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Quality scores in reverse auctions: Motivations, information sharing and credibility

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Abstract: Non-price attributes such as prior relationship, product quality, and reliability can be more important than bidding prices for the buyers when selecting the winner of a reverse auction. In this regard, in open-ended reverse auctions (also known as buyer-determined auctions), buyers usually evaluate and assign *'quality score'* (QS) to each supplier. Although QS enables the buyer to incorporate the above-mentioned non-price attributes in its sourcing decisions, it may have repercussions on the outcome of a reverse auction. On one hand, the QS influences the relative positions of the suppliers against each other in terms of the buyer's sourcing preference. On the other hand, this very characteristics of QS may invalidate its role among the suppliers as the buyer may become tempted to abuse it in order to regulate the competition towards its own benefits. In order to analyze the tug of war between the operational and strategic benefits of QS in a reverse auction setting, in this paper, we develop a bi-level supply chain between a buyer and two competing suppliers. One of the two suppliers is an incumbent with known track record, whereas the other one is untested. Ceteris paribus, the buyer calculates the (relative) QS of untested supplier with respect to the known one, and decides whether or not (and if yes, how) to credibly share this with the two suppliers possibly via advance minimum revenue guarantees. Analyzing pooling, separating, and semi-separating equilibria of resulting signalling game between the buyer and the two suppliers, we develop insights on the impact of QS on the buyer-determined reverse (procurement) auctions. Our results suggest that such advance guarantees should be offered only to the incumbent supplier when the difference between suppliers' quality scores is relatively low. In addition, the degree of price competition among suppliers increases when the degree of information asymmetry between the upstream and downstream levels of supply chain regarding the quality scores is sufficiently low. Moreover, when the number of qualified entrants in the auction increases, suppliers' price competition and buyer's signaling cost increase, which suggests the buyers not to share the quality scores information. We have provided the managerial implications of our findings.

Key Words: Reverse (procurement) auction, quality score, asymmetric information, credibility, signaling.

1 Introduction

An increasing number of buyers nowadays use (electronic) reverse auctions to make their sourcing decisions mostly because it is believed to save buyers considerable amounts of money by lowering prices. For instance, Furey (2009) reported that in the early 2010's, e-sourcing through Ariba, a company specialized in online procurement services, saved companies 5% to 7% of their procurement costs. Similarly, General Electric (GE) alone claimed a saving of about $680 million and a net saving of more than 8% in 2001 by using SourceBid, a reverse auction tool and a part of GE's Global Exchange Network (GEN). In addition, the U.S. General Services Administration attributed savings of 12%-48% to the use of procurement auctions (Sawhney 2003). FreeMarkets, one of the leading online auction software providers, reported that its customers saved approximately 20% on more than $30 billion in purchases between 1995 and 2001 (Anderson and Frohlich 2001).

Reverse auctions in practice are held in different formats. One popular format of reverse auctions corresponds to a buyer-determined auction (Jap 2002), in which suppliers compete on price and after the bidding is over, the buyer considers non-price attributes such as product quality, reliability, and timely delivery fulfillment, and perhaps some other financial concerns such as switching and contractual costs, to award the contract. This auctioning format has been used for a wide range of products and services such as machined metal components, printed circuit boards, marketing services, and legal services (Furey 2009). One of the best known benefits of these auctions is the flexibility of the buyer in that she does not need to award the contract to the supplier with the lowest price (Anderson and Frohlich 2001, Jap 2002). However, this lack of commitment can reduce the degree of price competition among suppliers because a supplier does not have to offer the lowest bid to win the contract. Our findings in fact support the validity of this concern, especially when the degree of information asymmetry regarding the buyer's evaluation of non-price attributes is high.

One of the commonly used procedures for determining the winner of the contract in buyer-determined auctions is that the buyer evaluates each supplier across several non-price dimensions and assigns to him a score that represents her estimation of suppliers' expected performance in non-price attributes, and then, adjusts the bidding prices of the suppliers using these scores. The supplier with the lowest final score (which includes both price and non-price attributes) is then awarded with the order.[1] Hence, for the purpose of this paper and in accordance with what is being done in practice, we group all the non-price attributes and label them as *quality* (Tunca et al. 2014, Engelbrecht-Wiggans and Katok 2007, and Haruvy and Katok 2013 use a similar approach). We also call the unique score for non-price attributes as *Quality Score (QS)* and denote the final adjusted price by *Generalized Price* (GP).

There is always an inherent uncertainty with respect to the buyer's understanding and evaluation of suppliers' quality scores, especially in buyer-determined procurement auctions. This uncertainty logically comes from two main sources: (1) lack of knowledge regarding the buyer's choice of critical no-price factors as well as her relative emphasis on different factors (attribute choice uncertainty), and (2) lack of knowledge on the procedure that the buyer takes to evaluate suppliers' score on each factor (procedural uncertainty). Depending on the degree of information asymmetry between buyer and suppliers, QS can be decomposed into a combination of known and hidden factors (KF, and HF, respectively) as follows:

$$QS = \lambda \times KF + (1 - \lambda) \times HF$$

Note that both forms of QS uncertainties can be represented with the above structure, where λ determines the visibility of buyer's evaluation scheme on QS to the suppliers. If, for instance, $\lambda = 1$, suppliers have no uncertainty on the buyer's selection of attributes as well as the specific procedure employed for evaluating QS. On the other hand, if $\lambda = 0$, the suppliers lack visibility because neither do they have access to the attributes on which QS is based on nor do they know how QS is calculated. Therefore, in the above expression, the

[1] According to a case study written by ITBID - a company specialized in e-sourcing services- (2012), in practice, the buyer usually assigns a quality score (QS) to each supplier based on her estimation of their capabilities in some critical non-price factors. The "considered price" is calculated by considering both "actual price" and "quality score". The supplier with the lowest *considered price* will be offered the contract.

relative value of $0 \leq \lambda \leq 1$ determines the suppliers' inability to predict the true value of QS that will be employed by the buyer to adjust the bid prices.[2]

Our results suggest that under asymmetric information scenario (i.e., $\lambda = 0$), since the suppliers decide on their prices solely based on their prior beliefs, this natural uncertainty can result in a significant reduction (resp. increase) in competition degree (resp. offered prices) in upstream channel level. Therefore, the buyer may be encouraged to fully or partially share the QS information with the suppliers (i.e., shift the value of λ towards 1). However, there are two important concerns that may preclude this information sharing.

The first concern is the credibility of the information sharing in a competitive supply chain environment. As proven in various studies, any type of information asymmetry among supply chain parties can create an incentive for information distortion in order to influence the decisions of uninformed parties. It is easy to observe that in a buyer-determined auction, the buyer has an incentive to signal distorted quality scores to force the actual winner to decrease his price due to the fear of losing the auction. This in turn makes the uninformed parties doubt in credibility of provided information. As a result, the buyer needs to take some costly actions in order to make the provided information credible for the suppliers. The next concern is just the opposite to the rationale behind buyer's sharing of QS information. Indeed, there is a fear that truthfully sharing the QS information can turn into a competitive disadvantage. For instance, Haruvy and Katok (2013) identifies situations in which decreasing visibility (increasing uncertainty) leads to more price competition in the auction which leads to lower costs for the auction holders. The reason is that if the suppliers know the true information, they can identify situations when they can win the auction by undercutting other suppliers' prices. In such situations, they intentionally reduce their price only to a level that ensures their success in the auction.

Even though the mechanism of buyer-determined auctions has been extensively explored in the literature, to the best of our knowledge, the credibility of QS information sharing and its impact on the upstream competition degree have not received attention in the reverse auction studies. Therefore, the objectives of this paper are threefold: (i) to examine *when* and *how* the QS information can be credibly shared from the buyer to the suppliers in buyer-determined reverse auctions; (ii) to identify how it influences the degree of price competition among the suppliers; (iii) to evaluate the impact of QS information sharing on decisions, profits, and cost of channel parties.

In order to address these issues, we develop a competitive bi-level supply chain model in which the downstream party uses a buyer-determined reverse auction for procurement. At the upstream level, there are two competing heterogeneous sellers (hereinafter refereed to as supplier N and supplier U) with different cost and quality specifications.[3] First, the buyer evaluates non-price attributes and assigns quality scores (QS) to the participating suppliers. The supplier N is the incumbent supplier, hence, his QS is publicly known. On the other hand, supplier U is the entrant supplier, whose QS is private information for the suppliers. Here, the buyer may signal the true value of QS via an advance minimum revenue guarantee. Afterwards, suppliers compete on prices and submit their bids (quotations). Finally, after assessing the final score/rank for each supplier from both their bids and the quality scores, the buyer makes her final order allocation decision.

We characterize equilibrium decisions and profit/costs of all the parties in the channel under both symmetric and asymmetric information setting (pooling, separating, and semi-separating). The comparison between pooling and separating equilibria allows us to evaluate when and how the QS information can be credibly shared from the buyer to the suppliers and how it affects the degree of price competition between suppliers and the profits and costs of channel parties.

First of all, the suppliers' incomplete information (either on the buyer's choice of non-price attribute or on the measurement procedure) leads to an uncertainty on their side regarding what price to charge, which,

[2]This notion of QS is widely used in keyword auctions (a generalized type of forward auctions) as well, as a way to incorporate non-price attributes in the bid of advertisers (Geddes 2014). Historically, in 2002, Google first introduced an auction design that ranks advertisers by adjusting their bid prices with click-through rates (CTR). Later, Google introduced both attribute choice uncertainty by defining ambiguous non-price factors such as the quality of the advertisement text, and procedural uncertainty by making the ranking formula hidden. Over the years, many search engines follow the footsteps of Google and this trend reduced the value of λ, which in turn increased the uncertainty of quality scores from the perspective of advertisers. Our results can provide some explanation for this phenomenon.

[3]Throughout the paper, we use feminine and masculine pronouns for the buyer and suppliers, respectively.

in turn, generates an incentive for the buyer to distort her QS information and invalidates the credibility of the provided information. We show that this distortion can be fixed via an advance minimum revenue guarantee *only if* it is offered to the incumbent supplier (supplier N) as long as the relative difference between quality scores of suppliers U and N is not too low. Secondly, in the symmetric information setting, when one of the suppliers undercuts his opponent's price, he does so by taking into account the true value of his opponent's QS. However, in the presence of incomplete QS information, the suppliers cannot credibly infer the true information, which enforces them to decide on their prices only based on their a priori beliefs. Our results show that the degree of price competition (resp. equilibrium unit prices) in the upstream level is a decreasing (resp. an increasing) function of the degree of the channel information asymmetry. The comparison between pooling and separating equilibria suggests that offering advance revenue guarantees to the incumbent supplier can indeed help the buyer to motivate higher competition levels between suppliers when the degree of information asymmetry is high. In contrast, when the degree of information asymmetry is sufficiently low, the maximum level of suppliers' competition will be obtained under asymmetric information (even higher than that under symmetric information setting). Next, the comparison of cost and profits under both pooling and separating equilibria enables us to evaluate when credible QS information sharing is more preferred [and when it is not] for each supply chain partner. Finally, the analysis of a general case with $n > 1$ suppliers with known QS and $m > 1$ new entrants with unknown QS allows us to assess how an increase in the number of participants in the auction can intensify both the price competition and the cost of credible information sharing.

This paper is organized as follows: in the next section, we review the related literature. In §3, we develop the model framework. In §4 and §5, we analyze the symmetric and asymmetric information settings. In §6, we evaluate the impact of information sharing on the equilibrium decisions, cost, and profits of the channel partners. As an extension, in §7, we analyze the reverse auction under the presence of multiple incumbent and entrant suppliers. We conclude in §8.

2 Related literature

Our work fits within the broad range of sourcing and e-sourcing literature. We refer the readers to Elmaghraby (2000) for a survey of the sourcing literature and to Elmaghraby (2004) and Elmaghraby (2007) for overviews of online markets and procurement auctions in practice. A recent review of the e-auctions literature is presented in Gupta et al. (2009). More specifically related to our work, in the heart of the procurement literature, there is an extent stream of research on understanding the mechanism of reverse auctions in terms of incorporating non-price attributes. For instance, Santamaría (2015) compares the performance of a scoring auction, in which suppliers compete on the adjusted bids or scores, with a buyer-determined auction, in which suppliers compete on the price, and the buyer adjusts a certain number of the bids with the non-price attributes after the auction to determine the winner; and argues that the choice of procurement auction depends mainly on the cost advantage of the incumbent and availability of non-price attributes. Chen-Ritzo et al. (2005) experimentally explores the performance of a multi-attribute auction where bidders can specify both a price and levels of non-price attributes (quality and lead time) and compares it with a price-only auction mechanism. Chen et al. (2005) incorporated transportation costs as another non-price attributes into reverse auctions. In fact, the choice of non-price attributes may differ based on the priority that the buyer puts on different economic or quality considerations.

Most of the papers in this stream focus on understanding the effect of auction mechanism on the buyer surplus and on the incumbent and entrant suppliers' actions in the bidding process. For instance, Tunca et al. (2014) empirically study multi-dimensional open-ended (buyer-determined) auctions. They provide evidence that what may be perceived as incumbency bias in reverse auctions can in fact be a revelation of preference for quality. Zhong and Wu (2006) use data from auctions in the high-tech industry to study bidding behavior in buyer-determined auctions, when the buyer has preferred and non-preferred suppliers due to the existence of non-price attributes. They find that preferred suppliers are more likely to win a contract than non-preferred suppliers, and that final bids from preferred and non-preferred suppliers differ significantly. They argue that these differences are consequences of the non-price attributes, which play a crucial role not only in the buyer's final decision, but also in the bidding strategies of the suppliers. Similarly, Santamaría (2015) shows that in buyer-determined auctions, suppliers markup their final bids depending on the distribution of the non-price attributes, and the markup depends on the type of supplier (entrant or incumbent). In line with the above papers, we provide an auctioning framework which enables us to characterize the difference between bidding prices of incumbent and entrant suppliers and the buyer's preference on credible information sharing given the distribution of non-price attributes (QS in our paper).

Given the essential role of quality score in our model, this paper is also related to the stream of research on keyword advertising auctions in which the auction-holder assigns a quality score to each bidder in order to capture critical non-price features (Liu and Chen 2006, Feng et al. 2007, Weber and Zheng 2007, Chen et al. 2009, Liu et al. 2010). For instance, Liu and Chen (2006) consider a static weighted unit-price auction where bidders bid on unit prices, and the winner is determined by their bids as well as their past performance. Chen et al. (2010) extends Liu and Chen's model by considering the dynamic effects of bidder performance evolution. Amaldoss et al. (2015) address some limitations of generalized second-price (GSP) auction for selling advertising slots and evaluate the performance of other new mechanisms used in practice (first-page bid estimate -FPBE- mechanism first developed by Google). To the best of our knowledge, no paper in this stream addresses the problem of credible quality score information sharing. Even though we do not consider a forward keyword auction in this analysis, our results is consistent to the actions taken in the practice with regard to the quality score information of the auctioneer.

Our paper is also related to the stream of research on information asymmetry in supply chain. The SCM literature on the effects and implications of information asymmetry can be divided into two general groups. A group of studies (Corbett and De Groote 2000, Ha 2001) focus on information asymmetry in production cost of the suppliers, and the other group (Cachon and Lariviere 2001, Li and Scheller-Wolf 2011, Wang et al. 2014, Gümüş 2014) focus on the demand forecast information asymmetry. In contrast, to the best of our knowledge, this work is the first one that focuses on the information asymmetry on the suppliers' quality scores evaluated by the buyer and credible information sharing in buyer-determined reverse auctions. In our analysis, we use a signaling game framework to model an informed principal who utilizes the advance revenue guarantee with the aim of credible QS information sharing (see Riley 2001 for extensive literature review on signaling games).

In practice, QS information is privately held by the downstream member of the supply chain. The buyer may use her information to decrease her costs even at the expense of the other parties. Hence, the suppliers are likely not to trust the provided information. This possibility impedes credible information sharing. Recent studies provide strong evidence that information asymmetry can have significant impacts on the auction's results and certainly affects the buyer surplus (Haruvy and Katok 2013, Mithas and Jones 2007). For instance, Haruvy and Katok (2013) using experimental analyses show that in on-line procurement auctions with open-bid format in which suppliers bid on price, but exogenous bidder quality affects winner determination, the buyer surplus significantly decreases when the information about bidders' quality is public. In line with this paper, we show that there is an optimal level for information asymmetry, and if the uncertainty increases above a threshold, the buyer surplus will be hurt.

The next important stream of research related to our study is about advance commitments in the supply chain. Similar to our use of advance guarantee for signaling quality score information, Klotz and Chatterjee (1995) use an advance quantity guarantee to the incumbent supplier in exchange for participating in the procurement auction. Cachon (2004) compares inventory risks under push, pull ,and advance purchase discount contracts between suppliers and retailers. Özer and Wei (2006) structure an advance purchase contract that enables credible forecast information sharing between a manufacturer and a retailer. Boyaci and Özer (2010) study when to start and stop advance selling in order to acquire enough demand information from retailers for capacity planning by a manufacturing company. Advance commitments also can be from a retailer to the consumers. Tang et al. (2004) analyze the application of advance purchase discounts from a retailer to the consumers. Yu et al. (2014) study the role of advance selling commitment from a seller to consumers in signaling product Quality. Similar to these works, we examine to find out whether the buyer can use advance revenue guarantees to signal her private information before holding the auction.

3 Model framework

In order to investigate the impact of information sharing in buyer-determined reverse auctions, we model a stylized decentralized supply chain consisting of one buyer and two suppliers. One of the suppliers (supplier N) is well known to the buyer as a local (incumbent) supply source, whereas the other one (U) is untested for the buyer.

With the aim of choosing the main supply source, the buyer considers a simple reverse (electronic) auction among the suppliers, in which the suppliers are asked to offer their bidding prices. She assigns a private quality score (QS) to each supplier based on her estimation of their capabilities in some critical factors such as product quality, reliability, and timely delivery fulfillment. As we will see below, the quality scores enable the buyer to incorporate *non-price* related factors in the deliberation decision of the winner. Let QS_i denote the quality score assigned by the buyer for the supplier i, where $i \in \{N, U\}$. For the sake of simplicity and without loss of generality, we normalize QS for supplier N (QS_N) to 1 and for supplier U (QS_U) to α, where $0 < \alpha \leq 1$. Since QS is of critical value in deciding the winner among the suppliers along with their bidding prices, we assume that the exact value of α is known only to the buyer. On the other hand, the suppliers hold a-priori belief on α denoted by F_α. For the sake of tractability, we assume that F_α is a uniform distribution function $\mathcal{U}$ defined between $\underline{\alpha}$ and $\bar{\alpha}$, where without loss of generality, $0 < \underline{\alpha} \leq \bar{\alpha} \leq 1$. Note that the width of the range $[\underline{\alpha}, \bar{\alpha}]$ denoted by $\Delta = \bar{\alpha} - \underline{\alpha}$ represents the degree of information asymmetry between the buyer and the suppliers.

Upon receiving the bid prices, the buyer then calculates a generalized price $GP_i(QS_i, p_i)$ for each supplier i. Since the higher the quality score or the lower the bid price is, the more likely the supplier would win the auction, this imposes certain restrictions on the sensitivity of $GP_i(QS_i, p_i)$ with respect to its arguments in the sense that it should increase and decrease in QS_i, and p_i, respectively. In this paper, we assume a fractional form[4] for $GP_i(QS_i, p_i) = \frac{p_i}{QS_i}$. First, note that it satisfies the above sensitivity conditions. Second, since GP_i is generalized price for each supplier and supplier N's quality score is assumed to be 1, it implies that in order for supplier U to win the contract against supplier N, he should offer a price that is strictly less than $\alpha \times 100\%$ of supplier N's bidding price p_N.

In addition to the quality scores, the suppliers also differ in terms of their cost structures. Let c_i denote the marginal cost of supplier i. Throughout the paper, we assume $c_U < c_N$, and discuss briefly how our results change otherwise. Also, we consider that there is a cap, i.e., *reserve price* on the bid prices offered by the suppliers. We denote the reserve price by p_r and assume that it is greater than c_N. One can interpret the reserve price as the maximum price the buyer is willing to pay to the suppliers or price in the spot market to which she has always access. To sum up, the buyer procures from the supplier whose generalized price GP_i is less than that of the other supplier GP_{-i} and reserve price p_r, where i and $-i \in \{U, N\}$ and $i \neq -i$. Also, in order to focus on the main research questions related to the quality score, we assume that buyer's demand is deterministic and equal to Q units.

