**Dynamic mid-term optimization
of a mining complex under uncertainty**

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Abstract: This study presents a production scheduling optimization method for a mining complex, which provides a flexible long-term plan for future investments and operational decisions. This strategic planning method uses an adapted two-stage SIP model which expands upon the two-stage framework by performing multiple recourse stages that are solved iteratively, allowing parallel designs in a scenario-tree structure. In this model, dynamic decisions are made sequentially over time, based on new information. A case study with options to invest over trucks and a secondary crusher show an increased expected NPV compared to the two-stage stochastic formulation.

1 Introduction

Mining complexes represent a continuous flow of material with several stages: multiple mines, transportation systems, processing streams, ports, etc., where the performance of each strongly depends on the other, and each entails nonlinear relations towards material characteristics and project value. Because of this, stages must be joined in a simultaneous optimization model that considers profits once the product is sold, and not when the rock is extracted from the ground. Mine planning optimization models have tackled this problem and focused on optimizing the whole mining complex jointly to obtain a plan that maximizes value given the annual targets, blending requirements, operational constraints, and configuration of the system. This joint optimization is referred to as "simultaneous optimization of mining complexes". [1]–[3] State of the art studies also consider stochasticity in variables such as geology or market. The latest research on stochastic simultaneous optimization of mining complex (SSOMC) uses a set of scenarios to represent the uncertainty and provide a unique life of mine extraction sequence and destination policy which perform best given the uncertainty. Montiel et al. [4] considered geological uncertainty to optimize the production schedule of a mining complex, where mining, blending, processing, and transportation decisions are defined in one model. Kizilkale and Dimitrakopoulos [5] optimize mining rates under market uncertainty in a mining complex using dynamic programming mechanism and show the advantages of simultaneous optimizations compared to the traditional method. Another example is given in Zhang and Dimitrakopoulos [6], who account for market uncertainty and develop a decomposition method to optimize both the mining schedule and the downstream material flow plan. Other examples can be found [7]–[10].

The mentioned SSOMC methods use a geological risk discounting parameter [11] to defer risk onto later periods where more information will be available. This risk discounting works by penalizing deviations from production targets more heavily on the initial periods and reducing this penalty towards the end of the life of the operation. This risk deferral mechanism is the starting point of the methodology proposed herein, which takes this concept one step further by allowing the scenarios to deviate in later periods as much as to define different designs. At the same time, it looks at controlling the number of branches, as, even though full flexibility does allow considering the decisions that uncertainties can cause, this may result in an exponentially increasing set of design solutions that make it hard for the operation to know exactly what to do [12]. Thus, this study develops a dynamic model which is a compromise between risk management and flexibility, providing an initially clear solution for the project to follow, but at the same time, allowing the long-term design of change. This, by integrating dynamic decision-making into the optimization and formulating it as a scenario tree, able to branch as more information is available.

Thus far, stochastic mine planning optimization has focused mainly on including the effects of geological and/or market uncertainty into the formulation. New research has aimed at extending the approach to consider geometallurgical aspects of the operation. [13] As stated by Dowd et al. [14], these uncertainties and how to integrate them into the optimization process, as well as ways to include and maximize flexibility in design, are some of the main challenges present in strategic mine planning nowadays. In this paper, geological and geometallurgical uncertainty will define design flexibilities, where the goal is to transform the planning of a mining complex into a dynamic mechanism that adapts to change. To do this, a set of equally probable simulations is used to represent the spatial variability of the deposit [15].

When considering geological uncertainty, it can be agreed that no new information will be acquired over the short-term scheduled deposit until after mining, and present decisions must be taken with the information at hand. On the other hand, the long-term plans that define what to extract in over 2–5 years tend to produce an illusion of certainty over the mine design that will surely change. Because of this information acquisition and due to reporting requirements, the standard practice of mining operations is to re-optimize their life of mine plan on an annual basis as a corrective measure. However, this is a passive mechanism which can prove to be suboptimal and very expensive (relocation costs, lost contracts, etc.), [16] for example, if price increases and infrastructure is placed in strategic spots where the pit could expand, it would require great relocation costs to take advantage of the opportunity. [17] Increasing project flexibility through strategic planning has proven to be very beneficial to the project's performance and value, by planning and preparing to react timely to change [18].

