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Optimal mining rates revisited: Managing mining equipment and geological risk at a given mine setup

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Abstract: This paper presents a mixed integer programming formulation dealing with the effective minimisation of risk incurred when optimizing mining production rates in such a way that production targets are met in the presence of geological uncertainty. This is developed through the concept of a "stable solution domain" that provides all feasible combinations of ore and waste extraction for the ultimate pit limit of a given deposit, independent of the geological risk. The proposed formulation provides an optimal annual extraction rate, together with the optimal utilization of a mining fleet and an equipment acquisition program. This solution eliminates unnecessary capital expenses and is feasible under all geological scenarios. The mathematical programing model is detailed and tested at a gold deposit. The results are used as input to a production schedule design and are compared to the schedule generated using a constant mining rate; the comparison shows that about 40% of equipment acquisition can be delayed for 7 years and mill demand still be met, thus maximizing profit and minimizing costs.

Key Words: Mine production rate, stable solution domain, mine planning.

1 Introduction

Production scheduling of open pit mines is a major aspect in terms of planning and production streamlining, asset valuation and operations. Production scheduling is a process leading to the determination of a sequence of extraction which involves the removal of at least two types of material: ore and waste. If the production schedule maximizes the project's overall profit, subject to technical, economic and environmental constraints, then it is said to be optimal. Two major technical constraints involved in the determination of such schedule are: (i) the feasible combinations of ore and waste production (stripping ratio), and (ii) the ore production rate that meets the mill feed requirements.

Optimization methods have long been used to improve mine design and life-of mine production schedules (Kim, 1979; Barbaro and Ramani, 1986; Dagdelen and Johnson, 1986; Whittle and Rozman, 1991; Tolwinski, 1998; Whittle, 1999; Godoy, 2003; Stone et al.; 2005; Jewbali, 2006; Menabde et al. 2007; Meagher, 2010; Godoy and Dimitrakopoulos, 2011). The common industry practice is to discretize the pit space in a sequence of nested pits (Whittle, 1999), which is accomplished through the repeated use of a parametric ultimate pit algorithm, by successively changing the commodity price. For lower prices, smaller pits are produced (Hustrulid and Kuchta, 1995) and will extend toward the area of highest grade and/or will have a very low stripping ratio. Since early cash flows are subject to less discounting, and thus contribute more to the Net Present Value (NPV), it is advantageous to bring income forward and delay expenditure as long as possible.

In dealing with the points raised above on stripping ratios and ore production rates that meet mill feed requirements, the optimization of mine production rates for ore and waste over the life of an open pit mine can only be done within a so-called physically "feasible" domain of solutions. This domain is based on early work (Rzhenevisky, 1968; Tan and Ramani, 1992) revisited in Godoy (2003), and it adopts concepts in the context of open pit scheduling based on nested pits and geological uncertainty. The current mine scheduling framework establishes the feasible domain based on two extreme cases of deferment of waste removal: the 'worst' and 'best' shown in Figure 1. The worst case corresponds to mining out each successive bench in a mine before starting the next. This schedule provides the maximum quantity of waste that can be removed from the pit to recover a certain amount of ore (i.e. the highest stripping ratio). This schedule does not perform well, given that waste is removed from early, and thus discounted little, whereas the income from mining ore at the bottom of the pit is delayed for later periods, and thus heavily discounted. The best case corresponds to the sequential mining of the nested pits, which is, mining each successive bench of the smallest pit possible, followed by the bench of the next pit and so on. This schedule removes the minimum necessary quantity of waste (lowest stripping ratio) that has to be removed to provide both the necessary working room and the safety of operations. In economic terms, this schedule then provides the highest NPV. Given the best and worst cases of mining, Figure 2(a) shows an example of a feasible solution domain of a gold deposit from Godoy (2003) in the form of a cumulative graph. The solution domain is bounded by the curves of cumulative tonnages of ore and waste of the best and worst mining cases and accounts for all the feasible

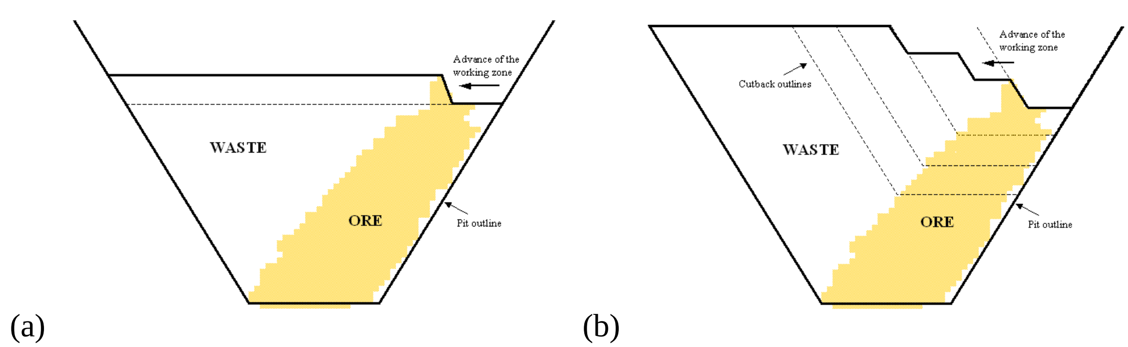


Figure 1: (a) Schematic representation of the (a) worst case mining schedule where each successive bench is mined out before starting the next; and (b) best case mining schedule where each successive bench of the smallest pit shell is mined sequentially and then each successive bench of the next pit and so on.