The timing of decisions and events is shown in Figure 1 and provided as follows.

1. Buyer assigns QS_i to supplier i.
2. Buyer decides whether or not to share the quality score information with the suppliers.
3. In response to the buyer's decision, suppliers update their prior beliefs and simultaneously submit their bids (unit prices).
4. Based on the unit prices and quality scores of the suppliers and the reserve price, the buyer decides on the supplier who wins the order allocation and satisfies the end-consumer demand Q.

Before we start the analysis, we summarize the list of notations used for parameters and decision variables in the paper in Table 1.

Throughout the paper, equilibrium profits/costs and decision variables are annotated with asterisks.

[4] Note that since what matters the most is the ranking of generalized prices between two suppliers, considering the different functional forms for GP-function does not alter our results. However, this specific functional form is also in accordance with the actions in practice (refer to Anderson and Frohlich 2001).

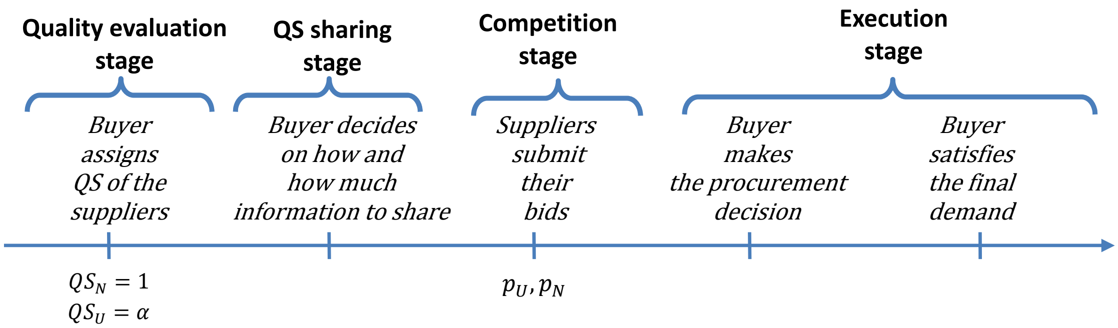


Figure 1: Timeline of decisions and events.

Table 1: Notation used for model parameters and decision variables.

Model parameters	
$c_U; c_N$	Marginal cost of suppliers U and N, respectively.
α	The quality score for U (QS for N is normalized to one)
$\mathcal{U}[\underline{\alpha}, \bar{\alpha}]$	Suppliers prior belief (uniform distribution) regarding α
π_U, π_N	Expected profit of the suppliers U and N, respectively.
TC_B	Expected cost of the buyer
Q	Total demand
p_r	Reserve price (spot market price)
Decision variables	
η_U, η_N	Advance order guaranteed to the suppliers U and N, respectively, at a fixed price (p_r).
$p_U; p_N$	Unit prices quoted by the supplier U and N, respectively.
$GP_U; GP_N$	Generalized prices calculated for the supplier U and N, respectively.
$q_U; q_N$	Buyer's order allocation decisions for the supplier U and N, respectively.

4 Symmetric information: Benchmark

To establish a benchmark, in this section, we consider the case where the true value of the quality score α is known to all the parties in the supply chain. The problem, therefore, transforms to a symmetric information Stackelberg game between two price-competing suppliers and the buyer. Given the suppliers' bid prices and the buyer's reserve price, it is then straightforward to show that the order would go to supplier i if and only if $GP_i < \min(p_r, GP_j)$, where $i \neq j$. Plugging this into the buyer's cost function TC_B, we can express:

$$TC_B(p_N, p_U, \alpha, p_r) = \begin{cases} Q \times p_N & p_N \leq \frac{p_U}{\alpha}, p_N \leq p_r \\ Q \times p_U & p_N > \frac{p_U}{\alpha}, p_U \leq p_r \\ Q \times p_r & otherwise \end{cases} \quad (1)$$

Given the order allocation and the bid price from supplier j, the best response for the supplier i is to bid in such a way that his GP_i is infinitesimally less than GP_j offered by his competitor provided that his bid price is above his marginal cost and below the reserve price. Hence, the best response function for suppliers N and U can be expressed as follows:

$$p_N^*(p_U) = min(max(c_N; \frac{p_U}{\alpha}); p_r) \quad (2)$$

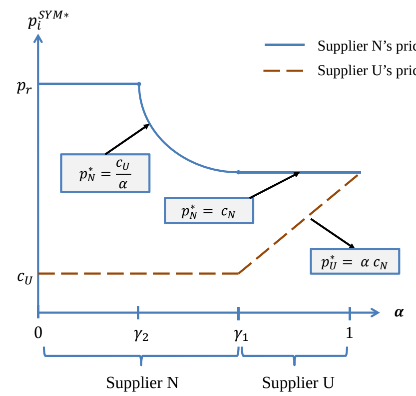
$$p_U^*(p_N) = min(max(c_U; \alpha p_N); p_r) \quad (3)$$

Solving the best response functions simultaneously leads to the equilibrium provided in the following proposition (proofs for all propositions are provided in the appendix).

Proposition 1 *Let $\gamma_1 = c_U/c_N$ and $\gamma_2 = c_U/p_r$ (where $\gamma_2 < \gamma_1$ as $p_r > c_N$). Under symmetric information equilibrium, the buyer orders from supplier U if $\alpha > \gamma_1$, and from supplier N if $\alpha \leq \gamma_1$. For when $\alpha < \gamma_2$, buyer orders from N at the reserve price (p_r). Complete characterization of equilibrium decisions, profits and costs for the supply chain parties under symmetric information is provided in Table 2.*

Table 2: Equilibrium decisions, profits, and cost under symmetric information.

Regions		$0 < \alpha < \gamma_2$	$\gamma_2 < \alpha < \gamma_1$	$\gamma_1 < \alpha < 1$
Prices (bids)	p_N^*	p_r	$\frac{c_U}{\alpha}$	c_N
	p_U^*	c_U		$c_N\alpha$
Order Alloc.	q_N^*, q_U^*	$Q, 0$		$0, Q$
Suppliers'	π_N^*	$Q(p_r - c_N)$	$Q(\frac{c_U}{\alpha} - c_N)$	0
Profits	π_U^*	0		$Q(\alpha c_N - c_U)$
Buyer's Cost	TC_B^*	Qp_r	$Q\frac{c_U}{\alpha}$	$Qc_N\alpha$



Note that when α is relatively high ($\gamma_1 < \alpha$), supplier U becomes the sole supplier for the buyer and sets a price p_U^* such that his quality-score adjusted unit price ($\frac{p_U}{\alpha}$) is infinitesimally less than supplier N's marginal cost c_N. Letting $\frac{p_U}{\alpha} = c_N$ and solving for p_U^* would yield the equilibrium price for supplier U. Likewise, when α is between γ_1 and γ_2, supplier N can set p_N^* in a similar fashion. Finally, when α is less than γ_2, supplier N always wins by charging infinitesimally less than the reserve price. In Table 2, one important note is that the supplier U and N's prices and profits are increasing and decreasing[5] in α, respectively, as stated in the following Corollary 1.

Corollary 1 *Under symmetric information setting,*

1. *the equilibrium bidding price and expected payoff of supplier N (p_N^*, π_N^*) are decreasing in α, while those of supplier U (i.e., p_U^* and π_U^*) are increasing in α.*
2. *information setting, by an increase in c_U for a fixed c_N, supplier U weakly decreases his unit price, while supplier N (weakly) increases his unit price.*

The first part of this corollary in fact explains the true meaning of relative QS ($\frac{QS_U}{QS_N} = \alpha$) in the buyer-determined reverse auctions, which basically means that by an increase in α, indeed, the buyer's preference in doing business with supplier U increases and, hence, supplier U can increase his bid and enjoy more expected profit in the auction. On the other side, supplier N's price and profit decreases as α increases. Therefore, α plays a very important role in determining not only the winner, but also the profit that they can make.

The second part of the corollary addresses the effect of relative cost efficiency of the supplier U on the equilibrium bid prices. The more cost-efficient supplier U is , the higher he would charge against his competitor.

5 Asymmetric information

In this section, we analyze the equilibrium under asymmetric information setting where the true value of α is known only to the buyer. We also assume that any information received from the buyer is non-verifiable by the suppliers. Non-verifiability refers to the situation where the suppliers update their prior belief only if the buyer has no incentive to manipulate the signal. Hence, in order to credibly share the true value of α with the suppliers, the buyer would need to provide a costly signal along with his information. In this paper, we consider a commonly used commitment contract called "minimum revenue guarantee". Basically, this contract enables the buyer to insure a supplier with a minimum level of revenue even if the guaranteed supplier loses the auction. We assume this guarantee is through a pre-determined proportion of total demand Q at a fixed external price that can be set in a negotiating process. Without loss of generality, we fix this

[5]Throughout the paper, we use "decreasing" and "increasing" in their weak senses unless otherwise is stated.

external price to p_r for both suppliers so that we can characterize the guarantee only by the proportion of the demand guaranteed. Let η_i be the guaranteed portion of the demand to the supplier i, $i \in \{N, U\}$. That means, the buyer commits to order a minimum of $\eta_i \times Q$ units to the supplier i in such a way that his revenue is higher than a pre-determined threshold. To clarify, if for instance, the advance guarantee contract is to be offered only to the supplier N, the buyer guarantees the revenue of $\eta_N Q p_r$ to him even if he loses the auction; therefore, in case if he loses the auction, the buyer orders $\eta_N Q$ to the supplier N at the negotiated price and procures the rest of the supply $(1 - \eta_N)Q$ from supplier U at p_U or from the spot market at p_r depending on the result of the auction; however, supplier N obtains the entire order of Q at p_N if he wins the auction ($p_N \leq p_r$; $p_N \leq \frac{p_U}{\alpha}$). Note that although the advance guarantees in practice are usually offered to the incumbent suppliers (supplier N in our model), we also consider the possibility of offering the guarantee to the unknown supplier U for the buyer in order to have a more comprehensive analysis.

The minimum revenue guarantee can be costly to the buyer if the chosen supplier (the supplier whom the buyer will offer the guarantee) fails the competition. Hence, choices of signaling tool and η_i ($i = N$ or U) should intuitively reflect the buyer's prediction of winning chances of suppliers. As in the above example where the guarantee is offered to supplier N, a buyer may be able to signal a low (resp. high) value of α by a high (resp. low) revenue guarantee η_N to supplier N to indicate that supplier N has higher (resp. lower) chance of winning the auction. Next, we examine when the advance guarantee to the suppliers can provide information about α to them.

Given a minimum revenue guarantee contract, the sequence of events is as follows: (1) Based on her private information on α, the buyer chooses the supplier that she wants to offer the guarantee ($i = U$ or N), and guarantees a revenue of $\eta_i Q p_r$ to him. (2) suppliers update their prior belief about the true value of α (recall that suppliers a priori believe that α is uniformly distributed between $\underline{\alpha}$ and $\bar{\alpha}$). (3) Suppliers decide on their bid (p_U and p_N) simultaneously. (4) The buyer calculates suppliers' generalized prices (GP), and then decides on order allocation based on suppliers' ranks and the realized reserve price. Recall that the buyer has to satisfy the entire demand (Q), and the ordering does not depend on the suppliers' bids or the quality scores. Four decisions need to be made in this asymmetric information game: the buyer's choice of signaling tool (whether N or U), the minimum revenue (the percentage of total quantity $\eta_i Q$ when the price is fixed) to be guaranteed to the chosen supplier, suppliers' bids, and the final procurement decision (order allocation).

The above sequence leads to a signaling game (Fudenberg and Tirole 1991). The buyer knows the true value of α and makes an advance guarantee minimizing her expected cost. The suppliers, hence, may be able to infer the buyer's private information from her offered guarantee. Full information sharing (exact inference) is possible only when a buyer with α guarantees an advance order of η_i that is different from η_i' of any other buyer with $\alpha' \neq \alpha$. Such equilibrium is referred to as a *separating* equilibrium. In contrast, the suppliers gain no new information if the buyer always offers the same revenue guarantee regardless of its private information α. Such equilibrium is referred to as a *pooling* equilibrium. Additionally, partial information sharing (non-exact inference) is also possible when a buyer with any type of α such that $\alpha \in [\alpha_L, \alpha_H]$ guarantees the same amount in advance η_i, while other types choose different signals such as $\eta_i' \neq \eta_i$. Such equilibrium is referred to as a *semi-separating* equilibrium.

Let i ($i = N$ or U) be the selected supplier that the buyer will offer a guarantee of η_i and $-i$ denotes the other supplier. If supplier i is guaranteed to be offered a minimum revenue of $\eta_i Q p_r$ in the auction, he will naturally increase his minimum bid from c_i to a price p_i^{low} in which he is indifferent between losing or winning the auction. Recall that suppliers always compete to take the order even at their lowest possible price. Hence, the only difference with the symmetric information setting is that their minimum price in the auction has increased from c_i to p_i^{low}. The increase in minimum price for the guaranteed supplier directly depends on the guaranteed revenue.

$$p_i^{low} = c_i + \eta_i(p_r - c_i) \text{ for } i = U \text{ and } N$$

To clarify with extreme examples, if $\eta_i = 1$, supplier i increases his price to p_r to make sure that he will get the whole order at p_r (the maximum allowed price); but, if $\eta_i = 0$, he has no advantage over the other supplier and should compete to take the order even at c_i, similar to symmetric information case. Obviously, there is no increase in the minimum price of the other non-guaranteed supplier ($p_{-i}^{low} = c_{-i}$), and he is willing to take the order even at his marginal cost (c_{-i}).

Given an advance guarantee offer to supplier i, the expected total cost for the buyer under asymmetric information is

$$TC_B(\eta_i, \alpha, p_N, p_U) = \begin{cases} Qp_i & \frac{p_i}{QS_i} \leq \frac{p_{-i}}{QS_{-i}}; p_i \leq p_r \\ (1-\eta_i)Qp_{-i} + \eta_i Qp_r & \frac{p_i}{QS_i} > \frac{p_{-i}}{QS_{-i}}; p_{-i} \leq p_r \\ Qp_r & otherwise \end{cases} \tag{4}$$

Recall that $QS_N = 1$ and $QS_U = \alpha$. In the buyer's cost function, first case refers to the situation when the guaranteed supplier wins the auction at p_i while the second case refers to the case when the chosen supplier fails in the competition. In the case of failure, the guaranteed supplier will have an order of $\eta_i Q$ at p_r and the other supplier takes the rest of demand at his proposed unit price. Third case refers to all other situations when the buyer has to procure all the demand at the market price p_r. In that case, all the demand will be ordered to supplier i or only $\eta_i \times 100$ percent of it to him and the rest to be satisfied from the spot market.

The suppliers' expected profits after observing the reserve price are as follows:

$$\pi_i(\eta_i, \underline{\alpha}, \bar{\alpha}, p_{-i}) = Q \times Emax_{p_i \leq p_r}\left((p_i - c_i)Prob(i\ wins); \eta_i(p_r - c_i)Prob(-i\ wins)\right) \tag{5}$$

$$\pi_{-i}(\eta_i, \underline{\alpha}, \bar{\alpha}, p_i) = (1-\eta_i)Q \times Emax_{p_{-i} \leq p_r}\left((p_{-i} - c_{-i})Prob(-i\ wins); 0\right) \tag{6}$$

where $Prob(i\ wins) = Prob(\frac{p_i}{QS_i} \leq \frac{p_{-i}}{QS_{-i}})$ and $Prob(-i\ wins) = Prob(\frac{p_i}{QS_i} > \frac{p_{-i}}{QS_{-i}})$. The expectations in both cases are with respect to α. The second term in π_i ensures that the supplier i ($i = UorN$) competes to win the auction only if he would get a payoff higher than the minimum guaranteed profit. Therefore, he never bids a price lower than p_i^{low}.

In the following two subsections we analyze the pooling and separating (and semi-separating) equilibria of the signaling game.

5.1 Pooling equilibrium: No information sharing

In a pooling equilibrium, the buyer provides the same guarantee $0 \leq \eta_i \leq 1$ to the chosen supplier $i = U$ or N regardless of the true value of α. Therefore, suppliers cannot update their belief on α; i.e., $(\alpha \mid \eta_i) \sim \mathcal{U}[\underline{\alpha}, \bar{\alpha}]$. In this situation, suppliers' bidding prices will be solely based on their prior belief.

Note again that p_i^{low} is the minimum price that a supplier ($i = U, N$) bids because we assume that for a lower price than this threshold the supplier prefers to lose the auction. Also, they will not go beyond the known reserve price p_r because otherwise they lose the auction.

The following lemma helps us in characterizing the equilibrium outcome of the game when suppliers cannot update their beliefs.

Lemma 1 *Under the least costly pooling equilibrium (i.e. $\eta_i = 0$ and $p_i^{low} = 0$), given the order allocation policy and the suppliers' belief that α is uniformly distributed between $\underline{\alpha}$ and $\bar{\alpha}$, the simultaneous price competition between suppliers leads to either of the following five different equilibrium points:*

- *PE-1: Internal solution of* $p_N^{Int} = \frac{\bar{\alpha}c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha}c_N c_U}}{4\underline{\alpha}}$ *and* $p_U^{Int} = \frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha}c_N c_U} + 4\underline{\alpha}c_U)}{8\underline{\alpha}}$.
- *PE-2: Boundary solution of* $p_N = p_r$ *and* $p_U = (\bar{\alpha}p_r + c_U)/2$
- *PE-3: Boundary solution of* $p_N = c_N$ *and* $p_U = (\bar{\alpha}c_N + c_U)/2$
- *PE-4: Boundary solution of* $p_N = \sqrt{\frac{c_N c_U}{\underline{\alpha}}}$ *and* $p_U = c_U$
- *PE-5: Boundary solution if* $p_N = p_r$ *and* $p_U = c_U$

Knowing the suppliers' optimal pricing policy, we can now investigate the existence of sustainable pooling equilibrium in different ranges of α. Note that in order for a pooling equilibrium to be sustainable η_N and η_U must be independent of realization of α, i.e. $\eta_i^*(\alpha) = \eta_i^*$ for $i = N, U$ for any $\alpha \in (\underline{\alpha}, \bar{\alpha})$. Besides, since there are multiple potential pooling equilibria, we only focus on the one that is *least costly* from the buyer's perspective. The following proposition characterizes all sustainable pooling equilibria of the game in each region.

Proposition 2 *Depending on the values of $\underline{\alpha}$ and $\bar{\alpha}$, the pooling equilibrium can be characterized in 5 different cases as below. Complete characterization of equilibrium decisions, profits and costs for the supply chain parties under asymmetric information for each case is provided in Table 3.*

Case PE_1: i.e. $\underline{\alpha} \leq \frac{((c_U + c_N\bar{\alpha}))}{2c_N}$,$\underline{\alpha} \leq \frac{c_N\bar{\alpha}^2}{c_U}$, and $\underline{\alpha} \geq \frac{c_N(c_U+\bar{\alpha}p_r)}{2p_r^2}$. The supplier's optimal prices will follow PE-1 (mentioned in Lemma 1). Let $\gamma_1 = (1/2)\frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U} + 4\underline{\alpha} c_U)}{(\bar{\alpha} c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U}}$. The buyer orders from supplier N if $\alpha \leq \gamma_1$, and from supplier U if $\alpha > \gamma_1$.

Case PE_2: i.e. $\underline{\alpha} \leq \frac{c_N(c_U+\bar{\alpha}p_r)}{2p_r^2}$ and $\underline{\alpha} \leq \frac{c_N\bar{\alpha}^2}{c_U}$. The suppliers' optimal price would follow PE-2. Let $\gamma_2 = (1/2)\frac{(\bar{\alpha}p_r + c_U)}{p_r}$. Supplier N gets the order at p_r if $\alpha \leq \gamma_2$, while supplier U would be considered as the main source if $\alpha > \gamma_2$.

