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COSMO Suite: A platform for optimizing mining complexes with uncertainty

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Abstract: Over the past several years, there has been substantial progress in developing new stochastic mine planning optimization models and computationally efficient solvers that are capable of managing risk throughout the entire mineral value chain. Recent models have focused on modelling the economic value of the products sold to customers, rather than attempting to evaluate the economic value of the individual mining blocks. This modelling shift permits the integration of many aspects that could not be previously considered, such as the ability to incorporate non-linear geometallurgical interactions in the processing streams. These advanced models have consistently shown the ability to generate a higher net present value than traditional deterministic optimization methods, which is a direct result of risk management. Despite these developments and consistent value-added results, existing commercial mine planning optimization tools have been reluctant to incorporate these new concepts in their platforms.

This paper provides an overview of the ongoing development of COSMO Suite, a platform designed to aid in the dissemination of new stochastic mine planning optimization algorithms, techniques and models to the mining industry. COSMO Suite provides a graphical user interface to create advanced multi-mine and multi-process models for mineral value chains, create tailored optimization models and perform risk analyses on existing mine designs. This software is driven by COSMO Suite Library, a C++ application programming interface that compartmentalizes the process of modelling the mineral value chain, the optimization objectives and constraints, and the solvers, which allows for the rapid development, testing, benchmarking and deployment of new stochastic optimization models and algorithms. As a result, users can benefit from the accelerated knowledge transfer from an academic setting to industrial use.

Keywords: Open pit production scheduling, stochastic optimization, software, development library

1 Introduction

Existing commercial long-term mine planning packages generally employ a combination of heuristic and exact optimization techniques to solve real-world deterministic mine design and production scheduling optimization models that have been in existence for several decades (Lerchs and Grossmann, 1965; Johnson, 1968; Picard, 1976; Dagdelen, 1985; Tolwinski and Underwood, 1996; Caccetta and Hill, 2003). As the performance of desktop computers has exponentially increased, these tools have gradually been adopted by the mining industry to quickly analyze solutions and unlock hidden value in a mining operation. Despite these computational advancements, most commercial tools have remained relatively stagnant in their ability to implement modern concepts and solving techniques. As a result, some groups have developed their own proprietary, in-house tools (Hoerger et al., 1999; Stone et al., 2007; Menabde et al., 2007), however, continued development of these tools is subjected to the cyclic nature of the mining industry and the inertia of long-used workflows.

Traditional mine planning tools consider a single model as input to the optimization process, thus do not consider the inherent uncertainty in the supply of materials (i.e., geological uncertainty), the technical performance (recoveries, throughput rates), and the demand for the final products (metal prices). As a result, traditional optimizers create a design that can be severely misleading when the performance of the design is tested with a set of simulations that better describe the uncertainty in the value chain (Ravenscroft, 1992; Dimitrakopoulos et al., 2002; Godoy and Dimitrakopoulos, 2004; Dimitrakopoulos et al., 2005). This issue is exacerbated by the fact that traditional geostatistical estimation techniques tend to smooth out the low- and high-grade materials and do not reproduce the spatial characteristics of the deposit. To overcome these severe limitations, stochastic optimization models have been developed to integrate geological uncertainty into the mine design and production scheduling process, with a particular emphasis on scheduling for open pit mines (Godoy, 2003; Ramazan and Dimitrakopoulos, 2004; Ramazan and Dimitrakopoulos, 2013). Using a set of conditional grade simulations (Journel, 1974; David, 1988; Goovaerts, 1997) as a group to describe the volumetric and grade uncertainty for a given deposit, these optimizers aim to generate a mine design that not only attempts to maximize the net present value (NPV) of the schedule, but also explicitly manages the risk of not meeting ore or metal production targets. As a result, these optimizers provide a higher level of confidence that production, hence financial, forecasts will be attained, particularly in the early years of production where there is a large emphasis on recouping capital investments, during which information about the deposit is limited. Despite consistently demonstrating that it is possible to create mine designs with higher NPV and also have substantially less risk (Godoy, 2003; Jewbali, 2006; Albor and Dimitrakopoulos, 2010; Benndorf and Dimitrakopoulos, 2013; Leite and Dimitrakopoulos, 2014), these developments have not yet been adopted by commercial mine design packages. Recently, Minemax (Butler, 2015) provides the ability to perform risk analysis to test the sensitivity of their deterministic designs using a set of simulated orebody models. This is a critical step on the road to making stochastic mine planning a standard industry practice, however, is still a long way from implementing the aforementioned developments.

More recently, stochastic optimizers have been expanded to deal with the global optimization of mining complexes (Whittle, 2007; Whittle, 2009). The global optimization of mining complexes aims to simultaneously optimize multi-mine extraction sequences, which define the distribution of materials over time, the destination policies, which define where extracted materials are sent, and the use of the various processing streams for processing, distribution and product marketing. This is generally considered to be an extremely difficult optimization problem to solve, particularly in conjunction with the scale of open pit production scheduling problems. This difficulty largely due to non-linearities in the optimization model, such as stockpiles (Bley et al., 2012; Moreno et al., 2015), grade-recovery curves and the complex multi-commodity flow through the various multi-tier processing streams. Stochastic global optimization for mining complexes attempts to maximize the NPV of the entire mineral value chain, while simultaneously minimizing the impacts of risk in the quantities and qualities of all products generated, from the mines through to the final customer. Goodfellow (2014) proposes a new form of multi-element destination policy to be used in lieu of traditional cut-off grades; this new destination policy is more appropriate for mining operations with multiple elements or strict product quality requirements. Moreover, the author develops a unified modelling approach for mining complexes, and considers advanced concepts, such as simultaneously optimizing capital expenditures under uncertainty, which may be used to decide on mine production rates and processing expansion decisions. Montiel (2014) investigates the use of operating and transportation alternatives to model the flexibility for processing streams to dynamically tune throughput rates, recoveries and alternative routings to minimize risk in the value chain. Additionally, this the author considers simultaneous open pit and underground mine production scheduling.

