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On the interplay between self-driving cars and public transportation: A game-theoretic perspective

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Abstract: Cities worldwide struggle with overloaded transportation systems and their externalities, such as traffic congestion and emissions. The emerging technology of autonomous transportation systems bears a high potential to alleviate these issues. At the same time, this technology might also introduce negative effects, in particular by disproportionately cannibalizing public transportation. A careful analysis of this trade-off requires modeling both modes of transportation within a unified framework. In this paper, we propose such a framework, which allows us to study the interplay among mobility service providers, public transport authorities, and customers, and in particular to analyze the effect of autonomous ride-hailing services on the demand for public transportation. This framework combines a graph-theoretic network model for the transportation system with a game-theoretic model whereby mobility service providers are profit-maximizers and customers select individually-optimal transportation options. We apply our modeling approach to data for the city of Berlin, Germany and present sensitivity analyses to study factors that mobility service providers or municipalities can act upon to strategically steer the overall system. We show that depending on market conditions and policy restrictions, autonomous ride-hailing systems may complement or cannibalize a public transportation system, and discuss the main factors behind such different outcomes as well as strategic design options available to policymakers.

Keywords: Autonomous mobility-on-demand, ride hailing, game theory, multimodal transport

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1 Introduction

All around the world, cities struggle with overstrained transportation systems whose externalities cause economic and environmental harm such as working hours lost in congestion or health dangers caused by particulate matters, NO_x, and stress (Frakt 2019). For example, in 2017, traffic-related externalities cost U.S. citizens 305 billion USD (INRIX 2019). Municipalities try to resolve these problems by improving existing transportation systems, but face several obstacles (Hu and Wang 2018, Hu 2019). Public transport not based on surface roads, e.g., subway lines, are often hindered by spatial limitations in urban areas, as well as long lead times of major infrastructure projects. Improving the road infrastructure often faces similar obstacles. Accordingly, a better utilization of current urban infrastructure by means of new technologies and mobility concepts is necessary to resolve the root cause of problems in today's transportation systems.

In recent years, various new mobility concepts emerged. However, all of them struggle with specific obstacles.

Car-sharing systems such as Zipcar offer the opportunity to reduce the number of individually owned cars in cities. However, given the small fleet sizes, customers are reluctant to use such vehicles due to inconvenient accessibility; vice versa, these concepts are often still not economically viable for mobility service providers (MSPs).

Ride-hailing services such as Uber or Lyft appear as an affordable alternative to conventional taxi services and decrease the need for individually owned cars in cities. However, the uncontrolled growth of such services often worsens congestion. For example, in Manhattan, an increase of 68,000 for-hire vehicles from 2013 to 2018 correlated with a decrease in average travel speed from 6.2 mph to 4.7 mph (Hu 2019).

Ride-pooling services can potentially reduce traffic in cities by pairing passengers with similar trips into a vehicle (see, e.g., Alonso-Mora et al. 2017, Ostrovsky and Schwarz 2019). However, such systems can only work efficiently when the underlying ride-hailing system operates efficiently. So far, customers appear to be reluctant to use ride-pooling due to unsatisfactory user experiences (Heid et al. 2017).

An emerging mobility concept, namely autonomous mobility-on-demand (AMoD), has the potential to help address the challenges outlined above. An AMoD system consists of a fleet of robotic self-driving cars, which transport passengers from their origins to their destinations. An operator controls the fleet by dispatching passenger trips to vehicles while simultaneously deciding on their routes. Such a system overcomes the limitations of free-floating car-sharing systems by remedying the limited accessibility of cars. Further, it overcomes the inherent inefficiencies of current ride-hailing systems as a single operator centrally decides on the fleets size and all operational actions. It also eliminates the largest cost component of ride-hailing systems: driver's time. This central coordination and information transparency also allows for more efficient ride-pooling with fair compensation schemes among customers.

While AMoD clearly has the potential of addressing the many challenges of urban transportation, it also introduces a major risk: it may excessively cannibalize large-scale public transportation: buses, trams, subways, and other similar modes of travel. The net effect on cities is largely an empirical question, whose answer depends on many specifics of various cities. A principled approach to answer this question entails developing a unifying framework that incorporates various modes of transportation, to study customers' travel behavior and choices. In this paper, we provide such a framework, which makes it possible to analyze the interactions among MSPs, public transport authoritys (PTAs) controlled by municipalities, and customers in today's and future transportation systems. In particular, we tailor this framework to the specific case of AMoD systems.

1.1 Related literature

Our work lies at the interface between economics, especially game theory, and transportation science. To keep our literature review concise, we will focus on the most recent and related works, with a special emphasis on game-theoretic approaches in mobility systems and transportation network modeling related to AMoD.

Two categories of game-theoretic approaches related to our problem exist.

Network pricing problems arise in the field of traffic management and congestion avoidance (Brotcorne et al. 2001, Bianco et al. 2016, Kuiteing et al. 2017). These problems are typically modeled as Stackelberg games and formalized as mathematical programs with equilibrium constraints (Patriksson and Rockafellar 2002) or as bilevel optimization problems (Colson et al. 2005, Labbé and Violin 2016). In these games, an upper-level player (e.g., a central authority) sets prices or tolls on some arcs in a network to maximize a given objective and the lower-level players (e.g., drivers) react accordingly. Compared with our problem setting, such games do not accommodate the non-cooperative interaction among multiple MSPs. Moreover, they focus on arc or path prices and do not directly allow for more general pricing schemes.

Mobility-on-demand related games have recently been extensively discussed. Banerjee et al. (2015), Bai et al. (2019), and Guda and Subramanian (2019) focused on the coordination of customer demand and driver supply. Bimpikis et al. (2019) and Wang et al. (2018) studied ride-sharing platforms, highlighting the impact of the demand pattern on the platforms' profits and consumers' surplus, while studying cost-sharing strategies between customers and drivers. Further works have focused on the societal costs of ride-hailing companies (Rogers 2015) and on the impact of mobility-on-demand systems on the taxi market (Wallsten 2015). Overall, these approaches do not sufficiently capture our problem characteristics as (*i*) they focus on a two-sided market with drivers and customers, not accounting for centrally-controlled autonomous vehicles, (*ii*) they do not consider multimodal or intermodal routes, and (*iii*) they do not provide a general and flexible game-theoretic framework that captures both the interactions among MSPs as well as among MSPs and customers.

As for transportation models for AMoD systems, previous papers have investigated queuing-theoretic models (Zhang and Pavone 2016), simulation-based models (Levin et al. 2017), and multi-commodity network flow models (Rossi et al. 2018). Microscopic studies expect autonomous vehicles to ease traffic management, e.g., via improved intersection clearing (Lee and Park 2012, Guler et al. 2014) and freeway merging (Zhou et al. 2017). Macroscopic studies have shown that AMoD systems contribute to more accessible, efficient, and sustainable transportation systems (Pavone 2015, Fagnant and Kockelman 2015). Mahmassani (2016) showed that autonomous vehicles increase the throughput of highway facilities and improve traffic stream stability. Salazar et al. (2019) analyzed the operation of an AMoD system in cooperation with public transport. Ostrovsky and Schwarz (2019) were the first to discuss the economics of AMoD systems, focusing on the effects of carpooling. Compared to our problem setting, all approaches but Ostrovsky and Schwarz (2019) imposed a central decision maker and neglected game-theoretic dynamics. Further, Ostrovsky and Schwarz (2019) neglected MSP behavior, essentially replacing MSPs with a perfectly competitive, zero-profit market. In contrast, one of the key ingredients in our work is the analysis of MSPs' behavior.

Concluding, to the best of the authors' knowledge, there exists no methodological framework capable of analyzing the dynamics among customers and multiple MSPs offering different mobility services, while considering the operational constraints of the system.

1.2 Contribution

To fill this research gap, we provide the first game-theoretic framework that captures the dynamics among multiple MSPs and customers while accounting for the operational constraints within the

system and for single MSPs. Specifically, our scientific contribution is fivefold. First, we develop a generic mathematical framework that allows analyzing the dynamics of complex transportation problems by combining graph-theoretic network models with game-theoretic approaches that consider the interconnections between a transportation network and its corresponding market place. Second, we tailor this framework to the specific case of an AMoD system interacting with public transport. Third, we develop a computationally tractable quadratic program which yields the equilibrium prices for this specific game. Fourth, we provide a real-world case study for the city of Berlin. Fifth, we present extensive numerical results and sensitivity analyses which yield managerial findings on the interaction between an AMoD system and public transport.

1.3 Organization

The remainder of this paper is structured as follows. We specify our problem setting in Section 2 and develop our methodology in Section 3. In Section 4, we tailor our methodology to study the interplay between an AMoD operator and a public transportation system. Section 5 details our experimental design, focusing on a real-world case study. We present results in Section 6. Section 7 concludes this paper by summarizing its main findings. In the appendices of this paper, we provide basic notation conventions and an overview of the used nomenclature (Appendix A), fundamentals of graph theory (Appendix B), and proofs for all propositions stated throughout the paper (Appendix C). When introducing a term defined in the appendix for the first time, we mark it with a dagger†.

2 Problem setting

In this work, we focus on intra-city passenger transportation, where an AMoD fleet substitutes the service of current taxi or ride-hailing fleets. In such a system, different MSPs interact with each other and with customers. We distinguish MSPs between commercial MSPs and municipalities and we focus on three stakeholder groups.

Mobility service providers offer transportation services to customers and aim at maximizing their profit and market share. To remain competitive, MSPs require cost-effective operations, ignoring the resulting externalities and their effect on the overall system or on other players.

Municipalities offer transportation services via a PTA to customers while aiming to sustain infrastructure services, accessibility, and quality of life in cities. While MSPs complement the transportation services offered by municipalities, they also cause externalities and dissatisfaction. Accordingly, municipalities try to influence MSPs, e.g., through subsidies or taxes.

Customers represent the demand side and request for transportation services. Customers can choose between different transportation modes or combine them to complete their ride. Each customer selects a trip in line with her preferences, e.g., minimizing her cost, travel time, or a combination of both.

To adequately capture the dynamics of such a system and the interactions between stakeholders, we model the city's transportation system on two different levels: a transportation network and its market place (see Figure 1).

Transportation market place: The interaction between the different stakeholders takes place in the system's *market place*, e.g., via a smartphone app. Here, MSPs and municipalities offer several types of transportation services to customers. Customers have different transportation demands and respond to these offers depending on their individual rationale. These interactions happen at the operational level in a short time horizon. At the strategic level, MSPs interact and therefore influence each other as their business models may interfere, i.e., a customer may substitute the service of one provider with that of another provider, depending on quality and price.

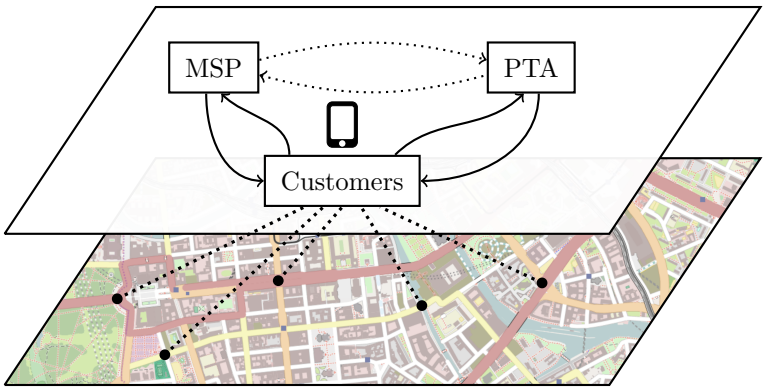


Figure 1: Schematic model of a transportation system with one commercial MSP and a PTA. The upper level is the transportation market place, where the different stakeholders interact. The lower level is the transportation network, where the realization of a demand and supply match takes place.

Transportation network: The realization of a demand and supply match between customers and mobility service providers takes place in the transportation network which consists of the city's road and public transportation networks. Accordingly, the transportation network imposes boundaries on offers in the transportation market place as it determines the available infrastructure and comprises externalities such as congestion and travel times.

In this paper, we focus specifically on the interaction between an AMoD operator and a municipality offering transportation service through a PTA. Here, we focus on a short-term perspective, i.e., the equilibrium for a snapshot of a day. Given this time horizon, the PTA does not change her prices at such an operational level, because the decision on public transport tariffs is taken strategically for a significantly longer time horizon. Conversely, the AMoD operator changes her prices at the operational level to maximize her profit. We consider the pricing decisions for the AMoD operator for a snapshot of the current system. When taking the pricing decision, the AMoD operator does not only consider potential profits from serving customers, but also additional costs which result from relocating vehicles after finished trips. To this end, rebalancing empty vehicles can be interpreted as a reorganization of vehicle positions to match anticipated demand.

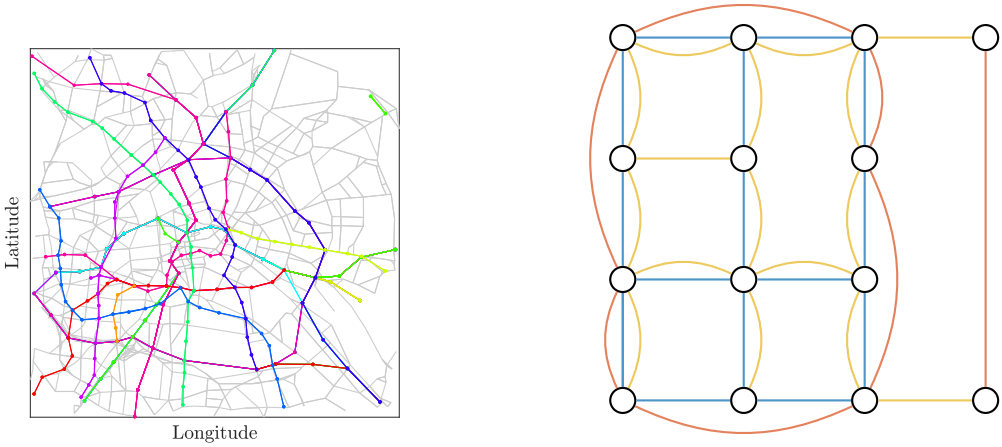
3 Methodology

We now develop the methodology to analyze the problem setting introduced in Section 2. We first show in Section 3.1 how the transportation network can be formalized through a graph-theoretic approach. We then introduce a fundamental game-theoretic model for a transportation market place in Section 3.2.

3.1 Graph representation of a multi-stakeholder transportation system

We represent a transportation network on a multigraph[†] $\mathcal{G} = (\mathcal{V}, \mathcal{A}, s_o, s_d)$ with a vertex set $\mathcal{V}$, an arc set $\mathcal{A}$, and identifiers $s_o : \mathcal{A} \to \mathcal{V}$ and $s_d : \mathcal{A} \to \mathcal{V}$ assigning each arc to its source and sink (see Figure 2). Each vertex $v \in \mathcal{V}$ denotes a location where a customer can start or end her trip. Each arc $a \in \mathcal{A}$ represents a certain transportation mode for a trip, e.g., a self-driving car or a subway line. Accordingly, multiple arcs may exist between any two vertices v_1, v_2 to model the available modes of transportation.

We define arc subsets $\mathcal{A}_j \subset \mathcal{A}$, each defining a subgraph $\mathcal{G}_j = (\mathcal{V}_j, \mathcal{A}_j, s_{o,j}, s_{d,j})$ with $\mathcal{V}_j := \bigcup_{a \in \mathcal{A}_j} \{s_o(a), s_d(a)\}$, $s_{o,j} := s_o|_{\mathcal{A}_j}$, and $s_{d,j} := s_d|_{\mathcal{A}_j}$ (i.e., the restrictions of s_o and s_d to $\mathcal{A}_j$). Each subgraph denotes a homogeneous mode of transportation, e.g., the subway network or the service



(a) Road map and railway transportation network (U-Bahn and S-Bahn) of the central urban area of the city of Berlin, Germany.

(b) Schematic abstraction of the city map as a multigraph. Blue arcs are roads, yellow arcs are walking arcs, and red arcs are public transport lines.

Figure 2: Road and part of the public transport networks for the central urban area of the city of Berlin, Germany, (left) and their schematic abstraction as a multigraph (right).

region of an AMoD system. Additionally, each subgraph $\mathcal{G}_j$ is controlled by an MSP (e.g., through price setting), from here on referred to as its operator. The subgraph $\mathcal{G}_0 = (\mathcal{V}_0, \mathcal{A}_0, s_{\mathrm{o},0}, s_{\mathrm{d},0})$ denotes a part of the overall network where customers can move free of (monetary) charge, e.g., by walking. Accordingly, for the subgraph $\mathcal{G}_0$ neither an operator nor prices exist.

