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A study of integrated capacity decisions**

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# Stochastic mining supply chain optimization: A study of integrated capacity decisions

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**Abstract:** The mining value chain comprises many inter-related components, from mines to transportation, to customers. When the individual components are optimized separately the value that can be generated from the enterprise suffers. Optimization of the mining value chain requires a shift away from conventional methods of optimization and towards the simultaneous optimization of all related aspects, including: the mine's extraction sequence, material destination decisions, material transport decisions, and equipment capacities. Further, if these decisions are to be robust, they must be made while considering sources of uncertainty and managing the related technical risk.

The contributions included in this paper are meant to assist strategic planning and evaluation of mining projects under uncertainty. Specifically, the simultaneous integration of capacity decisions in long-term scheduling is meant to provide a tool that generates a NPV-optimal mine sequence and destination policy that is also synchronized with equipment capacities selected while being robust to two sources of uncertainty, geological uncertainty and metal price uncertainty.

# 1 Introduction

The mining value chain is the route by which raw materials are extracted from within the earth and transformed into marketable products. A mining complex is a set of mines, processors, transport mechanisms and stockpiles that can be considered as a stand-alone unit within the broader supply network. The global mining supply chain is comprised of many such mining complexes that vary in their degree of complexity and inter-connectivity.

Typically, a mining complex is controlled by a managing entity with the goal of maximizing the net present value (NPV) of cash flows generated by the complex as a whole. Cash flows are estimated based on a model of the mine plan which is optimized under certain assumptions and subject to certain constraints. The conventional approach to optimization within the industry has focused on optimizing each component of the mining complex on its own (sequentially) rather than considering the enterprise as a whole (simultaneously). This approach ignores the important inter-dependence of components within the mining complex. The traditional approach also does not incorporate various sources of uncertainty prohibiting any notion of risk-resiliency in design considerations. Further, conventional mine planning ignores capital investments and their relationships to operational capacities, instead they consider fixed capacities as static constraints for the optimization.

This paper builds upon recent work in stochastic mine planning which focuses on the simultaneous global optimization of mining complexes under uncertainty (Goodfellow and Dimitrakopoulos, 2016; Montiel and Dimitrakopoulos, 2015). Within the mining complex framework, the task of creating an optimal mine plan involves determining the following: an extraction sequence for selective mining units (blocks) in each mine, a destination policy dependent on material type and attribute values, and the quantity of material flow between destinations. Notably, these decisions simultaneously contemplate material blending, stockpiling, and time value of money. In the stochastic context these decisions must also contemplate sources of uncertainty to manage technical risk. If the optimization is extended to consider capital expenditure decisions, these also must be incorporated within the model along with the corresponding adjustments to the relevant constraints.

Goodfellow (2014) and Montiel (2015) propose an integrated model of the mining complex that is able to consider the simultaneous stochastic optimization of the entire mineral value chain while including capacity decisions. Unlike conventional models, value is calculated based on products sold and not at the level of mining blocks. This unlocks the power of material blending and allows for complex revenue calculations to be considered. The model described by Goodfellow is extended in this paper to make three main contributions:

1. The simultaneous optimization of mining and processing capacities in the pre-production stage under geological uncertainty;
2. The incorporation of metal price uncertainty in optimization of second-stage material movement decisions;
3. The inclusion of various financial contracts in product-based revenue calculations.

The idea is to create a globally-optimal plan for the entire mineral value chain that will remain robust under uncertainty. The first-stage variables determine the decisions made in an uncertain environment, these decisions must remain robust over the distribution of possible outcomes. The values of the second-stage variables are determined after uncertainty has been unveiled, and the optimal value is chosen based on the revealed information.

One of the important contributions of Goodfellow and Montiel is the method by which material flow is modeled within the mining complex. The formulation moves away from calculating attributes at the block level and instead tracks attributes as they move through the value chain. As materials move from one destination to another, blending and other interactions can easily be accounted for at a level of detail that far exceeds what was accomplished in prior works. This ability allows for economic values, and other characteristics of interest, to be calculated where they are realized instead of assigning each block values before materials are extracted, stockpiled, blended, transported, processed and sold.

Capacity decisions represent a trade-off between the amount of capital outlay required and the operational constraints of the project. The most common approach to mine design is to set mining and processing capacities based on comparisons with existing projects with similar characteristics. This approach is not necessarily optimal since it does not tailor the investment decisions to the specific traits of the project at hand. This work will deal with integrating plant sizing decisions within the global mine optimization framework.

Only recently have there been attempts to incorporate capacity optimization in the stochastic mine planning framework. Montiel and Dimitrakopoulos (2015) allow the optimizer to alter the processing capacity between different operating modes whereby process throughput and recovery can change between two pre-set levels. This is a metaheuristic approach that enables better alignment between the mine schedule and the processor's capabilities under geological uncertainty, but it does not incorporate capital investment decisions. Goodfellow and Dimitrakopoulos (2016) implement dynamic mining capacities by allowing the optimizer the ability to purchase loading and hauling equipment but the approach does not integrate processing capacity decisions.

The limitations of these prior attempts at including capacity optimization in stochastic mine planning is that they approach the optimization problem from a pre-constrained starting point – either the processing or the mining capacity has already been fixed to some extent. Mining and processing capacities are inherently interrelated so it only makes sense to optimize them simultaneously; this is especially true for an operation that is in the planning stage while there is still the flexibility to make these important design choices. Setting mining capacity determines the maximum quantity of material (ore and waste) that can be extracted from the mines considered in the complex. The capital outlay required in order to produce at the desired mining capacity is generally related to the purchase of equipment such as trucks, shovels, loaders, etc. The amount of ore that can be mined under the mining capacity constraint will have an effect on the optimal processing capacity (and vice versa). Determining the processing capacity involves establishing the optimal trade-off between capital outlay and the size for each the (possibly many) processing streams. An approach presented here is to allow the optimizer to decide the optimal mining and processing capacities simultaneously as the LOM schedule is generated.

Another contribution made in this work is the inclusion of flexibility in revenue calculation. In many instances a mining company is forced to raise external funds to build their project, this is especially true for “junior” or “mid-tier” companies that do not have the cash reserves for a large capital outlay. These funds are invariably accompanied by additional encumbrances impacting the project's revenue stream. In terms of the mathematical model used in this paper, the value of attributes, may not be constant. Due to the restrictive use of block values in traditional formulations, it was not possible accommodate a change in revenue calculations. The formulation set forth by Goodfellow (2014) can accommodate complex revenue structures due to the generality of its mechanism of hereditary attribute calculation. This paper extends the mechanism to include attribute-and-variable-dependant revenue calculations. The contribution allows for a mining complex optimization that considers project encumbrances brought on by the financing of the asset. In summary, this work attempts to reconcile plant capacity decisions within the stochastic optimization of mining complexes while also incorporating metal price uncertainty. Additionally, it includes the ability to make revenue calculations flexible in order to properly manage various financial contracts that accompany mining projects.

In the following sections, first, the mathematical model utilized is presented. Then, mining project financing effects on optimization is addressed, and is followed by the integration of metal price uncertainty. The application of the proposed approach is then applied at a copper-gold mining complex with a precious metal streaming agreement. Conclusions and suggestions for future research follow.

## 2 Model formulation

The model is formulated as a two-stage mixed integer stochastic optimization (Birge and Louveaux, 2011); where the first-stage variables are scenario-independent and the second-stage (recourse) variables are scenario dependant. The second-stage variables are used to manage technical risk through penalizing deviations from pre-set targets.

