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Abstract: The open-pit mine production scheduling problem aims to optimize the net present value of a mining asset. Several solution methods have been proposed to find the most profitable mining sequence. Such methods entail determining which mining blocks from those used to represent the related mineral deposit should be extracted and when. However, little is reported in the technical literature that considers the material flow once mined and, more specifically, incorporating stockpiling as part of the mine scheduling strategy, thus adding technical intricacies due to the difficulty of correctly modeling the materials' blending once sent into a stockpile.

In this paper, a new model is provided to address the topic of open-pit mine production scheduling considering multiple destinations for the mined material, including stockpiles, and accounting for multi-element uncertainty. Unlike conventional models, the proposed model allows for an accurate estimation of the resulting grade of the stockpile without using unrealistic assumptions or non-linear constraints. A solution approach based on extending the Bienstock and Zuckerberg algorithm to the stochastic optimization and two heuristics is presented and applied to different real-size instances. Results show that this approach provides a feasible integer solution within less than 1.7% of optimality in a reasonable time. Properties and limitations of the model presented are also discussed, and recommendations for further research are made.

Keywords: Open-pit optimization, stochastic mathematical programming, Langrangian relaxation, stockpile, multi-element mineral deposit, long-term production planning, Tabu search

Résumé : Le problème de planification de la production des mines à ciel ouvert vise à optimiser la valeur actuelle nette d'un projet minier. Plusieurs méthodes de solution ont été proposées pour trouver la séquence minière la plus rentable. Ces méthodes consistent à déterminer quels blocs parmi ceux utilisés pour représenter le gisement minéral concerné doivent être extraits et à quel moment. Cependant, peu d'ouvrages dans la littérature technique s'intéressent au flux du minerai une fois extrait et, plus spécifiquement, l'incorporation du stockage dans la stratégie de planification de la mine, ajoutant ainsi de la complexité au problème et ce, dû à la difficulté de modéliser correctement la notion de mélange une fois les matériaux arrivés à la pile de stockage.

Dans cet article, un nouveau modèle est présenté dans le but de traiter le sujet de la planification de la production des mines à ciel ouvert, prenant en compte plusieurs destinations pour le matériau extrait, y compris les piles de stockage, et en tenant compte de l'incertitude multi-éléments. Contrairement aux modèles conventionnels, le modèle proposé permet une estimation précise de la teneur résultante du stock sans recourir à des hypothèses irréalistes ou à des contraintes non linéaires. Une méthode de résolution basée sur l'extension de l'algorithme de Bienstock et Zuckerberg à l'optimisation stochastique ainsi que deux heuristiques est présentée et appliquée à différentes instances de taille réelle. Les résultats montrent que cette approche fournit une solution entière réalisable avec moins de 1,7% d'optimalité dans un délai raisonnable. Les propriétés et les limites du modèle présenté sont également discutées et des recommandations pour des recherches ultérieures sont formulées.

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1 Introduction

The long-term production scheduling of open pit mines seeks to find an optimal mining sequence, which means determining when, if ever, to extract each portion of the mineral deposit and where to send it among a given set of possible destinations (waste dump, processing plants, leach pads, stockpiles, and so on), so that the total net present value (NPV) of the operation is maximized. This scheduling is subject to some restrictions such as limited resources, production requirements or spatial precedence constraints. This decision problem, also called the open-pit mine production scheduling problem (OPMPSP), has been studied during the last fifty years not only due to its complexity since it implies large data sets and multiple constraints, but also due to its direct impact on a mining project's profitability and success. Through the development of optimization methods, researchers are trying to propose models that are increasingly accurate and close to reality on the ground. However, efforts to include stockpiles in the optimization process are limited, as it is mathematically challenging to assess the grade of the material inside the stockpile appropriately.

1.1 Models without stockpiling

Early attempts to solve the OPMPSP do not consider stockpiles as part of the problem and focus instead on the extraction sequence. At that time, a three-dimensional model of blocks was already introduced to represent deposits and discretize them into mining blocks. Drilling operations allow associating a set of attributes to each block, such as metal grades, material type, density, etc. These attributes are estimated based on interpolation using available surrounding samples. Conventional approaches reinforce their inaccuracy by considering a single estimated value for each attribute and hence ignore both uncertainty and in-situ variability. Such methods are said to be deterministic in the sense that they consider all attribute values as exact, ignoring the related uncertainty and its sources. Johnson (1968) relaxes the integrity constraints and takes advantage of the particular structure of the problem to apply the Dantzig-Wolfe decomposition. Dagdeleen and Johnson (1986) propose an exact method based on the Lagrangian relaxation of resource constraints and use the sub-gradient method to adjust the multipliers. Many authors have since then proposed methods to make large instances computationally tractable. While Ramazan (2007) developed the Fundamental Tree Algorithm to optimally aggregates blocks into groups, Whittle (1988) and Tabesh and Askari-Nasab (2011) use different aggregation heuristics to reduce the size of the problem. However, aggregation often leads to infeasible solutions once the disaggregation is done and original variables are restored (Boland et al., 2009), and thus, compromises the usefulness itself of the methods. Topal (2003) eliminates some binary variables by defining the earliest and the latest time of extraction for each block. Caccetta and Hill (2003) use the ultimate pit limit for preprocessing and apply a branch-and-cut algorithm to solve the problem. Ramazan and Dimitrakopoulos (2004) relax the integrity of the variables associated with waste blocks. Using a sliding time window was also a popular method (Dimitrakopoulos and Ramazan, 2008; Cullenbine et al., 2011; Lamghari and Dimitrakopoulos, 2016). Other approaches rely on heuristics and metaheuristics. Denby and Schofield (1994) use a genetic algorithm, while Shishvan and Sattarvand (2015) present a metaheuristic based on an ant colony optimization. Lamghari et al. (2015) propose a hybrid method based on linear programming (LP) and variable neighborhood local search. Decomposition methods also seem to be particularly efficient when applied to OPMPSP (Tachefine, 1997; Chicoisne et al., 2012; Bienstock and Zuckerberg, 2009, 2010). The Bienstock-Zuckerberg (BZ) algorithm, in particular, proved its efficiency to solve optimally the LP relaxation of the OPMPSP. Taking advantage of the special structure of the problem, their decomposition method can tackle large instances with an arbitrary number of side constraints. Munoz et al. (2017) document proves the correctness of the Bienstock-Zuckerberg (BZ) algorithm and shows that it can be generalized to handle more problems arising in the context of mine planning.

Despite considerable advances in deterministic optimization methods, ignoring the uncertainty present in several parameters, especially in the metal content, severely compromises the robustness of these methods and directly affects the mining project's performance (Dowd, 1994; Baker and Giacomo, 1998). To address the geological uncertainty and represent the in-situ variability of the grades, multiple equally probable scenarios of the orebody's geological profile are used simultaneously as inputs for the OPMPSP (Goovarets, 1997; Boucher and Dimitrakopoulos, 2009). Accounting for metal uncertainty adds a level of complexity to the optimization process but presents also many benefits discussed, inter alia, by Ravenscroft (1992), Dowd

(1994, 1997), Godoy and Dimitrakopoulos (2004), Menabde et al. (2007), Albor and Dimitrakopoulos (2010), Ramazan and Dimitrakopoulos (2013), Marcotte and Caron (2013), and Behrang et al. (2014). Overall, the stochastic optimization increases the reliability of scheduling and its chances of meeting production targets through more realistic modeling and risk management. It also leads to higher NPVs (in the order of 15-30%) compared to those obtained by deterministic models (Dimitrakopoulos, 2011). Different approaches to handle metal uncertainty have been proposed in the literature. Formulations minimizing deviations from production targets over multiple orebody simulations have been introduced for example by Godoy (2003), Albor and Dimitrakopoulos (2009) and Goodfellow and Dimitrakopoulos (2013). Menabde et al. (2007) add to their model constraints to ensure that the production targets are met on average. Boland et al. (2008) propose a multistage stochastic programming approach. Benndorf and Dimitrakopoulos (2013) adopt a stochastic integer mathematical programming (SIP) to deal with multi-element uncertainty. The authors also manage to minimize deviation from production requirements. Lamghari and Dimitrakopoulos (2012) suggest an efficient metaheuristic solution approach based on a diversified Tabu search while Lamghari et al. (2014) propose a variable neighborhood descent heuristic and Albor and Dimitrakopoulos (2010) generate a set of nested pits, group these pits into pushbacks and then produce the long-term production scheduling based on the pushback designs obtained. Brika et al. (2018) propose an adaptation of the BZ algorithm to the stochastic optimization combined with two heuristics to solve efficiently an OPMPS problem under multi-element uncertainty and dealing with different destinations. The work presented herein is an extension of the later paper.