The development of these stochastic global optimization models has been made possible through the development of computationally efficient optimizers, such as metaheuristics. While these algorithms do not guarantee mathematical optimality, they are capable of solving large-scale, real-world and non-linear optimization mine planning models (Godoy, 2003; Albor and Dimitrakopoulos, 2009; de Freitas Silva et al., 2014; Goodfellow and Dimitrakopoulos, 2015; Montiel and Dimitrakopoulos, 2015) in a reasonable amount of time, and are often near-optimal (Lamghari et al., 2012; Senecal and Dimitrakopoulos, 2014; Lamghari et al., 2015). Lamghari and Dimitrakopoulos (*in this volume*) propose a hyper-heuristic optimization algorithm that is capable of dynamically tuning itself in order to efficiently solve a wide variety of mine production scheduling problems. This approach is extremely interesting for the future of mine planning optimization because all previous developments in mine planning optimization over the past several decades, both deterministic and stochastic, can be viewed as smaller heuristics to solve the global stochastic optimization problem. As a result, future research can focus on the rapid development of these heuristics in the context of an overarching solver that automatically learns when to apply the heuristic to maximize its effectiveness.

At the request of COSMO's mining consortium, a new, modern stochastic mine planning platform has been developed to help the industry to move towards a stochastic mine planning framework and to disseminate the rapid advances in optimization models and solution techniques. This platform provides access to modern mine planning tools for industrial purposes, and may also be used by developers to create their own optimization algorithms that can easily and consistently be benchmarked against existing methods. The purpose of this paper is to acquaint readers with the basic functionality of the software. First, an overview of *COSMO Suite*'s graphical user interface (GUI) and some of its features is provided. Following this, the backend C++ development library is briefly discussed, followed by conclusions.

2 Modelling and optimizing with COSMO Suite

This section provides an overview of the features of the *COSMO Suite* GUI. Figure 1 shows the home screen for the software. After authenticating the encrypted login credentials, a user may use this window to create, open, duplicate or save a project, and also quickly access COMSO's website. Once a project has been created or opened, it is possible to use the menu bar at the top to access all of the modelling, optimization, analysis and visualization features provided by the software, which are discussed in the subsequent sections. All of the relevant project parameters, including the model of the flow of materials through the mineral value chain, the optimization constraints and objectives, solver parameters and risk analysis formatting are stored in a standard, non-proprietary XML file format. While the C++ backend that powers the optimization algorithms has been developed specifically to read this format to create the relevant data structures (objects), existing software (Lamghari et al., 2012; Senecal and Dimitrakopoulos, 2014) can interact with *COSMO Suite* by modifying the code that reads the parameter files. As a result, it is not necessary to completely rewrite existing software to benefit from *COSMO Suite*'s modelling interface.

2.1 Customizing data sets

Prior to creating the model of the mineral value chain, it is first necessary to specify all of the relevant data sets (groups of files) that will be used in the optimization model. Users can input a wide variety of data into the models, such as orebody simulations, grade-recovery curves, metal price simulations, among others. Figure 2 shows an example of a window with data sets loaded. In this case, the model contains a set of 10 orebody simulations, where each file contains a list of blocks, which are defined by their spatial coordinates, material code, mass, copper and gold mass and zone codes. In its current form, the software is able to read data sets formatted as tables (space- or tab-delimited), GSLIB format and Whittle's PIL format. Over time, a wide variety of filters will be implemented to be able to read and automatically process other file formats.

2.2 Modelling mineral value chains

After the data sets have been loaded, it is then possible to create a model of the various locations in the mineral value chain to be considered (e.g., mines, stockpiles, mills, autoclaves, ports and customers). Each location stores a set of products (e.g., blocks, stockpile material, concentrate, metal sold to customers) that may be forwarded on to another location for further treatment. Value chain locations are classified into two groups: sources (mines) and destinations. The primary distinction between these two classes is that sources are, in the present implementation, unable to receive materials from other locations in the value chain (i.e., aspects such as in-pit dumping are not currently considered). Typically, blocks serve as the smallest mining unit for open pit mine production scheduling decisions. Future research will investigate the use of irregular grids or a combination with regular grids, which can be used for other advanced cases, such as underground stope or bench sequencing. Each product is associated with a set of attributes, which are quantities of interest to describe the product (e.g., metal grade and tonnage) or are of use to the optimization model (e.g., revenues, costs and total trucking hours to get the mined material).

Each location can be customized with its own name, image and years of active operation, and is assumed to produce a set of products that may be forwarded on to other value chain locations. After defining the materials at the various locations, the flow of materials inside and between the locations is defined by graphically linking them together in the value chain model window. Material will not flow between locations unless this link is defined. Figure 3 shows an example of a single copper-gold mining complex, where the mine produces a set of six materials. In this example, the sulphides can be sent to a stockpile or the mill directly, however, they cannot be sent to the oxide heap leach because they have not been connected. It is often beneficial to create separate "input" materials at a location to model the geometallurgical or cost attributes independently. For example, in Figure 3, the mine and stockpile materials received by the sulphide mill have been separated in order to provide accurate accounting for reclamation costs for the processed stockpiled material in each year.

Attributes are pieces of information of interest that quantify the characteristics of the materials (e.g., tonnage, metal content) or are of general interest to the optimization model (e.g., mining costs, total quantities mined, recoveries, throughput rates and energy consumption). There are four main types of attributes that can be defined:

i) Data set attributes, which relate to information stored in a file (e.g., block grades).
ii) User-defined expressions, which may include pre-defined parameters and be used to create customized non-linear functions of other attributes.
iii) Attributes from look-up tables, where an output value (e.g., recovery) is derived from a relational table given another input attribute (e.g., grade).
iv) Optimization attributes, where the optimizer decides the values dynamically (e.g., capital expenditures to decide how many trucks to purchase).

Figure 4 shows an example of an expression attribute (sulphide mill profits), which is defined as a function of the total gold and copper masses, their respective recoveries derived from grade-recovery curves, and simulated metal prices; all input attributes to the expression are referenced directly by their name provided by the modeller. The software automatically suggests the names of other relevant attributes as the user types, which substantially expedites the process of defining expressions and helps to reduce errors in the model. Finally, it is noted that the GUI serves as an important tool for understanding how modern stochastic global optimization models defer substantially from existing approaches for open pit planning: by modelling the flow of materials and the non-linear transformations of attributes in the processing streams, it can be visually seen that the software focuses on modelling the economic value of the products sold at the end of the value chain, rather than the individual blocks located at the mines (sources).

Figure 1: Home window for *COSMO Suite*.