Case PE_3: i.e., $\underline{\alpha} \geq \frac{((c_U + c_N\bar{\alpha}))}{2c_N}$. Under this equilibrium, the unit prices offered by suppliers N and U follow PE-3. In this region, the buyer always orders from supplier U .

Case PE_4: i.e. $\underline{\alpha} \geq \frac{c_N\bar{\alpha}^2}{c_U}$ and $\underline{\alpha} \geq \frac{c_U c_N}{p_r^2}$. Under this equilibrium, unit prices offered by suppliers N and U follow PE-4. Supplier N always wins the auction at this case .

Case PE_5: i.e. $\underline{\alpha} \geq \frac{c_N\bar{\alpha}^2}{c_U}$ and $\underline{\alpha} \leq \frac{c_U c_N}{p_r^2}$. Under this equilibrium, unit price offered by suppliers N and U follow PE-5. Supplier N always wins the auction at this case.

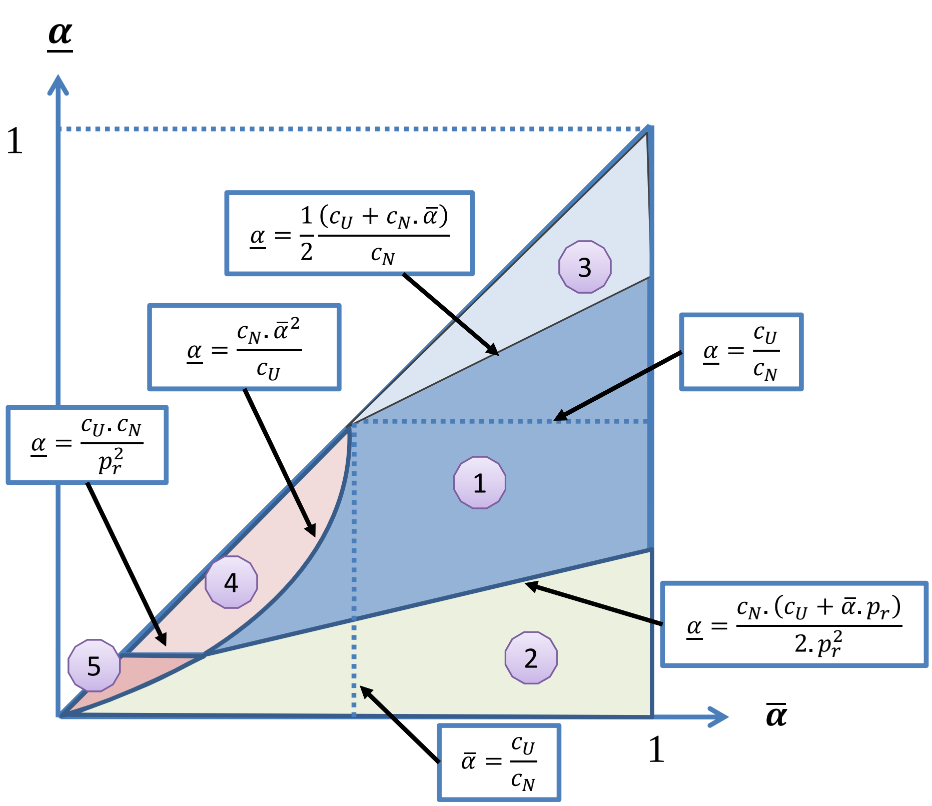


Figure 2: Equilibrium characterization under asymmetric information: Pooling equilibrium.

Table 3: Equilibrium characterization under asymmetric information: Pooling equilibrium.

Regions		$(\underline{\alpha}, \bar{\alpha}) \in PE_1$		$(\underline{\alpha}, \bar{\alpha}) \in PE_2$	
Range of α		$\alpha \leq \gamma_1$	$\alpha > \gamma_1$	$\alpha \leq \gamma_2$	$\alpha > \gamma_2$
Advance	η_N^*	$\eta_N^* \in \left[0, \frac{(c_N \bar{\alpha} + \sqrt{c_N^2 \bar{\alpha}^2 + 8\underline{\alpha} c_N c_U} - 4 c_N \underline{\alpha})}{4(\underline{\alpha}(p_r - c_N))}\right]$		$\eta_N^* \in [0, 1]$	
Guarantees	η_U^*	$\eta_U^* \in \left[0, \frac{(c_N \bar{\alpha}^2 + \bar{\alpha}\sqrt{c_N^2 \bar{\alpha}^2 + 8\underline{\alpha} c_N c_U} - 4 c_U \underline{\alpha})}{8(\underline{\alpha}(p_r - c_U))}\right]$		$\eta_U^* \in \left[0, (1/2)\frac{\bar{\alpha} p_r - c_U}{p_r - c_U}\right]$	
Prices (bids)	p_N^*	$\frac{\bar{\alpha} c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U}}{4\underline{\alpha}}$		p_r	
	p_U^*	$\frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U} + 4\underline{\alpha} c_U)}{8\underline{\alpha}}$		$\frac{c_U + p_r \bar{\alpha}}{2}$	
Order Alloc.	q_N^*, q_U^*	$Q, 0$	$0, Q$	$Q, 0$	$0, Q$
Suppliers'	π_N^*	$Q(\frac{\bar{\alpha} c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U}}{4\underline{\alpha}} - c_N)$	0	$Q(p_r - c_N)$	0
Profits	π_U^*	0	$\frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U} + 4\underline{\alpha} c_U)}{8\underline{\alpha}} - c_U)$	0	$Q(\frac{c_U + p_r \bar{\alpha}}{2} - c_U)$
Buyer's Cost	TC_B^*	$Q\frac{\bar{\alpha} c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U}}{4\underline{\alpha}}$	$Q\frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U} + 4\underline{\alpha} c_U)}{8\underline{\alpha}}$	Qp_r	$Q(\frac{c_U + p_r \bar{\alpha}}{2})$

Regions		$(\underline{\alpha}, \bar{\alpha}) \in PE_3$	$(\underline{\alpha}, \bar{\alpha}) \in PE_4$	$(\underline{\alpha}, \bar{\alpha}) \in PE_5$
Range of α		$\underline{\alpha} \leq \alpha \leq \bar{\alpha}$	$\underline{\alpha} \leq \alpha \leq \bar{\alpha}$	$\underline{\alpha} \leq \alpha \leq \bar{\alpha}$
Advance	η_N^*	$\eta_N^* = 0$	$\eta_N^* \in \left[0, \frac{\sqrt{c_N c_U/\underline{\alpha}} - c_N}{p_r - c_N}\right]$	$\eta_N^* \in [0, 1]$
Guarantees	η_U^*	$\eta_U^* \in \left[0, (1/2)\frac{\bar{\alpha} c_N - c_U}{p_r - c_U}\right]$	$\eta_U^* = 0$	$\eta_U^* \in [0, 1]$
Prices (bids)	p_N^*	c_N	$\sqrt{\frac{c_N c_U}{\underline{\alpha}}}$	p_r
	p_U^*	$\frac{c_U + c_N \bar{\alpha}}{2}$	c_U	c_U
Order Alloc.	q_N^*, q_U^*	$0, Q$	$Q, 0$	$Q, 0$
Suppliers'	π_N^*	0	$Q(\sqrt{\frac{c_N c_U}{\underline{\alpha}}} - c_N)$	$Q(p_r - c_N)$
Profits	π_U^*	$Q(\frac{c_U + c_N \bar{\alpha}}{2} - c_U)$	0	0
Buyer's Cost	TC_B^*	$Q\frac{c_U + c_N \bar{\alpha}}{2}$	$Q\sqrt{\frac{c_N c_U}{\underline{\alpha}}}$	Qp_r

Proposition 2 defines different values of η_N and η_U for which the suppliers cannot update their prior beliefs. One important note is that none of the least costly pooling equilibria can be eliminated using *intuitive criterion* (Cho and Kreps 1987) or *universal divinity* (Banks and Sobel 1987) because in fact no type of buyer in the defined ranges of η_i has no incentive to deviate to another outcome.

The emergence of regions 3, 4, and 5 where only one supplier wins the auction for the entire range of $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$ is due to the notion of minimum acceptable price and the assumption that the suppliers will stop decreasing their price at some break-even point. The common property of all the points in these regions is that they provide a low uncertainty to the suppliers and also show a clear preference from the buyer side in that the whole range of α is either higher or lower than the ratio c_U/c_N, the point where the buyer is indifferent between the suppliers if they both bid on their lowest possible costs (their marginal costs). Such case usually happens when the suppliers have a good understanding of the buyer's expectations, and the comparative performance of the suppliers is well known to each other. But in contrast, when the suppliers face a high degree of uncertainty in buyer's quality score assignment, they offer a premium price to maximize their expected profits. The following proposition proves the effect of uncertainty in belief and suppliers cost-homogeneity on the unit prices of the suppliers.

Proposition 3 *Under asymmetric information, the following statements are true with respect to the effect of parameters on the suppliers' optimal prices in pooling equilibria.*

1. *Both suppliers weakly increase their unit price in response to an increase in cost homogeneity of suppliers (captured as an increase in c_U for a fixed c_N). But this also increases the ratio of p_U/p_N which leads to lower chance for U to win the auction. Increasing suppliers' cost homogeneity (when c_U becomes closer to c_N) also leads to a smaller region 1 and a bigger surface for the combination of regions 2 and 3 in Figure 2.*
2. *The equilibrium prices p_N^* and p_U^* are both weakly increasing at $\bar{\alpha}$ (for fixed $\underline{\alpha}$) and weakly decreasing at $\underline{\alpha}$ (for fixed $\bar{\alpha}$)*

As Proposition 3 expresses, in contrast to symmetric information setting, an increase in cost homogeneity of suppliers leads to an increase in p_U, but, in fact, it does not translate to higher profit for him because on the other side, supplier N increases his price in such a way that decreases p_U/p_N (e.g. if supplier U have a %10 increase, supplier N may have %8 increase) and as a result, supplier U's chance of winning will decrease.

The second part of the proposition addresses the effect of asymmetry degree on the suppliers' prices: reducing uncertainty generally leads to a lower unit price from both suppliers. However, this is only true as long as there is small degree of uncertainty; in fact, there is a threshold for suppliers' uncertainty regarding the QS information in which the unit price under symmetric and asymmetric information scenarios become equal. The following proposition explores these thresholds and find the situations where asymmetric information setting can result in lower bid prices than symmetric information.

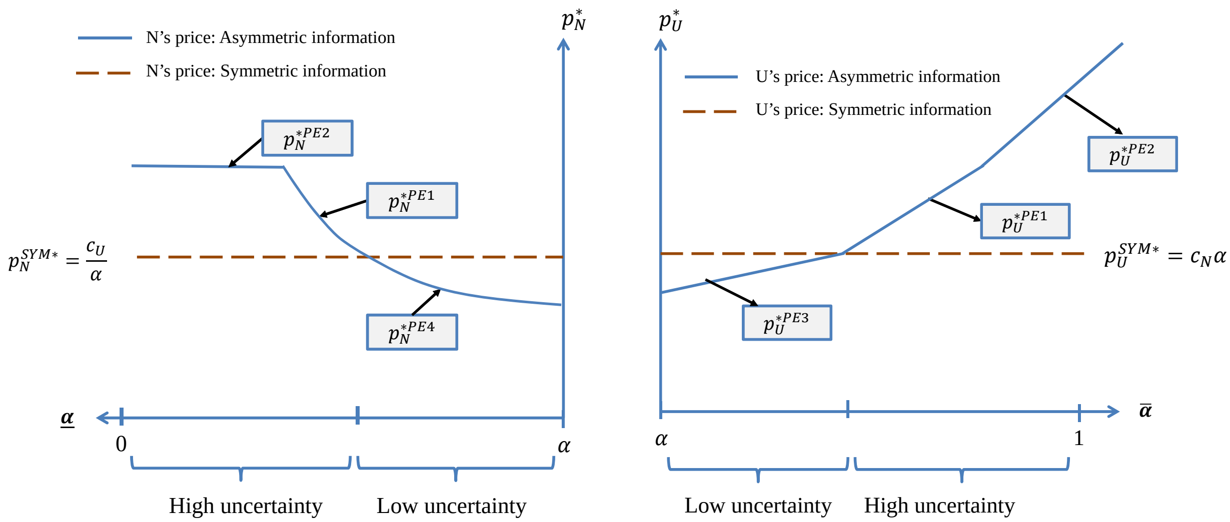
Proposition 4 *When the degree of uncertainty is sufficiently low, the winning price in the auction under pooling equilibrium can be lower than the winning price under symmetric information, i.e.*

- *for any $\alpha < \frac{c_U}{c_N}$, if $\bar{\alpha} < \frac{c_U}{c_N}$ and $\underline{\alpha} \geq \frac{c_N \bar{\alpha}^2}{c_U}$, then $p_N^{*PE} \leq p_N^{*SYM}$.*
- *for any $\alpha > \frac{c_U}{c_N}$, if $\bar{\alpha} > \frac{c_U}{c_N}$ and $\underline{\alpha} \geq \frac{((c_U + c_N \bar{\alpha}))}{2c_N}$, then $p_U^{*PE} \leq p_U^{*SYM}$.*

According to this proposition, low asymmetry between the buyer and the suppliers leads to higher price competition even compared to when there is no information asymmetry among channel partners.

5.2 Separating equilibrium

In the separating equilibrium, the buyer guarantees different levels of revenues in advance and, hence, suppliers U and N are able to correctly infer the true value of α. A separating equilibrium in this model must meet the following requirements so that the credible information sharing becomes possible. Firstly, given the fixed



(a) Supplier N's price for any $\alpha \le \frac{c_U}{c_N}$ (b) Supplier U's price for any $\alpha > \frac{c_U}{c_N}$

Figure 3: The impact of uncertainty on the equilibrium prices under symmetric information vs. pooling equilibrium.

selected price of p_r for the guarantee, the buyer's advance guarantee $\eta_i(\alpha) : [\underline{\alpha}, \bar{\alpha}] \to [0, 1]$ must be a one-to-one strategy for the informed buyer. This ensures that the buyer sends different signals for different QS information. Suppliers then correctly infer the true value of α when they observe the guarantee of $\eta_i(\alpha)$ that is expected in equilibrium. In that case, their posterior belief would be $\alpha = {\eta_i}^{-1}(\eta_i(\alpha))$ with probability one. Secondly, the choice of guarantee $\eta_i(\alpha)$ should be incentive compatible for the buyer so that the buyer has no incentive to deviate from the equilibrium. To examine if the second requirement meets, we find the best response of the buyer given the best response of the suppliers at the equilibrium. We conclude by verifying that the buyer has no incentive to deviate from equilibrium. To do so, we utilize buyer's cost function and suppliers' profit functions given at Equations 4, 5, and 6.

The next lemma characterizes the suppliers' optimal bids and the equilibrium cost and profits given an advance minimum revenue guarantee to supplier $i = N$ or U.

Lemma 2 *Let assume $\gamma_1^N = \frac{c_U}{p_N^{low}}$,$\gamma_2^N = \frac{c_U}{p_r}$,$\gamma_1^U = \frac{p_U^{low}}{c_N}$, and $\gamma_2^U = \frac{p_U^{low}}{p_r}$. In the separating equilibrium, after observing the guarantee η_i to supplier i ($i = U$ or N) and correctly inferring α, the suppliers' optimal bid would be as presented in Table 4.*

This lemma follows directly from the definition of p_i^{low} value for $i = N, U$ and the fact that after receiving the signal and correctly inferring α, the suppliers biding strategy would be very similar to the symmetric information case except that the minimum price would be p_i^{low} instead of c_i for $i = N, U$. Only in the cases when the guaranteed supplier fails in the competition, the buyer has to pay $\eta_i Q p_r$ and $(1 - \eta_i)Qp_{-i}$ to the guaranteed (i) and non-guaranteed $(-i)$ suppliers, respectively.

For the equilibrium to be separating, the buyer should have no incentive to mislead the suppliers. In other words, a buyer with true value of α must prefer, or at least should be indifferent to offer, the advance guarantee of $\eta_i(\alpha)$ to any other guarantee of $\eta_i(\alpha')$. The necessary condition for meeting this requirement in the general form is that $TC_B(\eta_i(\alpha), p_N(\alpha), p_U(\alpha), \alpha) \le TC_B(\eta_i(\alpha'), p_N(\alpha'), p_U(\alpha'), \alpha)$.

When α and α' are both in either range of $(0, \gamma_2^i)$, (γ_2^i, γ_1^i) or $(\gamma_1^i, 1)$ for $i = U, N$, where the total cost function of the buyer changes smoothly (continuous derivative with no breakpoint), this general condition is equivalent to the first-order condition

$$\frac{\partial TC_B(\eta_i(\alpha'), p_N(\alpha'), p_U(\alpha'), \alpha)}{\partial \alpha'} \Big|_{(\alpha'=\alpha)} = 0$$

Table 4: Best responses of the suppliers and the buyer after receiving the signal: Separating equilibrium.

		$0 \leq \eta_N \leq 1$		
Equilibrium decisions		$0 < \alpha \leq \gamma_2^N$	$\gamma_2^N \leq \alpha \leq \gamma_1^N$	$\gamma_1^N \leq \alpha \leq 1$
Prices (bids)	p_N^*	p_r	$\frac{c_U}{\alpha}$	$c_N + \eta_N(p_r - c_N)$
	p_U^*	c_U		$\alpha(c_N + \eta_N(p_r - c_N))$
Order Alloc	q_N^*, q_U^*	$Q, 0$		$\eta_N Q, (1-\eta_N)Q$
Suppliers'	π_N^*	$Q(p_r - c_N)$	$Q(\frac{c_U}{\alpha} - c_N)$	$\eta_N Q(p_r - c_N)$
Profits	π_U^*	0		$(1-\eta_N)Q(\alpha p_N^{low} - c_U)$
Buyer's Cost	TC_B^*	Qp_r	$Q\frac{c_U}{\alpha}$	$Q(\eta_N p_r + (1-\eta_N)\alpha p_N^{low})$

		$0 \leq \eta_U \leq 1$		
Equilibrium decisions		$0 < \alpha \leq \gamma_2^U$	$\gamma_2^U \leq \alpha \leq \gamma_1^U$	$\gamma_1^U \leq \alpha \leq 1$
Prices (bids)	p_N^*	p_r	$\frac{c_U + \eta_U(p_r - c_U)}{\alpha}$	c_N
	p_U^*	$c_U + \eta_U(p_r - c_U)$		αc_N
Order Alloc	q_N^*, q_U^*	$(1-\eta_U)Q, \eta_U Q$		$0, Q$
Suppliers'	π_N^*	$(1-\eta_U)Q(p_r - c_N)$	$(1-\eta_U)Q(\frac{p_U^{low}}{\alpha} - c_N)$	0
Profits	π_U^*	$\eta_U Q(p_r - c_U)$		$Q(\alpha c_N - c_U)$
Buyer's Cost	TC_B^*	Qp_r	$Q(\eta_U p_r + (1-\eta_U)\frac{p_U^{low}}{\alpha})$	$Q\alpha c_N$

† Note that $p_i^{low} = c_i + \eta_i(p_r - c_i)$ for $i = U$ and N only if the supplier i is offered a minimum revenue of $\eta_i Q p_r$.

assuming that the second order condition would be satisfied. This leads to an ordinary differential equation (ODE) that should be satisfied in any α. For instance, if the incumbent supplier is guaranteed ($i = N$) to have a minimum revenue of $\eta_N Q p_r$ at p_r if he loses, then for the cases of $\frac{c_U}{p_N^{low}} < \underline{\alpha} < \alpha < \bar{\alpha} < 1$, the following ODE should be satisfied at any α:

$$(c_N + \eta_N(\alpha)(p_r - c_N))(1 - \eta_N(\alpha)) + [p_r\alpha - 2\alpha c_N - 2\alpha\eta_N(\alpha)(p_r - c_N) + p_r]\frac{\partial \eta_N(\alpha)}{\partial \alpha} = 0$$

The following proposition proves the existence of separating equilibria under certain conditions.

