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# **A note on an induced subgraph characterization of domination perfect graphs**

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**Abstract:** Let  $\gamma(G)$  and  $\iota(G)$  be the domination and independent domination numbers of a graph  $G$ , respectively. Introduced by Sumner and Moorer [23], a graph  $G$  is domination perfect if  $\gamma(H) = \iota(H)$  for every induced subgraph  $H \subseteq G$ . In 1991, Zverovich and Zverovich [26] proposed a characterization of domination perfect graphs in terms of forbidden induced subgraphs. Fulman [15] noticed that this characterization is not correct. Later, Zverovich and Zverovich [27] offered such a second characterization with 17 forbidden induced subgraphs. However, the latter still needs to be adjusted.

In this paper, we point out a counterexample. We then give a new characterization of domination perfect graphs in terms of only 8 forbidden induced subgraphs and a short proof thereof. Moreover, in the class of domination perfect graphs, we propose a polynomial-time algorithm computing, given a dominating set  $D$ , an independent dominating set  $Y$  such that  $|Y| \leq |D|$ .

**Key Words:** Domination, independent domination, forbidden induced subgraphs.

# 1 Introduction

## 1.1 Basic definitions and notations

In this paper, graphs are undirected and simple. Standard notions are explained, for instance, by Diestel [11].  $V$  and  $E$  denote the vertex and edge sets of a graph  $G$ , respectively. For a given vertex  $v$ ,  $N(v)$  denotes the set of all neighbors (i.e. adjacent vertices) while, for a given vertex set  $X$ ,  $G[X]$  denotes the subgraph of  $G$  induced by  $X$ . Moreover, if  $G$  and  $H$  are two graphs, we say that  $G$  is  $H$ -free if  $H$  does not appear as an induced subgraph of  $G$ . Furthermore, if  $G$  is  $H_1$ -free,  $H_2$ -free,  $\dots$ ,  $H_k$ -free for some graphs  $H_1, H_2, \dots, H_k$ , we say that  $G$  is  $(H_i)_{i=1}^k$ -free.

A *dominating set* of a graph  $G = (V, E)$  is a set  $D$  of vertices such that every vertex  $v \in V \setminus D$  has at least one neighbor in  $D$ . The *domination number* of a graph  $G$ , denoted by  $\gamma(G)$ , is the minimum cardinality of a dominating set. A dominating set with such cardinality is called *minimum* while a dominating set is *minimal* if no proper subset is a dominating set.

A graph is *complete* if it contains all possible edges. A set  $D$  of vertices is *independent* (also called *stable*) if the subgraph induced by  $D$  has no edge. An independent set  $X$  of a graph  $G = (V, E)$  is *maximal* if for every vertex  $v \in V \setminus X$ ,  $X \cup \{v\}$  is not independent. A dominating set  $D$  of a graph  $G$  is called *independent* if  $D$  is independent. It is known [5, 6], that an independent dominating set is a maximal independent set, and conversely. The *independent domination number* of a graph  $G$ , denoted by  $\iota(G)$ , is the minimum cardinality of an independent dominating set in  $G$ . Thus, an independent dominating set is *minimum* if its cardinality is minimum.

Sumner and Moore [23] introduced the notion of *domination perfect graph*, as a graph  $G$  such that  $\gamma(H) = \iota(H)$ , for all induced subgraph  $H$  of  $G$ . A graph is said *minimal domination perfect* if the graph is not domination perfect but all proper induced subgraphs are.

## 1.2 Previous works

The class of domination perfect graphs has been studied. Looking for a characterization, many authors focused on special subclasses of graphs. We present here a brief survey on domination perfect graphs.

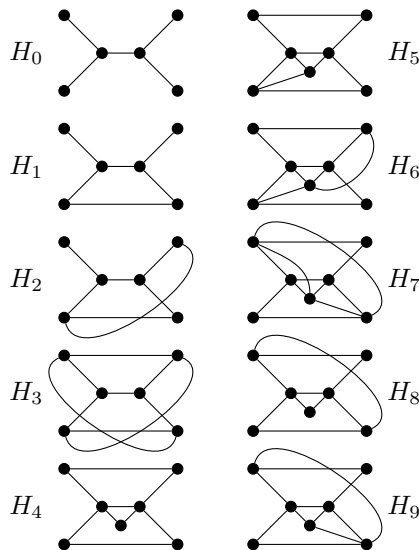


Figure 1: An illustration of graphs  $H_i$ , for  $i = 0, \dots 9$ .