## 2.1 Model components

Following the framework described by Goodfellow (2014), a mining complex is comprised of a set of mines  $m \in \mathbb{M}$ , and processing destinations  $\mathcal{P}$ . The mines are discretized into blocks  $b \in \mathbb{B}_m$  which are characterized by their material type and attribute values  $\beta_{p,b,s}$ , where  $p \in \mathbb{P}$  refers to primary attributes and  $s \in \mathbb{S}$  is one of the scenarios used to represent the deposit.

Material flow is modeled using a graph structure with nodes  $\mathcal{N}$  (sources/destinations in the mining complex) connected by arcs representing allowable incoming-outgoing ( $I(i) \subseteq \mathcal{N}$ ,  $\mathcal{O}(i) \subseteq \mathcal{S} \cup \mathcal{P}$ ) pairs. Each material type is subdivided into a number of different clusters, based on the values of the block's multiple attributes. Material types are clustered using the k-means++ algorithm as a preprocessing step (Arthur and Vassilvitskii, 2007). The Euclidian distance is used as the similarity metric and the number of clusters to use is a modelling decision.

The optimization model has four types of decision variables. A first-stage mine sequencing variable is what determines the period  $t \in \mathbb{T}$  in which each block is extracted. After a block is extracted a choice must be made regarding which destination it is sent to. Much like grade binning discussed in (Lane, 1964; Wooller, 2007), the pre-processing step of material clustering allows for robust destination policies to be established. Destination policy decisions are determined by the another first-stage variable which establishes where to send blocks belonging to each cluster in a given period. A second-stage variable determines the proportion of the tonnage held at each location  $i \in \mathcal{S} \cup \mathcal{P}$  is that is sent to location  $j \in \mathcal{S} \cup \mathcal{P}$  period  $t \in \mathbb{T}$  and under scenario  $s \in \mathbb{S}$ .

The variable  $v_{h,t,s} \in \mathbb{R}$  is used to account for the quantity/value of hereditary attributes, derived as a function of primary attributes  $v_{p,i,t,s} \in \mathbb{R}$  at locations within the mining complex. Notably, primary attributes must be additive (e.g., tonnage) to sum attributes over destinations in order to calculate the non-additive hereditary attributes (e.g., metal grade) as material moves through the value chain.

In order to do this both the cost, and the incremental increase contributed by the capacity decision must be considered in the objective function and constraints of the model. To accomplish this an additional variable is added to integrate capacity decisions within the optimization. The first-stage variable  $w_{k,t} \in \{L_{k,t}, U_{k,t}\}$  establishes the amount of extra capacity gained from capital expenditure option  $k \in \mathbb{K}$  which must be purchased at a price,  $p_{k,t}$ . The objective function can then be written as follows to account for the purchase of extra capacity. Notably this formulation assumes a linear relationship between incremental capacity and incremental cost, however  $p_{k,t}$  can be made to be a function of the number of capacity increases in order to account for economies of scale.

Objective Function:

$$\begin{aligned} \max \left\{ \frac{1}{|\mathbb{S}|} \sum_{s \in \mathbb{S}} \sum_{t \in \mathbb{T}} \sum_{h \in \mathbb{H}} p_{h,t} \cdot v_{h,t,s} \right. & \text{Expected NPV} \\ & - \sum_{t \in \mathbb{T}} \sum_{k \in \mathbb{K}} p_{k,t} \cdot w_{k,t} & \text{Capital Expenditure} \\ & \left. - \frac{1}{|\mathbb{S}|} \sum_{s \in \mathbb{S}} \sum_{t \in \mathbb{T}} \sum_{h \subseteq \mathbb{H}} c_{h,t}^+ \cdot d_{h,t,s}^+ + c_{h,t}^- \cdot d_{h,t,s}^- \right\} & \text{Deviation Penalties} \end{aligned} \quad (1)$$

The goal of mine complex optimization is to maximize NPV (identified by the first term in the above objective function). However, maximizing a project's NPV without regard for uncertainty, as is the case with the traditional mine planning approach, is misleading. The traditional approach does not account for uncertainty and thus cannot guarantee that the results it predicts are realistic. The third term in the objective function shown above is used to manage risk through the use of penalties that are assigned when deviations from targets occur. In this way the objective function of the stochastic optimization searches for a solution that has both high expected NPV and low risk of failing to meet targets. This gives the stochastic solution its robustness under uncertainty.

Part two of the objective function includes the simultaneous optimization of capacity decisions within the optimization. The capacity limits are allowed to be expanded for a cost. Adjustments to the model's constraints are required to allow the available capacity to grow after a capacity decision is made. Hereditary attribute constraints provide the mechanism by which deviations from upper  $U_{h,t}$  and lower  $L_{h,t}$  target ranges are calculated as shown in Equations (2) and (3). When capacity optimization is included in the model the target ranges can be adjusted by the increment of each capacity option  $k \in \mathbb{K}$ .

$$v_{h,t,s} - d_{h,t,s}^+ \leq U_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^t \kappa_{k,h} \cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (2)$$

$$v_{h,t,s} + d_{h,t,s}^- \geq L_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^t \kappa_{k,h} \cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (3)$$

Capital expenditure constraints ensure that a capital expenditure option is exercised only once for one-time options, and between the lower  $L_{k,t}$  and upper  $U_{k,t}$  limits for multiple-purchase options.

$$\sum_{t \in \mathbb{T}} w_{k,t} \leq 1 \quad \forall k \in \mathbb{K}^1 \subseteq \mathbb{K} \quad (4)$$

$$L_{k,t} \leq w_{k,t} \leq U_{k,t} \quad \forall k \in \mathbb{K}, t \in \mathbb{T} \quad (5)$$

By including the above constraints, the model is able to handle multiple capital expenditure options. These decisions are considered simultaneously with the other mine planning variables in the optimization. The desired outcome of the methods proposed in this paper is an integrated mine plan that includes: a block extraction sequence, a material destination policy, a material flow plan, and an optimal capital allocation, all optimized in sync with each other while being robust under uncertainty as represented by the set of scenarios  $s \in \mathbb{S}$ . Ideally such an optimization would be carried out at the feasibility, or detailed engineering, stage of a project when there is flexibility to establish the mine's design before construction begins.

## 2.2 Solution methods

Given that the mine complex optimization model is very large, metaheuristic methods are the necessary tools used in order to obtain a solution. Metaheuristics do not guarantee convergence to mathematical optimality, but instead they are powerful tools that can generate good-quality solutions in an acceptable amount of time. The solutions are better than those generated by traditional mine planning methods in that they are much better at meeting production forecasts (due to their ability to manage risk), and in that they achieve better value (Ramazan, 2013).

This paper employs the simulated annealing metaheuristic (Kirkpatrick, 1984) with a number of various perturbation mechanisms that helps the algorithm explore the solution space as thoroughly as possible. Capacity optimization makes the solution more difficult since capacity decisions have a significant effect on the remainder of other decision variables. For this reason, including capacity decisions makes it necessary to consider measures in order to ensure the solution does not get trapped in local optima (Cicirello, 2007). A further detailed discussion on metaheuristics and mathematical optimization methods inherent to the solution methods applied to the optimization performed are outside the scope of this work. For a comprehensive discussion on metaheuristics and solution methods applied to mine complex optimization the reader is referred to: Blum and Roli (2003); Caccetta and Hill (2003); Dimitrakopoulos and Montiel (2013); Goodfellow (2014); Lamghari and Dimitrakopoulos (2016).