To reflect customers choice options in a real transportation system, we assume $\mathcal{G}_0$ to be non-trivial (i.e., $\mathcal{V}_0 \neq \emptyset$) and strongly connected†. Further, the subgraphs' arc sets are disjoint: formally $\mathcal{A}_j \cap \mathcal{A}_l = \emptyset$, $\forall j \in \{0, 1, \ldots, N\}, l \in \{0, 1, \ldots, N\} \setminus \{j\}$, where N denotes the number of operators in the system. While these properties allow customers to choose between all available transportation modes, they do not prevent various operators from providing service on the same road, as the multigraph setting allows multiple arcs for each origin-destination pair.

We use a time-invariant network flow representation for customer movements and differentiate between transportation requests and transportation demands. A request refers to a single customer k and is well-defined by a pair $r_k = (o_k, d_k) \in \mathcal{V} \times \mathcal{V}$, which states her origin† o_k and her destination† d_k. Conversely, a demand aggregates identical requests from different customers and therefore refers to a customer flow.

Definition 1 (Demand) *A demand q_i is a triple $(o_i, d_i, \alpha_i) \in \mathcal{V} \times \mathcal{V} \times \mathbb{R}_{>0}$ uniquely defined by its origin o_i, its destination d_i, and a demand rate α_i, which results from all requests coinciding in o_i and d_i. For an arbitrary set of M demands with label set $\{1, \ldots, M\}$, we define $\mathcal{Q} \coloneqq \{q_1, \ldots, q_M\}$ as the set of all demands.*

To ensure that a customer is not forced to use a given transportation mode, each demand starts and ends on the subgraph $\mathcal{G}_0$. Formally, for all $q_i = (o_i, d_i, \alpha_i) \in \mathcal{Q}$ we have $o_i, d_i \in \mathcal{V}_0$. In general, a given demand may be satisfied by multiple paths†. With $\mathcal{P}(\mathcal{A})$ being the set of all paths in $\mathcal{G} = (\mathcal{V}, \mathcal{A}, s_{\mathrm{o}}, s_{\mathrm{d}})$, we can define the demand satisfying paths.

Definition 2 (Demand satisfying paths) *A set of L paths $\{p_1, \ldots, p_L\} \subseteq \mathcal{P}(\mathcal{A})$ satisfies a demand $q_i = (o_i, d_i, \alpha_i)$ if the origins $\chi_{\mathrm{o}}(p_j)$ and destinations $\chi_{\mathrm{d}}(p_j)$ of the paths and of the demand coincide, i.e., if $\chi_{\mathrm{o}}(p_j) = o_i$ and $\chi_{\mathrm{d}}(p_j) = d_i$ for all $j \in \{1, \ldots, L\}$. We denote by $\mathcal{S}(q_i) \subseteq \mathcal{P}(\mathcal{A})$ the set of all paths satisfying demand q_i.*

Then, according to the customers' choices, the demand induces flows along these paths.

Definition 3 (Flow) *A flow f is a pair $f = (p, \beta) \in \mathcal{P}(\mathcal{A}) \times \mathbb{R}_{>0}$, denoting the customer rate β that uses a path p. We introduce $\mathcal{F}(\mathcal{G})$ as the set of all flows on $\mathcal{G}$ and the projection operators $\chi_{\mathrm{p}}(f) = p$ and $\chi_{\mathrm{r}}(f) = \beta$ mapping a flow to its path p and its flow rate β, respectively.*

The flows on an AMoD operator's subgraph depict her fleet operations. We recall from Section 2 that the operator relocates empty vehicles to offer services in consecutive time steps and is thus interested in balanced flows.

Definition 4 (Balanced set of flows) *A family of L sets of flows $(\mathcal{F}_1, \ldots, \mathcal{F}_L)$ on a multigraph $\mathcal{G}_j = (\mathcal{V}_j, \mathcal{A}_j, s_{\mathrm{o},j}, s_{\mathrm{d},j})$ is balanced if the in-degree (deg_{in}) and out-degree (deg_{out}) of each vertex coincide. Formally,*

$$deg_{\mathrm{in}}(v) \coloneqq \sum_{i\in\{1,\ldots,L\}, f\in\mathcal{F}_i, a\in\chi_{\mathrm{p}}(f)} \chi_{\mathrm{r}}(f)\cdot 1_v(s_{\mathrm{d},j}(a)) \quad = \quad \sum_{i\in\{1,\ldots,L\}, f\in\mathcal{F}_i, a\in\chi_{\mathrm{p}}(f)} \chi_{\mathrm{r}}(f)\cdot 1_v(s_{\mathrm{o},j}(a)) =: deg_{\mathrm{out}}(v), \qquad \forall v \in \mathcal{V}_j,$$

with $1_v(\bar{v}) = 1$ for $\bar{v} = v$ and $1_v(\bar{v}) = 0$ for $\bar{v} \neq v$.

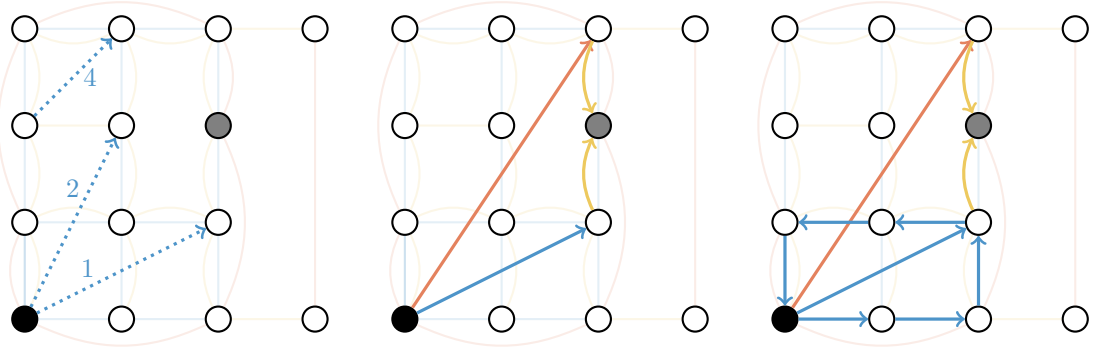
3.2 Game-theoretic setting

We focus on the interplay among (different) MSPs and customers: first, customers set requests in $\mathcal{G}_0$. Second, the MSPs decide on prices for their offered services on their subgraphs $\mathcal{G}_j$. Third, customers respond by choosing a transportation service which then results in transport flows on $\mathcal{G}$. We illustrate the game for the case of an AMoD MSP and a PTA controlled by a municipality in Figure 3. In the following, we formalize the customer and operator decisions, define the game equilibrium, and provide sufficient conditions for its existence.

3.2.1 Customers' reactions and operators' decisions

Formally, we focus on a simultaneous game between a finite set of operators $\mathcal{N} = \{\mathrm{o}_1, \ldots, \mathrm{o}_N\}$ with $N \coloneqq |\mathcal{N}| < +\infty$. Herein, customers act as non-strategic reactive players.

Customer reaction: Customers may travel free of charge by using arcs in the subgraph $\mathcal{G}_0$ (i.e., they walk) or may request an MSP's service. If they request such a service, they cannot influence the MSP's operations. Instead, they only ask for a mobility service between an origin and a destination. Formally,



(a) Prices for a subset of the origin-destination pairs for the AMoD MSP, operating on the road subgraph (blue subgraph).

(b) Possible paths serving the demand between the black origin vertex and the grey destination vertex.

(c) Possible AMoD vehicle flows serving demand and rebalancing the system. The AMoD MSP only serves arcs on her subgraph.

Figure 3: Schematic example of the different game stages: (a) the AMoD MSP sets the prices for all origin-destination pairs in her graph. (b) each demand splits over available paths. (c) the AMoD MSP operator serves her demand share and rebalances the system.

customers select arcs in the fully-connected† operators' subgraph denoted by $\bar{\mathcal{G}}_j = (\bar{\mathcal{V}}_j, \bar{\mathcal{A}}_j, \bar{s}_{\mathrm{o},j}, \bar{s}_{\mathrm{d},j})$. More specifically, customers select paths in the graphs' arc sets $\mathcal{A}_0 \cup \bigcup_{j=1}^{N} \bar{\mathcal{A}}_j$. As customers may choose different services, each demand q_i splits over its demand satisfying paths $\mathcal{S}(q_i) \subseteq \mathcal{P}(\mathcal{A}_0 \cup \bigcup_{j=1}^{N} \bar{\mathcal{A}}_j)$. To model this customer behavior, we use a reaction curve $\phi_i : \mathcal{S}(q_i) \to [0, \alpha_i]$ for each demand $q_i = (o_i, d_i, \alpha_i)$ which assigns a share of the total demand to each demand satisfying path. We require a reaction curve to be valid, i.e., to preserve the demand rate.

Definition 5 (Valid reaction curve) *Consider a demand $q_i = (o_i, d_i, \alpha_i)$. We say a reaction curve $\phi_i : \mathcal{S}(q_i) \to [0, \alpha_i]$ is valid if it preserves its demand q_i, i.e.,*

$$\sum_{p \in \mathcal{S}(q_i)} \phi_i(p) = \alpha_i.$$

Let $\Phi(q_i) \subset \mathbb{R}_{\geq 0}^{\mathcal{S}(q_i)}$ be the set of valid reaction curves for the demand q_i. We further constrain the reaction curve to be in the customers' action space $\mathcal{B}_{\mathrm{c},i} \subseteq \mathbb{R}_{\geq 0}^{\mathcal{S}(q_i)}$ and assume that each customer has at least one available action, i.e., $\mathcal{B}_{\mathrm{c},i} \cap \Phi(q_i) \neq \emptyset$. For instance, if demand q_i is not allowed to travel through the path $\bar{p} \in \mathcal{S}(q_i)$, then the corresponding action space reads $\mathcal{B}_{\mathrm{c},i} = \{\phi \in \mathbb{R}_{\geq 0}^{\mathcal{S}(q_i)} \mid \phi(\bar{p}) = 0\}$. Finally, each reaction curve is associated with a non-negative cost $J_i : \Phi(q_i) \times \Xi_1 \times \ldots \times \Xi_N \to \mathbb{R}_{\geq 0}$, with Ξ_j denoting the set of pricing strategies available to operator o_j as elaborated below. This cost, for instance, could represent the sum of the fares paid throughout the trip or the total cost, consisting of the fares paid and a customer's monetary value of time.

Operator decision: Each operator o_j serves customer demands, herein operating her fleet in order to maximize profit. Her profit depends on the customer fares and the operational costs, resulting from both customer transporting vehicles and operating empty vehicles which the operator relocates in the system to balance her flows.

Recall that customers move on the fully-connected version of the operators' subgraph and on the non-controlled graph $\mathcal{G}_0$. To satisfy the request for mobility service induced by the demand q_i on an operator's subgraph, each operator o_j selects a set of potentially active flows, in line with Definition 6.

Definition 6 (Potentially active set of flows) *Consider a demand q_i with reaction curve ϕ_i. A set of flows $\mathcal{F}_i$ on the operator o_j's subgraph $\mathcal{G}_j$ is potentially active if for each demand-satisfying path $p \in \mathcal{S}(q_i)$ with $\phi_i(p) > 0$, and for each arc $a \in p \cap \bar{\mathcal{A}}_j$ there exist some flows $\mathcal{F}_i^{a,p} \subseteq \mathcal{F}_i$ which satisfy the mobility service induced by demand q_i such that*

(i) their origins and destinations coincide with $\bar{s}_{\mathrm{o},j}(a)$ and $\bar{s}_{\mathrm{d},j}(a)$,
(ii) the sum of their rates matches the demand share on path p, i.e., $\sum_{f \in \mathcal{F}_i^{a,p}} \chi_{\mathrm{r}}(f) = \phi_i(p)$.

To prevent a flow from serving multiple rides, we require the flow sets $\{\mathcal{F}_i^{a,p}\}_{a,p}$ to be mutually disjoint and denote the set of potentially active flow sets by $\mathcal{H}_i(\phi_i)$.

The operator may additionally select a set of flows of rebalancing vehicles $\mathcal{F}_0$. To ensure that flows are balanced and that additional constraints (e.g., limited vehicles availability) are fulfilled, we introduce the non-empty and closed operator's action set $\mathcal{B}_{\mathrm{o},j} \subseteq 2^{\mathcal{F}(\mathcal{G}_j)} \times \ldots \times 2^{\mathcal{F}(\mathcal{G}_j)}$ and impose $(\mathcal{F}_1, \ldots, \mathcal{F}_M, \mathcal{F}_0) \in \mathcal{B}_{\mathrm{o},j}$. In this setting, the revenue of an operator reads

$$\sum_{i \in \{1,\ldots,M\}, p \in \mathcal{S}(q_i), a \in p \cap \bar{\mathcal{A}}_j} \phi_i(p) \cdot \xi_j(\bar{s}_{\mathrm{o},j}(a), \bar{s}_{\mathrm{d},j}(a)),$$

depending on the prices set by the operator, where we refer to a representation of all prices set by the operator as a pricing strategy ξ.

Definition 7 (Pricing strategy) *Consider a multigraph $\mathcal{G}_j = (\mathcal{V}_j, \mathcal{A}_j, s_{\mathrm{o},j}, s_{\mathrm{d},j})$. A pricing strategy ξ on $\mathcal{G}_j$ assigns a price $c \in \bar{\mathbb{R}}_{\geq 0} \coloneqq \mathbb{R}_{\geq 0} \cup \{+\infty\}$ to each origin-destination pair $(o,d) \in \mathcal{V}_j \times \mathcal{V}_j$:*

$$\begin{array}{rcl} \xi : \mathcal{V}_j \times \mathcal{V}_j & \to & \bar{\mathbb{R}}_{\geq 0} \\ (o,d) & \mapsto & c. \end{array}$$

As multiple pricing strategies can exist, we collect the feasible pricing strategies on a graph $\mathcal{G}_j = (\mathcal{V}_j, \mathcal{A}_j, s_{\mathrm{o},j}, s_{\mathrm{d},j})$ in the set $\Xi_j \subseteq \bar{\mathbb{R}}_{\geq 0}^{\mathcal{V}_j \times \mathcal{V}_j}$. In line with realistic constraints on pricing strategies, we assume Ξ_j to be closed, convex, and decoupled such that it is of the form

$$\Xi_j = \left\{ \xi \in \bar{\mathbb{R}}_{\geq 0}^{\mathcal{V}_j \times \mathcal{V}_j} \;\middle|\; g_j(\xi(o,d), o, d) \leq 0 \text{ for all } (o,d) \in \mathcal{V}_j \times \mathcal{V}_j \right\}$$

for some $g_j : \bar{\mathbb{R}}_{\geq 0} \times \mathcal{V}_j \times \mathcal{V}_j \to \mathbb{R}^l$, with $l \in \mathbb{N}$, expressing the constraints on the price $\xi(o,d)$. For instance, if prices are regulated by an upper bound $B \in \mathbb{R}_{\geq 0}$, the function g_j reads $g_j(\xi_j(o,d), o, d) = \xi_j(o,d) - B$.

To model the operator's costs (e.g., energy consumption, maintenance, and depreciation of the vehicles) we suppose that the cost corresponding to a set of flows $\mathcal{F}$ is $c_j(\mathcal{F}) \in \mathbb{R}_{\geq 0}$. Accordingly, the total cost is

$$\min_{\substack{\mathcal{F}_i \in \mathcal{H}_i(\phi_i), \mathcal{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_j)}, \\ (\{\mathcal{F}_i\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\mathrm{o},j}}} \sum_{i=1}^{M} c_j(\mathcal{F}_i) + c_j(\mathcal{F}_0).$$

Consequently, the operator's profit reads

$$U_j(\xi_j, \{\phi_i\}_{i=1}^M) \coloneqq \sum_{i \in \{1,\dots,M\}, p \in \mathcal{S}(q_i), a \in p \cap \bar{\mathcal{A}}_j} \phi_i(p) \cdot \xi_j(\bar{s}_{\mathrm{o},j}(a), \bar{s}_{\mathrm{d},j}(a)) - \min_{\substack{\mathcal{F}_i \in \mathcal{H}_i(\phi_i), \mathcal{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_j)}, \\ (\{\mathcal{F}_i\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\mathrm{o},j}}} \sum_{i=1}^{M} c_j(\mathcal{F}_i) + c_j(\mathcal{F}_0).$$

A few comments on this general setting are in order. First, we do not include direct interactions among customers, but our model can be extended to accommodate them. In particular, one can define customers as strategic players, interacting simultaneously with themselves and sequentially with MSPs. In line with this, we leave effects such as ride pooling to future research. Second, we assume without loss of generality that an operator serves all customer requests. However, an operator can technically drop a customer by imposing an artificially high transportation fare, which causes the customer to refuse to choose a ride. Third, we neglect the operator's fixed costs because they do not affect the operational decisions. Fourth, we consider a time-invariant transportation system. This assumption reflects the mesoscopic nature of our study, and holds true if transportation demands change slowly compared to the average trip travel time, as typically observed in densely populated urban areas (see Neuburger 1971).