1.2   Models with stockpiling

Stockpiles have long been excluded from the mine planning optimization process, despite their significant role in the material flow post-extraction and the different advantages they provide for mining companies. Indeed, it is mathematically challenging to correctly model the material-flow inside the stockpile considering the limitation of conventional optimization methods. Some mine planning software packages (Whittle, 2010; Mintec, 2013, MineMax, 2016) consider stockpiling as part of open pit mine scheduling. However, due to their modeling techniques, they do not provide optimal solutions.

In this context, academic researchers have proposed different models to remedy the identified deficiencies. To model the grade within the stockpile, it is typically assumed that material, once sent to the stockpile, is mixed automatically and homogeneously, and when reclaimed, it no longer has the quality that it entered the stockpile, but henceforth the current average quality of the stockpile. This assumption implies constraints to ensure the preservation of material regarding tonnage and quality grade and thus, often results in models with non-convex nonlinear constraints. However, it is important to stress that this assumption of homogeneous mixing is unrealistic; rather, the material is stacked into successive layers. Bley et al. (2012) propose exact algorithmic approaches to solve two different nonlinear integer models considering one stockpile and taking advantage of the special structure of such specific models. They relax the nonlinear stockpiling constraints and introduce an aggressive branching pattern to limit their violation to be arbitrarily close to zero. Then, they apply a primal heuristic to repair the solution and make it a fully feasible solution. Goodfellow and Dimitrakopoulos (2016, 2017) and Montiel and Dimitrakopoulos (2015) propose global optimization models that consider all the interrelated aspects of the mineral value chain simultaneously in a stochastic context. Their method combines different metaheuristics such as simulated annealing, particle swarm optimization, and differential evolutions. They also develop a policy to select the destination of each block once it is extracted. Comparisons with the deterministic equivalent of the proposed optimizer and a commercial mine planning software show the outperformance of their method.

Although correctly capturing the material flow inside a stockpile with respect to the homogenous mixing assumption requires nonlinear models, due to their inherent complexity, researchers prefer to have recourse to linear models at the expense of solution accuracy. Smith (1999) solves a short-term production scheduling considering stockpiling and blending. He approximates, with a piecewise linear formulation, the quadratic terms in the original model representing the product of the grade inside a stockpile and the quantity withdrawn from it in each period. Akaike and Dagdelen (1999) do not consider blending in stockpiles and assume that there is an infinity of stockpiles; in other terms, that each block has its associated stockpile and thus,

the trackability of each block is preserved from its extraction to its final destination, through to stockpile. This assumption is unrealistic, though it alleviates the nonlinearity. Later, Ramazan and Dimitrakopoulos (2013) and Smith and Wicks (2014) adopt the same assumption but in two different contexts. The first paper uses a stochastic framework to reflect uncertainty in the geological and economic input data. Instead, the second paper considers a deterministic context in which the model is solved using a sliding time window heuristic. Yarmuch and Ortiz (2011) define two stockpiles, one for the high-grade ore and another for the low-grade, and then solve the problem period by period, readjusting the grade inside the stockpiles at the end of each period. Tabesh et al. (2015) linearize the problematic constraints by defining a "sufficient" number of stockpiles, where each stockpile covers material within a tight grade range. Unfortunately, the authors do not provide any numerical results. Lamghari and Dimitrakopoulos (2015) introduce a new formulation to solve the OPMPSP under metal uncertainty and considers multiple destinations, including stockpiles. The nonlinearity is avoided by estimating the corresponding average grade for each stockpile and adjusting it successively. To manage geological risk, the surplus over ore production targets is indirectly penalized through the costs imposed by sending ore material to the stockpiles. Moreno et al. (2017) analyze different stockpiling models, most of which are aforementioned, and propose a new linear-integer model. The authors show that this model is a close approximation of the nonlinear-integer equivalent model presented by Bley et al. (2012). However, they also assume homogeneous mixing of the material in a single stockpile in each period.

In this context, the objective of this paper is to propose a new linear mixed integer model that can tackle large instances of the OPMPSP with stockpiling (OPMPSP+S) and under metal uncertainty. This model does not consider the homogeneous mixing assumption and does not require approximations to handle the blending constraints properly. The remainder of the paper is organized as follows: Section 2 presents an existing model to solve the OPMPS that does not incorporate stockpiles before introducing a new model with stockpiling. Section 3 outlines a reformulation of the proposed model to fit the Bienstock-Zuckerberg algorithm's framework. In Section 4, a three-step solution approach is described. Computational results of application on actual case studies are presented in Section 5. Section 6 explains the limitations of the proposed model and suggests recommendations to address them. Conclusions follow in Section 7.

2 Mathematical formulation

This section first presents a mathematical formulation of a model that does not incorporate stockpiling before introducing stockpiles in a new model.

2.1 Open pit mine production scheduling without stockpiling

The first subsection brings in notation and the following section introduces the mathematical model.

2.1.1 Notations

Sets:

- $\mathcal{B} = \{1 \ldots B\}$: set of blocks;
- $\mathcal{T} = \{1 \ldots T\}$: set of time periods, which discretize the life of the mine;
- $\mathcal{M} = \{1 \ldots M\}$: set of processing plants;
- $\mathcal{W} = \{1 \ldots W\}$: set of waste dumps;
- $\mathfrak{D} = \mathcal{M} \cup \mathcal{W} = \{1 \ldots D\}$: set of destinations including the waste dumps and the processing plants;
- $\mathfrak{R} = \{1 \ldots R\}$: set of ore properties (i.e. geological elements);
- $\mathcal{S} = \{1 \ldots S\}$: set of possible scenarios of the orebody. Each scenario has the same probability of occurrence and represents a different simulation of the geological profile of each block;
- Γ_b^+ and Γ_b^-: respectively the set of immediate successors and predecessors of block b.

Parameters:

- $p_{b,d,t,s}$ is the discounted profit obtained, if block b is sent to destination d in period t under scenario s. If the destination is a processing plant, the profit is equal to the value of the metal content recovered of the block less the processing and selling costs. If the block is sent to the waste dump, it is equal to 0. Note here that the mining cost of the block is computed separately;

- $g_{b,s}^r$ is the grade of block b for element r under scenario s;

- $Tmax_{m,t}$ and $Tmin_{m,t}$ are respectively the expected maximum and minimum ore tonnages sent to processing plant m at period t;

- $Gmax_{m,t}^r$ and $Gmin_{m,t}^r$ are respectively the expected maximum and minimum grades for resource r sent to processing plant m at period t;

- $mc_{b,t}$ is the discounted cost of extracting a block b at period t;

- cu_t and cl_t are respectively the discounted unit costs of upper and lower deviations from $Tmax_{m,t}$ and $Tmin_{m,t}$;

- cu_t^r and cl_t^r are respectively the discounted unit costs of upper and lower deviations from $Gmax_{d,t}^r$ and $Gmin_{d,t}^r$;

- Q_b is the tonnage of block b;

- M_{max} is the maximum mining capacity per period.

Variables:

- $y_{b,d,t}^{at}$ is a binary variable which takes 1 if the block b is completely extracted and sent to destination d at time t, 0 otherwise;

- $qu_{m,t,s}$ and $ql_{m,t,s}$ are continuous variables representing respectively the upper and lower deviation from ore tonnage production target at period t sent to processing plant m under scenario s;

- $qu_{m,t,s}^r$ and $ql_{m,t,s}^r$ are continuous variables representing respectively the upper and lower deviation from production target r at period t sent to processing plant m under scenario s.

2.1.2 Mathematical model

Following the description and the notation given in the previous sections, the problem can be formulated as a two-stage stochastic mixed integer programming model (Birge and Louveaux, 1997).