## 3 Project financing's effect on optimization

Another contribution made in this work is the inclusion of flexibility in revenue calculation. In many instances a mining company is forced to raise external funds to build their project, this is especially true for "junior" or "mid-tier" companies that do not have the cash reserves for a large capital outlay. These funds are



invariably accompanied by additional encumbrances impacting the project’s revenue stream. In terms of the mathematical model used in this paper, the value of attributes  $p_{h,t}$ , may not be constant.

The mining industry’s inability to manage uncertainty has been a major contributor in preventing it from generating attractive risk-adjusted rates of return (Ball and Brown, 1980; McClain et al. 1996; Tufano, 1998). This has forced mining companies to increasingly seek less traditional sources of capital as the availability of debt and equity has become more and more scarce (Dionne and Garand, 2003). Such sources include royalty, streaming and offtake agreements as outlined in Table 1.

**Table 1: Non-traditional sources of capital that impact project revenue**

Type	Description
<b>Royalties</b>	
Gross Revenue (GR)	The miner pays a percentage of gross (top-line) revenue to its royalty partner in exchange for an initial capital investment
Net Smelter Return (NSR)	The miner pays a percentage of net revenue to its royalty partner in exchange for an initial capital investment.
Net Profit Interest (NPI)	The miner pays a proportion of the project’s net profits to its royalty partner (usually only after it has recovered its capital costs) in exchange for an initial capital investment.
<b>Streaming Agreements</b>	
Metal Streams	Streams provide the right to purchase a proportion of production of one or more of the mine’s metals at a discounted price in exchange for an initial capital investment. Streams are well suited to mines with significant co-product production. Precious metals are the most common metals subject to streaming agreements; for example, a Cu-Au mine may wish to stream future gold production in order to fund production of the main metal, copper.
<b>Offtake Agreements</b>	
Metal Offtakes	Offtake agreements give the offtake buyer the right to purchase future metal or concentrate production from the mine in exchange for an upfront payment. The payment is usually intended either as pre-payment for a portion of the future metal delivery or to secure a joint-venture interest in the project.

Given that the financing alternatives outlined in Table 1 impact the revenue structure of the mineral complex, they must be accounted for in project optimization; the change in revenue calculation will impact the final LOM plan, resulting in different values for the decision variables.

## 4 Including metal price uncertainty in mine complex optimization

By including uncertainty in the input prices the optimization is able to manage market volatility by making decisions that can take advantage of opportunities during high-price periods. Conversely, in weaker periods, stockpiles can act as a buffer to shield the operation from selling in low-price environments. Without the inclusion of market price uncertainty within the optimization, these operational flexibilities are wasted.

Past attempts at including metal price uncertainty have focused on determining pushback, or ultimate pit designs (Castillo and Dimitrakopoulos, 2014; Meagher, Sabour, and Dimitrakopoulos, 2009) and have fallen short of generating an integrated LOM plan robust to price uncertainty. The main reason that price uncertainty is not considered in simultaneous stochastic mine optimization is the large number of simulations required to represent uncertainty which can be in the order of 100-1000 (Briggs et al., 2012). Including this many scenarios increases the solution time prohibitively.

This paper proposes a two-part optimization whereby the entire mine complex is optimized first under geological uncertainty, then this schedule is fixed and used as an input to a second optimization where only down-stream decisions are optimized under metal price uncertainty. This procedure reflects a mine’s operational reality in which the long-term LOM plan is optimized at a given starting point, and the down-stream decisions are taken in subsequent years when price uncertainty is revealed. Notably, this approach to incorporating metal price uncertainty assumes knowledge of the entire path of each metal price scenario, i.e. material movement decisions made in a given year are based on the (assumed-to-be known) paths of metal

prices in future years. Although there is a theoretical argument against this approach, in practical terms the approach suffices as long as autocorrelation is not the driving force driving prices. Thus, in this work the full set of metal price simulations is used and the model is solved as-is.

To model market uncertainty metal price simulations are generated using a stochastic reduced-form model. The simulation methods proposed are based on commonly used pricing models for each type of commodity (Schwartz, 1997). Due to the important influence of supply and demand, base metals can be simulated using a mean reverting process with Poisson jump diffusion. The typical model for precious metal prices is a trending Geometric Brownian Motion model with Poisson jump diffusion. The trend component is used in precious metal price modeling to account for a positive correlation with inflation.

Apart from the large increase in the size of the optimization brought about by the inclusion of another set of simulations, the difficulty in incorporating metal price uncertainty at the same time that the block-wise mining sequence is optimized is that the metal extracted is not necessarily being mined in the same period in which it is sold. For these reasons metal price uncertainty is included in a second stage of the optimization as follows. First, an optimal mining schedule, plant capacity, and destination policy is determined using the model described above. Then, the block extraction sequence and material destination policies are fixed while metal price uncertainty is included and the problem is re-optimized to establish the best material movement and transportation decisions in each period. This approach allows the optimization the ability to create a mine plan that is robust to geological uncertainty while also including the ability to “plan” material movement decisions that consider metal price volatility.

## 5 Case study: Application at a Cu-Au mine with a precious metal streaming agreement

The case study presented below illustrates an application of simultaneous stochastic capacity optimization within the mining complex framework. Geological uncertainty is incorporated through the use of ten orebody simulations and a second optimization is run in order to optimize downstream decisions under metal price uncertainty using 100 copper price simulations and 100 gold price simulations. Revenue is calculated based on a gold streaming agreement whereby a portion of the mine’s gold production is sold to a customer at a fixed price. The mining complex used in this case study produces two products, copper and gold. The complex comprises a single open pit mine that holds four main material types: oxide, sulphide, transition, and waste. Of these, the two materials containing the bulk of the mine’s profit – sulphide and transition – are further split into two categories based on the grade of the main metal of interest, copper. The mining complex has six processing destinations and one stockpile. The processing destinations include: a sulphide and oxide dump leach; an oxide, sulphide and transition heap leach; and a sulphide processing plant. Each of these processing destinations is fed directly from the mine, with the sulphide plant accepting additional feed from a stockpile as shown in the material flow diagram in Figure 1. Copper electrolyte solution is produced at the sulphide dump leach and the sulphide heap leach, this solution is then converted into cathode copper at the solvent extraction electrowinning (SX-EW) plant. Gold metal is recovered from leachate which is produced at the oxide dump leach, oxide heap and transition heap leach. The sulphide processing plant produces a copper-gold concentrate containing 30% copper and 5-30g/t gold. This product accounts for the greatest proportion of the mine’s revenue.

Non-linear grade-recovery functions are used at the process destinations. The curves allow a realistic modelling of the relationship between the head-grade into a particular process and the amount of metal that can be effectively output from the process. This is a benefit of the formulation that is impossible to consider all other models that rely on predetermined block values. The copper and gold recovery curves used in this study are provided. Looking at the copper grade-recovery curve, it is important to note that recoveries for grades in the range of 0-0.4% vary considerably between the different process destinations. The average copper grade of the project used in this case study is 0.3% which makes the grade-recovery relationship an important factor in the optimization of the LOM schedule and capacity decisions.