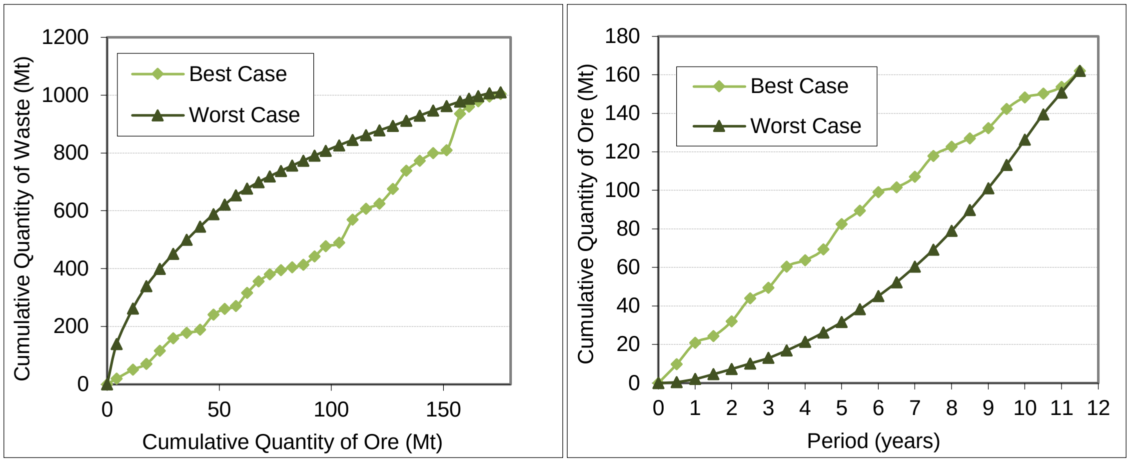


Figure 2: Feasible domain of (a) ore production and waste removal and (b) ore production (Godoy 2002).

combinations of stripping ratios for the given orebody being considered and over its life-of-mine. This reflects the possible number and spatial arrangement of simultaneous working zones.

The feasible domain, as presented in Figure 2, is a function of two factors: (i) the spatial distribution of ore and waste in the region contained by the ultimate pit limits, and (ii) a specific set of nested pits. The definition of these two factors is subject to a chain of interconnected factors such as geological, economic, technological and environmental. As Dagdelen and Johnson (1984) state, production scheduling can be seen as a prescription of a mine sequence which maximizes cumulative project NPV while satisfying four major constraints: (a) mill feed grade, (b) slope constraints, (c) milling capacity, and (d) mining capacity. The definition of (a) and (b) is all that is required for the derivation of the solution domain in the cumulative graph of ore production and waste removal. However, the specification of (c) and (d) account for the time aspect of the mining sequence formation and can thus further restrict the solution domain. This last aspect can be represented by the cumulative graph of the ore production with respect to time, where both extreme mining cases are presented as two separate ore production curves. These curves form the feasible domain of the possible time distribution of the ore production for a given processing capacity. Figure 2(b) illustrates this domain and was produced by assuming a constant mining capacity for the extreme cases presented in Figure 2(a). It is important to note that the cumulative graphs in Figure 2 can account for geological uncertainty (Boucher et al., 2014; Emery et al., 2014) and, in fact, can generate optimal mining rates for a given case which can always be met. This requires that the feasible domain of solution is calculated for individual stochastically generated scenarios of the orebody under consideration (Godoy, 2003); then, the common intersection of all individual feasible domains provides the 'stable' solution domain (SSD), or domain where the ore-waste combinations shown are always available, independent of geological risk. This is shown in the case study presented in a subsequent section.

The work presented herein builds on the work by Godoy (2003) and Godoy and Dimitrakopoulos (2004), which is limited in that the variables related to the increased and decreased mining capacity are defined as linear variables and, therefore, the optimization does not produce values of mining capacity that are necessarily multiple of the equipment's total capacity. Thus, the optimal solution may provide a fractional number of equipment, which is ultimately an unfeasible solution. While small differences may be accepted, high levels of equipment under-utilisation may be practically not viable. Note that while mining production rates are optimized, a physical mining sequence that meets those rates is not produced and must be subsequently generated based on them.

The current paper starts by presenting a mixed integer programming formulation, which is specific for a mine with one mill, a long-term stockpile, a waste dump, and a mine-owned fleet of mining equipment. Then, the formulation is applied at a gold mine and comparisons to conventional practices are made. Discussion and conclusions follow.

2 The optimization model

The determination of an optimal combination of ore and waste production consists of selecting a curve, from all the possible curves that fall inside the SSD, which maximizes the corresponding NPV. The optimization model delivers a life-of-mine schedule of ore production and waste removal, as well as a prescription for the formation of mining capacity and the acquisition of equipment, which maximizes total discounted cash flow for a set of economic and technological parameters. This production and acquisition plan is modeled to provide results specifically for a given mine configuration, which includes three material destinations, namely, mill, long-term stockpile (low grade ore to be processed after mining stops) and waste. Additionally, it is assumed that the mining fleet is owed by the operation and bought sequentially, equipment have a fixed life span and will be replaced, while mining rates must be stabilized for long periods of time (years). The mathematical model includes an objective function and constraints as follow.

2.1 Objective function

The objective function below consists of 5 parts. The first corresponds to the income obtained from high grade ore metal, considering mining, processing, operating, selling and marketing costs. The second section corresponds to the cost of mining low grade ore. In this case, as the low grade ore is destined to a long term stockpile there is no income considered from mining it during the evaluated time span. The third term corresponds to the cost of mining waste. The fourth term considers the purchase costs, i.e. the cost of adding a new equipment of a given type and model in a certain year, in order to increase the production capacity of the system. Finally, the fifth term corresponds to the ownership costs, such as the cost of unused equipment of a certain type and model, given that the production rate of that year is lower than the maximum available capacity. The objective function is:

$$\begin{aligned} \text{MAX} \ \sum_{i=1}^{n} d_i\,(1-R)\left[(S_i - C_i^{ma})\,\gamma_i - \left(C_{p_i}^{m} + C_{p_i}^{p} + C_{t_i}\right)(\alpha_{p_i})^{-1}\right] M_{p_i} - \\ - \sum_{i=1}^{n} d_i C_{s_i}^{m} (\alpha_{s_i})^{-1} M_{s_i} - \sum_{i=1}^{n} d_i C_{w_i} W_i - \\ - \sum_{k=1}^{K}\sum_{z=1}^{Z}\sum_{i=1}^{n} d_i H_{kzi} NE_{kzi} - \sum_{k=1}^{K}\sum_{z=1}^{Z}\sum_{i=1}^{n} d_i U_{kzi} DE_{kzi} \end{aligned} \tag{1}$$

where $i = 1, ..., n$ denotes the time periods to be considered in the production scheduling optimization. The definitions of the variables participating in the objective function and in the following constraints are given in Table 1, and the constants for the model in Table 2.