The next section of this paper describes the proposed methodology, mathematical model and solving procedure implemented. Application to a mining complex comprised of one copper-gold deposit and six destinations (four processing streams, one stockpile, and one waste dump) which has been adapted for confidentiality reasons and to aid in the discussion. Conclusions follow.

2   Dynamic stochastic optimization of the mining complex

As an initial step, this methodology will focus on capital investment (CAPEX) options (particularly the purchase of an extra crusher); however, future work will focus on extending it to operational modes at both mining and processing level. In the proposed method, the optimization will not get any more information to decide on next year's investments, so this period is optimized at its best with the representation of the uncertainty given by the simulations. However, executing this initial year plan provides new information that will be used for the re-optimization of further years, and decisions could change. The proposed method repeats this information acquisition mechanism iteratively every year and allows scenarios to differ in investment decisions if this adds value. Here, if a representative number of scenarios do, the optimization allows branching the design into parallel plans, modeling the system's design decisions as a tree, divided in annual stages (as shown in Figure 1 and presented in next section).

The solving process can be divided into three steps, which are applied iteratively. Given that each production period is represented by t, where $t \, \epsilon \, \{1,\dots, \, T\}$, and T is the final production period, first, the formulation is solved as an adapted two-stage SIP (as in [19]). In traditional two-stage optimization models, the block extraction sequence corresponds to first stage variables (i.e. unique decisions over all scenarios), and the processing stream decisions to second stage variables (i.e. scenario dependent decisions that help correct decisions that were made under uncertainty). In the current model, extraction and investment decisions will be first-stage decisions, but only for the current period (t) and, for the following periods (t+1 to T), all decision variables will be considered as "second-stage", so the problem is solved individually for each scenario. This initial step is represented in the leftmost section of Figure 1, where each square is an operational year of the project, the continuous line represents a unique solution, and the dotted lines are independent solutions for each scenario. It must be noted that these scenarios are all equi-probable representations of the deposit, which respect the hard data but at the same time show the spatial variability of the orebody. If each scenario was completely optimized separately, this would produce completely different schedules (none of which would individually represent reality or the optimal design); rather, they must be accounted for together to manage the risk related to the deposit's geology. In this case, they are jointly accounted for during the first year, but left free for following ones, not to provide an actual plan, but to provide a probabilistic guidance of future investments.
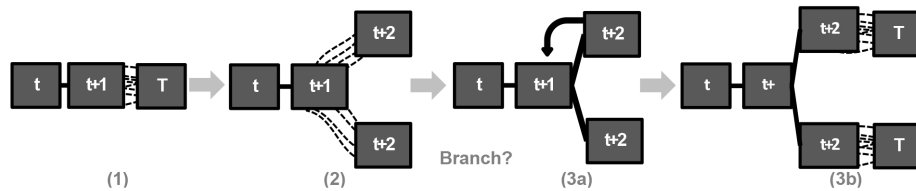

Figure 1: Steps of the solving mechanism for the proposed dynamic model

Second, the later periods (t+1,…, T) of each of the individual scenario-dependent solutions is analyzed and compared in terms of the CAPEX investment decisions taken. If a "representative number of scenarios" perform the same investment in a year (ex: set to 30% in the following case study), then the mine plan branches on that year (with and without the investment). Finally, each branch is re-optimized as in step one, fixing the initial investments per branch (as shown in step 3a of Figure 1), with extraction variables as first stage decisions during the initial year, and the following periods as scenario dependent (as presented in section 3b of Figure 1). Note that, if no representative amounts of scenarios decide to invest (or not invest), then the first period is frozen and the second period is solved robustly (i.e. with first stage decisions for t+1, and scenario dependent from t+2 to T). This process is repeated until period T.