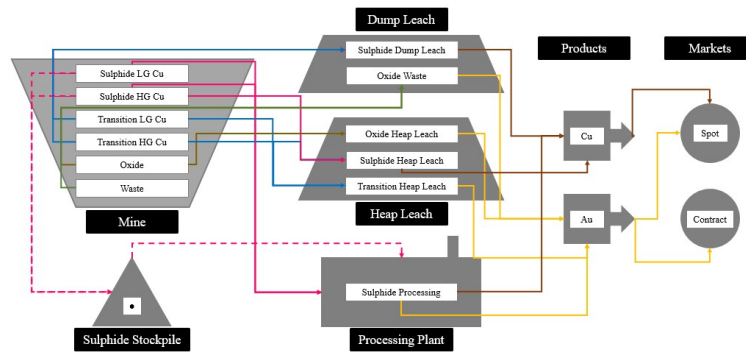


Figure 1: Material flow diagram for the mining complex used in the case study

The parameters used in this case study are presented in Table 2. Note that the mining and milling capacities outlined in the table are used as a base case for illustrative purposes, these will be optimized within the model.

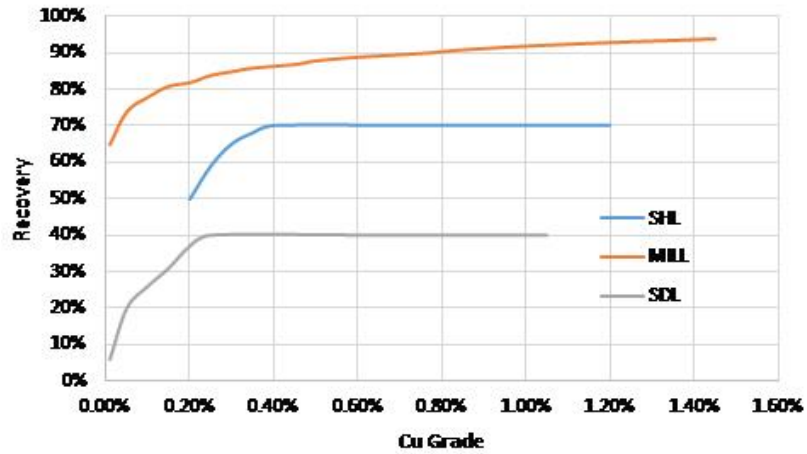


Figure 2: Grade-recovery curves for copper

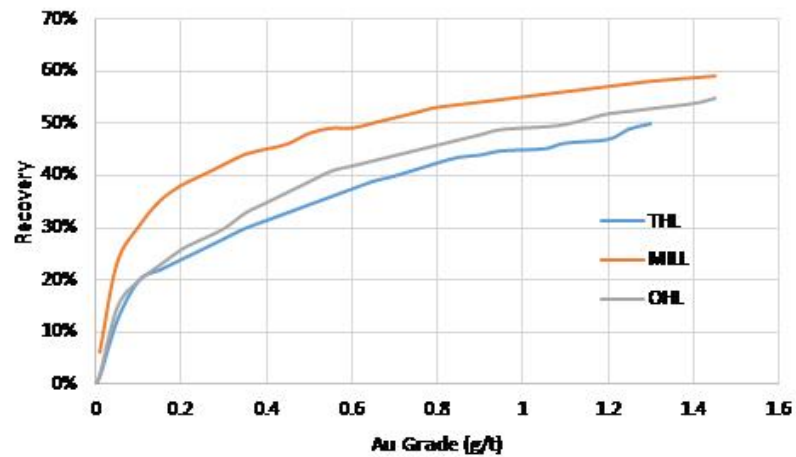


Figure 3: Grade-recovery curves for gold

**Table 2: Assumptions and inputs used in the mining complex optimization**

Type	Value
<b>Geological Model</b>	
Number of blocks	493,290
Block dimensions	25x25x10m
Metals of interest	Cu, Au
Block tonnage	13,000 – 15,000 tonnes/block
<b>Financial</b>	
Copper price	\$2.88/lb
Gold Price	\$1,480/oz
Discount rate	8%
Mining cost	\$2.60/tonne
Milling cost	\$29.37/tonne
Heap leach cost	\$5.35-7.74/tonne
Dump leach cost	\$4.87/tonne
<b>Optimization</b>	
Geological discount rate	10%
Deviation penalties	5-100 per unit deviation (depending on constraint)
Objective function	Max NPV
<b>Operational</b>	
Mining capacity	20,000,000 tonnes/year
Milling capacity	6,000,000 tonnes/year
Mill stockpile capacity	10,000,000 tonnes
Sulphide leach capacity	8,100,000 tonnes/year
Metal recovery	Variable, based on recovery curves

## 5.1 Stochastic capacity optimization results

For comparison purposes, a base case mine plan using a deterministic optimization model with fixed capacities (as shown in Table 2) was generated. The deterministic solution was generated using the same model as described above but only using a single estimated orebody model as input (no simulations). Figure 4 shows the resulting sulphide mill input tonnage for the deterministic optimization. Notably the fixed capacity of six million tonnes per year is not expected to be exceeded over the life of the operation. However, the risk analysis, created by running a number of simulations through the base case schedule, shows that these expectations are likely not to be met. Contrary to the deterministic schedule, the stochastic schedule, incorporating and managing geological uncertainty, is able to reliably meet production targets within the tight specified range.

Although the stochastic LOM plan shown in Figure 4 does a good job meeting production targets at the sulphide mill, the optimization is based on a fixed throughput level and mining capacity. By incorporating capacity decisions, the optimizer can seek to maximize the trade-off of sending increasing or decreasing these capacities and sending the material elsewhere. The “best” mill capacity is the optimal trade-off between the extra capital cost and the increased cash flow from the metal recovered by the larger mill. The results presented below cover the following operational decisions: scheduling (including pit limits), destination policy, material flow, and capital expenditure/operational capacity selection. In this case, the optimization was no longer forced to abide by the mining and processing constraints laid out in Table 2; instead the capacities are simultaneously optimized along with the rest of the mining complex. The milling capacity is modeled as a one-time decision made in year 1 with a two-year lag time. This allowed for a two-year pre-production period during which mill construction, stripping, stockpiling and leaching could occur, with milling only allowed to commence in year-3. During this 3-year ramp-up period mining capacity was allowed to increase, reaching its maximum level in year 3 at which point it is fixed for the remainder of the LOM. Table 3 provides the capital costs and capacity parameters used in the stochastic optimization model.

Using a 26-core 2.60GHz Intel Xeon CPU and 128GB of RAM the optimization took 37 hours and 7 minutes. Figure 5 shows the results from the simultaneous stochastic optimization with capacity integration. The optimized design called for a 4,800,000 tonne per year milling capacity at the sulphide mill. The resulting mining capacity started at 5,000,000 tonnes in year 1 as construction was allowed to commence, increasing to 18,000,000 tonnes in year-2, and reaching a maximum capacity of 25,000,000 tonnes in year-3 until the

