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# All-inclusive stochastic short-term production scheduling approach of an iron-ore deposit with future multi-element ore control data

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**Abstract:** Short-term mine production scheduling optimization is developed as a single formulation where mining considerations, production constraints, uncertainty in the orebody metal quantity and quality as well as fleet availability are evaluated together to define a well-informed sequence of mining that results in high performance during a mine's operation. A stochastic integer program is developed for the above. However, it is noted that partly- informed and ultimately costly decisions can be taken in the above development because of imperfect geological knowledge and information available during the actual ore control stage, affecting the performance of short-term schedules. To address this issue, orebody uncertainty models are updated by simulated future ore control data to account for local scale material type and grade variability affecting grade control classification of materials being extracted and sent to different ore/waste destinations. This updating leads to substantially improved short-term schedules accounting for better potential classifications of ore and waste materials. In general, the updating of multi-element orebody uncertainty models is based on the correlation of exploration data and past ore control data; the updated orebody uncertainty models are then used to optimize, while accounting for uncertainties, the short-term production scheduling, leading to better performance in terms of matching ore quality targets and delivering recoverable reserves anticipated. The above is demonstrated in a case study at an iron-ore deposit.

**Key Words:** Production scheduling, mining equipment fleet, stochastic simulation, future data, uncertainty in equipment availability.

## 1 Introduction

Short-term production scheduling defines a sequence of materials to be extracted over months, weeks, or days based on orebody models generated from exploration drilling data and accounts for mining considerations, production constraints, and so on. Typically, the optimization of the mining fleet available is a separate step. To improve this two-step approach and limited use of information, short-term mine production scheduling optimization is developed as a single formulation where mining considerations, production constraints, uncertainty in the orebody metal quantity and quality, as well as fleet availability are evaluated together. This aims to define a well-informed sequence of mining that results in high performance during a mine's operation. Additionally and more importantly, in order to account for potential future ore control information, past ore control data are used to simulate future ore control data and inform the definition of waste and ore categories assessed during the optimization of a short-term schedule.

Stochastic integer programming provides an optimization framework for formulating scheduling approaches that explicitly account for uncertainty in their parameters, as opposed to the traditional deterministic optimization models that are formulated assuming certainty in their input parameters. The stochastic short-term production scheduling mathematical programming formulation herein not only accounts for uncertainty in all related input parameters, but additionally proposes to update the orebody uncertainty with stochastically simulated future multi-element ore control data. At the same time, it accounts for all mining considerations and uncertainty in all related mining fleet parameters into their formulation.

The simulation of future grade control data is shown first in Guardiano et al. (1997) where the production schedule of a gold mine is assessed by simulating future grade control data. The latter data is generated by adding to a simulated realization of the deposit (based on exploration drilling) randomly generated errors from a distribution of blasthole errors assumed to represent the deposit. The variance of this error corresponds to the difference between exploration data nugget effect variogram and grade control data nugget effect variogram (Knudsen 1992). This technique is conceptually interesting but simplistic and arbitrary. Khosrowshahi et al. (2007) propose a simulation of the chain of mining to identify errors in every stage of the mining process to forecast the recoverable reserves during mining and with the expected short-term plans. Sampling and assaying errors, mining selectivity and movement due to blasting are incorporated into the evaluation of several chains of mining to determine the parameters that may match the current mining performance of the mine. Two sampling errors due to the shape of the blasthole cone and the impacts of the blasthole subdrill were used to define distributions of the errors. However, a drawback of this study is that a local normal distribution error is used to simulate future ore control data through the domain. Journel and Kyriakidis (2004) show that the difference between ore control data and exploration data are not constant errors through the deposit. A more elaborate study is presented in Peattie and Dimitrakopoulos (2013) based on the spatial variability of the blasthole error. The errors are calculated from the difference of estimation based on exploration data and estimation based on ore control at block scale. The spatial variability of these errors is modeled to simulate the error at the same grid where only an orebody model based on exploration data is available. The ore control data map is calculated from the orebody model available plus the error simulated.

The use of simulated future grade control data for long-term mine production scheduling is shown in Dimitrakopoulos and Jewbali (2013), who present an approach to forecast ore control data of a single element base on the spatial correlation between ore control data and exploration of a mined out sector. A pseudo cross variogram is used to evaluate the cross spatial variability of two data that are not at the exact same location. This pseudo cross variogram is used at the ore control sector where only exploration data is available to perform a co-simulation of grade control errors conditioned to the available data. The study also shows the performance improvement from integrating future ore control data, including meeting production expectations and higher NPV. Past approaches, such as the above, are implemented for single element deposits. To extend the approach to multi-element deposits, techniques available to simulate multiple correlated variables such as Minimum/Maximum Autocorrelation Factors (MAF) (Desbarats and Dimitrakopoulos 2000) can be used to simulate spatial multi-element errors at the mining sector considered.

In the following sections, the method used here is presented and includes (a) the simulation of future multi-element ore control data, and (b) the stochastic short-term scheduling formulation used. An application at an iron ore deposit shows the practical aspects of the proposed approach and conclusions are drawn thereafter

## 2 Method

The proposed approach herein considers possible short-scale information that better assists in the classification of the material in Stage 1, where the future data is simulated. In the second Stage, short-term mine production scheduling is stochastically optimized.

### 2.1 Stage 1: Future multi-element ore control data

The multivariate technique Minimum/Maximum Autocorrelation Factors (MAF) assists to simulate multi-element deposits by transforming correlated variables to de-correlated factors, simulating them and reconstructing the simulated original variables (Desbarats and Dimitrakopoulos 2000). The MAF approach considers the multivariate observation vector as $Y(x) = (Y_1(x), \ldots, Y_q(x))^T$ and their $q$ orthogonal lineal combinations or MAF $F_i(x) = a_i^T Y(x), i = 1, \ldots, q$. $Y(x)$ are de-correlated as follows:

- Decompose the variance-covariance symmetric matrix of $Y(x)$. Their spectral decomposition $HDH^T$ provides orthonormal eigenvectors matrix $H$ and diagonal matrix of eigenvalues $D$.
- Compute the conventional principal components factors as $W^T = HD^{-1/2}$.
- Calculate the variogram matrix $\Gamma_Y(\Delta)$ and their spectral decomposition into orthonormal eigenvectors $C$ matrix and eigenvalues $\Lambda$ diagonal matrix.
- Then, the MAF transformation matrix is $HD^{-1/2}C$.
- The MAF factors will be $F(x) = Y(x) \overbrace{HD^{-1/2}}^{W^T} C$.

The MAF non-correlated factors $F(x)$ are simulated independently where their spatial relationship is modeled and sequential Gaussian simulation is carried out. The output of the simulations is re-correlated using the inverse of the MAF transformation matrix $(W^T C)^{-1}$.

An approach to jointly simulate future multi-element ore control data featured herein is based on MAF. The approach considers the spatial relationship of the errors between exploration data and ore control data from a mined sector so as to forecast ore control data where only exploration data is available. It is assumed that mined sector $A$ has similar geology as sector $B$, where it is required to simulate future ore control data.

The joint simulation of errors requires a map of errors per element from the mined out sector $A$ and to then de-correlate these errors. Next, the spatial correlation of these de-correlated errors is modeled to be used in the joint simulation at the grid of sector $B$. The simulated maps of errors are added to the orebody realizations at sector $B$ to forecast the map of the future ore control data. The details of the approach are as follows:

- Calculate the error $e_{bh-ddh}$ between exploration data and ore control data per element.
- Standard normal score transformation per error for sector $A$, $Y_A(x)$.
- MAF transformation $F(x) = Y_A(x) W^T C$ is executed to de-correlate error variables of sector $A$ to permit simulating the error of the elements independently.
- The variogram $\gamma^A_{F(x)}(h)$ of the MAF non-correlated factors $F(x)$ are modeled.
- Simulation is performed for each MAF at sector $B$ using the modeled variograms $\gamma^A_{F(x)}(h)$ of mined sector $A$.
- The inverse of the MAF transformation matrix $(W^T C)^{-1}$ is applied to MAF variables simulation at Sector $B$ to recover the correlation among errors.
- The normal score simulated errors are back transformed into the original space.
- Error maps are added to the available realizations based on exploration data at sector $B$.

The approach requires modeling variograms as the number of elements is evaluated. The steps are shown in Figure 1.

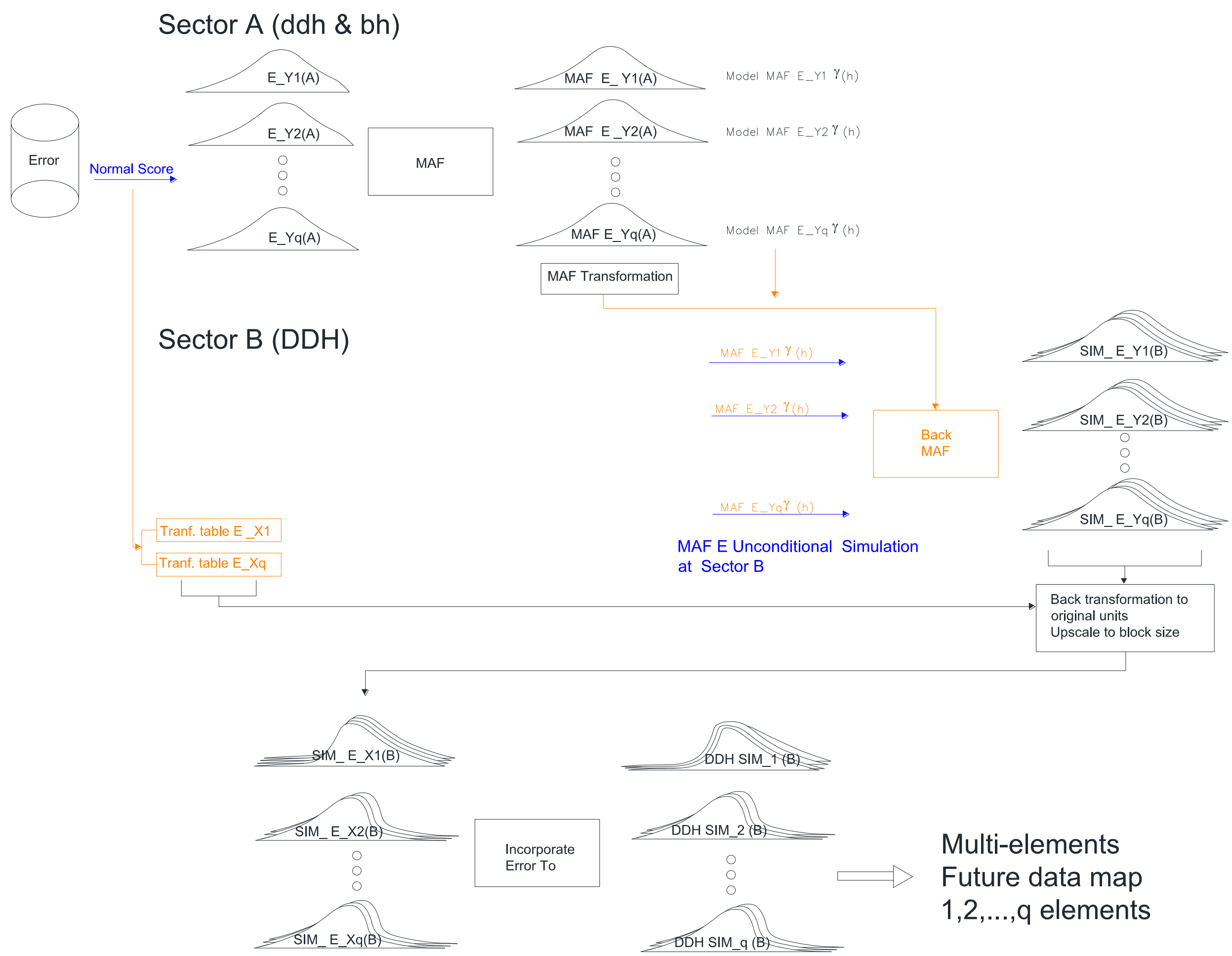


Figure 1: Schematic representation of simulated future multi-element ore control data.

The MAF transformation permits the resulting MAF non-correlated factors to be simulated independently. For the simulation, the sector $B$ is discretized by nodes and each node is simulated in the Gaussian space, conditioned to previous nodes simulated. These nodes are simulated sequentially following a path that changes from one realization to another. Once all the MAF non-correlated factors are simulated, back MAF transformation is applied to recover the correlation among variables. These variables are the errors and the simulation of multi-element map errors is added to the multi-element simulation at sector $B$ to obtain the maps of the future ore control data.

## 2.2 Stage 2: Stochastic short-term production scheduling

The short-term mine production scheduling discussed above is formulated as a stochastic integer programming model with recourse (Birge and Louveaux 1997), accounting for uncertainty in input parameters. The solution aims to minimize the total mining cost along with deviations from production targets, considers operational aspects, such as mining direction and minimum width, and maximizes fleet utilization. In the formulation presented herein, the first-stage decisions are made before the uncertainty is revealed, then the second-stage decisions or recourse actions are made after uncertainty is considered.