Note that the mentioned "representative number of scenarios" is a user defined parameter (referred to as representative ratio R_t in the following sections), which is used to filter and consider only options that have the potential to add value and at the same time, a significant probability of being applied. This is summarized in Equation 1, where the decision for branching the design is defined given a predefined R_t ratio, and considering $\omega_{k,t,s}$, the binary decision variable of investment in option k, for scenario $s \in S$,

$$
\begin{cases}
branch\ solution, & if\ \frac{\sum_{s \in S} \omega_{k,t,s}}{|S|} \in [R_t,\ 1 - R_t] \\
unique\ solution, & otherwise
\end{cases}
\tag{1}
$$

Investment decisions are divided into branching and non-branching options (k^* and k respectively). The first corresponds to unique investment decisions that will have a big impact on the mining schedule (such as a new plant), but it is not clear if the investment should be made. The second group has a relatively reduced impact and/or are multiple small decisions which would make branching unpractical (such as truck purchases). By doing this, it is possible to obtain a controlled design tree that shows the range of potential evolving designs of the project.

2.1 Stochastic integer programming formulation

The proposed model uses the formulation developed in [19], where the optimization is shifted from a block-value point of view to the value of the actual product sold. Here, the author aims at simultaneously optimizing multi-mine production schedules, destination policies and processing streams under uncertainty, including CAPEX options which allow the optimizer to adapt its capacity by acquiring new equipment; all this with the objective of maximizing project value. The objective function (OF) of this model and some of the main constraints are presented next, where S is the set of equally probable orebody models, T is the set of time periods, H are hereditary attributes which are the variables that are tracked along the value chain (such as grades, throughput, etc.). The complete model can be found in the referenced text.

$$
\max \underbrace{\frac{1}{|S|} \sum_{s \in S} \sum_{t \in T} \sum_{h \in H} p_{h,t} \cdot v_{h,t,s}}_{\text{Discounted revenues and costs}} - \underbrace{\sum_{t \in T} \sum_{k \in K} p_{k,t} \cdot \omega_{k,t}}_{\text{Capital expenditure costs}} - \underbrace{\frac{1}{|S|} \sum_{s \in S} \sum_{t \in T} \sum_{h \in H} c_{h,t}^+ \cdot d_{h,t,s}^+ + c_{h,t}^- \cdot d_{h,t,s}^-}_{\text{Risk discounted penalties for deviations}}
\tag{2}
$$

$$
v_{h,t,s} - d_{h,t,s}^+ \le U_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot \omega_{k,t'} \qquad h \in H,\ t \in T,\ s \in S
\tag{2.1}
$$

$$
v_{h,t,s} + d_{h,t,s}^- \ge L_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^{t} \kappa_{k,h} \cdot \omega_{k,t'} \qquad h \in H,\ t \in T,\ s \in S
\tag{2.2}
$$

The OF contains three parts,

  i) Maximize revenue obtained from selling at a discounted price (or cost) of $p_{h,t}$ a quantity $v_{h,t,s}$ of the hereditary attribute h in period t, scenario s.
 ii) Discounts CAPEX costs, where $p_{k,t}$ represents the price (or cost) of the option, and $\omega_{k,t}$ is the decision variable that defines the number of CAPEX options k that are exercised on period t. This way, the cost of flexibility obtained from new investments is directly accounted at the OF.
iii) Finally, the penalties for deviating from production target are minimized, where $d_{h,t,s}^{+/-}$ represents the deviations of attribute h, at time t, on scenario s, and $c_{h,t}^{+/-}$ are the cost of deviation, which defers risk to later periods by using geological risk discounting factors.

Here, CAPEX decisions are scenario independent. Constraints (2.1) and (2.2) define the bounds and measure the deviation from production targets for each hereditary attribute in each orebody simulation at time t, considering the minimum and maximum limits, as well as the per-unit capacity added ($\kappa_{k,h}$). λ_k is to the lifespan of the capacity increment, and τ_k is the lead time to delivery of the investment considered.

This model provides a unique solution that performs best under all scenarios. The proposed method aims at using this formulation as a starting point, by initially transforming decision variable $\omega_{k,t}$ into $\omega_{k,t,s}$. The detailed procedure to obtain this compound solution is presented next.