3.2.2 Game equilibrium

As a basis for the definition of game equilibria, we first introduce a customer demand's optimal reaction: A demand reacts optimally if its reaction curve minimizes its cost for given operators' pricing strategies.

Definition 8 (Optimal reaction) *Given the operators' pricing strategies $(\{\xi_j\}_{j=1}^N) \in \prod_{j=1}^N \Xi_j$, the reaction curve $\phi_i^\star \in \Phi(q_i) \cap \mathcal{B}_{\mathrm{c},i}$ is optimal for demand q_i if*

$$J_i(\phi_i^\star, \{\xi_j\}_{j=1}^N) \leq J_i(\phi_i, \{\xi_j\}_{j=1}^N), \quad \forall \phi_i \in \Phi(q_i) \cap \mathcal{B}_{\mathrm{c},i}.$$

Let $\mathcal{E}_i(\{\xi_j\}_{j=1}^N)$ be the set of all optimal reactions.

Since operators interact simultaneously, we say that the game is at equilibrium if none of the operators can increase her profit by unilaterally changing her pricing strategy, given that customers react optimally. To keep the definition of such an equilibrium concise, we assume without loss of generality that for all operators' pricing strategies $(\{\xi_j\}_{j=1}^N) \in \prod_{j=1}^N \Xi_j$ and all demands $i \in \{1,\dots,M\}$, the set $\mathcal{E}_i(\{\xi_j\}_{j=1}^N)$ is a singleton which ensures profit uniqueness for a pricing strategy. A relaxation of this assumption is however straightforward, e.g., by introducing a selection function for the sets $\mathcal{E}_i(\{\xi_j\}_{j=1}^N)$.

Denoting the operator's profit as $U_j(\xi_j, \{\mathcal{E}_i(\{\xi_j\}_{j=1}^N)\}_{i=1}^M)$ we state the game equilibrium.

Definition 9 (Game equilibrium) *The pricing strategies $(\{\xi_j^\star\}_{j=1}^N) \in \prod_{j=1}^N \Xi_j$ are an equilibrium of the game if no operator can increase her profit by unilaterally deviating from her pricing strategy. Formally, $(\{\xi_j^\star\}_{j=1}^N)$ is an equilibrium if for all $j \in \{1,\dots,N\}$*

$$U_j(\xi_j^\star, \{\mathcal{E}_i(\xi_1^\star,\dots,\xi_j^\star,\dots,\xi_N^\star)\}_{i=1}^M) \geq U_j(\xi_j, \{\mathcal{E}_i(\xi_1^\star,\dots,\xi_j,\dots,\xi_N^\star)\}_{i=1}^M), \quad \forall\, \xi_j \in \Xi_j.$$

Definition 9 provides a general definition of equilibrium for our game. In the special case of a single operator, the equilibrium condition reduces to $\xi_1^\star \in \arg\max_{\xi_1 \in \Xi_1} U_1(\xi_1, \{\mathcal{E}_i(\xi_1)\}_{i=1}^M)$. To conclude the analysis of the general setting, we provide sufficient conditions for the existence of an equilibrium.

Proposition 1 (Existence of equilibria) *Consider the set of pricing strategies $\bar{\Xi}_j(\{\xi_k\}_{k\neq j}) \coloneqq \{\xi_j \in \Xi_j \,|\, U_j(\xi_j, \{\mathcal{E}_i(\{\xi_j\}_{j=1}^N)\}_{i=1}^M) > -\infty\}$ and assume that:*

1. *The set of pricing strategies $\bar{\Xi}_j$ is closed, convex, upper and lower semicontinuous in all prices $\xi_k(v_1,v_2)$, $v_1,v_2 \in \mathcal{V}_k$ and $\xi_k \in \Xi_k$, and contained in a bounded set, i.e., there exists a bounded (w.r.t. some norm) set of pricing strategies $\tilde{\Xi}_j$ such that $\bar{\Xi}_j(\{\xi_k\}_{k\neq j}) \subseteq \tilde{\Xi}_j$ for all $\xi_k \in \Xi_k$.*
2. *The profit of each operator U_j is (i) jointly continuous in all prices $\xi_k(v_1,v_2)$, with $v_1,v_2 \in \mathcal{V}_k$ and $\xi_k \in \Xi_k$, and (ii) concave in the pricing strategy ξ_j in the sense that for all $j \in \{1,\dots,N_o\}$ and all $\lambda \in [0,1]$*

$$\begin{aligned} &U_j(\lambda\xi_j^1 + (1-\lambda)\xi_j^2, \{\mathcal{E}_i(\xi_1,\dots,\lambda\xi_j^1 + (1-\lambda)\xi_j^2,\dots,\xi_N)\}_{i=1}^M) \\ &\qquad \geq \lambda U_j(\xi_j^1, \{\mathcal{E}_i(\xi_1,\dots,\xi_j^1,\dots,\xi_N)\}_{i=1}^M) + (1-\lambda) U_j(\xi_j^2, \{\mathcal{E}_i(\xi_1,\dots,\xi_j^2,\dots,\xi_N)\}_{i=1}^M). \end{aligned}$$

Then, the game admits an equilibrium.

In other words, Proposition 1 ensures that regularity and convexity of the set of pricing strategies together with continuity and concavity of the operators' profit are sufficient to ensure the existence of an equilibrium. Indeed, we show in Section 4 that these assumptions are satisfied for the special case of the interplay among an AMoD operator and a public transportation system.

3.2.3 Basic example

To illustrate the basic concept of our methodology, we consider a simplified transportation network with a transportation demand of $\alpha_1 \in \mathbb{R}_{>0}$ customers per unit time from vertex v_1 to vertex v_2 (see Figure 4). Formally, $q_1 = (v_1, v_2, \alpha_1)$ and $M = 1$. Two operators o_1 and o_2 offer mobility services on their corresponding subgraph, and a pedestrian layer allows customers to walk free of charge between the two vertices. In line with our problem setting, the two operators have full information on the demand and simultaneously decide on their pricing strategies. Then, customers react accordingly.

Specifically, the following implications result for customers and operators.

Customers: Customers can reach their destination either using the service provided by the operators (paths $p_1 \coloneqq \{a_1\}$ and $p_2 \coloneqq \{a_2\}$) or by walking (path $p_3 \coloneqq \{a_3\}$). Formally, the set of demand satifying paths is $\mathcal{S}(q_1) = \{p_1, p_2, p_3\}$. Customers minimize the sum of the fares paid

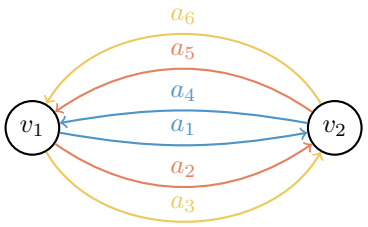


Figure 4: Multigraph representation of a simplified transportation network, consisting of two vertices with three different modes of transportation (two operators and a pedestrian layer, depicted in blue, red, and yellow).

throughout the trip and the monetary value of time. The cost associated with a valid reaction curve $\phi_1 : \mathcal{S}(q_1) \to [0, \alpha_1]$ (i.e., $\sum_{j=1}^{3} \phi_1(p_j) = \alpha_1$) reads

$$J_i(\phi_1, \xi_1, \xi_2) = \phi_1(p_1) \cdot (\xi_1(v_1, v_2) + V_{\mathrm{T}} \cdot t_1) + \phi_1(p_2) \cdot (\xi_2(v_1, v_2) + V_{\mathrm{T}} \cdot t_2) + \phi_1(p_3) \cdot V_{\mathrm{T}} \cdot t_3,$$

where $V_{\mathrm{T}} \in \mathbb{R}_{>0}$ is the customers' monetary value of time, $t_j \in \mathbb{R}_{>0}$ is the time required to reach the destination with operator o_j, $j \in \{1, 2\}$, and t_3 is the time required to walk to the destination. We stipulate that whenever the services of operators o_1 and o_2 are equally attractive (i.e., $\xi_1(v_1, v_2) + V_{\mathrm{T}} \cdot t_1 = \xi_2(v_1, v_2) + V_{\mathrm{T}} \cdot t_2$) customers always travel with operator o_1; similarly, whenever operator o_j's service and walking are equally attractive (i.e., $\xi_j(v_1, v_2) + V_{\mathrm{T}} \cdot t_j = V_{\mathrm{T}} \cdot t_3$) customers always opt for the operator's service.

Operators: Each operator o_j selects an arbitrary pricing strategy $\xi_j \in \Xi_j \coloneqq \bar{\mathbb{R}}_{\geq 0}^{\mathcal{V}_j \times \mathcal{V}_j}$, which results in the revenue $\xi_j(v_1, v_2) \cdot \phi_1(p_j)$. We posit that operators keep the vehicles flows balanced and that the cost related to a flow $f = (p, \beta)$ is $\bar{c}_j \cdot \beta \geq 0$; i.e., $c_j(\mathcal{F}) = \sum_{f \in \mathcal{F}} \bar{c}_j \cdot \chi_{\mathrm{r}}(f)$. To match the travel request induced by the demand q_1, operator o_j selects a flows set from the set of potentially active flows

$$\mathcal{H}_1(\phi_1) = \begin{cases} \{\{(p_j, \phi_1(p_j))\}\} & \text{if } \phi_1(p_j) > 0, \\ \emptyset & \text{if } \phi_1(p_j) = 0. \end{cases}$$

To ensure $(\mathcal{F}_0, \mathcal{F}_1)$ is balanced, the operator finally rebalances her fleet with the set of flows of empty vehicles $\mathcal{F}_0 = \{(\{a_{j+3}\}, \phi_1(p_j))\}$. Overall, operator o_j's profit reads

$$U_j(\xi_j, \phi_1) = \begin{cases} \xi_j(v_1, v_2) \cdot \phi_1(p_j) - 2\bar{c}_j \cdot \phi_1(p_j) & \text{if } \phi_1(p_j) > 0, \\ 0 & \text{if } \phi_1(p_j) = 0. \end{cases}$$

To ease the presentation, we suppose that operator o_1 serves customers faster than operator o_2 and that walking takes longer than the service of the operators (i.e., $t_1 < t_2 < t_3$) and we assume that $\bar{c}_1 < V_{\mathrm{T}}(t_2 - t_1)/2$. Then, the definition of equilibrium (Definition 9) leads to the following proposition.

Proposition 2 (Game equilibria) *At equilibrium, all customers use the mobility services offered by operator* o_1*, which generates positive profit. Formally,* $(\xi_1^\star, \xi_2^\star) \in \Xi_1 \times \Xi_2$ *is an equilibrium of the game if and only if*

$$\begin{aligned} &\xi_1^\star(v_1, v_2) = \min\{V_{\mathrm{T}}(t_2 - t_1) + \varepsilon_2, V_{\mathrm{T}}(t_3 - t_1)\}, \quad && \xi_1^\star(v_2, v_1) = \varepsilon_1, \\ &\xi_2^\star(v_1, v_2) = \varepsilon_2, && \xi_2^\star(v_2, v_1) = \varepsilon_3, \end{aligned}$$

for some $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in \mathbb{R}_{\geq 0}$*. The equilibrium results in the optimal reaction curve* $\phi_1^\star(p) = \alpha_1$ *for* $p = p_1$ *and* $\phi_1^\star(p) = 0$ *for* $p \in \{p_2, p_3\}$ *and in the operators' profits* $U_1 = \alpha_1(\xi_1^\star(1, 2) - 2\bar{c}_1) > 0$ *and* $U_2 = 0$*.*

4 The Interplay between an AMoD system and public transport

In the following, we tailor our general framework to the specific case where a single AMoD operator interacts with a public transport system, which we will then use for our managerial studies. While

this case neglects the existence of multiple self-driving car operators, we see it as a natural starting point to study practical situations. We expect this case to be realistic for many geographical areas once self-driving technology matures to the point of practical implementation, in particular due to technological and licensing constraints faced by the firms, and due to natural economies of scale.

Methodologically, this specific case allows us to highlight the tradeoffs faced by the PTA and the effects of its various policies in a particularly transparent manner. Moreover, the single-operator case is also informative about the more general multi-operator case: first, the presence of the public transport option itself disciplines the pricing behavior of the single operator, and brings it closer to the competitive world. Second, one may view the single-operator case as an upper bound on an AMoD operator's impact compared to the outcome of its competitive version with multiple AMoD operators.

For these reasons, we focus our analysis on the above mentioned setting and leave the step of formally investigating a multi-operator case as a naturally subsequent analysis for future research.

4.1 General setting

Formally, we focus on a game with two operators both providing service to customers, but with different pricing strategies. The municipality (Operator 2) operates a public transport system through the PTA. Here, prices are fixed for a medium-term time horizon. Hence, we treat the municipality's pricing strategy as fixed, i.e., $\Xi_2 = \{\xi_2\}$. This assumption, in essence, simplifies the game to an optimization problem faced by the single AMoD operator (Operator 1), offering mobility services on the road network. The operator takes customer requests and the PTA's prices (and potentially other policies) as given, and can also compute customers' optimal reaction curves for its own possible pricing strategies. Given this information, the AMoD operator picks the pricing strategy that will maximize her profit.

The AMoD operator's problem is as follows. The operator selects a short-term pricing strategy $\xi_1 \in \Xi_1 \coloneqq \bar{\mathbb{R}}_{\geq 0}^{\mathcal{V}_1 \times \mathcal{V}_1}$ to maximize her profit. She faces a transportation system in steady state and operates a fleet of N_{veh} vehicles to serve the transportation requests of the customers while conserving the vehicle flow at each vertex to account for future demand. Hence, the AMoD operator's action set comprises a set of balanced flows $(\mathcal{F}_1, \dots, \mathcal{F}_M, \mathcal{F}_0)$ and results to

$$\mathcal{B}_{\text{o},1} = \left\{ (\mathcal{F}_1, \dots, \mathcal{F}_M, \mathcal{F}_0) \,|\, \sum_{i \in \{1,\dots,M\}, f \in \mathcal{F}_i} \chi_{\text{r}}(f) \cdot t_{\text{AMoD},i} + \sum_{f \in \mathcal{F}_0} \chi_{\text{r}}(f) \cdot t(\chi_{\text{p}}(f)) \leq N_{\text{veh}} \right\},$$

where the number of vehicles corresponding to a flow results from the multiplication of its rate and travel time. Here, $t_{\text{AMoD},i}$ is the time required to serve the demand q_i, assumed to be known a priori, and $t : \mathcal{P}(\bar{\mathcal{A}}) \to \mathbb{R}_{\geq 0}$ is a function mapping each path to its corresponding travel time.

In the remainder of this section, we first specify the customer reaction (i.e., the route selection) and the AMoD operator decision (i.e., the pricing strategy selection) for our specific setting. We then prove the existence of equilibria and show that the game can be efficiently solved as a convex quadratic program. We note that we will show the existence of equilibrium based on Proposition 1, although we could also have demonstrated the existence of equilibrium in this game by showing that the reduction to the AMoD operator's optimization problem is well-behaved and satisfies the conditions for the existence of an optimal policy. We chose instead to base our proof on Proposition 1, both because the implication is very direct, and to illustrate how the assumptions of Proposition 1 manifest themselves in specific cases.