The notation used to formulate short-term scheduling follows. Note that indexes relate to the set of trucks, shovels, sectors, blocks, periods and realizations of uncertain parameters.

- $j$: a sector or bench, where $j = 1, \ldots, J$
- $i$: an shovel, where $i = 1, \ldots, I$
- $k$: a block at sector, where $k = 1, \ldots, K(j)$
- $l$: a truck model, where $l = 1, \ldots, L$
- $p$: a period of a production schedule, where $p = 1, \ldots, P$
- $\varepsilon$: an element grade of $k$ block that have economical value, where $\varepsilon = 1, \ldots, E$
- $\delta$: a deleterious element grade of $k$ block, where $\delta = 1, \ldots, D$
- $s$: simulated grade realization or scenario, where $s = 1, \ldots, S$
- $\alpha$: realization of shovel mechanical availability given historical data, where $\alpha = 1, \ldots, A$
- $r$: truck cycle trip and mechanical availability realization, where $r = 1, \ldots, R$

The parameters used at the fleet allocation, cost and penalties at objective function, production target and multi-element quality and tonnage are explained as follows:

- $h_{fleet}$: fleet operation hours by period $p$
- $\iota$: maximum number of shovels allowed by sector
- $Q_i^{sh}$: hourly production of shovel $i$
- $\omega_i(\mu_i, \sigma_i)$: mean and standard deviation of historical mechanical availability by shovel $i$
- $a_{ij'}^{p-1}$: binary parameter, if shovel $i$ is or not allocated to sector $j'$ at previous period $p-1$
- $c_{j'j}^{ExcM}$: cost of moving shovel from $p-1$ allocation sector $j'$ to new allocation sector $j$
- $c^{prodExc-}$: penalty cost for tonnage not produced regarding to the expected productivity
- $Q_l^{trk}$: capacity of truck $l$
- $\varphi_{jl}(\mu_{jl}, \sigma_{jl})$: mean and standard deviation of cycle time by truck $l$ at sector $j$
- $\psi_l(\mu_l, \sigma_l)$: mean and standard deviation of historical mechanical availability by truck $l$
- $c^{\varphi}$: time cycle cost per $\varphi$ units
- $c^{m-}, c^{m+}$: penalty cost for shortage and surplus total mining tonnage respect to the targets
- $c^{o-}, c^{o+}$: penalty cost for shortage and surplus ore mining tonnage respect to the targets
- $c^{\varepsilon-}, c^{\varepsilon+}, c^{\delta+}, c^{\delta-}$: penalty cost for deviation from main elements and contaminants limits
- $P^{\min}, P^{\max}$: minimum and maximum mining tonnage target
- $O^{\min}, O^{\max}$: minimum and maximum ore tonnage target
- $G^{\varepsilon-}, G^{\varepsilon+}, G^{\delta-}, G^{\delta+}$: quality or grade requirements for ore tonnage produced $\%Tol_{o-}$, $\%Tol_{o+}$, $\%Tol_{m-}$, $\%Tol_{m+}$, $\%Tol_{\varepsilon-}$, $\%Tol_{\varepsilon+}$, $\%Tol_{\delta-}$, $\%Tol_{\delta+}$: allowed percentage of tonnage and grade deviation from targets
- $B_{jk}$: block tonnage $k$ at sector $j$
- $bh, ddh$: Ore control data and exploration data at mined sector $A$
- $BH, DDH$: Ore control data and exploration data at not mined sector $B$
- $c^m$: mining cost by $B_{jk}$ unit
- $g_{jks}^{\varepsilon}, g_{jks}^{\delta}$: grade block $k$ of main elements and deleterious in scenario $s$ at sector $j$
- $O_{jks}$: binary parameter flagging the block $k$ at $j$ sector for scenario $s$ that has the minimum quality to be used at the blending process; otherwise, the block is flagged as waste.
- $\varphi_{rjl}$: truck cycle time $r$ of truck $l$ at sector $j$ given cycle time distribution
- $\theta_{rjl}$: maximum number of trips of truck $l$ at sector $j$ for cycle hauling realization $r$ and mechanical availability realization $r$

$$\theta_{jlr} = \frac{\psi_{lr} \times h_{fleet}}{\varphi_{jlr}} \quad \forall r = 1, \ldots, R, \quad \forall j = 1, \ldots, J, \quad \forall l = 1, \ldots, L$$

- $Q^{sh}_{i\alpha}$: maximum production rate of shovel $i$ per mechanic availability realization $\alpha$and each realization $\omega_{i\alpha}$is drawn from the available mechanical availability distribution, and it is

$$Q^{sh}_{i\alpha} = \omega_{i\alpha} \times h_{fleet} \times Q^{sh}_{i} \quad \forall\alpha = 1, \ldots, A, \quad \forall i = 1, \ldots, I$$

The decision variables used are as follows:

- $x^p_{jk}$: binary variable, if block $k$ at sector $j$ is mined or not at period $p$
- $e^p_{ij}$: binary variable, if shovel $i$ is or not allocated to sector $j$ at period $p$
- $n^p_{jilr}$: number of trips of truck $l$ to sector $j$, shovel $i$ at period $p$ for cycle time realization and mechanical availability realization $r$
- $f^p_{ji\alpha}$: deviation of shovel $i$ at sector $j$ from expected shovel production$Q^{sh}_{i\alpha}$
- $y^p_{jk}$: number of blocks that were not scheduled at period $p$ to mine block $k$ at sector $j$ to match mining width requirements.
- $d^{m-}_p, d^{m+}_p$: shortage tonnage to match lower production limit and surplus tonnage to match upper production limit at period $p$
- $d^{o-}_{sp}, d^{o+}_{sp}$: shortage of ore mining to match lower bound and the surplus to match upper bound at period $p$ accounting for grade scenario $s$
- $d^{\varepsilon-}_{sp}, d^{\varepsilon+}_{sp}$: deviation from $\varepsilon$ grade targets at period $p$ for grade scenario $s$
- $d^{\delta-}_{sp}, d^{\delta+}_{sp}$: deviation from $\delta$ deleterious grade targets at period $p$ for grade scenario $s$

## 2.3 Objective function

Decision variables $x^p_{jk}$, $y^p_{jk}$ and $e^p_{ij}$ are related with the first-stage and remaining decision variables are related with the second-stage. The first-stage decisions include minimizing the costs of extraction of materials, movement of shovels, production shortage, and matching mining width. In the second-stage, these costs are minimized over a range of possibilities of a recourse cost associated with deviations from ore production and quality targets, hauling cost, and lack of mining with maximum shovel productivity. The formulation of the stochastic short-term production scheduling considers eight components in the objective function. The first, fourth, fifth and eighth components depend on deterministic parameters. The cost of extracting every ton of the pit is reduced directly in the first term, and this cost does not include hauling cost. The hauling cost is covered and minimized in the second component in order to efficiently allocate the trucks. This component considers the possible fluctuations of two parameters: cycle trip and mechanic availability of each truck. The fourth component considers uncertainty in the mechanic availability of each shovel and the loading cost is indirectly reduced where the lack of mining expected digging rate is minimized to maximize the utilization of the shovels. The third component avoids inefficient excessive shovel movements and the fifth component avoids unrealistic short-term production patterns.

$$
\begin{aligned}
Minimize = & \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)} c^m B_{jk} x^p_{jk}}^{1st} + \overbrace{\frac{1}{R}\sum_{p=1}^{P}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{i=1}^{I} \varphi_{jlr} c^{\varphi} n^p_{jilr}}^{2nd} \\
& + \overbrace{\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{j'=1}^{J}\sum_{j=1}^{J}\left(c^{ExcM}_{j'j} e^p_{ij} a^{p-1}_{ij'}\right)}^{3rd} + \overbrace{\frac{1}{A}\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{\alpha=1}^{A}\left(c^{prodExc-} f^p_{ji\alpha}\right)}^{4th} + \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)}\left(c^{smoth-} y^p_{jk}\right)}^{5th} \\
& + \overbrace{\frac{1}{S}\left\{\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\varepsilon=1}^{E}\left(c^{\varepsilon-} d^{\varepsilon-}_{sp} + c^{\varepsilon+} d^{\varepsilon+}_{sp}\right) + \sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\delta=1}^{D}\left(c^{\delta+} d^{\delta+}_{sp} + c^{\delta-} d^{\delta-}_{sp}\right)\right\}}^{6th} \\
& + \overbrace{\frac{1}{S}\sum_{s=1}^{S}\sum_{p=1}^{P}\left(c^{o+} d^{o+}_{sp} + c^{o-} d^{o-}_{sp}\right)}^{7th} + \overbrace{\sum_{p=1}^{P}\left(c^{m-} d^{m-}_{p} + c^{m+} d^{m+}_{p}\right)}^{8th} \qquad (1)
\end{aligned}
$$

Components six and seven account directly for the updated orebody uncertainty; however, the short-scale information of the multi-elements may influence indirectly the rest of the decision variables of others components.

### 2.3.1 Constraints for Production and fleet allocation

The constraints below link the fleet allocation decision variables with mined block decision variables, to guarantee that the short-term production schedule accounts for fleet allocations and production targets.

$$\sum_{p=1}^{P} x_{jk}^{p} \leq 1, \quad \forall j = 1, \ldots, J, \quad \forall k = 1, \ldots, K(j) \tag{2}$$

Constraint (2) ensures that a block of material may be mined once at any period. The block is a selective mining unit that may be mined in one period assuming that the time period may be from weeks to months.

$$\sum_{i=1}^{I} e_{ij}^{p} \leq \iota, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J \tag{3}$$

$$\sum_{j=1}^{J} e_{ij}^{p} \leq 1, \quad \forall p = 1, \ldots, P, \quad \forall i = 1, \ldots, I \tag{4}$$

$$x_{jk}^{p} - \sum_{i=1}^{I} e_{ij}^{p} \leq 0, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J, \quad \forall k = 1, \ldots, K(j) \tag{5}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \varphi_{jlr} \times n_{jilr}^{p} \leq h_{fleet} \times \psi_{lr} \quad \forall p = 1, \ldots, P, \quad \forall l = 1, \ldots, L, \quad \forall r = 1, \ldots, R \tag{6}$$

$$n_{jilr}^{p} - \theta_{jlr} e_{ij}^{p} \leq 0, \quad \forall p = 1, \ldots, P, \quad \forall r = 1, \ldots, R, \quad \forall j = 1, \ldots, J, \quad \forall l = 1, \ldots, L, \quad \forall i = 1, \ldots, I \tag{7}$$

$$\sum_{l=1}^{L} \left( Q_{l}^{truck} \times n_{jilr}^{p} \right) - Q_{i\alpha}^{sh} \times e_{ij}^{p} + f_{ji\alpha}^{p} = 0 \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J, \quad \forall i = 1, \ldots, I, \quad \forall \alpha = 1, \ldots, A, \quad \forall r = 1, \ldots, R \tag{8}$$

$$\sum_{i=1}^{I} \sum_{l=1}^{L} \left( Q_{l}^{truck} \times n_{jilr}^{p} \right) - \sum_{k=1}^{K(j)} \left( B_{jk} \times x_{jk}^{p} \right) = 0, \quad \forall p = 1, \ldots, P, \forall j = 1, \ldots, J, \forall r = 1, \ldots, R \tag{9}$$

The mining equipment can be placed in a given number of locations. A possible path of the locations of each piece of equipment is provided as part of a short-term plan. Shovels are allocated to available sectors or remain in the current sector previously allocated. A sector must be mined at some period and a shovel must be allocated to the sector that has a lower cost of hauling and provides the material to match quality requirements. Constraints (3) ensure that each sector is allocated with less equal than $\iota$ shovels at sector $j$ per period $p$. The parameter $\iota$ is the maximum number of shovels that can be allocated in each sector. Constraint (4) ensures that each shovel $i$ may be assigned to one sector, while the cost of movement is minimized in the objective function to prevent excessive shovel movement among sectors. Inequality constraints are used for the fleet allocation because not all the available shovels or trucks are allocated in scenarios where there is more equipment than the production requires in accounting for hauling distance. Constraint (5) guarantees that a mining block in sector $j$ is mined only if a shovel is allocated to sector $j$.

Variable $n_{jilr}^{p}$ decides the optimal number of trips for truck $l$ to sector $j$ and shovel $i$ per period $p$, thus accounting for fluctuations of truck cycle time and mechanical availability. The number of trips decision variable $n_{jilr}^{p}$ also supports in the allocation of each truck $l$ to shovel $i$ to sector $j$ for mechanical availability and hauling realization $r$ per period $p$. The formulation considers that a truck can be allocated to more than one shovel at the same sector $j$ or different sectors. Constraint (6) limits the number of trips of a truck to its scheduled time per period as the operation progresses by extracting minerals and continuously extending the access. Indeed, the roads change dynamically. This implies uncertainty in the hauling time. The trip cycle time $\varphi_{jlr}$ of truck $l$ to sector $j$ is drawn from distribution $R$ times.