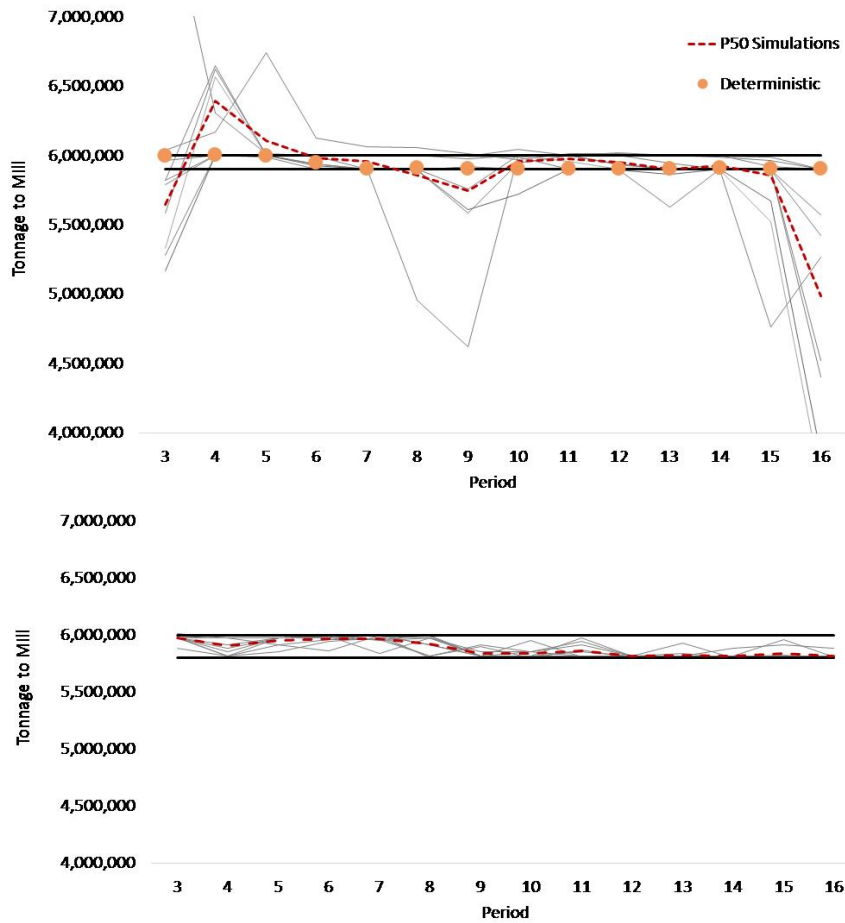


Figure 4: Top – risk analysis of deterministic optimization. Bottom – risk analysis of stochastic optimization incorporating geological uncertainty

Table 3: Capital costs assumed for incremental mining and milling capacities

Mining	Milling
<b>Incremental Capacity</b>	
1,500,000 tonnes/year	200,000 tonnes/year
<b>Cost per Increment</b>	
\$4,000,000	\$10,000,000
<b>Lead Time</b>	
0 years	2 years
<b>Life</b>	
LOM	LOM

remainder of the LOM in year-16. Notably the optimizer selected a higher mining rate and a lower milling rate than what otherwise would have been assumed for the capex-constrained model. This is due to the fact that the resulting optimal schedule is able to selectively feed the sulphide mill with high-grade material while making better use of the dump leach and heapleach facilities which have a lower operating cost. Notably, these decisions are being made in large part based on the grade-recovery relationship outlined in Figure 2 and Figure 3, something that is impossible if traditional block-based economic values are used.

The results of the optimization show that the stochastic schedule is robust to geological uncertainty and does a good job at feeding the mill within a narrow range. This avoids both financial losses due to missing

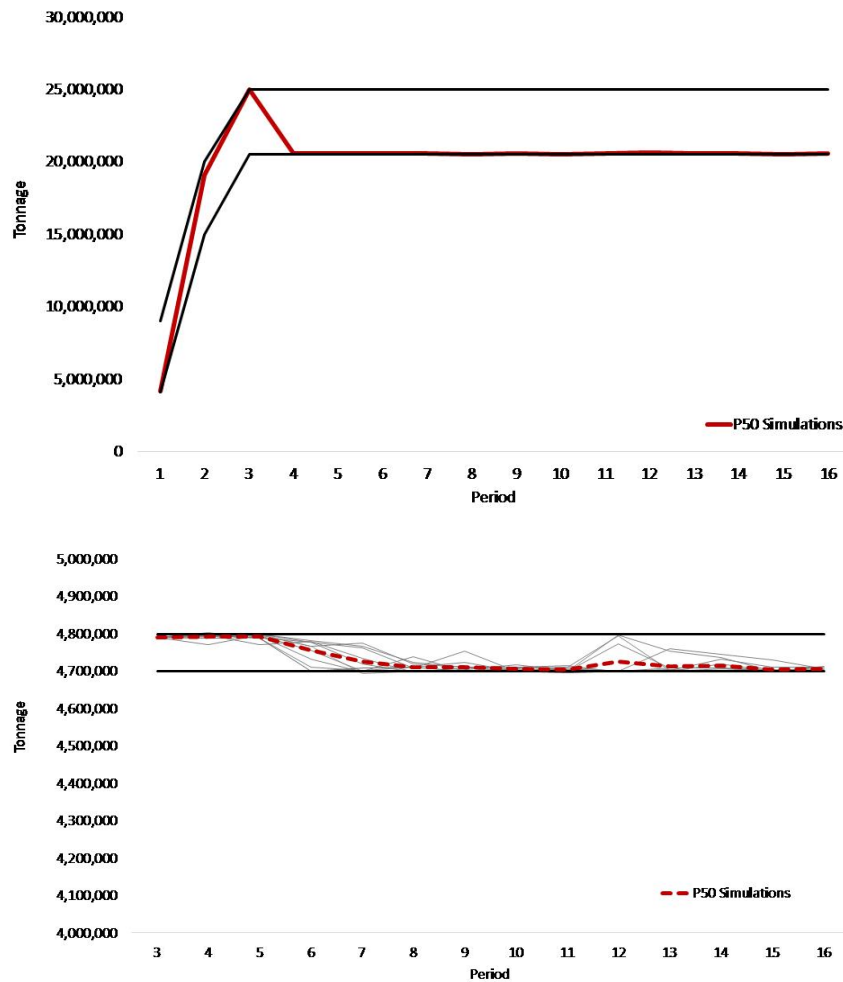


Figure 5: Mining capacity (top) and sulphide mill capacity (bottom) selection for stochastic optimization with integrated capacity decisions. (note the scale change on the vertical axis)

targets and extra costs associated with overusing the sulphide stockpile. These benefits can be seen when comparing the financial results of the schedules as shown in Figure 6.

The deterministic optimization used for the basis of comparison is performed using the same formulation but with only single estimated inputs. Based on the risk analysis, the NPV that the deterministic optimization predicts has an 80% chance of falling short of its estimate. The risk analysis forecasts an NPV that is 1% (\$20,000,000) lower than the one predicted using the estimated model. This is due to its inability to manage the grade-related risk inherent to the underlying geology. This causes the deterministic schedule to largely misclassify material which leads to a misguided schedule, destination policy and material flow decisions. The schedule that is optimized simultaneously with capacity decisions shows a 12% (\$290,000,000) increase over the stochastic schedule that does not integrate capacity optimization.

The pit design and extraction sequence generated by the optimization is presented in Figure 7. These designs can be compared to the stochastic schedule that did not integrate capacity optimization. Each period is represented by a different colour moving from cold to hot colours as years progress. The empty blocks at the bottom of the pit represent uneconomic material that was left behind. Also of note is that the final pit is larger than the one generated when the model was constrained to a fixed mining capacity of 20,000,000 tonnes per year. This again is due to the new schedule's ability to selectively send material to the sulphide