4.2 Customers' reactions and operators' decisions

Customer route selection: Customers select their preferred trip through a navigation app and can choose between an AMoD ride ($p_{\text{AMoD},i}$) and a public transport ride combined with walking ($p_{\text{PT},i}$):

$$p_{\text{AMoD},i} \coloneqq (a), \quad a \in \bar{\mathcal{A}}_1, \bar{s}_{\text{o},1}(a) = o_i, \bar{s}_{\text{d},1}(a) = d_i, \qquad p_{\text{PT},i} \in \mathcal{P}^*(o_i, d_i),$$

where o_i and d_i are the origin and the destination of the i^{th} demand $q_i = (o_i, d_i, \alpha_i)$, respectively. The public transport path $p_{\text{PT},i}$ results from the shortest path[†] on the union of the fully-connected public transport subgraph $\bar{\mathcal{G}}_2 = (\bar{\mathcal{V}}_2, \bar{\mathcal{A}}_2, \bar{s}_{\text{o},2}, \bar{s}_{\text{d},2})$ and the non-controlled subgraph $\mathcal{G}_0$, computed by weighing each arc with the sum of the monetary value of time (V_{T}) and its fare. Accordingly, the navigation app weighs each arc $a \in \mathcal{A}_0 \cup \bar{\mathcal{A}}_2$ with $V_{\text{T}} \cdot t(a)$ if $a \in \mathcal{A}_0$ and $\xi_2(\bar{s}_{\text{o},2}(a), \bar{s}_{\text{d},2}(a)) + V_{\text{T}} \cdot t(a)$ if $a \in \bar{\mathcal{A}}_2$. The AMoD path $p_{\text{AMoD},i}$ results from the arc in the fully-connected AMoD operator subgraph $\bar{\mathcal{G}}_1 = (\bar{\mathcal{V}}_1, \bar{\mathcal{A}}_1, \bar{s}_{\text{o},1}, \bar{s}_{\text{d},1})$ having the origin and the destination of the demand as the source and sink vertex, respectively. This definition holds without loss of generality as each vertex in the non-controlled subgraph, where demands are placed, can be associated to a vertex in the AMoD operator subgraph. Accordingly, the action space of the customers' reads

$$\mathcal{B}_{\text{c},i} = \left\{ \phi \in \mathbb{R}_{\geq 0}^{\mathcal{S}(q_i)} \;\middle|\; \phi(p) = 0 \,\forall\, p \in \mathcal{S}(q_i) \setminus \{p_{\text{AMoD},i}, p_{\text{PT},i}\} \right\}.$$

We consider rational customers who minimize their total cost, given by the sum of fares paid and their monetary value of time (V_{T}), so that the cost associated with a reaction curve $\phi \in \Phi(q_i) \cap \mathcal{B}_{\text{c},i}$, given the pricing strategies of the AMoD operator and of the municipality, reads

$$J_i(\phi, \xi_1, \xi_2) = (\xi_1(o_i, d_i) + V_{\text{T}} \cdot t_{\text{AMoD},i}) \cdot \phi(p_{\text{AMoD},i}) + (\xi_{\text{PT},i} + V_{\text{T}} \cdot t_{\text{PT},i}) \cdot \phi(p_{\text{PT},i}),$$

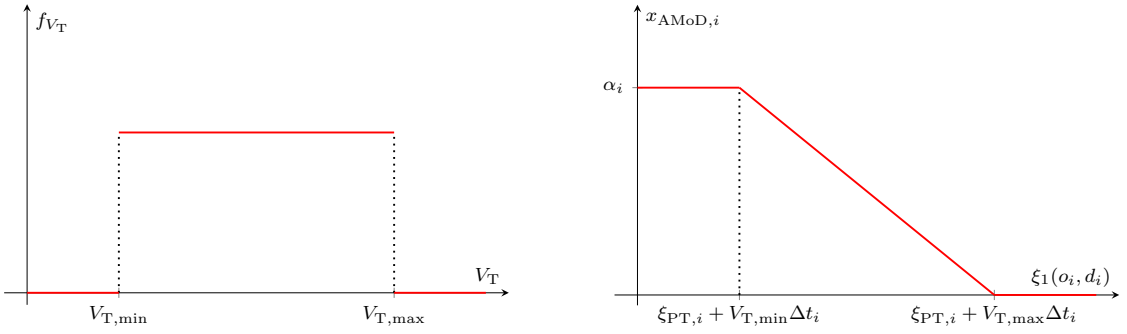
with $t_{\text{AMoD},i}$ and $t_{\text{PT},i}$ being the travel times for the demand q_i when choosing either an AMoD or a public transport ride, and $\xi_{\text{PT},i} \coloneqq \sum_{a \in p_{\text{PT},i} \cap \bar{\mathcal{A}}_2} \xi_2(\bar{s}_{\text{o},2}(a), \bar{s}_{\text{d},2}(a))$ being the price related to the path $p_{\text{PT},i}$. We assume $t_{\text{AMoD},i}$ and $t_{\text{PT},i}$ to be distinct, but allow them to be arbitrarily close. Then, with Definition 8, the reaction curve ϕ_i of a homogeneous demand is optimal if

$$\phi_i = \underset{\phi \in \Phi(q_i) \cap \mathcal{B}_{\text{c},i}}{\arg\min} \; (\xi_1(o_i, d_i) + V_{\text{T}} \cdot t_{\text{AMoD},i}) \cdot \phi(p_{\text{AMoD},i}) + (\xi_{\text{PT},i} + V_{\text{T}} \cdot t_{\text{PT},i}) \cdot \phi(p_{\text{PT},i}). \tag{1}$$

In reality, individual customers have different values of time such that demands become heterogeneous. We consider such heterogeneity by defining $V_{\text{T}} \sim \mathbb{P}$ to be dependent on some probability distribution $\mathbb{P}$ and modify Equation (1) to

$$\phi_i = \mathbb{E}_{V_{\text{T}}} \left[\underset{\phi \in \Phi(q_i) \cap \mathcal{B}_{\text{c},i}}{\arg\min} \; (\xi_1(o_i, d_i) + V_{\text{T}} \cdot t_{\text{AMoD},i}) \cdot \phi(p_{\text{AMoD},i}) + (\xi_{\text{PT},i} + V_{\text{T}} \cdot t_{\text{PT},i}) \cdot \phi(p_{\text{PT},i}) \right].$$

In this work, we proceed with uniformly distributed values of time (Figure 5a), such that the corresponding reaction curves (Figure 5b) are in line with the current literature on discrete choice models (see, e.g., Train 2009). Lemma 1 formalizes these reaction curves for a uniformly distributed value of time between $V_{\text{T,min}} \in \mathbb{R}_{\geq 0}$ and $V_{\text{T,max}} \in \mathbb{R}_{\geq 0}$, using the partition sets $\mathcal{I}_> \coloneqq \{i \in \{1, \dots, M\} \,|\, t_{\text{PT},i} > t_{\text{AMoD},i}\}$ and $\mathcal{I}_< \coloneqq \{i \in \{1, \dots, M\} \,|\, t_{\text{PT},i} < t_{\text{AMoD},i}\}$.



(a) Distribution of the value of time.

(b) Flow on the AMoD path $p_{\text{AMoD},i}$ for $t_{\text{PT},i} > t_{\text{AMoD},i}$. For brevity, we use $\Delta t_i \coloneqq t_{\text{PT},i} - t_{\text{AMoD},i} > 0$.

Figure 5: Distribution of the value of time and corresponding reaction curve, depicted in term of the flow on the AMoD path.

Lemma 1 (Optimal reaction curve) *If* $V_{\mathrm{T}} \sim \mathrm{Unif}(V_{\mathrm{T,min}}, V_{\mathrm{T,max}})$ *with* $V_{\mathrm{T,max}} > V_{\mathrm{T,min}}$*, the optimal reaction curve* $\phi_i \in \mathcal{E}_i(\xi_1, \xi_2)$ *of the demand* $q_i = (o_i, d_i, \alpha_i)$ *reads*

$$\phi_i(p_{\mathrm{AMoD},i}) = x_{\mathrm{AMoD},i} \quad \text{and} \quad \phi_i(p_{\mathrm{PT},i}) = \alpha_i - x_{\mathrm{AMoD},i},$$

where for $i \in \mathcal{I}_>$

$$x_{\mathrm{AMoD},i} = \begin{cases} 0 & \text{if } \xi_1(o_i,d_i) > \xi_{\mathrm{PT},i} + V_{\mathrm{T,max}}(t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}), \\ \frac{\alpha_i\left(V_{\mathrm{T,max}}(t_{\mathrm{PT},i}-t_{\mathrm{AMoD},i})+\xi_{\mathrm{PT},i}-\xi_1(o_i,d_i)\right)}{(V_{\mathrm{T,max}}-V_{\mathrm{T,min}})(t_{\mathrm{PT},i}-t_{\mathrm{AMoD},i})} & \text{if } \xi_1(o_i,d_i) \in \xi_{\mathrm{PT},i} + [V_{\mathrm{T,min}}, V_{\mathrm{T,max}}](t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}), \\ \alpha_i & \text{if } \xi_1(o_i,d_i) < \xi_{\mathrm{PT},i} + V_{\mathrm{T,min}}(t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}), \end{cases}$$

and for $i \in \mathcal{I}_<$

$$x_{\mathrm{AMoD},i} = \begin{cases} 0 & \text{if } \xi_1(o_i,d_i) > \xi_{\mathrm{PT},i} - V_{\mathrm{T,min}}(t_{\mathrm{AMoD},i} - t_{\mathrm{PT},i}), \\ \frac{\alpha_i\left(-V_{\mathrm{T,min}}(t_{\mathrm{AMoD},i}-t_{\mathrm{PT},i})+\xi_{\mathrm{PT},i}-\xi_1(o_i,d_i)\right)}{(V_{\mathrm{T,max}}-V_{\mathrm{T,min}})(t_{\mathrm{AMoD},i}-t_{\mathrm{PT},i})} & \text{if } \xi_1(o_i,d_i) \in \xi_{\mathrm{PT},i} - [V_{\mathrm{T,min}}, V_{\mathrm{T,max}}](t_{\mathrm{AMoD},i} - t_{\mathrm{PT},i}), \\ \alpha_i & \text{if } \xi_1(o_i,d_i) < \xi_{\mathrm{PT},i} - V_{\mathrm{T,max}}(t_{\mathrm{AMoD},i} - t_{\mathrm{PT},i}). \end{cases}$$

AMoD operator profit maximization: The AMoD operator aims to maximize her profit, given by the excess of revenue over costs. With the cost $c_{\mathrm{o},1}(a)$ for traversing arc $a \in \mathcal{A}_1$, the profit of the AMoD operator becomes

$$\begin{aligned} U_1(\xi_1, \{\mathcal{E}_i(\xi_1,\xi_2)\}_{i=1}^M) \coloneqq & \sum_{i=1}^M \phi_i(p_{\mathrm{AMoD},i}) \cdot \xi_1(\chi_{\mathrm{o}}(p_{\mathrm{AMoD},i}), \chi_{\mathrm{d}}(p_{\mathrm{AMoD},i})) \\ & - \min_{\substack{\mathcal{F}_i \in \mathcal{H}_i(\phi_i), \mathcal{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_1)}, \\ (\{\mathcal{F}_i\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\mathrm{o},1}}} \sum_{i \in \{1,\dots,M\}, f \in \mathcal{F}_i} \chi_{\mathrm{r}}(f) \sum_{a \in \chi_{\mathrm{p}}(f)} c_{\mathrm{o},1}(a) + \sum_{f \in \mathcal{F}_0} \chi_{\mathrm{r}}(f) \sum_{a \in \chi_{\mathrm{p}}(f)} c_{\mathrm{o},1}(a). \end{aligned}$$

Accordingly, solving Problem 1 yields an equilibrium.

Problem 1 *Since the municipality has only one available action,* $(\xi_1^\star, \xi_2^\star) \in \Xi_1 \times \Xi_2$ *is an equilibrium if and only if*

(i) $\xi_2^\star = \xi_2$ *and*
(ii) the profit of the AMoD operator is maximized; i.e.,

$$\xi_1^\star \in \arg\max_{\xi_1 \in \Xi_1} U_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2^\star)\}_{i=1}^M).$$

4.3 Problem decomposition

We now introduce an alternative problem formulation, where the optimal paths to serve customers and the optimal paths to rebalance empty vehicles remain decoupled.

Problem 2 *Define the modified profit* $\tilde{U}_1$ *as*

$$\begin{aligned} \tilde{U}_1(\xi_1, \{\mathcal{E}_i(\xi_1,\xi_2)\}_{i=1}^M) \coloneqq & \sum_{i=1}^M \phi_i(p_{\mathrm{AMoD},i}) \cdot \xi_1(\chi_{\mathrm{o}}(p_{\mathrm{AMoD},i}), \chi_{\mathrm{d}}(p_{\mathrm{AMoD},i})) \\ & - \min_{\substack{\mathcal{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_1)}, \\ (\{\mathcal{F}_i^\star\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\mathrm{o},1}}} \sum_{i=1}^M \chi_{\mathrm{r}}(f_i^\star) \sum_{a \in \chi_{\mathrm{p}}(f_i^\star)} c_{\mathrm{o},1}(a) + \sum_{f \in \mathcal{F}_0} \chi_{\mathrm{r}}(f) \sum_{a \in \chi_{\mathrm{p}}(f)} c_{\mathrm{o},1}(a), \end{aligned}$$

where

$$\mathcal{F}_i^\star \coloneqq \{f_i^\star\} = \begin{cases} \emptyset & \text{if } \phi_i(p_{\mathrm{AMoD},i}) = 0, \\ \{(\mathcal{P}^*(\chi_{\mathrm{o}}(p_{\mathrm{AMoD},i}), \chi_{\mathrm{d}}(p_{\mathrm{AMoD},i})), \phi_i(p_{\mathrm{AMoD},i}))\} & \text{if } \phi_i(p_{\mathrm{AMoD},i}) > 0, \end{cases}$$

where the shortest path computation bases on the weights $c_{\mathrm{o},1}(a)$ *for each arc* $a \in \mathcal{A}_1$*. Then,* $(\xi_1^\star, \xi_2^\star) \in \Xi_1 \times \Xi_2$ *is an equilibrium if and only if*

(i) $\xi_2^\star = \xi_2$ *and*

(ii) the profit of the AMoD operator is maximized; i.e.,

$$\xi_1^\star \in \arg\max_{\xi_1 \in \Xi_1} \tilde{U}_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2^\star)\}_{i=1}^M).$$

With Proposition 3, we show that this decoupled problem formulation is equivalent to Problem 1.

Proposition 3 (Problem equivalence) *Problem 1 and Problem 2 are equivalent.*

Given the generally non-linear shape of $\phi_i \in \mathcal{E}_i(\xi_1, \xi_2)$, Problem 2 remains non-convex. Nevertheless, we can exploit the properties of the optimal reaction curve to ease its analysis. Specifically, for $t_{\text{AMoD},i} < t_{\text{PT},i}$, we observe that prices below $\xi_{\text{PT},i} + V_{\text{T,min}}(t_{\text{PT},i} - t_{\text{AMoD},i})$ cannot be equilibrium prices as there exists $\varepsilon_i > 0$ such that the price $\xi_{\text{PT},i} + V_{\text{T,min}}(t_{\text{PT},i} - t_{\text{AMoD},i}) + \varepsilon_i$ leads to a higher revenue, but to the same costs (see Figure 5b). Furthermore, prices above $\xi_{\text{PT},i} + V_{\text{T,max}}(t_{\text{PT},i} - t_{\text{AMoD},i})$ cannot increase the profit, as revenues and costs remain constant. Proceeding analogously for $t_{\text{AMoD},i} > t_{\text{PT},i}$, we can state Lemmas 2 and 3.

Lemma 2 (Restricted set of pricing strategies) *The restricted set of pricing strategies*

$$\Xi_{\text{res},1} \coloneqq \left\{ \xi \in \Xi_1 \;\middle|\; \textit{If } (o,d,\alpha) \notin \mathcal{Q} \textit{ for all } \alpha \in \mathbb{R}_{\geq 0}, \textit{ then } \xi(o,d) = 0, \textit{ else for } q_i = (o,d,\alpha) \right.$$
$$\left. \xi(o,d) \in \begin{cases} [\xi_{\text{PT},i} + V_{\text{T,min}}(t_{\text{PT},i} - t_{\text{AMoD},i}), \xi_{\text{PT},i} + V_{\text{T,max}}(t_{\text{PT},i} - t_{\text{AMoD},i})] & \textit{if } i \in \mathcal{I}_>, \\ [\xi_{\text{PT},i} - V_{\text{T,max}}(t_{\text{AMoD},i} - t_{\text{PT},i}), \max\{0, \xi_{\text{PT},i} - V_{\text{T,min}}(t_{\text{AMoD},i} - t_{\text{PT},i})\}] & \textit{if } i \in \mathcal{I}_<, \end{cases} \right\}$$

is non-empty, compact, and convex, and for all $\xi_1 \in \Xi_{\text{res},1}$ the map $\xi_1 \mapsto \phi_i(p_{\text{AMoD},i})$, with $\phi_i \in \mathcal{E}_i(\xi_1, \xi_2)$, is affine.