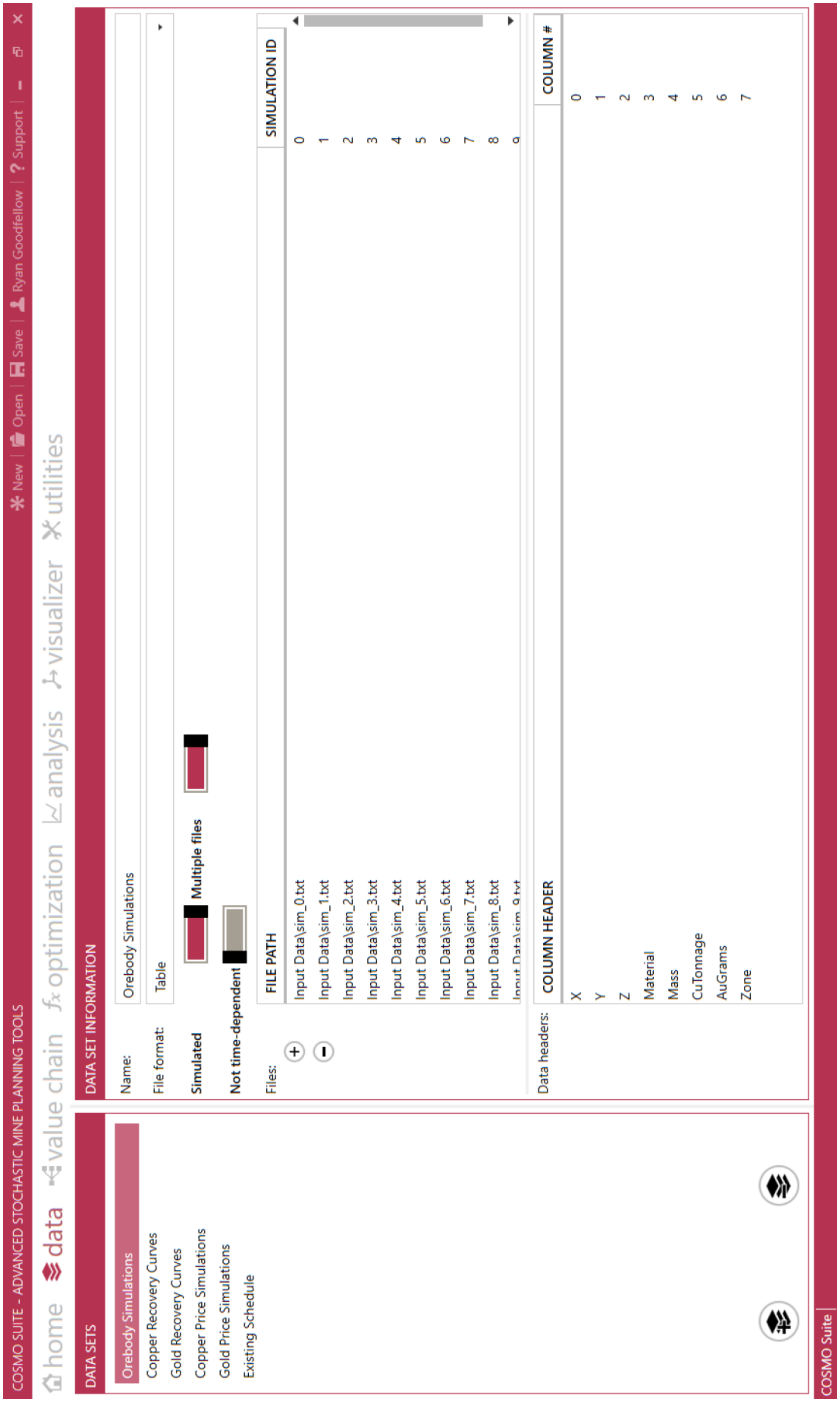

Figure 2: Setting up data sets for orebody simulations, grade-recovery curves and metal price simulations.

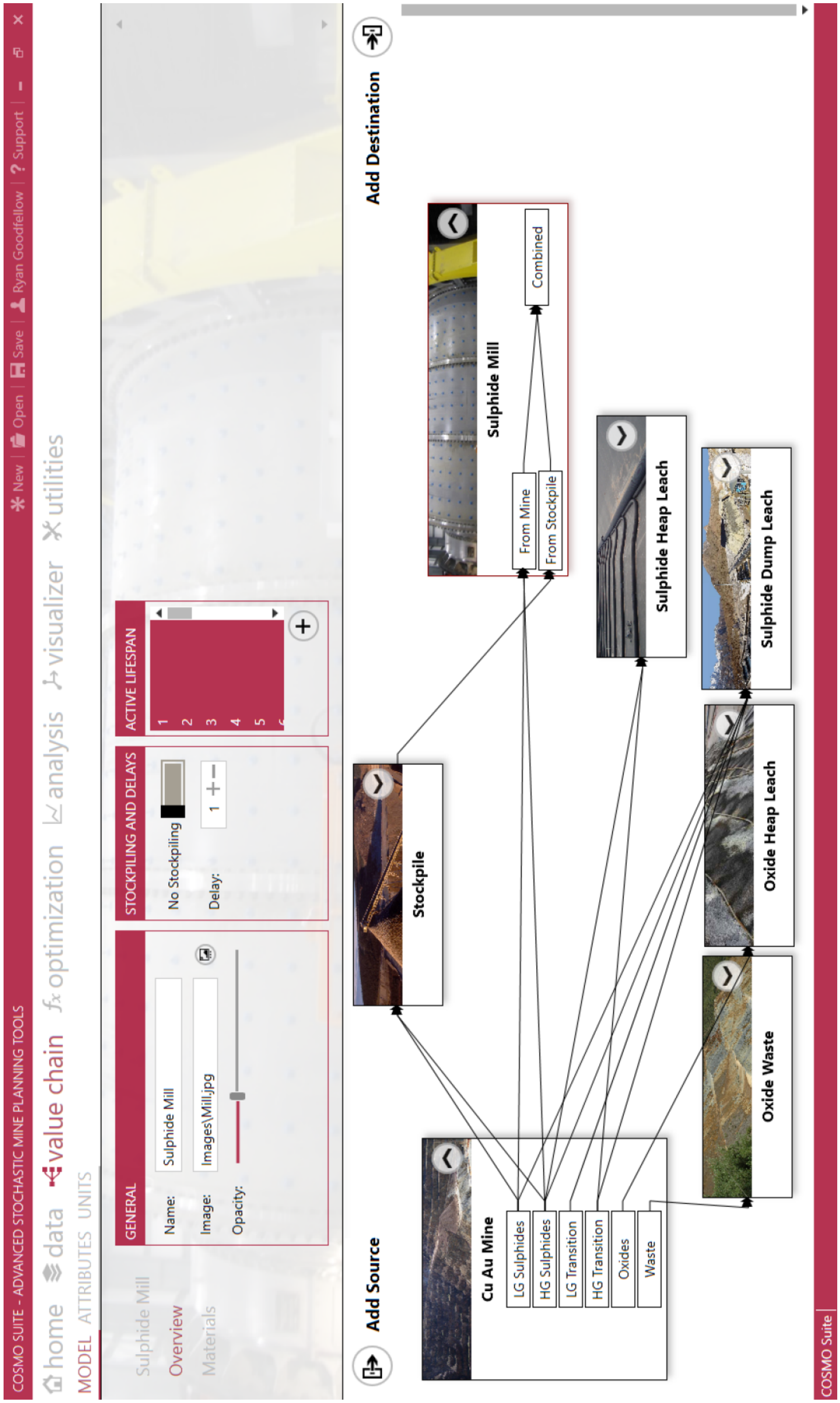

Figure 3: Interface for modelling the various processing streams for the materials produced in the value chain.