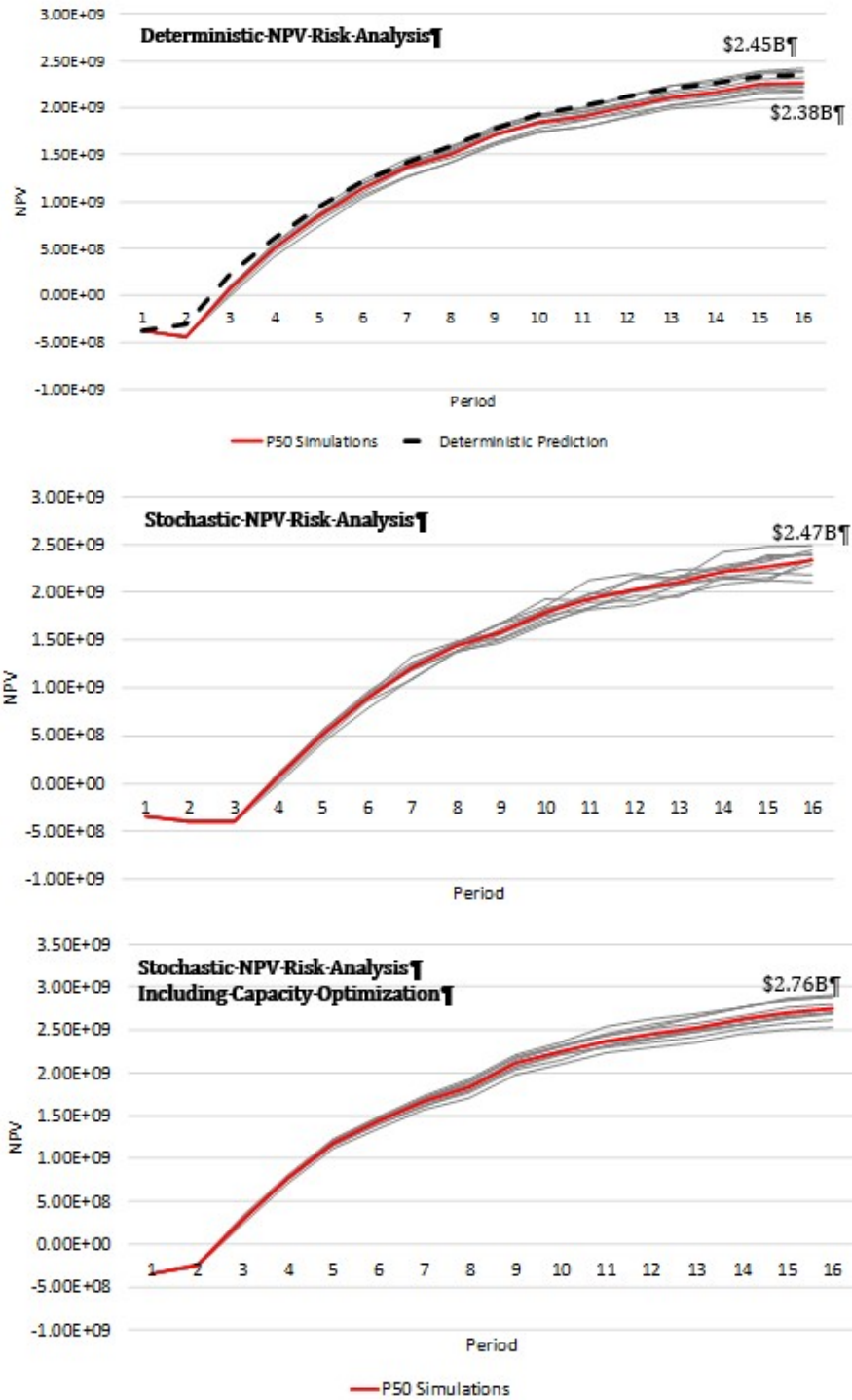
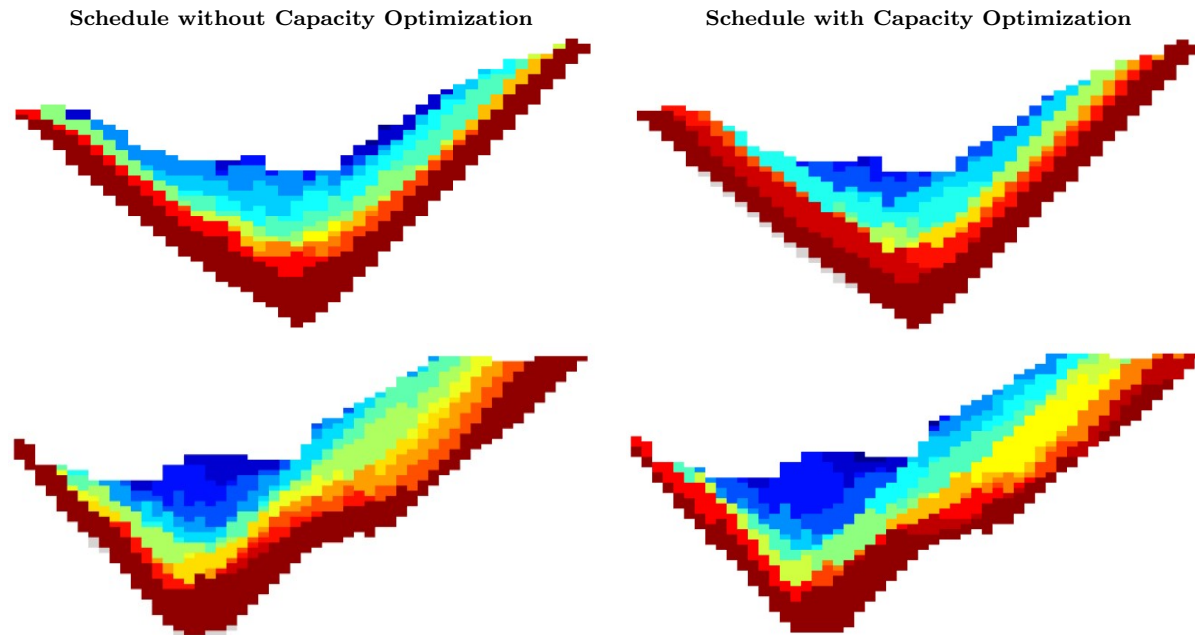


Figure 6: NPV results of capacity optimization applied to optimization of a mining complex under geological uncertainty

mill and make better use of the upgraded 25,000,000 tonne per year mining capacity by sending material to stockpiles and leach pads which do not have a strict capacity requirements like the mill.

Two other important aspects of the resulting LOM plan generated by the optimization are the material destination policy and the inter-destination decisions. As noted previously, the mine plan based on simultaneous capacity optimization decides to make more use of heap leach and stockpiling in order to favour higher grades at the mill. The impact of the optimization on these variables is shown in Appendix I.



**Figure 7: Cross-sections of the mining sequences and pit designs for long term mine schedules with and without capacity optimization under geological uncertainty (colours represent periods of extraction)Capital costs assumed for incremental mining and milling capacities**

## 5.2 Stochastic optimization with variable capacities, a precious metal streaming agreement, and two types of uncertainty

As noted above, selling contracts can be important encumbrances for many mining projects. If the commodities produced by a certain project are subject to such an agreement, it becomes necessary to account for the change in revenue calculation within the optimization. In this section the same mining complex is considered but a precious metal streaming agreement is applied to a proportion of the gold produced, and both geological and commodity price uncertainty is included.

