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# A game between two interconnected power utilities

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**Abstract:** This article studies the strategic withholding of physical transmission rights (PTRs) held by power producers in two different markets. Contrary to many previous studies, we assume that local markets are efficient but not the use of the interconnection. We model a two-settlement market, solved by backward induction. Our sensitivity analysis with respect to the allocation of PTRs brings insights on the strategic use of interconnections. Our main result is that PTR's withholding is an equilibrium strategy under quite soft conditions. This is an important consideration given the development of high-voltage direct current (HVDC) transmission lines to connect different markets.

**Key Words:** Game theory, power market, HVDC interconnection, physical transmission rights, withholding, regulation.

**Résumé:** Cet article étudie la rétention stratégique des droits de transmission physique (PTRs) détenus par des producteurs d'électricité sur deux marchés interconnectés. À la différence de nombreuses études précédentes, nous faisons l'hypothèse que chaque marché est localement efficient, mais pas l'utilisation de l'interconnexion entre eux. Nous résolvons par induction à rebours un modèle de marché à deux niveaux. L'analyse de sensibilité par rapport à l'allocation des PTRs donne un aperçu de son importance vis-à-vis de l'utilisation stratégique de l'interconnexion. Nous trouvons que, dans le cadre du modèle, la rétention de droits apparaît aisément. Cette considération revêt de l'importance dans la mesure où l'investissement dans les lignes de transmission à haute tension en courant continu (HVDC) a crû dans les dernières années.

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# 1 Introduction

Wholesale power market cannot be managed as any other. Power trade is exposed many market failures, such as non price-responsiveness of the demand-side, concentration of the supply-side, network externalities, expansive storage cost and load volatility. However, the liberalisation of power sector is still in the agenda of institution (e.g. the FERC and the European Commission). Such position is supported by the idea that regulation may preserves positive aspects of market - efficiency incentives - without potential dominant position abuses. For years, power market design research focused in the implementation of such efficient regulation on local market: internalisation of network externalities, bid based approach and efficient dispatch, scarcity signalling [9, 11, 18, 28, 30, 31].

Nevertheless, if trade between two nodes within a single jurisdiction has been widely treated in the literature, the effect of trade between two jurisdictions, with potentially different rules and associated incentives, has not been much studied. For example, [10] shows with a numerical simulation of Central Western Europe, that a centralized implicit auction design of transmission is always better than a decentralized explicit auction design. But this result may not apply so easily, since the level of coordination required to implement an integrated design is very important (joint optimization of every power flows in the network). Especially, it might be difficult and costly to coordinate in the case of a High Voltage Direct Current (HVDC) interconnections between two jurisdictions.

Efficiency of HVDC management raises many questions. Indeed, many aspects might impede their optimal use. Difference between markets rules could provide wrong incentives for producers [2, 3, 8]. Also, the lack of coordination between markets might cause inefficiency in the trading process [24]. Finally, allocation of transmission rights may provide dominant position in networks [6, 7, 21]. Empirical studies show evidences that interconnection's use between two control zones linked by a HVDC line is sub-optimal. It results in imperfect arbitrage of price differential between control zones. In other words, there still exists profit opportunities that traders would have otherwise taken advantage of in a contestable market. Such inefficiency could be explained by many factors: imperfect coordination [26, 29], incomplete information [29], withholding strategies where exchange is settled through physical transmission rights [5] or imperfect information [13]. However, to our knowledge, the only analytical article on interconnection between two power systems where a withholding strategy appears is in Joskow & Tirole [21]. Gilbert et al. [14] extends [21] in a game theoretical framework. However, they explicitly assume that the interconnection capacity is fully used. This assumption is indeed widely used in the numerical literature on market power in power networks. If this assumption may be justified in the case of an integrated treatment of energy and transmission through an implicit auction [7, 10, 17, 22, 23, 30], this may not be the case when energy and transmission are managed in separate mechanisms.

HVDC interconnections are increasingly built by merchant projects promoters, whose profits are based on the sale of interconnection access rights to power producers, retailers and traders. Allocation of physical rights to use this interconnection are in their vast majority to the discretion of the owner. For instance, the Champlain Hudson Power Express (CHPE) project plan to connect the south of the Québec province directly to New York City. In the case of CHPE, its owner - CHPE Incorporated - has been authorized by the Federal Energy Regulatory Commission (FERC) to charge negotiated rates for its transmission rights, with 75 percent of its capacity to be sold through bilateral negotiations and 25 percent through an open season [12]. Our intuition is that such allocation may bring significant dominant position if not monitored correctly. Finally, competitiveness of the trading process might be altered.

Our article aims at describing imperfect arbitrage between two nodes as a result of market power. We characterize withholding strategies of Physical Transmission Rights (PTR) on an HVDC interconnection between two nodes, each representing a power system. We assume that local regulation fosters marginal cost pricing on physical power markets. To do so, we develop a multi-stage model. The first stage correspond to an interconnection game, where players set their strategies to maximises their profit. The second stage corresponds to the physical market where production and consumption occur. This stage correspond to a local optimization by a system operator (SO) at each node, given the output of the interconnection stage.

Hence, the first stage corresponds to a financial market, in the sense that transactions at this stage does not result in immediate delivery, which will occur later, in the second stage.

In this model, and this is the main result of the article, we find that PTR's withholding is an equilibrium strategy under quite soft conditions. However, as long as a competitive fringe of traders owns a sufficient amount of rights, strategic behaviour of power producers is discouraged. In some cases, PTR's withholding maximises local welfare, which fits benevolent system operators best interests. But it is detrimental to them in many cases, as a SO internalizes only imperfectly the output of the interconnection market. Actually, we found that the equilibrium of this game is Pareto-Dominated by the perfect arbitrage situation from the local welfare point of view in many cases, even if one producer acts as a monopolist on the residual demand of the foreign market. Hence, a regulated market fails to provide on its own good incentives to maximises local welfare in interconnected networks.

Overall, the contribution of this article is manifold. First we propose to close the gap between the theoretical result of existence of withholding strategies of physical transmission rights found in [21] and the numerical literature by providing an implementable model of interconnection game where perfect arbitrage is not assumed *ex-ante*. Second, it brings new features in term of analytical characterization of withholding strategies. Hence, we provide necessary and sufficient conditions for existence and uniqueness of equilibrium. Finally, assuming functional forms, we characterize withholding strategy of players for any given level of ownership of transmission rights.

The article is organized as follows. Part two reviews the related literature. The general model is developed and discussed in part three. In the fourth part, we assume functional forms to fully characterize the equilibrium and to conduct comparative statics. Conclusive comments constitute the fifth part.

## 2 Literature review

An important part of the literature about strategic interactions in interconnected power systems is concerned with the impact of various levels of interconnection capacity on the local market power of generating firms. The mainstream idea of these studies is that an increase in interconnections capacity would permit to enlarge the relevant market size, thus increasing the competitiveness of local markets.

