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Stochastic global optimization of open pit mining complexes with capital expenditures: Application at a copper mining complex

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Abstract: Previous research related to the optimization of mining operations has predominantly focused on generating a life-of-mine production schedule that maximizes the discounted cash flows of the material extracted and products produced. Stochastic optimization models address the issue of integrating uncertainty into the decision-making, leading to mine designs and production schedules with higher value and better risk management, thus helping to ensure that the mining operation is capable of meeting production targets over time. More recent models address the challenge of stochastic global optimization, which aim to holistically optimize a mining complex, from the production schedule, through to the products created, marketed and sold. Existing stochastic formulations, however, assume that the bottlenecks in the mining complex, such as mine production and milling capacities, have been defined a-priori, thus ignore the impact that the quantity and timing of capital expenditures required to create these capacities may have on the overall profitability of the operation.

This work builds on previous developments in stochastic global optimization for mining complexes and integrates capital expenditure options in order to appropriately design the bottlenecks or constraints in the model. This formulation is solved using a combination of the particle swarm optimization and simulated annealing algorithms. An application for a copper mining complex demonstrates the ability to decide when to invest capital in order to increase the number of both trucks and shovels used. The results indicate that the stochastic optimizer is able to outperform its deterministic-equivalent by significantly reducing the risk associated with materials sent to the mill, in addition to an overall increase in net present value by 5.7%.

Key Words: Mining complexes, capital investment, uncertainty, bottlenecks.

1 Introduction

The primary objective of a mining enterprise is to maximize the value of its assets for its stakeholders. This requires optimizing many strongly interrelated components, such as the amount and timing of capital expenditures (CAPEXs) that are required to develop, maintain or expand an operation, the sequencing of extraction from the mines, and the use of the various processing streams to maximize the utility of the products mined and treated. Naturally, the amount of capital expenditure is strongly related to the rate of which a mining complex can produce, treat and sell materials. The global optimization of mining complexes addresses the challenge of integrating all relevant aspects of optimizing a mining enterprise. Existing methods have predominantly focused on aspects of mine production scheduling, the use of the processing streams that stockpile, blend, treat and transform the bulk mined material into refined products, and distribution networks that are used to deliver the products to customers (Hoerger et al., 1999a; Hoerger et al., 1999b; Urbaez and Dagdelen, 1999; Caccetta and Hill, 2003; Chanda, 2007; Stone et al., 2007; Whittle, 2007; Whittle, 2009; Topal and Ramazan, 2012; Sandeman et al., 2012; Bley et al., 2012; Epstein et al., 2012; Singh et al., 2014; Blom et al., 2014), while leaving strategic capital expenditure decisions outside of the optimization model in the form of what-if scenarios. Given the strong relationship between capital expenditures, capacities, operating costs, production scheduling and the use of processing streams, this scenario-wise design methodology leads to a sub-optimal use of capital and the non-renewable resource. Moreover, many of the existing attempts at global optimization for mining complexes ignore the compounded effects that uncertainty has on the performance of the mining complex, particularly the ability to fully utilize the capacities that are purchased with a significant capital cost. In order to truly maximize the value of the mining operation, it is necessary to optimize all aspects of the mining complex, including capital expenditures, and simultaneously manage the opportunities and risk that arise in the mining complex's various components.

Recent research as focused on integrating geological uncertainty into mine design and production scheduling optimization. Godoy and Dimitrakopoulos (2004) propose a sequential optimization methodology that first uses a modified linear programming model, based off work by Tan and Ramani (1992), to determine the optimal production rates (i.e. shovel and truck purchases) from an orebody while considering uncertainty in metal quantities. An improved version of this model is proposed by Godoy et al. (2015). A risk-based production scheduling algorithm is then used to find a single production schedule that minimizes the risk of not meeting ore and waste production targets over the life of the mine, which are governed by the previously determined mine production capacities. The authors demonstrate that the method is capable of not only substantially reducing the risk of the stochastic production schedule, but also generates a higher net present value (NPV).

Ramazan and Dimitrakopoulos (2013) propose a two-stage stochastic integer programming model (SIP) (Birge and Louveaux, 2011) that aims to generate a production schedule that not only maximizes the NPV of the design, but also reduce the risk of not meeting production targets (e.g. ore production capacity, total material movement capacity), metal quantities produced and blending targets. The authors introduce the concept of geological risk discounting, which is a time-dependent discount factor used to ensure that production targets are met at the beginning of the mine life, thus guaranteeing early cash flows, and deferring riskier material to later periods when more information is available. This model has been expanded upon and tested (Albor and Dimitrakopoulos, 2010; Benndorf and Dimitrakopoulos, 2013; Dimitrakopoulos and Jewbali, 2013; Leite and Dimitrakopoulos, 2013), and results consistently demonstrate the ability to not only generate a substantially higher NPV, but also minimize the risk of not meeting production targets, metal quantities and blending targets.

The previously mentioned methods for production scheduling with uncertainty, however, are limited by several assumptions. The formulations assume that ore and waste materials are classified a priori, hence are unable to simultaneously optimize cut-off grade decisions (Lane, 1988; Rendu, 2013) or mining complexes with multiple processing options. Despite the fact that the optimizer will seek to extract blocks with high economic value in early periods, a fixed ore-waste classification can result in low-grade ore that is sent for processing and deferring the processing of higher-grade material that may be readily available. Menabde et al. (2007) propose a production scheduling model that simultaneously generates a robust cut-off grade policy, however, it does not explicitly manage the upside or downside risk of not meeting production targets. Boland

et al. (2008) propose a multistage model that simultaneously generates an adaptive production schedule and scenario-dependent cut-off grade decisions. This, however, leads to overly optimistic destination decisions, as it assumes that the grades of the mined materials are known at the beginning of each period. Kumral (2013) proposes a model that attempts to simultaneously optimize the production schedule and define an ore-waste classification for each block. Scenario-independent block classifications have limited applicability for mining complexes that consider multiple material types because certain materials often cannot be treated with certain processing streams due to incompatible chemical reactions.

The aforementioned work in stochastic optimization for mine production scheduling attempts to meet production targets over the life of the mine, and reduce the risk associated with not being able to satisfy the targets. These models, however, fail to consider the timing and quantity of capital expenditures that permit the option to increase or decrease the target capacities. Recent work has sought to incorporate this additional level of decision-making directly in the optimizer. Groeneveld et al. (2011) propose a mixed integer program (MIP) model that schedules the mining of benches (a production schedule within predefined phases), optimizes destination decisions, and the timing and quantities of capital expenditures used to increase or decrease target capacities. By solving the optimization model for a set of metal price, cost and utilization simulations independently, which the authors refer to as a “flexible” design, it is possible to obtain a probability distribution for a capital expenditure that can be used to approximate the timing of the decision. This method, however, does not integrate uncertainty into the optimizer’s decision-making, and generates an overly optimistic solution that assumes perfect knowledge of uncertain events, i.e. a wait-and-see solution (Birge and Louveaux, 2011). Groeneveld et al. (2012) improve this model by forcing the optimizer to choose the same decisions at the beginning of the mine life across all scenarios. The authors note that geological uncertainty is not integrated in the models, and that a phase design is required prior to running their proposed model. Geological uncertainty can play a critical role when designing capacities because the uncertainty relates directly to the quantities that are available and sent. Giving the optimizer the ability to do detailed production scheduling can help manage the distribution of risk over time, thus providing a consistent quantity and quality of material at the appropriate capacity with controlled variability.

Goodfellow and Dimitrakopoulos (2014a) propose a generalized methodology for modelling and optimizing mining supply chains with geological uncertainty, including the ability to model non-linear transformations that occur in the processing streams. This method aims to generate robust destination policies, similar to cut-off grades, which define where materials are sent from the mine, and how to utilize the processing streams to maximize the utility of the materials extracted. The destination policies improve on cut-off grade policies because they can consider the blending of materials and complex, non-linear processing streams. Goodfellow and Dimitrakopoulos (2014b) improve on the method to consider the simultaneous optimization of multi-mine production schedules, destination policies and processing streams with uncertainty. This work expands the previously mentioned developments to include capital expenditure options, which permit the optimizer to change the target capacities in the mining complex (e.g. mine production and ore processing capacities). In the following section, a brief overview of the generalized modelling procedure is outlined. Following this, a mathematical model is given that may be tailored to suit the individual needs of each mining complex. A description of the proposed metaheuristics that are used to perform the optimization is provided. An application at an industry partner’s copper mining complex is then discussed. Finally, conclusions and future work are presented.

2 Flexible modelling of open pit mining complexes with uncertainty

2.1 Overview

2.1.1 Models of material and attribute uncertainty

In a mining complex, a *material* is a term used to define a physical product that is extracted from a mine (e.g., sulphide or oxide) or generated from blending and processing (e.g., tailings, concentrate, slag or refined metal). Materials often have unique mineralogical or geometallurgical properties that have varying impacts at the locations in a mining complex, which limit the choice of where they can be sent for further blending

or processing. An *attribute* is a generic term used to describe the property of a material that is of interest to the optimization model, such as metal mass or percent by weight (commonly referred to as “grade”), total mass, economic values from sale, costs, recoveries or mill residence time (among many other possibilities). It is useful to categorize the various attributes into one of two groups. Primary attributes are the fundamental variables of interest (e.g. metal content and mass) that are sent from one location in the mining complex to another, and are used to define the total quantity and quality of a single material (in its entirety) in a given period. Hereditary attributes are variables that are of interest for optimization models, and are derived using linear or non-linear expressions from primary attributes. In practice, these may be used to track information such as processing costs, revenues, throughput or energy consumption, among others.

