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Expanding simultaneous stochastic optimization of mining complexes to introduce geometallurgical constraints: Application at the Escondida mining complex, Chile

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Abstract: A mining complex is a mineral value chain that consists of multiple activities, starting from material extraction to a set of sellable products delivered to various customers and/or spot markets. Simultaneous stochastic optimization of mining complexes aims to simultaneously optimize mining, destination, blending, stockpiling, processing, transportation and logistic decisions for multiple mines to maximize the value of integrated mining business and minimize deviations from production targets. This paper expands simultaneous stochastic optimization of mining complexes to include geometallurgical constraints and performs a simultaneous optimization of extraction sequence, destination policy, and processing stream utilization decisions. Geometallurgical properties, such as SAG power index (SPI) and bond work index (BWI) influence the throughput, recovery and energy consumption of comminution circuits. Integrating geometallurgical constraints in the simultaneous stochastic optimization model helps to achieve higher and more stable throughput of comminution circuits. A multiple neighborhood simulated annealing algorithm is used to solve the large combinatorial optimization model. The performance of the model is tested at a multiple pit copper-gold mining complex (Escondida) and is compared to the conventional mine plan. Results indicate capabilities of the model to generate risk-resilient mine designs with improved expected NPV (cumulative discounted cash flows) by 267% and reduced risk of deviation from production targets, as compared to the conventional plan, which highlights the importance of the simultaneous stochastic optimization model of mining complexes to generate higher value with less risk for the integrated mining business.

Keywords: Escondida mining complex, simultaneous stochastic optimization, geo-metallurgy

1 Introduction

A mining complex is an integrated business (mineral value chain) starting from the extraction of material from multiple mines (open pit, underground), then to the blending of extracted material with multiple processing streams (stockpiles, crushers etc.), the processing of blended material using different processing paths (leachpads, mills, etc.) and alternatives (fine grain, medium grain, coarse grain, etc.), the transportation of final sellable products to the port/final stocks using one or multiple transportation and logistic schemes (trucks, pipes), and, finally, the marketing and sale of the final products (Pimentel et al., 2010; Montiel and Dimitrakopoulos, 2015, 2017; Goodfellow and Dimitrakopoulos, 2016, 2017). For instance, the Escondida multiple pit copper-gold mining complex in Chile consists of two mines (Escondida and Escondida Norte), and utilizes blending from/to multiple processing streams, multiple processing paths and alternatives, and multiple transportation strategies to generate multiple products (copper concentrate, copper cathode, gold concentrate, silver concentrate, molybdenum concentrate) for different customers.

Hoerger et al. (1999) introduces the concept of joint optimization of different components of a Newmont's Nevada operations. The model considers fixed long-term schedules and optimizes aspects of stockpiling and blending decisions. The model ignores supply/geological/grade uncertainty, a major source of technical risk in mine planning (Ravenscroft, 1992; Dowd, 1994; Dimitrakopoulos et al., 2002; Godoy, 2002). Stone et al. (2005) present BHP Billiton's advanced mine planning optimization software tool, "Blasor". Methods implemented optimizes long-term plans for both single mines and groups of multiple pits with blended-ore operations to maximize net present value. In the context of simultaneous optimization Blasor optimizes the proportion of material extracted annually from multiple pits, rather than generating an extraction sequence for multiple pits, while it aggregates mining blocks into aggregation units, and ignores supply uncertainty of mineral deposits. The stochastic version of Blasor (Menabde et al., 2005) has similar limitations, however, considers supply uncertainty. Whittle (2007) proposes a model to sequentially optimize several components of a mining complex and termed it global asset optimization. However, the model uses a sequential optimization approach, which first defines the extraction sequence of panels (aggregation of mining blocks), and then locally optimizes the blending and processing stream utilization strategies. Although the model considers multiple components of the mining complex, it does not optimize them simultaneously. Whittle (2010) presents the ProberB and ProberC algorithms to solve the global asset optimization model. The algorithm repeatedly creates a random feasible solution (a solution that satisfies mining constraints) and, then, finds the nearest local maximum using linear optimization software. Other limitations of the above global asset optimization include: (i) the aggregation of mining blocks into panels, (ii) the exclusion of all components of mining complexes during optimization, and (iii) the disregard of supply uncertainty of mineral deposits.

Simultaneous stochastic optimization of mining complexes is based on stochastic integer programming or SIP techniques (Birge and Louveaux, 2011) and past SIP models, for mine production scheduling that considers supply uncertainty (Dimitrakopoulos and Ramazan, 2004; Ramazan and Dimitrakopoulos, 2005; Ramazan and Dimitrakopoulos, 2013) and market uncertainty (Zhang and Dimitrakopoulos, 2017), motivated from (Ravenscroft, 1992; Dowd, 1994). Past models are limited to (i) single mine-to-destination configurations, rather than mineral value chain, and (ii) are based on the prior definition of ore and waste determined using economic value of blocks (please see review in Dimitrakopoulos, 2011).

Mineral value chain optimization models are large combinatorial optimization models that require substantial amounts of time to generate solutions with general purpose commercial solvers such as CPLEX. Metaheuristic algorithms based on tabu search (Lamghari and Dimitrakopoulos, 2012), variable neighborhood descent (Lamghari et al., 2014; 2015), progressive hedging (Lamghari and Dimitrakopoulos, 2016), simulated annealing (Godoy, 2002; Goodfellow and Dimitrakopoulos, 2016; Montiel and Dimitrakopoulos, 2015; 2017) have proven efficient for such large mine planning optimization models.

Pimentel et al., (2010) presents critical intricacies of an iron ore mining complex and highlights the importance of optimizing the mineral value chain with downstream (distribution planning) and upstream operations (production planning) simultaneously. Montiel and Dimitrakopoulos (2015) propose a model that incorporates multiple processing alternatives, multiple transportation alternatives, extraction sequence, processing stream utilization and destination policy decisions in the simultaneous stochastic optimization

of mining complexes under supply uncertainty. Results from a multi-pit copper operation with stringent blending requirements indicate that conventional plans generated using commercial mine planning software (Whittle, 2007) performs adversely when tested over simulated orebody representations of mineral deposits. Large and impractical deviation in terms of capacity (40% and 30%) and blending targets (11% and 22%) were observed. In the given case study, the simultaneous stochastic optimization model minimizes such large deviations from capacity targets to 1% and 3% and blending targets to 0.7% and 1.2% for two mills, while improving the value of the business to 5%, as compared to the conventional mine plan. Montiel and Dimitrakopoulos (2017) present an application of long-term simultaneous stochastic optimization of mining complexes that simultaneously optimizes the extraction sequence, destination policies, utilization of processing streams, and processing alternatives decisions. A case study at a copper mining operation indicates a large and impractical deviation in the capacity target (45%) when the conventional mine plan is tested over simulated mineral deposits. The proposed method minimizes the deviation to less than 1%, while increasing the value of the business to 30%, as compared to the conventional mine plan.

Goodfellow and Dimitrakopoulos (2016) propose a generalized model for long-term simultaneous stochastic optimization of mining complexes that integrates extraction sequence, cluster-based destination policies, and processing streams utilization decisions. The proposed model is general to be easily adapted to the simultaneous stochastic optimization of any mining complex. Materials are classified into scenario-independent clusters based on multivariate characteristics (e.g., copper grade, gold grade, silver grade) and destinations of such clusters are defined as scenario-dependent decisions. The concept of primary (metal tonnage, rock tonnage, cycle times, etc.) and hereditary attributes (grade, power consumption, ratios, etc.) is also introduced to model the flow of material in a mining complex. Results from a copper-gold operation indicate large deviations from capacity targets of sulphide leach pad and sulphide mill (40% and 31%) when a conventional mine plan is tested over simulated mineral deposits. In the given case study, simultaneous stochastic optimization of mining complexes better manages the technical risk associated with the supply of material and reduces the average deviation in capacity target for sulphide leach pad to 10% and sulphide mill to less than 1% in the initial 10 years to 12% near the end of life-of-mine, while maximizing the cumulative discounted value of the business by 22.6% over the conventional plan. Goodfellow and Dimitrakopoulos (2017) present an application of the model in Goodfellow and Dimitrakopoulos (2016) and outline the efficiency of the method at a nickel-laterite mining complex. The method simultaneously optimizes the cluster-based destination policies and processing stream utilization decisions and achieves a 3% higher value with an average deviation of less than 1% from capacity and blending targets, as compared to results from deterministic optimization on estimated mineral deposits using the proposed method.