Table 1: Model and constraints variables.

Variable	**Definition**
M_{p_i}	primary ore metal to be removed in year i
M_{s_i}	secondary ore metal to be removed in year i
W_i	waste quantity to be removed in year i
NE_{kzi}	new production equipment added of kth type, zth model (Integer ($k = 1$ for loading; $k = 2$ for haulage; $k = 3$ for drilling)
DE_{kzi}	decreased production equipment of kth type, zth model (Integer) ($k = 1$ for loading; $k = 2$ for haulage; $k = 3$ for drilling)
NC_{kzi}	new capacity added for kth type, zth model of production equipment ($k = 1$ for loading; $k = 2$ for haulage; $k = 3$ for drilling)
DC_{kzi}	capacity decrease for kth type, zth model of production equipment ($k = 1$ for loading; $k = 2$ for haulage; $k = 3$ for drilling)

Table 2: Model constants.

Constant	Definition
n	total number of time periods to be considered ($i = 1, \ldots, n$)
K	number of types of mine equipment
Z	number of models of production equipment per type
d_i	discount factor$d_i = (1+r)^{-i}$, where r is the interest rate
S_i	Selling price of metal
$C^m_{p_i}$	unit mining cost of primary ore type
$C^m_{s_i}$	unit mining cost of secondary ore type
$C^p_{p_i}$	unit processing cost of primary ore type
$C^p_{s_i}$	unit processing cost of secondary ore type
$C^m_{w_i}$	unit mining cost of waste removal
C^{ma}_i	marketing cost per unit payable metal
R	royalty as % of the net revenue
α_{p_i}	basic ore metal grade
α_{s_i}	secondary ore metal grade
γ_i	total recovery of the payable metal
C_{t_i}	time costs corresponding to operating costs of support services.
C^{max}_{kz}	capacity limit of kth type, zth model of production equipment
H_{kzi}	total purchase cost of kth type, zth model of mine equipment
U_{kzi}	total ownership cost of kth type, zth model of mine equipment

Equation (1) reflects the structure of the NPV of the mining project on the basis of discounted cash flow analysis, before taxation and without the treatment of the relevant depreciation and depletion allowances. The depreciation and depletion allowances represent constants in the mixed integer programming formulation so as not to affect the optimization of the production scheduling. The formulation represents an operating mine where the low grade ore is stockpiled and not processed. As a result, low grade ore metal doesn't provide revenue, and this is taken into account in expression (1).

2.2 Constraints

The constraints corresponding to expression (1) are presented next. The parameters used in this section are presented in Table 3.

1. Bounds of high grade ore metal production:

$$\sum_{j=1}^{i} M_{h,j} \leq PMX_i, \qquad \forall i \tag{2}$$

$$\sum_{j=1}^{i} M_{h,j} \geq PMN_i, \qquad \forall i \tag{3}$$

2. Bounds of low grade ore metal production:

$$\sum_{j=1}^{i} M_{l,j} \leq SMX_i, \qquad \forall i \tag{4}$$

$$\sum_{j=1}^{i} M_{l,j} \geq SMN_i, \qquad \forall i \tag{5}$$

Table 3: Constraints – constants and variables.

Constant	Definition
PMX_i	maximum cumulative quantity of primary ore metal
PMN_i	minimum cumulative quantity of primary ore metal
SMX_i	maximum cumulative quantity of secondary ore metal
SMN_i	minimum cumulative quantity of secondary ore metal
WMX_i	maximum cumulative quantity of waste
WMN_i	minimum cumulative quantity of waste
PMB_i	cumulative quantity of primary ore metal of the best case
PMW_i	cumulative quantity of primary ore metal of the worst case
ΔSR_{p_i}	stripping ratio between primary ore metal and waste
SMB_i	cumulative quantity of secondary ore metal of the best case
SMW_i	cumulative quantity of secondary ore metal of the worst case
ΔSR_{s_i}	stripping ratio of secondary ore metal
CE_{kz}	total capacity of production equipment type k, model z ($k = 1$ for loading; $k = 2$ for haulage; $k = 3$ for drilling)

3. Bounds of waste production:

$$\sum_{j=1}^{i} W_j \leq WMX_i, \qquad \forall i \tag{6}$$

$$\sum_{j=1}^{i} W_j \geq WMN_i, \qquad \forall i \tag{7}$$

4. Relationship between waste and high grade ore metal production:

if $PMB_i \geq PMW_i$ then

$$\Delta SR_{h,i} \sum_{j=1}^{i} M_{h,j} + \sum_{j=1}^{i} W_j \leq \Delta SR_{h,i} PMN_i + WMX_i, \qquad \forall i \tag{8}$$

if $PMB_i < PMW_i$ then

$$\Delta SR_{h,i} \sum_{j=1}^{i} M_{h,j} - \sum_{j=1}^{i} W_j \leq \Delta SR_{h,i} PMN_i - WMX_i, \qquad \forall i \tag{9}$$