Objective function:

$$(\mathcal{P})\ max\ Z= \frac{1}{S}\sum_{b=1}^{B}\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{s=1}^{S}(p_{b,d,t,s} - mc_{b,t})y_{b,d,t}$$

$$-\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{s=1}^{S}(cu_t qu_{m,t,s} + cl_t ql_{m,t,s}) - \frac{1}{S}\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R}\left(cu_t^r qu_{m,t,s}^r + cl_t^r ql_{m,t,s}^r\right) \quad (1)$$

Constraints:

$$\sum_{p=1}^{T}\sum_{d=1}^{D} y_{b,d,p}^{at} \leq 1 \qquad \forall b \in \mathcal{B} \qquad (2)$$

$$\sum_{p=1}^{t}\sum_{d=1}^{D} y_{b,d,p}^{at} \leq \sum_{p=1}^{t}\sum_{d=1}^{D} y_{a,d,p}^{at} \qquad \forall\,(a,b) \in \mathcal{B}^2 \text{ where } a \in \Gamma_b^-, t \in \mathcal{T} \qquad (3)$$

$$\sum_{b=1}^{B} Q_b \times y_{b,m,t,s}^{at} - qu_{m,t,s} \leq Tmax_{m,t} \qquad \forall b \in \mathcal{B}, m \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \qquad (4)$$

$$\sum_{b=1}^{B} Q_b \times y_{b,m,t,s}^{at} + ql_{m,t,s} \geq Tmin_{m,t} \qquad \forall b \in \mathcal{B}, m \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \qquad (5)$$

$$\sum_{b=1}^{B} Q_b \times \left(g_{bs}^r \ - \ Gmax_{m,t}^r \right) \times y_{b,m,t}^{at} \ - \ qu_{m,t,s}^r \le 0 \qquad \forall m \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S}, r \in \mathcal{R} \qquad (6)$$

$$\sum_{b=1}^{B} Q_b \times \left(g_{bs}^r \ - \ Gmin_{m,t}^r \right) \times y_{b,m,t}^{at} \ + \ ql_{m,t,s}^r \ge 0 \qquad \forall m \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S}, r \in \mathcal{R} \qquad (7)$$

$$\sum_{b=1}^{B} Q_b \times \left(\sum_{d=1}^{D} y_{b,d,t}^{at} \right) \le M_{max} \qquad \forall t \in \mathcal{T} \qquad (8)$$

$$y_{b,d,t}^{at} \in \{0,1\} \qquad \forall b \in \mathcal{B}, d \in \mathcal{D}, t \in \mathcal{T} \qquad (9)$$

$$qu_{m,t,s}, ql_{m,t,s} \ge 0 \qquad \forall m \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S} \qquad (10a)$$

$$qu_{m,t,s}^r, ql_{m,t,s}^r \ge 0 \qquad \forall m \in \mathcal{M}, t \in \mathcal{T}, s \in \mathcal{S}, r \in \mathcal{R} \qquad (10b)$$

The objective function (1) combines two goals. The first aims to maximize the discounted profit generated by the deposit exploitation over the simulated orebody models. The second one aims to minimize deviations from production targets considering all the scenarios.

The first constraint (2) ensures that each block cannot be extracted more than once. Constraint (3) represents the precedence constraints and ensures that a block cannot be mined if not all the overlaying blocks are already mined. Ore tonnage deviations from ore production targets are defined in inequations (6) and (7). Similarly, constraints (4) and (5) compute the grade deviations from respectively the maximum and minimum expected grades. The constraint (8) limits the mining capacity. Finally, constraint (9) and constraints (10) represent the integrity and the non-negativity constraints respectively.

2.2 A new linear model that considers stockpiling

In this section, a new model OPMPS+S that considers stockpiling is presented. Unlike conventional models, this model doesn't assume that material in the stockpile is automatically mixed and becomes homogeneous. Instead of that and due to new variables introduced in Section 2.2.1, each block is tracked from its extraction to its final destination. The same notation presented in Section 2.1 is used, and some additional notation is defined in what follows:

2.2.1 Notation

Parameters:

- $rc_{b,m,t}$ is the discounted cost of rehandling a block b from stockpiles to processing plant t at time t.

Variables:

- z_{b,m,t_0,t_1}^{at} is a binary variable that takes 1 if b extracted and sent to stockpile at time t_0 and then sent to processing plant t at time t_1 with $(t_0 < t_1)$.

With this new formulation, a stockpile is created for and associated with each period t_1 and each processing plant t. All the ore stocked in this stockpile during periods anterior to t_1 will be completely sent to processing plant t only in period t_1.

2.2.2 Mathematical model

The OPMPS+S model is obtained from the formulation in Section 2.1.2 by adding terms (written between "{}") associated with new variables. The numbers of equivalent constraints in both models are preserved. Those of the new model are differentiated by an apostrophe.

Objective function:

$$(\mathcal{P}^s) \ max \ \frac{1}{S} \sum_{b=1}^{B} \sum_{d=1}^{D} \sum_{t=1}^{T} \sum_{s=1}^{S} (p_{b,d,t,s} - mc_{b,t}) \times y_{b,d,t}$$

$$- \sum_{m=1}^{M} \sum_{t=1}^{T} \sum_{s=1}^{S} (cu_t qu_{m,t,s} + cl_t ql_{m,t,s})$$

$$- \frac{1}{S} \sum_{m=1}^{M} \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{r=1}^{R} (cu_t^r qu_{m,t,s}^r + cl_t^r ql_{m,t,s}^r) \tag{1'}$$

$$+ \left\{ \frac{1}{S} \sum_{b=1}^{B} \sum_{m=1}^{M} \sum_{t_0=1}^{T-1} \sum_{t_1=t_0+1}^{T} \sum_{s=1}^{S} (p_{b,m,t_1,s} - mc_{b,t_0} - rc_{b,m,t_1}) \times z_{b,m,t_0,t_1} \right\}$$

Constraints:

For each $b \in \mathcal{B}$:

$$\sum_{t_0=1}^{T} \sum_{d=1}^{D} y_{b,d,t_0}^{at} + \left\{ \sum_{t_0=1}^{T-1} \sum_{t_1=t_0+1}^{T} \sum_{m=1}^{M} z_{b,m,t_0,t_1}^{at} \right\} \leq 1 \tag{2'}$$

For each $(a,b) \in \mathcal{B}^2$ where $a \in \Gamma_b^-$ and $t \in \{1 \ldots T-1\}$:

$$\sum_{t_0=1}^{t} \sum_{d=1}^{D} y_{b,d,t_0}^{at} + \left\{ \sum_{t_0=1}^{t} \sum_{t_1=t_0+1}^{T} \sum_{m=1}^{M} z_{b,m,t_0,t_1}^{at} \right\} \leq \sum_{t_0=1}^{t} \sum_{d=1}^{D} y_{a,d,t_0}^{at} + \left\{ \sum_{t_0=1}^{t} \sum_{t_1=t_0+1}^{T} \sum_{m=1}^{M} z_{a,m,t_0,t_1}^{at} \right\} \tag{3a'}$$

For each $(a,b) \in \mathcal{B}^2$ where $a \in \Gamma_b^-$ and $t = T$:

$$\sum_{t_0=1}^{T} \sum_{d=1}^{D} y_{b,d,t_0}^{at} + \left\{ \sum_{t_0=1}^{T-1} \sum_{t_1=t_0+1}^{T} \sum_{m=1}^{M} z_{b,m,t_0,t_1}^{at} \right\} \leq \sum_{t_0=1}^{T} \sum_{d=1}^{D} y_{a,d,t_0}^{at} + \left\{ \sum_{t_0=1}^{T-1} \sum_{t_1=t_0+1}^{T} \sum_{m=1}^{M} z_{a,m,t_0,t_1}^{at} \right\} \tag{3b'}$$

For each $m \in \mathcal{M}$, $s \in \mathcal{S}$ and $t = 1$:

$$\sum_{b=1}^{B} Q_b \times y_{b,m,1}^{at} - qu_{m,1,s} \leq Tmax_{m,1} \tag{4a'}$$

$$\sum_{b=1}^{B} Q_b \times y_{b,m,1}^{at} + ql_{m,1,s} \geq Tmin_{m,1} \tag{5a'}$$