Lemma 3 (Equilibrium characterization) *Let $(\xi_1^\star, \xi_2^\star)$ be an equilibrium of the game with the restricted set of the pricing strategies, i.e., with $(\xi_1, \xi_2) \in \Xi_{\text{res},1} \times \Xi_2$. Then, it is an equilibrium in the game with the full set of pricing strategies, i.e., with $(\xi_1, \xi_2) \in \Xi_1 \times \Xi_2$.*

Based on Lemmas 2 and 3 and leveraging Theorem 1, we can find an equilibrium for the game with the restricted set of pricing strategies in which the reaction curve changes linearly with the price by solving the following quadratic problem

$$\underset{\rho, \bar{f}_0}{\text{maximize}} \; -\rho^\top Q\rho + b^\top \rho - c^\top(-Q\rho + b) - c_0^\top \bar{f}_0 \tag{2a}$$
$$\text{subject to } [\rho]_i \geq \xi_{\text{PT},i} + V_{\text{T,min}}(t_{\text{PT},i} - t_{\text{AMoD},i}) \qquad i \in \mathcal{I}_> \tag{2b}$$
$$[\rho]_i \leq \xi_{\text{PT},i} + V_{\text{T,max}}(t_{\text{PT},i} - t_{\text{AMoD},i}) \qquad i \in \mathcal{I}_> \tag{2c}$$
$$[\rho]_i \geq \xi_{\text{PT},i} - V_{\text{T,max}}(t_{\text{AMoD},i} - t_{\text{PT},i}) \qquad i \in \mathcal{I}_< \tag{2d}$$
$$[\rho]_i \leq \max\{0, \xi_{\text{PT},i} - V_{\text{T,min}}(t_{\text{AMoD},i} - t_{\text{PT},i})\}] \qquad i \in \mathcal{I}_< \tag{2e}$$
$$B^\top \left(\textstyle\sum_{i=1}^M (-Q\rho + b)\bar{f}_i^\star + \bar{f}_0 \right) = 0_{|\mathcal{V}_1|} \tag{2f}$$
$$\sum_{i=1}^M t_{\text{AMoD},i}[-Q\rho + b]_i + \sum_{a \in \mathcal{A}_1} t(a)\bar{f}_{0,a} \leq N_{\text{veh}}, \tag{2g}$$
$$\rho \in \mathbb{R}_{\geq 0}^M, \bar{f}_0 \in \mathbb{R}_{\geq 0}^{|\mathcal{A}_1|}, \tag{2h}$$

with prices ρ and rebalancing flows $\bar{f}_0$. Here, the rationale bases on two findings. First, Lemma 3 implies that it is sufficient to seek equilibria in the restricted set of pricing strategies, which is reflected in constraints (2b)–(2e). Second, Lemma 2 shows that in the restricted set of pricing strategies reaction curves are affine in the AMoD operator's prices. With $B \in \{0, \pm 1\}^{|\mathcal{V}_1| \times |\mathcal{A}_1|}$ being the incidence matrix of the AMoD operator's subgraph $\mathcal{G}_1$, constraint (2f) guarantees that vehicle flows are balanced. Constraint (2g) limits the number of vehicles. Finally, constraint (2h) ensures that prices and rebalancing flows are non-negative. Here, $Q \coloneqq \text{diag}(\{\alpha_i(V_{\text{T,max}} - V_{\text{T,min}})^{-1} | t_{\text{PT},i} -$

$t_{\text{AMoD},i}|^{-1}\}_i)$, $[b]_{i\in\mathcal{I}_>} := \frac{\alpha_i(V_{\text{T,max}}(t_{\text{PT},i}-t_{\text{AMoD},i})+\xi_{\text{PT},i})}{(V_{\text{T,max}}-V_{\text{T,min}})(t_{\text{PT},i}-t_{\text{AMoD},i})}$, $[b]_{i\in\mathcal{I}_<} := \frac{\alpha_i(-V_{\text{T,min}}(t_{\text{AMoD},i}-t_{\text{PT},i})+\xi_{\text{PT},i})}{(V_{\text{T,max}}-V_{\text{T,min}})(t_{\text{AMoD},i}-t_{\text{PT},i})}$, $[c]_i := \sum_{a\in\chi_{\text{p}}(f_i^\star)} c_{\text{o},1}(a)$, $\bar{f}^\star_{i,a} = 1$ if $a \in \chi_{\text{p}}(f_i^\star)$ and $\bar{f}^\star_{i,a} = 0$ else, and $c_{0,a} := c_{\text{o},1}(a)$ for some ordering of the arcs $\mathcal{A}_1$. The objective (2a) reflects the profit of the AMoD operator, considering the operator's revenue ($\rho^\top(-Q\rho+b)$), cost for providing service to the customers ($c^\top(-Q\rho+b)$), and cost for rebalancing the fleet ($c_0^\top \bar{f}_0$). With Q being positive semidefinite, the optimization problem (2) is convex and can be solved by off-the-shelf optimization algorithms.

Theorem 1 (Game equilibrium) *The game admits an equilibrium. Further, consider* $(\xi_1^\star, \xi_2^\star) \in \Xi_1 \times \Xi_2$ *with*

(i) $\xi_1^\star(o,d) = 0$ *if there is no demand from* o *to* d*, i.e.,* $(o,d,\alpha) \notin \mathcal{Q}$ *for all* $\alpha \in \mathbb{R}_{\geq 0}$*,*

(ii) $\xi_1^\star(o,d) = [\rho^\star]_i$ *if there is a demand from* o *to* d*, i.e.,* $q_i = (o,d,\alpha)$ *for some* $\alpha \in \mathbb{R}_{\geq 0}$*, whereby* $\rho^\star \in \mathbb{R}^M_{\geq 0}$ *results from solving* (2)*,*

(iii) $\xi_2^\star = \xi_2$*.*

Then, $(\xi_1^\star, \xi_2^\star)$ *is an equilibrium.*

While Theorem 1 provides a systematic way to find an equilibrium through a convex optimization problem, it does neither establish uniqueness nor characterize all the potential equilibria of the game. However, Proposition 4 ensures that all equilibria are equivalent such that the equilibrium found in Theorem 1 fully characterizes the system.

Proposition 4 (Equivalent equilibria) *The game with the restricted set of pricing strategies* $(\xi_1, \xi_2) \in \Xi_{\text{res},1} \times \Xi_2$ *(see Lemma 2) possesses a unique equilibrium. The game with the full set of pricing strategies* $(\xi_1, \xi_2) \in \Xi_1 \times \Xi_2$ *can possess multiple equilibria. These equilibria are equivalent in the AMoD operator's and the municipality's profit as well as in the demands' optimal reaction curves.*

5 Case study: Berlin, Germany

We base our case study on a real-world setting for the city center of Berlin, Germany. We derive the road network (Figure 6a) from OpenStreetMap data (Haklay and Weber 2008) and infer the public transport network, consisting of U-Bahn, S-Bahn, trams, and bus lines (Figure 6b), together with its schedules from the GTFS data (VBB 2019).

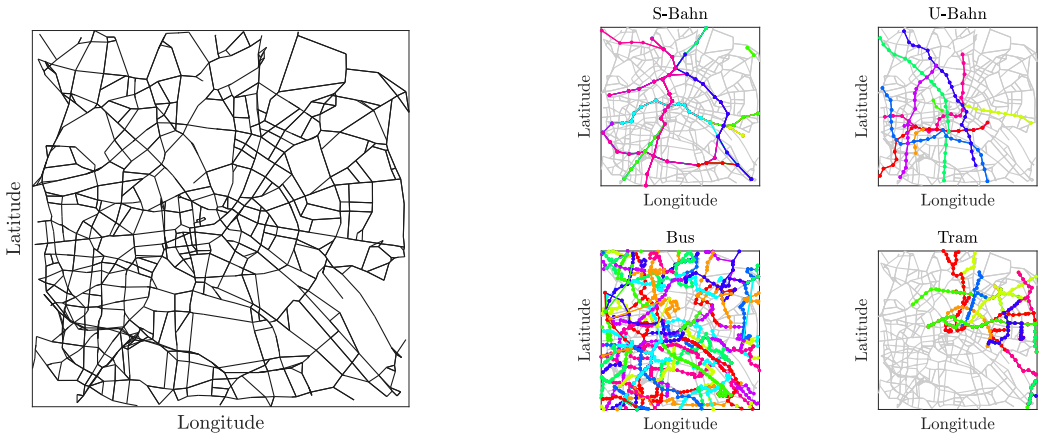
We use demand data from an existing case study (Horni et al. 2016, Ziemke et al. 2019). Since the authors considered only a representative ten percent sample of the population, we scale the rate of each demand by a factor of 10 such that our dataset consists of 129,560 travel requests, which equals a total demand rate of 18.0 customers per second for a two-hour time horizon. The average graph distance between origin and destination of a trip is 4.9 km.

We compute the AMoD service travel time $t_{\text{AMoD},i}$ for the demand q_i as

$$t_{\text{AMoD},i} = t_{\text{wait}} + \sum_{a\in\chi_{\text{p}}(f_i^\star)} t(a),$$

with an average waiting time t_{wait} of 3 min, according to today's waiting time for ride-hailing companies (Mosendz and Sender 2014). To account for congestion effects we increase the nominal travel time of each arc, given by the length over the free-flow velocity, by 56%, corresponding to the workdays evening peak congestion level in Berlin (TomTom 2019).

We consider an average walking velocity of 1.4 m/s. We compute public transport travel times based on the public transport schedules, whereby we consider a waiting time at a station of half the average time interval between two trips (5 min for U-Bahn and S-Bahn, 7 min for trams, and 10 min for buses) and 60 s walking-to-station and station-to-walking time for the U-Bahn and S-Bahn.



(a) Road network. **(b) Public transportation network.**

Figure 6: Road and public transportation network of the central urban area of Berlin, Germany. Different colors denote different public transportation lines.

To calculate costs for the AMoD operator, we consider autonomous electrified taxis with distance-based operation cost of $c_{\mathrm{d},1} = 0.34\,\mathrm{USD/km}$ (Bösch et al. 2018). This cost accounts for both variable costs, in terms of electrical energy, depreciation, and maintenance, and fixed costs, in terms of acquisition and insurance. We impose a maximum fleet size of 8,373 autonomous vehicles, reflecting the 8,373 taxi concessions released by the Berlin municipality in 2018 (Neumann 2019). Consistently with the fares in Berlin (BVG 2019), we set the price of a public transport ride to 2.80 €, corresponding to 3.12 USD (using the exchange rate of August 3rd 2019). In line with U.S. Department of Transportation (2016) and Wadud (2017), we assume the customers' value of time to be uniformly distributed between 10 USD/h and 17 USD/h.

Based on this case study, we study different settings in order to investigate the interplay between an AMoD system and the municipality as well as the impact of different interventions. Besides our basic setting (S1), we analyze potential AMoD operator strategies (S2–S4) to influence the equilibrium of the system. Settings (S5–S7) analyze how a municipality may counteract the AMoD operator. Specifically, the settings are as follows:

S1 - Basic setting: We analyze the basic setting of our case study.

S2 - Fleet size: We investigate the impact of the AMoD fleet size. To this end, we perform a parametric study, varying the *allowed* fleet size N_{veh} from 1,000 to 23,000 in intervals of 2,000 vehicles.

S3 - Vehicle characteristics: We evaluate the impact of vehicle autonomy by varying the operational costs according to Bösch et al. (2018). We consider non-autonomous vehicles with high-wage drivers ($c_{\mathrm{d},1} = 3.26\,\mathrm{USD/km}$) and with low-wage drivers ($c_{\mathrm{d},1} = 1.83\,\mathrm{USD/km}$).

S4 - Customers heterogeneity: We investigate the impact of price discrimination through group pricing. Specifically, we assume that the AMoD operator may set different prices for different customer groups (cf. Shapiro and Varian 1998). We distinguish three groups of customers: regular, young/students, and elderly. Table 1 shows the share of each group related to the total number of customers, and the range of its value of time. We determine the shares based on real demographic data (ASBB 2017) and assume the value of time of students and elderly to be 30% respectively 20% lower than that of regular customers. We impose that the price charged to students and elderly may not exceed the regular price.

S5 - Public transport price: We perform a parametric study to quantify the impact of the public transport price. Specifically, we vary the fares of the public transportation between 0 USD to 6 USD per ride, with an interval of 0.5 USD.

Table 1: Classes of customers with their corresponding parameters. Consistently with the demand data, we ignore customers younger than 18 years.

Customer Class	Percentage	Distribution Parameters $V_{T,min}$ [USD/h]	$V_{T,max}$ [USD/h]
Young/Students	11%	7.0	11.9
Elderly	22%	8.0	13.6
Regular	67%	11.1	19.0

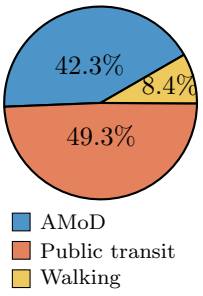
S6 - Public transportation infrastructure: We analyze the impact of public transport availability. To this end, we scale the service frequency of the public transportation by factors ranging from 1 (nominal service) to 3 (three times as frequent service), with a step size of 0.25.

S7 - AMoD service tax: In line with recent discussions regarding taxes on ride-hailing services, we analyze the impact of an additional, yet simple, percentage tax on the revenue of each trip served by the AMoD operator (Welle et al. 2018, Thadani 2019). This tax does not differ between trips that cover highly or low utilized roads. We consider taxes ranging from 0% to 100%, with a step size of 10%.

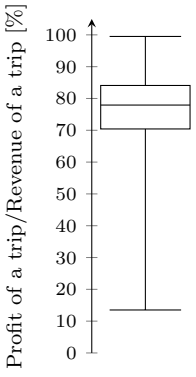
6 Results

Base case equilibrium

Figure 7 shows results for the base case equilibrium, detailing the modal split over all trips (Figure 7a) and the profitable split of the revenue of the individual trips served by the AMoD operator (Figure 7b). As can be seen, the usage of different transportation modes splits nearly equally between AMoD and public transport, while only a small share of customers opts to complete a trip solely by walking. The AMoD operator is able to operate the system very profitably on average: 73.9% of the revenue remain as profit, while 26.1% are used to cover the costs. Focusing on trips served by the AMoD operator, Figure 7b shows a similar trend: For 83% of the trips more than 65% of the fare paid by the customers remains as profit, and for 19% more than 85% of the fare remains profit. While Figure 7a shows that trips split nearly equally between public transport and AMoD services from a macroscopic perspective, Figure 8 shows that the opposite is the case when analyzing the solution from a microscopic perspective. The figure shows the modal split and the distribution of the resulting



(a) Modal share.



(b) Profitable percentage of the revenue of the trips.

Figure 7: Modal share (a) and statistics of the percentage of revenue of each trip which remains as profit for the AMoD operator (b).

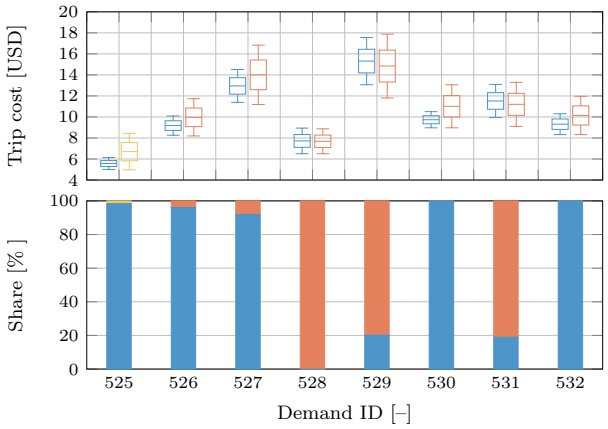


Figure 8: Cost of the trip and the related modal split for the individual demands. Recall that, since the value of time has a probabilistic representation, the cost of a trip, given by the fares and the monetary value of time, is probabilistic, too.

customer trip cost for a representative sample of origin-destination pairs. Often, either the AMoD service or the public transport provide a cost-optimal solution, independently of the customer's value of time, so that several origin-destination pairs are served completely by public transport (trip 528) or AMoD (trips 530, 532). In total, 22.0% of all trips are served solely by AMoD, 38.9% of all trips are served solely by public transport, and 4.4% of all trips are completed solely by walking. Only if the cost-optimal trip selection for customers varies between AMoD, public transport, and walking, depending on each customer's value of time, an origin-destination pair shows a modal split (trips 525, 526, 527, 529, 531). This is the case for 34.7% of the trips.

Table 2 further details the share of trips served by public transport, as a function of (*i*) the (graph) distance between origin and destination of the trip, and (*ii*) the station vicinity, defined as the sum of the (graph) distances between the origin and destination and their corresponding closest railway stations (i.e., U-Bahn or S-Bahn). As can be seen, the share of public transport trips increases for increasing trip lengths, which can be attributed to two main reasons. First, the distance-independent public transport price disincentives short trips in favour of long trips with the public transportation system. Second, the AMoD operator can gain more profit by serving shorter trips, because it can serve more trips (given a limited fleet size) and because the rebalancing costs are lower. Therefore, the AMoD operator adjusts her prices to make public transport less attractive for customers requesting short trips. Moreover, we observe that the public transport share generally increases with decreasing station vicinity.

AMoD related impact factors

Figure 9 shows the total modal split, the AMoD operator's revenue and profit, and the public transport revenue, depending on the AMoD fleet size (S2). As can be seen, the AMoD fleet size heavily influences the modal split and the AMoD operator's revenue and profit, resulting in AMoD shares that vary between 7.3% and 76.1%. The revenue of the AMoD operator increases steeper than the revenue of the public transport decreases, because the AMoD operator increases her prices to a maximum which is only slightly below the customer's trade-off to switch to public transport. As can be seen, the AMoD profit shows a decreasing utility margin as the operator serves the most profitable rides first, before accepting less profitable rides. With a fleet size exceeding 21,000 vehicles, no more additional requests are served by the AMoD system as the last 23.9% remain unprofitable due to high operational and rebalancing costs. Notably, the AMoD operator's profit begins to nearly stagnate already for fleet sizes above 15,000 vehicles.

