + & 1 \le n < \lceil \frac{h_p}{2(\theta\beta+v-p)} \rceil \\ \frac{2\theta\beta}{h_p}\Big[p-c_p-\theta\beta-(v-p)(p-c_o)\Big], & \\ \frac{(h_p^2-4n^2\theta\beta^2)(c_o-c_p-\theta\beta)}{4h_pn^2}, & \lceil \frac{h_p}{2(\theta\beta+v-p)} \rceil \le n < \lceil \frac{h_p}{2\theta\beta} \rceil \\ 0 & n \ge \lceil \frac{h_p}{2\theta\beta} \rceil \end{cases}$$

Therefore, the optimal n under the coupon policy is given by:

$$n^* = \begin{cases} \lceil \frac{h_p}{2\theta\beta} \rceil & \theta\beta \le c_o - c_p \\ \lfloor \frac{h_p}{2(\theta\beta+v-p)} \rfloor & \theta\beta > c_o - c_p \end{cases}$$

By comparing n^* under the fixed fee and coupon policies, we conclude that the online retailer prefers a larger number of pickup locations under the fixed fee policy.

We next evaluate how the profitability of the coupon policy depends on n. We let $\Delta\pi_o^{C-F}$ denote the difference between the online retailer's profit under the fixed fee and the coupon policies when $\alpha = \theta\beta$. We have

$$\Delta\pi_o^{C-F} = \begin{cases} \frac{2n\alpha}{h_p}\Big[p-c_p-\alpha-(v-p)(p-c_o)\Big], & 1 \le n < \lceil \frac{h_p}{2(\alpha+v-p)} \rceil \\ \frac{(h_p-2n\alpha)^2}{4nh_p}(c_p+\alpha-c_o) - \frac{n(v-p)(1-v+p)}{h_p}(p-c_p-\alpha), & \lceil \frac{h_p}{2(\alpha+v-p)} \rceil \le n < \lceil \frac{h_p}{2(v-p)} \rceil \\ +(1-\frac{n(v-p)}{h_p})\Big[p-c_p-\alpha-(v-p)(p-c_o)\Big] & \\ (\frac{h_p}{4n} - \frac{(h_p-2n\alpha)^2}{4nh_p})(c_o-c_p-\alpha), & \lceil \frac{h_p}{2(v-p)} \rceil \le n < \lceil \frac{h_p}{2\alpha} \rceil \\ \frac{h_p}{4n}(c_o-c_p-\alpha) & n \ge \lceil \frac{h_p}{2\alpha} \rceil \end{cases}$$

When $\alpha = \theta\beta \leq c_o - c_p$, we have $\Delta_o \pi^{C-F} \geq 0$. We next compute the first-order derivative:

$$\frac{\partial \Delta\pi_o^{C-F}}{\partial n} = \begin{cases} \frac{2\alpha}{h_p}\Big[p - c_p - \alpha - (v-p)(p-c_o)\Big] & 1 \leq n < \lceil \frac{h_p}{2(\alpha+v-p)} \rceil \\ \frac{h_p - 2n\alpha}{nh_p}\Big[\alpha + \frac{(h_p-2n\alpha)}{4n}\Big](c_o - \alpha - c_p) + & \lceil \frac{h_p}{2(\alpha+v-p)} \rceil \leq n < \lceil \frac{h_p}{2(v-p)} \rceil \\ \frac{(v-p)}{h_p}\Big[(v-p)(p-c_o) - (2-v+p)(p-c_p-\alpha)\Big] & \\ \Big[\frac{(h_p-2n\alpha)^2}{4n^2h_p^2} + \frac{\alpha(h_p-2n\alpha)}{nh_p} - \frac{h_p}{4n^2}\Big](c_o - c_p - \alpha) & \lceil \frac{h_p}{2(v-p)} \rceil \leq n < \lceil \frac{h_p}{2\alpha} \rceil \\ -\frac{h}{4n^2}(c_o - c_p - \alpha) & n \geq \lceil \frac{h_p}{2\alpha} \rceil \end{cases}$$

Therefore, when $\alpha = \theta\beta \leq c_o - c_p$, one can show that $\Delta\pi_o^{C-F}$ is increasing in n when the market is not saturated (i.e., $1 \leq n < \lceil \frac{h_p}{2(\alpha+v-p)} \rceil$) and is decreasing for $n > \lceil \frac{h_p}{2(\alpha+v-p)} \rceil$. In addition, when $n \to \infty$, we have $\Delta\pi_o^{C-F} \to 0$.