Proposition 5 *The following statements are true regarding the possibility of costly signaling for the buyer:*

1. *Regardless of the suppliers prior belief on α ($\underline{\alpha}$, $\bar{\alpha}$), the buyer will never be able to truthfully share the true value of α using a revenue guarantee to unknown supplier U.*
2. *The buyer can truthfully share her private information (α) using an advance minimum revenue guarantee to the known supplier (η_N) only when $c_U/c_N \leq \bar{\alpha} \leq 1$; in all other cases when $0 < \bar{\alpha} < c_U/c_N$ there is only pooling equilibrium as characterized in Proposition 2. In the separating equilibrium, the buyer with private information of α guarantees the incumbent supplier a minimum revenue of $\eta_N(\alpha)p_r$, as follows:*

$$\eta_N(\alpha) = \begin{cases} f(\alpha) & \max(\underline{\alpha}, \alpha^m) \leq \alpha \leq \bar{\alpha} \\ f(\alpha^m) & \underline{\alpha} \leq \alpha < \alpha^m \end{cases}$$

where $f(\alpha) = \frac{C_1\alpha p_r + C_1 p_r - 2C_1\alpha c_N - \sqrt{C_1^2\alpha^2 p_r^2 + 2C_1^2\alpha p_r^2 + C_1^2 p_r^2 - 4C_1\alpha}}{2C_1\alpha(p_r - c_N)}$, $C_1 = \frac{1}{c_N(\bar{\alpha}p_r - \bar{\alpha}c_N + p_r)}$ *and* $\alpha^m = \frac{(p_r - c_U)c_U}{c_N\bar{\alpha}p_r - \bar{\alpha}c_N^2 + c_N p_r - p_r c_U}$.

When the suppliers observe an advance guarantee η_N to supplier N, they update their belief about α according to the following function:

$$\alpha(\eta_N) = \begin{cases} \alpha, & \eta_N \in [\eta_N(\bar{\alpha}), \min(f(\underline{\alpha}), f(\alpha^m))] \\ \sim U(\underline{\alpha}, \alpha^m), & \eta_N > \min(f(\underline{\alpha}), f(\alpha^m)) \end{cases}$$

and their equilibrium bidding price would be:

$$(p_N^*, p_U^*) = \begin{cases} (c_N + \eta_N(p_r - c_N), \alpha(c_N + \eta_N(p_r - c_N))) & \eta_N \in [\eta_N(\bar{\alpha}), \min(f(\underline{\alpha}), f(\alpha^m))] \\ (c_N + \eta_N(\alpha^m)(p_r - c_N), c_U) & \eta_N > \min(f(\underline{\alpha}), f(\alpha^m)) \end{cases}$$

In this case, the buyer's total cost would be fixed equal to $\bar{\alpha}c_N$ for all values of α ($\underline{\alpha} \leq \alpha \leq \bar{\alpha}$).

Proposition 5 shows that the buyer cannot truthfully share the exact value of α with the suppliers by a revenue guarantee to the entrant supplier U, whereas she might be able to signal her private information by a guarantee to the known supplier N in cases if $\underline{\alpha}$ and $\bar{\alpha}$ are close enough. The intuitive reason why signaling α is not feasible using a revenue guarantee to the unknown supplier is that the buyer will be always better off to signal the lowest possible value of α (which cannot be lower than $\underline{\alpha}$) to make sure that the supplier N loses the auction (because he mistakenly increases his bidding price putting him in the failure position) and the supplier U wins the auction at its lowest possible price (c_U). In fact, the buyer would be better off to cheat by switching the winner. One may think that this result is only because we assumed the production cost of supplier U is lower than that of supplier $N(c_U < c_N)$. But our further analysis shows that even if $c_U > c_N$ the buyer is still unable to share the true value of α using an advance guarantee to the unknown supplier.

Proposition 6 ***If supplier N is more cost efficient than unknown supplier****, i.e. $c_N < c_U$, then regardless of the suppliers' prior belief on α ($\underline{\alpha}$ and $\bar{\alpha}$), the buyer will never be able to truthfully share the true value of α by a minimum revenue guarantee to either supplier U or N.*

This proposition proves that the buyer can use an advance guarantee to the incumbent supplier as an informative signal only if all the following requirements are satisfied: 1– The incumbent is assigned a higher

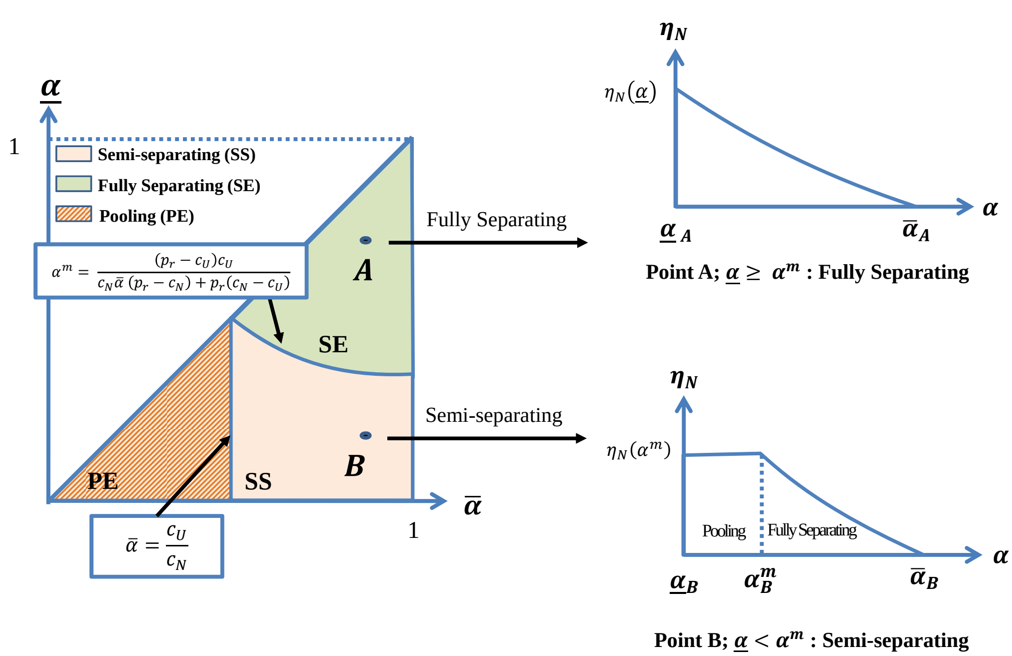


Figure 4: Characterization of separating equilibria.

quality score ($\alpha_N > \alpha_U$), and 2– he is less efficient that the other unknown supplier ($c_N > c_U$); otherwise, sharing information to the suppliers is not possible by guaranteeing supplier N.

The following proposition investigates the sensitivity of η_N and bid prices of the suppliers to the true value of α and the suppliers' belief about it under the separating equilibrium when the buyer commits to signal an advance minimum revenue guarantee.

Proposition 7 *Under asymmetric information when the buyer can truthfully share her private information:*

- *The equilibrium advance guarantee to supplier N (η_N^*) is increasing in $\bar{\alpha}$ and constant in $\underline{\alpha}$ for all values of $\alpha \in (\underline{\alpha}, \bar{\alpha})$.*
- *Supplier N's price (p_N^{*SE}) is decreasing in α (for $\alpha < \alpha^m$ is constant and then for $\alpha > \alpha^m$ becomes strictly decreasing).*
- *Supplier U's price (p_U^{*SE}) is increasing in α (for $\alpha < \alpha^m$ is constant and then for $\alpha > \alpha^m$ becomes strictly increasing).*

Interestingly, the buyer's signaling cost is only dependent to the highest-case scenario for the QS of unknown supplier ($\bar{\alpha}$) and not the absolute degree of uncertainty measured by $\bar{\alpha} - \underline{\alpha}$. Also, similar to the symmetric information scenario, the bid prices of the suppliers p_U^{SE} and p_N^{SE} increases and decreases, respectively, with respect to the true value of α under advance revenue guarantee.

6 The impact of credible information sharing

In this section, we compare the impact of buyer's choice of information sharing (whether pooling or separating equilibria) on the equilibrium decisions (suppliers' price and buyer's order allocation) and costs and profits.

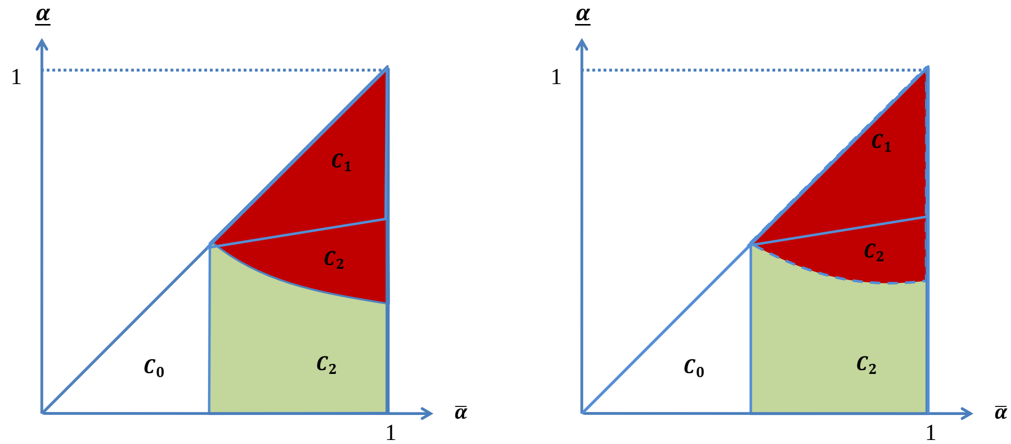
6.1 On equilibrium decisions: Prices/quantities

We do our best to characterize the impact of buyer's strategic decision of information sharing on the *expected* equilibrium unit prices of suppliers and the allocation of order quantities. When the suppliers are certain that the QS of unknown supplier is significantly low, i.e. $\underline{\alpha} < \alpha < \bar{\alpha} < \frac{c_U}{c_N}$ (region C_0), the buyer cannot truthfully signal her true type, i.e. $\eta_N = 0$. In contrast, the following proposition concentrate on a special case when the suppliers a priori believe that the unknown supplier' QS is high enough and the degree of asymmetry is low.

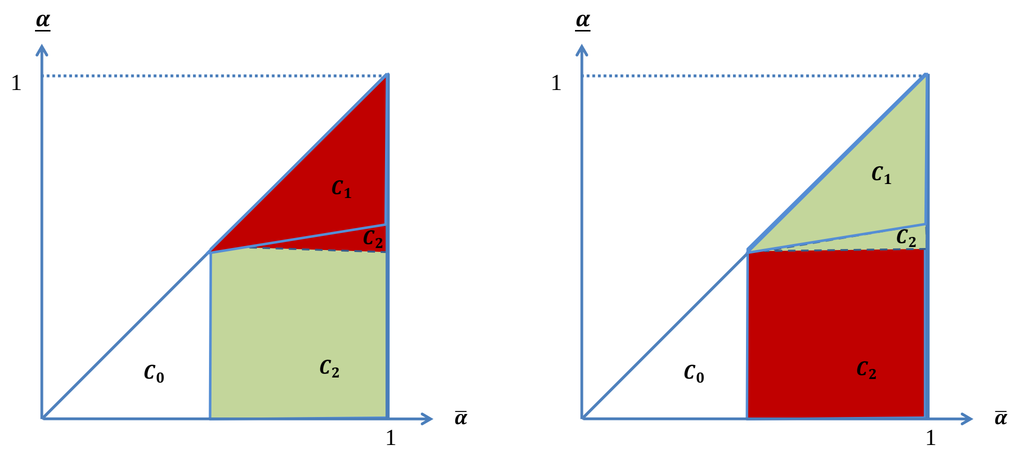
Proposition 8 *If the suppliers a priori believe that α is high and the degree of information asymmetry is sufficiently low, i.e. $\underline{\alpha} \geq \frac{c_U + c_N \bar{\alpha}}{2c_N}$ (this only can happen when $\underline{\alpha} \geq \frac{c_U}{c_N}$ at region C_1), then:*

- *The expected equilibrium unit prices are lower under pooling equilibrium than under separating equilibrium: $\bar{p}_U^{*PE} < \bar{p}_U^{*SE}$ and $\bar{p}_N^{*PE} < \bar{p}_N^{*SE}$.*
- *The expected equilibrium order quantity to supplier N (resp. U) is lower (resp. higher) under pooling equilibrium than under separating equilibrium: $\bar{q}_U^{*PE} > \bar{q}_U^{*SE}$ and $\bar{q}_N^{*PE} < \bar{q}_N^{*SE}$.*

Therefore, when the uncertainty is low and the suppliers believe that α is considerably high, hiding the information always leads to a lower expected unit price from both suppliers. Since the buyer has guaranteed supplier N, his order quantity would be higher under separating than pooling equilibrium. In region C_2 of Figure 5 where the uncertainty is medium or high, everything depends on the value of parameters. In the most general case, we have 4 different regions in C_2, depending on the combination of the possible cases of pooling (region 1 or 2 in Proposition 2) or separating (fully separating or partially separating) scenarios. Even though we can find in each subregion the break-even points where decision variables are equal under pooling and separating equilibria, it is analytically hard to fully characterize all the points in region C_2. Therefore, Figure 5 provides only a particular example when p_r/c_N is sufficiently large such that we only have three cases.



Regions	Supplier N's price	Supplier U's price
$(\underline{\alpha}, \bar{\alpha}) \in C_0$	$p_N^{PE} = p_N^{SE}$	$p_U^{PE} = p_U^{SE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_1$	$p_N^{PE} < p_N^{SE}$	$p_U^{PE} < p_U^{SE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_2$	$p_N^{PE} < p_N^{SE}$ if $\underline{\alpha} > \alpha^m$; $p_N^{PE} \geq p_N^{SE}$ o.w.	$p_U^{PE} < p_U^{SE}$ if $\Delta_{p_U}^{SE1} > 0$; $p_U^{PE} \geq p_U^{SE}$ o.w.



Regions	Supplier N's quantity	Supplier U's quantity
$(\underline{\alpha}, \bar{\alpha}) \in C_0$	$q_N^{PE} = q_N^{SE}$	$q_U^{PE} = q_U^{SE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_1$	$q_N^{PE} < q_N^{SE}$	$q_U^{PE} > q_U^{SE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_2$	$q_N^{PE} < q_N^{SE}$ if $\Delta_{q_N}^{SE1} > 0$; $q_N^{PE} \geq q_N^{SE}$ o.w.	$q_U^{PE} > q_U^{SE}$ if $\Delta_{q_U}^{SE1} > 0$; $q_U^{PE} \leq q_U^{SE}$ o.w.

Note. The different colored regions in the above figure denote the following impacts of pooling vs separating equilibria on decision variables: green (light shaded) regions – decision variable lower under pooling; red (dark shaded) regions – decision variable lower under separating; and, white regions – indifferent between pooling and separating equilibria. $\Delta_{p_U}^{SE1}$, $\Delta_{q_N}^{SE1}$, and $\Delta_{q_U}^{SE1}$ are characterized in the appendix.

Figure 5: Effects of pooling vs separating on price/quantities in asymmetric information setting.

6.2 On equilibrium profits/costs

This section establishes the comparative impact of pooling equilibrium on the costs/profits of channel parties with respect to the separating equilibrium.

In region C_0 where the suppliers believe that the buyer is low type ($\underline{\alpha} < \alpha < \bar{\alpha} < \frac{c_U}{c_N}$) there is no difference between pooling and separating (there is only possibility for pooling equilibrium). But, in other regions, the buyer's choice of information sharing can potentially change suppliers' profits and total costs of channel parties. The following proposition, fully characterizes this impact in a particular case when the suppliers a priori believe that the buyer is high type and there is low degree of asymmetry.

Proposition 9 *If the degree of information asymmetry is sufficiently low, i.e.* $\underline{\alpha} \geq \frac{c_U + c_N \bar{\alpha}}{2c_N}$ *(this only can happen when* $\underline{\alpha} \geq \frac{c_U}{c_N}$ *at region* C_1*), then:*

- *The equilibrium profit of supplier N is lower under pooling equilibrium than under separating equilibrium:* $\overline{\pi}_N^{*PE} \leq \overline{\pi}_N^{*SE}$.
- *The equilibrium profit of supplier U is lower under pooling equilibrium than under separating equilibrium only if* p_r *is sufficiently high, i.e.*

$$\overline{\pi}_U^{*PE} \leq \overline{\pi}_U^{*SE} \ \text{if}\ p_r \geq \frac{-c_N\underline{\alpha} + 4c_N\sigma^2_{\alpha,\eta_N} + 2c_N\mu_{\eta_N}\underline{\alpha} + 2c_N\mu_{\eta_N}\bar{\alpha} + c_U + 2E(\eta_N^2\alpha)c_N - 2c_U\mu_{\eta_N}}{2\sigma^2_{\alpha,\eta_N} + \mu_{\eta_N}\underline{\alpha} + \mu_{\eta_N}\bar{\alpha} + 2E(\eta_N^2\alpha)}$$

- *The unit cost of the buyer and the total supply chain' cost are lower under pooling equilibrium than under separating equilibrium:* $\overline{TC}_B^{*PE} \leq \overline{TC}_B^{*SE}$ *and* $\overline{TC}_{SC}^{*PE} \leq \overline{TC}_{SC}^{*SE}$.

In this situation, both suppliers are better off if the buyer chooses to share the QS information,

In region C_2, at the most general case, we have 4 different regions. Even though we can find in each subregion the break-even points where pooling=separating it is analytically hard to characterize it perfectly in region C_2.

Therefore, Figure 6 will provide only a particular example when p_r/c_N is sufficiently large such that we only have three cases. But in the appendix, we have captured all the break-even points but verifying whether they are effective or not is not easy.

7 Extension: Multiple suppliers in the auction

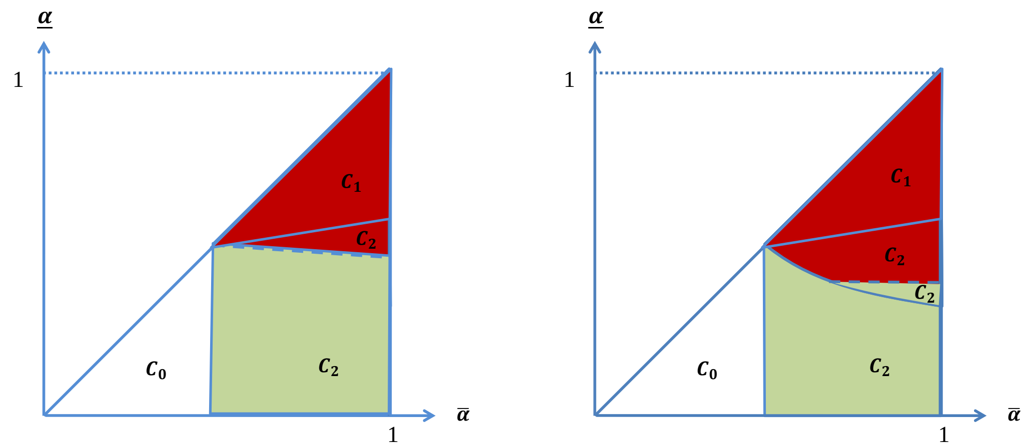
In order to see the effect of *competition* on the equilibrium decisions and buyer's choice of information sharing, in this section, we assume there are m untested suppliers and n known suppliers participating in the auction, where $m \geq 1$ and $n \geq 1$. To be consistent with the main model, we keep all the essential assumptions stated in §3 unchanged. We assume all the known suppliers are homogeneous in cost and quality score, i.e. all have a marginal cost of c_N and their quality score is normalized to be equal to one ($QS_N = 1$). The unknown suppliers are homogeneous in that their marginal cost is c_U, but they may differ in terms of their assigned quality scores. It is a common knowledge among the parties that untested suppliers' QS's are uniformly distributed between some publicly known $\underline{\alpha}$ and $\bar{\alpha}$.