The line graph  $L(T)$  of a tree  $T$  is always domination perfect [7, 20]. More generally, every line graph is domination perfect, proved by Allan and Laskar [2] and independently by Gupta (see Theorem 10.5 [17]). In fact, Allan and Laskar gave a sufficient condition in the following theorem.

**Theorem 1 (Allan and Laskar [2])** *Every claw-free graph is domination perfect.*

Topp and Volkmann [24] generalized their results to new classes of graphs.

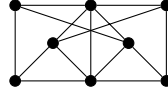


Figure 2: An illustration of the graph  $H_{10}$ .

**Theorem 2 (Topp and Volkmann [24])** *If  $G$  is  $H_{10}$ -free and  $(G_i)_{i=1}^{13}$ -free (see Figure 2 and Figure 3) then  $G$  is domination perfect.*

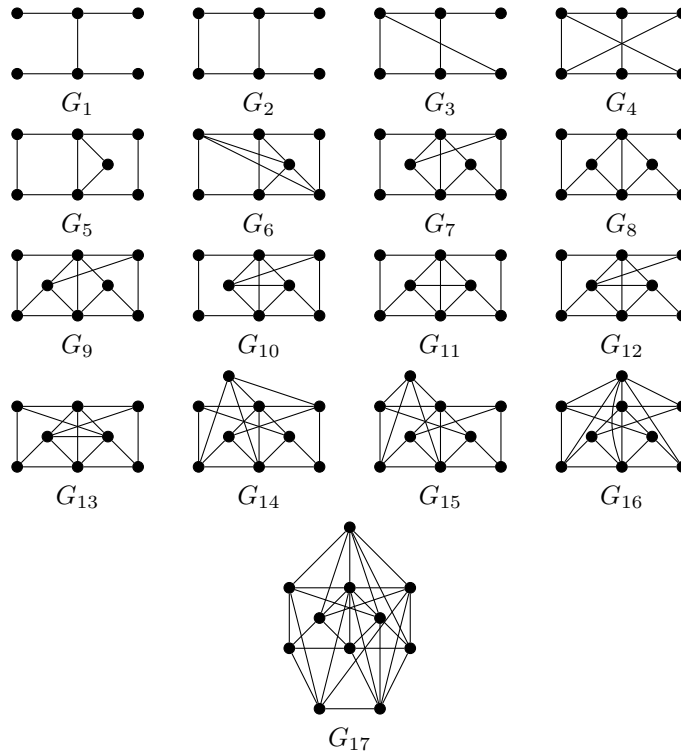


Figure 3: An illustration of graphs  $G_i$ , for  $i = 1, \dots, 17$ .

As observed in [27], the original version of this theorem in [24] was stated with two additional graphs, which were shown to be redundant.

Harary and Livingston [18] studied the class of domination perfect trees and offered a complex characterization of this class. Other characterizations of these particular trees are mentioned in [9, 13, 19]. Actually, determining a minimum dominating set and a minimum independent dominating set in trees can be achieved in linear time [7, 10, 14].

Sumner [22] gave a characterization of domination perfect graphs in the classes of chordal and planar graphs while Zverovich and Zverovich [26] tackled the case of triangle-free graphs. Consider the class  $\mathcal{S}$  of graphs defined by

$$\mathcal{S} = \{H \text{ graph on at most 8 vertices } \mid \gamma(H) = 2, \iota(H) > 2\}.$$

**Theorem 3**

- (Sumner [22]) *Let  $G$  be a chordal graph.  $G$  is domination perfect if and only if  $G$  is  $H_0$ -free.*

- (Sumner [22]) Let  $G$  be a planar graph.  $G$  is domination perfect if and only if  $G$  is  $\mathcal{S}$ -free.
- (Zverovich and Zverovich [26]) Let  $G$  be a triangle-free graph.  $G$  is domination perfect if and only if  $G$  is  $(H_i)_{i=0}^3$ -free.

where graphs  $H_i$  are drawn in Figure 1.