These studies, in a vast majority, assume a perfect arbitrage of the price differential, such that interconnection is always used at full capacity if there exists two different prices between two interconnected zones. Borenstein et al. [4] give an interesting analysis of such effect. They assume two interconnected nodes with one generator on each of them that set their quantity with respect to the price at their node settled by an Independent System Operator (ISO). They show that in that case even an infinitesimal capacity of interconnection bring a lot of welfare surplus as it keeps away the monopolistic equilibrium. They insist on the idea that the value of interconnection goes beyond its actual use, as its sole existence contains the threat of competition. They also show that the computation of such equilibrium - where each producer takes in account the reaction of the ISO - is difficult as it gives ways to a non-convex problem. This problem of non-convexity of transmission game in an electrical network was first found by Cardell et al. [7]. In this article, numerical insights are given about strategic interactions in a meshed three-nodes electrical network. Given some specific configuration on that network, one may find surprising results for a Cournot equilibrium: a Cournot firms may *increase* its generation level (w.r.t. the competitive level) in order to saturate the capacity constraint on one line, which limits the feasible set of reaction of a competitive fringe, hence raising the price settled at the consumption node. Several others numerical articles which deals with interconnected systems assume at least a perfect arbitrage of price differential, even if the latter may be gamed. This results in binding transmission constraints if price difference is non-zero.

Hobbs [16] compares the outcome of two Cournot-Nash models in a meshed network: no arbitrage versus perfect arbitrage. In the first case, only strategic producers sell power at different nodes, such that quantity is settled independently at each one. Hence the price differential between nodes does not reflect the opportunity cost of transmission. In the second model, price differentials between nodes reflect that cost, since transmission is managed by a central operator in a pool design. Metzler et al. [22], extends [16] with three

models assuming perfect arbitrage in a three-node meshed network with two generators and two technologies, differentiated by linear production costs. The two first models simulate decentralised trading. They differ in producers' perception of arbitrage between nodes. In the first one, they play à la Stackelberg w.r.t. arbitrageurs as they anticipate the effect of arbitrage on nodal prices. In the second model, they play à la Cournot, taking arbitrage quantity as given. The third model simulates a centralized pool, where the SO manages optimal dispatch of power plants at each node. Despite those differences, they found that the three models yields exactly the same price, producer output and profits. Hobbs et al. [17] extends the Cournot model in [22] to analyse the strategic behaviour of generating companies in a network model representative of Central Western Europe. They assume a separate market design for energy and transmission, power being traded bilaterally among nodes. They found relevant mark-up pricing that profits from loop-flows, especially in the Benelux area. Ehrenmann & Neuhoff [10] extends [7] to simulate and integrated market design, and compare their results with [17]. Bypassing non-convexities with heuristics, they found that the integrated market design, where SO proceeds to optimal dispatch and producers anticipate it. Such anticipation tend to increase competition between nodes. Consequently, the outcome of the integrated design is more competitive compared to the separated market design, where transmission rights are explicitly auctioned. But, to our knowledge, numerical simulations never assume that transmission capacity could be inefficiently used. Neuhoff et al. [23] provide a good survey of these numerical simulations. They assume that transmission contracts are never held by strategic generators, such that withholding is not considered, and price differential arbitrage is perfect.

However, empirical articles show strong evidences that, in the case of explicit auctioning, flows in interconnections don't follow this pattern. Each of the following articles emphasizes that capacity is far from being saturated when there are market price differences. Pineau & Lefebvre [26] links this effect to the transaction cost to use the infrastructure - a regulated fee to export and import to and from Québec -, and that price are settled differently in the two zones - average cost versus marginal cost pricing. They also emphasize the inefficiency created by a rule which commits Hydro-Québec, the regulated monopolist in Québec, to keep a certain amount of power in reserve for local demand. Turvey [29] and Bunn & Zachmann [5] found weak correlation between price differential and flow direction on the IFA interconnector between England and France. Turvey [29] links this to price uncertainty between the two markets, where gate closure happens at different time - hour-ahead in England and day-ahead in France -, and to a fixed fee of connection. Bunn & Zachmann [5] test more specifically the assumption that players behave strategically on their import and export decisions. Highlighting the facility for generators to by-pass the use-it-or-lose-it (UoL) rule, they found significant withholding of transmission rights, especially on the import direction. On the other side, they found over-export materializing by electricity flowing against price differential, which the authors attribute to a local dominant position strategy. Gebhardt & Höffler [13] analyse the day-ahead rights auction between Germany and The Netherlands in 2005, in term of informational content. They demonstrate that the poor efficiency of the interconnection is not such a matter of incomplete information, but more of imperfect one. In other words, the amount of intermittent information revealed between day-ahead auction closure and real-time commitment was not significant, compared to the amount of private information that was not revealed during auction because of non-participation of important players in importing markets. All these elements have in common explicit auction of transmission rights and pursuit of selfish local interests.

Some articles analyse theoretically the effect of imperfect arbitrage on equilibrium in a network. Smeers & Wei [27] analyse bilateral trading in a three-nodes meshed network example where power marketers have market power. Their access to other nodes is made by buying transmission rights in a market. If no line is congested, they found a Cournot-Nash equilibrium where price at production and consumption nodes differ. By increasing power marketers, their market power is diluted and they go progressively to a perfect arbitrage equilibrium resulting in lower price at consumption node. Joskow & Tirole [21] derive the best strategy for a strategic generator who has local market power for various mode of transmission rights allocation: bilateral trading, auction, and market. Especially, if rights are initially held by a single owner and traded bilaterally, then a monopolist's best strategy is to buy all of them, from which it will potentially withhold a part. They explain inefficiency of interconnection management by local market power and withholding of physical transmission rights. They show that these inefficiencies could be highly reduced with a use-it-or-lose-it rule, along with non-commitment of generators. In other words, if actors of neighbouring zones cannot contract

between themselves and unused rights are released in a day-head auction, then allocation of rights would have no negative effect on the total welfare. Gilbert et al. [14], extending [21], show that if rights are allocated in a uniform price auction (in a perfect information framework), then no market power enhancing rights would be held by strategic generators. It is noteworthy to mention that, by explicitly assuming that interconnection is always used at full capacity, the authors analyse only the case of financial transmission rights. In such case, no withholding strategy could occur - the price differential between the two nodes is perfectly arbitrated - but ownership of financial rights gives more incentives to create scarcity at the high-priced node.

Where the literature focus its analysis on export and import strategies as a mean to protect local market power, we look at those strategies from a more straightforward perspective. In our model, we analyse import and export strategies of power producers as a mean to gain extra-profit, given that local regulation does not permit them to earn extra-profits in their local market. In other words, generators are local price taker, selling their physical product at marginal cost, but they are price-maker on the foreign trade market, maximizing their marginal revenue on exports and imports. This approach simulates the case where each jurisdiction, each node of the model, only aims at maximizing local welfare, which is the sum of local consumer and producer's surplus. However, the extent to which generators could exert market power is limited by the rights they do not own. As in [14, 21], the rights not owned by generators are *de facto* allocated to a competitive fringe of traders that perfectly arbitrage the price differential, taking away any profit opportunity. Thus, the lesser rights this competitive fringe owns, the lower their ability to arbitrage price differences.

### 3 The model of two interconnected power systems

#### 3.1 Model overview

The model contains three types of players. Producers generate power for a given price and act strategic on the interconnection. System operators set price at their node such that supply equates demand in order to maximise consumer surplus. Traders arbitrate price differential between nodes. The model has two stages: 1) a financial stage where the interconnection game is played, and 2) a physical market where power production, exchange and consumption are settled. It is solved by backward induction: The second stage is solved, then the first one. Thus, decision variables which will have to be taken in following stages will be expressed as implicit functions of the first stage's decision variable of all agents: players as well as the competitive fringe.