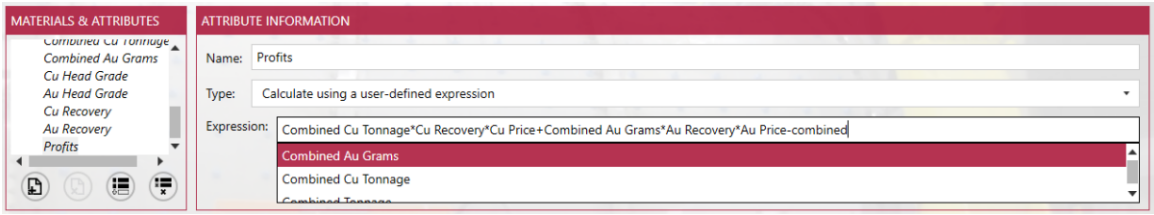

Figure 4: Attributes can be fully customized by creating them from user-defined expressions. The interface will automatically make suggestions based on what the user is typing.

2.3 Creating tailored optimization models

Conventional mine planning optimization tools typically provide the default objective of maximizing NPV, subject to mining, stockpiling, processing and metal production constraints. There is often a need for more advanced objectives and constraints that cannot be easily modelled using software, particularly for constraints on secondary elements, which can have a drastic effect on the performance of the optimization algorithm used and to the quality of the “optimized” solution. *COSMO Suite* provides a substantial amount of flexibility in creating a customized optimization model, including the objectives and constraints, in order to create a solution that satisfies the unique characteristics of the value chain being optimized.

After the model of the mineral value chain is complete, it is possible to impose constraints on any attribute in the value chain model. A constraint can be defined as being a less-than-or-equal-to, greater-than-or-equal-to or an equality constraint, where the bound may be a time-dependent (or simulated) value that is manually entered (e.g., specifying a maximum silica-to-magnesia ratio of 1.8), or the bound may be defined by the value of another attribute (e.g., the maximum number of trucking hours for a mine, which is a function of the number of trucks that the optimizer has decided to purchase). Additionally, it is possible to impose constraints on the amount of annual change for attribute values, which may, for example, be used to ensure mining capacities do not fluctuate wildly between periods. It is noted that, depending on the optimization technique, these constraints are not necessarily considered as hard constraints that can never be violated. In line with existing two-stage stochastic integer programming (SIP) models for mine planning with uncertainty (Birge and Louveaux, 1997; Ramazan and Dimitrakopoulos, 2013), constraints on value chain model attributes are often considered to be soft constraints, whereby any constraint violation in any scenario will be calculated and included as a penalty in the objective function. Optimization models can also incorporate different types of scheduling constraints, such as slope angles for open pit mines, which are treated as hard constraints (i.e., never violated), in addition to maximum sink rate and smoothness constraints, which are treated as soft constraints by penalizing schedules that go excessively deep or do not provide a practical mining shape. Alternatively, future versions will provide the ability to incorporate constraints defined in an input file, which will permit the ability to model advanced start, finish and relational constraints that are useful for modelling underground production scheduling problems.

After the constraints have been defined, it is then possible to model the objective function (Figure 5). Any attribute in the value chain model can be directly integrated in the objective function. Typical mine production scheduling models aim to maximize the NPV of the schedule, thus some attributes of interest are mining costs, stockpile reclamation costs, capital expenditures, processing costs and profits from the sale of the final products. Penalties for deviations for any of the value chain and scheduling constraints can also be included in the objective function. The impact of these penalties can be controlled through a penalty cost and a risk discount rate. Penalty costs allow the modeller to define the desired level of control on the risk profiles to ensure the various constraints are satisfied with minimal risk. The risk discount rate is used a tool to ensure that there is a high probability that constraints (e.g., ore production targets) are respected at the beginning of the mine life, and riskier material is deferred to later periods in the life of mine when more information is available (Benndorf and Dimitrakopoulos, 2013; Ramazan and Dimitrakopoulos 2013; Leite and Dimitrakopoulos, 2014).

As existing algorithms are adapted and new algorithms are developed using *COSMO Suite Library*’s C++ application programming interface (API), they can be integrated via the optimization window as additional plug-ins, which will allow for a seamless experience, from modelling through risk analysis. A user can select the optimizer they wish to use, and calibrate any of the relevant parameters in the plug-in window. An example of this interface is shown in Figure 6 for the simulated annealing algorithm that comes packaged with *COSMO Suite* by default. The progress of the optimizer can easily be viewed, paused or stopped from this window.

2.4 Risk analysis and visualization

After running the optimization, all results are stored in the project's working directory, and a risk profile for any attribute can be easily visualized (Figure 7) and saved to insert into documents. Many aspects, such as the graph and axes titles, font sizes and line styles can be modified, and the settings can be used as a template and applied to other risk profiles. Moreover, it is possible to plot the P-10/50/90 values to provide summary statistics, relevant constraints, and cumulative profiles to analyze the accumulating impact on risk on key attributes such as the discounted cash flows. By integrating this risk profile tool directly in the interface, it allows the user to rapidly analyze the quality of the solutions provided by the optimizer and calibrate the critical SIP parameters (penalty costs, risk discount rates) to achieve desirable risk profiles and project value.

Finally, COSMO Suite provides an integrated 3-D visualizer (Figure 8) that can be used to view any attribute stored on a regular grid, such as grades, tonnages and the optimized production schedule. Data can be filtered based on the values of any collocated attribute, which may be used, for example, to only visualize mined blocks in the first year. The visualizer also features a simple cross-section tool that allows for mixed cross-sectional and 3-D views (Figure 8).