where the stripping ratio of high grade ore metal $\Delta SR_{h,i}$ is:

$$\Delta SR_{h,i} = \frac{WMX_i - WMN_i}{PMX_i - PMN_i} \tag{10}$$

5. Relationship between waste and low grade ore metal production:

if $SMB_i \geq SMW_i$ then

$$\Delta SR_{l,i} \sum_{j=1}^{i} M_{l,j} + \sum_{j=1}^{i} W_j \leq \Delta SR_{l,i} SMN_i + WMX_i, \qquad \forall i \tag{11}$$

if $SMB_i < SMW_i$ then

$$\Delta SR_{l,i} \sum_{j=1}^{i} M_{l,j} - \sum_{j=1}^{i} W_j \leq \Delta SR_{l,i} SMN_i - WMX_i, \qquad \forall i \tag{12}$$

where the stripping ratio of low grade ore metal is:

$$\Delta SR_{l,i} = \frac{WMX_i - WMN_i}{SMX_i - SMN_i} \tag{13}$$

6. Equipment capacity limitation:

$$\sum_{i=1}^{n} NC_{kz,i} - \sum_{i=1}^{n} DC_{kz,i} \leq C_{kz}^{\max}, \qquad k = 1; \forall z \tag{14}$$

7. Distribution of new added capacity among different types of equipment:

$$\sum_{j=1}^{i} \sum_{z=1}^{Z} NC_{1z,j} - \sum_{j=1}^{i} \sum_{z=1}^{Z} NC_{kz,j} = 0, \qquad k = 2, ..., K; \quad \forall i \tag{15}$$

8. Distribution of capacity decrease among different types of equipment:

$$\sum_{j=1}^{i} \sum_{z=1}^{Z} DC_{1z,j} - \sum_{j=1}^{i} \sum_{z=1}^{Z} DC_{kz,j} = 0, \qquad k = 2, ..., K; \quad \forall i \tag{16}$$

9. Capacity disposal given available equipment:

$$NC_{kz,i} \leq NE_{kz,i} \cdot CE_{kz}, \qquad \forall k; \quad \forall z; \quad \forall i \tag{17}$$

$$DC_{kz,i} \leq DE_{kz,i} \cdot CE_{kz}, \qquad \forall k; \quad \forall z; \quad \forall i \tag{18}$$

10. Relationship between added capacity and capacity decrease:

$$\sum_{j=1}^{i} NC_{kz,j} - \sum_{j=1}^{i} DC_{kz,j} \geq 0, \qquad \forall k; \quad \forall z; \quad \forall i \tag{19}$$

11. Stable tonnage of material extracted:

$$M_{h,i}(\alpha_{h,i})^{-1} + M_{l,i}(\alpha_{l,i})^{-1} + W_i - \sum_{j=1}^{i} \sum_{z=1}^{Z} NC_{kz,j} + \sum_{j=1}^{i} \sum_{z=1}^{Z} DC_{kz,i} = 0 \qquad k = 1; \quad \forall i \tag{20}$$

12. Definition of variables:

$$M_{h,i} \geq 0, \qquad \forall i \tag{21}$$

$$M_{l,i} \geq 0 \qquad \forall i \tag{22}$$

$$W_i \geq 0 \qquad \forall i \tag{23}$$

$$NC_{kz,i} \geq 0 \qquad \forall k, z, i \tag{24}$$

$$DC_{kz,i} \geq 0 \qquad \forall k, z, i \tag{25}$$

$$NE_{kz,i} \geq 0, \ integer \qquad \forall k, z, i \tag{26}$$

$$DE_{kz,i} \geq 0, \ integer \qquad \forall k, z, i \tag{27}$$

Constraints 1, 2 and 3 present the bounds on cumulative primary and low grade ore metal and waste tonnage, which are limited by the feasible domain defined previously. Constraints 4 and 5 present the relationship between waste and metal extracted, which considers the different possible geometries of the working zone, dependent on the best and worst cases defined. These cases are the ones that bound the solution domain. Constraint 6 ensures that production capacity is no greater than the capacity limit available of loading equipment (type 1). Constraints 7 and 8 ensure that the capacity available for one type of equipment is also available for all other equipment types (hauling and drilling). Constraint 9 links the integer decision variables of new and decreased equipment with their corresponding capacities, which are continuous values. Constraint 10 ensures that the cumulative added capacity is higher than the total decreased capacity in each period, what prevents the production rate to get negative values. Finally, constraint 11 allows the total extracted rock (primary and low grade ore, plus waste), to equal the available capacity used, which is provided by the added equipment; and constraint 12 identifies the variables as integers or continuous.

2.3 Comments

The objective function presented in expression (1) shows that the main variables of the model are the time related high grade ore metal, low grade ore metal and waste. While the variable corresponding to the waste quantities allow for the optimization of the waste-ore relation over time, the metal variables allow for the optimization of metal quantities. The metal optimization accounts for the ore quality at different parts of the orebody. The remaining variables of the optimization model are the added capacity and capacity decrease of each type and model of mining equipment. The inclusion of these variables deals with the stabilization of the mining rate over time periods as a function of the capacity.

The economic parameters involved in the stabilization of the mining rate are the unit purchase and ownership costs of each type and model of mine equipment. The total purchase cost is determined by the value of the equipment, and, once it is bought, its whole capacity can be used as new added mining capacity of the system. The total ownership cost is the penalty for the capacity decrease that reflects the economic consequences for having idle equipment. In this context, the stabilization of the mining rate over time periods is determined as a search for the balance between the purchase and ownership costs of the production capacity. This represents a direct incorporation of the related capital investments in the production scheduling optimization.