Earlier models for optimization of mining complexes considers supply and material type uncertainty, however, this work incorporates geo-metallurgical constraints in the simultaneous stochastic optimization of mining complexes under supply and material type uncertainty. Critical intricacies of the influence of geo-metallurgical properties in mine planning are discussed in Coward et al. (2009), Dunham et al. (2011), Macfarlane and Williams (2014). Geo-metallurgical properties such as SAG power index (SPI) and bond work index (BWI) determines the residence time of the material in comminution circuits. For instance, hard materials with a high strength index value tend to have higher crushing and grinding times. Also, blending of material with different geo-metallurgical characteristics can introduce problems such as inconsistent throughputs, metal recovery, and energy consumption. Processing alternatives with comminution circuits determine coarse- or fine-grinding of material and is important for a mining operation. Inconsistent geo-metallurgical properties of the material fed to processing plants influence the processing alternative decision, resulting in higher energy consumption with lower throughputs. Earlier models for simultaneous stochastic optimization of mining complexes do not consider geo-metallurgical uncertainty and is accounted for in the proposed simultaneous stochastic optimization model for mining complexes. In the following sections, a detailed description of the method is provided followed by a description and demonstration of the performance of the method at the Escondida mining complex. Conclusions and future work follow.

2 Method

2.1 Notation

In a mining complex, material from mines is first extracted, and then processed through a set of processing and stockpiling facilities, and finally, sellable products are sent to ports using various transportation and logistics schemes. The value chain model is defined using a set of nodes ($\mathcal{N} = \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}$) (see Goodfellow and Dimitrakopoulos, 2016, 2017), which consists of clusters of blocks with similar characteristics (\mathcal{C}), destinations that can stockpile material over time (\mathcal{S}), and destinations that transform material without stockpiling (\mathcal{P}). The primary set (\mathbb{P}) is comprised of fundamental additive properties of interest, such as metal tonnage and total tonnage, which flows through the mining complex, while the hereditary set (\mathbb{H}) is comprised of non-additive properties, such as head grade calculated from primary properties. Supply uncertainty of material is captured with a set of stochastic simulations of mineral deposits (\mathbb{S}). For instance, fifteen stochastic simulations for two mineral deposits will result in two hundred twenty-five joint uncertainty scenarios. Joint uncertainty scenarios will be herein referred to as scenarios.

Multiple mines (\mathbb{M}) discretized into mining blocks ($b \in \mathbb{B}_m$) with simulated material types and attributes ($\beta_{p,b,s}$) $\forall p \in \mathbb{P}, s \in \mathbb{S}$, grouped into clusters for each material type using k-means++ clustering algorithm, provide bulk material to the mining complex. Mine extraction sequence variable ($x_{b,t} \in \{0, 1\}$) defines whether (1) or not (0) block $b \in \mathbb{B}_m$ is extracted in period $t \in \mathbb{T}$ (\mathbb{T} , being the set of periods over which the blocks are being scheduled). A block b is accessible to extract after the extraction of its overlying blocks, O_b . Destination policy variables $z_{c,j,t} \in \{0, 1\}$ define whether (1) or not (0) cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period $t \in \mathbb{T}$, where by $\mathcal{O}(c)$ represents the set of destinations that cluster (material) $c \in \mathcal{C}$ can be sent. Membership of blocks to different clusters is defined using the parameters $\theta_{b,c,s} \in \{0, 1\}$, which represent whether (1) or not (0) block $b \in \mathbb{B}_m$ is a member of cluster $c \in \mathcal{C}$ in scenario $s \in \mathbb{S}$. Clusters are defined as scenario-independent parameters, while block membership is a scenario-dependent parameter. The processing stream utilization variables, $y_{i,j,t,s} \in [0, 1]$, define the proportion of material sent from location (i) to subsequent destination ($j \in \mathcal{O}(i)$) in period t and scenario s (for instance, amount of material to be stockpiled, reclaimed from stockpile). Processing stream utilization decisions are second-stage decisions and are adaptive to uncertainty, meaning that after the material is sent to a destination and uncertainty is revealed, the mining complex can adapt appropriately. Different components of a mining complex have different production targets. $d_{h,i,t,s}^+$ and $d_{h,i,t,s}^-$ variables calculates the surplus and shortage deviations from such production targets for different components of a mining complex over different periods and scenarios under supply uncertainty of material. Material with additive properties represented by $v_{p,i,t,s}$ extracted from multiple mines, flows through the mining complex and is finally processed at processing mill, that recover the different metal ($v_{h,i,t,s} = r_{p,i,t,s} \cdot v_{p,i,t,s}$) with different recovery factor ($r_{p,i,t,s}$). The value of generated products in a mining complex is calculated by multiplying the price ($p_{h,i,t}$) of different metals with the recovered metal quantity ($v_{h,i,t,s}$).

2.2 Optimization model

2.2.1 Objective function

$$\begin{aligned}
 & \max \underbrace{\frac{1}{\|\mathbb{S}\|} \sum_{i \in \text{SUPUM}} \sum_{t \in \mathbb{T}} \sum_{s \in \mathbb{S}} \sum_{h \in \mathbb{H}} p_{h,i,t} \cdot v_{h,i,t,s}}_{\text{Part I}} \\
 & - \underbrace{\frac{1}{\|\mathbb{S}\|} \sum_{i \in \text{SUPUM}} \sum_{t \in \mathbb{T}} \sum_{s \in \mathbb{S}} \sum_{h \in \mathbb{H}} c_{h,i,t}^+ \cdot d_{h,i,t,s}^+ + c_{h,i,t}^- \cdot d_{h,i,t,s}^-}_{\text{Part II}} - \underbrace{\sum_{t \in \mathbb{T}} \sum_{m \in \mathbb{M}} \sum_{b \in \mathbb{B}_m} c_t^{\text{smooth}} \cdot \omega_{b,t}}_{\text{Part III}} \quad (1)
 \end{aligned}$$

$$p_{h,i,t} = \frac{p_{h,i,1}}{(1+d)^t} \quad (2)$$

$$c_{h,i,t}^+ = \frac{c_{h,i,1}^+}{(1+rd)^t} \quad (3)$$

The optimization model simultaneously optimizes the extraction sequence, destination policies and processing stream utilization decisions for mining complexes under supply and material type uncertainty. The objective function for the model is a two-stage SIP maximization function given by Equation 1 and is a modified model of Goodfellow and Dimitrakopoulos, (2016). Extraction sequencing and destination policies are first-stage decisions and are robust to fluctuations arising from supply and material type uncertainty. Processing stream utilization decisions and deviations from production targets are second-stage or recourse decisions to adapt first-stage decisions. Part I in the objective function represents the discounted profits of products generated in a mining complex.

Discounted profits are calculated using Equation 2. Recourse decisions $d_{h,i,t,s}^+$ and $d_{h,i,t,s}^-$ are penalized using discounted penalty costs $c_{h,i,t}^+$ and $c_{h,i,t}^-$, respectively, for surplus and shortage for different production targets and is shown in Part II. Penalty costs are discounted (Equation 3) using a geological risk discount rate (Dimitrakopoulos and Ramazan, 2004), to defer the risk of not meeting production targets to later years when more information is available. Part III motivated from past work (Benndorf and Dimitrakopoulos 2013) ensures that the extraction sequence is practical considering equipment access and movement restrictions. Variable $w_{b,t}$, calculates the number of mining blocks scheduled in multiple extraction periods within the required mining width and penalizes such decision with an associated discounted penalty cost c_t^{smooth} . This ensures the ability to mine of the generated extraction sequence considering equipment movement instead of schedule, which is not mineable from operational perspective.

2.2.2 Constraints

$$v_{h,i,t,s} - d_{h,i,t,s}^+ \leq U_{h,i,t} \quad \forall h \in \frac{\text{Hard material}}{\text{Soft material}}, i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \quad (4)$$

$$v_{h,i,t,s} + d_{h,i,t,s}^- \geq L_{h,i,t} \quad \forall h \in \frac{\text{Hard material}}{\text{Soft material}}, i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \quad (5)$$

$$|\text{Neigh}(b)| \cdot x_{b,t} - \sum_{n \in \text{Neigh}(b)} \sum_{t' \in t} x_{n,t'} \leq w_{b,t} \quad (6)$$

Geo-metallurgical properties such as SPI and BWI introduce the problem of inconsistent throughput, recovery, and energy consumption in comminution circuits. The blending of material with different strength properties, which are non-additive, further affects processing alternatives of comminution circuits. The modelling of geo-metallurgical properties is achieved through characterizing materials as soft and hard based on their strength properties and controlling proportion of such material types. Thus, the tonnage of hard and soft material (additive property) flows through the mining complex and the ratio of such material is calculated and controlled at the different comminution circuits. Equations 4 and 5 ensure the proportion of hard and soft material is controlled within upper ($U_{h,i,t}$) and lower ($L_{h,i,t}$) targets respectively with allowed surplus ($d_{h,i,t,s}^+$) and shortage ($d_{h,i,t,s}^-$) deviations over different scenarios and periods at different comminution circuits.