Figure 10 shows the resulting modal share and profits for non-autonomous vehicles, scaled with respect to the nominal case (S3). The results show that the viability of a ride-hailing system depends heavily on the vehicle autonomy: As can be seen, for non-autonomous fleets, the share of served trips decreases by a factor of 4 for low-wage drivers and by a factor of 40 for high-wage drivers.

Table 2: Public transport modal share (in %) as a function of the (graph) distance between the origin and destination and of the station vicinity, defined as the sum of the (graph) distances between the origin and destination and their corresponding closest railway stations.

		Distance between origin and destination [km]								
		All	< 2 km	2–4 km	4–6 km	6–8 km	8–10 km	10–12 km	12–14 km	> 14 km
Station vicinity [km]	All	49.3	12.0	32.6	57.8	76.8	88.5	94.0	97.8	99.5
	< 0.1 km	55.0	21.5	48.7	86.1	92.4	85.1	100.0	100.0	100.0
	0.1–0.5 km	50.6	10.0	35.7	67.0	88.8	93.7	97.0	98.5	100.0
	0.5–1.0 km	45.8	10.6	26.7	57.2	80.7	93.2	96.9	97.8	99.8
	1.0–1.5 km	47.4	9.2	30.8	54.5	73.3	87.7	96.4	98.9	99.3
	1.5–2.0 km	51.0	14.9	35.0	55.5	69.5	80.6	91.5	97.2	99.5
	2.0–2.5 km	59.5	25.6	45.8	61.3	74.3	80.2	82.8	95.4	99.0
	2.5–3.0 km	59.5	11.5	44.0	61.2	75.0	81.4	89.8	97.8	100.0
	> 3 km	48.8	17.0	32.9	53.4	71.6	90.4	93.1	98.0	99.0

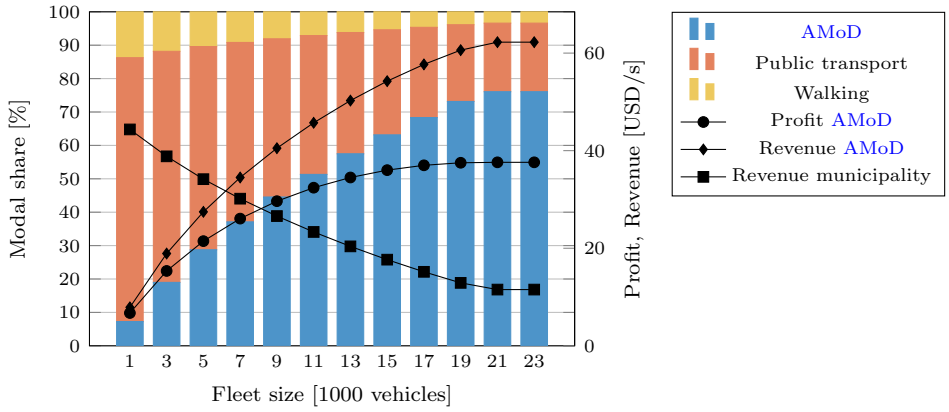


Figure 9: Impact of the fleet size on the modal share (left axis), on the profit and revenue of the AMoD operator, and on the revenue of the municipality (right axis).

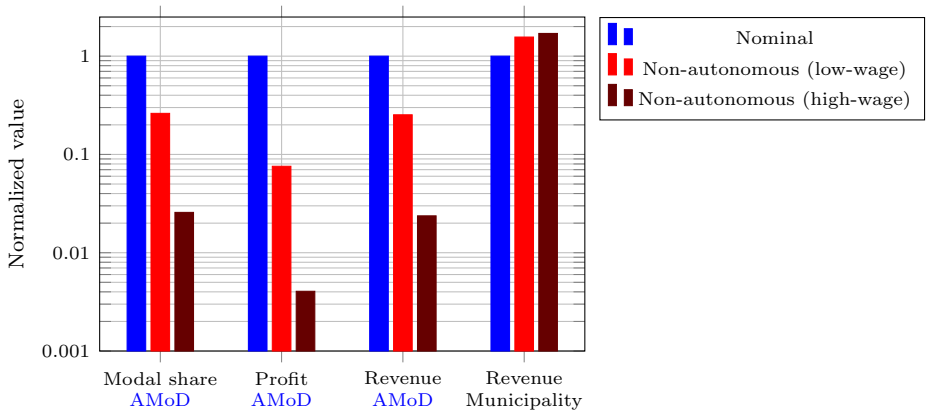


Figure 10: Impact of vehicle autonomy on the modal share, the profit and revenue of the AMoD operator, and the revenue of the municipality. The values are scaled with respect to the nominal case and depicted in logarithmic scale.

Table 3: Change in the modal share, profit, revenue, and cost incurred by the AMoD operator and the municipality between the nominal case (no group pricing) and the case where the AMoD operator may apply group pricing (group pricing), together with the corresponding relative difference.

	Modal share		Profit	Revenue		Cost
	AMoD	Public transport	AMoD	AMoD	Municipality	AMoD
No group pricing	42.3%	49.3%	28.6 USD/s	38.7 USD/s	27.7 USD/s	10.1 USD/s
Group pricing	42.1%	49.4%	28.7 USD/s	38.8 USD/s	27.7 USD/s	10.1 USD/s
Rel. difference	−0.4%	+0.1%	+0.3%	+0.2%	+0.1%	+0.0%

Table 3 details the impact of group pricing when considering heterogeneous customer groups (S4). As can be seen, group pricing at the AMoD operator's side causes only minor changes in the equilibrium: The profit of the AMoD operator increases by 0.3%, due to a slightly larger revenue while costs remain constant, and the modal share remains nearly unaffected. This result confirms that group pricing is indeed profitable for the AMoD operator, but it highlights its minor impact on the equilibrium.

Public transport impact factors

Figure 11 shows the development of the modal split based on the public transport price (S5). As can be seen, offering free or cheaper public transport can partly decrease the AMoD share in the system. In the latter case, the share of AMoD services in the system is about 35%. For a large range of prices

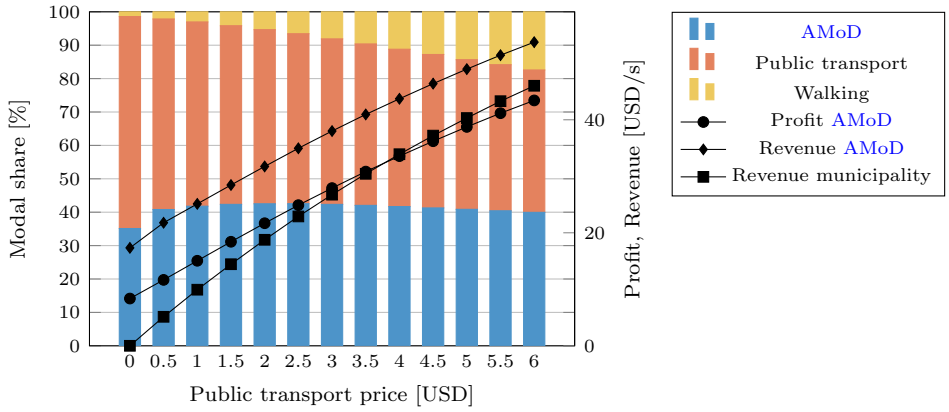


Figure 11: Impact of the public transport price on the modal share (left axis), on the profit and revenue of the AMoD operator, and on the revenue of the municipality (right axis).

(1 USD to 3.5 USD), the AMoD share remains constant, whereas it even decreases for public transport prices above 3.5 USD (which correlates with an increased average distance between the origin and the destination of the customers served). Indeed, further increased public transport prices do not shift customers from public transport to the AMoD system, but increases the share of walking customers. However, the AMoD operator's revenue and profit constantly increase with higher public transport prices. These effects occur because a higher public transport price allows the AMoD operator to raise the prices in the AMoD system as well, without affecting its operations. Similarly, the slight decrease in the AMoD modal share results from the AMoD operator focusing on less but more profitable trips in these settings.

One comment with these findings is in order. We currently do not model a customer's option to use no transportation mode at all, i.e., to stay home. In practice, customers would decline both services if prices increased too drastically, which would limit the revenue and profit increase for both the AMoD operator and the public transport. This behavior is partly reflected in an increasing shift to pure walking trips. However, here the model implies that a customer's individual price threshold to resign from a trip relates to her general value of time. Although the tendencies shown are not accurate for very high public transport fees, the dynamics shown for reasonable changes in public transport prices remain valid.

Figure 12 shows the impact of an increasing frequency of public transport services (S6). As can be seen, the modal split and the public transport revenue are slightly affected, while the AMoD operator's revenue and profit decrease. A more frequent public transport is more viable for the customers as the total cost of a public transport route decreases due to reduced waiting times. Accordingly, this forces the AMoD operator to reduce her prices to remain competitive. This shows that although an increased public transport service does not influence the AMoD share in the system, it may lower the prices for AMoD services, as the AMoD operator must reduce her prices to maintain her market share.

Figure 13 shows the impact of a service tax, imposed on the AMoD operator's revenue, i.e., on the revenue of each individual trip (S7). As can be seen in Figure 13a, a service tax up to 60% does not yield a change in the modal split but decreases the AMoD operator's profit. This effect results from the high profit share that the AMoD operator earns in the basic scenario. With a service tax above 60%, the modal split changes because the AMoD operator begins to refuse services that become unprofitable.

Focusing at the total revenue of the municipality, consisting of public transport fee revenues and tax revenues (Figure 13b), the municipality earns the highest total revenue when leveraging a fee of 70%. However, it remains questionable if a municipality could find acceptance for a service tax that is sufficiently high to impact the modal split. Still, a lower fee could be used to substitute other modes

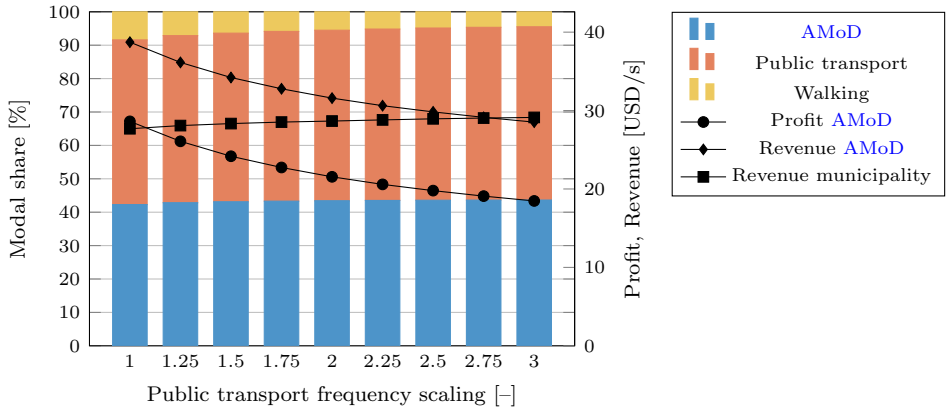
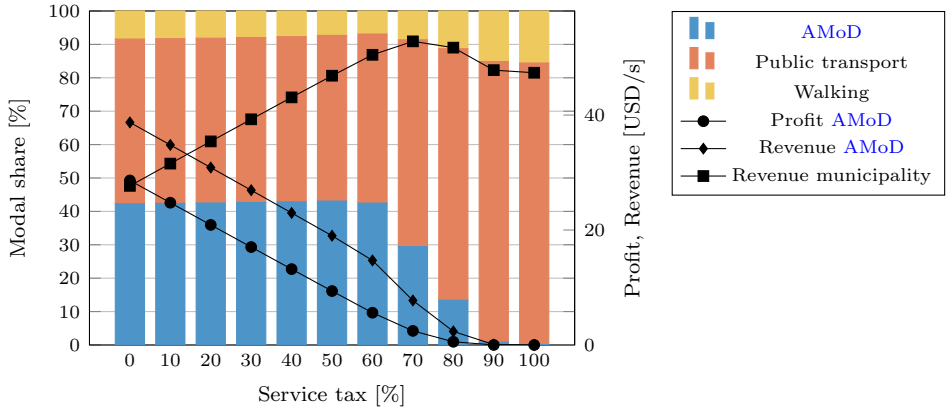
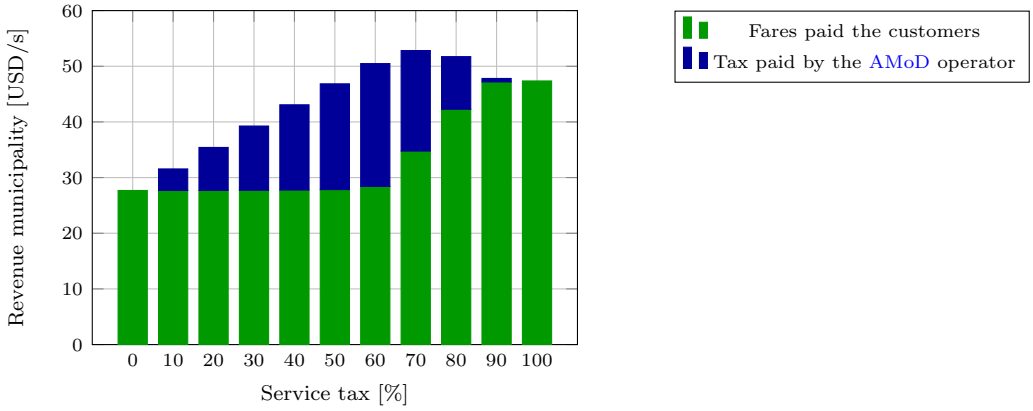


Figure 12: Impact of the public transport service frequency on the modal share (left axis), on the profit and revenue of the AMoD operator, and on the revenue of the municipality (right axis).

of public transport or to invest into additional infrastructure to increase the overall performance of the transportation system.



(a) Impact of a service tax on the modal share (left axis), profit and revenue of the AMoD operator, and revenue of the municipality (right axis).



(b) Composition of the revenue of the municipality, assuming that the taxes paid by the AMoD operator are entirely redirected to the municipality.

Figure 13: Impact of a service tax on the modal share and on the profit and revenue of the AMoD operator, on the revenue of the municipality (a), and on the composition of the revenue of the municipality (b).

7 Conclusions

With this work, we focused on the interplay between AMoD fleets and public transportation. To this end, we developed a general methodological framework to model interactions among MSPs and between MSPs and customers. Our framework combines graph-theoretic network flow models with a game-theoretic approach to capture both the interactions between MSPs and customers on the transportation market place and the constraints that result from the transportation network. We specified this framework for our application case, focusing on the interactions among an AMoD fleet operator, a municipality, and customers. We developed a computationally tractable quadratic program to find the equilibrium of the resulting game. We applied our methodology to a real-world case study for the city of Berlin, and we presented results for various settings to identify major impact factors. The results of our case study yield the following insights:

Autonomous mobility-on-demand systems can cannibalize public transportation. Our experiments reveal that the modal share splits nearly equally between AMoD system and public transport for the analyzed case study. If the AMoD fleet size is not limited, the share of customers taking the AMoD system can increase up to 76%.

Local cannibalization can deviate from the macroscopic effects. Our results show that nearly 22.5% of the trips are entirely served by the AMoD system, 38.9% are served solely by public transport, and only 34.7% show a modal split that correlates with the macroscopic findings.

Increasing the AMoD fleet size shows a diminishing utility margin. Our experiments indicate that the profit growth gained by enlarging the AMoD fleet diminishes as the fleet gets larger. The profit generated by the AMoD system reaches a plateau for a fleet of 15,000 vehicles, which is roughly 80% more than the number of taxi concessions released by the Berlin municipality.

Vehicle autonomy strongly influences the profitability of ride-hailing systems. Our studies showed that a (non-autonomous) mobility-on-demand system generates less than 10% of the profit and serves less than 25% of the customers compared to its autonomous counterpart.

Free public transport counteracts an AMoD system. Our results suggest that a free public transportation system reduces the AMoD modal share and profit by about 20% and 70%, respectively.

Improving the public transport availability does not impact the modal split. Our studies show that improving public transport availability through an increased service frequency does not shift the modal split towards public transport, but diminishes the profit of the AMoD operator.

Imposing high taxes on an AMoD system can impact the modal split. Our experiments revealed that a service tax imposed on the revenue of each AMoD trip lower than 60% diminishes the profit generated by the AMoD operator, yet it does not shift the modal split towards public transport. Only taxes above 60% shift the modal split towards public transport, e.g., decrease the AMoD share to 13.4% for a tax of 80%.

Three final comments are in order and open the field for future research directions. First, we did not allow customers to combine AMoD service and public transport in an intermodal fashion or to give up their trip. Incorporating these aspects may reveal further insights into the system behavior and into the exploitation of different transport modes. Second, we focused on a problem setting with a single AMoD operator. Extending this setting to multiple operators may yield additional insights. Finally, incorporating endogenous congestion and congestion pricing opens a variety of future research directions.