It is important to stress that the definition of proper limit values for the variables related to production capacity are essential in order to guarantee that the mining rates produced by the optimization formulation are physically mineable. The main reason for that is a possible lack of working space to accommodate large number of mining equipment, and the corresponding accessibility constraints. If the mining rates remain impractical after tightening the constraints related to maximum allowed capacity, an alternative is to redefine the physical pushbacks. In this case, production periods presenting deviations from the production targets can be flagged in the detailed mining sequence and be investigated further.

3 Case study at a gold mine

3.1 Generating optimal mining production rates

The present case study aims to demonstrate the technical and practical intricacies of the proposed model in Section 2 in a real operation. The case study considers a gold mine with the technological setup (or mining system) that the mathematical model represents. Here, high grade material is processed through a mill, and low grade material is taken to a long-term stockpile to be processed by the end of the mine's life (thus no profit is obtained from it in the evaluated time span). The deposit consists of an orebody of 170,000 blocks of 15×15×10 meters. The mill cut-off is fixed to 1.2ppm, which defines the high grade ore material as high grade gold material, and the low grade ore correspond to materials with grade higher than 0.9ppm and lower than 1.2ppm. Accordingly, there is approximately 170Mt of ore (destined to the mill), with an average grade of 2.36ppm, and 1,000Mt of waste material.

The two types of equipment included in the mathematical formulation correspond to haulage and loaders, which are owned by the mine. The "CAT 793C" model is considered for the former, while the latter case considers two models, the "PC8000" and "FEL 994". The costs and parameters used in this mining operation are presented in Table 4. Details of equipment's capacity, purchase and ownership costs, etc. are presented in Table 5.

Table 4: Operation's costs and technical parameters.

Parameter	Value
Mining Cost (US$/t)	3.00
Processing Cost (US$/t)	8.77
Capital Cost (US$/t)	3.65
Discount Rate (%)	10
Mill Recovery (%)	90
Mill cut-off grade (ppm)	1.2
Breakeven cut-off grade (ppm)	0.9

Table 5: Equipment parameters.

Type	**Loaders**		**Haulage**
Model	**PC8000**	**FEL_994**	**CAT_793C**
Purchase Cost (MUS$)	4.73	1.92	1.77
Ownership Cost (MUS$/year)	0.68	0.27	0.25
Capacity (Mt/year)	25.0	9.60	3.14
Maximum Availability (units)	5	4	34

The solution obtained from the optimization model will be referred to as "optimal". This optimal solution corresponds to a production schedule that maximizes the NPV within the SSD. This is unique, in the sense that the geological uncertainty has been effectively integrated into the optimization process by considering the intersection area of the extreme mining cases of 20 geological simulation models of the deposit. The mill feed demand targets used for the optimization model are presented in Figure 3. The mill demand variation between years 5 and 9 is caused by the mill being fed by other operations, leaving this presented annual capacity available for the processing of material from the current mine.

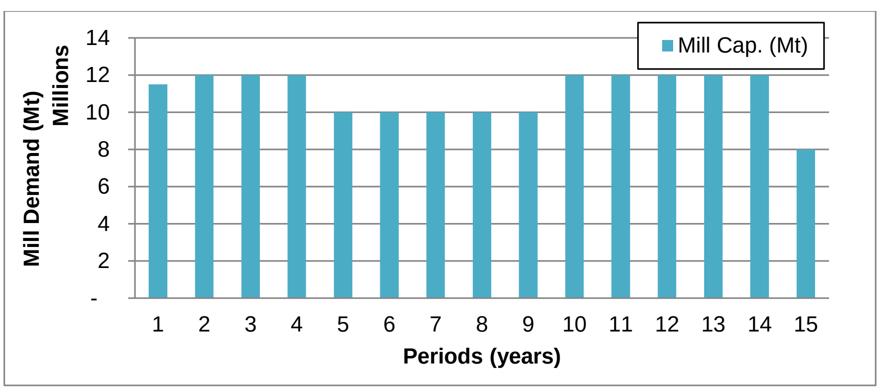


Figure 3: Mill annual available capacity.

The mining capacity required to meet the presented mill demand in the best and worst mining cases are presented in Figure 4. As the mill feed target must be met by the high grade ore, the worst case needs to remove excessively large amounts of material in the first periods (as mining is done bench-by-bench, so as to arrive to the in-depth ore). This causes high waste mining in the initial stages of the mine (over 90% of the

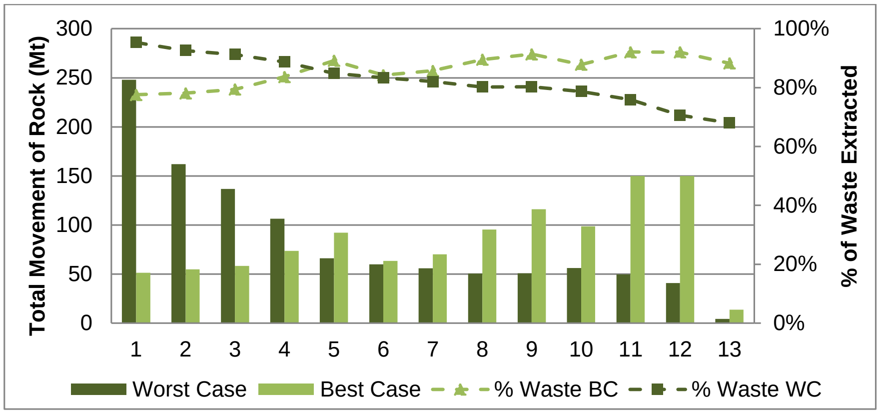
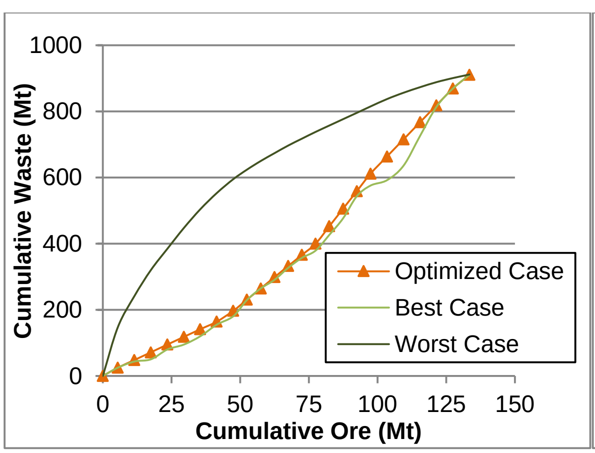