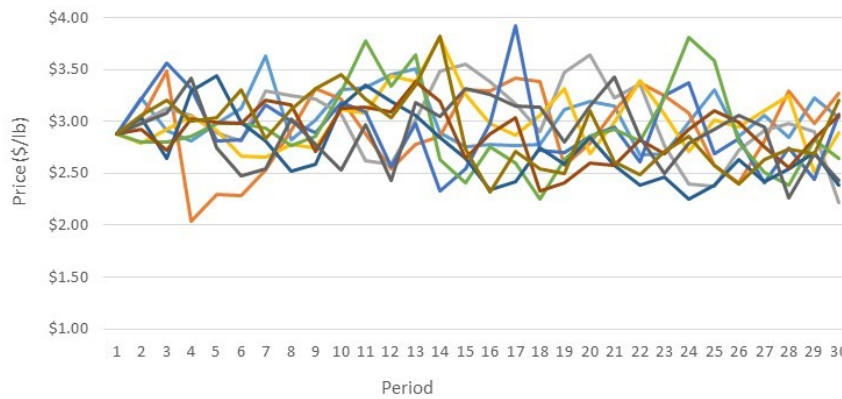
The parameters used to model copper and gold price uncertainty are given in Table 4. Copper price is modeled using a mean-reverting process and gold price is predicted with a trend model. Here metal price uncertainty is included in a second stage of the optimization. First, an optimal mining schedule, plant capacity, and destination policy is determined using the model described above. Then, the block extraction sequence and material destination policies are fixed while metal price uncertainty is included and the problem is re-optimized to establish the best material movement and transportation decisions in each period. This approach allows the optimization the ability to create a mine plan that is robust to geological uncertainty while also including the ability to “plan” material movement decisions that consider metal price volatility.

Using the parameters from Table 4 produces the simulation profiles for copper and gold shown in Figure 8 and Figure 9 respectively.



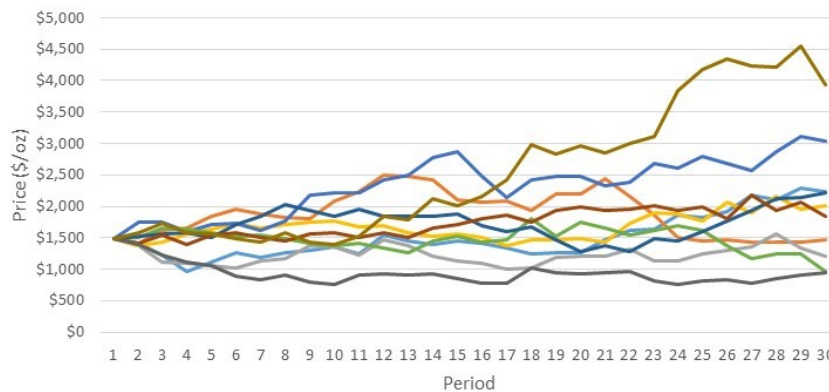
**Table 4: Parameters used to model metal price uncertainty**

Parameter	Value and Comments
<b>Copper</b>	
Initial price, $S_0$	US\$2.88/lb, the same as the reverting level
Reversion level, $\hat{S}$	\$2.88/lb, 5-yr real reverting level in 2015 dollars
Annual volatility, $\sigma$	9%, average annual volatility over 25 years
Mean reverting speed, $\alpha$	0.5,
Average jump frequency, $\mu_P$	2 per year, 25-yr average number of Cu price shocks
Average jump size, $\beta$	3%
<b>Gold</b>	
Initial price, $S_0$	US\$1480/oz, price assumption used by operation
Annual volatility, $\sigma$	5%, average annual volatility over 25 years
Annual drift, $\eta$	0.5%, 5-year moving average drift over 25 years
Average jump frequency, $\mu_P$	2 per year
Average jump size, $\beta$	5%



**Figure 8: Copper price simulations (periods represent years)**

The copper price simulations shown above show a clear pattern of mean reversion. This model is used for copper because the metal’s price is heavily influenced by supply-demand dynamics which are driven by the metal’s relatively stable marginal cost of production. Gold’s use as an inflation hedge means that the real growth rate of its price can be expected to match the pace of inflation. The simulations below use only a small trend value in order to ensure that the ratio of metal values (Cu/Au) remains relatively stable over the life of mine.



**Figure 9: Gold price simulations (periods represent years)**

The streaming agreement considered herein provides the miner with an upfront payment of \$800,000,000 in exchange for 80% of the mine’s LOM gold production at a price of \$620/oz. The remaining 20% of the mine’s gold belongs to the operation and is assumed to be sold at the market price of \$1,480/oz. The effect that this agreement has on the value resulting from the optimization can be seen in Figure 10.

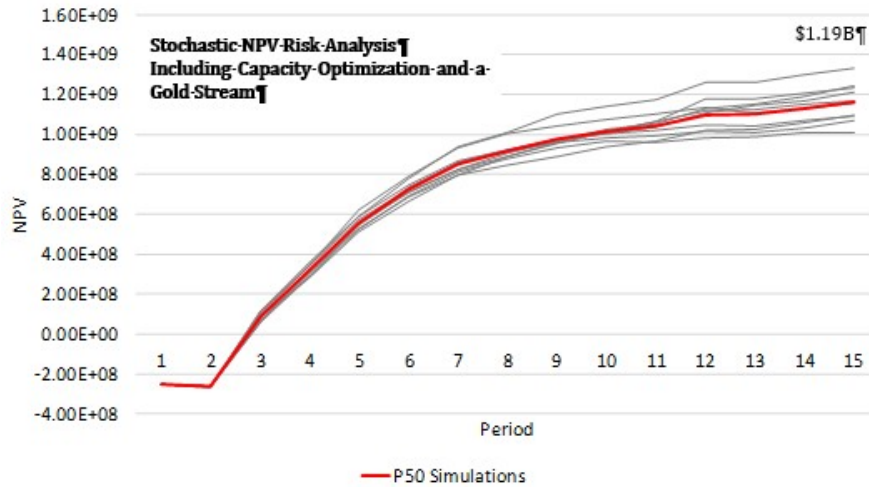


Figure 10: NPV risk analysis of optimization with gold stream under metal price and geological uncertainty

Notably, the expected NPV drops \$1.57B due to the stream and the mine life is shortened due to the corresponding reduction in the economic value of each block. This is reflected in the histogram shown in Figure 11 which illustrates the distribution of possible values of the gold stream based on the 200 (100 Cu and 100 Au) price simulations that used to optimize down-stream decision variables. Although the project has lost significant value through the gold stream contract, the inclusion of metal price uncertainty was able to shield a portion of the total potential loss. Without including metal price uncertainty, the stream would have accounted for an expected loss of \$1.68B compared to the same optimization excluding a stream. This benefit is due the optimizer’s ability to seek to recovery more metal during high-price periods and favor low-cost, lower-grade ore during low-price periods.

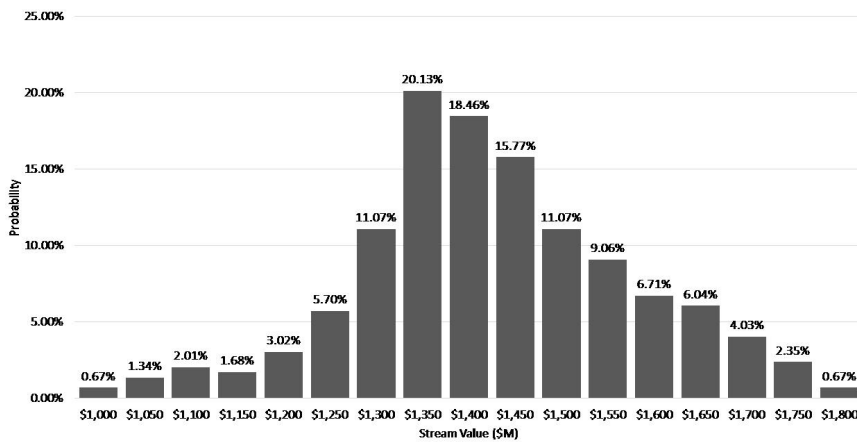


Figure 11: Distribution of values for the gold stream (to the stream owner) based on metal price simulations