Sumner and Moore [23] attempted to extend previous results to all graphs.

**Theorem 4 (Sumner and Moore [23])** *If  $G$  is  $\mathcal{S}$ -free and  $G$  is  $H_{10}$ -free then  $G$  is domination perfect, where  $H_{10}$  is depicted in Figure 2.*

Other sufficient conditions were found [15, 27], stated in the following theorems, where graphs  $H_i$ ,  $U_i$  and  $T_i$  are respectively represented in Figure 1, Figure 4 and Figure 5.

**Theorem 5 (Fulman [15])** *If  $G$  is  $(H_i)_{i=0}^4$ -free,  $H_7$ -free and  $(U_i)_{i=1}^2$ -free, then  $G$  is domination perfect.*

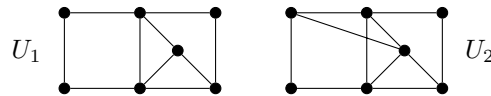


Figure 4: An illustration of graphs  $U_1$  and  $U_2$ .

**Theorem 6 (Zverovich and Zverovich [27])** *If  $G$  is  $(H_i)_{i=0}^3$ -free and  $(T_i)_{i=1}^2$ -free, then  $G$  is domination perfect.*

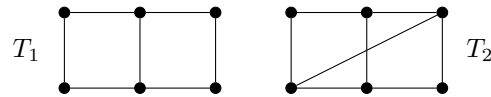


Figure 5: An illustration of graphs  $T_1$  and  $T_2$ .

Sumner [22] established that a graph is domination perfect if and only if  $\gamma(H) = \iota(H)$  only for all induced subgraph  $H$  with  $\gamma(H) = 2$ , and supposed impossible to provide a finite list of forbidden induced subgraphs characterizing domination perfect graphs. However, Zverovich and Zverovich [26] gave a first characterization with a list of 4 forbidden induced subgraphs. Nevertheless, Fulman [15] brought out a counterexample. Later, Zverovich and Zverovich [27] proposed another characterization with a list of 17 forbidden induced subgraphs.

**Theorem 7 (Zverovich and Zverovich [27])** *Let  $G$  be a graph. Then  $G$  is domination perfect if and only if  $G$  is  $(G_i)_{i=1}^{17}$ -free, where graphs  $G_i$  are depicted in Figure 3.*

In this paper, we point out a counterexample to Theorem 7 and we state a new characterization of domination perfect graphs. Moreover, we check the validity of results deduced from [27] and adapt some of them.

Zverovich [25] generalized the notion of domination perfect graphs by considering, instead of an equality of invariants, the difference between two invariants bounded by a constant. He called them  $k$ -bounded classes of dominant-independent perfect graphs. He found a characterization in terms of finite list of forbidden induced subgraphs for the  $k$ -bounded classes of independent-independent domination perfect graphs and the  $k$ -bounded classes of independent-domination perfect graphs. He conjectured that the  $k$ -bounded classes of independent domination-domination perfect graphs can be characterized by a finite list of forbidden induced subgraphs.

Besides, other graph invariants were investigated for comparison. The famous example is naturally perfect graphs [3, 4] whose chromatic number and clique number are equal for all induced subgraph. Furthermore, Gutin and Zverovich [16] studied the class of  $\Gamma$ -perfect graphs, graphs whose independence number and upper domination number are equal for all induced subgraph, and the similar class of IR-perfect graphs

with the independence number and the upper irredundance number. Later, Dohmen, Rautenbach and Volkmann [12] obtained different results on the corresponding  $k$ -bounded classes of graphs. Alvarado, Dantas and Rautenbach [1] investigated and characterized perfect graphs, according to all pairs of invariants among the total domination number, the paired domination number and the domination number while Rautenbach and Zverovich [21] characterized certain subclasses of perfect graphs when the pair of graph invariants is either the strong domination number and the domination number, or, the domination number and the independent strong domination number, or, the independent domination number and the independent strong domination number.