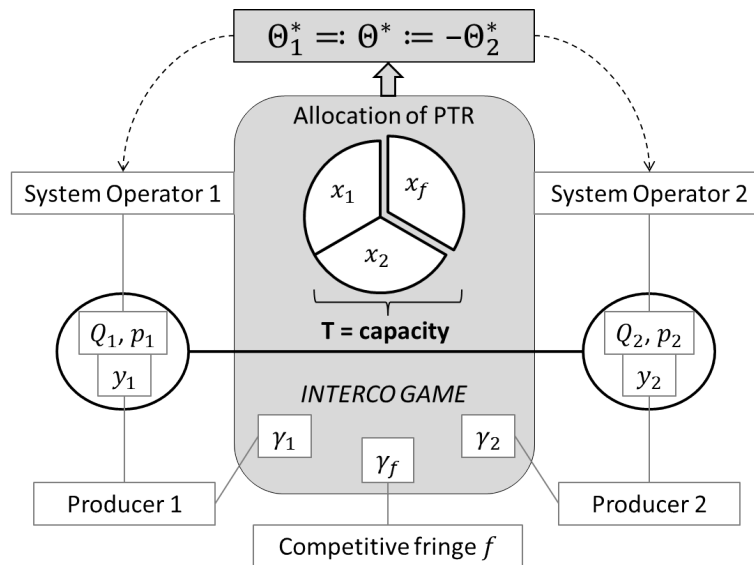


Figure 1: Model framework



The first stage corresponds to the interconnection game. Generators and traders set the level of export or import as a share  $\gamma_i$  of the amount  $x_i$  of PTRs they own. They maximise their profit considering price differential between the two nodes. Whereas competitive traders are price taker, strategic generators consider the willingness to pay of the demand at the other node. This stage is crucial as it sets the exporting/importing strategy as a function of the interconnection's ownership structure  $\mathbf{x}$ . At this stage, we show a sufficient condition on  $\mathbf{x}$  to avoid withholding strategy without assuming cost and utility functional forms. In a subsequent section, we assume functional forms which will permit use to define necessary and sufficient conditions for withholding to appears. At the end of this stage, the level of export  $\Theta$  is settled, and will be taken as given by system operators.

The second stage corresponds to the physical electricity market of electricity. At each node  $i$ , the system operator sets the quantity demanded  $Q_i$  that maximises local consumer surplus given the utility function  $U_i(Q_i)$ , under the constraint that supply equates demand, taking into account on the level on net export as a parameter. The dual variable of the equilibrium constraints being the shadow cost to supply one more unit of power, it could be interpreted as the market price. The local generator takes that price as given, and generates power up to the point where it is not profitable anymore. Hence, the second stage can be interpreted as a regulated market.

The main elements of the model are resumed as follows:

- Set
  - $\mathcal{I}$ : set of nodes,  $i = 1, 2$
- Functions
  - $U_i(\cdot)$ : Utility function of consumers at node  $i$
  - $C_i(\cdot)$ : generation cost function at node  $i$
  - $\Theta(\cdot)$ : net export function from node 1 to 2,  $\Theta(\cdot) \in [-T; T]$ 
    - $\theta_i$ : net export function from node  $i$ .
  - $\Delta(\cdot)$ : price differential function between node 2 and 1
- Decision variables
  - $y_i$ : level of generation at node  $i$ , in MW,  $y_i \geq 0$
  - $\gamma_i$ : share of PTRs which is effectively used by the generator at node  $i$ ,  $\gamma_i \in [0, 1]$
  - $\gamma_f$ : share of PTRs which is effectively used by the competitive of traders,  $\gamma_f \in [0, 1]$
  - $q_i$ : Load at node  $i$ .
- Dual variables
  - $p_i$ : shadow cost of equilibrium constraint on physical market at node  $i \equiv$  real-time price settled by the system operator
- Parameters
  - $T$ : capacity if the interconnection
  - $x_i$ : amount of PTRs owned by the generator at node  $i$ , in MW,  $x_i$  free
  - $x_f$ : amount of PTRs owned by the competitive fringe of traders, a positive amount is considered as export from node 1 to 2.

To note, PTRs could be positive as well as negative. A positive amount ( $x_i > 0$ ) corresponds to export rights, and a negative amount to imports. Also, we make the following assumptions:

1. The cost function  $C_i(y_i)$  is continuous and convex increasing:  $C'_i(y_i) > 0$ ,  $C''_i(y_i) \geq 0$
2. Consumer's utility function  $U_i(Q_i)$  is continuous and concave in  $Q_i$  ( $U'_i(Q_i) \leq 0$ ) with  $\lim_{Q_i \rightarrow 0} U'_i(Q_i) > 0$
3. Generators are price taker on local market, but price maker on foreign one.

4. Competitive fringe of traders are price-takers, hence they perfectly arbitrate the price differential with PTRs that are not owned by strategic generators..
5. PTRs are not subject to the Use-it-or-Lose-it (UoL) rule.

The two first assumptions are standards in the Neo-Classical Economic Theory. The three lasts are the main ingredients of the model. They are made to highlights the competitive situation at the local level, which allows a clear analysis of strategic interactions at the interconnected level. The third assumption is justified by the fact that physical markets, when they exist, are to the least strongly monitored. It is consequently difficult for generation companies to largely mark-up their marginal costs. Second, many power markets are not very concentrated due to unbundling, especially in the North-Eastern US.<sup>1</sup> Third, even if withholding may be a way to protect local market power, making this assumption permits to isolate the strategic effect of withholding rights to raise the marginal price on foreign market. This would be equivalent to a pivotal supplier that is in a monopolistic situation in order to supply the last MWs of demand. Fourth we could extend the analysis where, even if producers are local price takers on physical market, they could take into account the impact of their export and import strategy on the pricing at their zone. In this case, they would apply some indirect form of mark-up pricing at their node. The possibility of such strategy would therefore be a market design issue. The fourth assumption, existence of a competitive fringe, is common in the interconnection literature, and we do not question its reality. As the first stage is financial, and that such kind of market is not considered to have significant barrier to entry (or to leave), it is considered contestable: any profit opportunity would be taken away by a new entrant. However, as we settle access to the interconnection capacity through an allocation of PTRs *ex ante*, we limit such free-entry to the amount of rights which are not held by strategic generators. The fifth assumption may seem important, as [21] conclude that implementation of UoL rule for PTRs permits to avoid significant inefficiency. However, if it is efficient in theory, it is difficult to enforce in practice. Empirically, the definition of a UoL rule plays a critical role, and it is not unusual to see right's withholding despite its existence [5].

This set of assumptions is made to change the perspective in the analysis of market power in an electrical network. As previously seen in the literature on interconnection games, strategic generators exert market power at the node where generation take place, but not during trades on the interconnection market. Here, the generators cannot take advantage of their dominant position because of the local setting. But they can game the trading process settled at the interconnection by withholding their output. In the case where all rights are allocated to a strategic generator, the output of the game in term of foreign trade is similar in spirit to the model developped in [15] and [27]. On the other side, when all rights are allocated to the fringe, we obtain a competitive equilibrium both locally and on the interconnection. This equilibrium maximises the sum of local welfares.