Other constraints implemented in the model are capacity/quantity, blending/quality, destination policy, processing stream flow, reserve, slope, and mining width constraints that were adapted from the general model presented in Goodfellow and Dimitrakopoulos (2016). Capacity constraints such as metal tonnage and total tonnage are present with different components of the mining complex. Similarly, constraints on quality of material are present in a mining complex, which ensures maximum efficiency with operations and compliance with environmental standards. For instance, high Arsenic content could lead to environmental

problems; high Iron content might reduce recovery of copper, etc. Destination policy constraints ensure that the cluster of material is sent to only one destination over all scenarios in a scheduling period. The processing stream flow constraint ensures that material reclaimed from a processing stream must be equal to material available at a processing stream considering whether stockpiling is available or not. The reserve constraint states that a block can only be mined once over the scheduling horizon. The slope constraint ensures that a block can only be extracted if the blocks overlying it have been extracted first. The mining width constraint motivated from (Dimitrakopoulos and Ramazan, 2004; Benndorf and Dimitrakopoulos, 2013) calculates the number of blocks scheduled in multiple periods ($w_{b,t}$) inside a specified mining width ($|\text{Neigh}(b)|$) and is given by Equation 6. $w_{b,t}$ is penalized in the objective function (Equation 1) with a cost of c_t^{smooth} and, therefore, ensures that the blocks inside the mining width are extracted in the same scheduling period.

2.3 Solution approach

Simultaneous stochastic optimization of mining complexes is a large combinatorial optimization model with millions of binary decision variables. The simulated annealing algorithm (Metropolis et al., 1953; Kirkpatrick et al., 1983) with adaptive neighbourhood search (Ropke and Pisinger, 2006; Ribeiro and Laporte, 2012) of multiple neighbourhoods has shown an excellent performance in solving large stochastic optimization models of mining complexes (Goodfellow and Dimitrakopoulos, 2016) and is, therefore, used to solve the optimization model outlined in Section 2.2.

$$P(g(\Phi), g(\Phi'), Temp) = \begin{cases} 1, & \text{if } g(\Phi) \leq g(\Phi') \\ \exp\left(-\frac{|g(\Phi') - g(\Phi)|}{Temp}\right), & \text{otherwise} \end{cases} \quad (7)$$

Let Φ be the current solution with objective value of $g(\Phi)$. The algorithm starts by selecting one of three different neighborhoods: (i) which modifies the extraction sequence ($x \in \Phi$), (ii) which modifies the destination policy ($z \in \Phi$) and (iii) which modifies the processing stream utilization decision ($y \in \Phi$). An initial solution is generated using a greedy algorithm and then the solution is perturbed or modified selecting one of the neighborhoods among the three neighborhoods. The selection of the neighborhood is done as follows. Each neighborhood is given weights, which are initially equal. Using these weights, a neighborhood is selected out of the three neighborhoods. The selected neighborhood is used to generate a new solution Φ' . Rather than using a single temperature, as in a normal simulated annealing algorithm (Equation 7), a parameter $\delta \in [0, 1]$, which represents the probability of accepting a solution is used and the correct temperature for each neighborhood is derived from its cumulative probability distribution. The probability of accepting the new solution Φ' is based on the distribution given in Equation 7 using the correct temperature for different neighborhoods. "temp" is the annealing temperature which controls the acceptance of unfavorable solutions and is decreased gradually after certain number of iteration using a cooling factor as the algorithm progresses. For instance, higher the temperature higher is the probability of accepting unfavorable solution. Suppose that Φ' is better or non-improving than the current solution, then Φ' is accepted and the weights associated with acceptance are increased by $|g(\Phi') - g(\Phi)|$ or $\frac{1}{|g(\Phi') - g(\Phi)|}$ respectively. If not, then Φ' is rejected and the weights associated with rejection are increased by $\frac{1}{|g(\Phi') - g(\Phi)|}$. Diversification strategy is used to restart the algorithm with best known solution and weights and helps to explore the solution space. The algorithm stops when the stopping criterion is met.

3 Application at the Escondida Mining Complex

The performance of the simultaneous stochastic optimization model is tested at an actual highly-mechanized porphyry copper-gold mining complex (Escondida Mining Complex) located 170 km south-east of Antofagasta, Chile.

3.1 Overview of the operations

The Escondida mining complex consists of two mines named Escondida and Escondida Norte with 2.32 and 1.84 million blocks, respectively, of size $25 \times 25 \times 15 \text{m}^3$. The deposits consist of 8 different mine zones, 8 different

alterations and 10 lithologies each for the two mines. There are over 50,000 composite samples in each mine of length 15m each. The stratigraphic sequence of material is waste rock, followed by copper oxide, and mixed and copper sulphide. The mining complex produces copper concentrate and copper cathode as primary products and gold, silver and molybdenum concentrates as secondary products. The material extracted from both pits is classified into 11 different material types for each mine and can be sent to one of 9 different destinations (5 crushers, 2 stockpiles, 1 bio leach pad and 1 waste dump) as shown in Figure 1. Material from such destinations is further sent to processing mills and an oxide leach pad that supplies material to the port and to a cathode plant.

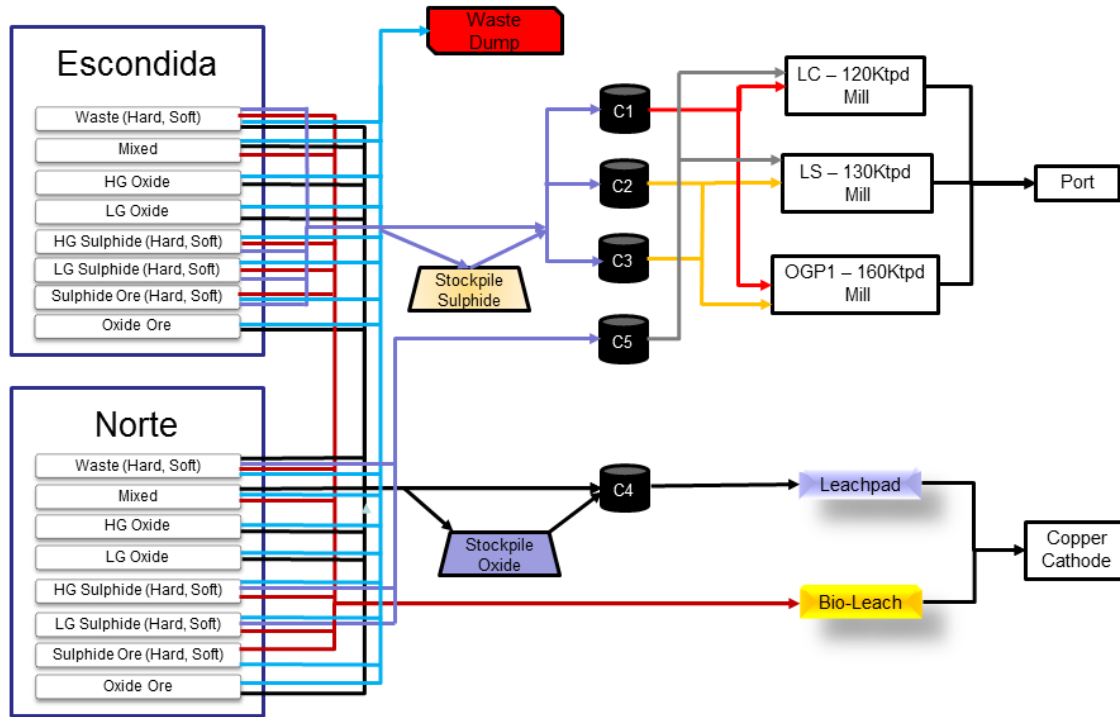


Figure 1: Flowchart of flow of material at Escondida Mining Complex.

The strategic long-term mine plan currently used at Escondida mining complex is optimized using a step-wise/sequential conventional optimization approach. First, extraction sequence of multiple mines is optimized independently, then the destination of the extracted material is decided based on cut-off grade policy, and finally, a separate optimization model defines the utilization of different processing streams. In addition, the optimization process is performed over estimated mineral deposits and ignores the supply uncertainty of materials mined. The long-term strategic mine planning at Escondida mining complex accounts only for copper products in the optimization model. Secondary products such as silver, gold, and molybdenum, concentrates are recovered at the processing mills but do not drive the optimization process. This strategic long-term plan of the Escondida mining complex will be referred to herein as “base case mine plan”. The performance of the base case mine plan is evaluated by testing it over different simulated scenarios of the two mineral deposits of the Escondida mining complex. Such analysis will present the risk associated with the base case mine plan with respect to materials mined and will be referred to as “risk analysis” of the base case mine plan.

The optimization model outlined in Section 2 overcomes the limitations of the approach used to generate the base case mine plan by: (i) simultaneously optimizing the extraction sequence, destination policies and the processing stream utilization decisions of the two pits at Escondida mining complex; (ii) incorporating supply and material type uncertainty for the two pits; (iii) considering multiple products (copper, gold, silver, and molybdenum) to drive the optimization process; (iv) integrating blending requirements and constraints for copper total and iron in the optimization model; and, lastly, (v) incorporating geometallurgical constraints

in the optimization model to ensure consistent performance of the processing mills. The strategic long-term stochastic mine plan generated with the method presented in Section 2 will be referred to as “stochastic mine plan”. Important to note that point (v) above includes SAG Power Index (SPI) and Bond Work Index (BWI) that influence the performance of comminution circuits; in addition, in the context of modelling geometallurgical properties, sulphide and waste material types are classified into hard and soft material based on respected SPI and BWI values and the proportions of such hard and soft materials being processed are decided by the optimization process.