2.5 Developing with COSMO Suite Library

The backend of *COSMO Suite* is based on a customized C++ library that has been designed for cross-platform portability and efficiency. Similar to the GUI, *COSMO Suite Library* compartmentalizes the models of the mineral value chain, optimization formulation and solvers. All aspects have been designed to be modular and provide flexibility for future research through the use of object factory design patterns, which can be used to automatically create new types of derived classes from base objects with minimal coding effort. For example, a developer can create new types of value chain locations, attributes, decision variable neighbourhoods, solution perturbations and algorithmic optimizers – all of which can be automatically registered and integrated into existing algorithms developed on the *COSMO Suite Library* platform. Using this approach, the developer only needs to focus on implementing the new features, without being concerned about the interactions between the various objects that occur in the rest of the model. Observer design patterns are also used to allow objects to subscribe to events (notifications) raised by other objects and adapt appropriately. This provides a powerful mechanism for the model to communicate when changes are made, and respond to these changes autonomously, such as when a change in a production schedule would alert all other downstream decisions that they need to be updated. This decoupled approach allows objects to be interchangeable, thus new ideas (e.g., a game theory-based destination policies, see del Castillo and Dimitrakopoulos, *in this volume*) can be tested and benchmarked against existing concepts (e.g., cut-off grade policies) without having to go through the process of identifying and rewriting tightly coupled code (e.g., block extraction sequence changes that force updates to destination decisions). Looking ahead, *COSMO Suite Library* will become a core aspect for developing new, smart optimizers that are capable of dynamically self-tuning their parameters and solution strategies (Lamghari and Dimitrakopoulos, *in this volume*) according to the structure of the mineral value chain and optimization formulation.

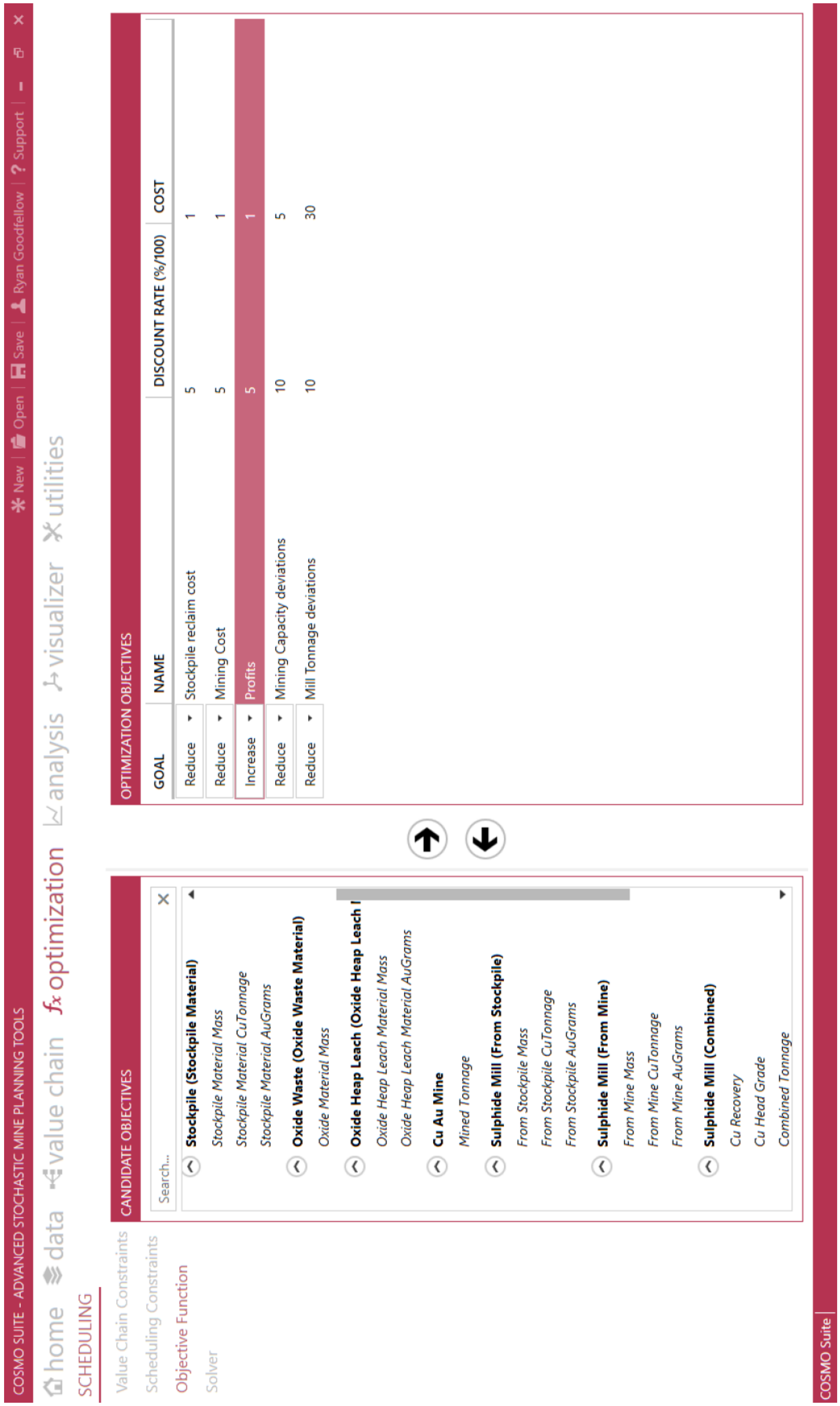

Figure 5: Creating an optimization model consists of defining the constraints in the value chain model and customizing the objective function for the unique needs of the operation.