## 3.2 Second stage: The Real-Time Power Market

### 3.2.1 Demand-side and supply-side

At each node, a SO acts as a central buyer on behalf of the demand. With this assumption, we focus the scope of the article on wholesale power pool. Such a pool market design is standard in many electricity markets (PJM, New York ISO, ISO New England, ERCOT, New-Zealand, etc.) Hence, each SO sets the total demand at its node such that consumer welfare is maximised under the constraint that supply equals demand at equilibrium:

$$\max_{q_i} U_i(q_i) \tag{1}$$

$$\text{s.t. } q_i + \theta_i = y_i \perp p_i \text{ free} \tag{2}$$

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<sup>1</sup>For example, Patton et al. [25], state that “*The Day-Ahead and Real-Time Markets provide competitive incentives for resources to perform efficiently and reliably*”. For PJM, “*The results of the energy market, the results of the capacity market and the results of the regulation market were competitive.*” [1]. For the New England power market, [20] states that “*Market concentration is low, and energy prices remain at levels consistent with the short-run marginal cost of production*”.

where  $p_i$  is the dual variable of the equilibrium of the market at node  $i$ . It could be interpreted as the cost to serve one more unit of demand.  $\theta_i$  is the value of net export from node  $i$ . This variable is defined as a function later in the first stage, and is currently taken as a parameter

Each generator at node  $i$  maximises its local profit, taking the local price  $p_i$ , settled by the SO as given.

$$\max_{y_i \geq 0} y_i p_i - C_i(y_i) \quad (3)$$

In this configuration, the model correspond to a utility model . A benevolent market operator reveals its willingness to pay for one more unit of power, the monopolist reacts to it by equalizing it with its marginal cost. He cannot play this price - he doesn't derive the corresponding inverse demand function - because regulation forces him to adopt a competitive behaviour at the local level. For a given level of  $\theta_i$ , the result of this stage would be the same if we were to replace monopolist and market operator by a benevolent social planner who set price as well as quantity. Likewise, one could replace the marginal cost of the monopolist by the supply function of a competitive market and obtain the same result.

### 3.2.2 Equilibrium conditions

Solving the Lagrangian of these constrained maximization problems, we obtain the First Order Conditions (FOC) for the second stage. For the regulated SO:

$$U'_i(q_i) - p_i = 0 \quad \perp \quad q_i \text{ free} \quad (4)$$

$$y_i - \theta_i - q_i = 0 \quad \perp \quad p_i \text{ free} \quad (5)$$

Condition 4 state that, at equilibrium, the marginal utility of consumers equals the marginal cost to supply demand for the system  $p_i$ . equation (5) is the market's equilibrium condition. From this, it is natural to define  $p_i$  as the price of market  $i$ .

For generators monopolist at each node  $i$ , we find  $y_i$  which maximises equation (3), under non-negativity constraint of  $y_i$ :

$$p_i - C'_i(y_i) \geq 0 \quad \perp \quad y_i \geq 0 \quad (6)$$

Condition 6 states that, at node  $i$ , production increases as long as the market price is greater or equal than the marginal cost of production.

One could remark, that these FOC are equivalent to a system of FOC where the physical market is managed by a benevolent social planner. As well, the equality between price and marginal cost is obtained in a competitive market.

### 3.2.3 Implicit function

We want to express production and consumption as implicit functions of the net export from node  $i$ ,  $\theta_i$ . Using the FOC, we know that at the optimum, we have:

$$q_i^* = y_i^* - \theta_i \quad (7)$$

$$U'_i(q_i^*) = p_i = C'_i(y_i^*) \quad (8)$$

These equations state intuitive equilibrium conditions for the regulated power market. First, at equilibrium, local demand must equalize total production minus export, which is equivalent to local supply. Second, the price of trade is equal to the marginal cost of the marginal unit at node  $i$ , which induces competitive equilibrium.

We look now at the impact of export  $\theta_i$  on local production  $y_i$  and consumption  $q_i$ . This will be useful later when we will highlight properties of our model. From equation (7) it is straightforward that  $\partial q_i(y_i, \theta_i)/\partial y_i > 0$  and  $\partial q_i(y_i, \theta_i)/\partial \theta_i < 0$ . By concavity of the utility function of the demand  $U_i(q_i)$ , it comes that:

$$\frac{\partial U'_i(q_i(y_i, \theta_i))}{\partial y_i} < 0 \quad \text{and} \quad \frac{\partial U'_i(q_i(y_i, \theta_i))}{\partial \theta_i} > 0$$

Using equation (7), we could derive  $q_i$  w.r.t.  $\theta_i$ :

$$\frac{dq_i(\hat{y}_i(\theta_i), \theta_i)}{d\theta_i} = \frac{d\hat{y}_i(\theta_i)}{d\theta_i} - 1$$

For this expression to be positive, one needs  $\frac{d\hat{y}_i(\theta_i)}{d\theta_i} \geq 1$ , a raise of exports by one unit increases production by at least one unit. This obviously cannot be correct, but let show it isn't. Assume correctness of this assertion, then:

$$\frac{d\hat{y}_i(\theta_i)}{d\theta_i} > 1 \implies \frac{dq_i(\hat{y}_i(\theta_i), \theta_i)}{d\theta_i} > 0$$

but, using the relationship described in equation (8), convexity of the cost function, and concavity of the utility function:

$$\theta_i \uparrow \implies y_i \uparrow \implies C'_i \uparrow \implies p_i \uparrow \implies U'_i \uparrow \implies q_i \downarrow$$

which is a contradiction. So we have:

$$\frac{d\hat{y}_i(\theta_i)}{d\theta_i} \leq 1 \tag{9}$$

which implies that

$$\frac{d\hat{q}_i(\hat{y}_i(\theta_i), \theta_i)}{d\theta_i} \leq 0$$

To prove that  $\frac{dy_i(\theta_i)}{d\theta_i} \geq 0$  is easy using the same method. Assume  $\frac{dy_i(\theta_i)}{d\theta_i} < 0$ , then we could describe the following relationship:

$$\theta_i \uparrow \implies y_i \downarrow \implies C'_i \downarrow \implies p_i \downarrow \implies U'_i \downarrow \implies q_i \uparrow$$

Which contradicts  $\frac{d\hat{q}_i(\hat{y}_i(\theta_i), \theta_i)}{d\theta_i} \leq 0$ . The case of neutrality is obtained if both cost and utility function are linear in  $y_i$  and  $q_i$  respectively. These results show that exports are a partial substitute to local consumption: a raise of one unit of export decrease by less than one unit the consumption. Because consumption decreases with a rise of exports, the marginal utility  $U'_i(q_i)$ , *i.e.* the price at node  $i$ , is increasing with export and decreasing with import. Wince we are in a two-node model, we could make the following remark.