A Notation

Let $\mathcal{X}$ and $\mathcal{Y}$ be sets. The cardinality of the set $\mathcal{X}$ is $|\mathcal{X}|$; we say that $\mathcal{X}$ is a singleton if $|\mathcal{X}| = 1$. The set of all functions mapping $x \in \mathcal{X}$ to $y \in \mathcal{Y}$ is denoted by $\mathcal{Y}^{\mathcal{X}}$. The set of all subsets of $\mathcal{X}$ is $2^{\mathcal{X}}$. The set of nonnegative real numbers is $\mathbb{R}_{\geq 0}$ and the set of strictly positive real numbers is $\mathbb{R}_{>0}$.

The nomenclature used throughout this paper holds as follows:

Graph-related symbols

$\mathcal{G}$	Multigraph
$\mathcal{V}$	Set of vertices
$\mathcal{A}$	Set of arcs
$s_{\mathrm{o}}(a)$	Source of arc a
$s_{\mathrm{d}}(a)$	Sink of arc a
v	Vertex
a	Arc
p	Path
$\mathcal{P}(\mathcal{A})$	Set of all paths for the arc set $\mathcal{A}$
$\chi_{\mathrm{o}}(p)$	Origin of path p
$\chi_{\mathrm{d}}(p)$	Destination of path p
$\mathcal{P}^*(o,d)$	Shortest path between o and d
f	Flow
$\mathcal{F}$	Set of flows
$\chi_{\mathrm{p}}(f)$	Path of flow f
$\chi_{\mathrm{r}}(f)$	Rate of flow f
$\deg_{\mathrm{in}}(v)$	In-degree of vertex v
$\deg_{\mathrm{out}}(v)$	Out-degree of vertex v

Operators-related symbols

o	Operator
$\mathcal{N}$	Set of operators
N	Number of operators
ξ	Pricing strategy
Ξ	Set of pricing strategies
U	Profit
$c(\mathcal{F})$	Cost of the set of flows $\mathcal{F}$
$c_{\mathrm{o}}(a)$	Cost of arc a
c_{d}	Distance-based cost
$\mathcal{B}_{\mathrm{o}}$	Operator's action space
$\mathcal{H}(\phi)$	Set of potentially active sets of flows for the reaction curve ϕ

Demands-related symbols

q	Demand
$\mathcal{Q}$	Set of demands
M	Number of demands
o	Origin vertex
d	Destination vertex
α	Demand rate
J	Cost related to a demand
$\mathcal{S}(q)$	Set of paths satisfying demand q
ϕ	Reaction curve
$\mathcal{B}_{\mathrm{c}}$	Customers' action space
$\Phi(q)$	Set of valid reaction curves for demand q
$\mathcal{E}$	Set of optimal reactions

Other symbols

N_{veh}	Number of vehicles
V_{T}	Value of time
$t(a)$	Travel time to traverse arc a
t_{AMoD}	Travel time of the AMoD ride
t_{PT}	Travel time of the public transport ride

B Graph theory

Definition 10 (Multigraph) *A (directed) multigraph $\mathcal{G}$ is a quadruple $(\mathcal{V}, \mathcal{A}, s_{\mathrm{o}}, s_{\mathrm{d}})$ such that $\mathcal{V}$ is the set of vertices, $\mathcal{A}$ is the set of arcs, $s_{\mathrm{o}} : \mathcal{A} \to \mathcal{V}$ assigns to each arc its source vertex, and $s_{\mathrm{d}} : \mathcal{A} \to \mathcal{V}$ assigns to each arc its sink vertex.*

Definition 11 (Path) *We refer to a path of length $L \in \mathbb{N}$ as a set of distinct arcs $\{a_1, \ldots, a_{L-1}\}$ for which there exists a set of exactly $L+1$ distinct vertices $\{v_1, \ldots, v_L\}$ such that $s_{\mathrm{o}}(a_i) = v_i$ and $s_{\mathrm{d}}(a_i) = v_{i+1}$ for all $i \in \{1, \ldots, L-1\}$. Note that by definition such a path cannot contain cycles. Let $\mathcal{P}(\mathcal{A})$ be the set of all paths.*

Definition 12 (Origin and destination) *Given a path $p = \{a_1, \ldots, a_{L-1}\}$ on $\mathcal{G} = (\mathcal{V}, \mathcal{A}, s_{\mathrm{o}}, s_{\mathrm{d}})$ we define the path origin and destination functions as $\chi_{\mathrm{o}} : \mathcal{P}(\mathcal{A}) \to \mathcal{V}$ and $\chi_{\mathrm{d}} : \mathcal{P}(\mathcal{A}) \to \mathcal{V}$.*

Definition 13 (Fully-connected) *A multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{A}, s_\mathrm{o}, s_\mathrm{d})$ is fully-connected if for all $v_1, v_2 \in \mathcal{V}$ such that $v_1 \neq v_2$ there exists an arc $a \in \mathcal{A}$ such that $s_\mathrm{o}(a) = v_1$ and $s_\mathrm{d}(a) = v_2$.*

Definition 14 (Fully-connected graph) *Consider $\mathcal{G} = (\mathcal{V}, \mathcal{A}, s_\mathrm{o}, s_\mathrm{d})$ with $\mathcal{V} = \{v_1, \ldots, v_{|\mathcal{V}|}\}$. Then, the fully-connected version of $\mathcal{G}$ is the quadruple $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{A}}, \bar{s}_\mathrm{o}, \bar{s}_\mathrm{d})$, where*

$$\begin{aligned} \bar{\mathcal{V}} &= \mathcal{V}, \\ \bar{\mathcal{A}} &= \{a_{1,2}, a_{1,3}, \ldots, a_{|\mathcal{V}|,|\mathcal{V}|-1}\}, \\ \bar{s}_\mathrm{o}(a_{i,j}) &= v_i, \\ \bar{s}_\mathrm{d}(a_{i,j}) &= v_j. \end{aligned}$$

Definition 15 (Shortest path) *Consider a multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{A}, s_\mathrm{o}, s_\mathrm{d})$ and a non-negative function $f : \mathcal{A} \to \mathbb{R}_{\geq 0}$. A path p is a shortest path between $v_1 \in \mathcal{V}$ and $v_2 \in \mathcal{V}$ if (i) $\chi_\mathrm{o}(p) = v_1$ and $\chi_\mathrm{d}(p) = v_2$ and (ii) it minimizes $\sum_{a \in p} f(a)$. We denote the set of shortest paths by $\mathcal{P}^*(v_1, v_2)$.*

C Proofs

Proof of Proposition 1. Since the graph has a finite number of nodes, the space of pricing strategies of a subgraph $\tilde{\mathcal{G}} = (\tilde{\mathcal{V}}, \tilde{\mathcal{A}}, \tilde{s}_\mathrm{o}, \tilde{s}_\mathrm{d})$ is isomorphic to $\bar{\mathbb{R}}^m_{\geq 0}$ with $m = |\bar{\mathcal{V}}|^2$, such that there exists an isomorphism $f : \bar{\mathbb{R}}^{|\tilde{\mathcal{V}}| \times |\tilde{\mathcal{V}}|}_{\geq 0} \to \bar{\mathbb{R}}^m_{\geq 0}$ uniquely mapping a pricing strategy to some $x \in \bar{\mathbb{R}}^m_{\geq 0}$. Then, with $f^{-1}(\bar{X}^j)$ being the bounded set containing all pricing strategies, Theorem 1 in Harker (1991) with $X^j = \bar{\mathbb{R}}^m_{\geq 0} \cap \bar{X}^j$ and $K^j(x) = f(\bar{\Xi}_j(\{\xi_k\}_{k \neq j}))$ establishes the sufficient condition for the existence of an equilibrium in the space of pricing strategies yielding a profit larger than negative infinity. Clearly, this is an equilibrium for the game with the full set of pricing strategies. □

Proof of Proposition 2. The proof follows directly from the definition of the optimal reaction curve (Definition 8) and the game equilibrium (Definition 9). □

Proof of Lemma 1. Recall that for a given value of time V_T the optimal reaction curve of the demand $q_i = (o_i, d_i, \alpha_i)$, denoted by $\tilde{\phi}_i$, reads $\tilde{\phi}_i(p_{\mathrm{AMoD},i}) = \tilde{x}_{\mathrm{AMoD},i}$ and $\tilde{\phi}_i(p_{\mathrm{PT},i}) = \alpha_i - \tilde{x}_{\mathrm{AMoD},i}$, where[1]

$$\tilde{x}_{\mathrm{AMoD},i}(V_\mathrm{T}) = \begin{cases} \alpha_i & \text{if } \xi_1(o_i, d_i) + V_\mathrm{T} t_{\mathrm{AMoD},i} \leq \xi_{\mathrm{PT},i} + V_\mathrm{T} t_{\mathrm{PT},i}, \\ 0 & \text{else.} \end{cases}$$

Then, the customer flow on the AMoD path $p_{\mathrm{AMoD},i}$, denoted by $x_{\mathrm{AMoD},i} \coloneqq \phi_i(p_{\mathrm{AMoD},i})$, is $x_{\mathrm{AMoD},i} = \mathbb{E}_{V_\mathrm{T}}[\tilde{x}_{\mathrm{AMoD},i}]$. For $i \in \mathcal{I}_>$, i.e., $t_{\mathrm{AMoD},i} < t_{\mathrm{PT},i}$, we have

$$\begin{aligned} x_{\mathrm{AMoD},i} = \mathbb{E}_{V_\mathrm{T}}[\tilde{x}_{\mathrm{AMoD},i}] &= \int_{V_{\mathrm{T,min}}}^{V_{\mathrm{T,max}}} \frac{1}{V_{\mathrm{T,max}} - V_{\mathrm{T,min}}} \tilde{x}_{\mathrm{AMoD},i}(v)\,\mathrm{d}v \\ &= \int_{\max\left\{V_{\mathrm{T,min}}, \frac{\xi_1(o_i,d_i) - \xi_{\mathrm{PT},i}}{t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}}\right\}}^{V_{\mathrm{T,max}}} \frac{\alpha_i}{V_{\mathrm{T,max}} - V_{\mathrm{T,min}}}\,\mathrm{d}v \\ &= \begin{cases} 0 & \text{if } \xi_1(o_i,d_i) > \xi_{\mathrm{PT},i} + V_{\mathrm{T,min}}(t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}), \\ \frac{\alpha_i\left(V_{\mathrm{T,max}}(t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}) + \xi_{\mathrm{PT},i} - \xi_1(o_i,d_i)\right)}{(V_{\mathrm{T,max}} - V_{\mathrm{T,min}})(t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i})} & \text{if } \xi_1(o_i,d_i) \in \xi_{\mathrm{PT},i} + [V_{\mathrm{T,min}}, V_{\mathrm{T,max}}](t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}), \\ \alpha_i & \text{if } \xi_1(o_i,d_i) < \xi_{\mathrm{PT},i} + V_{\mathrm{T,max}}(t_{\mathrm{PT},i} - t_{\mathrm{AMoD},i}). \end{cases} \end{aligned}$$

[1] Strictly speaking, the given $V_\mathrm{T} \mapsto x_{\mathrm{AMoD},i}(V_\mathrm{T})$ is not the *only* optimal reaction curve for a given value of time. However, all functions differ only at one point and, by basic properties of Lebesgue integration, their integrals coincide.

For $i \in \mathcal{I}_<$, i.e., $t_{\text{AMoD},i} > t_{\text{PT},i}$, we have

$$\begin{aligned} x_{\text{AMoD},i} = \mathbb{E}_{V_\text{T}}\left[\tilde{x}_{\text{AMoD},i}\right] &= \int_{V_{\text{T,min}}}^{V_{\text{T,max}}} \frac{1}{V_{\text{T,max}} - V_{\text{T,min}}} \tilde{x}_{\text{AMoD}}(v)\,\mathrm{d}v \\ &= \int_{V_{\text{T,min}}}^{\min\left\{V_{\text{T,max}}, \frac{\xi_{\text{PT},i} - \xi_1(o_i,d_i)}{t_{\text{AMoD},i} - t_{\text{PT},i}}\right\}} \frac{\alpha_i}{V_{\text{T,max}} - V_{\text{T,min}}}\,\mathrm{d}v \\ &= \begin{cases} 0 & \text{if } \xi_1(o_i,d_i) > \xi_{\text{PT},i} - V_{\text{T,min}}(t_{\text{AMoD},i} - t_{\text{PT},i}), \\ \frac{\alpha_i\left(-V_{\text{T,min}}(t_{\text{AMoD},i} - t_{\text{PT},i}) + \xi_{\text{PT},i} - \xi_1(o_i,d_i)\right)}{(V_{\text{T,max}} - V_{\text{T,min}})(t_{\text{AMoD},i} - t_{\text{PT},i})} & \text{if } \xi_1(o_i,d_i) \in \xi_{\text{PT},i} - [V_{\text{T,min}}, V_{\text{T,max}}](t_{\text{AMoD},i} - t_{\text{PT},i}), \\ \alpha_i & \text{if } \xi_1(o_i,d_i) < \xi_{\text{PT},i} - V_{\text{T,max}}(t_{\text{AMoD},i} - t_{\text{PT},i}). \end{cases} \end{aligned}$$

Since we assumed $t_{\text{AMoD},i} \neq t_{\text{PT},i}$ the sets $\mathcal{I}_>$ and $\mathcal{I}_<$ are indeed partition sets and $x_{\text{AMoD},i}$ is well-defined. This concludes the proof. □

Proof of Proposition 3. To show the equivalence of the two problems, we show that the modified profit equals the original profit. As revenues coincide, it is sufficient to show that costs are equivalent.

Specifically, a set of flows $\mathcal{F}_i$ is feasible (i.e., $\mathcal{F}_i \in \mathcal{H}_i(\phi_i)$) if and only if $\chi_\text{o}(\chi_\text{p}(f)) = \chi_\text{o}(p_{\text{AMoD},i})$, $\chi_\text{d}(\chi_\text{p}(f)) = \chi_\text{d}(p_{\text{AMoD},i})$ $\forall f \in \mathcal{F}_i$, and $\sum_{f\in\mathcal{F}_i} \chi_\text{r}(f) = \phi_i(p_{\text{AMoD},i})$. Consider $\mathcal{F}_i^\star, \bar{\mathcal{F}}_i \in \mathcal{H}_i(\phi_i)$, with $\mathcal{F}_i^\star \neq \bar{\mathcal{F}}_i$. First, by the definition of a shortest path, the cost related to $\mathcal{F}_i^\star$ imposes a lower bound on the cost of $\bar{\mathcal{F}}_i$:

$$\begin{aligned} \sum_{f\in\bar{\mathcal{F}}_i} \chi_\text{r}(f) \sum_{a\in\chi_\text{p}(f)} c_{\text{o},1}(a) &\geq \sum_{f\in\bar{\mathcal{F}}_i} \chi_\text{r}(f) \sum_{a\in\chi_\text{p}(f^\star)} c_{\text{o},1}(a) \\ &= \phi_i(p_{\text{AMoD},i}) \sum_{a\in\chi_\text{p}(f_i^\star)} c_{\text{o},1}(a) \\ &= \chi_\text{r}(f_i^\star) \sum_{a\in\chi_\text{p}(f_i^\star)} c_{\text{o},1}(a). \end{aligned}$$

Second, the operator space does not depend on paths in $\mathcal{F}_i$; therefore, $(\{\mathcal{F}_i^\star\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\text{o},1}$ implies $(\{\bar{\mathcal{F}}_i\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\text{o},1}$. Hence, the cost minimum can be attained with $\mathcal{F}_i = \mathcal{F}_i^\star$. This proves equivalence of the two profits and, consequently, of the two problems. □

Proof of Lemma 2. We prove non-emptiness, closedness, boundedness, convexity, and affinity of the map separately.
Non-emptiness: It suffices to observe that for $t_{\text{PT},i} > t_{\text{AMoD},i}$ there always exists a feasible price (since $V_{\text{T,min}} < V_{\text{T,max}}$) and that for $t_{\text{PT},i} > t_{\text{AMoD},i}$ $\xi_1(o,d) = 0$ is always feasible. *Closedness:* The set $\Xi_{\text{res},1}$ is the intersection of closed spaces and is therefore closed.
Boundedness: It is easy to bound the set (w.r.t. an arbitrary norm).
Convexity: Convexity follows directly from the well-known fact that the intersection of convex sets is again convex.
Affinity of the map: The result follows directly from Lemma 1.

This concludes the proof. □

To prove Lemma 3, we state Lemma 4.