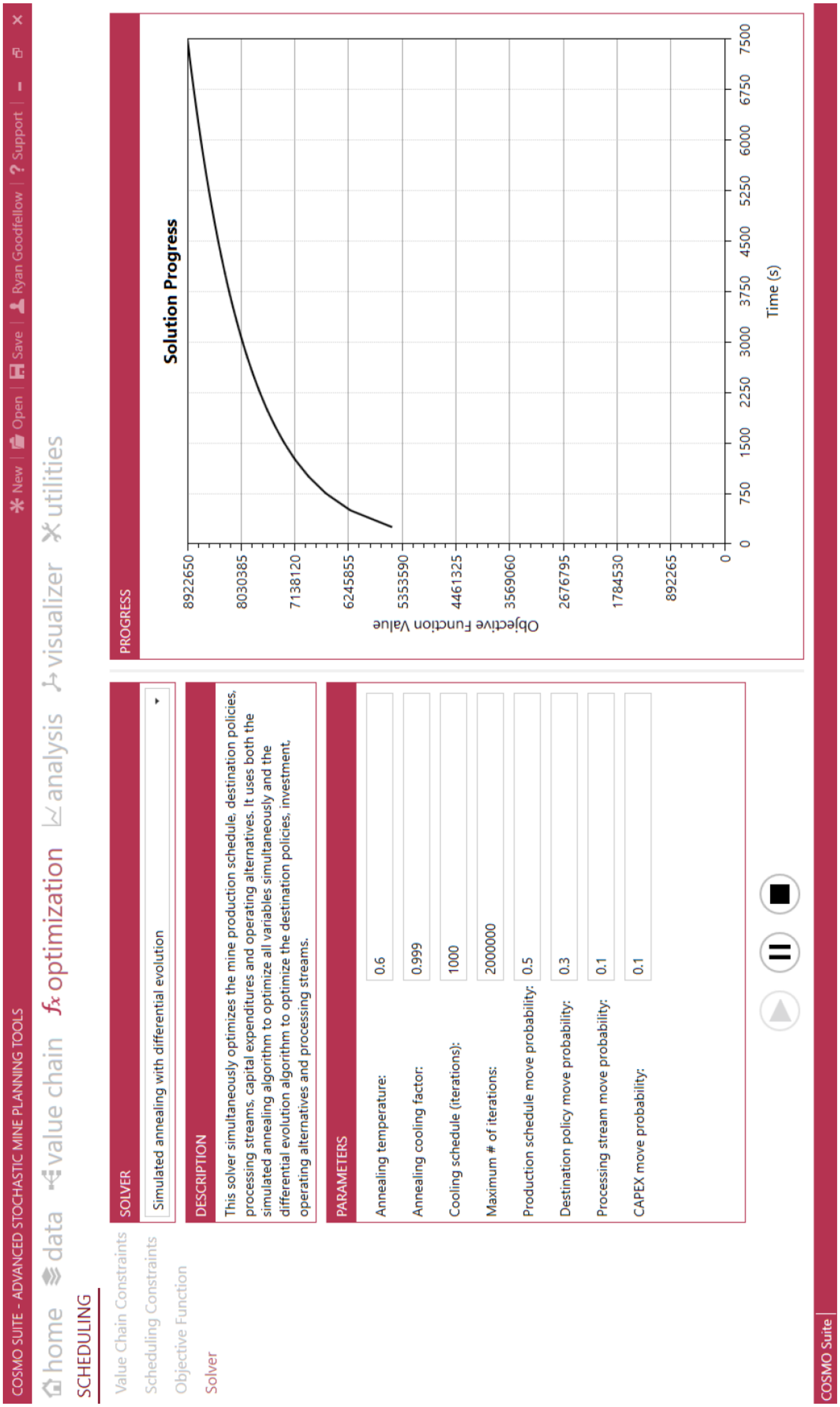

Figure 6: Solver selection, parameter configuration and solution progress.

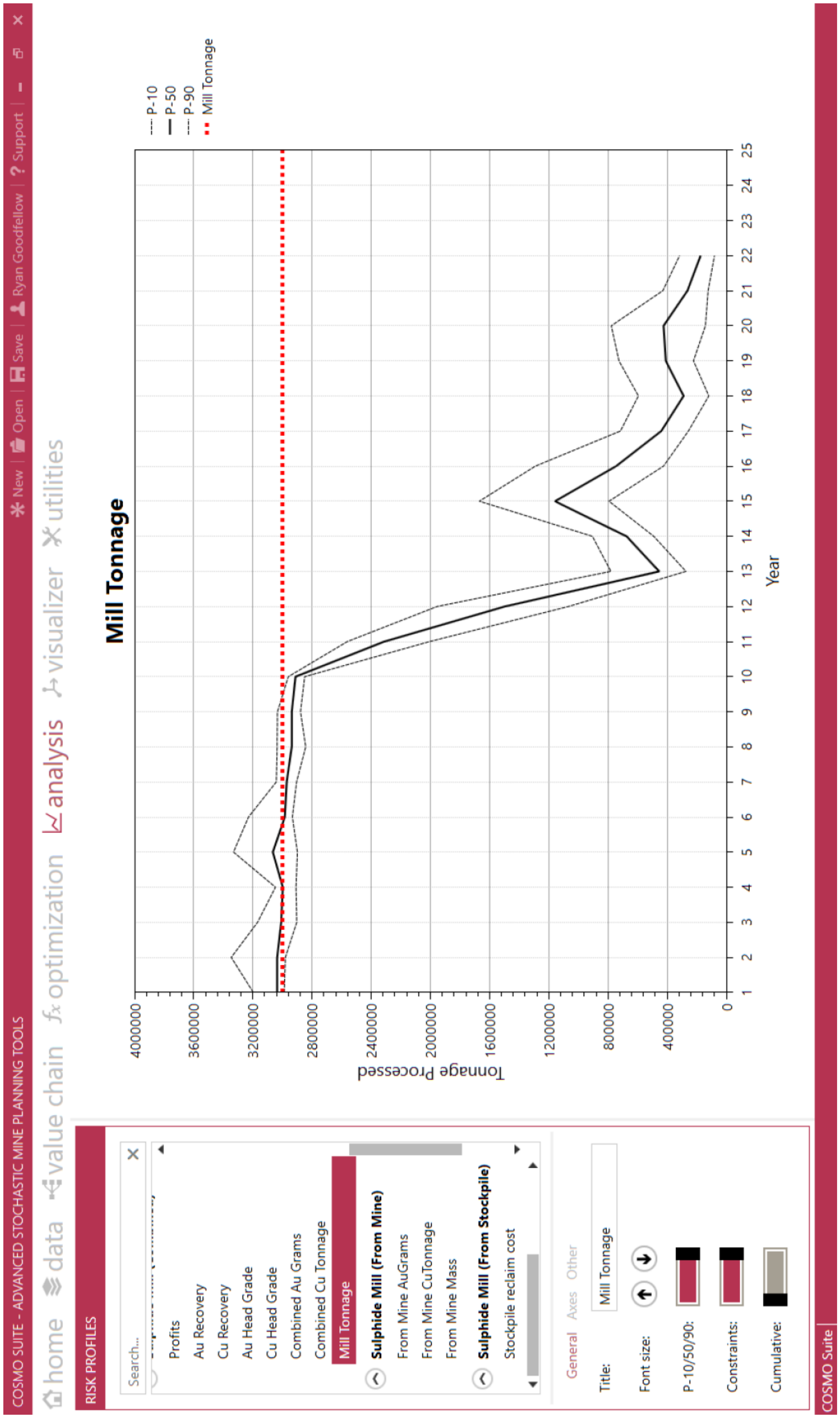

Figure 7: Risk profiles can be plotted to quickly identify risky aspects of the optimized design.