When $\alpha = \theta\beta > c_o - c_p$, since $\alpha \leq \frac{\bar{\beta}}{\theta} \leq \bar{\alpha} = p - c_p - \frac{v-p}{2-v+p}(p - c_o)$, we can conclude that $\Delta\pi_o^{C-F}$ is increasing in n when the market is not saturated (i.e., $1 \leq n < \lceil \frac{h_p}{2(\alpha+v-p)} \rceil$). When the market is saturated, the profitability of the coupon policy over the fixed fee policy decreases with n, and there exists a unique $n \in [\lceil \frac{h_p}{2(\alpha+v-p)} \rceil, \lceil \frac{h_p}{2(v-p)} \rceil]$, so that $\Delta\pi_o^{C-F}$ becomes negative. Then, for $n > \lceil \frac{h_p}{2(v-p)} \rceil$, $\Delta\pi_o^{C-F}$ starts to increase with n again, and when $n \to \infty$, we have $\Delta\pi_o^{C-F} \to 0$.

D.3 Total welfare

We find that when the objective is set to maximize the total welfare, the optimal policy is either the baseline policy when c_s is high or the coupon policy when c_s is low. To show this, we first prove that when $c_s > p + r - c_p - \frac{v-p}{2-v+p}[p - c_o + \frac{2(v-p)}{3} - 1]$, the total welfare under the baseline policy is higher than the total welfare under the fixed fee policy. Since the values of total welfare under the fixed fee and baseline policies are constant, this can be easily shown by comparing the total welfare under both policies. We let TW^F and TW^B denote the total welfare under the fixed fee and baseline policies, respectively. Then, TW^F and TW^B can be characterized as follows:

$$TW^B = (p - c_o)(v - p) + \frac{(v-p)^2}{2},$$

$$TW^F = (p - c_o)\Big[1 - \frac{(v-p)}{h_p}\Big](v-p) + (p + r - c_p - c_s)\frac{(v-p)(2-v+p)}{h_p} + \frac{(v-p)^2}{2}\Big[\frac{2}{h_p} + 1 - \frac{4(v-p)}{3h_p}\Big].$$

By comparing TW^B to TW^F, we can show that when $c_s > p + r - c_p - \frac{v-p}{2-v+p}(p - c_o + \frac{2(v-p)}{3} - 1)$, we have $TW^B \geq TW^F$.

We next show that when $c_s < p + r - c_p - \frac{v-p}{2-v+p}[p - c_o + \frac{2(v-p)}{3} - 1]$, the coupon policy always improves the total welfare relative to the fixed fee policy. To do so, we show that when $\beta = 0$, the total welfare is equal between the coupon and the fixed fee policies, and then the total welfare under the coupon policy increases with β. Since the total welfare under the fixed fee policy is constant, this will conclude the argument. We let $TW^C(\beta)$ denote the total welfare under the coupon policy. It is then enough to show that the first derivative of $TW^C(\beta)$ is positive. The total welfare under the coupon policy is given by:

$$TW^C(\beta)=\begin{cases}(p-c_o)\Big[1-\frac{(v-p+2\theta\beta)}{h_p}\Big](v-p)+(p+r-c_p-c_s)\Big[\frac{2\theta\beta+(2-v+p)(v-p)}{h_p}\Big]+ \\ \frac{(\theta\beta)^2+\theta\beta(v-p)(2-v+p)}{h_p}+\frac{(v-p)^2}{2}\Big[\frac{2}{h_p}+1-\frac{4(v-p)}{3h_p}\Big], & 0\le\beta\le\frac{h_p-2(v-p)}{2\theta} \\ (v+r-c_p-c_s)-\frac{(h_p-2\theta\beta)^2}{4h_p}(r+c_o-c_p-c_s)+ \\ \Big[\frac{(\theta\beta)^2}{2}-\frac{(\theta\beta)^3}{3h_p}+\theta\beta(1-\frac{h_p}{4})+h_p(\frac{h_p}{24}-\frac{1}{4})\Big], & \frac{h_p-2(v-p)}{2\theta}\le\beta\le\frac{h_p}{2\theta} \\ v+\theta\beta+r-c_s-c_p-\frac{h_p}{4}, & \beta\ge\frac{h_p}{2\theta}\end{cases}$$

It is clear that when $\beta=0$, $TW^C(\beta)=TW^F$. The first derivative of $TW^C(\beta)$ is given by:

$$\frac{\partial TW^C(\beta)}{\partial\beta}=\begin{cases}\frac{\theta}{h_p}\Big[2(p+r-c_p-c_s)-2(v-p)(p-c_o)+\theta\beta+(v-p)(2-v+p)\Big], & 0\le\beta\le\frac{h_p-2(v-p)}{2\theta} \\ \theta\Big[(\frac{h_p-2\theta\beta}{h_p})(r+c_o-c_p-c_s)+\theta\beta-\frac{(\theta\beta)^2}{h_p}+1-\frac{h_p}{4}\Big], & \frac{h_p-2(v-p)}{2\theta}\le\beta\le\frac{h_p}{2\theta} \\ \theta & \beta\ge\frac{h_p}{2\theta}\end{cases}$$