3.2 Optimization parameters

The solution approach for simultaneous stochastic optimization of mining complexes outlined in Section 2.3 requires various parameters which are summarized in Table 1. The related economic and operational parameters used for the Escondida mining complex is outlined in Table 2 and Table 3, respectively. Economic parameters have been scaled for confidentiality purposes. Penalty costs mentioned in Table 4 are set based on a trial and error mechanism with consideration of the magnitude of different targets to achieve acceptable technical risk with production targets. Figure 2(a) and 2(b) summarize the different quantity and quality targets used for the Escondida mining complex. Fifteen stochastic orebody simulations have been generated for each mine, which considers uncertainty in grade (Figure A3 in the Appendix), geometallurgical properties and material types (Figure A2 in the Appendix) using direct blocks simulation with minimum/maximum autocorrelation factor method (Boucher and Dimitrakopoulos, 2009) (Figure A1 in the Appendix). The existing pushback definition for the multiple mines was utilized as pit limits (120634 and 78355 blocks for Escondida and Escondida Norte respectively) in the optimization model to ensure that generated plans can be implemented with the current mine designs (Figure 3). Fixed recovery at the processing mills is considered in the optimization model. The leach pad recoveries are also considered as fixed. Material types from both mines are classified into 25 clusters of each material type for each mine for the cluster based destination policy decisions.

Table 1: Parameters used for the application of the proposed approach.

Parameter	Value
Initial annealing temperature	0*, 0.6
Cooling factor	0.95
Cooling after perturbations	1000
Perturbations before diversification	120,000
Number of diversifications	13

*Temperature used for first diversification for the algorithm to learn good perturbation

Table 2: Economic parameters used for Escondida mining complex case study.

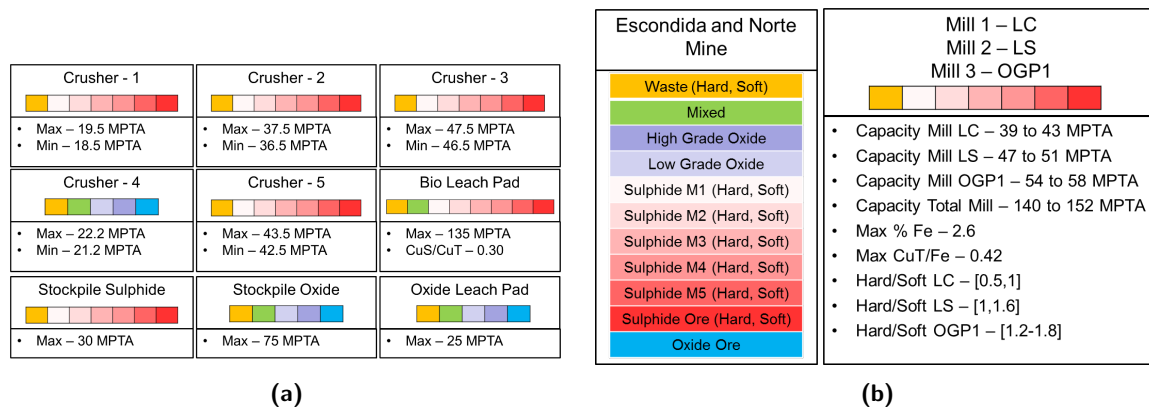
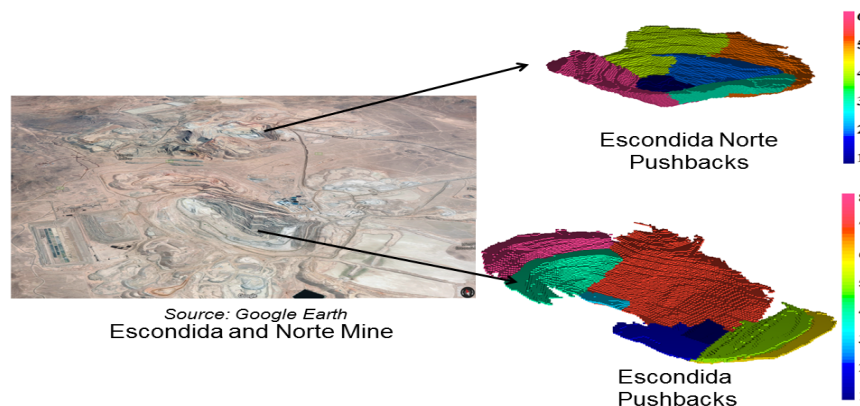
Attribute	Value
Discount rate (NPV)	8%
Geological discount rate	10%
Gold, silver, and molybdenum price (US M\$/Ton)	36, 0.44 and 0.011
Copper price (US \$/Ton.)	4960
Mining cost {excluding hauling cost} (US \$/Ton)	0.60
Hauling cost based on location (US \$/Ton)	Escondida: 0.4 to 1.27 Escondida Norte: 0.52 to 1.09
Milling cost including crushing (US \$/Ton)	5.7
Re-handling cost stockpile (US \$/Ton)	0.18
Oxide leach pad cost including crushing (US \$/Ton)	5.7
Bio leach pad cost (US \$/Ton)	1.65
Selling cost	Mill(US \$/ton) -514 Oxide leach pad (US \$/ton) - 496 Bio leach pad (US \$/ton) - 496

Table 3: Operational parameters used for the Escondida mining complex study.

Attribute	Value
Blocks for Escondida	120634
Blocks for Escondida Norte	78355
Scheduling period	8 years
Number of Pushbacks Escondida	8
Number of Pushbacks Escondida Norte	6
Number of clusters for the different material types	25
Mining width	200 m
Slope angle for Escondida	37
Slope angle for Escondida Norte	45
Recovery at mill for Cu (LC, LS, and OGP1)	0.80, 0.83 and 0.82
Recovery for minor elements at processing mills	0.20
Recovery oxide leach pad	0.65
Recovery bio-leach pad	0.27

Table 4: Penalty costs used for Escondida mining complex study.

Attribute	Value
Penalty cost - Capacity target (C1, C2, C3, C4, C5)	20, 15, 15, 20, 15 (US \$/Ton)
Penalty cost - Capacity target (Mill1, Mill2, Mill3)	15,15,15 (US \$/Ton)
Penalty cost - Mining width constraint	50000, 50000 (US \$/Ton)
Penalty cost - Stockpile capacity (Oxide, Sulphide)	100, 100 (US \$/Ton)
Penalty cost - Leach pad capacity (Oxide, Sulphide)	15, 15 (US \$/Ton)
Penalty cost - Ratio (CuT/Fe, CuS/CuT, Fe, hard/soft)	3000, 3000, 4000, 3500 (US \$/Ton)

**Figure 2: (a) Quantity and quality targets with crushers, stockpiles, and leach pads used for the Escondida mining complex study. (b) Quantity and quality targets with processing mills used for the Escondida mining complex study.****Figure 3: Pushbacks from Escondida and Escondida Norte used for the case study.**

3.3 Results

The results of the simultaneous stochastic optimization at the Escondida mining complex are presented in this section. Results are reported using probabilistic reporting, with the 10th, 50th and 90th percentiles (P10, P50 and P90, respectively) of the performance indicators of interest, and with respect to the simulated scenarios of all pertinent properties of the two mineral deposits of Escondida mining complex. The stochastic mine plan is also compared to the base case mine plan throughout its presentation and discussion, to highlight the differences in forecasted performance between the two mine plans and, in addition, show the added value by the stochastic framework where appropriate. Please note that the reporting of the base case mine plan in the respective figures reports the both risk profiles (the P10, P50 and P90) and the direct forecast from the conventional optimization approach used.

Figure 4(a)–(h) presents the risk profiles of meeting capacity targets for Crushers 1, 2, 3, and 5. The figures show that the stochastic mine plan (Figure 4(a), 4(c), 4(e), and 4(g)) has a higher probability of meeting the capacity targets, while reducing the average deviations from the related capacity targets. The base case mine plan and its risk analysis present large and impractical deviations (Figure 4(b), 4(d), 4(f), and 4(h)) from crusher capacity targets. In addition, a major difference in risk profile is observed for Crusher 5. The stochastic mine plan for Crusher 5 (Figure 4(g)) respects the capacity target over all periods compared to base case mine plan (Figure 4(h)) that do not process material in Crusher 5 in year 1 and 6.

Figure 5(a)–(h) represents the risk profiles of capacity targets for Mill 1, 2, 3 and Crusher 4. Larger probabilities of deviation from capacity target is observed for Mill 1 (Figure 5(a)), Mill 2 (Figure 5(c)), and Crusher 4 (Figure 5(g)) for the stochastic mine plan. Such large deviation originates from the processing of materials from two pits, which results in relatively high supply and material type uncertainty. Similar behavior is observed for capacity targets at the base case mine plan for Mill 1 (Figure 5(b)), Mill 2 (Figure 5(d)), and Crusher 4 (Figure 5(h)). Mill 3, which process material from one pit, shows small deviations from the capacity targets in the stochastic mine plan (Figure 5(e)) when compared to the large deviations for the base case mine plan (Figure 5(f)).