Traditional mine production scheduling optimization frameworks consider only a single representation of the spatial distribution of materials and their attributes, such as metal content. Often these models are generated by kriging (Matheron, 1965; Journel and Huijbregts, 1978; David, 1977), a geostatistical method used to estimate the values of the attributes at points or volumes of interest. These estimation methods are known to over-smooth the distributions of the attributes, resulting to less high- and low-grade materials, which ultimately leads to inaccurate financial and production forecasts (Ravenscroft, 1992; Dimitrakopoulos et al., 2002). Geostatistical simulation methods (Journel, 1974; Goovaerts, 1997; Chilès and Delfiner, 1999) are able to overcome the limitations of conventional estimation techniques. They offer the possibility to generate an infinite number of equally probable realizations of the geological conditions, which may be used as a group to quantify the geological uncertainty in each mineral deposit of the mining complex, and also better represent the geological variability (high- and low-values) of the attributes of interest. Several geostatistical simulation techniques exist, which are capable of generating simulations for both material types and multiple attributes (Journel and Alabert, 1989; Strebelle, 2002; Desbarats and Dimitrakopoulos, 2000; Godoy, 2003; Zhang et al., 2006; Arpat and Caers, 2007; Boucher and Dimitrakopoulos, 2009; Honarkhah and Caers, 2010; Mariethoz et al., 2010; Mustapha and Dimitrakopoulos, 2010; Mustapha and Dimitrakopoulos, 2011; Chatterjee et al., 2012; Mueller et al., 2012; Boucher et al., 2014). Figure 1 shows an example of a cross section for both material type and attribute (copper) simulations at the copper mine used in the case study; it is noted that the simulations for material types provides discrete geological units, whereas the copper grade attribute is a continuous variable.

Let \mathcal{S} represent a set of equally probable *scenarios*, whereby a scenario is a joint sampling from all sources of uncertainty considered in the optimization model. In the case of multi-mine operations, each scenario is indexed, and the number of scenarios is the product of the number of simulations for each orebody model. For example, if two mines are considered, each having 20 geological simulations, $\mathcal{S} = \{1, \dots, 400\}$. Naturally, as the number of independent sources of uncertainty (geology, prices, recoveries, costs, etc.), the size of the optimization model grows exponentially.

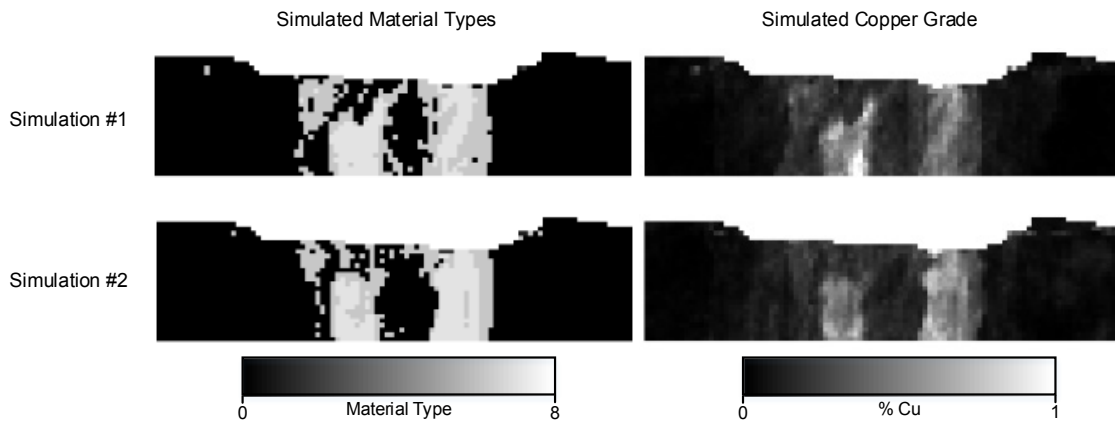


Figure 1: Example of simulated material types and copper grades at the copper mine used in the case study.

2.1.2 Material and attribute flow through a mining complex

In order to develop a model for the global optimization of open pit mining complexes, it is first necessary to establish some fundamental terminology. Tables 1, 2 and 3 provide the relevant sets, variables and parameters used in the optimization models, respectively. A mining complex is comprised of a set of mines (\mathbb{M}), stockpiles (\mathcal{S}) and processors (\mathcal{P}). For simplicity, this work will consider mining complexes where all mines and locations within operate during a fixed set of periods \mathbb{T} ; the more general case where the operating periods of mines and processing streams varies with time (Topal and Ramazan, 2012; Pimentel et al., 2013) is omitted. Mines are assumed to be the only sources of materials for the mining complex. Each mine $m \in \mathbb{M}$ is comprised of a set of discrete volumes (\mathbb{B}_m), referred to as *blocks*. Each block $b \in \mathbb{B}_m$ has simulated attributes $\beta_{p,b,s}$ for the primary attributes of interest ($p \in \mathbb{P}$), which are assumed to be inputs to the optimization model. Stockpiles are locations in the mining complex that are capable of storing incoming materials (and their attributes) over time and distributing them to subsequent locations when desired. Stockpiles are useful in practice because they can be used to blend materials together, thus creating a more homogenous product, and may also be used to store marginally valuable material that is treated at a later time when the opportunity cost of deferring more valuable is lower (i.e. the cut-off grade is lower). A processor is a generic term used to describe all other locations in the mining complex, which may, but not necessarily be used to transform an incoming bulk product into a purer form, for example, concentrators, smelters, refineries, leach pads. Additionally, in this definition, the set of processors may also contain other elements, such as modes of transport (rail, trucks, ports), which are useful for the optimization model. One of the primary distinctions between a stockpile and a processor, in the generic modelling sense, is that a processor does not store material over time; all material that is produced is sent out to subsequent destinations, if possible.

The primary and hereditary attributes at the stockpiles and processors are tracked in the optimization models using state variables. Let $v_{p,t,s}$ represent the value of attribute $p \in \mathbb{P}$ at location $i \in \mathcal{S} \cup \mathcal{P}$. Additionally, let the state variable $v_{h,t,s}$ represent the value of hereditary attribute $h \in \mathbb{H}$, which is calculated using a (non-) linear function, $f_h(p, i, k)$, of the primary attributes $p \in \mathbb{P}$ at location i and capital expenditure option $k \in \mathbb{K}$. These functions may be used, for example, to calculate non-linear recoveries (Goodfellow and Dimitrakopoulos, 2014b), mill throughputs, profits and costs, among others. It is noted that this definition, unlike that of Goodfellow and Dimitrakopoulos (2014a, 2014b), defines hereditary attributes as global functions that may be calculated using the primary attributes p from multiple locations $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ in a single equation. These equations are defined by the modeller, and may be a function of the level of capital expenditures, which is useful when modelling variable operating costs as a function of equipment

Table 1: Sets used to define material and attribute flow through a mining complex.

Set	Description
\mathbb{P}	Primary attributes.
\mathbb{H}	Hereditary attributes.
\mathbb{T}	Time periods.
\mathcal{S}	Joint scenarios for all sources of uncertainty.
\mathbb{M}	Mines.
\mathbb{B}_m	Blocks at mine $m \in \mathbb{M}$.
\mathbb{O}_b	Blocks that overly $b \in \mathbb{B}_m$ that must be extracted prior to b .
\mathbb{K}	All capital expenditure options.
\mathbb{K}^1	One-time capital expenditures ($\mathbb{K}^1 \subseteq \mathbb{K}$).
\mathcal{C}	Sub-groupings (clusters) of blocks with similar attributes.
\mathcal{S}	Stockpile destinations.
\mathcal{P}	Processors in the mining complex that must forward all products generated to the subsequent destinations, if available.
\mathcal{N}	Set of nodes that describe the mining complex, i.e. $\mathcal{N} = \mathcal{C} \cup \mathcal{S} \cup \mathcal{P}$.
$\mathcal{I}(i)$	A set of nodes that destination $i \in \mathcal{N}$ receives materials from (incoming).
$\mathcal{O}(i)$	A set of nodes that destination $i \in \mathcal{N}$ can send material to (outgoing).

Table 2: Variables used in the optimization model.

Decision variables	
$x_{b,t} \in \{0, 1\}$	Decision variable that defines whether or not block $b \in \mathbb{B}_m$ is extracted in period $t \in \mathbb{T}$.
$z_{c,j,t} \in \{0, 1\}$	Decision variable that defines whether or not cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period $t \in \mathbb{T}$.
$y_{i,j,t,s} \in [0, 1]$	Decision variable that defines the proportion of output material sent from $i \in \mathcal{N}$ to $j \in \mathcal{S} \cup \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$w_{k,t} \in \mathbb{Z}$	Decision variable that defines how many capital expenditure options $k \in \mathbb{K}$ are exercised in period $t \in \mathbb{T}$.
State variables	
$v_{p,i,t,s} \in \mathbb{R}$	State variable for attribute $p \in \mathbb{P}$ at location or cluster $i \in \mathcal{N} \cup \mathbb{M}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$v_{h,i,t,s} \in \mathbb{R}$	State variable for attribute $h \in \mathbb{H}$ at location $i \in \mathcal{N} \cup \mathbb{M}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$r_{p,i,t,s} \in [0, 1]$	Recovery factor for attribute $p \in \mathbb{P}$ at location $i \in \mathcal{S} \cup \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.
$d_{h,i,t,s}^+, d_{h,i,t,s}^-$	Surplus and shortage variables, respectively, from a deviation target for attribute $h \in \mathbb{H}$ at location $i \in \mathcal{S} \cup \mathcal{P}$, in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$.

Table 3: Parameters used in the optimization model.