**Remark 1** *Under assumptions 1-3, price (settled at the marginal utility) at node  $i$  is non-negatively correlated to exports from node  $i$  and imports from node  $j$ .*

$$\frac{d\hat{U}'_i(\theta_i)}{d\theta_i} \geq 0 \quad ; \quad \frac{d\hat{U}'_i(\theta_i)}{d\theta_j} \leq 0 \tag{10}$$

As we have characterized production and inverse demand as function of net export, we could move to the first stage of the game where the rate of use of PTRs is settled. The aim of this subsection is to characterize interior solution equilibrium, *i.e.* imperfect arbitrage of price differential, even when regulation is efficient at the local level. It is important to note that, in the subsequent section, we will often interchange the price argument  $p_i$  with the marginal utility function  $U'_i(\cdot)$  after deriving the FOC. As we have seen, those two elements are equivalent in real-time, because the physical market is "efficient". However, the use of  $p_i$  is made to highlight the fact that a player considers the price as fixed in certain situation.

### 3.3 First stage: The interconnection game

At this stage, producers and traders decide how much power they will export/import to/from the other market. This decision is expressed as a share  $\gamma_i$ ,  $\gamma_i \in [0, 1]$ , of the amount of PTRs  $x_i$  owned by player  $i$ . The collection of  $\gamma_i$  are grouped in a vector  $\Gamma = \{\gamma_1, \gamma_2, \gamma_f\}$ . Allocation  $\mathbf{x} = \{x_1, x_2, x_f\}$  is given exogenously and cannot be higher than the interconnection capacity  $T$  in absolute value. Any portion of interconnection capacity that is not entitled to one of the two monopolists will be managed efficiently. In other words, the competitive fringe of traders owns the residual part of PTRs.<sup>2</sup>

$$|x_1| + |x_2| + |x_f| = T \tag{11}$$

<sup>2</sup>To note, constraint (11) takes in account the netting flow even if, as we will see later, such netting is useless in this model.

Also, we could assume without loss of generality that, *ceteris paribus*, net export from  $i$ ,  $\theta_i$  is linear in  $\Gamma$  and has the following properties:

$$\frac{\partial \theta_i(\Gamma)}{\partial \gamma_i} = x_i \quad \frac{\partial \theta_i(\Gamma)}{\partial \gamma_j} = -x_j \quad i \neq j; i, j = 1, 2 \quad (12)$$

These properties just state that an export from monopolist at node  $i$  increases the total export from this node, as well as an import from monopolist at node  $j$ . The linearity come naturally that for  $\mathbf{x}$  and  $\gamma_j$  fixed, a raise of  $\gamma_i$  increase  $\theta_i$  by  $x_i$ , whatever is  $i$ .

### 3.3.1 Power producers

Producers are still price taker at their local node, so they do not consider the second stage in their profit maximizations, nor do they consider the change of local price due to their trading behaviour. This could be interpreted as an integrated system, where a producer sells its power at a regulated price, and the regulator do not care of the export-import policy as long as it does not negatively impact local supply. In term of market design it could also be interpreted as a regulation which links financial settlement in day-ahead and physical delivery in real-time.<sup>3</sup> In other words, this assumption is consistent with a regulator enforcing an efficient regulation at the local level in order to avoid any gaming from the incumbent, in order to preserve local consumers' surplus. Producer at node  $i$  sets  $\gamma_i$ ,  $\gamma_i \in [0, 1]$ , to maximise its profits at the exchange stage  $\Pi_i^E$ , given by:

$$\Pi_i^E = \hat{y}_i(\theta_i(\Gamma))p_i - C_i(\hat{y}_i(\theta_i(\Gamma))) + \gamma_i x_i (\hat{U}'_j(\theta_i(\Gamma)) - p_i) \quad (13)$$

One could note that even if generator  $i$  takes into account the impact of exports (imports) on physical production, we still assume that it is price-taker. Assuming an interior solution and considering properties of  $\theta_i$  given in (12), the first order condition yields :

$$\frac{d\hat{y}_i(\theta_i(\Gamma))}{d\theta_i(\Gamma)} \frac{\partial \theta_i(\Gamma)}{\partial \gamma_i} (p_i - C'_i) + x_i \left( \hat{U}'_j(\theta_i(\Gamma)) - p_i + \gamma_i x_i \frac{d\hat{U}'_j(\theta_i(\Gamma))}{d\theta_i(\Gamma)} \right) = 0 \quad (14)$$

The price-taking assumption on local market allows us to cancel the first term since, real-time price is equal to system's marginal cost. If we furthermore assume that for each  $i$ ,  $x_i \neq 0$ , we can rewrite the FOC for each  $i$ :

$$\begin{cases} \gamma_i = 1 & \text{if } \hat{U}'_j(\theta_i(\Gamma)) - p_i + \gamma_i x_i \frac{d\hat{U}'_j(\theta_i(\Gamma))}{d\theta_i(\Gamma)} \\ \gamma_i = 0 & \text{if } \hat{U}'_j(\theta_i(\Gamma)) - p_i + \gamma_i x_i \frac{d\hat{U}'_j(\theta_i(\Gamma))}{d\theta_i(\Gamma)} \\ \gamma_i \in [0, 1] & \text{otherwise} \end{cases} \quad (15)$$

The following Proposition gives necessary and sufficient conditions for the existence and uniqueness of interior solution.

**Proposition 1** *Under Conditions 1-5, the problem of generator  $i$  described by the system (15) admits a unique Cournot-Nash equilibrium  $\gamma_i^*$  if, for any  $i, j$ ,  $j \neq i$ :*

- i)  $x_i \geq 0$  and  $\frac{d^2 \hat{U}'_j(\theta_i)}{d^2 \theta_i} \leq 0$ , or
- ii)  $x_i \leq 0$  and  $\frac{d^2 \hat{U}'_j(\theta_i)}{d^2 \theta_i} \geq 0$

**Proof.** Assuming interior solution and considering properties of  $\theta_i$  given by (12), the second order condition (SOC) yields,  $\forall i = 1, 2$ ,  $i \neq j$ .

$$2x_i \frac{d\hat{U}'_j(\theta_i(\Gamma))}{d\theta_i(\Gamma)} + \gamma_i x_i^2 \frac{d^2 \hat{U}'_j(\theta_i(\Gamma))}{d^2 \theta_i(\Gamma)}$$

By Remark 1, we know that,  $\forall i, j$ ,  $j \neq i$ ,  $\frac{d\hat{U}'_j(\theta_i(\Gamma))}{d\theta_i(\Gamma)} \leq 0$ .

<sup>3</sup>Like the Two-Settlement of the *Standard Market Design* [19].

Hence, two cases may happen:

- If  $x_i > 0$ , the first term of the SOC is negative for any  $i$ . The second term has to be negative to insure existence of a global interior maximum, which implies  $\frac{d^2 \hat{U}'_i(\theta_i(\Gamma))}{d^2 \theta_i(\Gamma)} \leq 0$ .
- If  $x_i < 0$ , then the first term of the SOC is positive, thus yielding a global interior minimum when marginal utility function is convex in net export  $\theta_i$ .

□

When  $x_i$  is positive, player at node  $i$  maximises its profits from the sale of energy to the other node. The marginal utility function, which is equivalent to the inverse demand one, is a revenue in the profit function, and must be concave to insure existence of an equilibrium in finite terms. On the other side, when  $x_i$  is negative, it imports power from the other node and thus have to minimize the cost of furniture. In this case, the other node's price function appears as a cost in the profit function. As so it must be convex to admit a solution in finite terms.