The decision variable $n^p_{jilr}$ is also subject to the maximum number of trips that a truck $l$ can haul from each sector $j$. The maximum number of trips $\theta_{jlr}$ per truck $l$ is a preprocessed parameter because its components are not decision variables. Then, the number of total trips to each sector is restricted to a maximum number of trips times the $e^p_{ij}$ binary decision variable. The decision variable $e^p_{ij}$ is relevant in the Constraints (7) because not all the sectors will be allocated with a shovel and a sector without a shovel cannot have number of trips. Decision variables $n^p_{jilr}$ and $e^p_{ij}$ are linked. The inequality Constraint (7) also ensures that only an allocated sector with a shovel is assigned with trucks, and not all trucks are allocated at some scenarios. The link of truck $l$, shovel $i$ and sectors $j$ in the constraints ensure that all assignment possibilities for the trucks, shovel and sectors are taken into account.

There are capacity limits for each truck $Q^{trk}_l$ and shovel $Q^{sh}_i$. The available fleet and their respective capacity are included in the formulation. The production of each shovel assigned to sector $j$ is constrained to the maximum production of each shovel $Q^{sh}_{i\alpha}$. The $e^p_{ij}$ binary decision variable helps to formulate the shovel capacity constraints (8) because not all of the shovels may be allocated. The lack of expected production by each shovel is stored by the decision variable $f^p_{ji\alpha}$, which is minimized at the objective function.

There are $J$ sectors and each sector has $K(j)$ blocks to be evaluated. The tonnage of block $k$ is $B_{jk}$ and each block may be hauled from an in-situ location to a blending area or waste dump, taking into account the fleet capacity constraints. The decision variables at operational and production constraints are linked to fleet allocation constraints. Indeed, Constraint (9) links the number of trips $n^p_{jilr}$ of truck $l$ from sector $j$ and shovel $i$ given mechanic availability and hauling time realization $r$ with the mined block decision variable $x^p_{jk}$. The hauling tonnage by the trucks from sector $j$ for mechanical availability and hauling time realization $r$ must be equal to scheduled blocks tonnage at sector $j$.

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J} Q^{trk}_l \times n^p_{jilr} \geq M^{\min} \quad \forall p=1,\ldots,P, \quad \forall r=1,\ldots,R \tag{10}$$

$$0 \leq d^{m-}_p \leq \%Tol_{m-} \times M^{\min} \quad \forall p=1,\ldots,P \tag{11}$$

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J} Q^{trk}_l \times n^p_{jilr} - d^{m+}_p \leq M^{\max} \quad \forall p=1,\ldots,P, \quad \forall r=1,\ldots,R \tag{12}$$

$$0 \leq d^{m+}_p \leq \%Tol_{m+} \times M^{\max} \quad \forall p=1,\ldots,P \tag{13}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(O_{jks} \times B_{jk} \times x^p_{jk}\right) + d^{o-}_{sp} \geq O^{\min} \quad \forall p=1,\ldots,P, \quad \forall s=1,\ldots,S \tag{14}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(O_{jks} \times B_{jk} \times x^p_{jk}\right) - d^{o+}_{sp} \leq O^{\max} \quad \forall p=1,\ldots,P, \quad \forall s=1,\ldots,S \tag{15}$$

$$0 \leq d^{o-}_{sp} \leq \%Tol_{o-} \times O^{\min} \quad \forall p=1,\ldots,P, \quad \forall s=1,\ldots,S \tag{16}$$

$$0 \leq d^{o+}_{sp} \leq \%Tol_{o+} \times O^{\max} \quad \forall p=1,\ldots,P, \quad \forall s=1,\ldots,S \tag{17}$$

Production per time period $p$ is constrained to the production targets (10). The production includes ore tons plus the waste tons. The ore tonnage is the material that has a positive economic value, while the waste tonnage is the material without a positive economic value that needs to be extracted to allow access to ore and ensure the continuity of ore production in the following periods. The number of trip decision variables and truck capacities are used to calculate the total tonnage extracted per period. The proposed model considers strict constraints for early periods and can be relaxed for the latest periods. To relax the production constraints, the shortage $d^{m-}_p$ with respect to the target planned is considered, along with their respective tolerance of deviation (11).

Traditionally, an upper bound is not used in production formulation because the cost of mining will limit overproduction; however, in the current formulation the production must be limited because the capacity shovel constraints maximize the production by sector to increase the utilization of the shovel (12). The upper bound limits this maximization to keep close to the production targets. The deviation $d_p^{m+}$ with respect to the upper bound total production is penalized in the objective function and their tolerance is considered (13). As a production constraint, the ore tonnage should match the target ore production given by long-term production schedules (14, 15). The shortage $d_{sp}^{o-}$ respects the target planned and the surplus $d_{sp}^{o+}$, respects the upper bound ore processing and are penalized in the objective function. The deviations are limited by a percentage of ore production $\%Tol$ (16, 17). The upper bound is directly related to the ore tonnage scheduled plus the maximum capacity pile of ore next to the delivering location. The exceeding material from the upper bound may be considered as material that goes to the stockpile, and its tonnage is penalized by the corresponding re-handled cost.

Ore production must match certain quality constraints, that is, the expected grades or quality of the material at the end of the week or month must fit into specific ranges. This range depends on long-term production schedule specifications. To meet this demand, a block $x_{jk}^p$ is mined only if their grade helps to satisfy the required quality given the available fleet. Assuming that the study case has $E$ elements that have economic value and $D$ elements as deleterious elements, $2(E+D)$ quality constraints are needed to meet quality conditions. The grade of the main commodity for ore tonnage should satisfy the constraints (18, 19) and the quality deviations have tolerance (20, 21) to ensure a production schedule with low variable average quality.

Ore production cannot have more than the required limits of contaminants because this contains $D$ deleterious elements. The constraints (22, 23) ensure that the ore delivered by period given $S$ scenarios of the grades have average grades less than $G^{\delta+}$ and more than $G^{\delta-}$ for deleterious element $\delta = 1, \ldots, D$. The quality deviations related with contaminants are also constrained to tolerance (24, 25) to ensure production schedule with low variable average quality.

Blending of ore from sectors is carried out based on cutoffs that define the minimum quality that a block $k$ must have to be included in the blending process. If a block $k$ has the chance of being used for blending $O_{jks} = 1$; otherwise, the block $k$ is allocated to the waste dump directly $O_{jks} = 0$. The quality constraints are satisfied when the total ore production meets the required quality conditions set as targets.

### 2.3.2 Constraints for operational considerations

Operational considerations relate to the size of the equipment and accessibility restrictions that may require feasible (in a mining sense) production schedule patterns that allow the available equipment to work efficiently and streamline movements for safety reasons. The first operation consideration is the mining direction that facilitates access to the sectors to be mined and it is:

$$x_{jk}^p - \sum_{\tau=1}^{p} x_{jk'}^{\tau} \leq 0, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J, \quad \forall k = 1, \ldots, K(j), k' \in \Omega_{k'} \tag{18}$$

where $\Omega_{k'}$ is the set of indexes representing blocks that are horizontal predecessors which must be mined before block $k$ to match the mining direction. A sector could be mined following eight directions, as shown in Figure 2.

The second operational consideration is the mining width, which relates to the minimum width the patterns of a short-term schedule period has that permits fleet access to the orebody and materials to be extracted. Production schedules that do not account for mining width may deliver schedule patterns with singular blocks of early periods surrounded by blocks from later period, as shown in Figure 3. This production scheduling cannot be implemented as the blocks scheduled for Period 1, (blue squares) cannot be mined before some blocks belonging to Period 2 (orange squares) are extracted.

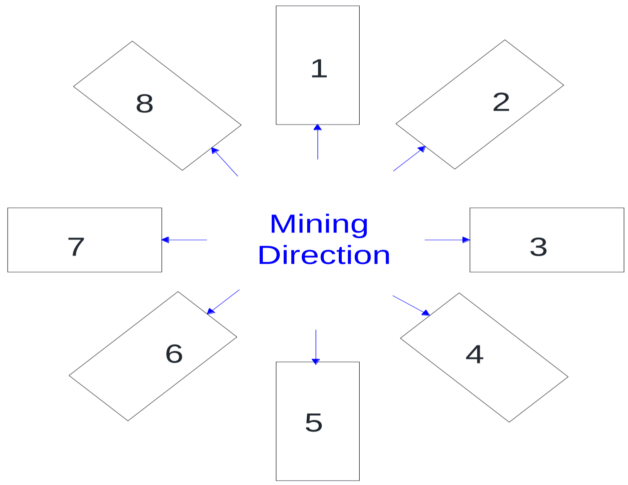


Figure 2: Eight mining directions considered by the formulation.

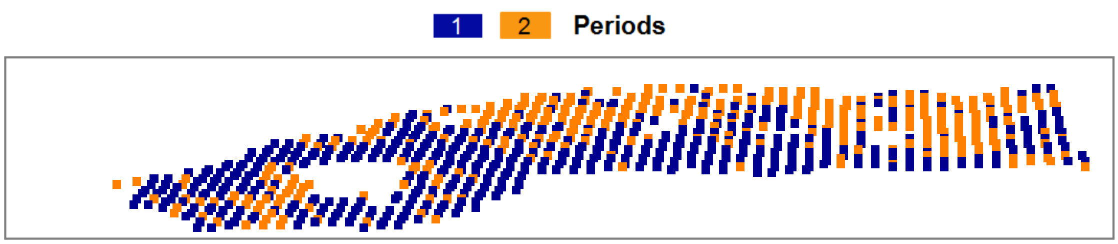


Figure 3: Production scheduling without mining width constraints.

The following mining width constraints account for feasible extraction patterns and may force the mining of some blocks before a given block $k$ as shown in Figure 3.

$$-2 \times x^p_{jk'} - x^p_{jk''} + (2 \times \upsilon + \upsilon) \times x^p_{jk} - y^p_{jk} \leq 0, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J \quad , \forall k = 1, \ldots, K(j)$$
$$k^{'} \in \Psi_{k'}, k^{''} \in \Psi_{k''} \qquad (19)$$

The mining width is discretized into $\upsilon$ blocks where $\Psi_{k''}$is its set of indexes. To mine a block $k, \upsilon$ blocks may be mined at the same period or have been mined at previous periods. $\Psi_{k'}$ is the set of indexes representing the adjacent blocks and priority of mining adjacent blocks $\nu$ is considered to avoid single blocks from some periods being surrounded by blocks from different periods. Indeed, the blocks $\nu$ that surround block $k$ must be mined with twice the priority than the second term at Constraints (27) to avoid infeasible mining patterns. The adjacent $\nu$ blocks belong to the inner window and the $\upsilon$ blocks belong to the outer window in smooth constraints (Dimitrakopoulos and Ramazan 2004). These smooth mining constraints are linked to mining width to provide feasible mining sequences that the fleet requires to operate efficiently. It is important to remark that $\upsilon$ number of blocks that match mining width are variable through the sector. The blocks that are located close to the border will require less $\upsilon$ blocks to be moved because some blocks were already mined or are 'air' (non-physically existing) blocks.

The mining width constraints are relaxed because at some locations feasible solutions will require mining only some $\upsilon$ blocks. The discrete decision variable $y^p_{jk}$ will store the lack of mining blocks that match the mining width considerations. This decision variable is penalized and minimized at the objective function.

# 3 Application at an iron ore mine

The implementation of the proposed stochastic short-term mine production schedule formulation accounting for future ore control data is the goal of this case study. The future ore control data is simulated, the orebody uncertainty is updated with simulated future ore control data, and the influence of short-scale information in the mine production scheduling and in the fleet allocation are evaluated. The proposed formulation is applied at an iron deposit. Iron ore deposits are typical examples of a multi-element environment, where the main production objective is to satisfy the customer quality requirement at a lower cost by optimally blending the different sectors of a mine. More specifically, when the iron content is evaluated and must be within customer specified limits there are also specific restrictions on the content of the so-called deleterious elements, such as phosphorous (P), silica ($SiO_2$), alumina ($Al_2O_3$) and the water and organic content measured as "loss on ignition" (LOI). These deleterious elements influence the physical and chemical properties of the iron ore product, significantly varies from customer to customer and contractual agreement to be met, and the performance of the process it will be used for. For instance, phosphorous affects steel quality (added cost), high silica and high alumina affect furnace efficiency, and the LOI affect fuel use and water in a hot furnace for steel making.

The stochastic long-term production scheduling (SLTPS) is a relevant input parameter used to define the target production of the short-term production scheduling (SSTPS). From the given long-term production schedule of five periods (years), the sector that correspond to the first period will be used to schedule the short-term mining sequence. This period or annual production is optimized into short time periods of twelve months and the quality targets and tonnage of the SSTPS are given in Table 1.