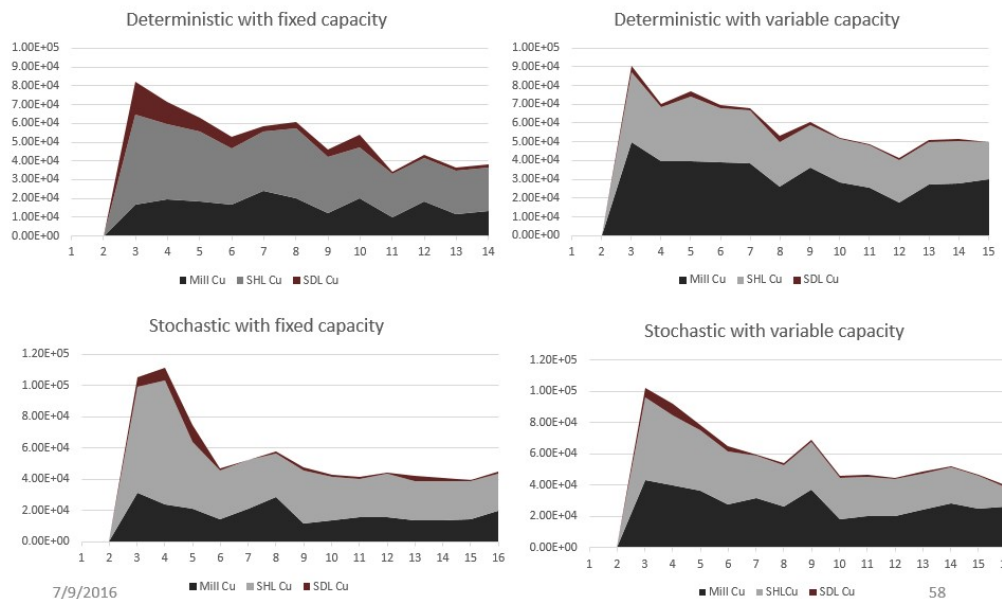
## 6 Conclusions

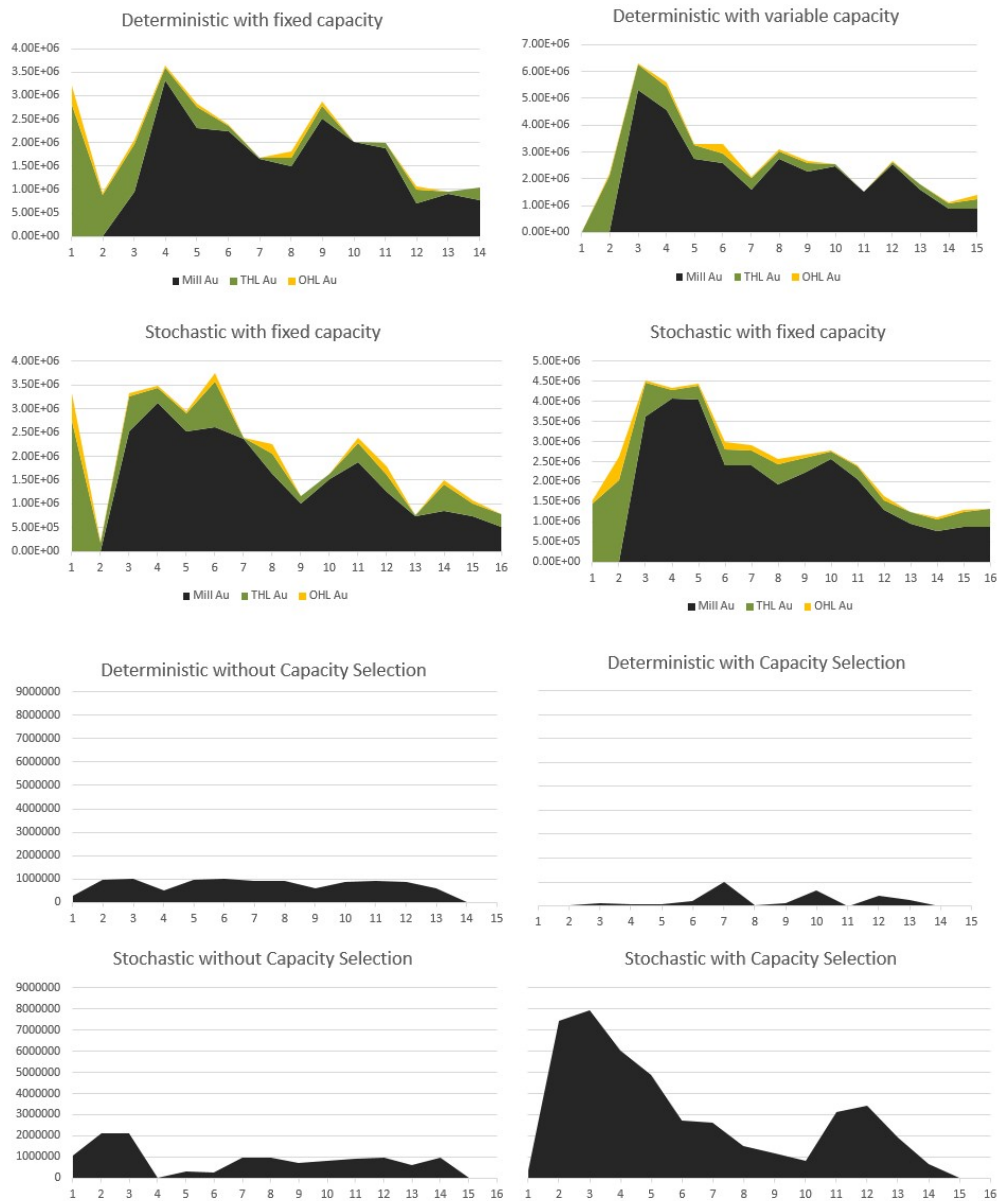
This paper addresses stochastic optimization and simultaneous integration of inter-related capacity decisions within the mining complex framework. Specifically, the mining complex is considered from the perspective of the planning stage and the optimizer is allowed to select both mining and milling capacities. The approach shows that eliminating arbitrary capacity restrictions at the outset of a project can unlock significant value that may otherwise have been lost. This study was conducted under geological uncertainty through the use of multiple orebody simulations in order to create a risk-robust schedule with higher probability of meeting the optimizer-set capacity targets. The result is a mine plan that has higher expected value with lower expected risk.

Throughout this study it was discovered that progressively reducing the flexibility of the capacity decision within a narrowing realistic range greatly improved the stability of the final solution. However, this approach means that the optimization has to be attempted a number of times in order to evaluate different options and to ensure an adequate approximation of optimality. In addition, this paper considered the impact that project financing can have on revenue calculation. A streaming contract was considered in the case study and the change in revenue calculation was included in the optimization. The resulting schedule was able to shield some of the potential losses to the stream owner by favouring high metal production in high-price periods and seeking low-cost, lower-grade tonnes during low-price periods.

In addition to mining and processing capacities, the capacities of auxiliary components of the mining complex can be incorporated in the optimization. These are typically components that are not critically related to production, but still play a part in the overall profitability of the operation. In most hard rock operations, the processing bottleneck is milling capacity, but other constraints of interest may include be: leach pad height, water use, down-stream transport, acid consumption, emission limits, fleet size fluctuation, equipment limits in various pits, or any combination of these factors. Regardless of the bottleneck, the model can incorporate it, and its corresponding capital cost, in the optimization.

## Appendix I: Material flow of optimized schedules (Tonnes)





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