This irregularity of the marginal utility function may pose problems for someone who wants to design a general model for analysis purpose. Indeed, the general case where a node export and import during several periods works only if the inverse demand function is linear in  $\theta_i$ . Other functional forms may be assumed for specific cases analysis as long as they respect sufficient conditions described in Proposition 1. At the end of the day, these conditions satisfy the neo-classical axiom of preference's convexity.

### 3.4 The competitive fringe of traders

In this section, we evaluate the impact of the fringe's access to the interconnection on the outcome of the game in a general framework. As formerly stated, net export from node  $i$ ,  $\theta_i$  is settled at the first stage, during the Interconnection Game. During this stage the net export is normalized from node 1 perspective for modelling purpose: it allows to fix a functional form. Hence we have:

$$\theta_1 = -\theta_2 = \Theta(\Gamma, \mathbf{x}) = \gamma_1 x_1 - \gamma_2 x_2 + \gamma_f x_f \quad (16)$$

$\theta_1 = -\theta_2$  corresponds to the fact that the net export from node 1 correspond to the net import from node 2's perspective. The vector  $\Gamma$  contains players and the competitive fringe's PTRs rate of use. The vector  $\mathbf{x}$  contains the volume of PTRs by entity. It is fixed exogenously.<sup>4</sup> As in [14, 21], we assume that the market is deep in that the competitive fringe of traders, market makers  $f$ , arbitrage away any profit opportunity, as long as the quantity they trade is not greater than the volume of PTRs  $x_f$  they own.

The price differential function  $\Delta$  corresponds to the difference of price between nodes 2 and 1. Hence, a positive  $\Delta$  is explicitly interpreted as “the marginal cost of supply at node 2 is higher than the one at node 1”. Given that  $\hat{U}'_i(\cdot) = p_i$  for all  $i$ , the price differential function could also be defined as depending on decision variables ( $\Gamma$ ) and ownership structure of the interconnection ( $\mathbf{x}$ ) :

$$\Delta(\Theta(\Gamma, \mathbf{x})) = \hat{U}'_2(\Theta(\Gamma, \mathbf{x})) - \hat{U}'_1(\Theta(\Gamma, \mathbf{x}))$$

The competitive fringe of traders maximises its profit, taking prices at both nodes as given. Thus, the fringe maximises the following program:

$$\max_{0 \leq \gamma_f \leq 1} \Pi_f^E = \sum \gamma_f x_f (p_2 - p_1) \quad (17)$$

which yields the complementarity condition:

$$x_f \Delta(\Theta) = 0 \quad \perp \quad 0 \leq \gamma_f \leq 1 \quad (18)$$

<sup>4</sup>Here we don't assume any rights allocation mechanism. We analyse the game whatever this allocation is. In practice, there are many ways to allocate rights: auction, negotiation, market; each of them giving a different outcome [21]. Later in this article, we will assess the impact of rights ownership through comparative statics. This gives indication of how players evaluate rights.

Given the competitive behaviour of the fringe, one would expect that PTRs owned by them are fully used, *i.e.*  $\gamma_f = 1$ , except when  $\Delta(\Theta^*) = 0$ .

$$\begin{cases} \gamma_f = 1 & \text{if } x_f \Delta(\Theta) > 0 \\ \gamma_f = 0 & \text{if } x_f \Delta(\Theta) < 0 \\ \gamma_f \in [0, 1] & \text{if } x_f \Delta(\Theta) = 0 \end{cases} \quad (19)$$

This problem is a linear program in a convex and compact set, it admit a unique solution.

The vector  $\mathbf{x}$  is of a relatively large dimension, and not every part of this vector space is worthy to analysis. Indeed, it is interesting to remark that, by checking respective FOC of players 1 and 2,  $x_1$  and  $x_2$  must be of opposite sign, otherwise at least one of the two must be null. More formally:

**Remark 2** *if  $x_1 x_2 > 0$  then  $\gamma_1 \gamma_2 = 0$*

Considering  $x_f$  fixed, we know that half of the parameter set  $(x_1, x_2)$  is not worth of analysis. Hence we may reduce the set of vector  $\mathbf{x}$  to a subset  $\mathcal{V}$  called *arbitrage direction*. It will be shown that, in this model, we could assume that  $\mathbf{x} \in \mathcal{V}$  without any loss of generality.

**Definition 1** *Let  $\mathcal{V} \subset \mathbb{R}^3$  be the set of arbitrage direction.  $\mathcal{V}$  is defined by the sign of  $\Delta(0)$ :*

- *if  $\Delta(0) > 0$  then the arbitrage direction is from node 1 to 2.*
- *if  $\Delta(0) < 0$  then the arbitrage direction is from node 2 to 1.*

where  $\Delta(0)$  is the price differential when interconnection capacity is not used, in other words the prices difference in the autarky situation:

$$\Delta(0) = \hat{U}'_2(0) - \hat{U}'_1(0) \quad (20)$$

Hence, normalizations of  $\Theta$  and  $\Delta$ , and the fringe's ownership of the residual part of the interconnection capacity imply the following definition.

**Definition 2**  *$\mathbf{x}$  is defined in the arbitrage direction, *i.e.*  $\mathbf{x} \in \mathcal{V} \subset \mathbb{R}^3$ , if:*

$$\begin{aligned} \Delta(0) > 0 &\Rightarrow x_1 \geq 0, x_2 \leq 0 \\ \Delta(0) < 0 &\Rightarrow x_1 \leq 0, x_2 \geq 0 \end{aligned} \quad (21)$$

$$x_f = \begin{cases} T - x_1 + x_2 & \text{if } \Delta(0) \geq 0 \\ -T - x_1 + x_2 & \text{if } \Delta(0) < 0 \end{cases} \quad (22)$$

Hence, if the price differential was arbitrated in the good direction, then  $\Theta$  would always be positively correlated with  $\Delta$ .

From first order conditions of the competitive fringe and strategic generators, respectively equation (18) and (15), it comes that if any part of  $\mathbf{x}$  is not defined in the arbitrage direction, then the associated part of  $\Gamma$  is null. In other words, counterflow to the arbitrage direction never happens as an outcome of the model. Thus we may assume for the remaining analysis:

**Assumption 1**  *$\mathbf{x} \in \mathcal{V}$  without loss of generality*

Also it is straightforward that, by comparing the FOC of traders (18) and of producers (15):

$$\gamma_f^* < 1 \implies \gamma_i^* = 0$$

The intuition here is that an interior  $\gamma_f$  implies a perfect arbitrage equilibrium. If  $\gamma_f$  would be interior and there still would be a non-null price differential, then the fringe would have the latitude (and the interest) to raise the quantity traded, until the point where  $\gamma_f$  being bounded above by one or that  $\Delta^* = 0$ . Hence, strategic generators always maximise their profit over the demand netted off the decision of the competitive fringe. When they do so, the price differential never reaches zero.

We now show that for a sufficient amount of rights owned by the competitive fringe, the equilibrium of the game will be a *perfect arbitrage* one.

**Definition 3** A *perfect arbitrage equilibrium* is an equilibrium at the exchange stage characterized by:

$$\Delta(\Theta^*). (T - |\Theta^*|) = 0 \quad (23)$$

By construction, an *imperfect arbitrage equilibrium* does not respect this definition. Also we define the minimum quantity necessary to cancel prices difference between the two nodes.