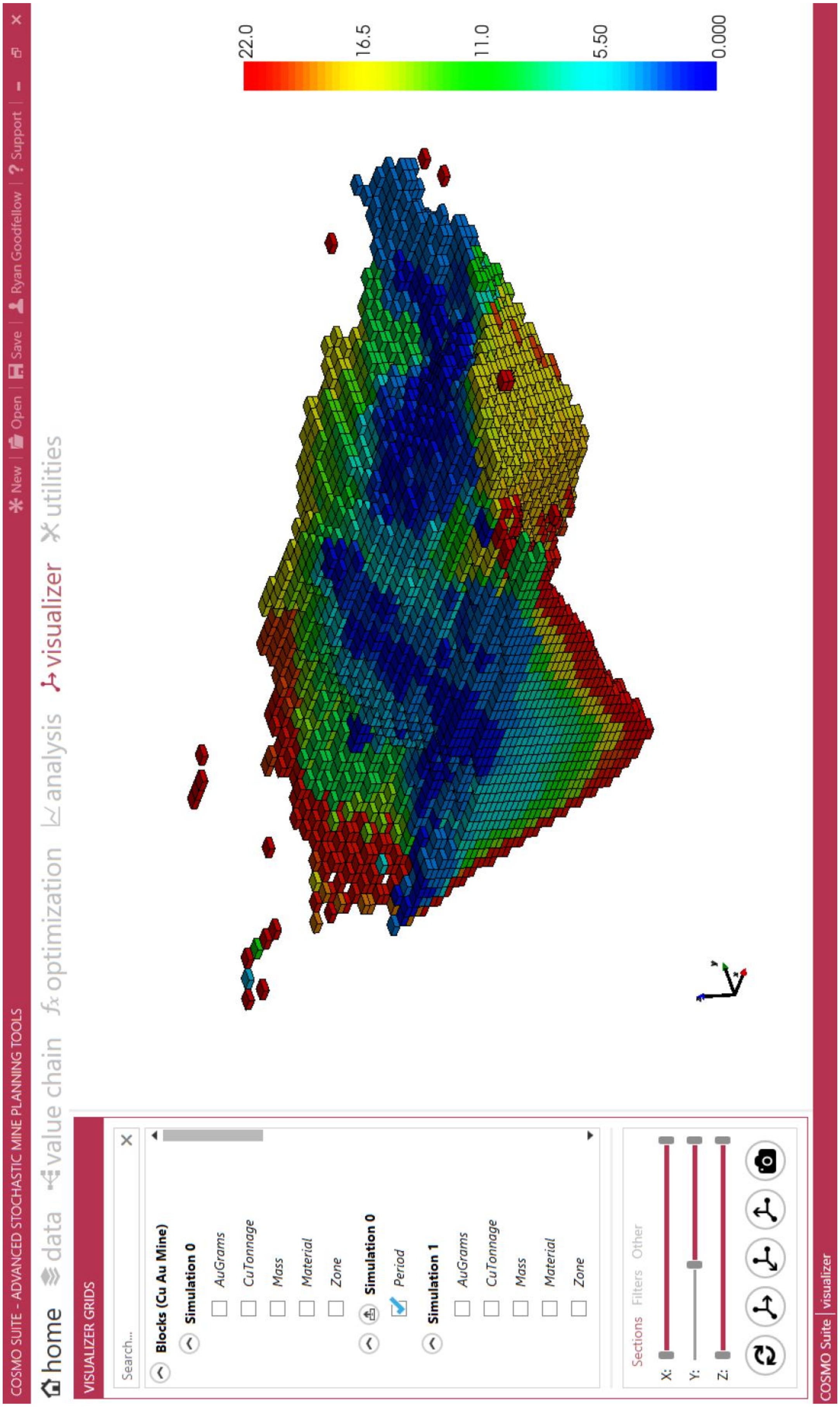

Figure 8: The visualizer may be used to plot cross-sections and 3-D views of the orebody simulations and schedules.

3 Conclusions

For over a decade, stochastic optimization models and computationally efficient solvers have been developed for mine design and production scheduling with uncertainty. Despite growing interest and demand, these technologies have not yet been widely adopted and tested in an industrial setting because of the lack of commercial tools. Given the rapid developments in the field of stochastic global optimization, which aims to optimize a mineral value chain, from the pits through to the customer, there is an increasing need to rapidly disseminate these concepts and technologies for practical use. This paper provides a general overview of *COSMO Suite*, a new tool that provides a means for these recent developments to be widely used. This software aims to simplify the process of creating complex models of mineral value chains and optimization formulations, calibrating parameters, analyzing risk profiles and visualizing the results. The backend C++ development library provides a substantial amount of flexibility, which is a core strategy for continued development and support. This software is free for industrial sponsors, and will be available for testing in an industrial setting in the very near future.

References

Albor Consuegara, F. and Dimitrakopoulos, R. (2009) Stochastic mine design optimization based on simulated annealing: pit limits, production schedules, multiple orebody scenarios and sensitivity analysis. IMM Transactions, Mining Technology, 118(2) : 79–90.

Albor Consuegara, F. and Dimitrakopoulos, R. (2010) Algorithmic approach to pushback design based on stochastic programming: method, application and comparisons. IMM Transactions, Mining Technology, 119(2) : 88–101.

Benndorf, J. and Dimitrakopoulos, R. (2013) Stochastic long-term production scheduling of iron ore deposits: Integrating joint multi-element geological uncertainty. Journal of Mining Science, 49(1) : 68–81.

Bley, A., Gleixner, A. M., Koch, T., Vigerske, S. (2012) Comparing MIQCP solvers to a specialized algorithm for mine production scheduling. Modeling, Simulation and Optimization of Complex Processes: 25–39.

Birge, J. R., Louveaux, F. (1997) Introduction to stochastic programming. Springer Series in Operations Research, 2nd edition, Berlin.

Butler, J. (2015) Managing uncertainty and risk in mining projects, online article: http://www.mineoptimization.com/managing-uncertainty-and-risk-in-mining-projects/, Accessed: Feb. 5, 2016

Caccetta, L. and Hill, S. P. (2003) An application of branch and cut to open pit mine scheduling. Journal of Global Optimization, 27(2-3) : 349–365.