## 2 Main results

### 2.1 Counterexample to characterization in [27]

Under Theorem 7, a graph  $G$  is domination perfect if and only if  $G$  does not contain any of  $G_1, \dots, G_{17}$  in Figure 3 as an induced subgraph. Nonetheless, graphs  $H_5$  and  $H_6$  in Figure 1 are counterexamples. Indeed, none contains any  $(G_i)_{i=1}^{17}$  as an induced subgraph, as the system GraphsInGraphs [8] confirmed. And none is domination perfect, since  $\gamma(H_5) = \gamma(H_6) = 2 \neq 3 = \iota(H_5) = \iota(H_6)$ .

Observe that every graph  $H_i$  is isomorphic to certain  $G_i$  or is an induced subgraph of certain  $G_i$ :  $H_0 \cong G_1, H_1 \cong G_2, H_2 \cong G_3, H_3 \cong G_4, H_4 \cong G_5, H_7 \cong G_6$  and  $H_5$ , respectively  $H_6$ , is an induced subgraph of  $G_7, G_8, G_{10}, G_{11}, G_{12}, G_{14}$  and  $G_{15}$ , respectively  $G_9, G_{13}, G_{16}$  and  $G_{17}$ . Since every graph  $G_i$  contains at least one subgraph isomorphic to a certain  $H_i$  as an induced subgraph, the class of  $(H_i)_{i=0}^9$ -free graphs is included in the class of  $(G_i)_{i=1}^{17}$ -free graphs.

Notice also that  $H_1$ , respectively  $H_2$ , is an induced subgraph of  $H_8$ , respectively  $H_9$ . Accordingly, the class of  $(H_i)_{i=0}^9$ -free graphs is exactly the same as the class of  $(H_i)_{i=0}^7$ -free graphs. From now, we consider both classes equivalently.

### 2.2 A new characterization of domination perfect graphs

**Lemma 1** *Let  $G$  be a  $(H_i)_{i=0}^7$ -free graph on  $n$  vertices, where graphs  $H_i$  are drawn in Figure 1, and  $D$  a dominating set of  $G$ . Then Algorithm 1 gives in  $\mathcal{O}(n^4)$  an independent dominating set  $Y$  such that  $|Y| \leq |D|$ .*

**Proof of Lemma 1.** Clearly, Algorithm 1 performs in  $\mathcal{O}(n^4)$  since the loop ‘while’ is applied at most  $\mathcal{O}(n^2)$  times, a minimal dominating set can be computed in  $\mathcal{O}(n^2)$ , the same worst time for computing private neighbors (at worst for instance with a breadth-first search), checking the completeness of a subgraph or finding a missing edge, and other operations take constant time.

Let  $D$  be a dominating set of  $G$ . Consider initially  $Y = D$ . We show that each iteration of the loop ‘while’ decreases strictly the number of edges in  $G[Y]$  and the resulting set  $Y$  is still a dominating set. Notice that in each case, the cardinality of  $Y$  is not greater than that of  $D$  (see lines 6, 8, 12, 15, 21 and 24 in Algorithm 1) and lines 1 and 28 in Algorithm 1 assure the minimality of  $Y$  at the beginning of the loop ‘while’. Let  $d_0d_1$  be an arbitrary edge in  $G[Y]$ . We consider  $N_i = \{x \in V(G) \setminus Y \mid N(x) \cap Y = \{d_i\}\}$ , the set of private neighbors of  $d_i$  according to  $Y$ , for  $i = 0, 1$ . Because  $Y$  is minimal and  $d_0d_1 \in E(G)$ , no  $N_i$  is empty, otherwise  $D \setminus \{d_i\}$  would be a smaller dominating set. We distinguish different cases, depending on the cardinality of  $N_0$  and  $N_1$ .