Material flow parameters and attribute transformation functions	
$\theta_{b,c,s} \in \{0, 1\}$	Defines whether or not block $b \in \mathbb{B}_m$ belongs to cluster $c \in \mathcal{C}$ in scenario $s \in \mathbb{S}$.
$\beta_{p,b,s}$	Simulated value for attribute $p \in \mathbb{P}$, block $b \in \mathbb{B}_m$ and scenario $s \in \mathbb{S}$.
$f_h(p, i, k)$	A (non)-linear function that is used to calculate the value of attribute $h \in \mathbb{H}$ using attributes $p \in \mathbb{P}$ from locations $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ and capital expenditure options $k \in \mathbb{K}$.
Optimization model parameters	
$U_{h,i,t}, L_{h,i,t}$	Upper and lower bounds (or initial) capacity for attribute $h \in \mathbb{H}$ at location $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ in period $t \in \mathbb{T}$.
$p_{h,i,t}$	Price of attribute $h \in \mathbb{H}$ at location $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ (note that this can be negative) in period $t \in \mathbb{T}$.
$c_{h,i,t}^+, c_{h,i,t}^-$	Unit surplus and shortage costs associated with deviations from bounds for attribute $h \in \mathbb{H}$ at location $i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}$ in period $t \in \mathbb{T}$.
Capital expenditure parameters	
$p_{k,t}$	Discounted purchase price for capital expenditure $k \in \mathbb{K}$ in period $t \in \mathbb{T}$.
$\kappa_{k,h,i}$	The per-unit increment for a constraint that capital expenditure $k \in \mathbb{K}$ has on attribute $h \in \mathbb{H}$.
λ_k	The life of capital expenditure $k \in \mathbb{K}$ (e.g. truck life before replacement).
τ_k	Lead time before capital expenditure $k \in \mathbb{K}$ is built or delivered.
$L_{k,t}, U_{k,t}$	Minimum and maximum purchase requirements, respectively, for option $k \in \mathbb{K}$ in period $t \in \mathbb{T}$.

purchases (i.e. economies of scale from expanding the mill capacity or mine production). Additionally, a recovery variable, $r_{p,i,t,s} \in [0, 1]$, may be used to define the quantity of attribute p recovered at a location $i \in \mathcal{S} \cup \mathcal{P}$ for each time period and scenario.

The flow of materials and their respective attributes through a mining complex is defined by three sets of decision variables (Figure 2), namely the mine production schedule decisions, destination policies and processing stream decisions. Production scheduling decision variables, $x_{b,t} \in \{0, 1\}$, determine whether (1) or not (0) a block b is extracted in period t ; these decision variables define the initial quantities of attributes for each material that is available in each time period. For open pit mines, a set of overlying blocks, \mathbb{O}_b is defined for each block $b \in \mathbb{B}_m$, which are the blocks that must be extracted prior to b in order to ensure slope stability and safety. These sets are generated via a pre-processing step that looks at the overlying

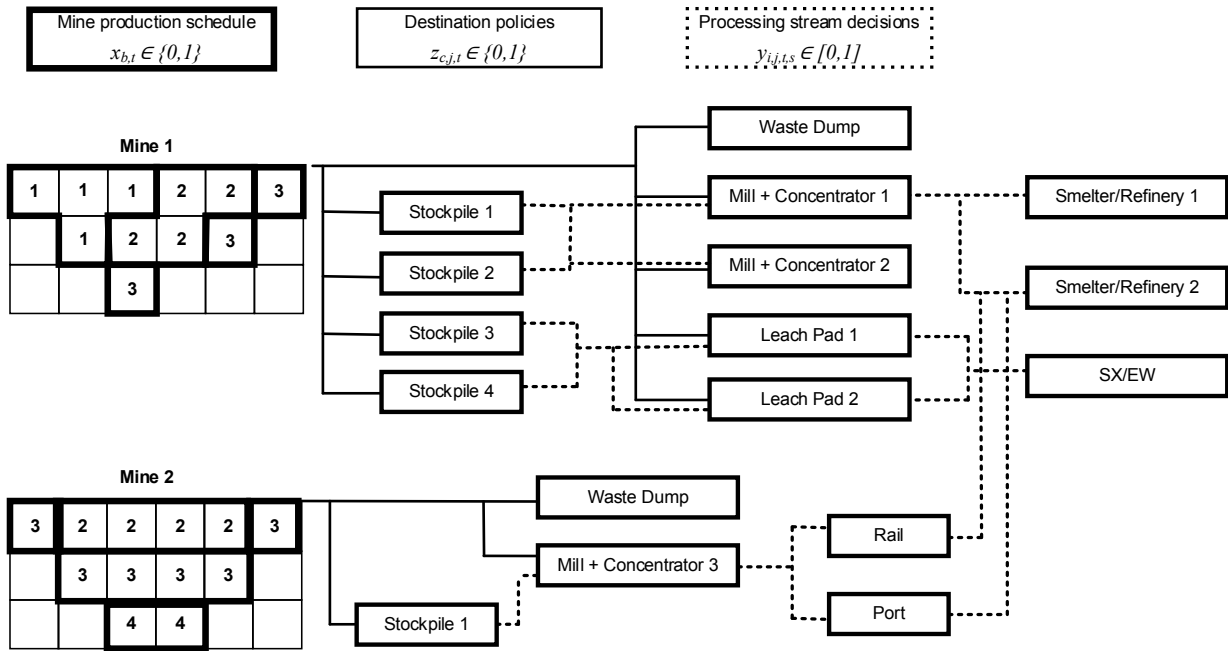


Figure 2: Decision variables related to material flowing through a mining complex.

blocks within an inverted cone (Khalokakaie et al., 2000), and are of minimum cardinality to avoid excessive memory usage.

After extraction, it is necessary to decide where to send the extracted materials. One method commonly used in the mining industry is a cut-off grade policy, which is a threshold value that defines where material above or below the prescribed threshold is sent. Simulated block attributes, $\beta_{p,b,s}$, are often sampled from continuous distributions, which complicates the decision of where to send extracted materials because it leads to non-linear formulations. In order to avoid these complex models, some research has instead focused on methods that use binary decision variables to define where material is sent. There are several ways to model the destination decisions, such as scenario-dependent block destinations (Boland et al., 2008; Kumral, 2011) and robust block destinations (Kumral, 2013, Montiel Petro and Dimitrakopoulos, 2013). The former, however, leads to overly optimistic solutions that don't integrate uncertainty into the decisions, whereas, the latter may lead to sending blocks to destinations that are incompatible with a block's simulated material type. Rather than attempting to make destination decisions on the block-scale, Menabde et al. (2007) propose a discretization of the continuous attribute into *bins* to define cut-off grade policies. Cut-off grade policies, however, are often not useful for global optimization models that require complex blending constraints and multiple attributes (e.g. multiple metals or deleterious elements). Goodfellow and Dimitrakopoulos (2014a) propose a generalization of this concept that is useful for mining complexes that consider the impacts of multiple attributes on the entire system. In this method, a set of multivariate bins (\mathcal{C}), referred herein as *clusters*, are created in a pre-processing step by clustering (Lloyd, 1982; Arthur and Vassilvitskii, 2007) the primary block attributes $\beta_p, b, s \forall p \subseteq \mathbb{P}, b \in \mathbb{B}_m, m \in \mathbb{M}, s \in \mathbb{S}$ for each material type and for each the mine. Let $\theta_{b,c,s} \in \{0,1\}$ represent a pre-processed parameter that defines whether (1) or not (0) block b belongs to a cluster $c \in \mathcal{C}$ in scenario $s \in \mathbb{S}$. Additionally, let the decision variable $z_{c,j,t} \in \{0,1\}$ decide whether (1) or not (0) cluster $c \in \mathcal{C}$ is sent to destination $j \in \mathcal{O}(c)$ in period $t \in \mathbb{T}$. These variables effectively form a robust destination policy that decides where to send all blocks with similar attributes (e.g. high iron content, medium silica, medium phosphorus) and material types, rather than deciding on the destination of individual blocks. Given that a block's simulated material type and grades may vary between simulations, the membership to a given cluster c for a scenario s may also vary accordingly. As a result, the destination of a block may vary between scenarios, according to its membership c distribution.

Finally, after material is received at the first set of destinations directly from the mines, the material flow through the remainder of the mining complex is governed by a set of processing stream decision variables. Let $y_{i,j,t,s} \in [0, 1]$ define the proportion of a material (product) sent from destination $i \in \mathcal{S} \cup \mathcal{P}$ to destination $j \in \mathcal{O}(i) \subseteq (\mathcal{S} \cup \mathcal{P}) \setminus i$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. It is noted that these scenario-dependent decision variables are designed to let the optimizer take recourse decisions (Birge and Louveaux, 2011) after the uncertainty has been revealed at the first set of destinations. For future reference, the set of locations in the mining complex that send material to $i \in \mathcal{S} \cup \mathcal{P}$ is denoted by $\mathcal{I}(i)$.