The formulation aims at modeling the system's design decisions as a tree, divided by period, group of events, and events, as presented in Figure 2. The first define each stage of the tree, the second correspond to groups of parallel designs with the same "ancestor", and finally, each event corresponds to the design of that branch in a period. Here, the design and extraction sequence is unique within a branch.
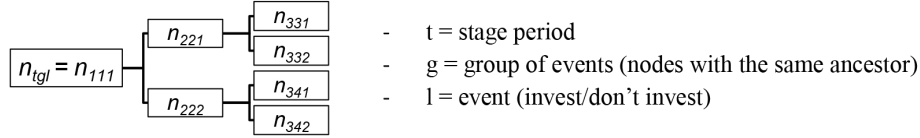

- t = stage period
- g = group of events (nodes with the same ancestor)
- l = event (invest/don't invest)

Figure 2: Tree structure used to define the branching mechanism of the proposed model

The mathematical formulation uses the same objective function presented in Equation 2, as well as the constraints (Equations 2.1 and 2.2), with some adaptations, mainly, that after the first period, 1st stage decisions also have a scenario component (as parallel designs can be created by branching). To present the formulation, the definition of the sets, list of decision variables and dynamic constraints is presented.

Sets		
K	Set of flexibilities and system options, indexed by k, where $K^*{\subset}K$ set of options that require branching over the design, and $K^o{\subset}K$ set that don't require branching.	
Ω	Set of scenarios, indexed by s = 1, ..., S, (i.e. $\int_{\forall s}\Omega_s{=}\Omega$) where $\Omega_{tgl}{\subseteq}\Omega$ Set of scenarios in node t,g,l (period t, group g, event l)	
D	Set of locations in the mining complex	
$\Theta(j)$	Set of destinations which can receive material from location $j{\in}D$.	
$z_{c,j,t,s}{\in}\{0,1\}$	Defines whether cluster c is sent to destination $j{\in}\Theta(c)$ in period t, scenario s	
$w_{k,t,s}{\in}\{0,1\}$	Defines whether investment option $k^*{\in}K^*$ is executed in period t, scenario s	
$q^{tgl}_{k*,t}{\in}\{0,1\}$	Branch design over investment option $k^*{\in}K^*$ in node tgl, period t	

Decision variables		
$x_{b,t,s}{\in}\{0,1\}$	Defines whether a block b is extracted at period t, scenario s	
$y_{i,j,s}{\in}[0,1]$	Defines proportion of material sent from i to j in period t, scenario s	
$z_{c,j,t,s}{\in}\{0,1\}$	Defines if a cluster of blocks c is sent to destination $j{\in}\Theta(c)$ in period t, scenario s	
$w_{k,t,s}{\in}\{0,1\}$	Defines whether investment option $k^*{\in}K^*$ is executed in period t, scenario s	
$q^{tgl}_{k*,t}{\in}\{0,1\}$	Defines whether to branch design over option $k^*{\in}K^*$ in node tgl, period t	

Dynamic constraints always enforced except when branching q is "activated".

Extraction decisions can be different between set of scenarios

$$\left(1-\sum_{\forall k*\in K} q_{k*,tgl}\right)\left(x_{b,(t+1),s}-x_{b,(t+1),s'}\right)=0 \quad \forall s\in\Omega_{tgl};\ \forall s'\in\Omega_{tgl'};\ \Omega_{tgl},\Omega_{tgl'}\subset\Omega;\ \Omega_{tgl}\bigcap\Omega_{tgl'}=\emptyset$$

Destination decisions can be different between set of scenarios

$$\left(1-\sum_{\forall k*\in K} q_{k*,tgl}\right)\left(z_{c,j,(t+1),s}-z_{c,j,(t+1),s'}\right)=0 \quad \forall s\in\Omega_{tgl};\ \forall s'\in\Omega_{tgl'};\ \Omega_{tgl},\Omega_{tgl'}\subset\Omega;\ \Omega_{tgl}\bigcap\Omega_{tgl'}=\emptyset$$