For the first part, we need to show that for $\beta\in[0,\frac{h_p-2(v-p)}{2\theta}]$, we have

$$\frac{\theta}{h_p}\Big[2(p+r-c_p-c_s)-2(v-p)(p-c_o)+\theta\beta+(v-p)(2-v+p)\Big]\ge 0.$$

Therefore, it is enough to show that

$$p+r-c_p-c_s-(v-p)(p-c_o)+\frac{(v-p)(2-v+p)}{2}\ge 0 \implies c_s\le p+r-c_p+\frac{(v-p)(2-v+p)}{2}-(v-p)(p-c_o).$$

Since we assume that $c_s\le p+r-c_p-\frac{v-p}{2-v+p}[p-c_o+\frac{2(v-p)}{3}-1]$, it is enough to show that

$$p+r-c_p+\frac{(v-p)(2-v+p)}{2}-(v-p)(p-c_o)\ge p+r-c_p-\frac{v-p}{2-v+p}\Big[p-c_o+\frac{2(v-p)}{3}-1\Big] \quad \text{(E.1)}$$

or equivalently

$$\frac{v-p}{2-v+p}\Big[1-\frac{2(v-p)}{3}-p+c_o\Big]\le\frac{(v-p)(2-v+p)}{2}-(v-p)(p-c_o).$$

Note that $[1-\frac{2(v-p)}{3}-p+c_o]>0$, $v-p>0$, and $2-v+p>1$, so we have

$$\frac{v-p}{2-v+p}\Big[1-\frac{2(v-p)}{3}-p+c_o\Big]\le(v-p)\Big[1-\frac{2(v-p)}{3}-p+c_o\Big]$$
$$\le(v-p)\Big[1-\frac{v-p}{2}-p+c_o\Big]=\frac{(v-p)(2-v+p)}{2}-(v-p)(p-c_o).$$

For the second part, we need to show that for $\beta\in[\frac{h_p-2(v-p)}{2\theta},\frac{h_p}{2\theta}]$, we have

$$\theta\Big[(\frac{h_p-2\theta\beta}{h_p})(r+c_o-c_p-c_s)+\theta\beta-\frac{(\theta\beta)^2}{h_p}+1-\frac{h_p}{4}\Big]\ge 0.$$

The above equation is a continuous concave quadratic function of β and, thus, it is enough to show that this function is positive at both threshold values (i.e., $\frac{h_p-2(v-p)}{2\theta}$ and $\frac{h_p}{2\theta}$). For $\beta=\frac{h_p}{2\theta}$, we have

$$\frac{h_p}{2}-\frac{1}{h_p}(\frac{h_p}{2})^2+1-\frac{h_p}{4}=1>0,$$

and for $\beta = \frac{h_p - 2(v-p)}{2\theta}$, we have

$$\theta\Big[\frac{2(v-p)}{h_p}(r + c_o - c_p - c_s) + \frac{h_p}{2} - (v-p) - \frac{1}{4h_p}(h_p^2 + 4(v-p)^2 - 4h_p(v-p))\Big] \geq 0$$

$$\frac{2(v-p)}{h_p}(r + c_o - c_p - c_s) - \frac{(v-p)^2}{h_p} + 1 \geq 0 \implies c_s \leq r + c_o - c_p - \frac{v-p}{2} + \frac{h_p}{2(v-p)}.$$

Since we assume $c_s \leq p + r - c_p - \frac{v-p}{2-v+p}(p - c_o + \frac{2(v-p)}{3} - 1)$, it is enough to show that

$$r + p - c_p + \frac{v-p}{2-v+p}\Big[1 - \frac{2(v-p)}{3} - p + c_o\Big] \leq r + c_o - c_p - \frac{v-p}{2} + \frac{h_p}{2(v-p)}.$$

By using Inequality (E.1), it is enough to show that

$$p + r - c_p + \frac{(v-p)(2-v+p)}{2} - (v-p)(p-c_o) \leq r + c_o - c_p - \frac{v-p}{2} + \frac{h_p}{2(v-p)}$$

$$\implies \frac{h_p}{2(v-p)} - \frac{v-p}{2} - p + c_o \geq (v-p)\Big[1 - \frac{v-p}{2} - p + c_o\Big],$$

where the last inequality follows from $v - p \leq 1$ and $\frac{v-p}{h_p} \leq \frac{1}{2}$, and hence concludes the proof.

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