Figure 4: Mining capacity required to meet mill demand in the best and worst case.

total material extracted), and mostly pure ore during the last years, which are heavily discounted. On the other hand, in the best case ore is made available in the initial periods by mining pit shell by pit shell, and during the last years, when the cash flow is more heavily discounted, the total movement of rock is higher (especially waste, with around 90% of the total extraction in the last period, as the stripping ratio increases for the deepest ore).

Figure 5(a) presents the stable solution domain for the pit limit defined for this deposit, created by the intersection of the areas of the cumulative quantities of ore and waste from the "Best Case" and "Worst Case" of 20 orebody simulations, such as the ones presented in Figure 4. In this case, the simulations where obtained by direct block simulation (Godoy, 2003; Godoy and Dimitrakopoulos, 2004). The solution obtained from the optimization model described in Section 2 is also presented in the interior of this domain referred to as "Optimized Case". As mentioned earlier, the optimized mining rate is completely inside of the SSD, what shows that the obtained result is a feasible extraction rate program. In addition, the "optimal" mining rate schedule is very close to the "Best Case", particularly before the first 75Mt of extracted ore. The later separation of the optimal case form the best case limit is likely caused because, as the depth of the deposit increases, the stripping ratio rises and more waste must be extracted to obtain one ton of ore. Figure 5(b) shows the annual extraction rate defined by the optimizer, where the operation starts with a capacity of 63Mt per year, with minor increases in capacity by year 4 and 5, and a major mining rate expansion by year 8, finishing with 100Mt by year 13, when the pit limit is reached. The optimizer aims to maximize profit by: i) delaying waste extraction as much as possible while ensuring that a smooth evolution of mining rates is



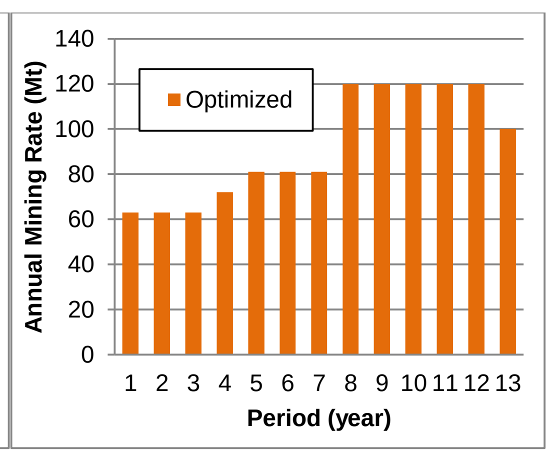


Figure 5: (a) Stable solution domain created by the extreme mining cases, with the mining rate solution obtained by the proppposed model. (b) Annual production rate plan.

maintained; ii) avoiding extreme changes in production rates in consecutive periods; and iii) ensuring that there is no mining capacity missing or left unused given the existing equipment availability.

This smooth production rate evolution is confirmed by the equipment acquisition program presented in Figure 6, where for each type of equipment (loaders and haulage), and for each model of a particular type, the equipment required per year for the "Optimal" production rate is shown. Here, it is shown that 3 haulage trucks are acquired in year 4, and 6 more trucks are added to the fleet in year 8, almost doubling the initial fleet. The PC8000 loader fleet is kept constant along the life of mine, and the capacity increase is achieved by acquiring FEL 994 loaders, more than doubling the initial fleet by year 8, thus delaying capital expenses in order to maximize the project's NPV.

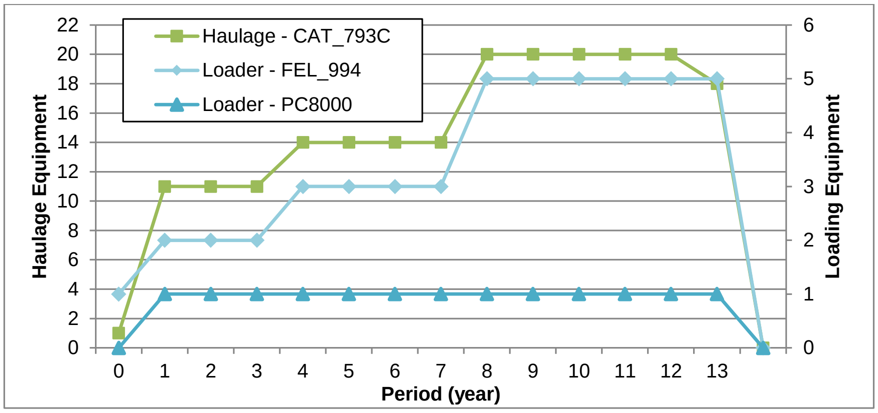


Figure 6: Equipment type and model acquisition schedule per period.

3.2 Using the optimal production rates for scheduling

To further explore the benefits of the proposed optimization model to generate optimal mining rates, a comparison is done between two schedules produced by Milawa Balanced algorithm (Whittle, 1999) available in Whittle Software. One life-of-mine production schedule is based on the mining rates defined by the optimizer presented in Section 2, and the other uses a constant mining rate (the traditional approach also practiced at the mine discussed here), equal to the average production of the optimized solution which is 93Mt per year. All of the remaining parameters used are identical in both cases, as is the high grade ore demand destined to the mill. The annual mining rate for each case is presented in Figure 7.