Investment decisions are the same if branching is not "activated".

$$\left(1-\sum_{\forall k*\in K} q_{k*,tgl}\right)\left(w_{k*,(t+1),s}-w_{k*,(t+1),s'}\right)=0, \quad \forall s\in\Omega_{tgl};\ \forall s'\in\Omega_{tgl'};\ \Omega_{tgl},\Omega_{tgl'}\subset\Omega;\ \Omega_{tgl}\bigcap\Omega_{tgl'}=\emptyset$$

Modeling a real size mining complex with multiple mines and processing streams under geological uncertainty entails more than a million binary variables, with over a million constraints. [9] If also investment decisions are

considered (result in scenario dependent extraction variables), then the problem grows even further. Thus, it is unfeasible to consider any exact solving methods, making it necessary to use an efficient heuristic mechanism. An adaptive neighborhood search simulated annealing mechanism is used to develop a good solution in a manageable amount of time. [20] Here, each decision variable defines a neighborhood, and the solution is perturbed iteratively (ex: adding one or multiple trucks if the fleet purchase neighborhood is selected), where the probability of selection of a perturbation is adapted depending on its historical performance in improving the objective function's value.

3 Case study: CU-AU mine

3.1 Mine complex configuration

The following case study corresponds to a Cu-Au mining complex with one mine, five processing destinations, a stockpile, and a waste dump. Figure 3 presents a diagram of the mining complex, specifying the products and material types allowed per destination. Each destination has variable recoveries, function of the feed head grade. Table 1 shows the mining, processing, and economic parameters, normalized by the mining cost "$x", for confidentiality purposes. In this case, 10 orebody simulations with variable copper and gold grade where used to generate a dynamic optimization of the mining complex. As an initial approach, 10 simulations seemed enough to show the method's implementation and potential, however, further studies should be made to ensure convergence.

Two options are considered (i) Invest on truck fleet (increasing extraction capacity), starting from an initial base fleet. (ii) Invest on a 2ry crusher that increases the capacity at the Sulphide Mill (which treats both Cu and Au). Further details of each CAPEX option are provided in Table 2.
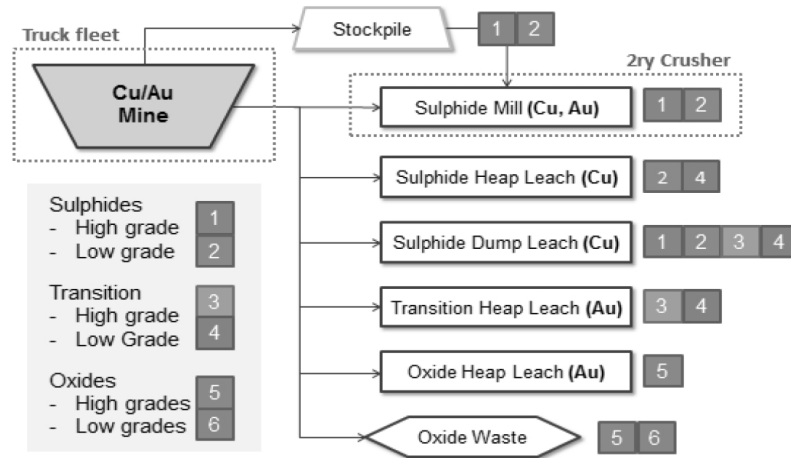

Figure 3: Diagram of the different processing streams available and the material types accepted

Table 1: Mining and economical parameters of the copper/gold mine

Mining Complex Parameters		Processing Cost Parameters		Economic Parameters	
Mining Cost	$1.00 * x	Sulfide Mill	$11.3 * x	Copper Price	$2.9/lb
Initial Mining Cap.	6 Mt	Sulfide Heap Leach	$2.98 * x	Gold Price	$1450/oz
Initial Processing Cap.	2 Mt	Sulfide Dump Leach	$1.87 * x	Discount rate	10%
Mining width	100m	Transition Heap Leach	$2.15 * x		
		Oxide Heap Leach	$2.06 * x		