First, in order to establish a benchmark, we consider the equilibrium under *symmetric information* where all the suppliers know their exact QS assigned by the buyer. Assume unknown suppliers' QS's $\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ are labeled in descending order such that $\underline{\alpha} \leq \alpha_m \leq \ldots \alpha_2 \leq \alpha_1 \leq \bar{\alpha}$. In the equilibrium, if there are multiple number of known suppliers ($n > 1$), they all keep undercutting each other's price upto their marginal cost level (c_N) regardless of the unknown suppliers' actions; otherwise if $n = 1$, supplier N only takes into account the value of α_1 and offers $\min(p_r, \max(c_N, \frac{c_U}{\alpha_1}))$. On the other side, if there are multiple number of unknown suppliers ($m > 1$), all except the one with the highest QS will offer their marginal cost, i.e. $p_U^{i*} = c_U$ for $i = 2, \ldots, m$, and the one with the highest QS takes a pricing policy as

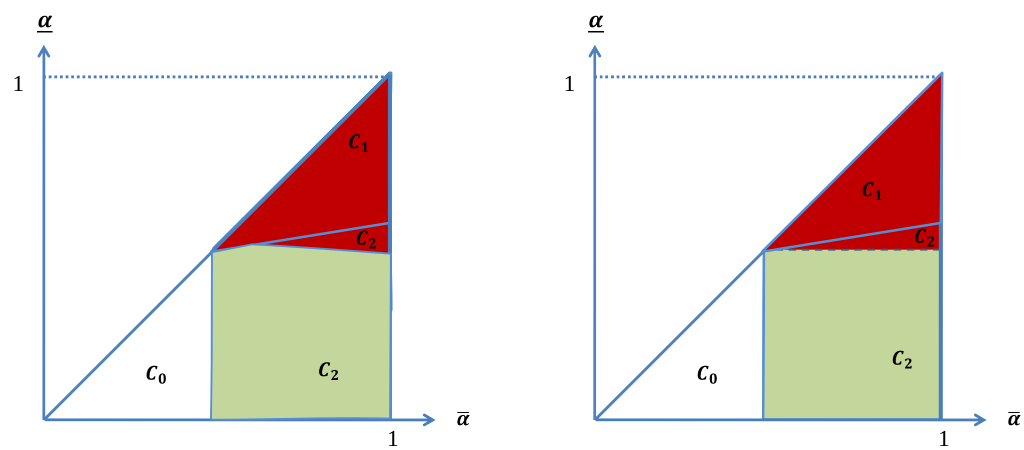
$$p_U^{1*} = \max(c_U, \min(c_N\alpha_1, \frac{c_U\alpha_1}{\alpha_2})),$$

where the terms $c_N\alpha_1$ and $\frac{c_U\alpha_1}{\alpha_2}$ ensure that his price is low enough to be competitive to the prices offered by known and other entrant suppliers, respectively. Note that the first-ranked supplier only considers α_2, the value of QS for the second-rank entrant supplier, since he only needs to secure himself against the next strongest competitor. In the equilibrium, for $\alpha_1 \leq \frac{c_U}{c_N}$, known suppliers win the auction (the buyer may choose to allocate the order equally among them) and for $\alpha_1 \geq \frac{c_U}{c_N}$, the entrant supplier with the highest QS (α_1) wins the auction.

As observed from the equilibrium unit prices under symmetric information setting, in the most general case, there are only three pieces of information that are probably worth to be shared: (1) the highest QS (α_1),



Regions	Supplier N's profit	Supplier U's profit
$(\underline{\alpha}, \bar{\alpha}) \in C_0$	$\pi_N^{PE} = \pi_N^{PE}$	$\pi_U^{PE} = \pi_U^{PE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_1$	$\pi_N^{PE} < \pi_N^{PE}$	$\pi_U^{PE} < \pi_U^{PE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_2$	$\pi_N^{PE} < \pi_N^{SE}$ if $\Delta_{\pi_N}^{SE1} > 0$; $\pi_N^{PE} \geq \pi_N^{SE}$ o.w.	$\pi_U^{PE} < \pi_U^{SE}$ if $\Delta_{\pi_U}^{SE1} > 0$; $\pi_U^{PE} \geq \pi_U^{SE}$ o.w.



Regions	Buyer's cost	Total supply chain cost
$(\underline{\alpha}, \bar{\alpha}) \in C_0$	$TC_B^{PE} = TC_B^{SE}$	$TC_{SC}^{PE} = TC_{SC}^{PE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_1$	$TC_B^{PE} < TC_B^{SE}$	$TC_{SC}^{PE} < TC_{SC}^{PE}$
$(\underline{\alpha}, \bar{\alpha}) \in C_2$	$TC_B^{PE} < TC_B^{SE}$ if $\Delta_{TC_B}^{SE1} > 0$; $TC_B^{PE} \geq TC_B^{SE}$ o.w.	$TC_{SC}^{PE} < TC_{SC}^{SE}$ if $\Delta_{TC_{SC}}^{SE1} > 0$; $TC_{SC}^{PE} \geq TC_{SC}^{SE}$ o.w.

Note. The different colored regions in the above figure denote the following impacts of pooling vs separating equilibria on decision variables: green (light shaded) regions - costs/profits lower under pooling; red (dark shaded) regions - costs/profits lower under separating; and, white regions - indifferent between pooling and separating equilibria. $\Delta_{\pi_N}^{SE1}$, $\Delta_{\pi_U}^{SE1}$ $\Delta_{TC_B}^{SE1}$, and $\Delta_{TC_{SC}}^{SE1}$ are characterized in the appendix.

Figure 6: Effects of pooling vs separating on supply chain partners' profit/costs in asymmetric information setting.

(2) the second rank QS (α_2), and (3) the supplier who possesses the highest rank. In fact, if all the channel partners have symmetric information regarding these three elements, even full information on the QS of other suppliers has no extra value and cannot influence the equilibrium outcome of the competition. Hence, we only focus on possible ways through which the buyer can signal these pieces of information. However, this needs a complicated three-dimensional signaling analysis. Rather, for the simplicity sake and in order to be able to study the possibility of information sharing using a signal of two dimensions (η_N, η_U), let assume that suppliers' ranking is a priori known to all the parties.[6] This assumption enables us to eliminate the need for sharing the third dimension of hidden information and simplify the problem to a two-dimensional signaling

[6] As a matter of fact, it is sufficient to assume that only first-rank supplier is known to all.

game. Note that even though supplier's relative rank in terms of QS is assumed to be commonly known, the exact scores is still unknown to all parties except the buyer.

Under asymmetric information scenario, similar to our main analysis with two suppliers, we only characterize pure (not-mixed) equilibria and specifically focus on symmetric actions from homogeneous parties (when possible) to ease the complexity of the analysis. The main objective of this section is simply to find out: (1) the equilibrium decisions under pooling equilibrium, (2) whether the buyer can signal the required information truthfully using advance minimum revenue guarantees to known suppliers ($\eta_N Q p_r$) and unknown suppliers ($\eta_U Q p_r$), and finally, (3) the possible impacts of increasing competition (increasing n and m) on the buyer's choice of information sharing.

Pooling equilibrium: We characterized the equilibrium decisions under pooling equilibrium when $n = m = 1$ in Section 5.1. When only the number of known suppliers increases ($n > 1, m = 1$), both suppliers decrease their prices to $p_N^{i*} = c_N$ for $i = 1, \ldots, n$ and $p_U^* = \frac{c_U + c_N \bar{\alpha}}{2}$. But, when $m > 1, n \geq 1$ finding a short closed-form solution is almost impossible; however, as a general rule, suppliers take the following policy when offering their prices:

$$p_U^{i*} = \min(p_r, \max[c_U, f_U^i(p_N^*, p_U^{1*}, \ldots, p_U^{i-1*}, p_U^{i+1*}, \ldots, p_U^{m*})]) \quad \forall i = 1, \ldots, m$$

$$p_N^* = \begin{cases} c_N & n > 1 \\ \min(p_r, \max[c_N, f_N(p_U^{1*}, \ldots, p_U^{m*})]) & n = 1 \end{cases}$$

where the procedure for finding $f_U^i(p_N^*, p_U^{1*}, \ldots, p_U^{i-1*}, p_U^{i+1*}, \ldots, p_U^{m*})$ and $f_N(p_U^{1*}, \ldots, p_U^{m*})$ is described in the appendix.

Separating equilibrium: The case of two competing suppliers $m = n = 1$ is characterized at Proposition 5. It is easy to prove that in the case of multiple known suppliers with one new entrant ($n > 1, m = 1$), the buyer can still signal the true value of α using a guarantee of $\frac{\eta_N}{n}$ to each known supplier only when $\bar{\alpha} > \frac{c_U}{c_N}$. Similar to the results of Proposition 5, the buyer's total cost would be $Q c_N \bar{\alpha}$.

In the most general case where $m > 1, n \geq 1$, as been observed in symmetric information setting, the buyer only needs to signal the two highest values of suppliers' QS's, i.e. α_1 and α_2 when $\alpha_1 > \frac{c_U}{c_N}$.[7] It is easy to observe that the buyer is never able to signal a low value of α_1 using a minimum revenue guarantee to known or unknown suppliers when the suppliers a priori believe that it is low ($\bar{\alpha} < \frac{c_U}{c_N}$), as we proved in Proposition 5. Therefore, we only focus on the possibility of signaling when $\bar{\alpha} > \frac{c_U}{c_N}$. Let assume $\frac{\eta_N}{n} Q p_r$ and $\frac{\eta_U}{m-1} Q p_r$ are the advance revenue guarantee to each known and unknown supplier, respectively.[8]

In order to have a separating equilibrium under which credible information sharing becomes possible, first, there should be a one-to-one projection from the signals, advance guarantees (η_N, η_U), to the hidden information (α_1, α_2), i.e.

$$(\eta_N, \eta_U)_{(\alpha_1, \alpha_2)} : ([\underline{\alpha}, \bar{\alpha}]^2 \mid \alpha_2 < \alpha_1) \to [0, 1]^2.$$

This ensures that the buyer sends different signals for different tuple of (α_1, α_2). Suppliers then correctly infer the true value of α_1 and α_2 when they observe the guarantee of $\eta_N(\alpha_1, \alpha_2)$ and $\eta_U(\alpha_1, \alpha_2)$ that are expected in equilibrium. In addition, the choice of guarantee should be incentive compatible for the buyer so that the buyer has no incentive to deviate from the equilibrium. This condition leads to solving the following system of *partial differential equations* (PDEs):

[7] In a situation where $\alpha_1 \leq \frac{c_U}{c_N}$, the buyer only needs to share α_1 because knowing the exact value of α_2 will not affect the outcome of the auction when the suppliers know that even the best entrant cannot compete with incumbent suppliers. Note that in this case, sharing α_1 makes sense only when there is one known supplier, $n = 1$, since in case of multiple incumbent suppliers $n > 1$, there is no point in signaling low quality of unknown suppliers as all the known suppliers in the equilibrium offer c_N and win the auction.

[8] As indicated in the model framework, this is an external price that should motivate suppliers to participate in the auction. For the sake of notational simplicity and without loss of generality, we assume here that the buyer offers the guarantee to both incumbent and entrant suppliers at the same price equal to the reserve price p_r.

$$\frac{\partial TC_B(\alpha_1,\alpha_2)}{\partial \alpha_i} = 0 \qquad \forall i \in \{1,2\}$$

where $TC_B(\alpha_1, \alpha_2)$ denotes the total cost of the buyer given her signal is credible to the suppliers.

The following proposition characterizes decision variables in separating equilibrium under asymmetric information when there are multiple unknown suppliers.

Proposition 10 *In the presence of $m \geq 2$ unknown suppliers in the auction, if $\underline{\alpha} > \frac{c_U}{c_N}$, the buyer can signal (α_1, α_2) with an equal minimum revenue guarantee to known suppliers $\frac{\eta_N}{n} Q p_r$ and another equal guarantee to unknown suppliers $\frac{\eta_U}{m-1} Q p_r$ as follows:*

$$\eta_N(\alpha_1,\alpha_2) = \frac{\alpha_1 p_r - p_r \alpha_2 m + p_r \alpha_2 + \alpha_1 c_U m - 2\alpha_1 c_U}{\alpha_1 (p_r - c_U)}$$

$$\eta_U(\alpha_1,\alpha_2) = \frac{(m-1)(p_r \alpha_2 - \alpha_1 c_U)}{\alpha_1 (p_r - c_U)}$$

When the suppliers observe advance guarantees of η_N and η_U to known and unknown suppliers, respectively, they update their belief about α_1 and α_2 accordingly and their equilibrium bidding price would be:

$$(p_N^*, p_U^{i>1*}, p_U^{1*}) = \begin{cases} (c_N + \frac{\eta_N}{n}(p_r - c_N) \\ (c_U + \frac{\eta_U}{m-1}(p_r - c_U))) \\ \frac{\alpha_1}{\alpha_2}(c_U + \frac{\eta_U}{m-1}(p_r - c_U)) \end{cases}$$

In this case, the buyer's total cost would be fixed equal to p_r for all values of α_1 and α_2 ($\underline{\alpha} \leq \alpha_2 < \alpha_1 \leq \bar{\alpha}$).

Impact of information sharing: According to the above proposition, when the number of new entrants is more than one, signaling the required information to the suppliers becomes very costly for the buyer compared to the case where $m = 1$. On the other hand, under the pooling equilibrium, by increasing the number of new entrants in the auction m, competition effect increases among suppliers and all the suppliers weakly decrease their prices. Therefore, it seems that from the buyer's perspective, pooling equilibrium is more preferred than separating equilibrium in the presence of multiple new entrants and she will be better off to hide the QS information from the suppliers when $m \geq 2$.[9] In fact, when the advance guaranteed price in case of failure is greater than the marginal cost of the least efficient parties, i.e. $p_r \geq c_N$, and in the presence of multiple known and unknown suppliers ($n \geq 2$ and $m \geq 2$), the buyer is always better off under pooling equilibrium because the unit price under pooling equilibrium, i.e. the winning price, is less than c_N while the unit cost for the buyer under separating equilibrium is p_r.

8 Conclusion

In this paper, we analyze how and when a buyer can credibly share her private QS information with her upstream suppliers, and how it impacts the equilibrium decisions and the profit/costs of channel parities. To address these questions, we develop a decentralized supply chain model with a buyer and two heterogeneous suppliers competing for the buyer's order quantity. We analyze the use of advance revenue commitment from the buyer to the suppliers as a signaling tool for transferring information on suppliers' quality scores. These

[9] This fact can offer an explanation for the recent phenomenon in keyword auctions that many search engines, including Google and Yahoo!, have made their quality ranking scheme more ambiguous to the participants. For instance, Google started with a simple ranking system in which all the advertisers could see their own QS immediately (symmetric information), but now it introduces new known and unknown attributes without informing the bidders of their scores. However, it seems that Google prefers the participants to be in partial asymmetry because within the current practice, they can see some scores immediately while their final quality ranking remains unknown. This in fact corresponds to our result that "low asymmetry is the key success factor for the buyer in buyer-determined auctions". We refer the readers to Geddes (2014) for a detailed survey on Google Adwords.

quality scores denote the buyer's estimation of the quality of suppliers in non-price factors and basically reflect the buyer's preferences over suppliers that will be used to rank them. After observing the advance guarantee, the suppliers update their beliefs on the true QS, and then compete on prices to earn the order from the buyer.

We first characterize the equilibrium under symmetric information and show that the comparison between costs and quality scores of U and N determines the winner of the reverse auction. Specifically, the untested supplier (U) is likely to win the auction if his relative quality score ($\frac{QS_U}{QS_N}$) is higher than his relative cost advantage, while the known incumbent supplier (N) is preferred when the buyer is more concerned with the non-price attributes and puts more emphasis on the incumbent's quality performance compared to an untested supplier. The analysis under asymmetric information setting comes with new twists to every aspect of the problem, especially to the credibility of QS information. First, knowing that the buyer would distort her private information on QS in order to influence the bid prices of the suppliers to her benefit, the suppliers simply ignore the information shared by the buyer unless she offers an advance minimum revenue guarantee to the supplier N before the auction starts. We show that this makes buyer's QS information sharing credible in the eyes of suppliers however, it leads to extra cost for the buyer. As the signaling cost increases in the relative differences between quality scores of suppliers U and N, this information sharing strategy is sustainable for the buyer as long as the relative quality scores of the suppliers U and N is not very low.

Secondly, in the presence of incomplete information, the suppliers decide on their prices based on their a priori beliefs on the true values of quality scores. Our results show that depending on the choices made by the suppliers, the strength of price competition (resp. equilibrium price) decreases (resp. increases) by an increase in the degree of uncertainty. Thirdly, a comparison of the expected cost/profit implications between sharing and not sharing the QS information enables us to evaluate when each strategy is more preferable (and when it is not) for each channel partner. Finally, a comprehensive analysis of the auction under the presence of multiple known and unknown suppliers reveals the new challenges and costs of sharing the QS information.

The above-mentioned results give some managerial insights on the feasibility of credible information sharing in competitive supply chains: First, when the degree of information asymmetry is sufficiently low, the total cost of the buyer and the whole supply chain under incomplete information can surprisingly be even lower than that under symmetric information. In contrast, when the degree of uncertainty increases, it leads to higher unit prices from the suppliers. This suggests that the buyers should not disclose quality score evaluation scheme to the bidders when they are relatively similar to each other in order to foster the degree of competition amongst them. However, when the suppliers are heterogeneous in terms of quality scores, the buyer should take necessary actions in order to decrease the degree of asymmetry regarding non-price attributes.

Next, the buyer has to take some costly action in order to make the information sharing credible for the suppliers. Advance commitments that are very common in supply chain interactions can serve as a credible signal in reverse auctions. Furthermore, in order for it to be credible, advance guarantee should be given to the incumbent only when suppliers believe that unknown supplier's QS should not necessarily be very low. This costly action is justifiable only if the degree of information asymmetry is high such as in cases where the suppliers are not sure of the buyer's preferences and concerns or when the non-price attributes are not easy to assess. In those cases, it is worth to incur an additional cost and share the QS information before the auction.

One important note regarding the advance commitment to suppliers is that no information can be transferred using a guarantee to unknown suppliers. In fact, provision of revenue guarantee to an unknown supplier when there is an incumbent with better expected performance in non-price attributes does not convey any credible information as the buyer has incentive to distort the information in order to reduce her total cost. This suggests that the buyer should not commit to an untested supplier before the auction even if she does not pay attention to the non-price factors in ranking of the suppliers.

Finally, when the number of participating entrants (untested suppliers U) increases, the need for signaling QS via advance guarantees decreases. This is because the degree of competition intensifies and the signaling cost incurred by the buyer increases as the number of tested and/or untested suppliers increases. Consequently, even when the degree of information asymmetry is high, the signaling cost dominates the competition effect, hence, the buyer is better off not to share her private information. That could offer an explanation for why Google initially started its keyword auctions with fully symmetric information and then gradually moved to more uncertain settings by introducing the hidden relevance score.

The model presented in this paper can be extended in different directions. First possible extension is to add other sources of information asymmetry to the model. For instance, in a situation where the buyer faces uncertain demand, she may have better forecast information than the suppliers. Incorporating forecasting asymmetry would enable us to see the impact of different types of asymmetry on the contracting decisions. In addition, suppliers' production cost and reliability are usually precisely known only to themselves. Including these factors in the model would provide a more realistic setting, analytically complicated though, that enables us to see how information richness in two different streams of supply chain can influence the contractual interactions and the degree of competition.

Moreover, we assumed here that all the parties are risk-neutral. One possibility would be to introduce risk-aversion into the players' objective functions. This extension would allow us to see the impact of the risk characteristics on the choice of sharing or not sharing the information.

Lastly, we believe that the analysis of the possibility and profitability of credible QS information sharing in buyer-determined auctioning contracts presents fruitful research opportunities and hope that this model will fuel future research in this direction.