Table 1: First year production quantity and quality requirements.

| **Period** | **Ore Tonage** | **$Fe_2O_3$ (%)** | **P (%)** | **$SiO_2$ (%)** | **$Al_2O_3$ (%)** | **LOI (%)** |
|---|---|---|---|---|---|---|
| 1.00 | 14,000,000 | 57.1–59.4 | 0.032–0.038 | 4.6–5.2 | 0.9–1.05 | 9.5–11 |

Note: Ore/Waste cus-off grade is Fe>= 56%

From the first year tonnage in Table 1, the mine must produce iron ore of about 1.16 million iron tons each month. The average grade of the related elements per month may be in the intervals of the first year long-term ore quality plan; however, the spatial variability of these grades varies when monthly increments are considered and along the mining direction and width. Ore quality intervals correspond to the upper bound and lower bound per element over the total year. In practice, the spatial variability of the grade, the mining direction and mining width make these quality targets hard to match on a monthly basis; however, the total annual production should satisfy these upper and lower bounds per element. The twelve months production may be extracted from 734 blocks of 25 meters by 25 meters of 3,525 to 21,150 tons, located at three consecutive benches of twelve meters height each. The orebody uncertainty based on exploration data is an input data for the sector to be extracted (sector $B$). $S = 10$ equally probable scenarios of the multi-element grades are considered and filtered inside the limits of the given long-term first year production schedule.

Additionally, two sets of data are provided from another sector that has similar geological domains and was mined out (sector $A$), for exploration data and ore past control data. The information of the exploration data contains 922 drillholes, the depth of which fluctuate from 6 meters to 108 meters and has the median space distance of 48 meters. The information of ore control data has 18,181 blastholes with lengths of 6 m and the median space distance of approximately 15 meters.

Besides the input data sets used to simulate future ore control data, the parameters related to the fleet are required. The size of the fleet, mechanical availability and hauling time from in situ orebody to destination are parameters used to allocated shovels and trucks at orebody sectors. For this case study $i = 2$ shovels and $l = 10$ trucks are given as a fleet. The hour productivity or digging rate of each shovel fluctuates between 1 180 and 1 400 tonnes and each truck can haul from 100 to 136 tonnes each trip. The mechanical availability of the fleet is considered as an uncertain parameter into the short-term production schedule formulation. The distribution of mechanical availability per truck and per shovel is given as input parameters from historical

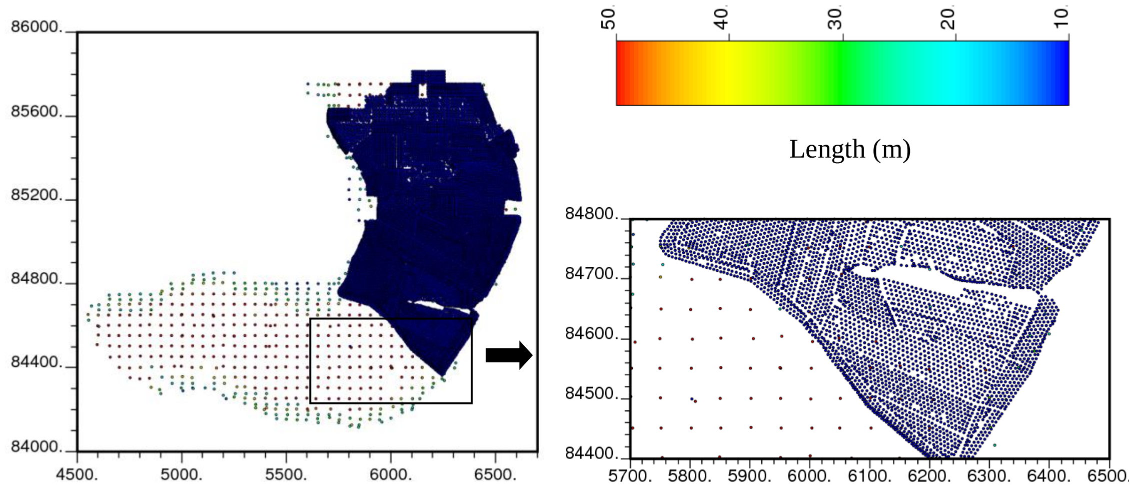


Figure 4: Location of drillholes (sparse data) and blastholes (dense data) at sector $A$.

data. Short-term production scheduling has the advantage of having some relevant information related to daily production that long-term production scheduling does not have. The hauling distance and the speed per truck are important parameters that are available and support the allocation of the trucks; the truck hauling time for each sector to the blending pad location or another destination may be calculated and are considered as uncertain parameters into the short-term production schedule formulation. The shovel model, digging rate and mechanical availability parameter distributions are given along with the track model, capacity and mechanical availability parameter distribution per truck.

Table 2: Shovel model and mechanical availability parameter distribution.

| | | | **Mechanical Availability (%)** | |
|---|---|---|---|---|
| **Model** | **Shovel ($i$)** | **Production (Tonnes/hour)** | **Mean** | **Std.Dev.** |
| HS6020 | 1 | 1180 | 83 | 4.5 |
| HS6030 | 2 | 1400 | 83 | 4 |

Table 3: Truck model and mechanical availability parameter distribution.

| | | | **Mechanical Availability (%)** | |
|---|---|---|---|---|
| **Model** | **Truck ($l$)** | **Tonnes** | **Mean** | **Std.Dev.** |
| Cat785D_501 | 1 | 136 | 83 | 5 |
| Cat785D_502 | 2 | 136 | 83 | 4 |
| | : | : | : | : |
| Cat785D_510 | 8 | 136 | 83 | 4 |
| Cat77G_511 | 9 | 100 | 83 | 5 |
| Cat77G_512 | 10 | 100 | 83 | 5 |

Short-term evaluation has the advantage of accounting for additional short-term information such as the hauling distance that is available at the short-term evaluation. This supports the allocation of trucks because the past records of speed per truck, truck hauling time per sector in a mine and blending pad location are available. Additionally, the parameter distribution of the time that spends $l$ truck from the sector $j$ to the destination is calculated as shown in Table 4.

Table 4: Trucks cycle time and parameter distribution ($\varphi_{jlr}$).

| | | Cycle time (minutes) | |
|---|---|---|---|
| **Sector ($j$)** | **Truck ($l$)** | **Mean** | **Std.Dev.** |
| 1 | 1 | 32 | 2.8 |
| 1 | : | : | : |
| 1 | 10 | 32 | 3.3 |
| 2 | 1 | 25 | 2.6 |
| 2 | : | : | : |
| 2 | 10 | 25 | 3.1 |
| 3 | 1 | 20 | 2.5 |
| 3 | : | : | : |
| 3 | 10 | 20 | 3 |

The cycle time $\varphi_{jlr}$ from sector $j$ to destination will be drawn $r$ times from the respective distribution and the maximum trips are calculated given the mechanical availability per truck and finally the target production used at the short-term constraints formulation is given in Figure 5.

| **Production Target** | **Parameter** | **Value** | **Unit** | **Penalty** |
|---|---|---|---|---|
| | Max production | 1,210,000 | Tonnes | 160 |
| | Min production | 1,100,000 | Tonnes | 160 |
| | Max Ore production | 1,210,000 | Tonnes | 16 |
| | Min Ore production | 1,000,000 | Tonnes | 4 |
| | Allowed deviation tolerance | <=10 | % | |
| **Quality Requirement** | Iron Ore(Fe2O3) | 57.1- 59.4 | % | 1 |
| | Phosphorous | 0.032 - 0.038 | % | 10 |
| | Silica | 4.6 - 5.2 | % | 10 |
| | Alumina | 0.9 - 1.05 | % | 10 |
| | Loss on ignition | 9.5 - 11 | % | 1 |
| | Allowed deviation tolerance | <=10 | % | |

| **Ore Definition** | **Parameter** | **Value** | **Unit** |
|---|---|---|---|
| | Fe2O3 | >= 56 | % |

| **Economic Parameters** | **Parameter** | **Value** | **Unit** |
|---|---|---|---|
| | Mining Cost* | 40 | $/Tonne |
| | Cycle time Cost | 120 | $/hour |
| | Shovel Moving Cost | 1000 | $/100 meters |

* Not include hauling cost

Figure 5: Target month production and parameters.

The total tonnage to be mined after twelve months of production is approximately 14,400,000 iron ore tons, and given the ore cut-off $>= 56\%$ $Fe_2O_3$ almost all the material will be mined as ore. The targets of production and actual ore production are quite similar. Note that a high penalty is applied to the lack of mining from the expected monthly production because all material scheduled for the twelve months must be mined to align the short-term production with the long-term planning expectations.

## 3.1 Joint-simulation of future ore control data

The mined out sector $A$ has exploration data and past ore control data, used to joint-simulate the future ore control data at sector $B$ where only an orebody model based on exploration data is available. The six meters exploration data composites may be located at different locations than the dense ore control data, the height of which is six meters. However, some composites of the exploration data may coincide at the same locations of the ore control data, or be close enough to be considered "co-located". The ore control data that intersect or are located <= 1 meter close to the exploration data location is filtered, 411 "co-located" samples are found.

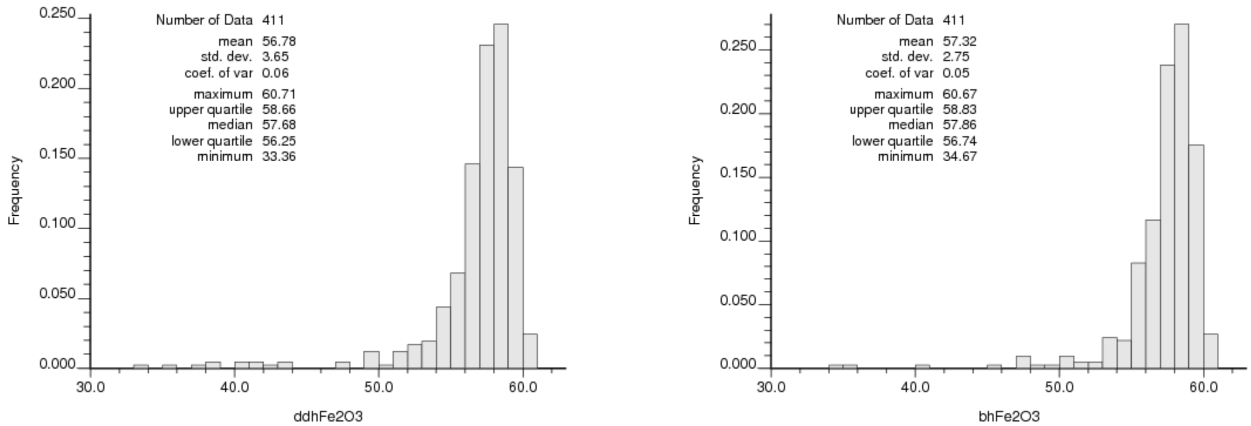


Figure 6: Iron histogram of exploration data (left) and ore control data (right).

The previous distributions show the $Fe_2O_3$ % data of the mined out sector $A$ where exploration data and ore control data is available. The exploration data contains $Fe_2O_3$ from 33.36% to 60.71%, P from 0.018% to 0.043%, $SiO_2$ from 1.86% to 21.51%, $Al_2O_3$ from 0.23% to 16.61% and LOI from 9.07% to 12.69%. On the other hand, the ore control data contains grades of $Fe_2O_3$ that fluctuate from 34.67% to 60.67%, P from 0.016% to 0.05%, $SiO_2$ from 1.9% to 23.01%, $Al_2O_3$ from 0.26% to 14.23% and LOI from 9.37% to 11.84%. The spatial location of the co-located data is shown in Figure 7.