Figure 6(a)–(h) presents the risk profiles of capacity targets for oxide leach pad, bio-leach pad (sulphide leach pad), sulphide stockpile and oxide stockpile. The stochastic mine plan better respects the capacity restrictions with the oxide leach pad (Figure 6(a)) and the sulphide stockpile (Figure 6(e)) compared to the large and impractical deviations of the base case mine plan (Figure 6(b), and 6(f)). The base case mine plan violates the capacity restriction of sulphide stockpile in year 2 (Figure 6(f)). In addition, the stochastic mine plan presents lower deviations in the initial two years, as compared to higher deviations in the later period for oxide leach pad capacity target, originating from geological risk discounting implemented with the optimization model. Capacity targets with bio-leach pad and sulphide stockpile is respected in stochastic mine plan (Figure 6(c) and 6(g)) and base case mine plan (Figure 6(d), and 6(h)).

Blending targets associated with different destinations at the Escondida mining complex (Figure 2(b)) are analyzed in Figures 7 and 8. Figure 7(a)–(f) represents the risk profiles for copper total to iron ratio target (CuT/Fe) for Mill 1, Mill 2, and Mill 3. The stochastic mine plan shows almost no deviations for blending targets for Mill 1, Mill 2 and Mill 3 (Figure 7(a), 7(c), and 7(e)). No deviations are observe for the base case mine plan (Figure 7(b), 7(d), and 7(f)). Figure 8(a)–(h) represents the risk profiles for iron grade (Fe) target for Mill 1, Mill 2, and Mill 3. The stochastic mine plan has small deviations for iron grade target for Mill 1 (Figure 8(a)) in year 3, 4, 5 and 6. Mill 2 (Figure 8(c)) presents small deviations in year 4, 5, and 6 for iron grade target in the stochastic mine plan. Mill 3 (Figure 8(e)) presents no deviations for iron grade target in the stochastic mine plan. The base case mine plan presents deviations in year 1, 2, 3, and 4 for Mill 1 (Figure 8(b)) for iron grade target. Mill 2 (Figure 8(d)) presents deviations in year 1, 2, and 3 for iron grade target in the base case mine plan. Mill 3 (Figure 8(f)) presents deviations in year 1, and 2 for iron grade target in the base case mine plan.

Geometallurgical constraints controlling proportions of hard and soft materials at the processing plants and part of the simultaneous stochastic optimization approach used to generate the stochastic mine plan of Escondida mining complex. Figure 9(a)–(f) displays the risk profiles of geometallurgical targets (hard/soft ratios) for Mill1, Mill 2, and Mill 3. The stochastic mine plan presents consistent ratio of hard and soft

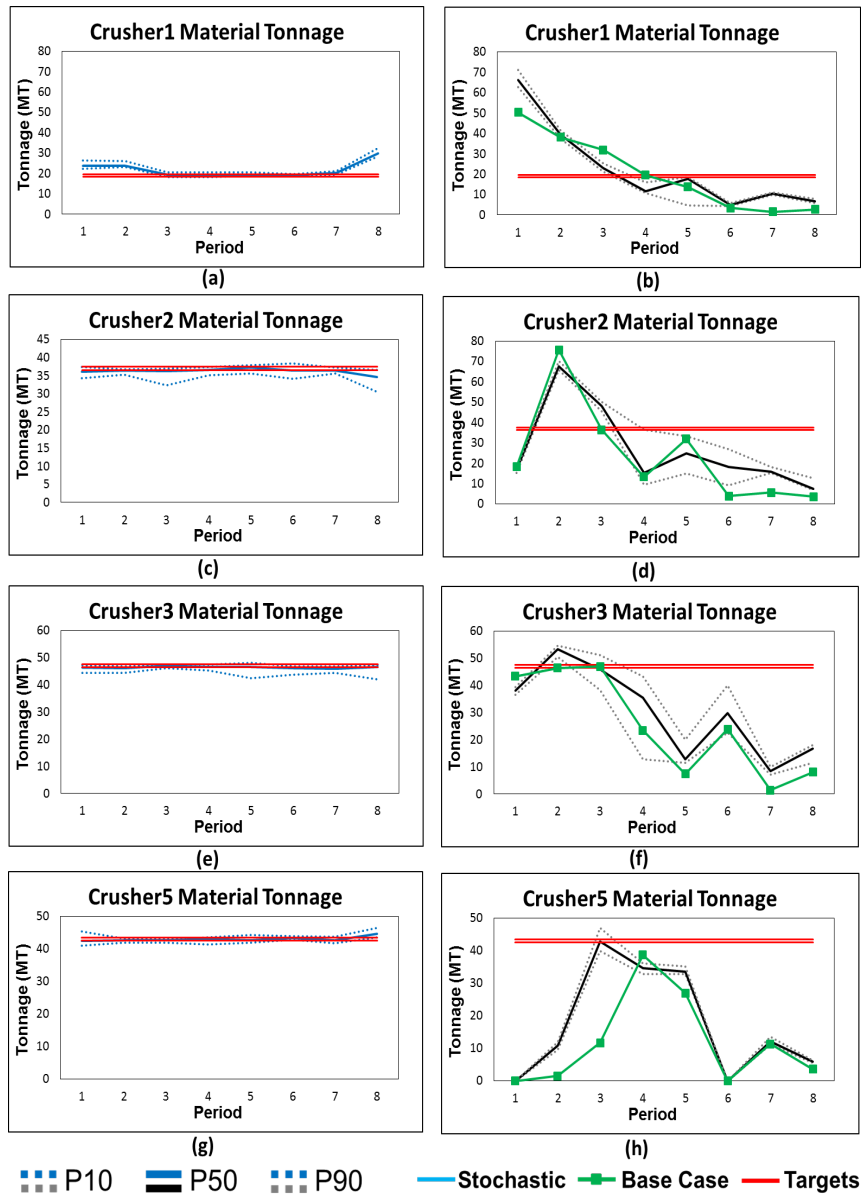


Figure 4: Forecasts for Crusher 1, 2, 3, and 5 capacity targets for stochastic mine plan compared to the base case mine plan.

material for different processing mills (Figure 9(a), 9(c), and 9(e)). The base case mine plan is not optimized to maintain such ratios and shows impractical fluctuations of hard and soft material at the three mills (Figure 9(b), 9(d), and 9(f)).

Figure 10(a)–(f) shows the risk profiles of the recovered secondary products (silver, gold and molybdenum) in the Escondida mining complex. The stochastic mine plan accounts for secondary products, along with copper products, in the optimization process and capitalizes on the associated value of such secondary products to increase the overall value of the Escondida mining complex. Note that the base case mine plan does not consider secondary products in the optimization process and only recovers secondary products. Higher amounts of silver, gold and molybdenum metal is recovered with the stochastic mine plan (Figure 10(a), 10(c), and 10(e)), as compared to the base case mine plan (Figure 10(b), 10(d), and 10(f)), which clearly outlines the advantage considering secondary products in the optimization of strategic mine plan.

Figure 11 represents the risk profile for cumulative discounted cash flows forecasts. The values are scaled with respect to the base case mine plan (the base case being 100 %) for confidentiality purposes. Risk