3 Optimization of mining complexes with capital expenditures

Given the flexibility required to accurately model an individual mining complex, a generalized optimization model is proposed, which can be configured to satisfy the needs of the decision-maker. Using the sets, variables and parameters outlined in Tables 1 to 3, respectively, it is possible to define a generalized two-stage stochastic integer program (Birge and Louveaux, 2011) that is used to optimize mining complexes with capital expenditures. In this model, the first-stage decisions, which must be made before the uncertainty is revealed, are the mine production schedule(s), destination policies and capital expenditures. The recourse variables, which adapt the optimization model to information garnered after uncertainty is revealed, include the processing stream decisions ($y_{i,j,t,s}$) and penalties related to deviations from production targets. Notably, these penalties are used to manage the upside and downside risk, and may be penalized using time-discounted, monotonically decreasing factors ($c_{h,t}^+$ and $c_{h,t}^-$, respectively) that forces riskier materials to be mined in later periods (*geological risk discounting*) (Ramazan and Dimitrakopoulos, 2013). These penalty costs, and the associated geological risk discount rate, may be determined experimentally by testing different values, re-optimizing and analyzing the resulting risk profiles. It is noted that it is often necessary to balance the orders of magnitudes for these penalty costs to force the optimizer to consider the differences in order of magnitudes between the constraints (e.g. millions of tonnages compared to a grade measured as a percentage). The optimization formulation is as follows:

Objective function:

$$\max \frac{1}{|\mathbb{S}|} \underbrace{\sum_{s \in \mathbb{S}} \sum_{t \in \mathbb{T}} \sum_{h \in \mathbb{H}} p_{h,t} \cdot v_{h,t,s}}_{\text{Discounted revenues and costs}} - \underbrace{\sum_{t \in \mathbb{T}} \sum_{k \in \mathbb{K}} p_{k,t} \cdot w_{k,t}}_{\text{Capital expenditure costs}} - \frac{1}{|\mathbb{S}|} \underbrace{\sum_{s \in \mathbb{S}} \sum_{t \in \mathbb{T}} \sum_{h \in \mathbb{H}} c_{h,t}^+ \cdot d_{h,t,s}^+ + c_{h,t}^- \cdot d_{h,t,s}^-}_{\text{Risk-discounted penalties for deviations}} \quad (1)$$

Subject to:

- I. *Mine reserve and slope constraints*, which guarantee that a block is only mined once, if at all, and obeys slope stability requirements.

$$\sum_{t \in \mathbb{T}} x_{b,t} \leq 1 \quad \forall b \in \mathbb{B}_m \quad (2)$$

$$x_{b,t} \leq \sum_{t'=1}^t x_{u,t'} \quad \forall b \in \mathbb{B}_m, m \in \mathbb{M}, u \in \mathbb{O}_b, t \in \mathbb{T} \quad (3)$$

- II. *Destination policy constraints*, which ensure that the clusters of materials $c \in \mathcal{C}$ are each only sent to a single destination.

$$\sum_{j \in \mathcal{O}(c)} z_{c,j,t} = 1 \quad \forall c \in \mathcal{C}, t \in \mathbb{T} \quad (4)$$

III. *Processing stream constraints*, which calculate the quantities of the primary attributes for each period and ensure mass balance in the mining complex.

$$v_{p,j,(t+1),s} = \underbrace{v_{p,j,t,s} \cdot \left(1 - \sum_{k \in \mathcal{O}(j)} y_{i,k,t,s}\right)}_{\text{Leftovers from previous period}} + \underbrace{\sum_{i \in \mathcal{I}(j) \setminus \mathcal{C}} r_{p,i,t,s} \cdot v_{p,i,t,s} \cdot y_{i,j,t,s}}_{\text{Incoming from other locations}}$$

$$+ \underbrace{\sum_{c \in \mathcal{I}(j) \cap \mathcal{C}} \left(\sum_{m \in \mathbb{M}} \sum_{b \in \mathbb{B}_m} \theta_{b,c,s} \cdot \beta_{p,b,s} \cdot x_{b,(t+1)} \right) \cdot z_{c,i,(t+1)}}_{\text{Incoming from mines}} \quad \forall p \in \mathbb{P}, j \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \quad (5)$$

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} = 1 \quad \forall i \in \mathcal{P}, t \in \mathbb{T}, s \in \mathbb{S} \quad (6)$$

$$\sum_{j \in \mathcal{O}(i)} y_{i,j,t,s} \leq 1 \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \quad (7)$$

IV. *Attribute calculation constraints*, which are used to calculate the value of the state hereditary attributes and quantities of interest at the mine level (e.g. per-period tonnages). Recall that the function $f_h(p, i, k)$ is defined by the modeler, and is not necessarily linear.

$$v_{h,t,s} = f_h(p, i, k) \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (8)$$

$$v_{p,m,t,s} = \sum_{b \in \mathbb{B}_m} \beta_{p,b,s} \cdot x_{b,t} \quad \forall m \in \mathbb{M}, p \in \mathbb{P}, t \in \mathbb{T}, s \in \mathbb{S} \quad (9)$$

V. *Hereditary attribute constraints*, which may be used to track the deviations for variables $h \in \mathbb{H}$ from upper- and lower-bound capacities (e.g. mining, stockpiling, processing, grade blending capacities). It is noted that these capacities may be increased or decreased by $\kappa_{k,h}$ by investing in the capital expenditure option $k \in \mathbb{K}$, and consider the lifespan of the capacity increment (λ_k) and the lead time to delivery or construction (τ_k).

$$v_{h,t,s} - d_{h,t,s}^+ \leq U_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^t \kappa_{k,h} \cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (10)$$

$$v_{h,t,s} + d_{h,t,s}^- \geq L_{h,t} + \sum_{t'=t-\lambda_k+\tau_k}^t \kappa_{k,h} \cdot w_{k,t'} \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathbb{S} \quad (11)$$

VI. *Recoveries*, which are constant for stockpiles and may be equal to the value of hereditary attributes for processors; this may, in turn, be a static recovery or a value from a grade-recovery curve.

$$r_{p,i,t,s} = 1 \quad \forall p \in \mathbb{P}, i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \quad (12)$$

$$r_{p,i,t,s} = v_{h,t,s} \quad \forall p \in \mathbb{P}, i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \quad (13)$$

VII. *End-of-year stockpile attribute constraints*, which may be used to calculate and track the quantity of an attribute that remains in a stockpile at the end of the production period.

$$v_{h,t,s} = v_{p,i,t,s} \cdot \left(1 - \sum_{j \in \mathcal{O}(i)} y_{i,j,t,s}\right) \quad \forall i \in \mathcal{S}, t \in \mathbb{T}, s \in \mathbb{S} \quad (14)$$

VIII. *Capital expenditure constraints* for one-time investments (e.g. expanding mill capacity), which ensure that the option is exercised once, if at all.

$$\sum_{t \in \mathbb{T}} w_{k,t} \leq 1 \quad \forall k \in \mathbb{K}^1 \subseteq \mathbb{K} \quad (15)$$

IX. *Variable definitions*

$$L_{k,t} \leq w_{k,t} \leq U_{k,t} \quad \forall k \in \mathbb{K}, t \in \mathbb{T} \quad (16)$$

$$x_{b,t} \in \{0, 1\} \quad \forall b \in \mathbb{B}_m, t \in \mathbb{T} \quad (17)$$

$$z_{c,j,t} \in \{0, 1\} \quad \forall c \in \mathcal{C}, j \in \mathcal{O}(j), t \in \mathbb{T} \quad (18)$$

$$y_{i,j,t,s} \in [0, 1] \quad \forall i \in \mathcal{S} \cup \mathcal{P}, j \in \mathcal{O}(i), t \in \mathbb{T}, s \in \mathcal{S} \quad (19)$$

$$r_{p,i,t,s} \in [0, 1] \quad \forall p \in \mathcal{P}, i \in \mathcal{S} \cup \mathcal{P}, t \in \mathbb{T}, s \in \mathcal{S} \quad (20)$$

$$v_{p,i,t,s} \geq 0 \quad \forall p \in \mathcal{P}, i \in \mathcal{S} \cup \mathcal{P} \cup \mathbb{M}, t \in \mathbb{T}, s \in \mathcal{S} \quad (21)$$

$$v_{h,t,s} \in \mathbb{R} \quad (22)$$

$$d_{h,t,s}^+, d_{h,t,s}^- \geq 0 \quad \forall h \in \mathbb{H}, t \in \mathbb{T}, s \in \mathcal{S} \quad (23)$$

4 Algorithmic optimization with metaheuristics

4.1 Hybrid optimization with simulated annealing and particle swarm optimization

The stochastic optimization model presented in Section 3 is computationally challenging to solve using conventional optimization methods; single orebody models often consist of hundreds of thousands of blocks, which can result in millions of binary decision variables. Moreover, given the flexibility that a modeller has to design these optimization formulations, these models may be non-linear, making exact optimization methods infeasible for realistic-sized mining complexes. While many authors have proposed mine production scheduling models and heuristics that are tailored to be solved using mathematical optimizers (Caccetta and Hill, 2003; Boland et al., 2008; Bienstock and Zuckerberg, 2010; Bley et al., 2010; Cullenbine et al., 2011; Bley et al., 2012; Lambert and Newman, 2013), the scale of the formulations poses a formidable challenge. Moreover, these methods often are limited by various simplifying assumptions, such as the use of aggregates (reduces scale of decision-making), not being able to accommodate lower bounds or blending constraints (generally required for mining complexes), linearity (to garner information using duality theory) and time-separability properties (precludes the use of stockpiles and complex processing streams). Metaheuristics are a class of optimization algorithms that do not necessarily give mathematical optimality, however, have been used as a tool to generate high-quality optimization solutions (when compared to commercial mine design software and optimization solvers) within a reasonable amount of time (Godoy, 2003; Ferland et al., 2007; Albor and Dimitrakopoulos, 2009; Lamghari and Dimitrakopoulos, 2012; Kumral, 2013; Lamghari et al., 2014a; Lamghari et al., 2014b).

The generalized formulation for optimizing mining complexes with uncertainty (Section 3) is optimized using a combination of the simulated annealing (Metropolis et al., 1953; Kirkpatrick et al., 1983; Geman and Geman, 1984) and particle swarm optimization (Kennedy and Eberhart, 1995) algorithms. The simulated annealing algorithm has been demonstrated in the past to be capable of optimizing large-scale mine production scheduling models with good results (Godoy, 2003; Albor and Dimitrakopoulos, 2009; Lamghari et al., 2014a). This method, however, is somewhat limited in its ability to handle a large number of continuous variables (i.e. processing stream decision variables); evaluating the objective function for the optimization of mining complexes is generally computationally demanding, and existing simulated annealing algorithms for continuous variables generally work with only a single variable at a time. Through experimental testing in Chapter 4, it was noted that using the simulated annealing algorithm is not particularly effective for optimizing processing stream decisions. As a result of this limitation, the particle swarm optimization algorithm is used because of its inherent ability to modify all continuous variables at each iteration, leading to more changes in the variables per objective function evaluation. Let $\Phi = [\mathbf{x}, \mathbf{z}, \mathbf{w}, \mathbf{y}]$ represent a solution

vector that is used to store the production schedules ($\mathbf{x}=[x_{b,t}]$), destination policies ($\mathbf{z}=[z_{c,j,t}]$), capital expenditure options ($\mathbf{w}=[w_{k,t}]$) and processing stream variables ($\mathbf{y}=[y_{i,j,t,s}]$). The simulated annealing (SA) algorithm is used to optimize the discrete variables ($\mathbf{x}, \mathbf{w}, \mathbf{z}$), and, after a specified number of iterations (e.g. 10), the particle swarm optimization (PSO) algorithm optimizes the continuous variables (\mathbf{y}). The methods are used interchangeably to avoid getting trapped in local optima. An initial schedule can be generated using either industry-standard planning software as input, or start from nothing by setting all blocks to being un-mined. The remaining variables for destination policies, processing stream values and capital expenditures can be initialized by randomly generating values that obey their constraints.