For each $m \in \mathcal{M}$, $s \in \mathcal{S}$ and $t \in \{2 \ldots T\}$:

$$\sum_{b=1}^{B} Q_b \times \left(y_{b,m,t}^{at} + \left\{ \sum_{t_0=1}^{t-1} z_{b,m,t_0,t}^{at} \right\} \right) - qu_{m,t} \leq Tmax_{m,t} \tag{4b'}$$

$$\sum_{b=1}^{B} Q_b \times \left(y_{b,m,t}^{at} + \left\{ \sum_{t_0=1}^{t-1} z_{b,m,t_0,t}^{at} \right\} \right) + ql_{m,t} \geq Tmin_{m,t} \tag{5b'}$$

For each $m \in \mathcal{M}$, $t = 1$, $s \in \mathcal{S}$ and $r \in \mathcal{R}$:

$$\sum_{b=1}^{B} Q_b \times (g_{bs}^r - Gmax_{m,t}^r) \times y_{b,m,1}^{at} - qu_{m,1.s}^r \leq 0 \tag{6a'}$$

$$\sum_{b=1}^{B} Q_b \times (g_{bs}^r - Gmin_{m,t}^r) \times y_{b,m,1}^{at} + ql_{m,1.s}^r \geq 0 \tag{7a'}$$

For each $m \in \mathcal{M}$, $t \in \{2 \ldots T\}$, $s \in \mathcal{S}$ and $r \in \mathcal{R}$:

$$\sum_{b=1}^{B} Q_b \times (g_{bs}^r \; - \; Gmax_{m,t}^r) \times \left(y_{b,m,t}^{at} + \left\{ \sum_{t_0=1}^{t-1} z_{b,m,t_0,t}^{at} \right\} \right) \; - \; ql_{m,t} \leq 0 \tag{6b'}$$

$$\sum_{b=1}^{B} Q_b \times (g_{bs}^r \; - \; Gmin_{m,t}^r) \times \left(y_{b,m,t}^{at} + \left\{ \sum_{t_0=1}^{t-1} z_{b,m,t_0,t}^{at} \right\} \right) \; + \; ql_{m,t,s}^r \geq 0 \tag{7b'}$$

For each $t \in \{1 \ldots T-1\}$:

$$\sum_{b=1}^{B} Q_b \times \left(\sum_{d=1}^{D} y_{b,d,t}^{at} + \left\{ \sum_{p=t+1}^{T} \sum_{m=1}^{M} z_{b,d,t,s}^{at} \right\} \right) \leq M_{max} \tag{8a'}$$

For $t = T$:

$$\sum_{b=1}^{B} Q_b \times \sum_{d=1}^{D} y_{b,d,T}^{at} \leq M_{max} \tag{8b'}$$

For each $b \in \mathcal{B}$, $d \in \mathcal{D}$, $t \in \mathcal{T}$:

$$y_{b,d,t}^{at} \in \{0,1\} \tag{9a'}$$

For each $b \in \mathcal{B}$, $m \in \mathcal{M}$, $t_0 \in \{1 \ldots T-1\}$ and $t_1 \in \{t_0+1, \ldots T\}$.

$$z_{b,m,t_0,t_1}^{at} \in \{0,1\} \tag{9b'}$$

For each $m \in \mathcal{M}$, $s \in \mathcal{S}$ and $t \in \mathcal{T}$:

$$qu_{m,t,s}, ql_{m,t,s} \geq 0 \tag{10a'}$$

For each $m \in \mathcal{M}, t \in \mathcal{T}$, $s \in \mathcal{S}$ and $r \in \mathcal{R}$:

$$qu_{m,t,s}^r, ql_{m,t,s}^r \geq 0 \tag{10b'}$$

The objective function (1') remains the same but the profit generated by the blocks sent from the stockpile to the processing plants is added, and rehandling costs are also subtracted. The reserve constraint (2') considers now the possibility of sending the block to the stockpile. Constraints (4')–(7') have two different expressions to embrace the fact that rehandling material from the stockpile cannot occur in the first period. Similarly, constraints (3') and (8') have two expressions since sending materiel to the stockpile in the last period is impossible. Constraint (9'b) also forces the new variables z_{b,m,t_0,t_1}^{at} to be binary. Constraints (10') remain unchanged.

3   Reformulation

In this section, a reformulation of the model described in Section 2.2 is presented. This step is essential to apply the Bienstock-Zuckerberg algorithm that will be discussed in the next section. Two new binary variables are defined as follows:

- $y_{b,d,t}^{by} = \sum_{p=1}^{t} y_{b,d,p}^{at}$. This variable takes 1 if block b is extracted and sent to destination d by time t (at t or earlier), 0 otherwise.
- $z_{b,m,t_0,t_1}^{by} = \sum_{p=1}^{t_0} z_{b,m,p,t_1}^{at}$. By definition, it takes the value of 1 if block b is extracted and sent to stockpile by time t_0 (at t_0 or earlier) and then sent to processing plant m at time "t_1" with ($t_0 < t_1$), 0 otherwise.

Then, variables y^{at} and z^{at} can be eliminated and replaced respectively by variables y^{by} and z^{by} considering the following system of equalities

$$y_{b,1,1}^{at} = y_{b,1,1}^{by} \qquad\qquad \forall b \in \mathcal{B}$$

$$y_{b,1,t}^{at} = y_{b,1,t}^{by} - z_{b,M,t-1,\ T}^{by} \qquad\qquad \forall b \in \mathcal{B}, t = 2 \ldots T$$

$$y_{b,d,t}^{at} = y_{b,d,t}^{by} - y_{b,d-1,t}^{by} \qquad\qquad \forall b \in \mathcal{B}, d = 2 \ldots D, t = 1 \ldots T$$

$$z_{b,1,t_0,t_0+1}^{at} = z_{b,1,t_0,t_0+1}^{by} - y_{b,D,t_0}^{by} \qquad\qquad \forall b \in \mathcal{B}, t_0 = 1 \ldots T - 1$$

$$z_{b,1,t_0,t_1}^{at} = z_{b,1,t_0,t_1}^{by} - z_{b,M,t_0,t_1-1}^{by} \qquad\qquad \forall b \in \mathcal{B}, t_0 = 2 \ldots T - 1, t_1 = t_0 + 2 \ldots T$$

$$z_{b,m,t_0,t_1}^{at} = z_{b,m,t_0,t_1}^{by} - z_{b,m-1,t_0,t_1}^{by} \qquad\qquad \forall b \in \mathcal{B}, m = 2 \ldots M, t_0 = 1 \ldots T - 1, t_1 = t_0 + 1 \ldots T$$

Using this transformation, an equivalent formulation can be obtained. In what follows and for reasons of brevity, only the reformulations of reserve constraints and slope constraints are shown. The substitution of the objective function and remaining constraints is trivial and will not, therefore, be detailed herein. Also, henceforth, the "by" index will be omitted to simplify notation and y and z will refer to y^{by} and z^{by} respectively,

$$z_{b,M,t-1,T} \leq y_{b,1,t} \qquad\qquad \forall b \in \mathcal{B}, t = 2 \ldots T \tag{11a}$$

$$y_{b,d-1,t} \leq y_{b,d,t} \qquad\qquad \forall b \in \mathcal{B}, d = 2 \ldots D, t = 1 \ldots T \tag{11b}$$

$$y_{b,D,t} \leq z_{b,1,t,t+1} \qquad\qquad \forall b \in \mathcal{B}, t = 1 \ldots T - 1 \tag{11c}$$

$$z_{b,m-1,t_0,t_1} \leq z_{b,m,t_0,t_1} \qquad\qquad \forall b \in \mathcal{B}, m = 2 \ldots M, t_0 = 1 \ldots T - 1, t_1 = t_0 + 1 \ldots T \tag{11d}$$

$$z_{b,M,t_0,t_1-1} \leq z_{b,1,t_0,t_1} \qquad\qquad \forall b \in \mathcal{B}, m = 2 \ldots M, t_0 = 1 \ldots T - 1, t_1 = t_0 + 2 \ldots T \tag{11e}$$

Constraints (11a)–(11e) correspond to reserve constraint (11') in the original formulation. Slope constraints (3') are replaced by constraints (12a) and (12b).

$$z_{b,M,t,T} \leq z_{a,M,t,T} \qquad\qquad \forall (a,b) \in \mathcal{B}^2, a \in \Gamma_b^-, t = 1 \ldots T - 1 \tag{12a}$$

$$y_{b,D,T} \leq y_{a,D,T} \qquad\qquad \forall (a,b) \in \mathcal{B}^2, a \in \Gamma_b^- \tag{12b}$$

Due to the reformulation introduced above, the reserve constraints (11) and the slope constraints (12) can be represented as precedence relationships in a directed graph $G = (\mathcal{V}, \mathcal{A})$ where the set of nodes $\mathcal{V}$ corresponds to the decision variables "y" and "z" and an arc $(a,b) \in \mathcal{A}$ means the value of the decision variable associated with node "a" should be equal or lower than that of node "b". Figure 1 illustrates the different type constraints for a given time of extraction t and a pair of blocks (a,b) where the block b is a predecessor of the block a.