Appendix: Proofs of propositions

Proof of Proposition 1: Under symmetric information, the problem can be analyzed backward by starting from the buyer's order allocation in the last stage. Let supplier N and U propose p_N and p_U per unit and their quality score assigned by the buyer are $QS_N = 1$ and $QS_U = \alpha$, respectively. Then, the buyer's total cost can be expressed as follows.

$$TC_B(p_N, p_U, \alpha, p_r) = \begin{cases} Q \times p_N & p_N \leq \frac{p_U}{\alpha}, p_N \leq p_r \\ Q \times p_U & p_N > \frac{p_U}{\alpha}, p_U \leq p_r \\ Q \times p_r & else \end{cases} \tag{7}$$

So, the buyer identifies supplier with the highest generalized price (GP) and if his unit price is less than or equal to the reserve price (spot market price), she orders the whole demand to the winner, otherwise she procures from external sources. Therefore, the buyer's optimal allocation policy can be specified as:

$$(q_N^*(p_N, p_U, \alpha, p_r), q_U^*(p_N, p_U, \alpha, p_r)) = \begin{cases} (Q, 0) & p_N \leq \frac{p_U}{\alpha}, p_N \leq p_r \\ (0, Q) & p_N > \frac{p_U}{\alpha}, p_U \leq p_r \\ (0, 0) & else \end{cases} \tag{8}$$

Given this order allocation policy, the two suppliers engage in a Bertrand price competition in which each supplier undercuts the other one until one of them reaches to his minimum price (marginal cost). The suppliers' expected profit after observing the reserve price is as follows:

$$\pi_N(\alpha, p_U) = Q \times max_{p_N \leq p_r} \left((p_N - c_N) Prob[p_N \leq \frac{p_U}{\alpha}]; 0 \right)$$
$$\pi_U(\alpha, p_N) = Q \times max_{p_U \leq p_r} \left((p_U - c_U) Prob[p_N > \frac{p_U}{\alpha}]; 0 \right)$$

The second term in the profit functions of suppliers U and N, zero, ensures that $p_i \geq c_i$ for $i = N$ and U.

Each supplier wants to maximize his expected profit given the action of the other supplier and the allocation policy of the buyer. This Bertrand competition results in an equilibrium in which the first-ranked supplier undercuts the second-ranked by just epsilon (i.e. infinitesimal) amount and wins the auction. In addition, they both know that the maximum unit price they can offer is p_r.

Given the profit functions of suppliers, their best response functions against each other would be as follows:

$$p_N^*(p_U) = min(max(c_N; \frac{p_U}{\alpha}); p_r)$$
$$p_U^*(p_N) = min(max(c_U; \alpha p_N); p_r)$$

Based on the above pricing schemes, in equilibrium one of the following will be true:

1. If $c_U/c_N \leq \alpha \leq 1$ the buyer procures from supplier U. This is because supplier N cannot offer a lower price than c_N in this range and supplier U can win the auction at a price infinitesimally lower than $c_N\alpha$.
2. If $0 < \alpha \leq c_U/c_N$ the buyer procures from supplier N. This is because supplier U will be undercut until he reaches his marginal cost c_U while supplier N can win the auction at a price infinitesimally lower than $min(c_U/\alpha; p_r)$. Therefore, when $c_U/p_r \leq \alpha \leq \frac{c_U}{c_N}$ supplier N's price would be c_U/α and when $0 < \alpha < \frac{c_U}{p_r}$ he will offer p_r.

Based on the supplier's optimal bidding policies, we can fully characterize the optimal order allocation, suppliers' profits, and buyer's cost under symmetric information in three regions of $0 < \alpha < \frac{c_U}{p_r}$, $\frac{c_U}{p_r} \leq \alpha \leq \frac{c_U}{c_N}$, and $\frac{c_U}{c_N} \leq \alpha \leq 1$ as presented in Table 2. □

Proof of Corollary 1: In order to prove this corollary, one should consider the value of bid prices and profits of suppliers provided in Table 2. To analyze the effect of α, for $\alpha > c_U/c_N$ the expected bid and profit of supplier U ($c_N\alpha$ and $Q[c_N\alpha - c_U]$ respectively) are strictly increasing, while $p_N^* = c_N$, $\pi_N^* = 0$ stay unchanged. In contrast, when $c_U/p_r \leq \alpha \leq c_U/c_N$, the expected bid and profit of supplier N (c_U/α and $Q[c_U/\alpha - c_N]$ respectively) strictly increase and $p_U^* = c_U$,$\pi_U^* = 0$ remain unaffected. Finally, in the last region of $\alpha < c_U/p_r$, the expected bids and profits of both suppliers stay unchanged ($p_U^* = \pi_U^* = 0$;$p_N^* = p_r$ and $\pi_N^* = Q(p_r - c_N)$). The first statement follows by considering the effect of α in each region and the continuity of the price and profit functions of suppliers.

The same procedure applies for proving the effect of cost heterogeneity. By fixing c_N and increasing c_U, the second part of the corollary would easily follow. □

Proof of Lemma 1 In order to find the optimal pricing policy of the profit-maximizing suppliers under the least costly pooling equilibrium ($\eta_i = 0$), first, assume there is no limit on their prices. Their profit function, then, would be as follows:

$$\pi_N = Q(p_N - c_N)\int_{\underline{\alpha}}^{p_U/p_N} f(\alpha)d\alpha$$
$$\pi_U = Q(p_U - c_U)\int_{p_U/p_N}^{\bar{\alpha}} f(\alpha)d\alpha$$

where $f(\alpha)$ is their prior belief regarding the true value of α. Since we mainly focus on the uniform distribution, $f(\alpha)$ for $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$ would be $f(\alpha) = \mathcal{U}[\underline{\alpha}, \bar{\alpha}] = \frac{1}{\bar{\alpha}-\underline{\alpha}}$. After plugging $f(\alpha)$ in the suppliers' profit function and taking the first derivative, the best response function of the suppliers will be given as: $p_N^*(p_U) = \sqrt{\frac{c_N p_U}{\underline{\alpha}}}$ and $p_U^*(p_N) = \frac{\bar{\alpha}p_N + c_U}{2}$.

By solving this system of equation, we find the internal equilibrium (PE-1) of the game when other constraints are not binding, as follows:

$$p_N^{Int} = \frac{\bar{\alpha}c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha}c_N c_U}}{4\underline{\alpha}} \text{ and } p_U^{Int} = \frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha}c_N c_U} + 4\underline{\alpha}c_U)}{8\underline{\alpha}}$$

This internal point will be the equilibrium only if it satisfies the upper and lower pricing bounds. Considering the maximum and minimum prices allowed, the best response of the suppliers is as follows:

$$p_N^*(p_U) = min(p_r, (max[\sqrt{\frac{c_N p_U}{\underline{\alpha}}}, c_N]))$$
$$p_U^*(p_N) = min(p_r, (max[\frac{\bar{\alpha}p_N + c_U}{2}, c_U]))$$

Therefore, based on these response functions, the optimal pricing would be either of:

- PE-1: Internal solution of p_N^{Int} and p_U^{Int} if both prices are in the allowed ranges.
- PE-2: Boundary solution of $p_N = p_r$ and $p_U = (\bar{\alpha}p_r + c_U)/2$, in case if $p_N^{Int} > p_r$; it is easy to see that $c_U \leq p_U = (\bar{\alpha}p_r + c_U)/2 \leq p_r$.
- PE-3: Boundary solution of $p_N = c_N$ and $p_U = (\bar{\alpha}c_N + c_U)/2$, in case if $p_N^{Int} < c_N$; it is easy to see that $c_U \leq (\bar{\alpha}c_N + c_U)/2 \leq p_r$.
- PE-4: Boundary solution of $p_N = \sqrt{\frac{c_N c_U}{\underline{\alpha}}}$ and $p_U = c_U$, in case if $p_U^{Int} < c_U$ only if $\sqrt{\frac{c_N c_U}{\underline{\alpha}}} \leq p_r$ (otherwise PE-5 happens).
- PE-5: Boundary solution if $p_N = p_r$ and $p_U = c_U$, in case if $p_U^{Int} < c_U$ and $\sqrt{\frac{c_N c_U}{\underline{\alpha}}} > p_r$.

Obviously, the following pairs of (p_N, p_U) cannot be the outcomes of the game:

- (p_r, p_r): because if N offers p_r the supplier U's optimal response should be $(\bar{\alpha}p_r + c_U)/2$ which is less than p_r.
- (c_N, p_r): because by definition supplier U's optimal price should be always lower than supplier N, i.e. $p_U^*(p_N) = \frac{\bar{\alpha}p_N + c_U}{2} < p_N$.
- $(\sqrt{\frac{c_N p_r}{\underline{\alpha}}}, p_r)$: because even if supplier U offers p_r in the equilibrium, supplier N will not go beyond p_r, i.e. $p_r < \sqrt{\frac{c_N p_r}{\underline{\alpha}}}$.

But (c_N, c_U) can be the equilibrium only at one specific point of $(\underline{\alpha} = \frac{c_U}{c_N}, \bar{\alpha} = \frac{c_U}{c_N})$ which is basically the common corner of regions 1, 3, and 4 in Figure 2. □

Proof of Proposition 2: In a pooling equilibrium, suppliers do not infer the true type of the buyer and hence rely on their a-priori belief to decide on their prices. Given an advance revenue guarantee $\eta_i Q p_r \geq 0$ their optimal pricing policy would be as:

$$p_N^*(p_U) = min(p_r, (max[\sqrt{\frac{c_N p_U}{\underline{\alpha}}}, p_N^{low}]))$$
$$p_U^*(p_N) = min(p_r, (max[\frac{\bar{\alpha}p_N + c_U}{2}, p_U^{low}]))$$

where p_i^{low} for $i = U, N$ denotes the minimum price offered by the supplier i, which is either c_i (if he is not offered any guarantee) or $c_i + \eta_i(p_r - c_i)$ if he is offered a guarantee of η_i.

A least costly pooling equilibrium should meet the following conditions: 1– For the whole range of α, the buyer should send a fixed signal (a fixed level of revenue should be guaranteed) regardless of her true type so that no specific information transfers; 2– given the suppliers' optimal response, the buyer should have no incentive to deviate from her original signal.

Now, let consider two different signaling scenarios for the buyer: she can make an advance guarantee to either supplier N or U.

First, assume $i = N$ ($\eta_U = 0, \eta_N \geq 0$): Here we find all possible values of η_N such that it leads to the same outcome as when $\eta_N = 0$. Given $\bar{\alpha}, \underline{\alpha}, c_U, c_N$, and η_N, if $p_N^{low} > p_N^{Int}$ where $p_N^{Int} = \frac{\bar{\alpha}c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U}}{4\underline{\alpha}}$, the equilibrium price for supplier N would be $p_N^* = p_N^{low}$. In that case supplier U wins the auction at $p_U = \frac{\bar{\alpha}p_N^{low} + c_U}{2}$. Therefore, the buyer's unit cost would be a direct function of p_N^{low}: $TC_B = Q.(\eta_N p_r + (1 - \eta_N)(\frac{\bar{\alpha}p_N^{low} + c_U}{2}))$. Consequently, choice of η_N has a direct impact on the buyer's total cost. Therefore, the buyer always prefers to decrease η_N as much as possible. The buyer's tendency to deviate holds for any value of $\eta_N > 0$. Therefore, a pooling equilibrium with $\eta_N > 0$ fails to exist in regions where PE-3 is the outcome of the least costly game with $p_N^{low} = c_N$: i.e. where $\underline{\alpha} \geq \frac{c_U + c_N\bar{\alpha}}{c_N}$.

But if $p_N^{low} \leq p_N^{Int}$ any other equilibrium outcome except PE-3 may prevail, i.e. p_N can be either p_r, p_N^{Int}, or $\sqrt{\frac{c_N c_U}{\underline{\alpha}}}$. Total cost of the buyer, therefore, is either Qp_N (if N wins) or $Q(\eta_N p_r + (1-\eta_N)p_U)$ (if U wins). In either case, the total cost of the buyer is independent from the choice of η_N. Hence, the buyer is indifferent between any value of

- $0 \leq \eta_N \leq \frac{(c_N\bar{\alpha} + \sqrt{c_N^2\bar{\alpha}^2 + 8\underline{\alpha}c_N c_U} - 4c_N\underline{\alpha})}{4(\underline{\alpha}(p_r - c_N))}(\eta : p_N^{low}\ (\eta) = p_N^{Int})$ when PE-1 is the outcome of the game.
- $0 \leq \eta_N \leq 1$ $(\eta : p_N^{low}(\eta) = p_r)$ when PE-2 or PE-5 are the outcome of the game.
- $0 \leq \eta_N \leq \frac{\sqrt{c_N c_U/\underline{\alpha}} - c_N}{p_r - c_N}$ $(\eta : p_N^{low}(\eta) = \sqrt{c_U c_N/\underline{\alpha}})$ when PE-4 prevails.

By the choice of η_N as mentioned above, the suppliers acquire no extra information and the buyer has no incentive to deviate.

Second, assume $i = U$ **(**$\eta_U \geq 0, \eta_N = 0$**):** Similarly, here we find all possible values of η_U such that it leads to the same outcome as when $\eta_U = 0$. Given $\bar{\alpha}, \underline{\alpha}, c_U, c_N$, and η_U, if $p_U^{low} > p_U^{Int}$ where $p_U^{Int} = \frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha}c_N c_U} + 4\underline{\alpha}c_U)}{8\underline{\alpha}}$ the equilibrium price for supplier U would be $p_U^* = p_U^{low}$ and for supplier N $min(p_r, \sqrt{p_U^{low} c_N/\underline{\alpha}})$. In the case when supplier N wins the auction at $p_N = \sqrt{p_U^{low} c_N/\underline{\alpha}}$, the buyer's unit cost would be a direct function of p_U^{low}: $TC_B = Q.(\eta_U p_r + (1-\eta_U)(\sqrt{p_U^{low} c_N/\underline{\alpha}}))$. Consequently, choice of η_U has a direct impact on the buyer's total cost. Therefore, the buyer always prefers to decrease η_U as much as possible. The buyer's tendency to deviate holds for any value of $\eta_U > 0$. Therefore, a pooling equilibrium with $\eta_U > 0$ fails to exist in regions where PE-4 is the outcome of the least costly game with $p_U^{low} = c_U$: i.e. where $\underline{\alpha} \geq \frac{c_N\bar{\alpha}^2}{c_U}$.

But if $p_r < \sqrt{p_U^{low} c_N/\underline{\alpha}}$, supplier N's price as the winning price would be p_r and the total cost of the buyer would be $\eta_U p_r + (1-\eta_U)p_r = p_r$. As a result, the buyer is indifferent to choose any η_U between 0 and 1.

For other cases, if $p_U^{low} \leq p_U^{Int}$ any other equilibrium outcome except PE-4 and PE-5 may prevail, i.e. p_U can be either p_U^{Int}, $(\bar{\alpha}c_N + c_U)/2$, or $(\bar{\alpha}p_r + c_U)/2$. Total cost of the buyer, therefore, is either Qp_U (if U wins) or $Q(\eta_U p_r + (1-eta_U)p_N)$ (if N wins). In either case, the total cost of the buyer is independent from the choice of η_U. Hence, the buyer is indifferent between any value of

- $0 \leq \eta_U \leq \frac{c_N\bar{\alpha}^2 + \bar{\alpha}\sqrt{c_N^2\bar{\alpha}^2 + 8\underline{\alpha}c_N c_U} - 4c_U\underline{\alpha}}{8\underline{\alpha}(p_r - c_U)}$ $(\eta : p_U^{low}(\eta) = p_U^{Int})$ when PE-1 is the outcome of the game.
- $0 \leq \eta_U \leq \frac{\bar{\alpha}p_r - c_U}{2(p_r - c_U)}$ $(\eta : p_U^{low}(\eta) = \frac{\bar{\alpha}p_r + c_U}{2})$ when PE-2 prevails.
- $0 \leq \eta_U \leq \frac{\bar{\alpha}c_N - c_U}{2(p_r - c_U)}$ $(\eta : p_U^{low}(\eta) = \frac{\bar{\alpha}c_N + c_U}{2})$ when PE-3 are the outcome of the game.

By the choice of η_U as mentioned above, the suppliers acquire no extra information and the buyer has no incentive to deviate.

Now, let find different ranges of $\underline{\alpha}$ and $\bar{\alpha}$ where different outcomes of the least costly games ($\eta_i = 0$) may happen. Note that we use c_i instead of a general p_i^{low} because η_i can be any value between zero and $\eta_i^{max} > 0$ (found previously) such that it should not affect the equilibrium prices of the suppliers in the competition compared to the case when $\eta_i = 0$; otherwise, the buyer will have an incentive to deviate from equilibrium as discussed above.

1. To find the region where internal solution (PE-1) is the equilibrium, we should have $c_N \leq p_N^{Int} \leq p_r$ and $c_U \leq p_U^{Int} \leq p_r$. By solving these equations, we get the range of $\underline{\alpha} \leq \frac{c_U + c_N\bar{\alpha}}{2c_N}$, $\underline{\alpha} \leq \frac{c_N\bar{\alpha}^2}{c_U}$, and

$\underline{\alpha} \geq \frac{c_N(c_U+\bar{\alpha}p_r)}{2p_r^2}$. In this range, the internal equilibrium outcome would happen. Supplier U wins the auction if $0 < \alpha < \frac{p_U}{p_N} = \frac{(\bar{\alpha}^2 c_N + \bar{\alpha}\sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U} + 4\underline{\alpha} c_U)}{2(\bar{\alpha} c_N + \sqrt{\bar{\alpha}^2 c_N^2 + 8\underline{\alpha} c_N c_U})}$ and in the rest of the range supplier N wins.

2. To find the region where PE-2 happens, we should have $p_r < p_N^{Int}$ and $c_U \leq p_U(p_r) \leq p_r$. By solving these equations, we get the range of $\underline{\alpha} \geq \frac{c_N(c_U+\bar{\alpha}p_r)}{2p_r^2}$ and $\underline{\alpha} \leq \frac{c_N\bar{\alpha}^2}{c_U}$. Supplier U wins the auction if $0 < \alpha < \frac{p_U}{p_N} = \frac{\bar{\alpha}p_r + c_U}{2p_r}$ and in the rest of the range supplier N wins.
3. In the region where PE-3 happens, we should have $p_N^{Int} < c_N$ and $c_U \leq p_U(c_N) \leq p_r$. By solving these equations we get the range of $\underline{\alpha} \geq \frac{c_U + c_N\bar{\alpha}}{2c_N}$. In this region, the buyer always orders from supplier U because $\underline{\alpha} \geq \frac{p_U}{p_N} = \frac{\bar{\alpha}c_N + c_U}{2c_N}$.
4. In order to have PE-4 as the equilibrium outcome, we should have $p_U^{Int} < c_U$ and $c_N \leq p_N(c_U) \leq p_r$, which leads to a region of $\underline{\alpha} \geq \frac{c_N\bar{\alpha}^2}{c_U}$ and $\underline{\alpha} \geq \frac{c_U c_N}{p_r^2}$. In this region, the buyer always orders from supplier N because $\bar{\alpha} \leq p_U/p_N = \sqrt{\frac{c_U\underline{\alpha}}{c_N}}$.
5. PE-5 can be the outcome of the game if $p_U^{Int} < c_U$ and $p_r < p_N(c_U)$, which only occur at a region of $\underline{\alpha} \geq \frac{c_N\bar{\alpha}^2}{c_U}$ and $\underline{\alpha} \leq \frac{c_U c_N}{p_r^2}$. Like case 4, in this region also the buyer always orders from N because $\bar{\alpha} \leq p_U/p_N = c_U/p_r$. □

Proof of Proposition 3: In order to prove these statements, we focus on the equilibrium prices of both suppliers when the buyer's advance guarantee conveys no information (Table 3). The effect of cost heterogeneity of suppliers (captured by increasing c_U at a fixed c_N) and lower and upper bounds of suppliers' belief can be easily verified by taking the first-order derivative of profit functions with respect to c_U, $\underline{\alpha}$, and $\bar{\alpha}$, respectively. It is skipped because of simplicity. □

Proof of Lemma 2: Assume supplier $i = U, N$ is to be offered an advance revenue guarantee of $\eta_i Q p_r$. Similar to the symmetric information setting, the buyer's cost function is as follows:

$$TC_B(p_N, p_U, \alpha, p_r, \eta_i) = \begin{cases} Q \times p_i & p_i/QS_i \leq \frac{p_{-i}}{QS_{-i}}, p_i \leq p_r \\ Q \times p_{-i} & p_i/QS_i > \frac{p_{-i}}{QS_{-i}}, p_{-i} \leq p_r \\ Q \times p_r & else \end{cases} \tag{9}$$

It implies that the buyer will surely give the supplier i an order of at least $\eta_i Q$ even if he loses the competition. This cost function leads to the following allocation policy.

$$(q_i^*(p_N, p_U, \alpha, p_r, \eta_i), q_{-i}^*(p_N, p_U, \alpha, p_r, \eta_i)) = \begin{cases} (Q, 0) & p_i/QS_i \leq \frac{p_{-i}}{QS_{-i}}, p_i \leq p_r \\ (\eta_i Q, (1-\eta_i)Q) & p_i/QS_i > \frac{p_{-i}}{QS_{-i}}, p_{-i} \leq p_r \\ (\eta_i Q, 0) & else \end{cases} \tag{10}$$

Similar to the symmetric information setting (Proposition 1), we can find the best response of the suppliers as following.

$$p_N^*(p_U) = min(max(p_N^{low}; \frac{p_U}{\alpha}); p_r)$$
$$p_U^*(p_N) = min(max(p_U^{low}; \alpha p_N); p_r)$$

where $p_i^{low} = c_i + \eta_i(p_r - c_i)$ and $p_{-i}^{low} = c_i$ for $i = U$ or N as the guaranteed supplier and $-i$ as the other supplier.