In the classic SA algorithm, a *perturbation* to the current solution vector is proposed (see Section 4.2). Let $g(\Phi)$ represent the objective function value (Eq. (1)) for the current solution vector, Φ , and let $g(\Phi')$ represent the objective function value for a perturbed solution vector, Φ' . For an objective function that is being maximized, a proposed perturbation is accepted or rejected according to the following probability distribution:

$$P(g(\Phi), g(\Phi'), \delta) = \begin{cases} 1 & \text{if } g(\Phi') \leq g(\Phi) \\ \exp\left(-\frac{|g(\Phi')-g(\Phi)|}{\delta}\right) & \text{otherwise} \end{cases} \quad (24)$$

where δ is commonly referred to as an *annealing temperature*, which is initially defined as an input parameter for the first iteration, $\delta(0)$, and is gradually cooled as the algorithm progresses using a cooling factor, $cf \in [0, 1]$, by applying $\delta = \delta \cdot cf$ every n_i iterations. If a perturbation is accepted, $\Phi = \Phi'$. If the perturbation is the best found by that iteration, the global best solution vector, Φ^g is updated, i.e. $\Phi^g = \Phi$. One of the difficulties of the classic SA algorithm is related to the use of multiple *neighbourhoods* of perturbations, where a neighbourhood refers to a class of perturbation in solution vector (\mathbf{x} , \mathbf{w} , or \mathbf{z}). A perturbation for each neighbourhood may have a drastically different impact on the objective function value. Figure 3 shows an example of a cumulative probability distribution with respect to rejected neighbourhood perturbations $g(\Phi') - g(\Phi) < 0$. In the classic SA algorithm, for a given δ , Eq. (24) will likely reject all capital expenditure perturbations (CAPEX), however accept most production schedule and destination policy perturbations. Naturally, there is a strong relationship amongst these variables, and this phenomenon may result in SA tunneling into a local optimum.

Alternatively, it is possible to modify the simulated annealing algorithm to deal with these multiple neighbourhoods independently. Rather than a fixed δ that applies to all neighbourhoods, the proposed method uses variable annealing temperatures, δ_x , δ_w , and δ_z , which are calculated using an *annealing probability temperature* and a distribution function similar to Figure 3. This distribution is first constructed by sampling 1000 perturbations for each neighbourhood prior to the full-scale annealing using Eq. (24). An initial annealing probability temperature, $\rho(0)$ is specified as an input parameter (e.g. $\rho(0) = 0.8$), and the appropriate annealing temperatures (δ_x , δ_w , δ_z) are derived. Equation (24) then uses the appropriate annealing

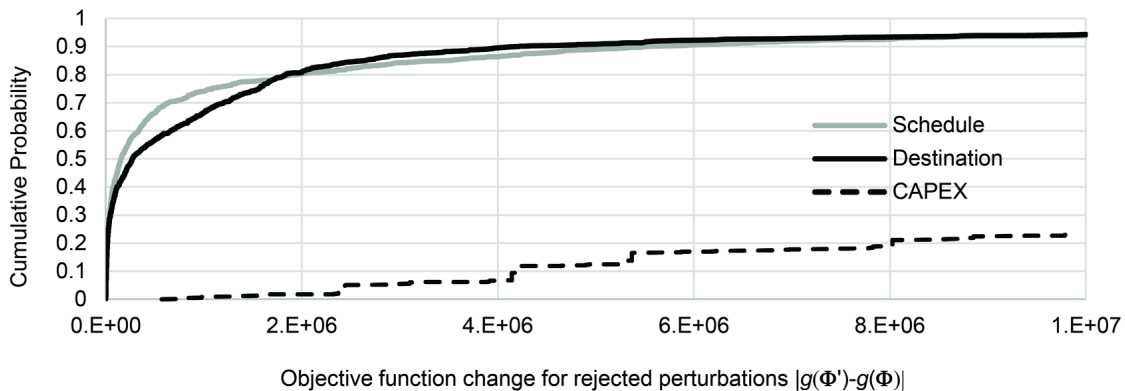


Figure 3: Cumulative distribution of change in objective function values for rejected perturbations for production scheduling, destination policy and capital expenditures.

temperature according to the neighbourhood that the proposed perturbation belongs. This method has the added advantage that the modeller does not need to spend excessive time calibrating the initial temperature $\delta(0)$ for the classic SA algorithm. Similar to the annealing temperatures, the annealing probability temperature is updated by $\rho = \rho \cdot cf$ every n iterations. As the algorithm progresses and sub-optimal perturbations are discovered ($g(\Phi') - g(\Phi) < 0$), the cumulative distributions that define $\delta_{\mathbf{x}}$, $\delta_{\mathbf{w}}$, and $\delta_{\mathbf{z}}$ are updated accordingly. This helps to ensure that the appropriate neighbourhood annealing temperatures are always updated to the local solution space, defined by Φ .

It is noted that the proposed SA algorithm does not perturb the processing stream variables $\mathbf{y} \in \Phi$. PSO (Kennedy and Eberhart, 1995) is a population-based metaheuristic that can optimize both discrete and continuous variables. In the proposed method, PSO is used to optimize the processing streams after a number of defined iterations of the SA algorithm. Unlike the previous work (Goodfellow and Dimitrakopoulos, 2014a; Goodfellow and Dimitrakopoulos, 2014b), which uses PSO in conjunction with the global best solution vector, Φ^g , this method uses the working solution vector, Φ , and focuses solely on the processing stream decisions $\mathbf{y} \in \Phi$. A *particle* is a data structure that stores a temporary processing stream solution vector, $\mathbf{y}_i \in \Phi$, a particle best solution vector, \mathbf{y}_i^b and a velocity vector, \mathbf{v}_i for all particles $i \in \{1, \dots, N^P\}$, where N^P is a parameter that defines the total number of particles. Additionally, a vector \mathbf{y}^g is used to store the processing stream solution vector. At each iteration ($\alpha + 1$) of PSO, the particles (solution vectors) are updated as follows:

$$\mathbf{v}_i(\alpha + 1) = c_1 \cdot \mathbf{v}_i(\alpha) + c_2 \cdot r_1 \cdot (\mathbf{y}_i^b - \mathbf{y}_i) + c_3 \cdot r_2 \cdot (\mathbf{y}^g - \mathbf{y}_i) \quad (25)$$

$$\mathbf{y}_i(\alpha + 1) = \mathbf{y}_i(\alpha) + \mathbf{v}_i(\alpha + 1) \quad (26)$$

$$\mathbf{y}_i^b = \mathbf{y}_i(\alpha + 1) \quad \text{if } g([\mathbf{x}, \mathbf{w}, \mathbf{z}, \mathbf{y}_i(\alpha + 1)]) \geq g([\mathbf{x}, \mathbf{w}, \mathbf{z}, \mathbf{y}_i^b]) \quad (27)$$

$$\mathbf{y}^g = \mathbf{y}_i^b \quad \text{if } (g([\mathbf{x}, \mathbf{w}, \mathbf{z}, \mathbf{y}_i^b]) \geq \max \left\{ g([\mathbf{x}, \mathbf{w}, \mathbf{z}, \mathbf{y}^g]), g([\mathbf{x}, \mathbf{w}, \mathbf{z}, \mathbf{y}_j^b]) \right\}) \quad \forall j \in \{1, \dots, N^P\} \quad (28)$$

where c_1 , c_2 and c_3 are inertia coefficients (parameters), and r_1 and r_2 are random uniform numbers between 0 and 1. In the previous equations, $g([\mathbf{x}, \mathbf{w}, \mathbf{z}, \mathbf{y}_i(\alpha + 1)])$ is used to denote the objective function value (Eq. (1)) for a solution vector $[\mathbf{x}, \mathbf{w}, \mathbf{z}, \mathbf{y}_i(\alpha + 1)]$, where $\mathbf{x}, \mathbf{w}, \mathbf{z} \in \Phi$ is from simulated annealing. It is noted that in the event that Eqs. (6) and (7) are violated when updating Eq. (26), the processing stream variables are need to be re-normalized prior to evaluating Eq. (27). The PSO algorithm is iterated until all particles converge on an optimum (approximately 0.1%) or after a specified number of iterations (e.g. 100). The processing stream portion of the solution vector is updated upon termination of PSO, i.e. $\{\mathbf{y} \in \Phi\} = \mathbf{y}^g$.