- **Case 1.** The first easy case is  $|N_0| = 1$  or  $|N_1| = 1$ . Without loss of generality, we can assume that  $|N_0| = 1$ . Thus,  $(Y \setminus \{d_0\}) \cup N_0$  is trivially a dominating set with fewer edges than  $Y$ .
- **Case 2.** Suppose that  $|N_0| \geq 2$  and  $|N_1| \geq 2$ .
  - **Case 2.1.** If there exists  $i$  such that  $G[N_i]$  is complete, say  $N_0$ , then  $(Y \setminus \{d_0\}) \cup \{t\}$ , where  $t$  is an arbitrary vertex in  $N_0$ , is a dominating set with fewer edges than  $Y$ .



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**Algorithm 1** Polynomial-time algorithm to find an independent dominating set with at most the cardinality of a given dominating set.

---

**Require:**  $G = (V, E)$  a  $(H_i)_{i=0}^7$ -free graph

**Require:**  $D$  a dominating set of  $G$

**Ensure:**  $Y$  an independent dominating set of  $G$  such that  $|Y| \leq |D|$

```

1:  $Y \leftarrow$  a minimal dominating set included in  $D$ 
2: while  $G[Y]$  contains an edge do
3:   Let  $d_0d_1$  be an edge of  $G[Y]$ 
4:   Let  $N_i$  be the set of private neighbors of  $d_i$ , for  $i = 0, 1$ 
5:   if  $|N_0| = 1$  then
6:      $Y \leftarrow (Y \setminus \{d_0\}) \cup N_0$ 
7:   else if  $|N_1| = 1$  then
8:      $Y \leftarrow (Y \setminus \{d_1\}) \cup N_1$ 
9:   else
10:    if  $G[N_0]$  is complete then
11:      Let  $t$  be an arbitrary vertex in  $N_0$ 
12:       $Y \leftarrow (Y \setminus \{d_0\}) \cup \{t\}$ 
13:    else if  $G[N_1]$  is complete then
14:      Let  $t$  be an arbitrary vertex in  $N_1$ 
15:       $Y \leftarrow (Y \setminus \{d_1\}) \cup \{t\}$ 
16:    else
17:      Let  $u_0v_0$  be a missing edge in  $N_0$ 
18:      Let  $u_1v_1$  be a missing edge in  $N_1$ 
19:      if  $G[\{u_0, v_0, u_1, v_1\}]$  is a perfect matching then
20:        Let  $u_0u_1, v_0v_1$  be edges of the perfect matching
21:         $Y \leftarrow (Y \setminus \{d_0, d_1\}) \cup \{u_0, v_1\}$ 
22:      else
23:        Let  $u_1u_0v_1v_0$  be the path on 4 vertices
24:         $Y \leftarrow (Y \setminus \{d_0, d_1\}) \cup \{u_1, v_0\}$ 
25:      end if
26:    end if
27:  end if
28:   $Y \leftarrow$  a minimal dominating set included in  $Y$ 
29: end while
30: return  $Y$ 

```

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- **Case 2.2.** Let  $u_0v_0$ , respectively  $u_1v_1$ , be a missing edge in  $N_0$ , respectively in  $N_1$  (see Figure 6). Look at the adjacency between the 4 vertices  $u_0, u_1, v_0, v_1$ . Because  $G$  is  $(H_0, H_1, H_2, H_3)$ -free,

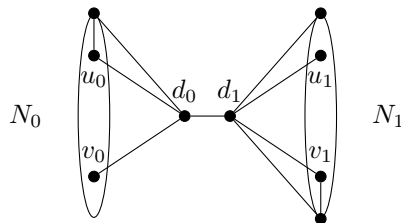


Figure 6: The edge  $d_0d_1$  with two non-complete private neighborhoods.

without loss of generality, we may assume that either only  $u_0u_1$  and  $v_0v_1$  are edges in  $G$ , or only  $u_0u_1, u_0v_1$  and  $v_0v_1$  are edges in  $G$ . We discern each case.