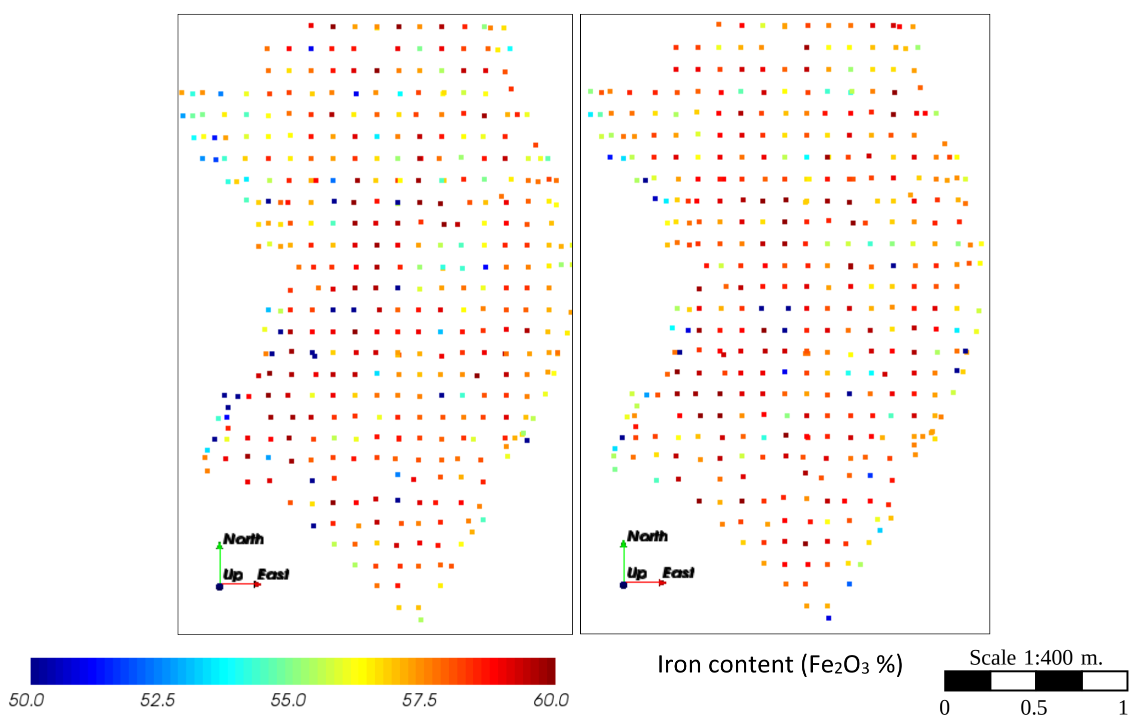


Figure 7: Co-located iron element of exploration data (right) and ore control data (left).

The difference between ore control data and exploration data per element is modeled and used to simulate future ore control data at similar geological domains. These differences will be named errors in this approach. The univariate distributions of these differences or errors are illustrated in Figure 8. The source of these errors are variable. For instance, the collar coordinates of the exploration data is usually well-measured; however, the survey angle that defines the location of each sample through the drillhole could be miss-measured. In consequence, the exploration data may differ from the ore control data at closest location. The other source of difference between both data is the quality of sampling; the core sample of exploration data have more precise information about the structures, mineralogy, grade and alterations. On the other hand, the sampling of blastholes for ore control data cannot provide precise samples because the reverse circulation drilling may

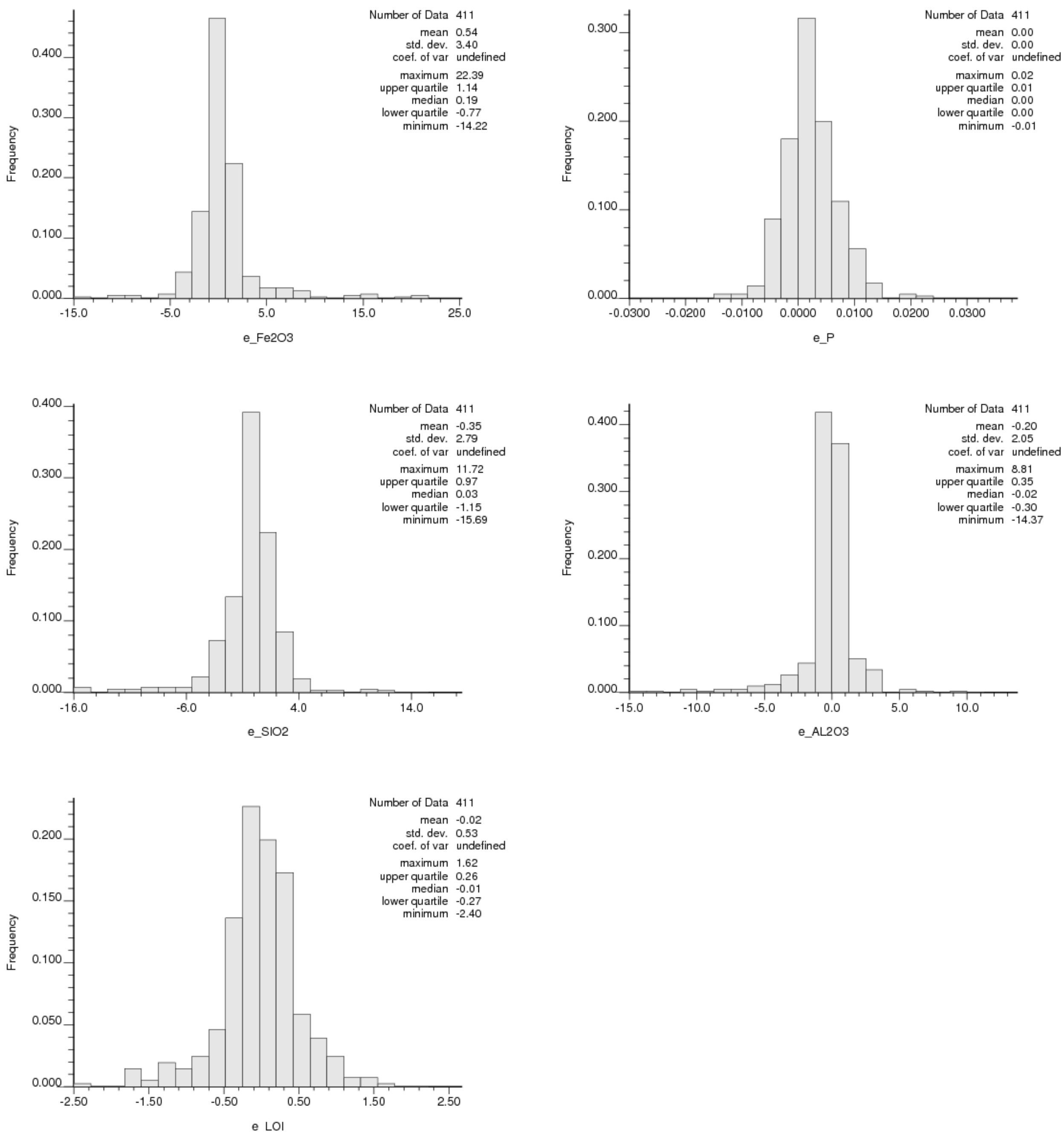


Figure 8: Univariate distribution of errors at mined out sector $A$.

contaminate the samples and the quality of the collection of samples is subject to the team expertise and time sampling available (Pitard 2008).

The errors are transformed into standard normal score and their correlation matrix for lag zero show that only three sets of errors have reasonable correlation. These correspond to $Fe_2O_3$, $SiO_2$ and $Al_2O_3$ elements; otherwise, the set of errors that correspond to P and LOI elements show low correlation between them as well as to the previous set of elements.

Figure 9 shows the illustration of the correlation between elements, the shadow part of Table 5 is considered by the scatter plot graphs which are in original units.

Table 5: Correlation matrix of the multi-element error.

| | **NS e_$Fe_2O_3$** | **NS e_P** | **NS e_Si02** | **NS e_$Al_2O_3$** | **NS e_LOI** |
|---|---|---|---|---|---|
| **NS e_$Fe_2O_3$** | 1 | 0.1757 | -0.9135 | -0.7373 | -0.1677 |
| **NS e_P** | 0.1757 | 1 | -0.2859 | 0.0596 | 0.0756 |
| **NS e_Si02** | -0.9135 | -0.2859 | 1 | 0.4941 | -0.092 |
| **NS e_$Al_2O_3$** | -0.7373 | 0.0596 | 0.4941 | 1 | 0.278 |
| **NS e_LOI** | -0.1677 | 0.0756 | -0.092 | 0.278 | 1 |

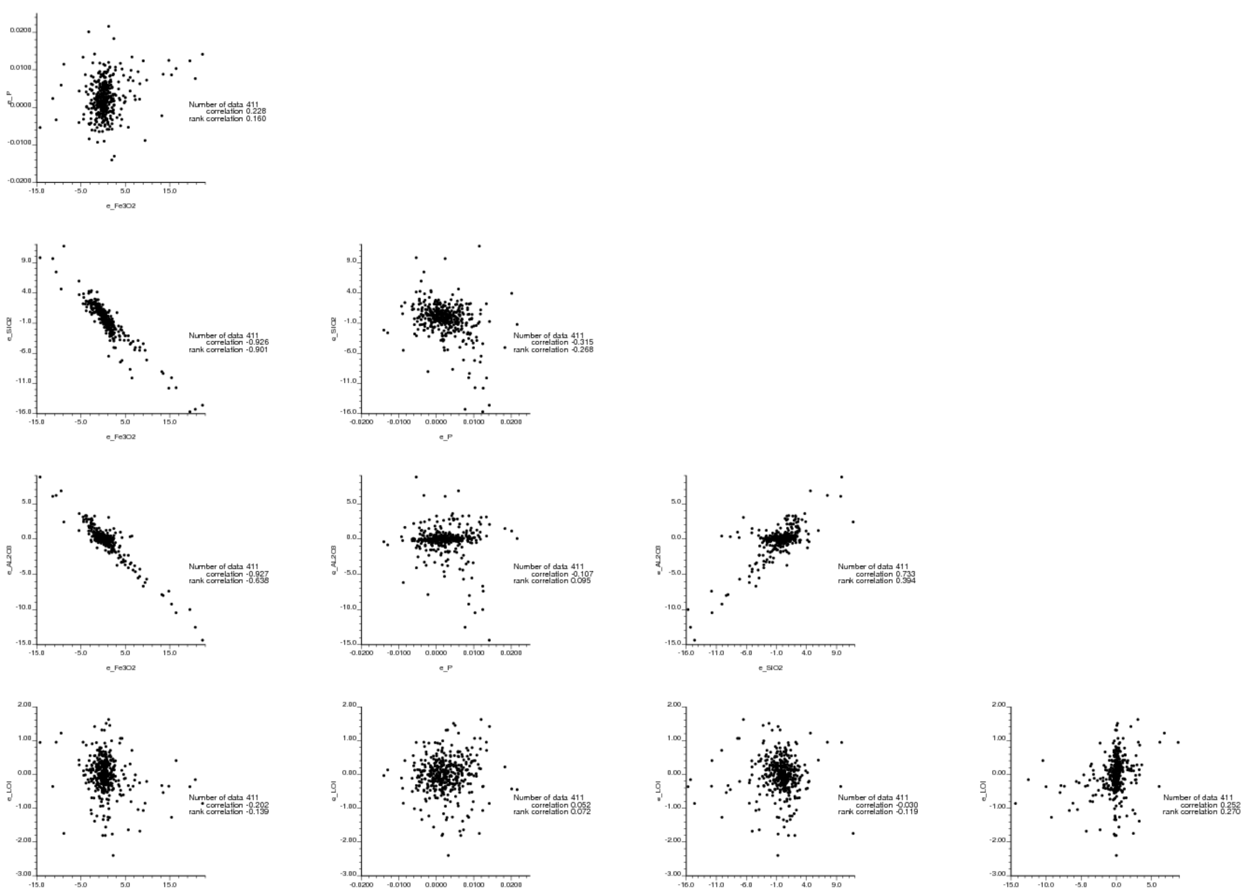

Figure 9: Scatter between errors.