Lemma 4 *We consider a pricing strategy $\xi_1 \in \Xi_1$ and define its projection $\xi_{\text{proj},1}$ in the restricted set of pricing strategies $\Xi_{\text{res},1}$ (cf. Lemma 2) as $\xi_{\text{proj},1}(o,d) \coloneqq 0$ if there is no demand from o to d, i.e., if $(o,d,\alpha) \notin \mathcal{Q}$ for all $\alpha \in \mathbb{R}_{\geq 0}$. If there is demand from o to d, i.e., if $q_i = (o,d,\alpha) \in \mathcal{Q}$ for some $\alpha \in \mathbb{R}_{\geq 0}$, we define it as*

$$\xi_{\text{proj},1}(o,d) \coloneqq \begin{cases} \max\{\xi_{\text{PT},i} + V_{\text{T,min}}(t_{\text{PT},i} - t_{\text{AMoD},i}), \\ \qquad \min\{\xi_{\text{PT},i} + V_{\text{T,max}}(t_{\text{PT},i} - t_{\text{AMoD},i}), \xi_1(o,d)\}\} & \text{if } i \in \mathcal{I}_>, \\ \max\{\xi_{\text{PT},i} - V_{\text{T,max}}(t_{\text{AMoD},i} - t_{\text{PT},i}), \\ \qquad \min\{\xi_{\text{PT},i} - V_{\text{T,min}}(t_{\text{AMoD},i} - t_{\text{PT},i}), \xi_1(o,d), 0\}\} & \text{if } i \in \mathcal{I}_<. \end{cases}$$

Then,

i) $\xi_{\text{proj},1} \in \Xi_{\text{res},1}$ *and* $\xi_{\text{proj},1} = \xi_1$ *if* $\xi_1 \in \Xi_{\text{res},1}$,
ii) $\mathcal{E}_i(\xi_{\text{proj},1}, \xi_2) = \mathcal{E}_i(\xi_1, \xi_2) \, \forall \, i \in \{1, \dots, M\}$, *and*
iii) $U_1(\xi_{\text{proj},1}, \{\mathcal{E}_i(\xi_{\text{proj},1}, \xi_2\}_{i=1}^M) \geq U_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2\}_{i=1}^M)$.

Proof of Lemma 4. Properties (i) and (ii) follow directly from the definition of projection and Lemma 1. To prove (iii), we use the definition of profit and the equivalence of the reaction curves. Specifically, for $\phi_i \in \mathcal{E}_i(\xi_1, \xi_2) = \mathcal{E}_i(\xi_{\text{proj},1}, \xi_2)$ for all $i \in \{1, \dots, M\}$ we have

$$\begin{aligned} &U_1(\xi_1, \{\phi_i\}_{i=1}^M) - U_1(\xi_{\text{proj},1}, \{\phi_i\}_{i=1}^M) \\ &\qquad = \sum_{i=1}^M \phi_i(p_{\text{AMoD},i}) \cdot (\xi_1(\chi_{\text{o}}(p_{\text{AMoD},i}), \chi_{\text{d}}(p_{\text{AMoD},i})) - \xi_{\text{proj},1}(\chi_{\text{o}}(p_{\text{AMoD},i}), \chi_{\text{d}}(p_{\text{AMoD},i}))) \\ &\qquad \geq 0. \end{aligned}$$

Then, the desired inequality follows from $\phi_i(p_{\text{AMoD},i}) = 0$ whenever $\xi_1(\chi_{\text{o}}(p_{\text{AMoD},i}), \chi_{\text{d}}(p_{\text{AMoD},i})) \geq \xi_{\text{proj},1}(\chi_{\text{o}}(p_{\text{AMoD},i}), \chi_{\text{d}}(p_{\text{AMoD},i}))$. Rearranging the equation above yields

$$U_1(\xi_{\text{proj},1}, \{\mathcal{E}_i(\xi_{\text{proj},1}, \xi_2\}_{i=1}^M) \geq U_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2\}_{i=1}^M).$$

This concludes the proof. □

Proof of Lemma 3. The proof is based on the definition of equilibrium (cf. Definition 9). Consider an arbitrary pricing strategy $\xi_1 \in \Xi_1$ and its related projection $\xi_{\text{proj},1}$, as defined in Lemma 4. Then, we have

$$\begin{aligned} U_1(\xi_1^\star, \{\mathcal{E}_i(\xi_1^\star, \xi_2^\star)\}_{i=1}^M) &\geq U_1(\xi_{\text{proj},1}, \{\mathcal{E}_i(\xi_{\text{proj},1}, \xi_2^\star)\}_{i=1}^M) && ((\xi_1^\star, \xi_2^\star) \text{ is an eq. in } \Xi_{\text{res},1} \times \Xi_2) \\ &\geq U_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2^\star)\}_{i=1}^M). && (\text{Lemma 4}) \end{aligned}$$

As ξ_1 is arbitrary, we conclude that $U_1(\xi_1^\star, \{\mathcal{E}_i(\xi_1^\star, \xi_2^\star)\}_{i=1}^M) \geq U_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2^\star)\}_{i=1}^M)$ for all $\xi_1 \in \Xi_1$, implying that $(\xi_1^\star, \xi_2^\star)$ is indeed an equilibrium in the full set of pricing strategies. □

We state and prove Lemma 5 to proceed with the proof of Theorem 1.

Lemma 5 *The set of pricing strategies* $\xi_1 \in \Xi_{\text{res},1}$ *for which* $U_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2)\}_{i=1}^M) > -\infty$ *is non-empty, closed, bounded, and convex.*

Proof of Lemma 5. We prove non-emptiness, boundedness, closedness, and convexity separately.
Non-emptiness: It suffices to consider the pricing strategy $\bar{\xi}_1$ that yields $\phi_i(p_{\text{AMoD},i}) = 0$ for all $i \in \{1, \dots, M\}$.
Boundedness: The result follows directly from the set being a subset of the bounded set $\Xi_{\text{res},1}$ (cf. Lemma 2).
Closedness: Since $\mathcal{B}_{\text{c},1}$ is closed, the set is closed too.
Convexity: The profit equals negative infinity for $\xi_1 \in \Xi_{\text{res},1}$ if and only if the set $\bar{\mathcal{F}}(\xi_1) \coloneqq \{\mathcal{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_j)} \,|\, (\{\mathcal{F}_i^\star(\xi_1)\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\text{o},1}\}$ is non-empty. Let ξ_1^1 and ξ_1^2 and assume that the sets $\bar{\mathcal{F}}_0(\xi_1)$ and $\bar{\mathcal{F}}_0(\xi_2)$ are non-empty. Consider $\bar{\xi}_1 = \lambda \xi_1^1 + (1-\lambda)\xi_1^2$ for $\lambda \in (0,1)$. We aim to show that the profit is finite for $\bar{\xi}_1$. Take $\mathcal{F}_0^1 \in \bar{\mathcal{F}}(\xi_1^1)$ and $\mathcal{F}_0^2 \in \bar{\mathcal{F}}(\xi_1^2)$, and consider

$$\bar{\mathcal{F}}_0 \coloneqq \left\{ f \in \mathcal{F}(\mathcal{G}_1) \;\middle|\; \begin{array}{l} f \in \lambda f^2 \oplus (1-\lambda) f^2 \text{ for } (f^1, f^2) \in \mathcal{F}_0^1 \times \mathcal{F}_0^2 \text{ sharing the same path,} \\ \quad \vee \\ f \in \{\lambda f^1, (1-\lambda) f^2\} \text{ for } (f^1, f^2) \in \mathcal{F}_0^1 \times \mathcal{F}_0^2 \text{ not sharing the same path with other flows.} \end{array} \right\},$$

where $\lambda f \coloneqq (\chi_\mathrm{p}(f), \lambda \cdot \chi_\mathrm{r}(f))$ for $\lambda > 0$ and

$$f^1 \oplus f^2 \coloneqq \begin{cases} \{(\chi_\mathrm{p}(f^1), \chi_\mathrm{r}(f^1) + \chi_\mathrm{r}(f^2))\} & \text{if } \chi_\mathrm{p}(f^1) = \chi_\mathrm{p}(f^2), \\ \{f^1, f^2\} & \text{else.} \end{cases}$$

Since $\bar{\phi}_i(p_{\mathrm{AMoD},i}) = \lambda\phi_i^1(p_{\mathrm{AMoD},i}) + (1-\lambda)\phi_i^2(p_{\mathrm{AMoD},i})$ (see Lemma 2), $(\{\mathcal{F}_i^\star\}_{i=1}^M, \bar{\mathcal{F}}_0)$ is balanced and fulfils the fleet size constraint. Hence, the set $\bar{\mathcal{F}}(\bar{\xi}_1)$ is non-empty, meaning that the profit is finite, showing the required convexity property.
This concludes the proof. □

Proof of Theorem 1. We first prove the existence of an equilibrium. We then show that the quadratic program suffices in finding it.

Existence of Equilibrium: To prove existence of an equilibrium we use Proposition 1. Consider the game with the restricted set of pricing strategies, i.e., with $(\xi_1, \xi_2) \in \Xi_{\mathrm{res},1} \times \Xi_2$. If this game possesses an equilibrium, then the game with the full set of pricing strategies, i.e., with $(\xi_1, \xi_2) \in \Xi_1 \times \Xi_2$, has an equilibrium according to Lemma 3. First, we note that Ξ_1 is trivially upper and lower semicontinuous in all prices $\xi_2(v_1, v_2)$ with $v_1, v_2 \in \mathcal{V}_2$ because Ξ_2 is a singleton. Second, the set of pricing strategies for which the profit is finite is closed, bounded (and therefore contained in a bounded set), and convex according to Lemma 5. Third, concavity of the profit of the AMoD operator (which implies continuity) follows from:

1. $\phi_i(p_{\mathrm{AMoD},i}) \cdot \xi_1(\chi_\mathrm{o}(p_{\mathrm{AMoD},i}), \chi_\mathrm{d}(p_{\mathrm{AMoD},i}))$ is quadratic in the prices $\xi_1(o,d)$ for $o,d \in \mathcal{V}_1$ (cf. Lemma 2).
2. $\min_{\mathcal{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_j)}, (\{\mathcal{F}_i^\star\}_{i=1}^M, \mathcal{F}_0) \in \mathcal{B}_{\mathrm{o},1}} \chi_\mathrm{r}(f_i^\star) \sum_{a \in \chi_\mathrm{p}(f_i^\star)} c_{\mathrm{o},1}(a) + \sum_{f \in \mathcal{F}_0} \chi_\mathrm{r}(f) \sum_{a \in \chi_\mathrm{p}(f)} c_{\mathrm{o},1}(a)$ is convex in $\phi_i(p_{\mathrm{AMoD},i})$ (cf. Chapter 5 of Bertsimas and Tsitsiklis 1997). Since the map $\xi_1 \mapsto \phi_i(p_{\mathrm{AMoD},i})$, with $\phi_i \in \mathcal{E}_i(\xi_1, \xi_2)$, is affine (cf. Lemma 2) and the composition of a convex and an affine function is convex, the cost is convex in the prices $\xi_1(o,d)$ for $o,d \in \mathcal{V}_1$.

Further, the profit of the AMoD operator is concave in the pricing strategy ξ_1, because the function $h(x) = f(x) - g(x)$ is concave for concave $f : \mathcal{X} \to \mathcal{Y}$ and convex $g : \mathcal{X} \to \mathcal{Y}$. Hence, Proposition 1, together with the municipality being an inactive player, establishes the existence of an equilibrium.

Quadratic Programming Formulation: Again, we show that the given pricing strategy is an equilibrium in the restricted set of pricing strategies and leverage Lemma 3 to conclude that it is an equilibrium also in the game with the full set of pricing strategies. It is easy to see that prices of origin-destination pairs with no demand are irrelevant. Formally, if there is no demand from $\tilde{o} \in \mathcal{V}_1$ to $\tilde{d} \in \mathcal{V}_2$ and $\bar{\xi}_1$ is an equilibrium pricing strategy, then

$$\tilde{\xi}_1(o,d) \coloneqq \begin{cases} \gamma & \text{if } o = \tilde{o}, d = \tilde{d}, \\ \bar{\xi}_1(o,d) & \text{else,} \end{cases}$$

is an equilibrium pricing strategy for any $\gamma \in \bar{\mathbb{R}}_{\geq 0}$. This allows us to focus on origin-destination pairs corresponding to some demand. Lemma 2 allows to express $\phi_i(p_{\mathrm{AMoD},i}) = [-Q\rho + b]_i$ for the given Q and b, yielding the revenue $\rho^\top(-Q\rho + b)$ and the cost $c^\top(-Q\rho + b)$. For arc-based costs we can always map $\bar{f}_0 \in \mathbb{R}_{\geq 0}^{|\mathcal{A}_1|}$ to a set of flows and vice versa. Hence, the rebalancing cost is $c_0^\top \bar{f}_0$ and the balanced set constraint can be expressed through the incidence matrix. The fleet size constraint follows analogously.

It remains to prove that we can jointly optimize prices *and* rebalancing vehicles. This follows from $\max_{x \in \mathcal{X}} \max_{y \in \bar{\mathcal{Y}}(x) \subseteq \mathcal{Y}} f(x,y) = \max_{(x,y) \in \mathcal{X} \times \mathcal{Y}, y \in \bar{\mathcal{Y}}(x)} f(x,y)$ and $-\min_{x \in \mathcal{X}} g(x) = \max_{x \in \mathcal{X}} -g(x)$ for finite functions $f(x,y) : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$ and $g(x) : \mathcal{X} \to \mathcal{Z}$. Hence, the optimization problem (2) yields an equilibrium. This concludes the proof. □

Proof of Proposition 4. As the optimization problem (2) is strictly convex in ρ, there exists a unique maximizer $\rho^\star$. Hence, the equilibrium in the restricted set of pricing strategies is unique (cf. Theorem 1).

One may easily create examples of games with the full set of pricing strategies possessing multiple equilibria (e.g., by perturbing the prices of origin-destination pairs without demand). To prove that all equilibria result in the same reaction curves and in the same profits consider two arbitrary non-equal equilibrium $(\xi_1^\star, \xi_2^\star)$ and $(\bar{\xi}_1, \bar{\xi}_2)$ and assume without loss of the generality that $\xi_1^\star \in \Xi_{\text{res},1}$ and $\bar{\xi}_1 \notin \Xi_{\text{res},1}$ (of course, $\xi_2^\star = \bar{\xi}_2 = \xi_2$).

First, the profit of the AMoD operator at both equilibria, given by $U_1(\xi_1^\star, \{\mathcal{E}_i(\xi_1^\star, \xi_2^\star\}_{i=1}^M)$ and $U_1(\bar{\xi}_1, \{\mathcal{E}_i(\bar{\xi}_1, \bar{\xi}_2\}_{i=1}^M)$, must coincide; else, the equilibrium yielding the lower profit would not be an equilibrium (cf. Problem 1).

Second, we show that both equilibria result in the same optimal reaction curves. We assume for the sake of contradiction that at least one reaction curve differs, i.e., $\mathcal{E}_i(\xi_1^\star, \xi_2^\star) \neq \mathcal{E}_i(\bar{\xi}_1, \bar{\xi}_2)$ for some $i \in \{1, \dots, M\}$. Define $\bar{\xi}_{\text{proj},1} \in \Xi_{\text{res},1}$ for $\bar{\xi}_1$ as in Lemma 4. By the lemma, $\mathcal{E}_i(\bar{\xi}_{\text{proj},1}, \xi_2) = \mathcal{E}_i(\bar{\xi}_1, \xi_2)$ and $U_1(\xi_{\text{proj},1}, \{\mathcal{E}_i(\xi_{\text{proj},1}, \xi_2\}_{i=1}^M) \geq U_1(\xi_1, \{\mathcal{E}_i(\xi_1, \xi_2)\}_{i=1}^M)$. We consider two cases:

1. If $\xi_1^\star = \bar{\xi}_{\text{proj},1}$, we clearly have $\mathcal{E}_i(\xi_{\text{proj},1}, \xi_2) = \mathcal{E}_i(\bar{\xi}_1, \xi_2)$ and, by transitivity, the optimal reaction curves of the two equilibria coincide, leading to a contradiction.
2. If $\xi_1^\star \neq \bar{\xi}_{\text{proj},1}$, uniqueness of the equilibrium in the restricted set of pricing strategies implies $U_1(\xi_1^\star, \{\mathcal{E}_i(\xi_1^\star, \xi_2^\star)\}_{i=1}^M) > U_1(\bar{\xi}_{\text{proj},1}, \{\mathcal{E}_i(\bar{\xi}_{\text{proj},1}, \xi_2)\}_{i=1}^M)$ and, thus, $U_1(\xi_1^\star, \{\mathcal{E}_i(\xi_1^\star, \xi_2)\}_{i=1}^M) > U_1(\bar{\xi}_1, \{\mathcal{E}_i(\bar{\xi}_1, \xi_2)\}_{i=1}^M)$, contradicting $(\bar{\xi}_1, \bar{\xi}_2)$ being an equilibrium.

Hence, the optimal reaction curves coincide.

Third, since the reaction curves coincide and the municipality is an inactive player, the profit of the municipality coincides, too. This concludes the proof. □

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