4.2 Neighbourhood perturbations for simulated annealing

As the SA algorithm progresses, it is necessary to find valid perturbations to the solution vector, Φ . Given the inclusion of capital expenditure options, where a perturbation can drastically increase or decrease a capacity constraint, the optimizer is likely to cycle or converge on a local optimum as it attempts to find a large number of production schedule perturbations that satisfy large changes in the constraints. To avoid the chances of this occurring, a combination of both small and large production scheduling changes is used. The following neighbourhood perturbation mechanisms are used to modify an existing solution, Φ , during the simulated annealing algorithm:

- i. *Destination policy perturbations* ($\mathbf{z} \in \Phi$) are generated by randomly selecting a cluster $c \in \mathcal{C}$ for a period $t \in \mathbb{T}$, which is currently sent to destination $j \in \mathcal{O}(c)$ and sending it to $j' \in \mathcal{O}(c)$.
- ii. *Random capital expenditure perturbations* ($\mathbf{w} \in \Phi$) are generated by randomly selecting a capital expenditure option $w_{k,t} = n$ and choosing a new value, i.e. $w_{k,t} = n' \in [L_{k,t}, U_{k,t}]$. If the option is a one-time decision ($k \in \mathbb{K}^1$), all other variables $w_{k,t'} \forall t' \in \mathbb{T} \setminus t$ are set to 0.
- iii. *Delayed capital expenditure perturbations* ($\mathbf{w} \in \Phi$) are generated by randomly selecting a capital expenditure option $w_{k,t} = n$ for any $k \in \mathbb{K} \setminus \mathbb{K}^1$ and deferring the purchase of one unit by setting $w_{k,t} = w_{k,t} - 1$ and setting $w_{k,(t+1)} = w_{k,(t+1)} + 1$.
- iv. *Small production schedule perturbations* ($\mathbf{x} \in \Phi$) are generated by randomly selecting a block $b \in \mathbb{B}_m$ on the boundary between two periods and advancing or delaying its extraction to a period $t' \in \mathbb{T}$ of a

randomly selected, directly adjacent block (i.e. above, below or the four adjacent blocks on the same elevation). When advancing the extraction period of block b , slope constraint violations are corrected by searching for blocks $b' \in \mathcal{O}_b$ where $x_{b',t''} = 1$ and $t' < t''$, which are also moved to period t' . A similar procedure is used when delaying a block b 's extraction period by searching for slope constraint violations below b .

- v. *Conical production schedule perturbations* ($\mathbf{x} \in \Phi$) are similar to small production schedule perturbations, however do not require the condition that the block $b \in \mathbb{B}_m$ lie on the boundary between two periods. The extraction period of the block is advanced or delayed by randomly selecting a new $t' \in \mathbb{T}$ (i.e. not considering the directly adjacent blocks). Any slope constraint violations are corrected in a similar manner, however, rather than moving all violating blocks into a single period, which may incur drastic penalty costs for deviations from targets, the blocks are moved into several periods. For example, if the mine can sink two benches (levels in elevation) per period, and a block b 's extraction period is advanced, any blocks that lie two or three levels above will be extracted in period $t' - 1$. Similarly, any blocks that lie four or five levels above are extracted in period $t' - 2$. This helps to split a large schedule change into several periods, which is likely to have less of an impact on the penalties in the objective function.
- vi. *Bench-wise production schedule perturbations* ($\mathbf{x} \in \Phi$) are generated by first labelling the blocks in each bench (vertical level) of the schedule $\mathbf{x} \in \Phi$ using a connected component labelling algorithm (Cormen et al., 2009). This algorithm assigns a unique label to all blocks that are mined within the same period and are spatially connected (Figure 4). Rather than randomly selecting a single block that forms an apex of a cone, a component of a bench is then randomly selected (e.g. Figure 4B, component “K”) to delay or advance the extraction period.

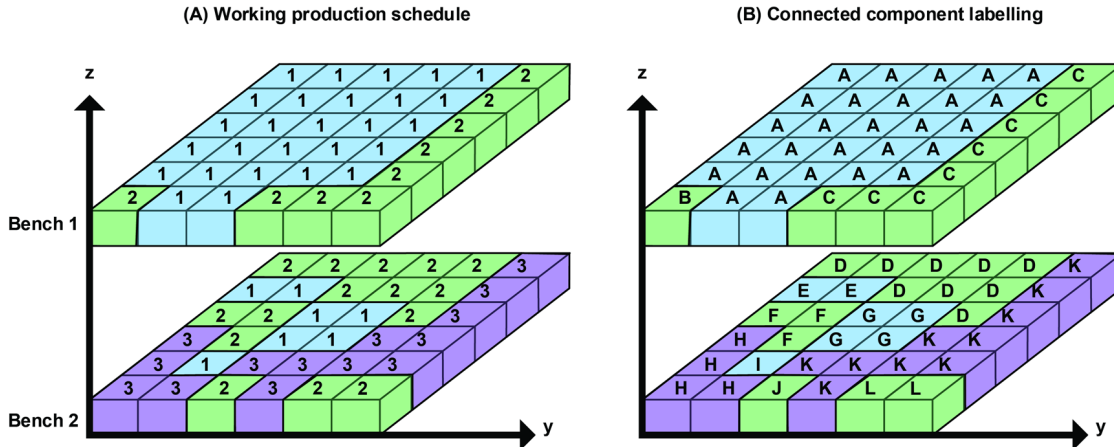


Figure 4: Example of labelling a production schedule $\mathbf{x} \in \Phi$ as connected components for bench-wise production schedule perturbations. Spatially connected blocks in (B) are selected for advancing or delaying their period of extraction.

5 Application at a copper mining complex

The proposed method for the global optimization of an open pit mining complex with capital expenditures is applied to a copper mining complex that is supplied by an industrial partner. The name of this mining operation and some of the relevant modelling parameters are withheld for confidentiality purposes.

5.1 Overview

The mining complex under study consists of a single mine that primarily produces copper, a group of stockpiles, a mill and concentrator processing stream, a leach pad and a waste dump (Figure 5). The mine

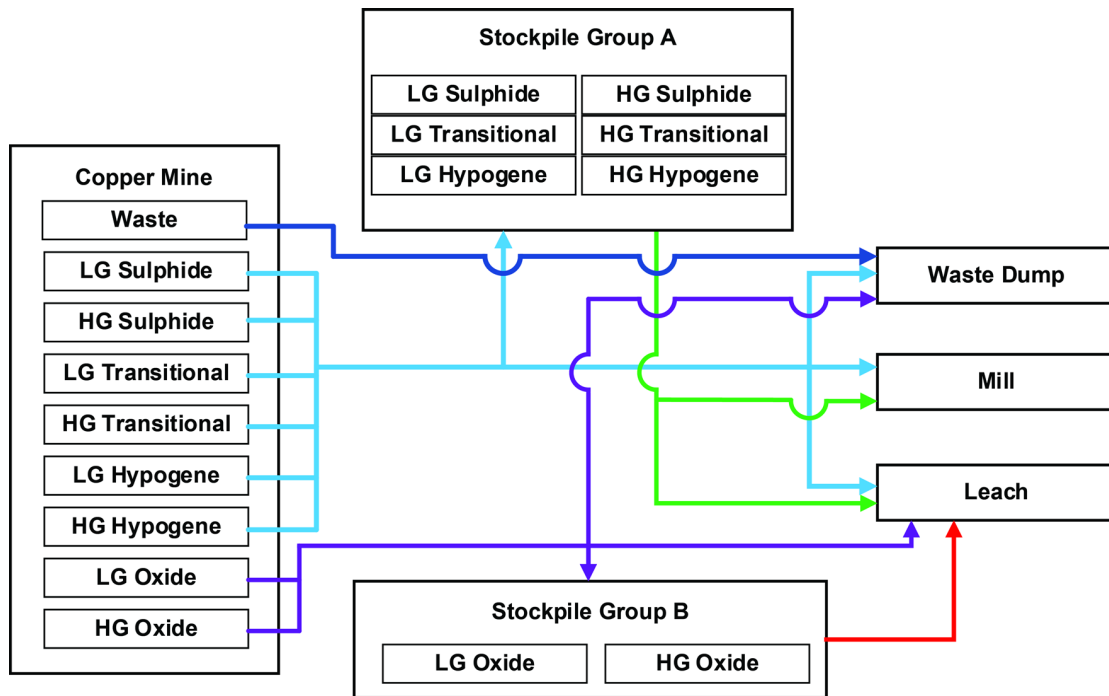


Figure 5: Material flow diagram for the copper mining complex.

produces five main material classes, specifically waste, supergene sulphide, transitional, oxide and hypogene materials. With the exception of the waste material, all material classes are divided into two materials types that are used for modelling the mining complex based on being above or below a pre-defined threshold. It is noted that this threshold is defined only to better differentiate the materials that are stockpiled to improve selectivity. The low- and high-grade sulphide, transitional and oxide materials have the option of being sent to a unique stockpile (according to material type), a mill, a leach pad or a waste dump. The stockpiles for these materials, in turn, each sends material to either the mill or the leach pad. The low- and high-grade oxide materials may not be treated at the mill, thus have the option of being stockpiled, sent to the leach pad or the waste dump. The low- and high-grade oxide stockpiles feed only the leach pad. The waste materials are automatically sent to the waste dump.

This case study considers two types of capital expenditures: (i) shovels, which are used to extract material from the ground; and (ii) trucks, which are loaded by shovels and haul material from the mine to the various processing streams. The optimizer, therefore, has full control over the production rates from the mine. Table 4 provides an overview of the relevant parameters for the shovel and truck capital expenditures. To provide more consistent production rates, the decision to purchase or replace shovels or trucks only occurs every 5 and 4 years, respectively. While it may be interesting to allow the optimizer to make these decisions annually, the result is a series of fluctuations in fleet size, which, in turn, would result in cycles with excessive amounts of hiring or laying off employees. It is interesting to note that the shovel production rates depends on the block's classification of ore and waste (destination policy variables $z_{c,j,t}$). For selectivity reasons, the shovel is able to load at only 3100 tons per hour for material that is sent to the stockpiles, leach pad or mill, rather than the 4200 tons per hour for material that is sent to the waste dump.

Table 5 provides an overview of the key parameters required to model of the mining complex. The mine has provided an estimated orebody model and a set of 50 geological simulations. During as the metaheuristic optimization progresses, simulations are added gradually until a stable solution is obtained (i.e. the objective function value does not change by adding more scenarios). In this case study, this is achieved by starting with 5 simulations, optimizing, then incrementally increasing the number of simulations by 5 and continuing the optimization process. In this case, it was found that the objective function and risk profiles remain stable

Table 4: Capital expenditure options for the copper mining complex.