The scatter plot maps show a negative slope, or negative correlation, between $Fe_2O_3$ and the deleterious elements $SiO_2$ and $Al_2O_3$, that is, more iron quality by tonnage imply less silica and alumina quality by tonnage. The MAF procedure de-correlates this normal score errors at all lags and the back MAF transformation matrix is used to re-correlate these errors. Then, the simulation of these MAF non-correlated factors are performed independently. The lag used is 48 meters which corresponds to the median spacing among the data location. The spatial correlation of the MAF non-correlated factors is modeled to perform joint simulation at sector $B$ where only exploration data is available. The set of simulations per error element recover their correlation by using the back MAF transformation matrix. The scatter plots of the errors at sector $B$ after the joint simulation are as follows:

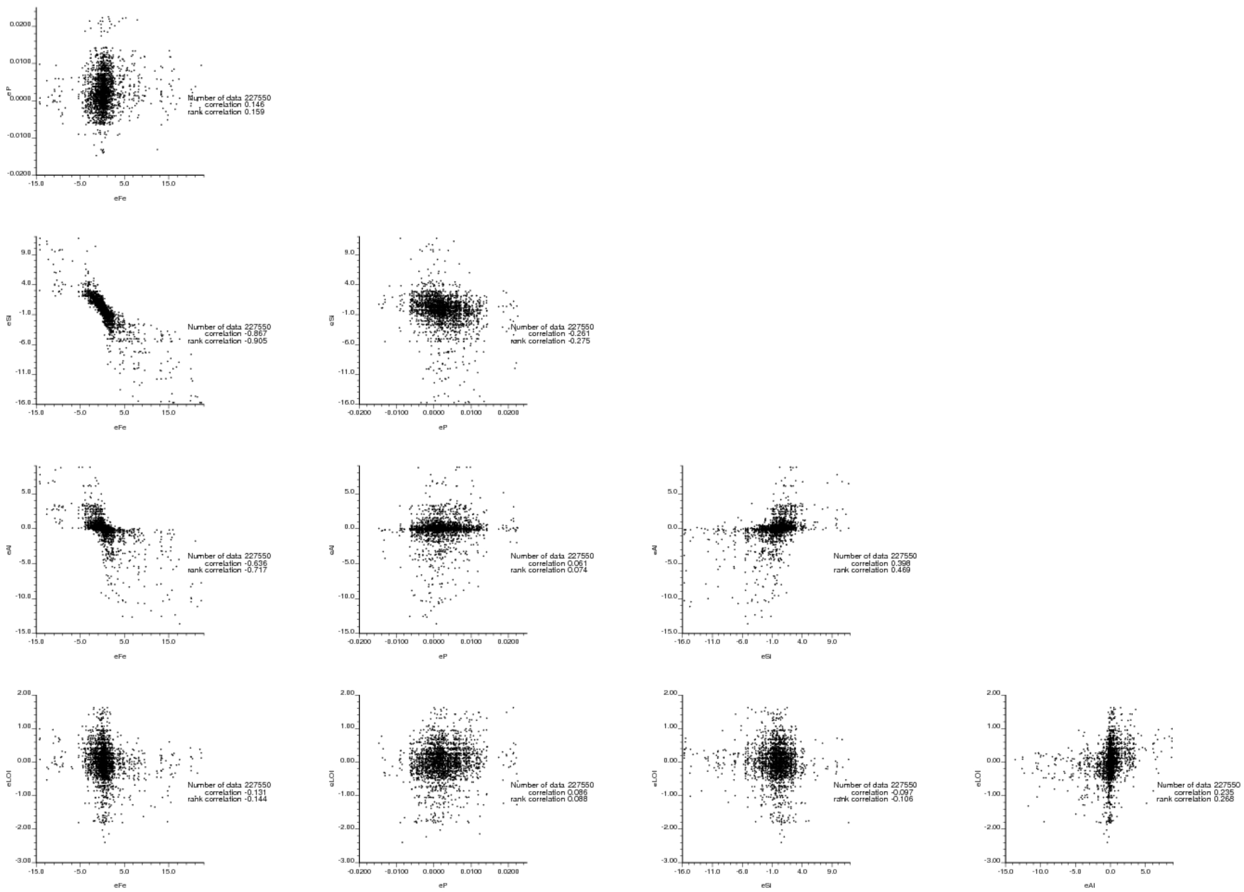

Figure 10: Scatter between errors corresponding to realization 1.

The comparison of the input scatter plot between the errors of mined sector $A$ (Figure 9) and the scatter plot between the joint simulation errors of sector $B$ (Figure 10) show that the input correlation between errors are reproduced by the MAF simulation. Indeed, the spatial correlation between multi-element errors of exploration data and ore control data of historical data was used to generate the possible error that may happen in the sector $B$ where only exploration data is available.

The joint simulation of errors at nodes is scaled up to the block size. These error maps are added to the simulation based on exploration data used to generate the map of possible ore control data at sector $B$ per element and update the orebody uncertainty. The simulation based on exploration data and the updated orebody simulation that considers the correlation of the errors of mined sector are compared. The future ore control data orebody model shows higher entropy than exploration data orebody model.

The mean of the iron increases slightly from 58.58 to 58.85 % and the future iron ore control data distribution is wider than the available simulation based on exploration data. The mean of the phosphorous increases from 0.032% to 0.034% and the resulting future phosphorous ore control data distribution obtains a longer right tail of 0.047% than 0.038% for the available simulation based on exploration data. The silica grade mean decreases from 5.12% to 5.03% and the silica future ore control data distribution becomes wider than the available simulation based on exploration data. The alumina grade mean decreases from 0.96 to

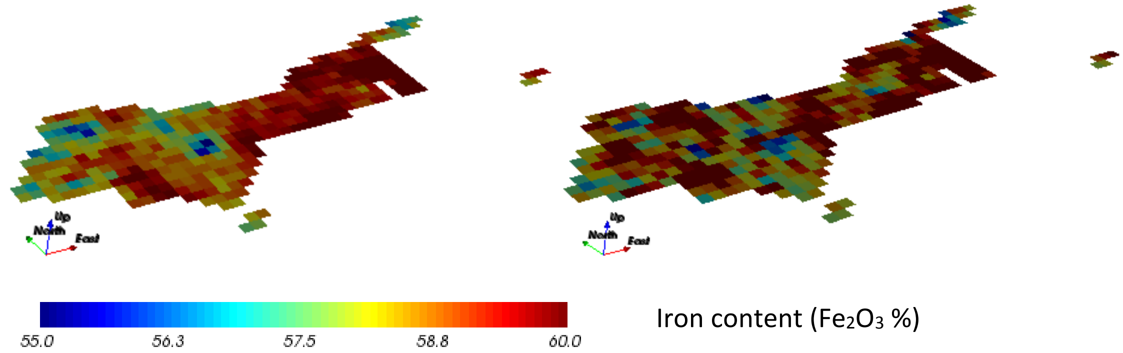


Figure 11: Realization 1 upper bench $Fe_2O_3$ map of sector $B$ base of exploration data (right) and future ore control data map (left).

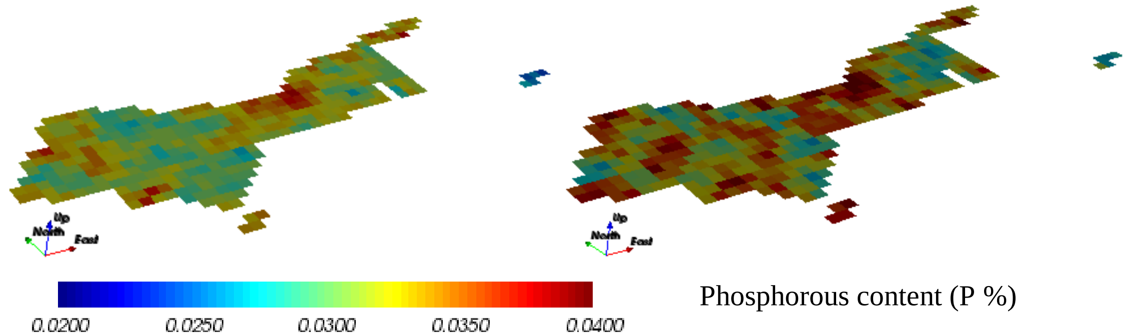


Figure 12: Realization 1 upper bench phosphorous map of sector $B$ base of exploration data (right) and future ore control data map (left).

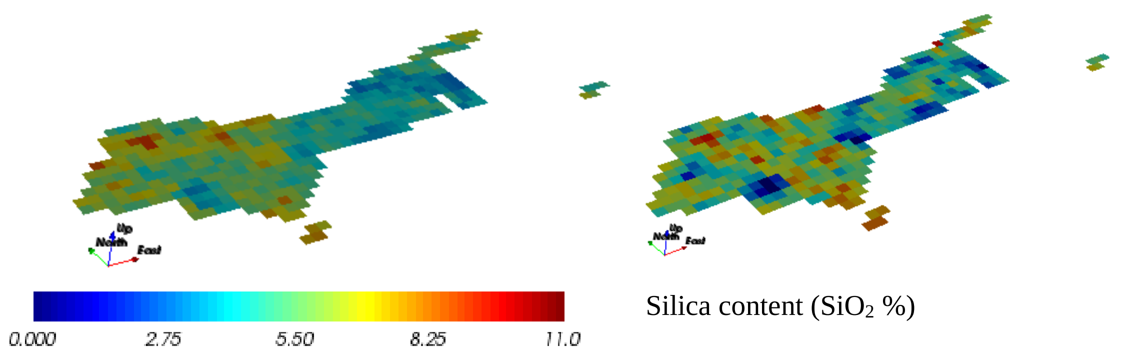


Figure 13: Realization 1 upper bench $SiO_2$ map of sector $B$ base of exploration data (right) and future ore control data map (left).

0.91% and the alumina future ore control data distribution becomes wider than the available simulation based on exploration data. The grade mean of the loss of ignition increases from 9.99% to 10.02% and their distribution becomes wider than the available simulation based on exploration data.

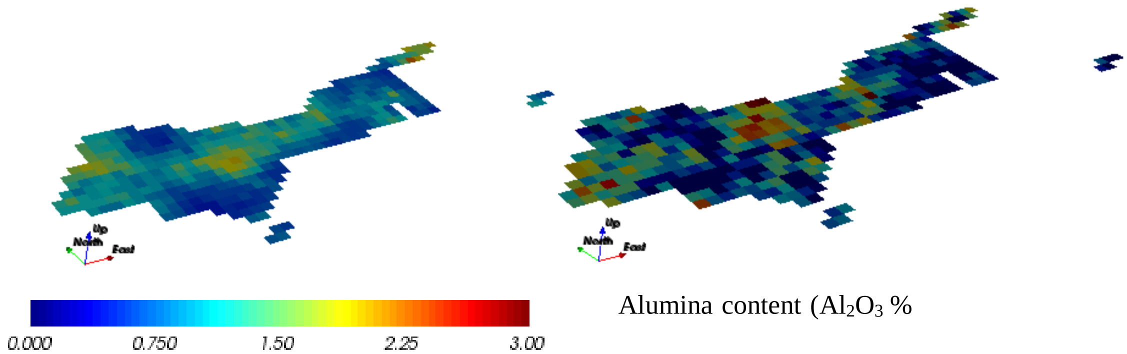


Figure 14: Realization 1 upper bench $Al_2O_3$ map of sector $B$ base of exploration data (right) and future ore control data map (left).

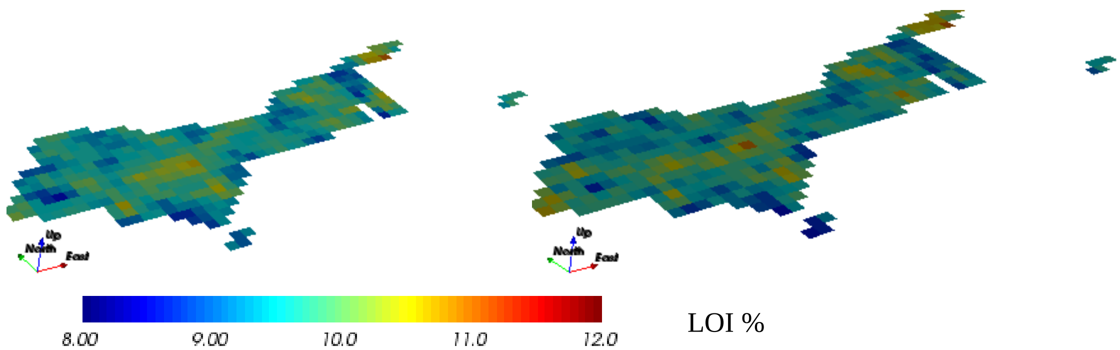


Figure 15: Realization 1 upper bench LOI map of sector $B$ base of exploration data (right) and future ore control data map (left).

### 3.2 Simulated future multi-element ore control data in stochastic short-term production scheduling and risk analysis

The short-term production scheduling proposed incorporates operational considerations in order to deliver recoverable reserves for each period. Besides these considerations, the orebody model used in the short-term production evaluation must account for grade control process prior to production.

The short-term production scheduling must consider relevant short-scale information to efficiently match target production. The orebody uncertainty based on sparse exploration data is updated using future multi-element ore control data. Then, ten simulations of the future multi-element ore control data and the e-type of these realizations per element are used as the input data for the monthly production scheduling. The influence of possible short-scale information in the stochastic short-term production scheduling is evaluated.

The patterns of the production schedule based on exploration data orebody uncertainty is quite different from the production schedule patterns based on the future ore control data orebody uncertainty, that is, a change in the quality of the multi-elements made a relevant impact on the decision of which sector to mine per month, as expected.

It is important to remark that the stochastic production scheduling considers four source of uncertainty: orebody uncertainty, shovel mechanic availability uncertainty, truck mechanic availability uncertainty and hauling time uncertainty. The iron monthly average of the six first months does not match the upper bound; however, the remaining six months do match the upper bound. The other two elements of phosphorous and loss on ignition satisfy both upper bounds and lower bounds. Variable production schedule monthly average grade is observed for all the elements because the future ore control data map simulations show an increase of entropy or spatial variability of the grades.