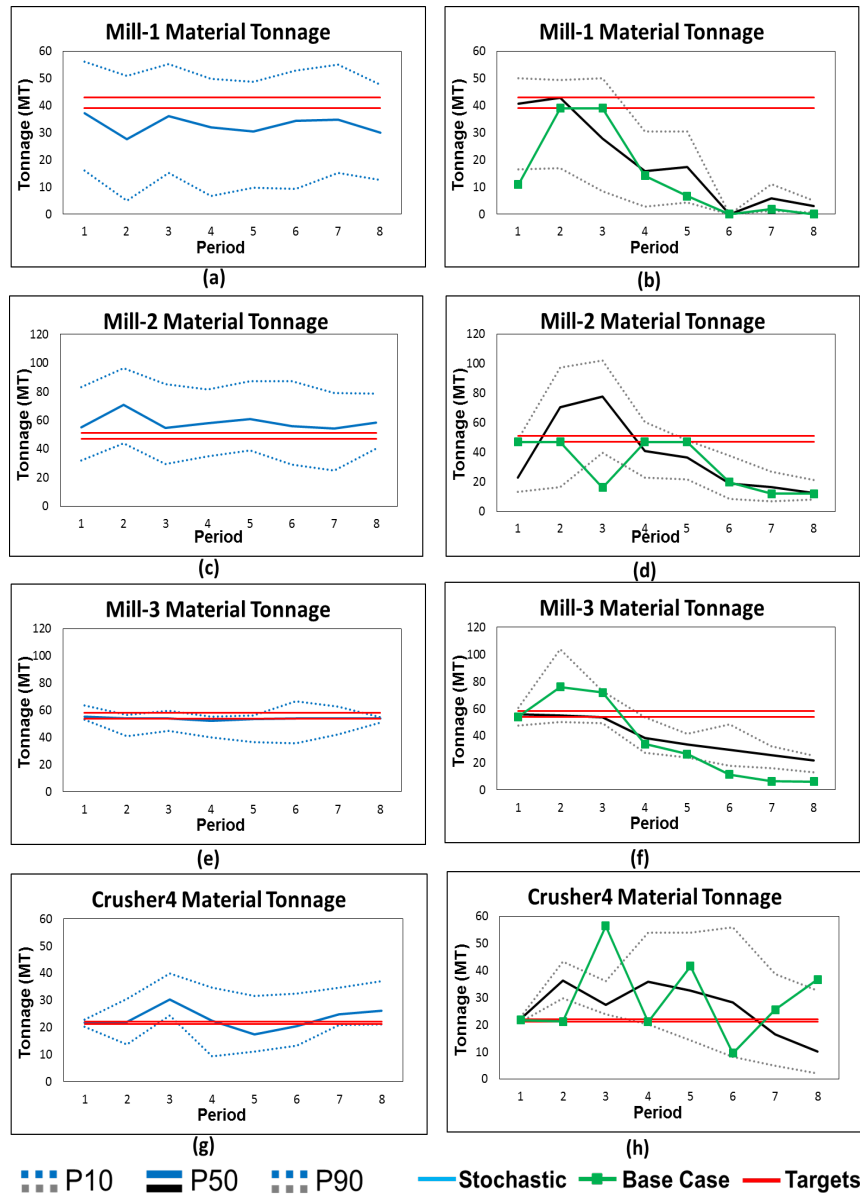


Figure 5: Forecasts for Mill 1, 2, 3 and Crusher 4 capacity target for stochastic mine plan compared to the base case mine plan.

analysis of the base case mine plan shows an increase of 46 % over the base case NPV and clearly explains the typical smoothing effects (overestimation of low grade and underestimation of high-grade metal) from estimating metal content and pertinent properties of mineral deposits. The stochastic mine plan presents a 267 % higher NPV compared to the base case mine plan, for several compounding reasons. The very large difference in the NPV forecast of the stochastic mine plan compared to the base case mine plan is due to the:

- (i) simultaneous optimization of extraction sequence, destination policies, and processing stream utilization under uncertainty;
- (ii) ability of stochastic optimization to capitalize of the variability and manage uncertainty of grades and properties of the materials being mined, as described by the simulated scenarios of the orebodies, including major effects on blending;
- (iii) integration of different blending and capacity restriction in the simultaneous stochastic optimization model allows for a more rational utilization of mills and crushers through the blending of materials from multiple mines (see Figure 4(g) for Crusher 5). The base case mine plan, on the other hand,

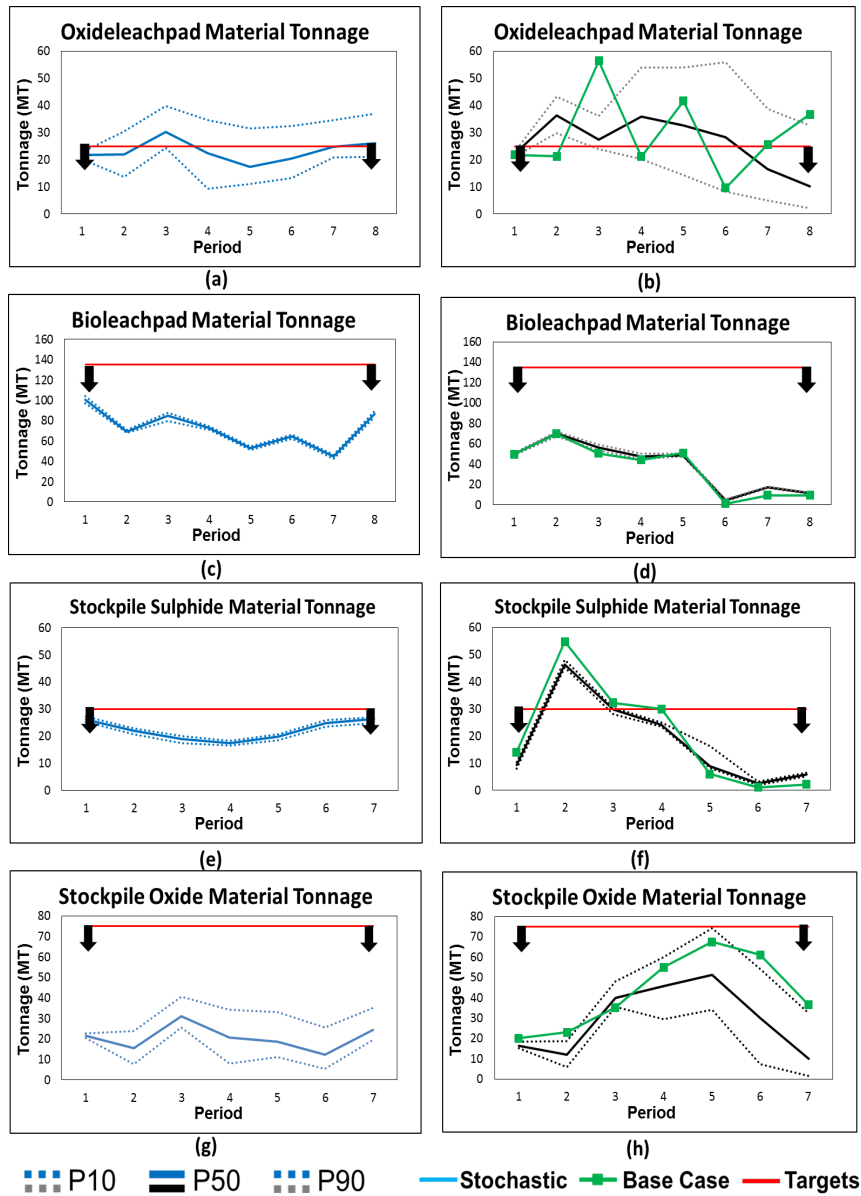


Figure 6: Forecasts for oxide leach pad, bio-leach pad, sulphide and oxide stockpile capacity target for stochastic mine plan compared to the base case mine plan.

optimizes extraction sequence for multiple mines independently of each other, and then uses a separate optimization model to define processing stream utilization decisions to satisfy blending restrictions. Such independent and stepwise optimization can lead to underutilization of mills and crushers to meet blending restrictions (Figure 4(h));

- (iv) utilization of secondary products in the optimization process.

Figure 12 displays the cross-sectional and top view of the extraction sequence of the Escondida mining complex. The smoothing constraints implemented with the model help to generate reasonably mineable shapes for the extraction sequence in stochastic mine plan. The stochastic mine plan extracts material in different proportions and areas from the multiple mines compared to the base case mine plan to achieve the required production targets with Escondida mining complex.

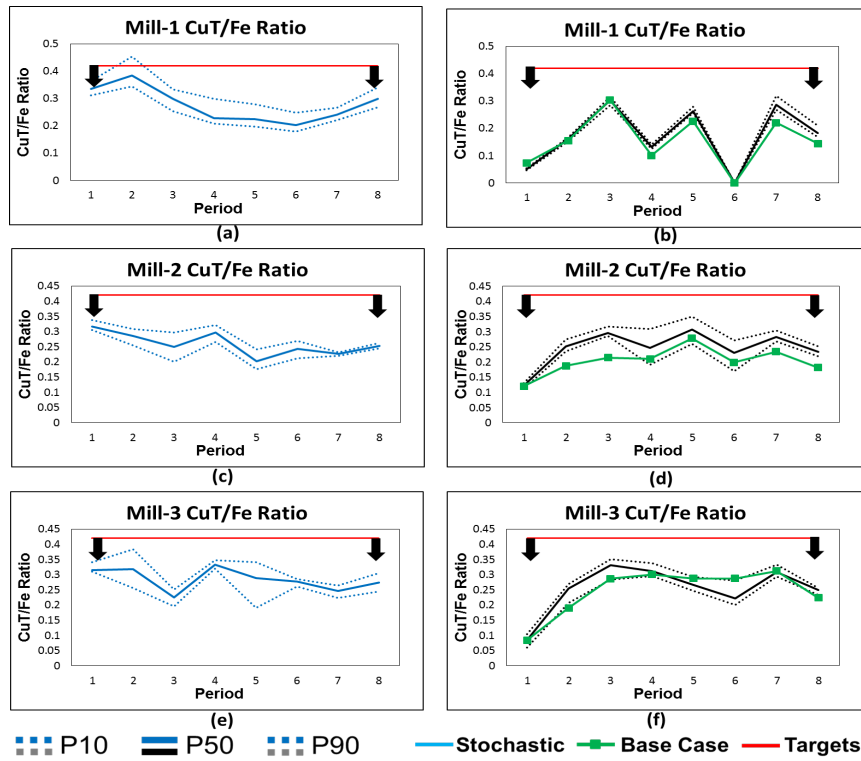


Figure 7: Forecasts for copper total and iron ratio for Mill1, 2, and 3 for stochastic mine plan compared to the base case mine plan.