Dagdelen, K. (1985) Optimum multi-period open pit mine production scheduling. PhD Thesis, Colorado School of Mines, Golden, CO, USA.

David, M. (1988) Handbook of applied advanced geostatistical ore reserve estimation. Developments in geomathematics, Elsevier, 232 p.

De Freitas Silva, M., Dimitrakopoulos, R., Lamghari, A. (2015) Solving a large SIP model for production scheduling at a gold mine with multiple processing streams and uncertain geology. Mining Technology, 124(1) : 24–33.

Del Castillo, M. F. and Dimitrakopoulos, R. (2015) A multivariate destination policy for geometallurgical variables in mineral value chains using coalition-formation clustering. COSMO Report No. 9, *in this volume*.

Dimitrakopoulos, R., Farrelly, C.T., Godoy, M. (2002) Moving forward from traditional optimization: grade uncertainty and risk effects in open pit design. Transactions of Institute of Mining and Metallurgy (Section A: Mining Technology), 111 : A82–A88.

Dimitrakopoulos, R., Martinez, L., Ramazan, S. (2005) Optimising open pit design with simulated orebodies and Whittle Four-X – a maximum upside/minimum downside approach. In proceedings, Orebody modelling and strategic mine planning, AusIMM Spectrum Series: 181–186.

Godoy, M. (2003) The effective management of geological risk in long-term production scheduling of open pit mines. PhD thesis, University of Queensland, Brisbane, Australia.

Godoy, M. and Dimitrakopoulos, R. (2004) Managing risk and waste mining in long-term production scheduling. SME Transactions, 316 : 43–50.

Goodfellow, R., (2014) Unified modelling and simultaneous optimization of open pit mining complexes with supply uncertainty. PhD Thesis, McGill University, Montreal, QC, Canada.

Goodfellow R. and Dimitrakopoulos R. (2015) Global asset optimization of open pit mining complexes under uncertainty. Applied Soft Computing, 40 : 292–304.

Goovaerts, P. (1997) Geostatistics for natural resources evaluation. Oxford University Press, 496 p.

Hoerger, S., Hoffman, L., Seymour, F. (1999) Mine planning at Newmont's Nevada Operations, Mining Engineering, 51(10) : 26–30.

Jewbali, A. Modelling geological uncertainty for stochastic short-term production scheduling in open pit metal mines. PhD Thesis, University of Queensland, Brisbane, QLD, Australia.

Johnson, T.B. (1968) Optimum open pit mine production scheduling. PhD Thesis, University of California, Berkeley, CA, USA.

Journel, A. G. (1974) Geostatistics for conditional simulation of ore bodies. Economic Geology, 49 : 673–687.

Lamghari, A. and Dimitrakopoulos, R. (2012) A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. European Journal of Operational Research, 222(3) : 642–652.

Lamghari, A., Dimitrakopoulos, R., Ferland, J. A. (2015) A hybrid method based on linear programming and variable neighborhood descent for scheduling production in open-pit mines. Journal of Global Optimization, 63(3) : 555–582.

Lamghari, A. and Dimitrakopoulos, R. (2015) Hyper-heuristic approaches for solving stochastic optimization formulations of mineral value chains, COSMO Report No. 9, *in this volume*.

Leite, A. and Dimitrakopoulos, R. (2014) Stochastic optimization of mine production scheduling with uncertain ore/metal/waste supply. International Journal of Mining Science and Technology, 24(6) : 755–762.

Lerchs, I. and Grossmann, I. F. (1965) Optimum design of open pit mines. CIM Bulletin, 58 : 47–54.

Menabde M., Froyland G., Stone P., Yeates, G. (2007) Mining schedule optimization for conditionally simulated orebodies. In proceedings, Orebody modelling and strategic mine planning: Uncertainty and risk management models, AusIMM Spectrum Series 14, 2nd Edition: 379–384.

Montiel, L. (2014) On globally optimizing a mining complex under supply uncertainty: Integrating components form deposits to transportation systems. PhD Thesis, McGill University, Montreal, QC, Canada.

Montiel, L. and Dimitrakopoulos, R. (2015) Optimizing mining complexes with multiple processing and transportation alternatives: an uncertainty-based approach. European Journal of Operational Research, 247(1) : 166–178.

Moreno, E., Ferreira, F., Goycoolea, M., Espinoza, D., Newman, A., Rezakhah, M. (2015) Linear programming approximations for modeling instant-mixing stockpiles. In proceedings, 37th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Society for Mining, Metallurgy & Exploration: 582–587.

Picard, J.-C. (1976) Maximal closure of a graph and applications to combinatorial problems. Management Science, 22(11) : 1268–1272.

Ramazan, S. and Dimitrakopoulos, R. (2004) Uncertainty-based production scheduling in open pit mining. SME Transactions, 316 : 106–112.

Ramazan, S. and Dimitrakopoulos, R. (2013) Production scheduling with uncertain supply: a new solution to the open pit mining problem. Optimization and Engineering, 14(2) : 361–380.

Ravenscroft, P. J. (1992) Risk analysis for mine scheduling by conditional simulation. IMM Transactions, Section A, Mining Industry, 101 : 104–108.

Senecal, R. and Dimitrakopoulos, R. (2014) Parallel implementation of Tabu Search procedure for stochastic mine scheduling. In proceedings, Orebody Modelling and Strategic Mine Planning: Integrated mineral investment and supply chain optimization, AusIMM: 405–414.

Stone, P., Froyland, G., Menabde, M., Law, B., Pasyar, R., Monkhouse, P. H. L. (2007) Blasor – blended iron ore mine planning optimization at Yandi, Western Australia. In proceedings, Orebody modelling and strategic mine planning: Uncertainty and risk management models, AusIMM Spectrum Series 14, 2nd Edition: 133–136.

Tolwinski, B. and Underwood, R. (1996) A scheduling algorithm for open pit mines. IMA Journal of Mathematics Applied in Business & Industry, 7 : 247–270.

Whittle, G. (2007) Global asset optimization. In proceedings, Orebody modelling and strategic mine planning: Uncertainty and risk management models, AusIMM Spectrum Series 14, 2nd Edition: 331–336.

Whittle, J. (2009) The global optimizer works – what next?. In proceedings, Advances in orebody modelling and strategic mine planning: Old and new dimensions in a changing world, AusIMM Spectrum Series 17, 1st Edition: 3–5.