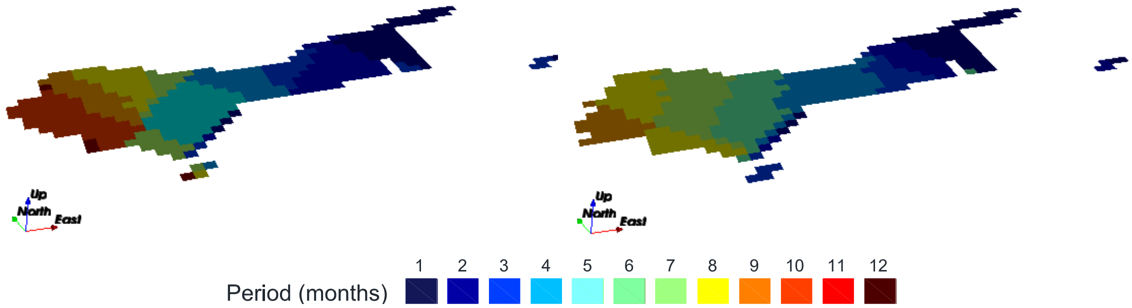


Figure 16: Stochastic production schedule upper bench accounting for ore body uncertainty based on exploration data (left) and based on future ore control data (right).

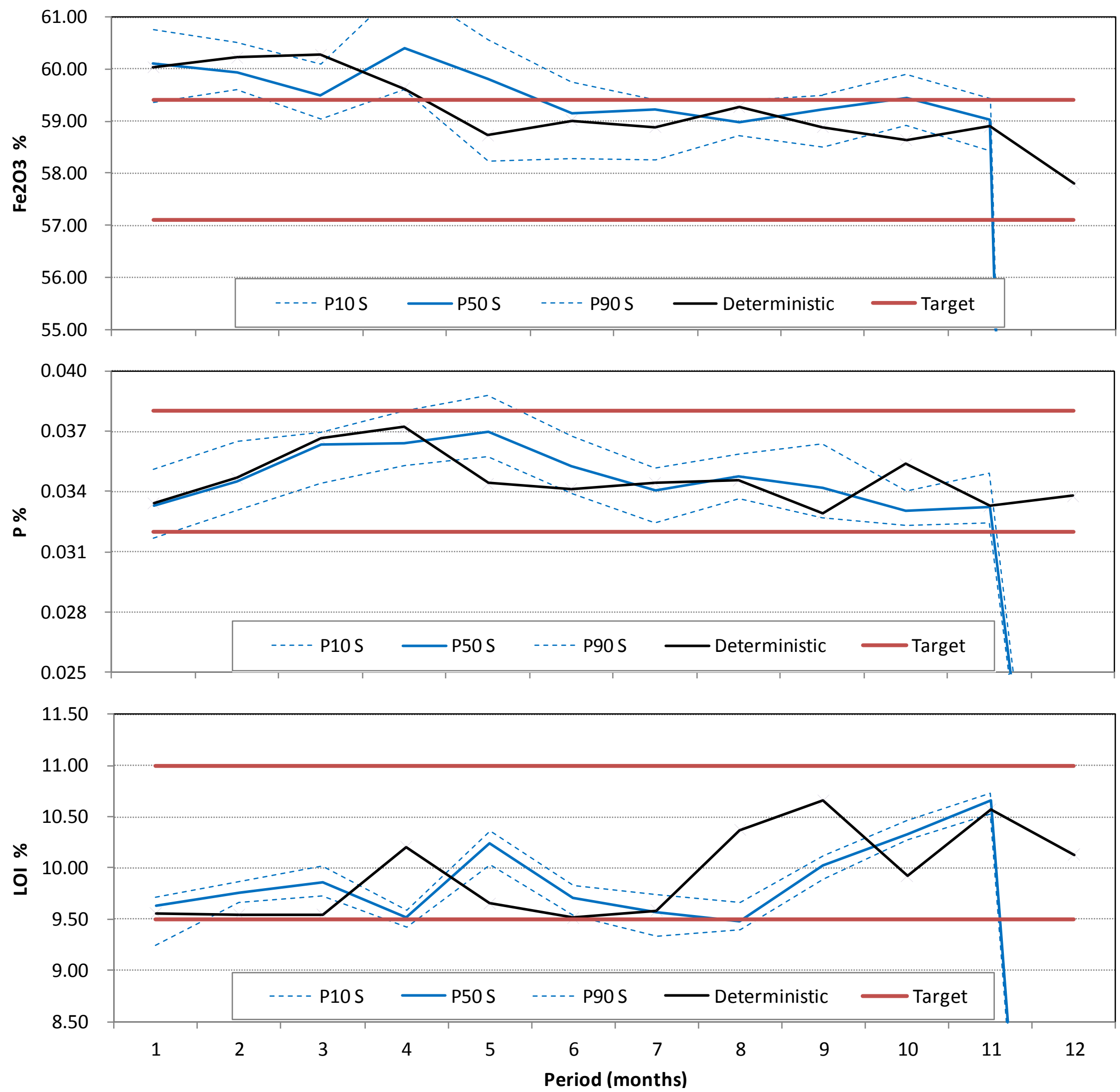


Figure 17: Short-term schedule solution for iron (top), phosphorous (middle) and LOI (bottom) where (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile).

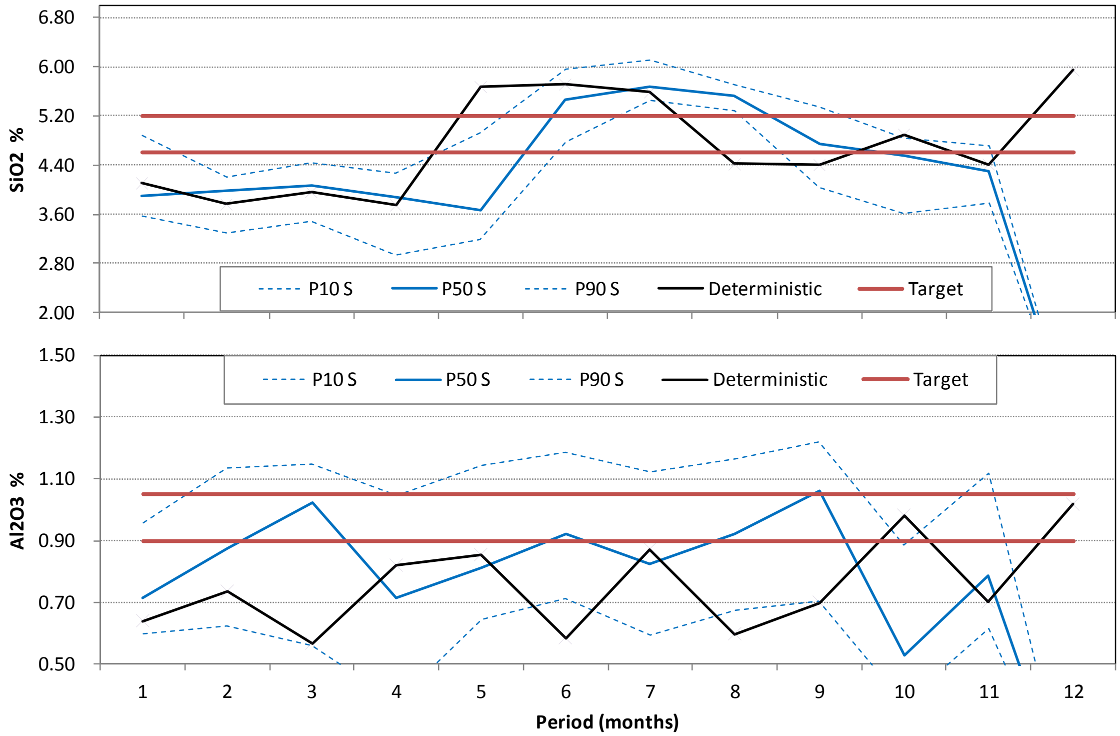


Figure 18: Short-term schedule solution for silica (top) and alumina (middle) where (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile).

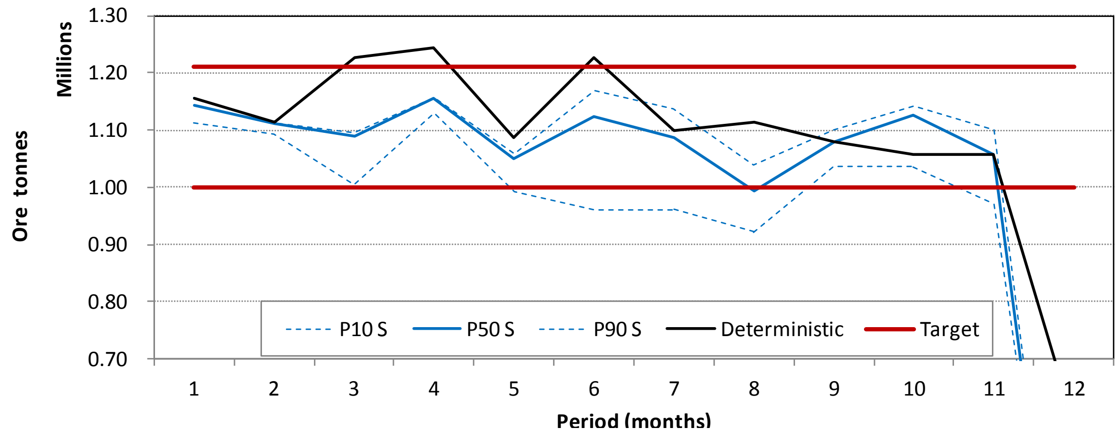


Figure 19: Short-term schedule solution for ore tonnes (red lines: upper and lower bounds; black line: deterministic solution; blue lines stochastic solution and risk profile).

The silica and alumina quality of the monthly production schedule barely match their quality targets. The same effect was observed by the production schedule based on exploration data orebody uncertainty with the same elements; however, the average grade of the monthly production taking into account future data is more variable.

The monthly ore tonnage of the stochastic production schedule accounting for future ore control data orebody uncertainty and fleet parameters source of uncertainty are less variable than the deterministic one that does not consider any source of uncertainty. The possible lack of matching quality targets in some periods is expected since the upper and lower bounds provided by the long-term production schedule was determined based on the exploration of sparse data orebody uncertainty where the short-scale information was not taken

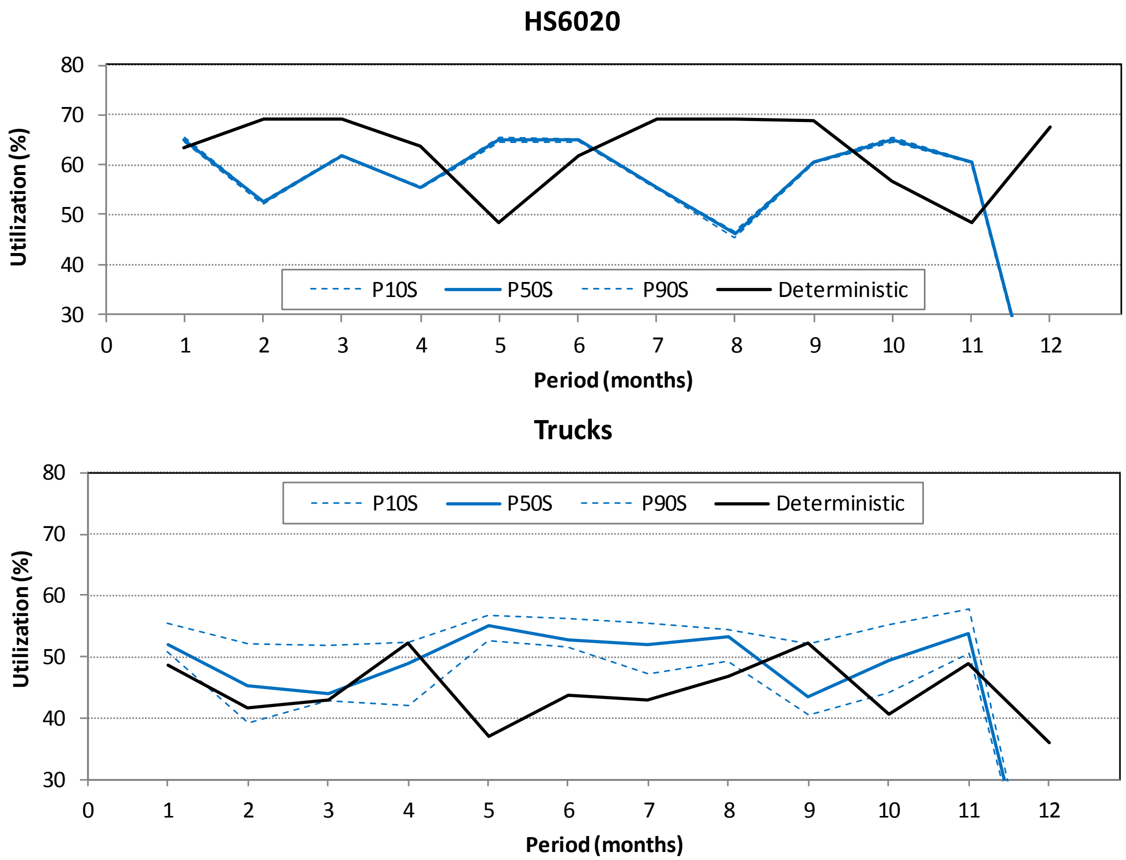


Figure 20: The risk profiles of the stochastic production schedule fleet utilization (blue) and the deterministic production schedule fleet utilization (black).

into account. The utilization of the fleet when the source of uncertainty related to the multi-element grade considers the future ore control data is showed in Figure 20.