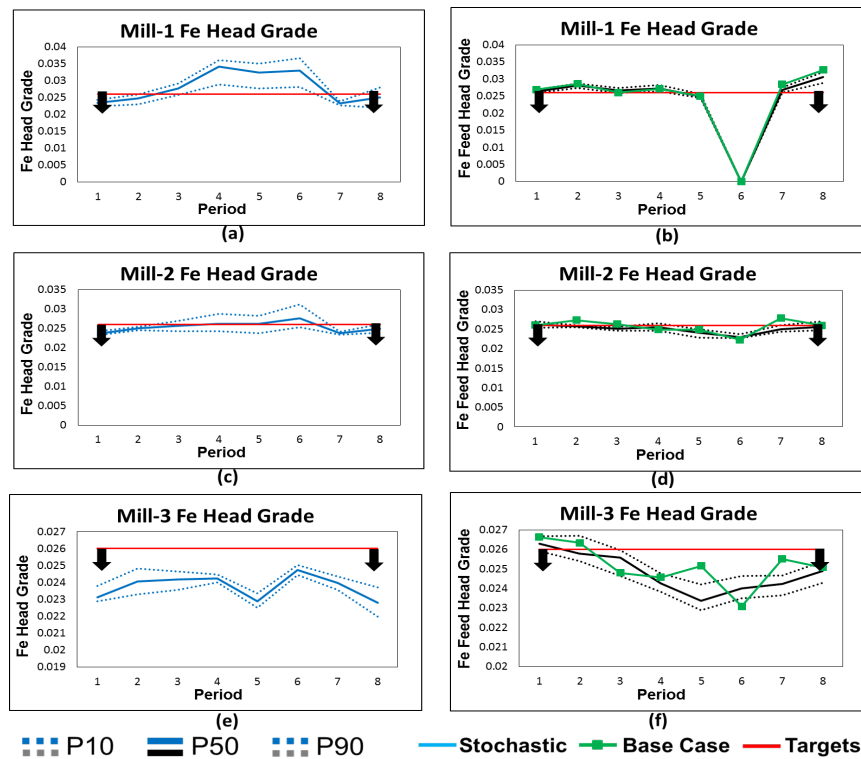


Figure 8: Forecasts for iron grade for Mill 1, 2, and 3 for stochastic mine plan compared to the base case mine plan.

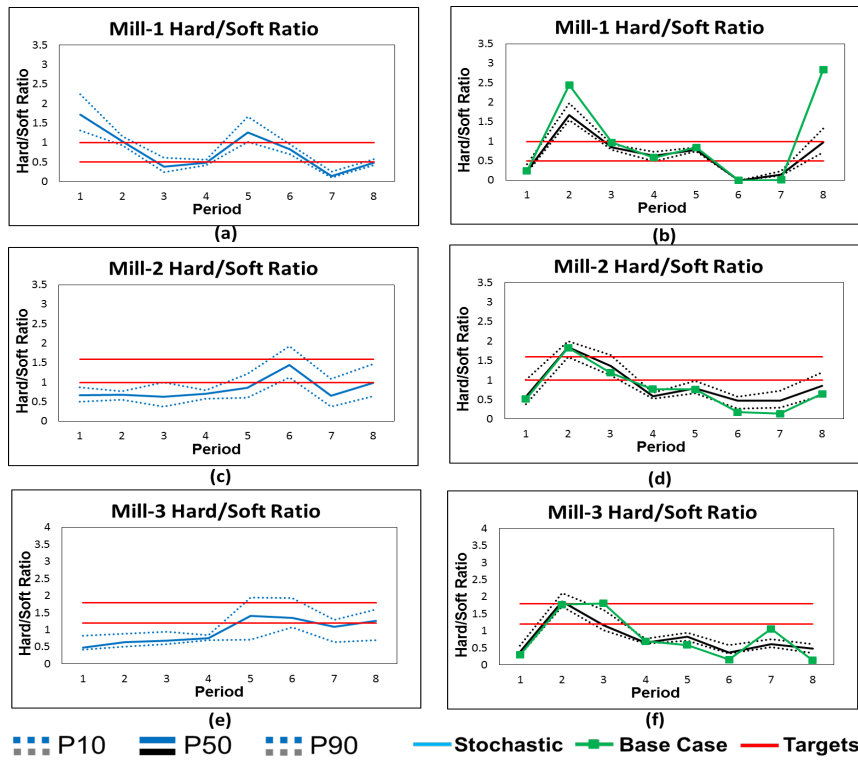


Figure 9: Forecasts for hard/soft ratio for Mill 1, 2, and 3 for stochastic mine plan compared to the base case mine plan.

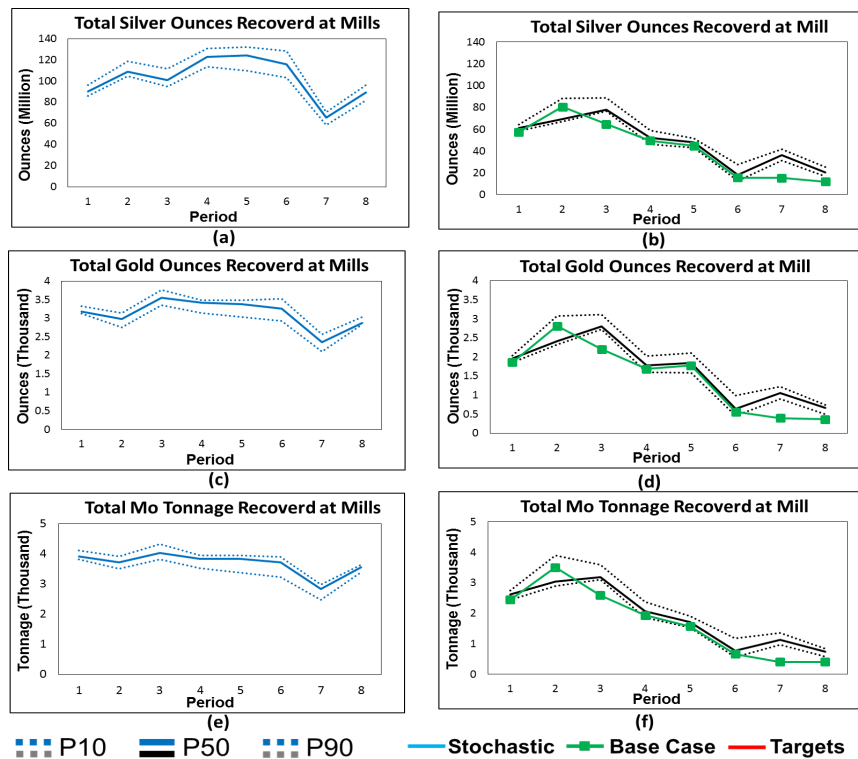


Figure 10: Forecasts for silver, gold, and molybdenum recovered at Mill 1, 2, and 3 for stochastic mine plan compared to the base case mine plan.

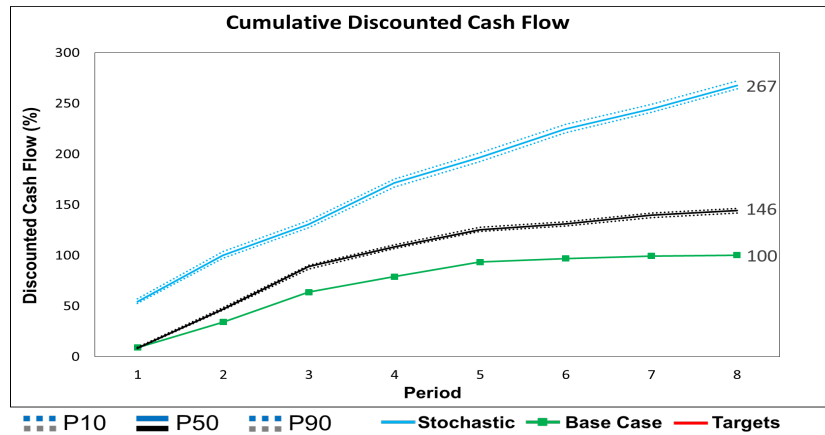


Figure 11: Cumulative discounted cash flows forecasts for stochastic mine plan compared to base case mine plan.