Parameters	Shovels	Trucks
Undiscounted cost (\$USD)	30,000,000	5,000,000
Life (years)	10	8
Purchase or replacement decision frequency (years)	5	4
Minimum, maximum purchased per period	0,4	0,40
Lead time to delivery (years)	1	1
Capacity increment ($?_{k,t}$)	8670 h	8670 h
Combined productivity and utilization factor	0.58	0.65
Production capacity*	4200 tph (waste) 3100 tph (ore)	230 t

*Shovel production rate (tons per hour) depends on destination policies that classify material as ore or waste.

Table 5: Modelling parameters for the copper mining complex.

Orebody model	
Number of blocks	128 946
Life (years)	33
Simulations used during optimization	30
Discount rate	6%
Geological risk discount rate	5%
Pit slope angle	39°
Economic parameters	
Copper price	\$3.00/lb
Selling cost (mill)	\$0.40/lb recovered Cu
SX/EW cost (leaching)	\$0.30/lb recovered Cu
Mining cost*	\$1.60/ton
Stockpile rehandling cost	\$0.50/ton
Leach cost (sulphides)	\$8.80/ton
Leach cost (oxides, transitional, hypogene)	\$10.00/ton
Processing cost (mill processing stream)	\$10.50/ton
Copper recovery parameters	
Mill processing stream recovery (variable)	82%–90%
Leach recovery (sulphides)	78%
Leach recovery (oxides)	70%
Leach recovery (transitional and hypogene)	58%
Truck cycle times (return trip)**	
In-pit travel time (variable with block depth)	3-42 mins.
Mine to waste dump	42 mins.
Mine to stockpile	29 mins.
Mine to leach pad	25 mins.
Mine to mill	46 mins.
Stockpile to mill	17 mins.
Stockpile to leach pad	9 mins.

* Before applying a mining cost adjustment factor (costs increase with pit depth).

** Truck cycle times have been adjusted to account for the combined productivity and utilization of the equipment.

when using 30 simulations, and adding more does not drastically alter the net present value or risk profiles. Gradually adding simulations (scenarios) to the model has two advantages, when compared to starting with all scenarios: i) it is possible to see how many scenarios are required to obtain a stable design – by starting with all at once, it is not possible to see whether or not more should be added; and ii) the computational time is reduced because fewer simulations are used to converge on a relatively good solution before increasing the computational load by adding more simulations and continuing the optimization process. For an in-depth discussion related to the number of simulations required, the reader is referred to Albor and Dimitrakopoulos (2009). Both the estimated and simulated orebody models contain information for each block related to the

copper grade and tonnage (Figure 1), recovery if treated in the mill processing stream, in-pit travel time (i.e. the round-trip time required by a truck to access the block from surface), a mining cost adjustment factor (used to increase the operating costs with depth), block tonnages and material types (Figure 4). With the exception of the in-pit travel time and the mining cost adjustment factor, which only relate to the spatial location of blocks, all variables have been simulated. Table 6 summarizes the relevant information related to clustering the simulations to generate the destination policy variables. It is noted that the quantities of the oxide and transitional materials is small, relative to the quantity of hypogene material, hence fewer clusters are used for these materials to form destination policies. This assumption is made for the sole purposes of reducing the size of the optimization model.

Table 6: Clustering parameters used to form destination policies.

Material type	Destination policy parameters	Number of clusters per period
Waste	Copper tonnage	All: 1
Sulphide	Recoverable copper per mill hour	LG: 20, HG: 20
Transitional	Recoverable copper per mill hour	LG: 5, HG: 5
Hypogene	Recoverable copper per mill hour	LG: 10, HG: 10
Oxide	Copper tonnage	LG: 20, HG: 20
Total number of destination policy variables (all periods):		3 663

The objective of the optimization model is to maximize the net present value of the cash flows from mining, processing and selling copper, while considering the capital expenditures required to produce at an optimal production rate. Table 5 shows the relevant economic parameters used to calculate the revenues associated with the sale of copper concentrate for the mill and the copper from the leaching processing streams. Table 7 shows the constraints used to penalize deviations from truck, shovel, mill and leach capacities. It is noted that the capacity constraints are expressed in hours of operation, rather than tonnages, which is typically used in production scheduling models. Given that certain materials are harder than others, the residence time in the mill can vary according to how long it takes to grind material down to a finer size for the concentrator. The shovel and truck constraints are also expressed in hours to more accurately model the shovel's adaptive production rates and the dynamic truck cycle times. The stockpiles are assumed to have unlimited capacity. It is noted that, during the first two years of production, the cash flows are substantially higher than the rest. As a result, the optimizer naturally seeks to extract an infeasible amount of material, and an additional set of constraints for shovel production is used with a high penalty cost to force the optimizer to generate a feasible solution. Additionally, the upper bounds differ for the mill in the deterministic model from the stochastic model (to be discussed in Section 5.2). The upper bound for the deterministic model is the bound used in practice, however, in the stochastic case, the optimizer avoids having an average production at this limit because of the excessive penalties that are incurred for the simulations that produce above this capacity. This would result in an overly conservative design that will rarely fill the mill up to its capacity. As a result, the stochastic optimization model uses a relaxed upper bound to permit the optimizer to create a design that is able to fully utilize the mill (on average). Finally, given that the optimizer is able to decide how many

Table 7: Optimization model constraints.

Constraint Description	Lower, Upper Bounds	Lower, Upper Penalties (Undiscounted) ^a
Truck hours ^b	-, 0 ^c	-, 600
Shovel hours (all periods) ^b	-2 190, 0 ^c	3 000, 7 000
Shovel hours (periods 1 and 2)	-, 2000 ^c	-, 2 000 000
Mill hours (deterministic) ^d	8 000, 8 345	10 000, 40 000
Mill hours (stochastic) ^d	8 000, 8 450	10 000, 40 000
Leach capacity (tons)	-, 21 900 000	10

^a A geological discount rate of 5% is applied to the penalties to manage the distribution of risk over time.

^b Truck and shovel hour calculations are adjusted for productivity and utilization (Table 4).

^c Constraint bounds are adjusted by the number of trucks or shovels purchased (Table 4).

^d Mill requires a ramp-up to full production in the first four years of production (Figure 6). The same penalty costs are used for these periods. The lower-bound constraints are removed after period 28.

trucks and shovels to purchase, the lower- and upper-bounds for trucks and shovels, outlined in Table 7, are starting points. As the optimizer purchases more equipment, the bounds are changed dynamically using the increments shown in Table 4.

5.2 Comparison of deterministic-equivalent and stochastic designs

Using the parameters defined in Section 5.1, a comparison can be made between the deterministic and stochastic designs. First, a deterministic-equivalent design is generated using a single, estimated orebody model that is generated using kriging. Recall that estimated models do not depict the spatial and volumetric uncertainty of material types, and estimated methods tend to smooth out the distributions for the attributes of interest (e.g. copper grades). It is worthwhile to note that existing commercial mine design and production scheduling software is not able to incorporate many of the key details that are required in this model, such as variable throughputs, shovel production rates and truck cycle times that depend on ore/waste classification, and targeting production in hours rather than tonnages. It is therefore not possible to provide a benchmark against other methods, as was done by Goodfellow and Dimitrakopoulos (2014b). The deterministic design discussed herein is referred to as the *deterministic-equivalent* design, because it uses the proposed modelling and optimization methodology with a single scenario (i.e. $\mathbb{S} = \{1\}$). In this sense, it is possible to highlight the differences between deterministic and stochastic models, with all other details being the same. For this study, the primary focus of the optimization model is to maximize the net present value (NPV), provide a consistent feed of materials to the mill processing stream and obey mine production targets, which the optimizer decides by purchasing or replacing shovels and trucks.

All testing is performed using Amazon’s EC2 cloud computing platform with Windows 2012-based virtual machines that use Intel Xeon E5-2670 v2 processors (32 virtual CPUs) and have 244 GB RAM. Both the deterministic and stochastic solutions are generated without the use of an initial schedule. The deterministic design is generated in 20 hours, whereas the stochastic solution requires 49 hours. It is noted that for the stochastic design, four orebody simulations are used at the beginning of the algorithm, and are gradually added; this aids to reduce the computational load at the beginning of the algorithm when trying to find an adequate solution. In this example, the SA algorithm parameters are $\rho(0) = 0.8$, $c_f = 0.999$ and n_i ; the algorithm is run for 600,000 iterations and the parameters are reset and the optimization is re-run. This aids in diversifying the solution to ensure that the solution is not trapped in a local optimum. The PSO algorithm is run every 100 iterations of the SA algorithm, with 15 particles. The PSO inertia parameters, c_1 , c_2 and c_3 are set to 0.8, 0.4 and 1.2, respectively. The PSO algorithm terminates after the objective function values for all particles lie within 0.1% of the best-found value. The SA+PSO algorithm is terminated by the user; while this is not ideal for comparing the computational performance, the objective is to obtain a high-quality solution that satisfies the modeller. Admittedly, the computational performance of the method is hindered by the generalized modelling methodology, which requires the use of maps and expression tree data structures to dynamically evaluate the current solution. If one is concerned with the computational performance, it is possible to adapt the optimization methods proposed with specially tailored models of the mining complex that avoid these data structures.