The utilization of the trucks and shovels accounting for future ore control data orebody uncertainty and fleet parameters source of uncertainty is slightly higher and less variable than the utilization of the deterministic production schedule that does not account for any source of uncertainty. When the local grade variability of the multi-elements is higher, the stochastic production schedule formulation allocates more efficiently than the deterministic one, which allocates one or three more trucks per month with lower utilization. The allocation of an optimal number of trucks will reduce the overall mining cost.

Testing performance of the stochastic short-term production scheduling that account for future short-scale information (blue) against the production scheduling without any source of uncertainty (black).

The fleet utilization of the stochastic production schedule which account for future ore control data orebody uncertainty is higher and less variable than the stochastic production schedule fleet utilization that accounts for exploration data orebody uncertainty. The stochastic production scheduling seems more robust than the deterministic one in handling high local variability of the multi-elements that was provided by the simulation of the future ore control data. Less variable monthly cost is observed.

The cumulated cost of the stochastic monthly production schedule is around 15 million CAD dollars less than that of the production schedule without any source of uncertainty accounted for. This proportion of difference was also observed for a stochastic production schedule that accounts for orebody uncertainty based on exploration data.

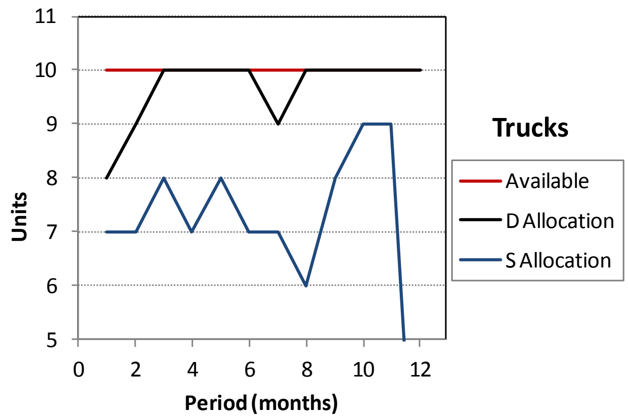


Figure 21: Available trucks (red), number of trucks allocated accounting for four sources of uncertainty (blue) and without accounting for uncertainty (black).

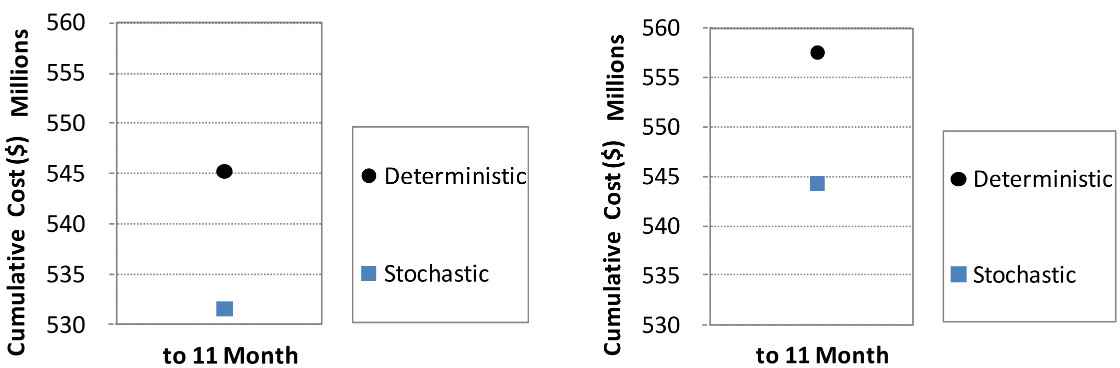


Figure 22: Cumulative cost of the short-term production scheduling that account for orebody uncertainty based on exploration data (left) and orebody uncertainty based on future ore control data (right).

# 4 Conclusions

The orebody uncertainty is updated by simulated future ore control data to account for short-scale information. The alternative, based on the errors between exploration data and historical ore control data used to simulate future ore control data at a similar domain where only exploration data is available, was implemented. The updated orebody uncertainty was used to optimize the short-term production to better match the ore quality targets at the time of production.

The local spatial variability or entropy of the future ore control data orebody model maps is higher than the exploration data orebody model maps. When the spatial variability of the multi-element grade is higher, the proposed stochastic production scheduling approach delivers high fleet utilization, more efficient truck allocation and low variable production monthly average in: grade, ore tonnage and cost than the deterministic production scheduling, which does not account for any source of uncertainty.

# References

Alarie, S., and M. Gamache. Overview of Solution Strategies Used in truck Dispatching Systems for Open Pit Mines. International Journal of Surface Mining, Reclamation and Environment, 16(1), 2002, 59–76.

Benndorf, J. Efficient Sequential Simulation Methods with Implications to Long Term Production Sheduling. MPhil Thesis. The University of Queensland, 2005.

Birge, J.R., and F. Louveaux. Introduction to Stochastic Programming. Springer, New York, 1997.

David, M. Geostatistical Ore Reserve Estimation. Elsevier Scientific Publishing Company, Netherland, 1977.

Desbarats, A.J., and R. Dimitrakopoulos. Geostatistical simulation of regionalized pore-size distribution using min/max autocorrelation factors. Mathematical Geology, 2000, 919–941.

Deutsch, C.V., and A. Journel. GSLIB: Geostatistical Softare Library and User's Guide. 2nd Edition. Oxford University Press, New York, 1998.

Dimitrakopoulos, R., and A. Jewbali. Joint stochastic optimisation of short and long term mine production planning: Method and application in a large operating gold mine. Mining Technology, 122(2), 2013, 101–109.

Dimitrakopoulos, R., and S. Ramazan. Stochastic integer programming for optimizing long-term production schedules of open pit mines: Methods, application and value of stochastic solutions. IMM Transactions, Mining Technology, 117, 2008, 155–160.

Dimitrakopoulos, R., and S. Ramazan. Uncertainty-based production scheduling in open pit mining. Transactions of the Society for Mining, Metallurgy, and Exploration, Inc., 316, 2004, 106–112.

Dowd, P.A. Risk in minerals projects: Analysis, perception and management. Transactions of the Institution of Mining and Metallurgy, Section A Mining Technology, 106, 1997, 9–18.

Eivazy, H., and H. Askari-Nasab. A mixed integer linear programming model for short-term open pit mine operation scheduling. Transactions of the Institution of Mining and Metallurgy, 121, 2012, 97–108.

Fytas, K., and P.N. Calder. A computerized model of open pit short and long range production scheduling. 19th International Symposium 1986 – Application of Computers and Operations Research. SME, 1986, 11.

Gershon, M.E. Mine scheduling optimization with mixed integer programming. Society of Mining Engineers of AIME, 82_324, 1982, 1–6.

Godoy, M., and R. Dimitrakopoulos. Managing risk and waste mining in long-term. SME Transactions, 2004, 43–45.

Goovaerts, P. Geostatistics for Natural Resources Evaluation. Oxford University Press, New York, 1997.

Guardiano, F.B., H. Parker, and E.H. Isaaks. Recoverable reserves and conditional simulation – A case study at Fort Knox gold project. MRDI, 1997.

Hustrulid, W., and M. Kuchta. Open Pit Mine Planning & Design. Taylor & Francis/Balkema, London, 1995.

IBM. http://publib.boulder.ibm.com. 2010. (accessed April 19, 2012)

Isaaks, E.H., and R.M. Srivastava. An introduction to Applied Geostatistics. Oxford University Press, New York, 1989.

Jewbali, A. Modeling Geological Uncertainty for Stochastic Short-Term Production Scheuling in Open Pit Metal Mines. PhD. Thesis, University of Queensland, 2006.

Johnson, R.A., and D.W. Wichern. Applied Multivariate Statisticsl Analysis. Pearson Education, Inc., Toronto, 2007.

Johnson, T.B. Optimum Open-Pit Mine Production Scheduling. International Symposium on Computer Applications and Operations Research in the Mining Industry. American Institute of Mining, Metallurgical, and Petroleum Engineers, New York, 1969, 24.

Journel, A.G. Resampling from Stochastic Simulation. Environmental and Ecological Statistics, 1, 1994, 63–91.

Journel, A.G., and P.C. Kyriakidis. Evaluation of Mineral Reserves: A Simulation Approach. Oxford University Press, Inc., New York, 2004.

Kahle, M.B., and F.J. Scheafter. Open Pit Mine Planning and Design. AIME, 1979.

Khosrowshahi, S., W.J. Shaw, and G.A. Yeates. Quantification of risk using simulation of the chain of mining – Case study at Escondida copper Chile. Orebody modelling and strategic mine planning: Uncertainty and risk management models, Spectrum Series 14, The Australasian Institute of Mining and Metallurgy, 2007, 33–41.

Knudsen, H.P. Blathole samples – A source of bias? Mining Engineering, 1992, 251–253.

Kumral, M., and P. Dowd. Short term mine production scheduling for industrial minerals using multi-objective simulated annealing. SME Transactions, 2002, 731–742.

Lane, K.F. The Economic Definition of Ore: Cut Off Grades in Theory and Practice. Mining Journal Books, London, 1991.

L'Heureux, G., M. Gamache, and F. Soumis. Mixed integer programming model for short-term planning in open-pit mines. Mining Technology, 122(2), 2013, 101–109.

Peattie, R. Future data and recoverable reserve estimation. MPhil Thesis, University of Queensland, 2007.

Peattie, R., and R. Dimitrakopoulos. Forecasting recoverable ore reserves and their uncertainty at Morila Gold Deposit, Mali: An efficient simulation approach and future grade control drilling. Mathematical Geosciences, 45(8), 2013, 1005–1020.

Pitard, F. Blathole sampling for grade control-The many problems and solutions. Sampling Conference. Perth, 2008, 15–21.

Ramazan, S., and R. Dimitrakopoulos. Recent applications of operations research in open pit mining. SME Transaction, 316, 2004, 73–78.

Ramazan, S., and R. Dimitrakopoulos. Traditional and new MIP models for production scheduling with in-situ grade variability. International Journal of Surface Mining, Reclamation and Environment, 18, 2004, 85–98.

Ravenscroft, P.J. Risk analysis for mine scheduling by conditional simulation. Trans. Instn. Min. Metall., 1992, 104–108.

Rendu, J-M. Orebody modelling, mine planning, reserve evaluation and the regulatory environment. In proceedings, Orebody modelling and strategic mine planning: Uncertainty and risk management models, Spectrum Series 14, 117, AusIMM, 2007, 385–392.

Schleifer, J. Short range planning: from scheduling to sequencing. 26th Proceedings of the Application of Computers and Operations Research in the Mineral Industry, 1996, 183–188.

Schrijver, A. Theory of Linear and Integer Programming. John Wiley & Sons, Inc., Chichester, 1986.

Souza, M.J.F., I.M. Coelho, S. Ribas, H.G. Santos, and L.H.C. Merschmann. A hybrid heuristic algorithm for the open pit mining operational planning problem. European Journal of Operational Research, 207(2), 2010, 1041–1051.

Strunk Jr., W., and E.B. White. The Elements of Style. Fiftieth Anniversary. Pearson Education, Inc., New York, 2009.

Switzer, P., and A.A. Green. Min/Max autocorrelataion factors for multivariate spatial imaging. Technical Report No 6, Dept. of Statistics, Stanford University, 1984, 14 p.

Topal, E., and S. Ramazan. A new MIP model for mine equipment scheduling by minimizing maintenance cost. European Journal of Operational Research, 207(2), 2010, 1065–1071.

Topal, E., and S. Ramazan. Mining trucks scgeduling with stochastic maitenance cost. Journal of Coal Science & Engineering, 18(3), 2012, 313–319.

Vallée, M. Mineral resource + engineering, economic and legal feasbility = ore reserve. CIM Bulletin, 93(1038), 2000, 53–61.

Vargas, M., N. Morales, E. Rubio, and P. Mora. Optimal open pit short term planning under uncertainty and blending constraints. University of Chile, Santiago, Internal Report, 2008.

Wackernagel, H. Multivariate Geostatistics: An Introduction with Application. Springer-Verlag, Berlin, Heidelberg, 2003.

Wilke, F.L, and W. Woehrle. A model for short-range planning and monitoring of mining in potassium deposits of level formation. 16th Application of Computers and Operations Research in the Mineral Industry – 1979, SME, 1979, 304–312.

Wilke, F.L., and Th. Reimer. Optimization the short term production schedule for an open pit iron mining operation. 15th APCOM Symposium, Brisbane, Australia, 1977, 425–433.

Wolsey, L.A. Integer Programming. John Wiley & Sons, Inc., 1998.