4   Solution approach

The solution approach adopted herein is a three-step method. It is an adaptation of the algorithm presented in Brika et al. (2018) to handle the stockpiling. In what follows, a general description of the methodology is presented. Section 4.1 describes the first step which is an extension of the algorithm introduced in Bienstock and Zuckerberg (2009) to the stochastic optimization. It aims to solve optimally the linear relaxation of the problem. Then, a greedy heuristic illustrated in Section 4.2 is applied to round the fractional solution previously obtained and make it integer. Finally, in Section 4.3, a Tabu search heuristic allows to improve the quality of this new integer solution. Since no major changes are added to the method introduced in Brika et al. (2018), only a brief description of the three algorithms will be presented. For further details, the reader is referred to the latter paper.

4.1   Solving the linear relaxation

The mathematical formulation of the problem described in Section 3 can be illustrated in a more compact form as follows

$$Z = \max p^t y + c^t q$$

$$s.t. \; y_i \leq y_j \qquad\qquad\qquad \forall \, (i,j) \in \mathcal{A} \qquad\qquad (13)$$

$$My + Hq \leq d \qquad\qquad\qquad\qquad (14)$$

$$y \in \{0,1\}^n, q \geq 0 \qquad\qquad\qquad\qquad (15)$$

where "p" represents the vector of the discounted profit, "c" the vector of the discounted unit cost of deviation, "y" both the "y" and the "z" variables for the sake of simplicity and "q" the deviation variables. Constraints (13) represent all the precedence constraints (both slope and reserve constraints), while constraints (14) regroup the different side constraints (blending constraints, mining capacity constraints, etc.). The efficiency of the Bienstock-Zuckerberg algorithm lies on this particular structure. Indeed, once the resource constraints are relaxed, the problem boils down to a maximum closure problem, which can be solved in polynomial time (Hochbaum, 2001). Nevertheless, there is one critical issue remaining: relaxing the side constraints is insufficient since it leads to infeasible solutions. Instead of that, the algorithm uses the Lagrangian relaxation that limits the violation of the relaxed constraints by adding appropriate penalties in the objective function. These penalties are continuously adjusted until the algorithm converges. More formally, at each iteration, a maximum closure problem is solved, and the solution provides a new partition that, when intersected with the previous partition, allows one to model some constraints of the LP as one. Indeed, the algorithm forces the variables in the same group of the partition to be equal, thereby considerably reducing the size of the problem. It can be compared to an aggregation, but the main difference is the fact that individual properties of the blocks over all scenarios are preserved. The solution of the LP, in turn, aims to update the penalties by replacing them with the new dual variables of the side constraints, and so on, until the algorithm converges and an optimal solution is obtained. However, this solution is fractional and needs to be rounded.
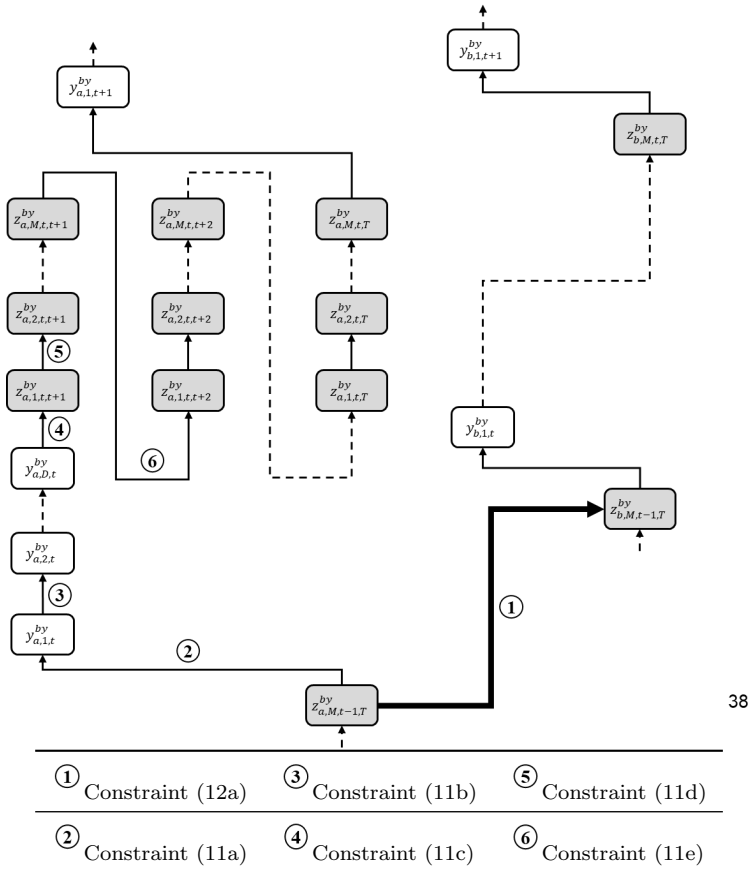

Figure 1: Precedence constraints

4.2   Rounding heuristic

Once a fractional solution is obtained, a rounding heuristic is applied to make it integer. It is a greedy two-step heuristic that takes up ideas from the TopoSort heuristic proposed in Chicoisne et al. (2012) and extends the rounding heuristic introduced in Brika et al. (2018) to handle the stockpiling. The first step consists in sorting the blocks according to a topological ordering with respect to a weight vector $\boldsymbol{w}$. This ordering defines a feasible extraction sequence b_1, b_2, $\ldots, b_n$ where b_i represents a mining block and where a block b_i will appear before b_j in the sequence, if it satisfies either b_i is a predecessor of b_j or $w_i < w_j$. To calculate the weights, a function equivalent to the one proposed by Chicoisne et al. (2012) is adopted. For each $b \in \mathcal{B}$, the weight is estimated as follows

$$w_b = \overline{z}_{b,M,1,T} + \sum_{t=2}^{T-1} t \times \left(\overline{z}_{b,M,t,T} - \overline{z}_{b,M,t-1,T} \right) + T \times \left(\overline{y}_{b,D,T} - \overline{z}_{b,M,T-1,T} \right) + (T+1) \times \left(1 - \overline{y}_{b,D,T} \right),$$

where $\overline{y}$ and $\overline{z}$ are the values of the LP solution obtained by the decomposition method. The weight w_b can be interpreted as the estimated extraction time of the block b. A weight equal to $T + 1$ corresponds to a block that will not be extracted. The second step consists in rounding the fractional solution following the weighted topological ordering. From this point, the heuristic proposed herein is completely different by nature from the one proposed in Chicoisne et al. (2012). The latter one is a constructive heuristic suitable only for a deterministic context where there are only hard constraints that represent upper bounds to satisfy. For its part, the current stochastic model handles soft constraints by allowing their violation and contains both upper and lower bounds for each production target. Also, instead of starting from an empty solution, the algorithm starts from an initial feasible solution obtained by simply rounding the fractional LP solution. The blocks that were completely extracted in the same period are by now fixed. Only the extraction and processing times and the destinations of the remaining blocks can be modified. Then, the algorithm tries to improve the current solution by moving a non-fixed block at once following the topological ordering. The only difference with the algorithm introduced in Brika et al. (2018) is the fact that the option of sending a block to different stockpiles should now be considered.