Figure 6 (left) shows the results from the deterministic-equivalent design. With the exception of the first year, the deterministic-equivalent design indicates that the optimizer is able to consistently feed materials to the mill up to its capacity for the first 15 years (Figure 6A). Afterwards, the design is capable of meeting the minimum bounds that are imposed on the mill’s operating hours. In a pure NPV maximization approach, where the lower bound on mill hours does not exist, the optimizer would choose to send fewer quantities to the mill. This result can be seen after year 28, where the number of operating hours is drastically reduced. This result, however, is not ideal for long periods of time because of indirect costs that are incurred when not fully utilizing the mill. A minimum bound penalty is used to approximate these indirect costs when production dips below a specified threshold. As a result, the optimizer obeys the lower bound constraint by blending low- and high-grade materials. It is possible to test the sensitivity and risk associated with the deterministic-equivalent design using a set of geological simulations by taking the deterministic-equivalent decision variables (production schedule, destination policies and capital expenditures) and testing how the simulations react to the design. Figure 6A also shows the exceedance probabilities (P-10, P-50, P-90), which

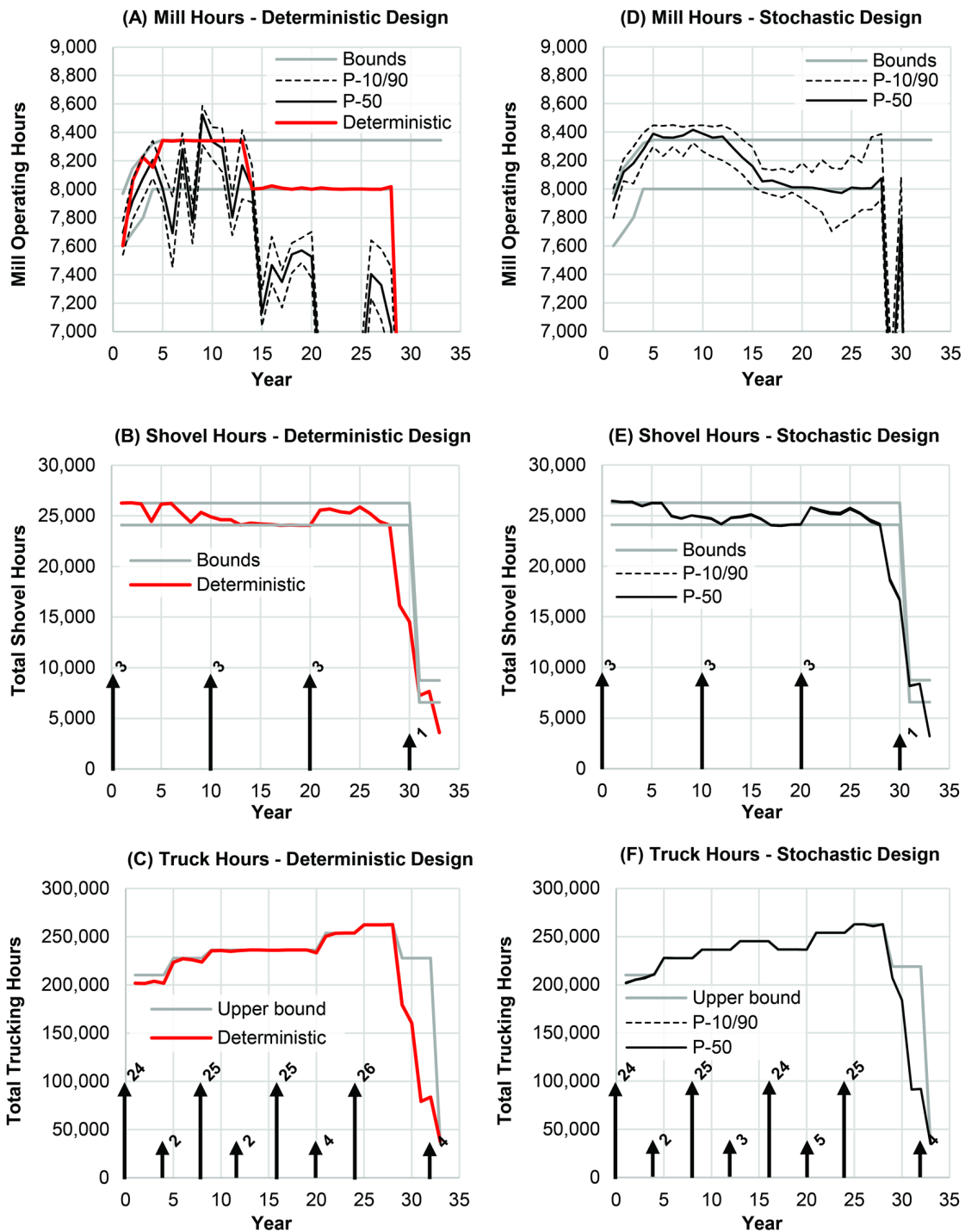


Figure 6: Comparison of risk profiles for the deterministic-equivalent and stochastic copper mining complex designs.

are used to quantify the risk for the design. Using the simulations, these probabilities represent the value for which 10%, 50% and 90% of the scenarios lie above in any given period. Unlike what the estimated model indicates with the deterministic design, there is a large amount of risk associated with the use of the mill. The risk analysis indicates that there are large fluctuations in the utilization of the mill during the first 15 years of production (where the estimated model indicates it is filled to capacity). After year 15, there aren't

sufficient quantities of material to feed the mill using the deterministic-equivalent destination policy (defined in terms of recoverable copper per mill hour). This indicates that the deterministic-equivalent design is a knife's edge solution, where it performs very well for the estimated model, but does not perform well when the uncertainty related to spatial locations, volumes and metal quantities are considered. Figure 6B shows the number of hours used by the shovels. In this example, the optimizer has chosen to use three shovels, and replaces the shovels every 10 years. It is clear that the optimizer is capable of staying within the bounds of their production capabilities for the life of the mine. Similarly, Figure 6C shows the number of truck hours for the deterministic design. It is noted that as the mine extracts increasingly deep material, the number of trucks increases accordingly to compensate for the increase in cycle times to bring material to surface.

A stochastic optimizer considers all geological simulations simultaneously to generate a single production schedule, destination policy and capital expenditure strategy that manages the risk associated with uncertainty. Figure 6D shows the risk profiles for the stochastic design for mill utilization. The optimizer is capable of fully utilizing the mill during the first 13 years, and the utilization begins to decline thereafter. Unlike the deterministic-equivalent solution (Figure 6A), the stochastic design is able to provide enough material to the mill to satisfy the minimum bound, which is a much more practical solution. Additionally, it is noted that the stochastic optimizer successfully manages the distribution of risk over time; at the beginning of the mine life, the distance between the P-10 and P-90 profiles is relatively small, with an average of 186 hours over the first 10 years, which implies that the stochastic design has a high probability of consistently feeding the mill and helps to guarantee early cash flows. Later in the mine life, the distance between the P-10 and P-90 profiles widens up to an average of 372 hours for the last 10 years, which is a result of the optimizer deferring the extraction of riskier material through time using the geological risk discount rate. This is an improvement over the risk analysis from the deterministic-equivalent design that randomly distributes risk over time, which is indicated by an average of 283 and 391 hours between the P-10 and P-90 values for the first and last 10 years, respectively. Not only is the stochastic design providing a more consistent feed to the mill, but the risk associated with the materials sent is also reduced. The shovel and truck production rates (Figures 6E and 6F, respectively) for the stochastic design are similar to those of the deterministic-equivalent solution (Figures 6B and 6C), however there are some minor differences. Notably, the stochastic optimizer chooses to extract more material between periods 12 and 16, which is indicated by the slight increase in shovel and truck hours. This is a result of the optimizer needing to uncover more material during these periods in order to provide enough material to remain above the mill's minimum bound in the later years of the mine's life. In order to achieve this, the optimizer decides to purchase an additional truck in year 12.

Given the inability to consistently feed the mill up to the desired capacity using the deterministic-equivalent design, and the high risk associated with the quantities sent, a risk analysis of the deterministic solution indicates a 1.7% lower NPV than the deterministic-equivalent design originally indicated (based on the P-50 value). This minor impact on the NPV, given the inconsistent feed, is a result of the simulations having a higher metal content above the cut-off grade than what the estimated (smoothed) model indicates. The stochastic design is not only able to provide a consistent feed of material to the mill and reduce the risk associated with the quantities sent, but is also able to attain a 5.7% increase in NPV over the deterministic design. While this is not a drastic increase, it is also necessary to also consider the fact that the mill is consistently fed with materials, thus does not incur the large overhead and opportunity costs that are incurred by under-utilizing the mill.

6 Conclusions

This chapter proposes a global optimization modelling and optimization methodology for open pit mining complexes that aims to manage risk in the production and processing of mined materials, and, additionally, the capital expenditures required to maximize the value of the operation. The generalized and flexible modelling procedure that is outlined permits the ability to model very large mining complexes with a high-degree of detail, including non-linearities that are typically ignored in practice due to computational and theoretical limitations of conventional mathematical optimizers. Previous formulations for mine production scheduling with uncertainty have attempted to manage risk around a fixed target, such as mine production and mill capacities; this a priori definition of capacities or bottlenecks leads to sub-optimal use of both the depletable

natural resource and the large investments required to produce and process material. By incorporating capital expenditure decisions in the model, the optimizer is able to control aspects such as optimal mine production rates and the timing of opening new processing streams that has previously been ignored in mine production scheduling models. The proposed method uses a combination of simulated annealing and particle swarm optimization to generate multi-mine production schedules, destination policies, capital expenditure strategies, and the use of the available processing streams in order to maximize the performance of the mining complex.

The method is tested on a large-scale, real-world copper mining complex, provided by an industrial partner. Experimental results indicate that the optimizer is able to successfully create a production schedule, destination policy and capital expenditure strategy that manages risk associated with fully utilizing a mill, and also simultaneously decides the timing of purchases or replacement of shovels and trucks. The result is a risk-based design with a 5.7% higher NPV than a deterministic-equivalent design that does not consider risk. Moreover, the stochastic design ensures the smooth operation of the mill. It is noted that, in this example, the complexity of the optimization formulation surpasses the capabilities of commercially available production scheduling methods, thus a comparison cannot be provided at the time of this study. Future work will seek to test the method in cases where both multi-mine production rates and sizing of the mill are considered, and to develop new methods and models that can be used as a basis of comparison for this method.

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