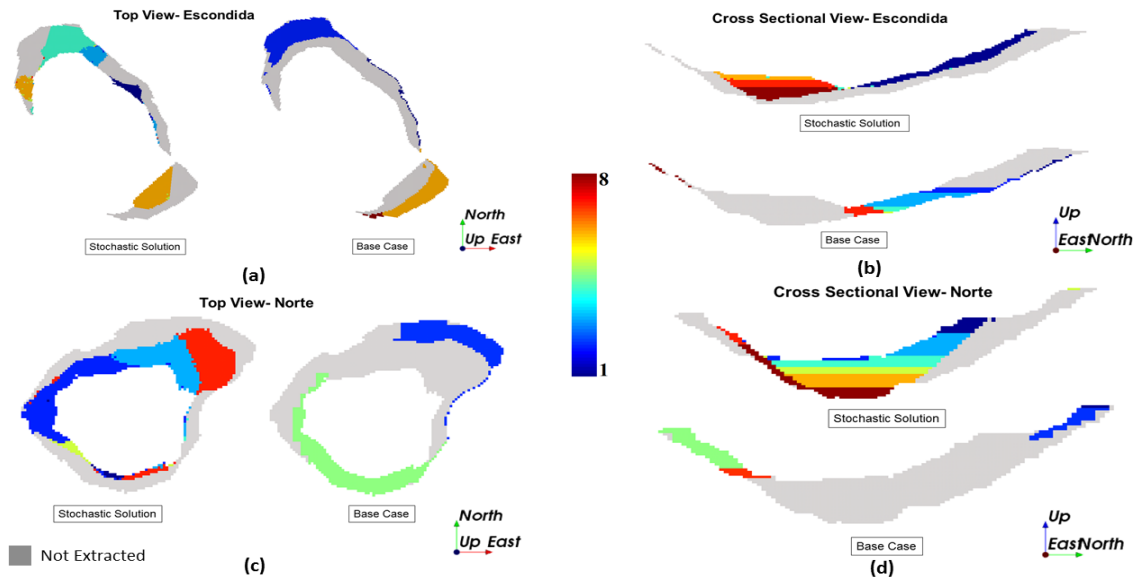


Figure 12: Generated schedule (a) top view Escondida, (b) cross-section view Escondida, (c) top view Escondida Norte, (d) cross-section view Escondida Norte.

4 Conclusions

This work expands the simultaneous stochastic optimization of mining complexes to include geometallurgical constraints and shows the efficiency of the method through an application at a large copper-gold mining complex named Escondida. The model simultaneously optimizes the extraction sequence, destination policies and processing stream utilization decisions for multiple mines under supply and material type uncertainty. Supply and geometallurgical uncertainty of the mineral deposits is captured by generating stochastic orebody models using direct block simulation with minimum/maximum autocorrelation factor. The prohibitively large model is solved using a multi-neighbourhood simulated annealing with adaptive neighbourhood search algorithm. The ability of simultaneous stochastic optimization models to (i) focus on the value of generated products, (ii) blend of material from multiple mines and processing streams to achieve production targets, (iii) capitalize on variability of material supply and synergies of simultaneous optimization helps to achieve different production targets with and increased value of 267%, as compared to the base case mine plan. Risk analysis of the base case plan shows a 46% higher expected NPV and outlines the undervaluation of the NPV with the base case mine plan that uses estimated mineral deposit representation. Risk profile for capacity

targets for mills presents a high variability in achieving targets, therefore precise planning at short-term scale and grade control is required to achieve the long-term yearly targets.

Future work will consider integrating the optimization of processing alternative (coarse, medium and fine grinding) of material at different mills, the utilization of different mills and transportation alternatives available at Escondida mining complex in the simultaneous stochastic optimization of mining complexes to further improve the performance of the simultaneous stochastic optimization model. Geo-metallurgical modelling in the present study is achieved through controlling the proportion of hard and soft material at the mills to ensure consistence throughput, however, future work could emphasize integrating such uncertainties (SPI, BWI, recoveries) in the optimization model.

Appendix

This section briefly describes the method and results of the simulation algorithm utilized to generate stochastic orebody models of the mineral deposits in the Escondida mining complex.

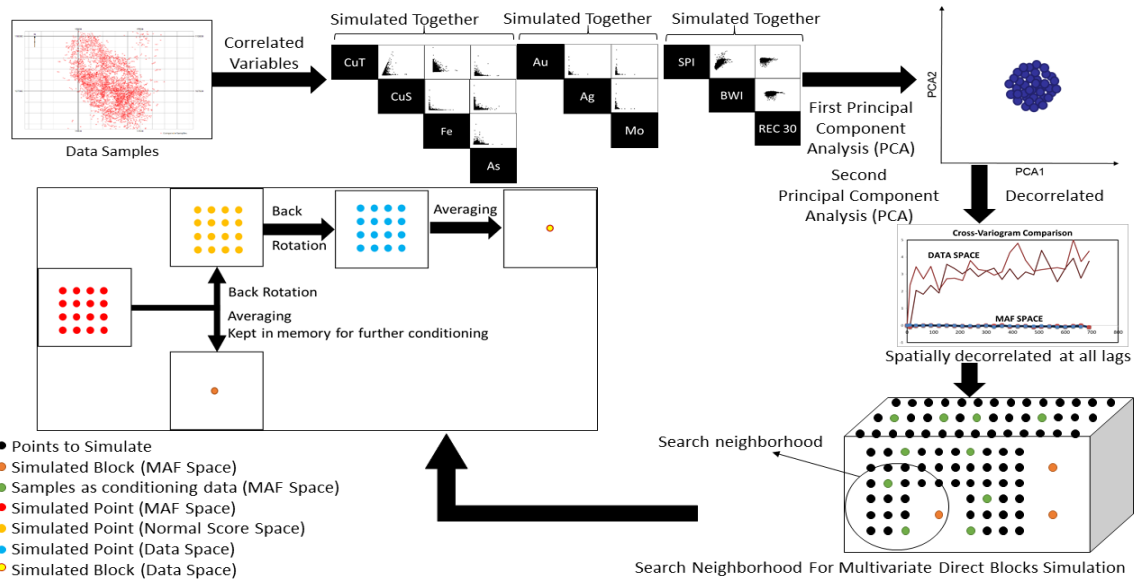


Figure 13: Direction block simulation with minimum/maximum autocorrelation factor method utilized for the case study.

Boucher and Dimitrakopoulos (2009, 2012) presented a direct block simulation method (Godoy, 2002) with minimum/maximum autocorrelation factor (Switzer and Green, 1984) for the simulation of multivariate distribution at block support scale for mining deposits. The method consists of discretizing blocks into point support scale, transforming the attributes from data space to normal score space, utilizing two subsequent PCA transformation to orthogonalize the variables defined as MAF variables for independent simulation, averaging point support scale values to generate block support scale values for further conditioning concurrently back transforming the point support simulated values to data space and averaging to generated simulated values at block support scale (Figure A1).

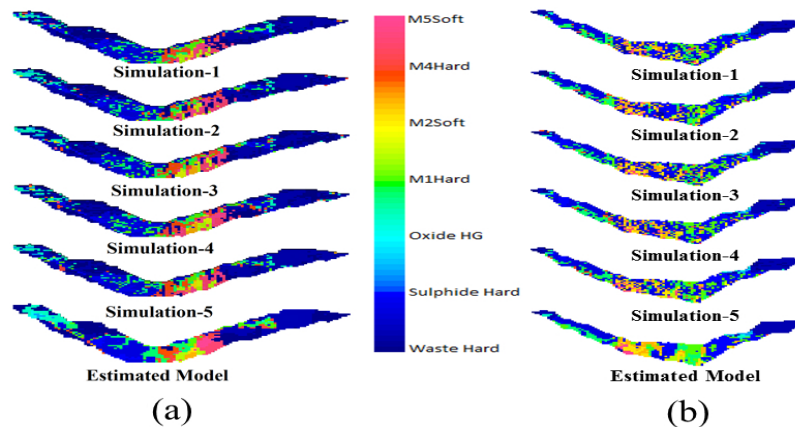


Figure 14: (a) representing the cross section of Escondida, (b) representing cross section of Escondida Norte depicting material type variability.

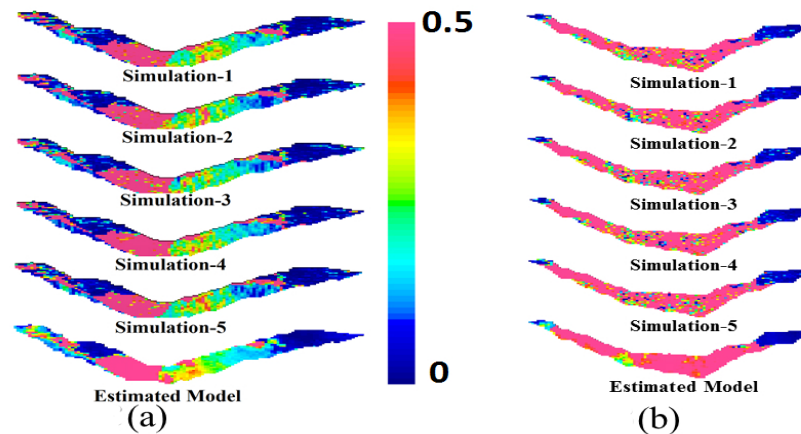


Figure 15: (a) representing cross section of Escondida and (b) cross section of Escondida Norte for grade variability in the deposit.

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