4.3   Tabu search

The last step of the method consists in applying successively T times a Tabu search to improve the quality of the integer solution obtained by the Rounding Heuristic. Starting from the first period, once iteration t is reached, all blocks scheduled in earlier periods (i.e. $\{ b \in \mathcal{B} | t_0^b < t \}$). are considered fixed, and only the remaining blocks can be rescheduled. A neighborhood is then formed by all the solutions obtained from the current solution by either postponing to $t + 1$ the extraction of one block originally scheduled in period t or pushing forward to t the extraction of one block originally scheduled in period $t + 1$. However, it is important to stress the fact that only candidates that do not violate the precedence constraints are retained.

5   Numerical results

This section presents the numerical experiments achieved to assess the efficiency and robustness of the solution approach introduced in this paper. This method has been tested on six instances with sizes ranging between 4,734 blocks and five periods and 132,672 blocks and 12 periods. For each one of them, ten equiprobable scenarios are used to represent their geological profiles, and thus, illustrate the in-situ variability and metal uncertainty. In what follows, the instances and the different parameters used in the tests are first described. Then, the computational results are provided.

5.1   Instances and parameters

5.1.1   Benchmark instances

Table 1 and Table 2 provide an overview of the instances and the main differences between them. The six instances are grouped into three benchmark datasets. The first set S1 comprises three small to large

size instances from multi-element iron deposits that contain two processing plants each. All must satisfy minimum and maximum expected grades for each geological element. The second set S2 consists of two instances representing two different actual deposits: a copper (Cu) deposit and a gold (Au) deposit. These instances consider two processors: one for the high-grade and another one for the low-grade. As opposed to the first set, only one geological element is considered, and for each processor, there is a minimum expected grade to be satisfied. The third and last set S3 also consists of one medium-size copper deposit. The only difference with the second set is the fact that the instance considers a single processor.

Table 1: Overview of the five instances

Instances		Number of blocks	Number of periods (T) in years	Destinations	Nature of the deposit
	I1	33,168 Block size: 25 x 25 x 2 meters Block weight: 3125 tonnes	5	2 processors, 1 waste dump	Multi-element iron ore deposits: iron content (Fe), silica content (SiO2), alumina content (Al2O3), phosphorus content (P), and the loss on ignition (LOI)
S1	I2	4,734 Block size: 25 x 25 x 12 meters Block weight: 18750 tonnes	5	2 processors, 1 waste dump	
	I3	132,672 Block size: 25 x 25 x 2 meters Block weight: 3125 tonnes	5	2 processors, 1 waste dump	Iron ore deposit considering a contaminant SiO2
S2	C1	22,549 Block size: 20 x 20 x 10 meters Block weight: 10000 tonnes	12	2 processors, 1 waste dump	Copper deposit
	G1	48,821 Block size: 15 x 15 x 10 meters Block weight: 5625 tonnes	10	2 processors, 1 waste dump	Gold deposit
S3	C2	20,626 Block size: 20 x 20 x 10 meters Block weight: 10800 tonnes	10	1 processors, 1 waste dump	Copper deposit

Table 2: Overview of the constraints for each instance

Instances		Mining Capacity	Ore production target		Side constraints		Number of side constraints	Number of arcs in the maximum closure problem
			Lower Bound	Upper Bound	Lower Bound	Upper Bound		
	I1	✓	✓	✓	✓	✓	1025	3,588,082
S1	I2	✓	✓	✓	✓	✓	1025	326,646
	I3	✓		✓	✓	✓	420	12,144,028
S2	C1	✓		✓	✓		276	8,777,675
	G1	✓		✓	✓		230	13,913,619
S3	C2	✓		✓	✓		120	3,591,254

5.1.2 Parameters

The economic parameters used to calculate the objective function coefficients are presented in Table 3. They were discussed a priori with industrial partners. Those used to calculate the penalties for deviating are presented in Table 4 and were chosen after trying several combinations so that there is a balance between a very high selectivity and a permissive one that would affect the products' homogeneity. Furthermore, no cut-off grade is used, therefore the solver is free to consider a block as ore or waste.

5.2 Implementation

All algorithms were coded in C++ and the tests were run on an Intel (R) Core(TM) i5-8250U CPU computer (1.60 GHz) with 8 GB of RAM operating under Windows 10. Recall that, before obtaining an integer solution, the first step consists in applying an extended version of the BZ algorithm (ExtBZ) to solve iteratively a maximum-flow subproblem then a reduced LP. The pseudoflow algorithm of Hochbaum (2008) is used for solving maximum-flow subproblems, and the LP is solved using Cplex 12.7 with the default settings. The second step consists in simply rounding (SR) the fractional solution as described above in Section 4.2 and then using the integer solution obtained as an initial solution for the rounding heuristic (RH). The later heuristic does not have any parameters. Finally, for the Tabu search (TS), two parameters were defined based on preliminary tests. The number of iterations during which a move remains tabu is fixed at 0.6N where N is the number of blocks that can be moved at each iteration. The second parameter is the stop criterion. It represents the maximum number of successive non-improving iterations and was fixed to 0.3N.

Table 3: Economic parameters

		Iron	Copper	Gold
Mining cost		$5/tonne	$1/tonne	$1/tonne
Low-grade processor	Processing cost	$5/tonne	$2.25/tonne	$6/tonne
	Recovery	100 percent	55 percent	45 percent
	Cost of taking ore from the stockpile	$0.50/tonne	$0.45/tonne	$0.45/tonne
High-grade processor	Processing cost	$6/tonne	$9/tonne	$15/tonne
	Recovery	100 percent	90 percent	95 percent
	Cost of taking ore from the stockpile	$0.5/tonne	$0.45/tonne	$0.45/tonne
Metal revenue		$26/tonne for the low-grade and $30/tonne for the high-grade	$1.7/pound	$28.2/gram
Discount rate		10 percent	10 percent	10 percent

Table 4: Costs of deviation from production targets ($/tonne)

	Fe	SiO2	Al2O3	P	LOI	Cu	Au	Ore production
S1	10	20	20	10	1			25
S2 and S3						20	2,000	25 for Copper/100 for Gold deposit

5.3 Numerical results

In this section, the performance of the proposed method is tested on six benchmark instances described in Section 5.1.1. Table 5 provides a summary of the running times and a comparison of $\%Gap$ used to assess the quality of solutions. The measure $\%Gap_{final}$ is the gap calculated with respect to the upper bound provided by ExtBZ: $\%Gap_{final} = \frac{Z_{LR} - Z^*}{Z_{LR}}$ where Z^* and Z_{LR} are respectively the value of the solution provided by the algorithm evaluated and the linear relaxation optimal value obtained by ExtBZ. Another measure $\%Diff$ is used to represent the percent difference between the value of the solution produced by the initial solution used as input and produced by the previous heuristic, and that produced by each improvement heuristic X (SR, RH and TS) to assess their efficiencies.

Results in Table 5 indicate that, for all instances except the largest one, ExtBZ was able to solve the LP relaxation to optimality in a few minutes. For the largest instance S1:I3, after a certain point, the convergence went very slowly, and it took one hour and a half to reach $10^{-5}\%$ of optimality, and then, more than four hours to reach optimality. On the other hand, the time required to run RH after getting the optimal LP relaxation solution is relatively negligible. Despite its greedy nature, the heuristic performs very well (all instances were within $1,7\%$ of optimality) even when the initial solution provided by SRH is very bad. Finally, the running time of TS is large compared to its efficiency to improve the solution provided by RH. This is because evaluating the solution value of all the neighbors using this method is very time-consuming,

especially for the instances that have a long lifetime of mine. The improvement obtained by TS is limited but still merits consideration. Overall, the table shows that, combined, the two heuristics do a very good job of improving the solution obtained by ExtBZ.