- \* **Case 2.2.1.** We have a perfect matching between the vertices  $u_0, v_0, u_1, v_1$ , i.e.  $u_0u_1$  and  $v_0v_1 \in E(G)$ . Clearly,  $(Y \setminus \{d_0, d_1\}) \cup \{u_0, v_1\}$  has fewer edges than  $Y$ . Suppose that  $Z = (Y \setminus \{d_0, d_1\}) \cup \{u_0, v_1\}$  is not a dominating set. Hence, there exists a vertex  $t \in V(G) \setminus Y$

which is not adjacent to  $Z$ , especially to  $u_0$  and  $v_1$ . Since  $Y$  is a dominating set,  $t$  must be adjacent to  $d_0$  or  $d_1$  or both. Because  $G$  is  $(H_4, H_5, H_6)$ -free,  $t$  can not be adjacent to  $d_0$  and  $d_1$ . Therefore  $t$  is only adjacent to  $d_0$  or  $d_1$ , say  $d_0$  (see Figure 7). Thus  $G[\{d_0, d_1, t, u_0, v_1, u_1\}]$  is an induced  $H_1$  or  $H_2$ , depending on the adjacency between  $t$  and  $u_1$ , a contradiction.

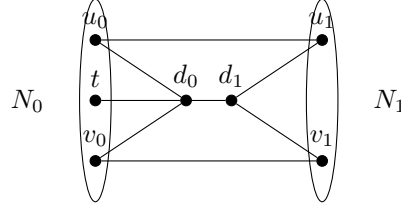


Figure 7: An illustration of Case 2.2.1.

\* **Case 2.2.2.** In the last case, only  $u_0u_1$ ,  $u_0v_1$  and  $v_0v_1$  are edges in  $G$ , i.e. they induce a path on 4 vertices. Obviously  $(Y \setminus \{d_0, d_1\}) \cup \{u_1, v_0\}$  has fewer edges than  $Y$ . Now, assume that  $Z = (Y \setminus \{d_0, d_1\}) \cup \{u_1, v_0\}$  is not a dominating set. Thus, there exists a vertex  $t \in V(G) \setminus Y$  which is not adjacent to  $Z$ , especially to  $u_1$  and  $v_0$ . Since  $Y$  is a dominating set,  $t$  must be adjacent to  $d_0$  or  $d_1$  or both. Because  $G$  is  $(H_7, H_8, H_9)$ -free,  $t$  is only adjacent to one of  $d_i$ , say  $d_0$  (see Figure 8). Therefore  $G[\{d_0, d_1, t, v_0, v_1, u_1\}]$  is an induced  $H_1$  or  $H_2$ , depending on the adjacency between  $t$  and  $v_1$ , a contradiction.

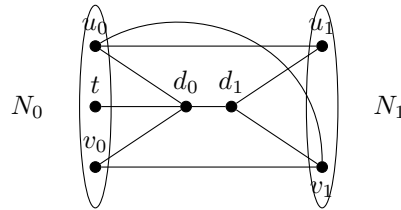


Figure 8: An illustration of Case 2.2.2.

□

**Theorem 8** Let  $G$  be a  $(H_i)_{i=0}^7$ -free graph, where graphs  $H_i$  are drawn in Figure 1. Then

$$\iota(G) = \gamma(G).$$

**Proof.** Let  $D$  be a minimum dominating set of  $G$ . By Lemma 1, we obtain an independent dominating set  $Y$  such that  $|Y| \leq |D|$ , which implies that  $\iota(G) \leq \gamma(G)$ . But the cardinality of every minimum dominating set is a lower bound on the cardinality of any independent dominating set, i.e.  $\gamma(G) \leq \iota(G)$ . Thus,

$$\gamma(G) = \iota(G).$$

□

Because  $\iota(H_i) = 3 \neq 2 = \gamma(H_i)$ , for  $i = 0, \dots, 9$ , we deduce from the last theorem the following one which gives a new characterization of domination perfect graphs.

**Theorem 9** The following assertions are equivalent for every graph  $G$  :

- $G$  is  $(H_i)_{i=0}^7$ -free.
- $G$  is  $(H_i)_{i=0}^9$ -free.
- For every induced subgraph  $H$  of  $G$ , it holds that  $i(H) = \gamma(H)$ .