Table 2: Information and purchase parameters of each investment option

	Truck (non-branching option)	2ry Crusher (branching option)
Undiscounted cost ($US)	4,000,000	26,000,000
Life of equipment (years)	6	25
Periodicity of decision (years)	2 years	once per LOM
Lead time (years)	1 year	2 years
Maximum purchase	8 units	1 unit
Initial Capacity available	15 Mt, 1st period (5 trucks)	2,500,000 tons
	6 Mt, 2nd-6th period (2 trucks)	
Tonnage constraint increment / unit	230 tons	300,000 tons

3.2 Results

3.2.1 Base case

This corresponds to the standard stochastic optimization of the mining complex [20], where a set of scenarios is used to define a unique solution. Both truck and 2ry crusher purchase options are included in the model but as 1st stage decisions. The solution provides the period each block is extracted, where it is sent, how many trucks are purchased per year (thus the annual production capacity), and if a crusher is purchased, respecting the parameters of Table 2. Figure 4 shows, on the left, the number of trucks purchased per year and the cumulative amount available (left axis), and the total extraction capacity available (full line), as well as the actual amount of material extracted per year (dotted line). In this case, the optimizer decided the 2ry crusher was not profitable. The right side of Figure 4 presents the cumulative discounted cash flow (CDCF) per scenario, ranging from M$700 to M$800 (P50 of MUS$756). This base case will be used to compare the new method proposed.
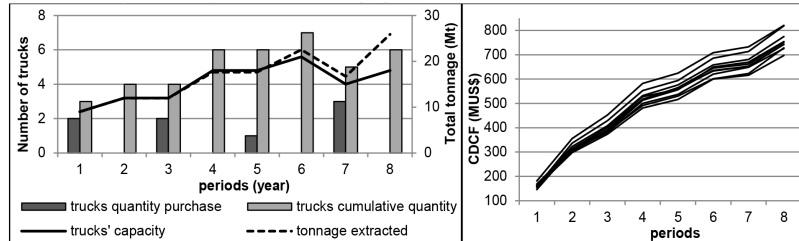

Figure 4: (left) Truck purchase (left axis) and total mining capacity (right axis). (right) CDCF per simulation
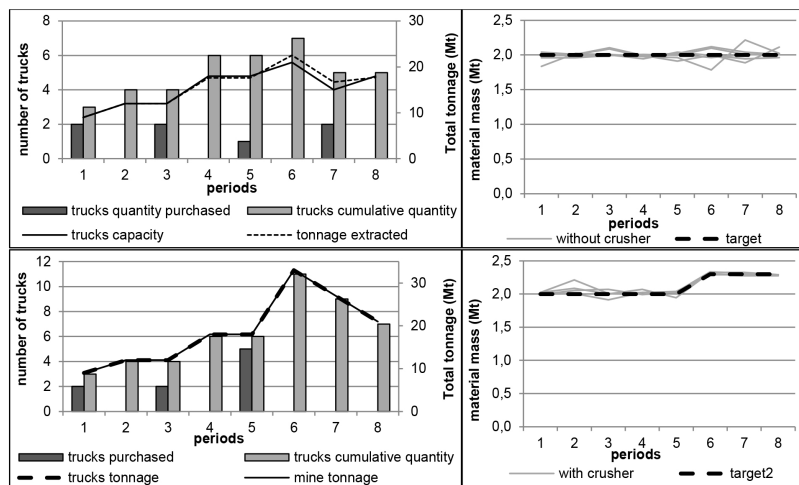

Figure 5: Truck purchase plan and mining capacity for option without (top-left) and without (bottom-left) 2ry crusher; and corresponding mill feed per period for each scenario (top and bottom right)

3.2.2 Proposed dynamic case

Step 1 – 1st stage optimization First, the model is optimized keeping the first period as 1st stage decisions, with an equal solution over all scenarios, and relaxing these decisions for the following years, leaving the solution scenario dependent. Second, the branching investment decisions of each scenario are compared and, if a representative number of scenarios ($R_t = 30\%$ in this case) decide differently over an investment on a set of years, then these scenarios are grouped and the design is branched. For the initial periods (where investment decisions are equal over all scenarios), the solution is re-optimized setting extraction and investment decisions as scenario independent, to obtain a unique design. In this case, 40% of the scenarios chose to invest in a 2ry crusher by period 4. As this 40% is above the defined $R_t = 30\%$, the project branches and the three first periods are re-optimized. Results show an investment of 2 trucks in year 1 and 2 in year 3. These investments are frozen and the final periods are re-optimized for each design branch.