It is interesting to note the amount of high grade ore tonnage actually being extracted in each schedule for the obtained mining rates, as it would be expected that the "Traditional" operation manages to initially produce more ore due to its higher initial rates, however, it is shown next that this is not the case. The mill feed demand target is presented in the black dotted line in Figure 8. The actual mill utilization for each of the two generated schedule is presented in this figure by the bars, showing the amount high grade ore material extracted in each period for the traditional and optimal case. It can be observed that both schedules manage to meet mill feed demand in every year, suggesting that the traditional schedule has an increased mining rate during the first years only to mine waste, which increases the operation's costs and doesn't generate any profit.

The "Traditional" case has a steady extraction rate along the whole project, but from Figures 7 and 8, it is possible to see that this causes the operation to invest in unnecessary capital during the initial years (in order to obtain this steady rate), even if this is not really necessary to meet mill demand, and only results in early waste mining and equipment acquisition. This reduces the profit of the initial years, which are less discounted and thus, has a strong effect over the project's NPV. In comparison, the "Optimal" production rates obtained by the proposed formulation show that during the first half of the mine life there is a lower capacity required, maximizing the profit by meeting mill demand, minimizing waste mining and delaying capital expenses as much as possible.

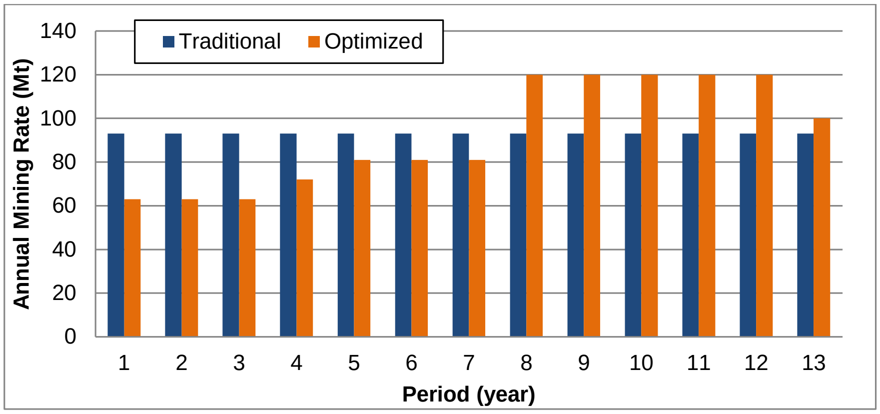


Figure 7: Total rock movement for schedules based on the traditional and optimal rates.

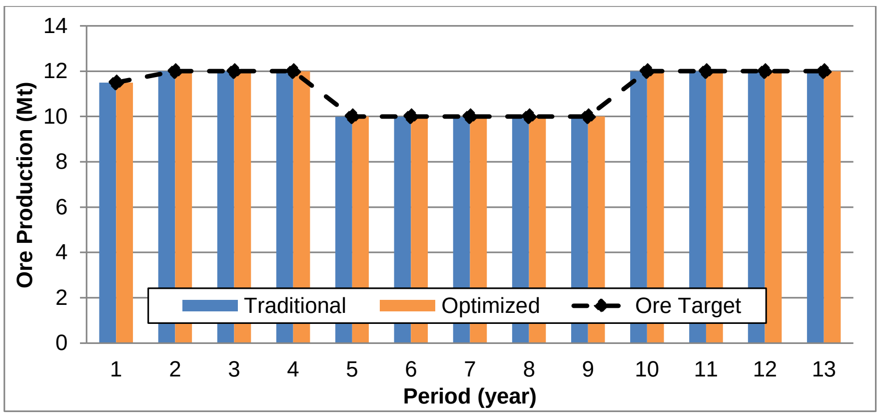


Figure 8: Mill's annual available capacity and primary ore extracted per period for traditional and optimized cases.

The previous analysis proves that the optimization model proposed here looks to maximize the net present value of the project by delaying unnecessary expenses and investments, and maximizing the metal production. This can be seen in Figure 9, where the cumulative discounted cash flow (DCF) for the "Optimized" as well as for the "Traditional" cases is presented, assuming that mill capacity as well as the mining rates are perfectly met. By periods 7 and 8, the optimal case incurs on high expenses to increase the equipment fleet and raise the mining rate of the operation; these costs cause a slight decrease in the cumulative DCF, and, as more waste rock must be removed at this point, the cumulative cash flow curve flattens in comparison to the traditional case. However, this also allows meeting ore demand, obtaining a 20.7% higher NPV than the stable mining rate case, which decides to extract more waste at the initial years, punishing the cash flow from the beginning of the operation.

The SSD presented in Figure 10 clearly illustrates the differences between the traditional and the optimized schedules obtained from Whittle. This figure shows that both extraction sequences are located inside the SSD, proving that they are both feasible mining rates independent of the encountered geology of the deposit. However, the "Traditional" case is consistently further apart from the "Best Case" in comparison with the "Optimized" case, which demonstrates that the traditional mining rates tend to extract higher amounts of waste earlier in the life of mine, only to obtain a fixed, stable mining rate.