By Theorem 9, we observe that Algorithm 1 actually computes, given a dominating set  $D$  of any domination perfect graph  $G$ , an independent dominating set  $Y$  such that  $|Y| \leq |D|$ , that implies if  $D$  is minimum then  $Y$  is also minimum.

### 2.3 Validity or adaptation of consequences of [27]

Based on their characterization, Zverovich and Zverovich [27] proposed two corollaries that we improve. To be self-contained, we include a slightly adapted proof of the first corollary. Consider  $\overline{G}$  the complement of the graph  $G = (V, E)$ , defined by  $\overline{G} = (V, (V \times V) \setminus E)$ .

**Corollary 1** *If a graph  $G$  is  $(\overline{H}_i)_{i=0}^7$ -free, where  $H_i$  is depicted in Figure 1, with diameter at least 3 and minimum degree at least 1, then  $G$  has an edge that does not belong to any triangle.*

**Proof.** Let  $\overline{G}$  be the complement of  $G$  (on  $n$  vertices). Clearly  $\overline{G}$  is  $(H_i)_{i=0}^7$ -free. If  $\gamma(\overline{G}) = 1$ , then  $\overline{G}$  has a dominating vertex, i.e. a vertex of degree  $n - 1$ . Then,  $G$  has an isolated vertex, i.e. a vertex of degree 0, a contradiction. Assume that  $\gamma(\overline{G}) \neq 2$ . For every pair of vertices  $x$  and  $y$ , there exists a vertex  $z$  which is not adjacent in  $\overline{G}$  to  $x$  and  $y$ . This implies in  $G$  that for every pair of vertices  $x$  and  $y$ , there is a vertex  $z$  adjacent to  $x$  and  $y$ , i.e. the diameter of  $G$  is at most 2, a contradiction. Thus,  $\gamma(\overline{G}) = 2$  and by Theorem 8, we deduce that  $\iota(\overline{G}) = 2$ , i.e. the graph  $G$  has an edge that does not belong to any triangle.  $\square$

Consider  $\mathcal{L}$  the class of graphs  $G$  satisfying all following conditions:

1.  $G$  is planar,
2.  $G$  is bipartite,
3.  $G$  has maximum degree 3,
4.  $G$  has girth  $g(G) \geq k$ , where  $k$  is fixed.

**Corollary 2** *Decision problems associated to the domination problem and the independent domination problem are both NP-complete in the class  $\mathcal{L}$ .*

The proof in [27] of the previous corollary is still valid if we consider the list of graphs  $(H_i)_{i=0}^7$  as forbidden induced subgraphs.

Moreover, Fischermann, Volkmann and Zverovich [14] characterized the class  $\mathcal{I}$  of graphs  $G$  such that for every induced subgraph  $H$  of  $G$ ,  $H$  has a unique minimum irredundant set if and only if it has a unique minimum dominating set.

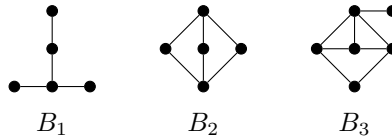


Figure 9: An illustration of graphs  $B_1$ ,  $B_2$  and  $B_3$ .

**Theorem 10 (Fischermann, Volkmann, Zverovich [14])** *A graph  $G$  belongs to  $\mathcal{I}$  if and only if  $G$  is  $(B_i)_{i=1}^3$ -free (see Figure 9).*

Their proof is based on the characterization of domination perfect graphs from Zverovich and Zverovich [27]. In order for the proof to remain correct, we need to check in the sufficient condition that every  $(B_i)_{i=1}^3$ -free graph  $G$  is domination perfect by using our characterization. Indeed, each of the graphs  $H_0$ ,  $H_1$ ,  $H_2$ ,  $H_4$  and  $H_5$  has an induced subgraph isomorphic to  $B_1$ . The graph  $H_3$  contains  $B_2$  as an induced subgraph. Finally,  $B_3$  is an induced subgraph of  $H_6$  and  $H_7$ . Because  $G$  is  $(B_i)_{i=1}^3$ -free,  $G$  is also  $(H_i)_{i=0}^7$ -free. By Theorem 9,  $G$  is thus domination perfect. The end of the proof in [14] remains.

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