It is easy now to see that the winner of the auction in the equilibrium is

- supplier U for $0 < \alpha < \frac{p_U^{low}}{p_N^{low}}$
- supplier N for $\frac{p_U^{low}}{p_N^{low}} < \alpha \leq 1$

Now by knowing the suppliers' optimal pricing and the buyer's optimal order allocation policy, we can characterize the equilibrium as presented in Table 5. □

Table 5: Best responses of the suppliers and the buyer after receiving the signal: Separating equilibrium ($c_N < c_U$).

Advance Guarantee (buyer's signal)		$0 \leq \eta_N \leq 1$			$0 \leq \eta_U \leq 1$	
Range of α		$0 < \alpha \leq \gamma_2^N$	$\gamma_2^N \leq \alpha \leq \min(1, \gamma_1^N)$	$\min(1, \gamma_1^N) \leq \alpha \leq 1$	$0 < \alpha \leq \gamma^U$	$\gamma^U \leq \alpha \leq 1$
Prices (bids)	p_N^* p_U^*	p_r c_U	$\frac{c_U}{\alpha}$ c_U	p_N^{low} αp_N^{low}	p_r p_U^{low}	$\frac{p_U^{low}}{\alpha}$ p_U^{low}
Order Alloc	q_N^*, q_U^*	$Q, 0$	$Q, 0$	$\eta_N Q, (1-\eta_N)Q$	$(1-\eta_U)Q, \eta_U Q$	$(1-\eta_U)Q, \eta_U Q$
Suppliers' Profits	π_N^* π_U^*	$Q(p_r - c_N)$ 0	$Q(\frac{c_U}{\alpha} - c_N)$ 0	$\eta_N Q(p_r - c_N)$ $(1-\eta_N)Q(\alpha p_N^{low} - c_U)$	$(1-\eta_U)Q(p_r - c_N)$ $\eta_U Q(p_r - c_U)$	$(1-\eta_U)Q(\frac{p_U^{low}}{\alpha} - c_N)$ $\eta_U Q(p_r - c_U)$
Buyer's Cost	TC_B^*	Qp_r	$Q\frac{c_U}{\alpha}$	$Q(\eta_N p_r + (1-\eta_N)\alpha p_N^{low})$	Qp_r	$Q(\eta_U p_r + (1-\eta_U)\frac{p_U^{low}}{\alpha})$

† Note that $p_i^{low} = c_i + \eta_i(p_r - c_N)$ for $i = U$ and N only if the supplier i is offered an advance revenue guarantee of $\eta_i Q p_r$.

Proof of Proposition 5: In order to characterize seperating equilibria, we analyze each signaling tool separately.

First, assume $i = U$, i.e. the buyer offers the guarantee to supplier U ($\eta_U \geq 0$). Given the suppliers' optimal response to the buyer's signal (Lemma 2), let consider the following scenarios for the prior belief of the suppliers ($\underline{\alpha}$, $\bar{\alpha}$):

1. $\frac{p_U^{low}}{c_N} < \underline{\alpha} < \alpha < \bar{\alpha} < 1$: suppliers cannot infer any information from η_U offered and, for example, supplier U can always take $p_U = c_N\underline{\alpha}$ and win the auction with no need for more information.
2. $0 < \underline{\alpha} < \alpha < \bar{\alpha} < \frac{p_U^{low}}{p_r}$: in this situation, the buyer has no incentive to signal the information, because in any case her final unit cost would be p_r which is already accessible before holding the auction.
3. $\frac{p_U^{low}}{p_r} < \underline{\alpha} < \alpha < \bar{\alpha} < \frac{p_U^{low}}{c_N}$: In the equilibrium, when the buyer chooses $\eta_U(\hat{\alpha})$, the suppliers infer $\hat{\alpha}$ and respond accordingly by their price choice. We should find $\eta_U(\alpha)$ in such a way that the buyer never finds any incentive to cheat, i.e. $TC_B(\eta_U(\alpha), p_N(\alpha), p_U(\alpha), \alpha) \leq TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha)$.

The buyer's profit if her real type is α and she signals $\hat{\alpha}$ would be:

$$TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha) = \begin{cases} \eta_U(\hat{\alpha})Qp_r + (1-\eta_U(\hat{\alpha}))Q\frac{p_U^{low}}{\hat{\alpha}} & \hat{\alpha} = \alpha \\ \eta_U(\hat{\alpha})Qp_r + (1-\eta_U(\hat{\alpha}))Q\frac{p_U^{low}}{\hat{\alpha}} & \hat{\alpha} > \alpha \\ Qp_U^{low} & \hat{\alpha} < \alpha \end{cases}$$

In order to prevent the buyer with a true type of α to signal an $\hat{\alpha} > \alpha$, it is easy to see that $\eta_U(\alpha)$ should be increasing in α and take the highest value at $\alpha = \bar{\alpha}$. But no matter how $\eta_U(\alpha)$ is chosen, the buyer is always motivated to signal an $\hat{\alpha} < \alpha$ because in that case $TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha) = Qp_U^{low}$ which is strictly lower than $TC_B(\eta_U(\alpha), p_N(\alpha), p_U(\alpha), \alpha) = \eta_U(\alpha)Qp_r + (1-\eta_U(\alpha))Q\frac{p_U^{low}}{\alpha}$. Therefore, the second condition for a separating equilibrium cannot be satisfied.

Given the impossibility of signaling α by η_U in all the above scenarios, information sharing is not feasible in any other possible range of $[\underline{\alpha}, \bar{\alpha}]$ because it should include a subset of the ranges discussed above in which signaling fails.

Second, assume $i = N$, i.e. the buyer offers the guarantee to supplier N ($\eta_N \geq 0$). Likewise, the following scenarios can be examined:

1. $0 < \underline{\alpha} < \alpha < \bar{\alpha} < \frac{c_U}{p_r}$: the buyer has no incentive to signal the information, because in any case her final unit cost would be p_r which is accessible even without the auction.

2. $\frac{c_U}{p_r} < \underline{\alpha} < \alpha < \bar{\alpha} < \frac{c_U}{p_N^{low}}$: suppliers cannot infer any information from η_N offered and, for example, supplier N can always take $p_U = c_U/\bar{\alpha}$ and win the auction with no need for more information. In fact, in this case, the guarantee is not enough to change suppliers' beliefs.
3. $\frac{c_U}{p_N^{low}} < \underline{\alpha} < \alpha < \bar{\alpha} < 1$: In the equilibrium, when the buyer chooses $\eta_N(\hat{\alpha})$, the suppliers infer $\hat{\alpha}$ and respond accordingly by their price choice. We should now find $\eta_N(\alpha)$ in such a way that the buyer never finds any incentive to cheat, i.e. $TC_B(\eta_N(\alpha), p_N(\alpha), p_U(\alpha), \alpha) \leq TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha)$.

The buyer's profit if her real type is α and she signals $\hat{\alpha}$ would be:

$$TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha) = \begin{cases} \eta_N(\hat{\alpha})Qp_r + (1-\eta_N(\hat{\alpha}))Q\hat{\alpha}p_N^{low} & \hat{\alpha} = \alpha \\ Qp_N^{low} & \hat{\alpha} > \alpha \\ \eta_N(\hat{\alpha})Qp_r + (1-\eta_N(\hat{\alpha}))Q\hat{\alpha}p_N^{low} & \hat{\alpha} < \alpha \end{cases}$$

For this to be an equilibrium, the buyer should always be better off (or at least indifferent) to signal her true type instead of any other wrong value. First, let find the optimal revenue guarantee for which the buyer never signals a lower value of α. In that case, the minimum value of $TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha) = \eta_N(\hat{\alpha})Qp_r + (1-\eta_N(\hat{\alpha}))Q\hat{\alpha}p_N^{low}$ should be always $\hat{\alpha} = \alpha$, or equivalently, first order condition should be satisfied at $\hat{\alpha} = \alpha$.

$$\frac{\partial(\eta_N(\hat{\alpha})Qp_r + (1-\eta_N(\hat{\alpha}))Q\hat{\alpha}p_N^{low})}{\partial\hat{\alpha}} \Big|_{\hat{\alpha}=\alpha} = 0.$$

This leads to the following differential equation:

$$(c_N + \eta_N(\alpha)(p_r - c_N))(1-\eta_N(\alpha)) + [p_r\alpha - 2\alpha c_N - 2\alpha\eta_N(\alpha)(p_r - c_N) + p_r]\frac{\partial\eta_N(\alpha)}{\partial\alpha} = 0.$$

This equation has to hold for all $\alpha \leq \bar{\alpha}$, therefore $\eta_N(\alpha)$ should be of the following form:

$$\eta_N(\alpha) = \frac{C_1\alpha p_r + C_1 p_r - 2C_1\alpha c_N - \sqrt{C_1^2\alpha^2 p_r^2 + 2C_1^2\alpha p_r^2 + C_1^2 p_r^2 - 4C_1\alpha}}{2C_1\alpha(p_r - c_N)}$$

where C_1 is a constant.

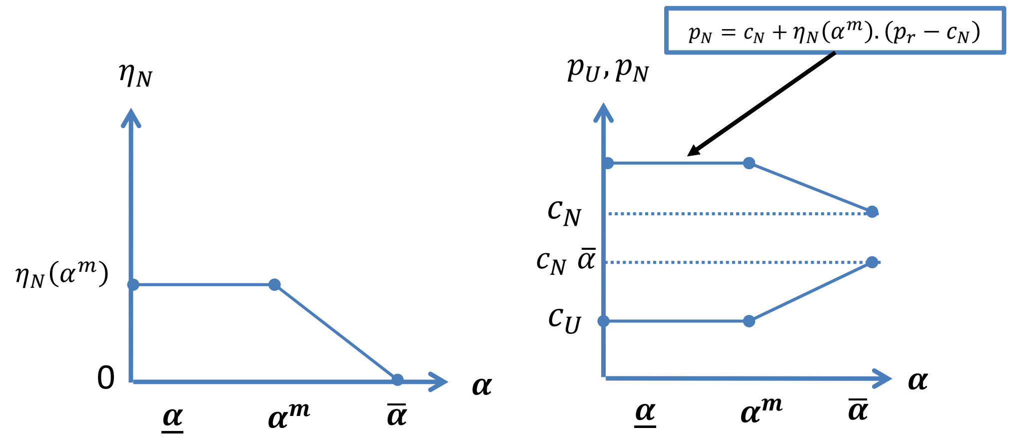
This $\eta_N(\alpha)$ transforms the buyer's cost function to a fixed constant value equal to $TC_B = \frac{1-C_1c_Np_r}{C_1(p_r-c_N)}$. Therefore, the problem switches to finding the minimum C_1 such that $\eta_N(\alpha)$ is - first, between 0 and 1 for all values of $\underline{\alpha} < \alpha < \bar{\alpha}$; -second, one-to-one (because the cost function is continuous in this range, it is sufficient to have $\eta_N'(\alpha)$ non-negative or non-positive for all the values of α). It is easy to see that $C_1 = \frac{1}{c_N(\bar{\alpha}p_r - \bar{\alpha}c_N + p_r)}$ satisfies all the conditions and gives the most efficient signaling tool: $\eta_N(\bar{\alpha}) = 0$; $0 < \eta_N(\underline{\alpha}) < 1$ (even for $\alpha = 0$, $\lim_{\alpha\to 0}\eta_N(\alpha) = \frac{\bar{\alpha}c_N}{p_r} < 1$); also $\eta_N(\alpha)$ is strictly decreasing in α and takes the highest value at $\alpha = \underline{\alpha}$. The buyer's total cost would be then $c_N\bar{\alpha}$.

We found a minimum revenue guarantee that prevents the buyer to signal a lower value of α than her true type. Interestingly, the buyer will never choose to signal a higher value of $\hat{\alpha} > \alpha$ because in that case: $TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha) = Qp_N^{low} > c_N\bar{\alpha}$. By utilizing this costly signaling tool, supplier U's price is increasing in α while supplier N's is decreasing until it reaches c_N.

There is only one note here: supplier U will not offer a price lower than c_U. Therefore, if for any value of α, the value of $\alpha(c_N + \eta_N(\alpha)(p_r - c_N)$ becomes lower than c_U (this happens when $\alpha < \alpha^m = \frac{(p_r - c_U)c_U}{c_N\bar{\alpha}p_r - \bar{\alpha}c_N^2 + c_Np_r - p_rc_U}$), the buyer will not guarantee more than a threshold $\eta_N(\alpha^m)$ where α^m is the point in which $\alpha(c_N + \eta_N(\alpha)(p_r - c_N) = c_U$. This is because the buyer becomes worse off by continuing to increase the guarantee for α lower than α^m as the supplier U does not decrease his price anymore, while supplier N increases his price.

Therefore, signaling becomes possible only when $\alpha^m < \bar{\alpha}$ or equivalently, when $\bar{\alpha}(c_N + \eta_N(\bar{\alpha})(p_r - c_N) \geq c_U$, which translate to a situation where $\bar{\alpha} > \frac{c_U}{c_N}$.

Since the buyer has no incentive to manipulate this signal, the suppliers will update their belief in the following fashion. If they observe $0 \leq \eta_N \leq \min(\eta_N(\underline{\alpha}), \eta_N(\alpha^m))$ they update their belief using $\eta_N^{-1}(\eta_N(\alpha))$;



but if they observe any value $\eta_N > \min(\eta_N(\underline{\alpha}), \eta_N(\alpha^m))$ they believe that $\underline{\alpha} \leq \alpha < \alpha^m$. In the former case, the equilibrium price by the suppliers is $p_N = c_N + \eta_N(p_r - c_N)$ and $p_U = \alpha(c_N + \eta_N(p_r - c_N))$ while in the latter, $p_N = c_N + \eta_N(p_r - c_N)$ and $p_U = c_U$. □

Proof of Proposition 6: Before analyzing the separating equilibrium, it is worthwhile to study the equilibrium under the symmetric information setting (as benchmark) when $c_U > c_N$. Under symmetric information, the equilibrium bids would be $p_U^* = c_U$ and $p_N^* = \min(c_U/\alpha, p_r)$ for $0 < \alpha \leq 1$. In that case, in equilibrium supplier N always wins at a price of $\min(c_U/\alpha, p_r)$ and makes some profit ($p_N^* > c_N$).

As we discussed in our previous analysis (Proposition 5), under asymmetric information, if a supplier is guaranteed to have at least an order of $\eta_i Q$, he may feel less pressure on him to win the auction at very low prices. As a result, supplier i never bids below $p_i^{low} = c_i + \eta_i(p_r - c_i)$ where $p_i^{low} > c_i$ for $i = U$ or N. The following lemma characterizes the suppliers' optimal bids and the equilibrium profits if $c_U > c_N$ and they receive truthful information regarding α.

Lemma 3 *Let assume $\gamma^U = p_U^{low}/p_r$,$\gamma_1^N = c_U/p_N^{low}$,and $\gamma_2^N = c_U/p_r$. In the separating equilibrium, after observing the revenue guarantee $\eta_i Q p_r$ to supplier i (i=U or N) and correctly inferring α, the suppliers' optimal bid would be as presented in the Table 5.*

The proof of this lemma directly follows from the definition of p_i^{low} and the similarity to the symmetric information setting when the suppliers become informed of the true type.

A separating equilibrium must satisfy all the requirements mentioned before: a one-to-one signal by a buyer who should have no incentive to deviate. We first, assume an interior separating equilibrium exists. Then, according to the Lemma 3, the suppliers' best response p_U and p_N and order allocation can be characterized depending on the buyer's choice of signal. In the equilibrium, when the buyer chooses $\eta_i(\hat{\alpha})$, the suppliers infer $\hat{\alpha}$ and bid according to $\hat{\alpha}$. Following, we analyze each signaling tool separately.

1) Let $i = U$, i.e. the buyer guarantees a minimum revenue of $\eta_U Q p_r$ to supplier U: In the equilibrium, when the buyer chooses $\eta_U(\hat{\alpha})$, the suppliers infer $\hat{\alpha}$ and respond accordingly by their price choice. We should find $\eta_U(\hat{\alpha})$ in such a way that the buyer never finds any incentive to cheat, i.e. $TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha) \leq TC_B(\eta_U(\alpha), p_N(\alpha), p_U(\alpha), \alpha)$.

The buyer's profit would be

$$TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha = \hat{\alpha}) = \eta_U(\hat{\alpha})Qp_r + (1 - \eta_U(\hat{\alpha}))Qp_N(\hat{\alpha})$$
$$TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha < \hat{\alpha}) = \eta_U(\hat{\alpha})Qp_r + (1 - \eta_U(\hat{\alpha}))Qp_N(\hat{\alpha})$$
$$TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha > \hat{\alpha}) = Qp_U$$

For this to be an equilibrium, the buyer's total cost should be minimized at $\hat{\alpha} = \alpha$ if the true type of the buyer is α. First, let find the optimal advance guarantee in which the buyer never signals a higher value of α.

In that case, the following first order condition should be satisfied at $\hat{\alpha} = \alpha$:

$$\frac{\partial TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha)}{\partial \hat{\alpha}} \,|_{\hat{\alpha}=\alpha=0} \,.$$

And, it leads to the following guarantee function:

$$\eta_U(\alpha) = \frac{-\alpha p_r + 2c_U - p_r + \sqrt{\alpha^2 p_r^2 - 4c_U \alpha p_r + 2\alpha p_r^2 + p_r^2 + 4p_r C_1 \alpha - 4c_U C_1 \alpha}}{2(-p_r + c_U)}$$

where C_1 is a constant and should be chosen such that $\eta_N(\alpha)$ satisfies all the required conditions and gives the most efficient signaling tool. It is easy to see that $\eta_N(\alpha)$ should be increasing in α and takes the highest value at $\alpha = \bar{\alpha}$. By Using this guarantee, if the buyer sticks to the truthful signaling of $\alpha = \hat{\alpha}$, his total cost would be fixed equal to $\min(p_r, \frac{c_U}{\underline{\alpha}}) > c_U$. But, using this signaling tool, the buyer always prefer to guarantee $\eta_U(\hat{\alpha})$ to signal a QS of $\hat{\alpha}$ far lower than α. In fact by $\eta_U \simeq 0$, the buyer signals that α is very low close to $\underline{\alpha}$ to receive bids of $p_U = c_U$ and $p_N = min(p_r, c_U/\underline{\alpha})$. In that case, supplier N loses the auction while supplier U wins at its lowest possible price c_U. Therefore, $TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha > \hat{\alpha}) < TC_B(\eta_U(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha = \hat{\alpha})$ which makes the separating equilibrium fail to exist and work.