It must be noted that, even though the schedule was obtained with the estimated model of the deposit, the mining rates were obtained considering the stable solution domain created from the intersection of the solution domains of 20 different geological simulations of the deposit, generating a feasible domain of ore and waste combinations, and not the solution domain formed by the estimated orebody model. Figure 11

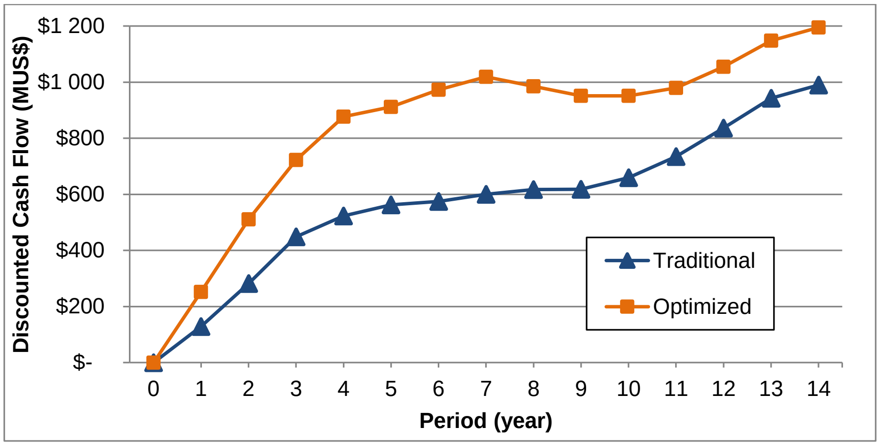


Figure 9: Cumulative discounted cash flow for the optimal and traditional cases.

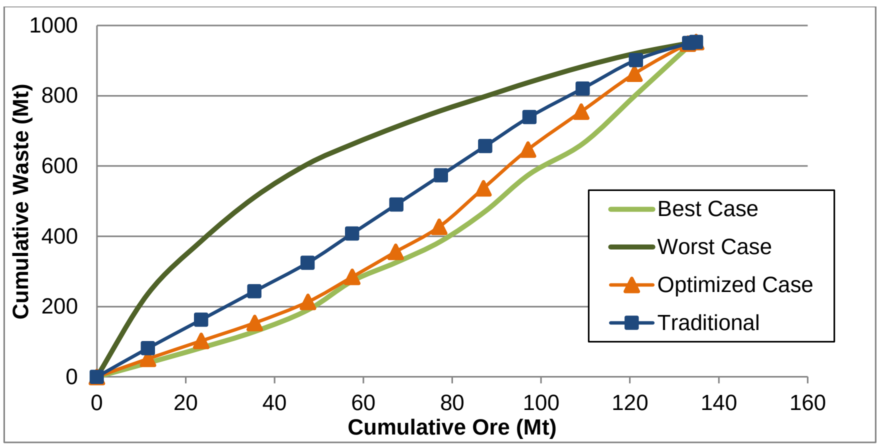


Figure 10: Cumulative extraction of ore and waste for the schedules obtained from traditional and optimized mining rates.

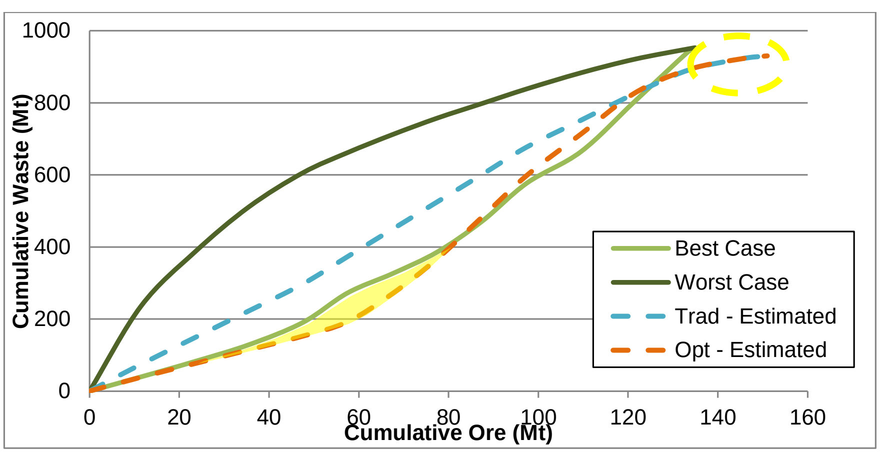


Figure 11: Comparisons of traditional and optimized schedules obtained over the SD, and the feasible SSD.

presents the effect of not taking into account the geological uncertainty to define the stable solution domain, and carrying the optimization process over the solution domain defined by the estimated model. In this case, both sequences obtained for the traditional and optimized mining rates (referred to as "Trad – Estimated" and "Opt – Estimated" respectively in Figure 10) present infeasible combinations of ore and waste extractions (highlighted in yellow in the graph), documenting once again that not considering geological uncertainty in the optimization process results in infeasible mine plans and impossibility to meet the expected mill demand.

4 Conclusions

In conclusion, it has been shown that the proposed MIP model not only provides a feasible mining rate which considers equipment acquisition and delaying of capital expenses, but also that this mining rate schedule presents clear benefits when used as a starting point for planning a mining schedule. This was shown when using the Milawa Balanced algorithm from Whittle Software over a gold mine case study to generate two schedules, with and without optimized mining rates, where the optimal mining rate case presented a 20% increase in NPV. However, it must be noted that this optimization's objective is to define an optimal mining rate, not an actual mining schedule. The scheduling problem is considered as a separate, complex problem, where the output of the model proposed in this study may be used as starting point for the design of an optimal long term mining schedule.

An important limitation of the current formulation is that the process is based on the solution domain, which, even though can consider geological uncertainty, it assumes that the ultimate pit limit is defined and fixed. This is not accurate, as after the scheduling is done, and subject to the different uncertainties that govern a mining operation, it is highly possible that the ultimate pit limit will change. Together with this, the different ownership and purchasing costs are assumed constant along the whole life of the equipment, which, once again, is an important simplification in the model.

Further work may focus on extending the proposed formulation to the mining scheduling optimization. Together with this, efforts could be made on increasing the complexity of the mining system considered, i.e. include multiple mines and deposits containing multiple elements, as well as considering the income obtained from the stockpile or other processing streams. Additionally, it would be important to consider different types of uncertainty, including commodity price, costs and equipment variability.

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