Table 5: Summary of the average gaps and running times of the solutions found by Cplex, the Bienstock-Zuckerberg extension and the different heuristics

Instances		ExtBZ		SR			RH			TS		
		Time	Gap(%)	Time	Gap(%)	Diff (%)	Time	Gap(%)	Diff (%)	Time	Gap (%)	Diff (%)
S1	I1	23 mn	0	3 s	45.3	-	6 s	0.6	44.7	12 mn	0.2	0.4
	I2	3 mn 44 s	0	1 s	2.2	-	2 s	0.4	1.8	4 mn 30 s	0.3	0.1
	I3	1h 30 mn*	10^{-5}	1 mn	18.1	-	2 mn	0.3	17.7	45 mn	0.2	0.1
S2	C1	13 mn	0	9 s	12.7	-	8 s	1.6	11.1	15 mn	1.2	0.4
	G1	28 mn 48 s	0	20 s	55.9	-	22 s	4.6	51.3	7 min	1.7	2.9
S3	C2	4 mn 33 s	0	3 s	98.8	-	5 s	13.9	84.9	5 mn 15 s	1.5	12.4

6   Limitation and recommendation

The model proposed considers neither a homogeneous mixing of the material inside a stockpile nor approximations of its grade. However, the current formulation can give rise to questions about its practicality. It allows having several stockpiles in operation at the same period. For example, if T is the lifetime of the mine and M the number of processing plants, the number of stockpiles by the end of period t with $(t < T)$ can reach a maximum of $M \times (T - t)$. Even if some mines can hold a large number of stockpiles simultaneously, it is not necessarily always the case. This section proposes three solutions to address this issue, which can be adopted either jointly or separately.

The first option is appropriate for a context in which the material degradation is important. Indeed, the material within the stockpile undergoes time-dependent changes that affect its properties and decrease its value (Rezakhah and Newman, 2018). To reduce the degradation, the model can be modified so that the duration of stockpiling for each block is limited. Indeed, the variables associated with stockpiles presented in Section 3 have two indexes for the time; one for the extraction time (t_0) and another for the processing time (t_1). Removing variables associated with (t_0, t_1) where $t_1 - t_0 > D_{max}$ implicitly forces the blocks to not be stockpiled longer than D_{max} periods. This solution significantly reduces the size of the problem and, therefore, facilitates its resolution.

The second option would consist of limiting the number of stockpiles in operation per period. For this option, some constraints and variables should be added to the model presented in Section 3. Let N_{max}^t be the maximum number of stockpiles that can be open simultaneously by the end of period t where $(1 \leq t \leq T-1)$ and w_{m,t_0,t_1} be a binary variable that takes 1 if the stockpile corresponding to the material intended to be sent at period t_1 to processing plant m is in operation during period t_0, 0 otherwise. The following constraints are added

$$w_{m,t_0-1,t_1} \leq w_{m,t_0,t_1} \qquad \forall m = 1 \ldots M, t_0 = 1 \ldots T-1, t_1 = t_0 + 1 \ldots T \qquad (16)$$

$$\sum_{m=1}^{M} \sum_{t=t_0+1}^{T} w_{m,t_0,t} \leq N_{max}^{t_0} \qquad \forall t_0 = 1 \ldots T-1 \qquad (17)$$

$$\sum_{b=1}^{B} \tilde{z}_{b,m,t_0,t_1} \leq N \times w_{m,t_0,t_1} \qquad \forall m = 1 \ldots M, t_0 = 1 \ldots T-1, t_1 = t_0 + 1 \ldots T \qquad (18)$$

where N is a big number and

$$\tilde{z}_{b,m,t_0,t_1} = \begin{cases} z_{b,m,t_0,t_1} - z_{b,m-1,t_0,t_1} & \forall b \in \mathcal{B}, m = 2 \ldots M, t_0 = 1 \ldots T-1, t_1 = t_0 + 1 \ldots T \\ z_{b,1,t_0,t_1} - z_{b,M,t_0,t_1-1} & \forall b \in \mathcal{B}, m = 1, t_0 = 2 \ldots T-1, t_1 = t_0 + 2 \ldots T \\ z_{b,1,t_0,t_0+1} - y_{b,D,t_0} & \forall b \in \mathcal{B}, m = 1, t_0 = 1 \ldots T-1, t_1 = t_0 + 1 \end{cases}$$

The constraints (16) can be interpreted as precedence constraints and constraints (17) and (18) as side constraints. There will be $(T-1)$ constraints (17) and $M \times \sum_{t=1}^{T-1} (T-t) = M \times \frac{(T-1) \times T}{2}$ constraints (18). However, if T is big, adopting this option is no more interesting since the side constraints' number can quickly become too large and question the proposed method's usefulness.

The third option would consist of limiting to N_{max} the number of the stockpiles that have been opened during the lifetime of the mine no matter if they were in operation simultaneously or not. For that, new binary variables $w_{m,t}$ are introduced. They take 1 if the stockpile of material intended to be sent to processing plant m at period t has been opened, 0 otherwise. The following constraints are also added

$$\sum_{m=1}^{M} \sum_{t=2}^{T} w_{m,t} \leq N_{max} \tag{19}$$

$$\sum_{b=1}^{B} \sum_{t_0=1}^{t-1} \tilde{z}_{b,m,t_0,t} \leq N \times w_{m,t} \qquad \forall m = 1 \ldots M, t = 2 \ldots T \tag{20}$$

Unlike the second option, the number of the additional side constraints remains limited. There will be only one constraint of type (19) and $M \times (T-1)$ constraints of type (20).

7 Conclusions

The paper proposes a new method for integrating stockpiles into strategic mine planning. This approach is completely different from those proposed in the literature. It has the advantage of maintaining the program linear. Moreover, the proposed modeling allows to preserve the structure of a precedence graph, which makes the use of the BZ algorithm possible. By introducing new variables, the formulation proposed bypasses the unrealistic assumption of homogeneous mixing of the material in a single stockpile in each period, and the non-linearity of the most precise models thus far in the literature. A solution approach has also been presented, it consists in applying first an adaptation of the Bienstock-Zuckerberg (BZ) algorithm to obtain an optimal solution of the linear relaxation, and then applying successively a greedy rounding heuristic (RH) and a Tabu search (TS). The method is an extension of the one recently proposed in Brika et al. (2018) for another variant of the problem, without stockpiling. Numerical results show that the proposed method managed to solve all the instances within 1,7% of optimality in a reasonable time and in a notably shorter period required by CPLEX to solve only the linear relaxation of the problem. Recommendations were provided to reduce further the running times and respond to some limitations of the model. An interesting step for future research would involve integrating some preprocessing techniques to fix some variables and using stabilization methods to accelerate the convergence of the BZ algorithm, thereby reducing significantly the running time.

References

Akaike, A., Dagdelen, K., A strategic production scheduling method for an open pit mine. Proceedings, 28th International APCOM symposium; Computer applications in the minerals industries; Golden, CO, 729–738 (1999).

Albor, F., R. Dimitrakopoulos, R., Stochastic mine design optimization based on simulated annealing: Pit limits, production schedules, multiple orebody scenarios and sensitivity analysis, Transactions of the Institution of Mining and Metallurgy, Section A: Mining Technology, 118(2), 79–90. https://doi.org/10.1179/037178409X12541250836860 (2009).

Albor, F., Dimitrakopoulos, R., 2010. Algorithmic approach to pushback design based on stochastic programming: method, application and comparisons. IMM Transactions, Mining Technology 119(2), 88–101 (2010).

Baker, C.K., Giacomo, S.M., Resource and reserves: their uses and abuses by the equity markets. Ore reserves and Finance: A Joint Seminar between Australasian Institute of Mining and Metallurgy and ASX, Sydney (1998).

Benndorf, J., Dimitrakopoulos, R., Stochastic long-term production scheduling of iron ore deposits: integrating joint multi-element geological uncertainty. Journal of Mining Science, 49(1), 68–81. https://doi.org/10.1134/S1062739149010097 (2013).

Bienstock, D., Zuckerberg, M., A new LP algorithm for precedence constrained production scheduling. Optimization Online (2009).

Bienstock, D., Zuckerberg, M., Solving LP relaxations of large-scale precedence constrained problems. Proceedings from the 14th conference on Integer Programming and Combinatorial Optimization (IPCO). Lecture Notes in Computer Science 6080, 1–14 (2010). https://doi.org/10.1007/978-3-642-13036-6_1.

Birge, J.R., Louveaux, F., Introduction to Stochastic Programming. Springer (1997).

Bley, A., Boland, N., Froyland, G., and Zuckerberg, M., Solving mixed integer nonlinear programming problems for mine production planning with stockpiling. Journal of Mining Science, 47(3), 338-359 (2012). http://www.optimization-online.org/DB_HTML/2012/11/3674.html.

Boland, N., Dumitrescu, I., Froyland, G., Gleixner, A., LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. Computers and Operations Research, 36(4), 1064–1089 (2009).