2) Let $i = N$, i.e. the buyer guarantees a minimum revenue of $\eta_N Q p_r$ to the supplier N: In the equilibrium, the guarantee to supplier N, $\eta_N Q p_r$, cannot be lower than what makes $p_N^{low} = c_U/\bar{\alpha}$, otherwise it transforms no information as supplier N can always takes $p_N = c_U/\bar{\alpha}$ and wins the auction with no need for more information. By a similar reasoning as previous part, we can show that $\eta_N(\alpha)$ should be decreasing at α to make sure that the buyer never signals a $\hat{\alpha}$ lower than α.

$$TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha = \hat{\alpha}) = \eta_N(\hat{\alpha}) Q p_r + (1 - \eta_N(\hat{\alpha})) Q p_U(\hat{\alpha})$$

$$TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha > \hat{\alpha}) = \eta_N(\hat{\alpha}) Q p_r + (1 - \eta_N(\hat{\alpha})) Q p_U(\hat{\alpha})$$

$$TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha < \hat{\alpha}) = Q p_N$$

But, in the same fashion as before, with decreasing signal, the buyer would be always motivated to signal a higher value of α (by a lower guarantee) to mislead both suppliers in a way such that supplier U offers a higher price than expected in the equilibrium while supplier N offers his lowest possible price and wins the auction at $p_N = \frac{c_U}{\bar{\alpha}}$. Therefore, $TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha < \hat{\alpha}) < TC_B(\eta_N(\hat{\alpha}), p_N(\hat{\alpha}), p_U(\hat{\alpha}), \alpha = \hat{\alpha})$, which makes the separating equilibrium fail to exist and work. □

Proof of Proposition 7: To prove that $p_N^{*SE} = c_N + \eta_N(p_r - c_N)$ is decreasing (which is equivalent to showing that the signal η_N is decreaing at α), we take the first derivative of p_N^{*SE} with respect to α and show that it is always negative given our assumptions on parameters α, $\underline{\alpha}$, $\bar{\alpha}$, $\frac{c_U}{c_N}$, and p_r.

$$\frac{\partial p_N^{*SE}}{\partial \alpha} = -\frac{\sqrt{C_1^2 p_r^4 (1+\alpha)^2 - 4C_1 \alpha p_r^2} - C_1 p_r^2 (1+\alpha) + 2\alpha}{2\alpha^2 \sqrt{C_1^2 p_r^2 (1+\alpha)^2 - 4C_1 \alpha}} \tag{1}$$

First, let find the acceptable domain, where the above derivative is meaningful. Domain is the range of parameters where $C_1 p_r^2 (1+\alpha)^2 \geq 4\alpha$, which includes any point of α and any upper and lower bound such that $0 < \underline{\alpha} \leq \alpha \leq \bar{\alpha} \leq 1$. This is because if we plug the real value of C_1, in the meaningful region, we should have $\frac{p_r^2 (1+\alpha)^2}{c_N [p_r + \bar{\alpha}(p_r - c_N)]} \geq 4\alpha$. The lowest value for the left-hand-side at any given α occurs at $\bar{\alpha} = 1$. And, it is easy to show that $p_r^2 (1+\alpha)^2 \geq 4\alpha c_N (2p_r - c_N)$ since $p_r^2 \geq c_N (2p_r - c_N)$ (for any $c_N \leq p_r$) and $(1+\alpha)^2 \geq 4\alpha$ (for any $0 < \alpha \leq 1$).

Since the denumerator in (1) is always positive; it is sufficient to show that the numerator is always negative, or

$$\sqrt{C_1^2 p_r^4 (1+\alpha)^2 - 4C_1 \alpha p_r^2} - C_1 p_r^2 (1+\alpha) + 2\alpha > 0. \tag{2}$$

Let denote $A = C_1 p_r^2 (1+\alpha)$ and $B = 4C_1 \alpha p_r^2$; hence we should show that

$$\sqrt{A^2 - B} - A + 2\alpha > 0. \tag{3}$$

And, this holds only if

$$B < 4A\alpha - 4\alpha^2. \tag{4}$$

If we replug the original values at (4), we find that the previous inequality (3) holds only if $C_1 p_r^2 > 1$, or equivalently when

$$\frac{p_r^2}{c_N[p_r + \bar{\alpha}(p_r - c_N)]} > 1. \tag{5}$$

Given $0 < \bar{\alpha} \leq 1$ the minimum of the fraction (the maximum of the denumerator) would be at $\bar{\alpha} = 1$. Therefore, we should have

$$\frac{p_r^2}{c_N(2p_r - c_N)} > 1 \tag{6}$$

which is always the case, since the maximum amount of $c_N(2p_r - c_N)$ would be at $c_N = p_r$ and for any $c_N < p_r$ we have $p_r^2 > c_N(2p_r - c_N)$. If we go back from statement (6) to (1), we have proved that p_N^{*SE} is strictly decreasing at α everywhere. But, considering the result of Proposition 5, supplier N's price indeed would be constant for $\alpha < \alpha^m$ (α^m given at Proposition 5) and strictly decreasing for values of $\alpha \geq \alpha^m$.

A simlar approach applies to prove that p_U^{*SE} is increasing at α.

$$\frac{\partial p_U^{*SE}}{\partial \alpha} = \frac{\sqrt{C_1^2 p_r^4 (1+\alpha)^2 - 4C_1 \alpha p_r^2} - C_1 p_r^2 (1+\alpha) + 2}{2\sqrt{C_1^2 p_r^2 (1+\alpha)^2 - 4C_1 \alpha}}$$

It is sufficient to show that the numerator is always positive, or $\sqrt{C_1^2 p_r^4 (1+\alpha)^2 - 4C_1 \alpha p_r^2} - C_1 p_r^2 (1+\alpha) + 2 > 0$.

Therefore, by the same transformation of variables to A and B, we should show that $\sqrt{A^2 - B} - A + 2 > 0$. This holds only if $B < 4A - 4$. If we replug the original values, we find that the previous inequality holds only if $C_1 p_r^2 > 1$, or equivalently when $\frac{p_r^2}{c_N[p_r + \bar{\alpha}(p_r - c_N)]} > 1$. And, this is always true as we proved it in the previous part. Therefore, supplier U's price would be increasing at α in the accepatable domain which includes $\bar{\alpha} > \frac{c_U}{c_N}$: in fact it remains fixed for $\alpha < \alpha^m$ and strictly increases for $\alpha \geq \alpha^m$. □

Proof of Proposition 8: First, we characterize the expected value of equilibrium decisions in pooling and separating equilibria. The expectation in both cases is only with respect to α. The equilibrium decisions does not depend on α under the pooling equilibrium. Therefore, we can easily capture them from Proposition 2 as presented again in Table 6.

Table 6: Expected value of equilibrium decisions under pooling and separating equilibria for any point in region C_1.

Equilibrium		Pooling	Separating
Prices (bids)	$\bar{p}_N^*$	c_N	$c_N + \mu_{\eta_N}(p_r - c_N)$
	$\bar{p}_U^*$	$\frac{c_U + c_N\bar{\alpha}}{2}$	$(c_N\mu_\alpha + (p_r - c_N)[\sigma^2_{\alpha,\eta_N} + \mu_\alpha \mu_{\eta_N}])$
Order Alloc	$\bar{q}_N^*$	0	$Q\mu_{\eta_N}$
	$\bar{q}_U^*$	Q	$Q(1 - \mu_{\eta_N})$

For the separating equilibrium, we first show that in all the points of $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$ such that $\underline{\alpha} \geq \frac{c_U + c_N\bar{\alpha}}{2c_N}$ fully separating is possible (Figure 4). To do so, it is sufficient to show that the region $\underline{\alpha} \geq \frac{(p_r - c_U)c_U}{c_N\bar{\alpha}p_r - \bar{\alpha}c_N^2 + c_N p_r - p_r c_U}$ (the locus of points where fully separating is possible) always include the whole region of $\underline{\alpha} \geq \frac{c_U + c_N\bar{\alpha}}{2c_N}$; and it is enough to verify that the line $\underline{\alpha} = \frac{(p_r - c_U)c_U}{c_N\bar{\alpha}p_r - \bar{\alpha}c_N^2 + c_N p_r - p_r c_U}$ is always lower than $\underline{\alpha} \geq \frac{c_U + c_N\bar{\alpha}}{2c_N}$ for $\frac{c_U}{c_N} < \bar{\alpha} < 1$ (Figure 4, Proposition 2). Both lines are continous and start from the point ($\underline{\alpha} = c_U/c_N, \bar{\alpha} = c_U/c_N$).; but the former and the latter are strictly decreasing and increasing in $\bar{\alpha}$, respectively; i.e.,

$$\underline{\alpha} = \frac{(p_r - c_U)c_U}{c_N\bar{\alpha}p_r - \bar{\alpha}c_N^2 + c_N p_r - p_r c_U} \qquad \frac{\partial \underline{\alpha}}{\partial \bar{\alpha}} = -\frac{(p_r - c_U)c_U(c_N p_r - c_N^2)}{(c_N\bar{\alpha}p_r - \bar{\alpha}c_N^2 + c_N p_r - p_r c_U)^2} < 0$$

$$\underline{\alpha} \geq \frac{c_U + c_N\bar{\alpha}}{2c_N} \qquad \frac{\partial \underline{\alpha}}{\partial \bar{\alpha}} = \frac{1}{2} > 0$$

Hence, the region $\underline{\alpha} \geq \frac{(p_r - c_U)c_U}{c_N\bar{\alpha}p_r - \bar{\alpha}c_N^2 + c_N p_r - p_r c_U}$ always fully include the whole region of $\underline{\alpha} \geq \frac{c_U + c_N\bar{\alpha}}{2c_N}$.

Therefore, under separating equilibrium (Proposition 5), the expected value of equilibrium decisions w.r.t α is as expressed in Table 6.

Observing the values in the above table and considering that $\mu_{\eta_N} > 0$, it is easily verifiable that $\bar{p}_N^{*PE} < \bar{p}_N^{*SE}$, $\bar{q}_N^{*PE} < \bar{q}_N^{*SE}$, and $\bar{q}_U^{*PE} > \bar{q}_U^{*SE}$. Also, we know that $\bar{p}_U^{*SE} > \frac{c_N(\bar{\alpha}+\underline{\alpha})}{2}$ and since in the region C_3 in poolig equilibrium (Proposition 2) $\underline{\alpha} > \frac{c_U}{c_N}$, then we always have $\bar{p}_U^{*PE} < \bar{p}_U^{*SE}$. □

Proof of Proposition 9: We first characterize the expected value of equilibrium decisions with respect to α, according to Propositions 2 and 5, as expressed in the following table.

Table 7: Expected value of equilibrium profits/costs under pooling and separating equilibria for any point in region C_1.

Equilibrium		Pooling	Separating
Suppliers' Profits	$\bar{\pi}_N^*$	0	$Q\mu_{\eta_N}(p_r - c_N)$
	$\bar{\pi}_U^*$	$Q(\frac{c_U + c_N\bar{\alpha}}{2} - c_U)$	$Q[c_N\frac{\bar{\alpha}+\underline{\alpha}}{2} + (p_r - 2c_N)(\sigma^2_{\alpha,\eta_N} + \mu_\alpha\mu_{\eta_N}) + (p_r - c_N)E(\eta_N^2\alpha) + c_U\mu_{\eta_N} - c_U]$
Buyer's Cost	$\overline{TC}_B^*$	$Q\frac{c_U + c_N\bar{\alpha}}{2}$	$Qc_N\bar{\alpha}$
Total Supply Chain Cost	$\overline{TC}_{CS}^*$	Qc_U	$Q(\mu_{\eta_N}c_N + (1-\mu_{\eta_N})c_U)$

Based on the table and given that $\mu_{\eta_N} > 0$, it is easy to verify that $\bar{\pi}_N^{*PE} < \bar{\pi}_N^{*SE}$, $\overline{TC}_B^{*PE} < \overline{TC}_B^{*SE}$ (because $\bar{\alpha} > \frac{c_U}{c_N}$) and $\overline{TC}_{SC}^{*PE} < \overline{TC}_{SC}^{*SE}$. But for supplier U, since $q_U^{*SE} < q_U^{*PE}$ and $p_U^{*SE} > p_U^{*PE}$, his profit can be lower or higehr in pooling compared to separating equilibrium.

Pooling equilibrium in the presence of multiple suppliers It is easy to observe that the best response of the suppliers in a pooling equilibrium where they cannot infer any new information regarding the quality scores, would be as follows:

$$p_U^{i*} = \min(p_r, \max[c_U, f_U^i(p_N^*, p_U^{1*}, \ldots, p_U^{i-1*}, p_U^{i+1*}, \ldots, p_U^{m*})]) \qquad \forall i = 1, \ldots, m$$

$$p_N^* = \begin{cases} c_N & n > 1 \\ \min(p_r, \max[c_N, f_N(p_U^{1*}, \ldots, p_U^{m*})]) & n = 1 \end{cases}$$

To establish the internal solution of the bid prices in pooling equilibrium, first let assume that $n \geq 2$. In this case, all the incumbent known suppliers will offer $p_N^* = c_N$ because in a non-cooperative game they just undercut each other to a level where they cannot decrease their price anymore. Then, the profit function for an entrant supplier with the rank $i \in \{1, \ldots, m\}$ is as follows:

$$\pi_U^i = (p_i - c_U) \int_{p_i/c_N}^{\bar{\alpha}} \frac{1}{\bar{\alpha} - \underline{\alpha}} d\alpha_i \prod_{j\in\{1,\ldots,m\}, j>i} \int_{\underline{\alpha}}^{\bar{\alpha}} \int_{\frac{\alpha_j p_i}{p_j}}^{\bar{\alpha}} \frac{2}{(\bar{\alpha} - \underline{\alpha})^2} d\alpha_i d\alpha_j$$
$$\prod_{j\in\{1,\ldots,m\}, j<i} \int_{\underline{\alpha}}^{\bar{\alpha}} \int_{\underline{\alpha}}^{\frac{\alpha_i p_j}{p_i}} \frac{2}{(\bar{\alpha} - \underline{\alpha})^2} d\alpha_j d\alpha_i \quad \forall i \in 1, \ldots, m \quad (11)$$

As above, there are three integration terms in the suppliers' profit function. They, respectively, represent the probability of having a better QS-adjusted price (QSAP) than known suppliers, suppliers with lower QS's, and suppliers with higher QS ranks. Suppliers' difference in their QS ranking brings a source of asymmetry in their pricing decisions. By taking the first derivative of the profit functions of suppliers with respect to their prices, we come up to a non-linear system of equation with m variables $\{p_U^1, \ldots, p_U^m\}$:

$$\frac{\partial \pi_U^i}{\partial p_i} = 0 \quad \forall i \in \{1, \ldots, m\}.$$

There are some established mathematical and evolutionary algorithms to solve such non-linear system of equations. For instance, we used *fsolve* function in MATLAB that can use three different algorithms of ‘trust-region-dogleg’ (default), ‘trust-region-reflective’, and ‘levenberg-marquardt’. Solving this system of equation will provide the internal solution for the pooling equilibrium, which should be adjusted with the boundary conditions.

Now, assume $n = 1$, in this case the profit function of the known supplier and entrant suppliers is as follows.

$$\pi_N = (p_N - c_N) \prod_{i \in \{1,\dots,m\}} \int_{\underline{\alpha}}^{p_i/p_N} \frac{1}{\bar{\alpha} - \underline{\alpha}} d\alpha_i$$

$$\pi_U^i = (p_i - c_U) \int_{p_i/p_N}^{\bar{\alpha}} \frac{1}{\bar{\alpha} - \underline{\alpha}} d\alpha_i \prod_{j \in \{1,\dots,m\}, j>i} \int_{\underline{\alpha}}^{\bar{\alpha}} \int_{\frac{\alpha_j p_i}{p_j}}^{\bar{\alpha}} \frac{2}{(\bar{\alpha} - \underline{\alpha})^2} d\alpha_i d\alpha_j$$
$$\prod_{j \in \{1,\dots,m\}, j<i} \int_{\underline{\alpha}}^{\bar{\alpha}} \int_{\underline{\alpha}}^{\frac{\alpha_i p_j}{p_i}} \frac{2}{(\bar{\alpha} - \underline{\alpha})^2} d\alpha_j d\alpha_i \qquad \forall i \in 1,\dots,m$$

The only difference between π_U^i when $n > 1$ with it when $n = 1$ is that in the latter untested suppliers should consider an internal price $p_N^* \geq c_N$ for the incumbent supplier. Again, we should take the first derivative of profit function of suppliers w.r.t their prices and then solve the non-linear system of equations with $m + 1$ variables $\{p_N, p_U^1, \dots, p_U^m\}$. □

Proof of Proposition 10: The buyer’s cost function in case if all the parties infer her true type, would be:

$$TC_B = (\eta_N(\alpha_1,\alpha_2) + \eta_U(\alpha_1,\alpha_2))p_r + (1 - \eta_N(\alpha_1,\alpha_2) - \eta_U(\alpha_1,\alpha_2))[c_U + \eta_U(\alpha_1,\alpha_2)(\frac{p_r - c_U}{m})]\frac{\alpha_1}{\alpha_2}$$

In order for this unit cost to be minimized, the first order condition should be satisfied w.r.t. both α_1 and α_2. This leads to two partial differential equations (PDEs) as follows:

$$\begin{aligned}\frac{\partial TC_B}{\partial \alpha_1} &= \left(\frac{\partial \eta_N(\alpha_1,\alpha_2)}{\partial \alpha_1} + \frac{\partial \eta_U(\alpha_1,\alpha_2)}{\partial \alpha_1}\right) p_r + \left(-\left(\frac{\partial \eta_N(\alpha_1,\alpha_2)}{\partial \alpha_1}\right)\right) - \left(\frac{\partial \eta_U(\alpha_1,\alpha_2)}{\partial \alpha_1}\right)\\ &\quad \alpha_1 \big(c_U + \eta_U(\alpha_1,\alpha_2)(p_r - c_U)/m\big)/\alpha_2 + \big(1 - \eta_N(\alpha_1,\alpha_2) - \eta_U(\alpha_1,\alpha_2)\big)\big(c_U + \eta_U(\alpha_1,\alpha_2)(p_r - c_U)/m\big)\\ &\quad /\alpha_2 + \big(1 - \eta_N(\alpha_1,\alpha_2) - \eta_U(\alpha_1,\alpha_2)\big)\alpha_1 \left(\frac{\partial \eta_U(\alpha_1,\alpha_2)}{\partial \alpha_1}\right)(p_r - c_U)/(m\alpha_2)\end{aligned}$$

$$\begin{aligned}\frac{\partial TC_B}{\partial \alpha_2} &= \left(\frac{\partial \eta_N(\alpha_1,\alpha_2)}{\partial \alpha_2} + \frac{\partial \eta_U(\alpha_1,\alpha_2)}{\partial \alpha_2}\right) p_r + \left(-\left(\frac{\partial \eta_N(\alpha_1,\alpha_2)}{\partial \alpha_2}\right)\right) - \left(\frac{\partial \eta_U(\alpha_1,\alpha_2)}{\partial \alpha_2}\right)\\ &\quad \alpha_1 \big(c_U + \eta_U(\alpha_1,\alpha_2)(p_r - c_U)/m\big)/\alpha_2 + \big(1 - \eta_N(\alpha_1,\alpha_2) - \eta_U(\alpha_1,\alpha_2)\big)\alpha_1 \left(\frac{\partial \eta_U(\alpha_1,\alpha_2)}{\partial \alpha_2}\right)(p_r - c_U)\\ &\quad /(m\alpha_2) - \big(1 - \eta_N(\alpha_1,\alpha_2) - \eta_U(\alpha_1,\alpha_2)\big)\alpha_1 \big(c_U + \eta_U(\alpha_1,\alpha_2)(p_r - c_U)/m\big)/\alpha_2^2\end{aligned}$$

Solving this system of equation, the most efficient function for η_N and η_U would be as follows.

$$\eta_N(\alpha_1,\alpha_2) = \frac{\alpha_1 p_r - p_r \alpha_2 m + p_r \alpha_2 + \alpha_1 c_U m - 2\alpha_1 c_U}{\alpha_1(p_r - c_U)}$$
$$\eta_U(\alpha_1,\alpha_2) = \frac{(m-1)(p_r \alpha_2 - \alpha_1 c_U)}{\alpha_1(p_r - c_U)}$$

If we plug these two guarantees in total cost of the buyer, we get $TC_B = p_r$, which shows in the most efficient way the buyer has to take a cost of p_r which is accessible even without holding the auction. □

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