Step 2 – Branching over the design Once the initial stage of the optimization is done, the design options considering and not considering the 2ry crusher investment are explored. To do this, first the blocks that were already scheduled in stage 1 are removed from the orebody model, and the model is re-solved within the remaining blocks. Also, as there was a truck purchase in the last period of stage 1 (period 3), truck purchases are not allowed for the first period of the second stage (period 4).

1. <u>Branch 1 - No investment in 2ry crusher</u>: In this case, the 2ry crusher option is removed from the SIP model, and the problem is re-optimized over periods 4 to 8. Results are shown in the top section of Figure 5, where the truck purchase plan is presented on the left side, with one truck in period 5 and two in period 7, and the mill feed is shown on the right side, presenting a stable feed of around 2Mt, with slight deviations during the last periods.

2. <u>Branch 2 - Invest in 2ry crusher</u>: In this case, the cost of the purchase is included in period 4, but the extra capacity is only available at period 6, (2-year lead time in Table 2). This can be seen on the dotted line at the bottom right side of Figure 5, which shows the mill feed target. The bottom-left side presents the truck's purchase plan, with 5 trucks bought in period 5, available just in time for when the extra mill capacity is obtained.

The joint cumulative NPV ranges between MUS$700 and MUS$900 (MUS$100 over the initial case), as presented in Figure 6, with a P50 of MUS$793, showing an increase in project value of almost MUS$40 over the initial base case and entails a tailored design that allows maximizing the project's potential.
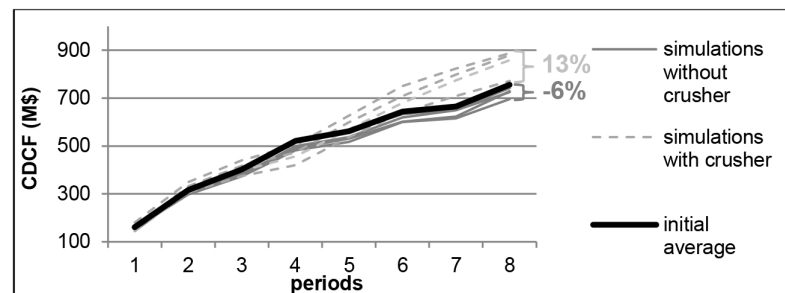

Figure 6: Cumulative discounted cash flow for each simulation considering the option of investing in a 2ry crusher (dotted blue line) or not investing (continuous red line)

4 Conclusions and future work

A dynamic SIP model was developed to include flexibility into the mine planning optimization problem. This was done by a tree structure solving mechanism which allows developing different solution designs. A case study over a copper-gold open pit mine with six possible destinations was presented, including two flexibility CAPEX investment options, (i) truck fleet to manage production capacity, and (ii) a 2ry crusher to increase

the mill's processing capacity. Results showed that 40% of the scenarios decided to invest in the crusher by year 4, representing an overall increase in NPV of M$40 compared to the initial two-stage SIP solution.

In conclusion, by applying this flexible formulation, it is possible to identify and actively include interesting options that might not be profitable now, but could be valuable in the future, easing the transition to change and allowing the project to be better prepared for it. This, to have the flexibility to alter the mine plan as more information is obtained, allowing the operation to be better prepared for uncertainty, and take full advantage of opportunities while hedging risk. Future applications will focus initially in performing a more in-depth study over the number of scenarios required to ensure convergence of the solution. Also, coming work will concentrate on including more complex variables affecting the system, such as hardness (i.e. SPI, BWi), recovery and throughput.

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