Boucher, A., Dimitrakopoulos, R., Block simulation of multiple correlated variables. Mathematical Geosciences 41(2), 215-237 (2009).

Brika, Z., Gamache, M., Dimitrakopoulos, R., Multi-Product Mine Scheduling Optimization under Multi-Element Geological Uncertainty. Cahier du GERAD. G–2018–72 (2018).

Caccetta, L., Hill, S. P., An application of branch and cut to open pit mine scheduling. Journal of Global Optimization, 27(2–3), 349–365 (2003).

Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., Rubio, E., A new algorithm for the open-pit mine production scheduling problem. Operations Research, 60(3), 517–528 (2012).

Cullenbine, C., Wood, R.K., Newman, A., A sliding time window heuristic for open pit mine block sequencing. Optim Lett, 5(3), 365–377. http://dx.doi.org/10.1007/s11590-011-0306-2 (2011).

Dagdelen, K., Johnson, T.B., Optimum open pit mine production scheduling by lagrangian parameterization. Proceedings of 19th International APCOM Symposium, Littleton, CO, 127–142 (1986).

Denby, B., Schofield, D., Open-pit design and scheduling by use of genetic algorithms. Transactions of the Institutions of Mining and Metallurgy l. 103, A21–A26 (1994)

Dimitrakopoulos, R., Stochastic optimization for strategic mine planning: A decade of developments. Journal of Mining Science, 84, 138–150 (2011).

Dimitrakopoulos, R., Farrelly, C. T., Godoy, M. C., Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design, Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology, 111 (2002).

Dimitrakopoulos, R., Ramazan, S., Stochastic integer programming for optimizing long-term production schedules of open pit mines: methods, application and value of stochastic solutions. Min. Technol. 117(4), 155–160 (2008). https://doi.org/10.1179/174328609X417279.

Dowd, P., Risk assessment in reserve estimation and open-pit planning. Transactions of the Institution of Mining and Metallurgy (Section A: Mining Industry), 103, A148–A154 (1994).

Dowd, P., Risk in minerals projects: analysis, perception and management. Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology, 106 (1997).

Godoy, M., The effective management of geological risk. Ph.D. thesis, University of Queensland, Australia (2003).

Godoy, M., Dimitrakopoulos, R., Managing risk and waste mining in long-term production scheduling. SME Transactions. 316, 43–50 (2004).

Goodfellow, R., Dimitrakopoulos, R., Global optimization of open pit mining complexes with uncertainty. Applied Soft Computing. 40, 292–304 (2016). https://doi.org/10.1016/j.asoc.2015.11.038.

Goodfellow, R., Dimitrakopoulos, R., Simultaneous stochastic optimization of mining complexes and mineral value chains. Mathematical Geosciences, 49(3), 341–360 (2017).

Hochbaum, D., A new-old algorithm for minimum-cut and maximum-flow in closure graphs. Networks, 37(4), 171–193, http://dx.doi.org/10.1002/net.1012,~MR~1837196 (2001).

Hochbaum, D., The pseudoflow algorithm: a new algorithm for the maximum flow problem. Operations Research 58, 992–1009 (2008)

Hoerger, S., Seymour, F., Hoffman, L., Mine planning at Newmont's Nevada operations. Mining Engineering, 51(10), 26–30 (1999).

Johnson, T.B., Optimum open pit mine production scheduling. Ph.D. thesis, University of California, Berkeley (1968).

Lamghari, A., Dimitrakopoulos, R., A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. European Journal of Operational Research, 222(3), 642–652 (2012).

Lamghari, A., Dimitrakopoulos, R., Ferland, J., A Hybrid Method Based on Linear Programming and Variable Neighborhood Descent for Scheduling Production in Open-Pit Mines. Journal of Global Optimization 63(3): 555–582 (2015). http://dx.doi.org/10.1007/s10898-014-0185-z.

Lamghari, A., Dimitrakopoulos, R., Progressive hedging applied as a metaheuristic to schedule production in open-pit mines accounting for reserve uncertainty. European Journal of Operational Research, 253, 843–85 (2016).

Marcotte, D., Caron, J., Ultimate open pit stochastic optimization. Computers & Geosciences, 51, 238–246 (2013).

Menabde, M., Froyland, G., Stone, P., Yeates, G., Mining schedule optimization for conditionally simulated orebodies, in: R. Dimitrakopoulos (Ed.), Orebody Modelling and Strategic Mine Planning: Uncertainty and Risk Management Models, AusIMM Spectrum Series, 2007, 379–384 (2007).

MineMax. MineMax planner. Retrieved from: http://www.minemax.com/. Visited in: 07.09.18 (2016).

Mintec, Inc. Minesight. Retrieved from: http://www.minesight.com/. Visited in 05.09.18 (2013).

Montiel, L., Dimitrakopoulos, R., Optimizing mining complexes with multiple processing and transportation alternatives: An uncertainty-based approach. European Journal of Operational Research, 247(1), 166–178 (2015). https://doi.org/10.1016/j.ejor.2015.05.002.

Moreno, E., Rezakhah, M., Newman, A., Ferreira, F., Linear Models for Stockpiling in Open-pit Mine Production Scheduling Problems. European Journal of Operational Research, 260(1), 212–221 (2017). http://dx.doi.org/10.1016/j.ejor.2016.12.014.

Munoz, G., Espinoza, D., Goycoolea, M., Moreno, E., Queyranne, M., Rivera, O., A study of the Bienstock-Zuckerberg algorithm. Applications in Mining and Resource-Constrained Project Scheduling (2017).

Ramazan, S., A new pushback design algorithm in open pit mining. MSc thesis, Department of Mining Engineering, Colorado School of Mines, Golden, Colorado, USA (1996).

Ramazan, S., The new fundamental tree algorithm for production scheduling of open pit mines. European Journal of Operational Research, 177(2), 1153–1166 (2007).

Ramazan, S., Dimitrakopoulos, R., Recent applications of operations research in open pit mining. Trans. Soc. Min. Metall. Explor. 316, 73–78 (2004).

Ramazan, S., Dimitrakopoulos, R., Production scheduling with uncertain supply: a new solution to the open pit mining problem. Optimization and Engineering, 14(2), 361–380 (2013). http://dx.doi.org/10.1007/s11081-012-9186-2.

Rezakhah, M., Newman, A., Open pit mine planning with degradation due to stockpiling, Computers and Operations Research. https://doi.org/10.1016/j.cor.2018.11.009 (2018).

Shishvan, M.S., Sattarvand, J., Long-term production planning of open pit mines by ant colony optimization. European Journal of Operational Research, 240(3), 825–836 (2015).

Smith, M. L., Optimizing inventory stockpiles and mine production: an application of separable and goal programming to phosphate mining using AMPL/CPLEX. CIM Bulletin, 92 (1030), 61–64 (1999).

Smith, M. L., Wicks, S. J., Medium-term production scheduling of the Lumwana mining complex. Interfaces, Special Issue: Operations Research in Mining, 44(2), 176–194 (2014).

Tabesh, M., Askari-Nasab, H., Two-stage clustering algorithm for block aggregation in open pit mines. Mining Technology, 120, 158–169 (2011).

Tabesh, M., Askari-Nasab, H., Peroni, R., A comprehensive approach to strategic open pit mine planning with stockpile consideration. In Proceedings of the Thirty-seventh international symposium on applications of computers and operations research in mineral industry, 326–332. Society for Mining, Metallurgy and Exploration (2015).

Tachefine, B., Méthode d'optimisation pour la planification de la production dans une mine à ciel ouvert. Ph.D. thesis, Ecole polytechnique de Montréal, QC (1997).

Topal, E., Advanced underground mine scheduling using mixed integer programming. Ph.D. thesis, Colorado School of Mines, Golden, CO (2003).

Whittle, J., Beyond optimization in open pit design. Proc Canadian Conf on Computer Applications in the Mineral Industries, Rotterdam, pp 331–337 (1988).

Whittle Programming. Gemcom Whittle User Help v4.4, Section: Stockpiles – General discussion (2010).

Yarmuch, J., Ortiz, J., A novel approach to estimate the gap between the middle- and short-term plans. In Proceedings, 35th APCOM symposium 2011 - Application of Computers and Operations Research in the Minerals Industry, 24–30 September 2011, University of Wollongong, WA, Australia. (2011).