**Definition 4** A *copper plate interconnection*  $T^*$  is the minimum capacity of the interconnection that would permit the price differential between the two nodes to equate zero with an efficient management. Formally:

$$\text{for } \mathbf{x} \in \mathcal{V}, |\Theta^*| = T^* \iff \Delta(\cdot) = 0$$

We could now show that for  $\mathbf{x} \in \mathcal{V}$ ,  $|x_f| \geq T^*$  is a sufficient condition for a perfect arbitrage equilibrium.

**Proposition 2** For  $\mathbf{x} \in \mathcal{V}$ ,  $|x_f| \geq T^* \iff \Delta^*(\cdot) = 0$

**Proof.** Let  $\mathbf{x} \in \mathcal{V}$ , hence

1. assume  $|x_f| \geq T^*$ , from FOC (18),  $\Delta^*(\cdot) = 0$
2. assume  $\Delta^*(\cdot) = 0$ , from FOC (15)  $\gamma_i^* = 0$ . If  $|x_f| < T^*$ , then we would have  $\gamma_f^* = 1$  and  $\Delta^* \neq 0$ , a contradiction.

□

Hence, assuming  $T^*$  being computable, we obtain a threshold which enforces perfect arbitrage since prices at both nodes equate. The another part of the perfect arbitrage equilibrium definition states that if a price difference exists, then the interconnection capacity constraint must be binding. This might be the case even if  $|x_f| < T^*$ . The trivial case happens for  $T < T^*$  and  $|x_f| = T$ . But it may also happens for any  $x_i \neq 0$ ,  $x_i \in \mathcal{V}$ . Indeed for some high differences between the two markets fundamentals (relatively to the interconnection capacity), strategic players may have interest to fully export, occasioning a *Constrained Nash Equilibrium*. However, if  $|x_f| < T^*$ , price differential never reaches zero.

In order to dig deeper in the analysis of this interconnection game, we need to assume functional forms to the utility and cost functions which follow the sufficient conditions of existence and uniqueness as defined in Proposition 1. This will allow to fully characterize the export/import strategies in closed form solution as an implicit function of any vector  $\mathbf{x}$ .

## 4 An example using functional forms

On top of the other hypothesis, we now assume:

- Quadratic cost function :  $C_i(y_i) = \frac{c_i}{2} y_i^2$ ,  $c_i \geq 0$
- Quadratic concave utility function :  $U_i(q_i) = q_i(a_i - \frac{b_i}{2} q_i)$ ,  $a_i > 0$ ,  $b_i \geq 0$ .

From this utility function, we could derive the marginal utility function of the aggregated demand,

$$U_i'(q_i) = a_i - b_i q_i$$

which acts as market  $i$ 's inverse demand function. So  $a_i$  could be interpreted as the reservation price of market  $i$ , and  $b_i$  is the price sensitivity of demand. The higher is  $b_i$ , the lower is the demand's ability to react to price.<sup>5</sup> Similarly, the higher is  $c_i$ , the lower is the ability of the monopolist at node  $i$  to react to market price's variations. Hence,  $c_i$  could be seen as the price sensitivity of node  $i$ 's total supply

<sup>5</sup>In the case of power market, one would expect very high  $b_i$ .



#### 4.1 Second stage: The real-time power market

Solving backward, we obtain competitive local electricity market output. Monopolist at node  $i$  choose the production  $y_i$  to maximise its profit, considering the vector of trades  $\theta_i$  as given. Regulation enforces competitive outcome, such that marginal utility equals marginal cost.

$$U'_i(y_i^*, \theta_i) = C'_i(y_i^*)$$

after some simple calculations we obtain  $y_i$ ,  $i = 1, 2$ , as a function of the net export of node 1.

$$\hat{y}_i(\theta_i) = \frac{a_i + b_i \theta_i}{b_i + c_i} \Rightarrow \frac{d\hat{y}_i(\theta_i)}{d\theta_i} > 0$$

which implies market  $i$ 's price:

$$p_i = C'_i(\hat{y}_i(\theta_i)) = \hat{U}'_i(\theta_i) = \frac{c_i(a_i + b_i \theta_i)}{b_i + c_i}$$

#### 4.2 First stage: The interconnection game

We first calculate the price differential as a function of the normalized  $\Theta$ . Also,  $\Theta$  is itself a function of the export actions of players and the competitive fringe, represented by the vector  $\Gamma$ , and the ownership structure of the interconnection, represented by the vector  $\mathbf{x}$ :

$$\Delta(\Theta(\Gamma, \mathbf{x})) = \hat{U}'_2(\Theta(\Gamma, \mathbf{x})) - \hat{U}'_1(\Theta(\Gamma, \mathbf{x})) = \frac{A - B\Theta(\Gamma, \mathbf{x})}{(b_1 + c_1)(b_2 + c_2)} \quad (24)$$

where:

$$\begin{aligned} A &= a_2 c_2 (b_1 + c_1) - a_1 c_1 (b_2 + c_2) \\ B &= b_2 c_2 (b_1 + c_1) + c_1 b_1 (b_2 + c_2) \end{aligned}$$

Hence, the price differential function is of linear form, and depends on the differences between the two local markets.  $A$  is the difference between weighted reservation price at each node. Those weights correspond to local supply sensitivity parameter times the sum of foreign supply and demand sensitivity parameter. Hence,  $A$  can be interpreted as the reservation price of the interconnection market defined from node 2 to 1, and  $B$  its price sensitivity to quantities. To note,  $A$  is defined on  $\mathbb{R}$  whereas  $B$  is positive (assuming  $b_i$  and  $c_i$  strictly positive for at least one node). However,  $\mathbf{x}$  being defined in the arbitrage direction without loss of generality (see Section 3.4),  $\Theta$  is always of same sign than  $A$ . So, as export increases in the arbitrage direction, the price differential always decreases. Also, given this functional form, we know that

$$T^* = \frac{|A|}{B}$$

The competitive fringe of traders and strategic generators maximise their profit given respectively by equations (17) and (13). Assuming interior solutions and  $x_1, x_2, x_f \neq 0$ , it gives the following system of three equations and three unknowns:

$$\begin{cases} \gamma_f &= \frac{A - B(\gamma_1 x_1 - \gamma_2 x_2)}{B x_f} \\ \gamma_1 &= \frac{A - B(\gamma_f x_f - \gamma_2 x_2)}{(B + D_2) x_1} \\ \gamma_2 &= -\frac{A - B(\gamma_f x_f + \gamma_1 x_1)}{(B + D_1) x_2} \end{cases} \quad (25)$$

where:

$$\begin{aligned} D_1 &= c_1 b_1 (b_2 + c_2) > 0 \\ D_2 &= c_2 b_2 (b_1 + c_1) > 0 \\ &\Rightarrow B = D_1 + D_2 \end{aligned}$$

One difficulty is that solution depends on parameters  $A$  and  $\mathbf{x}$  which could be positive as well as negative. However by assuming that the physical transmission rights are settled in the arbitrage direction, *i.e.*  $\mathbf{x} \in \mathcal{V}$ , we could bypass this difficulty. Indeed, assuming this,  $\mathbf{x}$  is sufficiently well-defined in term of direction (positive or negative), such that one could think about its length in term of absolute value.<sup>6</sup> Hence, following Definition 2, the system of equations (25) is equivalent to:

$$\begin{cases} \gamma_f &= \frac{|A| - B(\gamma_1|x_1| + \gamma_2|x_2|)}{B|x_f|} \\ \gamma_i &= \frac{|A| - B(\gamma_f|x_f| + \gamma_j|x_j|)}{(B + D_j)|x_i|}, \quad i = 1, 2, \quad j \neq i \end{cases}$$

Considering  $\gamma_f = 1$ , the Cournot-Nash strategy of player  $i$  in closed form is:

$$\gamma_i = \frac{D_i(|A| - B|x_f|)}{(B + D_i)(B + D_j)|x_i|} \quad (26)$$

Another difficulty in the characterisation of equilibrium strategies is that decision variables are bounded above and below. Thus, the strategy of each player taken individually is highly sensitive to the size of parameters, and notably to the ownership structure of the interconnection  $\mathbf{x}$ . For example, for relatively low value of  $x_i$ , but different from zero,  $\gamma_i$  is bounded above by 1. But player  $j$ ,  $j \neq i$ , takes this constrained strategy of player  $i$  into account, and then, in some cases, could act as a monopolist on the residual demand of the interconnection market, whereas when  $\gamma_i$  is interior, player  $j$  plays à la Cournot. So, strategies of players could follow different regimes given the size of the vector  $\mathbf{x}$ . The following Proposition provide such characterisation of equilibrium strategies.

**Proposition 3** For  $x_i \neq 0$ , the equilibrium strategy of player  $i$ ,  $\gamma_i^*$ , could follow different regimes, depending on the allocation  $\mathbf{x}$ :

$$\text{Monopolistic} : \gamma_i^* = \frac{|A| - B(T - |x_i|)}{|x_i|(B + D_j)} \iff \mathbf{x} \in S_i^M \quad (27)$$

$$\text{Cournot-Nash} : \gamma_i^* = \frac{D_i(|A| - B|x_f|)}{|x_i|(B^2 + D_i D_j)} \iff \mathbf{x} \in S^D \quad (28)$$

$$\text{Constrained} : \gamma_i^* = 1 \iff \mathbf{x} \in S_j^M \cup |x_i| \leq \sigma_i^M \quad (29)$$

$$\text{Non-participating} : \gamma_i^* = 0 \iff |x_f| \geq T^* \quad (30)$$

where

$$S_i^M = \{\mathbf{x} \in \mathcal{V} : |x_i| > \sigma_i^M \cap |x_j| \leq \sigma_j^D \cap x_f < T^*\} \quad (31)$$

$$S^D = \{\mathbf{x} \in \mathcal{V} : |x_i| > \sigma_i^D \cap |x_j| > \sigma_j^D \cap x_f < T^*\} \quad (32)$$

and for any  $i, j \neq i$

$$\sigma_i^M = \frac{|A| - BT}{D_j} \quad (33)$$

$$\sigma_i^D = \frac{D_i(|A| - B(T - |x_j|))}{D_j(B + D_i)} \quad (34)$$

**Proof.** Assume  $\mathbf{x} \in \mathcal{V}$ , such that  $\gamma_f, \gamma_i \geq 0$ ,  $i = 1, 2$ . From Proposition 2, for  $x_f \geq T^*$ , we have  $\Delta = 0$ , and consequently  $\gamma_i = 0$ ,  $i = 1, 2$ . So we focus our attention to the case where  $x_f < T^* \implies \gamma_f = 1$ .

$\Leftarrow$  : Assume first that  $x_j = 0$ . Then player  $i$  is a monopolist who equates its marginal cost with the marginal revenue he gets from the interconnection market's residual demand. In this case he just has to solve equation (25) where  $\gamma_f = 1$  and  $|x_j| = 0$ . Using equation (11), we change  $|x_f|$  by  $(T - |x_i|)$ , and we obtain that  $\gamma_i \geq 1$  if:

$$|x_i| \leq \frac{|A| - BT}{D_j} = \sigma_i^M$$

<sup>6</sup>See Appendix A.1 for more explanations.

Using equation (26), we compute the threshold value  $\sigma_i^D$  of  $|x_i|$  for which  $\gamma_i \geq 1$ . Using equation (11), we change  $|x_f|$  by  $(T - |x_i| - |x_j|)$  and we obtain that  $\gamma_i \geq 1$  if:

$$|x_i| \leq \frac{D_i |A| - B(T - |x_j|)}{B + D_i} = \sigma_i^D$$

For  $|x_i| > \sigma_i^D$ , player  $i$  plays à la Cournot as defined by equation (26). But for  $|x_i|$  below that threshold, its strategy get constrained. The other player takes this in account and acts as a monopolist over the residual demand. In other words, player  $j$  considers that player  $i$  is a part of the competitive fringe if  $|x_i| \leq \sigma_i^D$ . Thus, he solves equation (25), considering  $\gamma_f = \gamma_i = 1$ . This bring us naturally to the definition of  $S_i^M$  and  $S^D$ .

$\implies$  : Comparing  $\sigma_i^M$  and  $\sigma_i^D$  we obtain that

$$\sigma_i^M \geq \sigma_i^D \iff |x_j| \leq \sigma_j^D$$

So  $\mathbf{x} \in S^D$  and  $|x_j| \leq \sigma_j^D$  is a contradiction : player  $i$  cannot play à la Cournot if  $\mathbf{x} \in S_i^M$ . Similarly, he never plays monopolistic strategy when  $\mathbf{x} \in S^D$ . So, thresholds are well defined and constrained strategy only occurs when  $x_i \notin S_i^M \cup S^D$ .  $\square$

This Proposition characterizes the whole space of strategies of players at the Nash equilibrium of the game. It is interesting that a player's strategy does not directly depend on the other player's amount of rights, but rather on the competitive fringe's allocation.<sup>7</sup> However, player  $j$ 's allocation influences the regime of player  $i$  strategy, whether it is monopolistic or duopolistic.<sup>8</sup>

According to this strategy characterization, the interconnection market may follow four different regimes. A duopoly market regime appears when both players own a sufficient amount of rights to withhold them, and then act upward on the price. A monopoly regime of player  $i$ ,  $i = 1, 2$  appears when player  $i$  owns a relatively large amount of rights and player  $j$  owns too little to represent a credible threat for player  $i$ . In this case monopolist  $i$  maximises its revenue against the residual demand curve. Finally, the market may be competitive, in the sense that no withholding strategy appears, because of insufficient ownership of rights amount for strategic players ( $\gamma_i^* = 1, \forall i$ ), or sufficiently high allocation to competitive players ( $\gamma_i^* = 0$ ). The figure below show an example of how the strategy space defines the market regime space.

It is remarkable that a bigger interconnection capacity  $T$  does not necessarily means a more competitive market in this model. Indeed, the area where the interconnection market is competitive is decreasing as capacity  $T$  increases, up to the point  $T = |A|/B$  where it is null, and finally increasing with  $T$ . For a relatively low value of  $T$  compared to the demand on the interconnection market, price is high such that it is not profitable to withhold much rights. Up to the point where  $T$  is so low compared to the demand, *i.e.*

$$T \leq \frac{A}{B + D_i} \quad \forall i$$

the only strategy that maximises profit is to use all producer's rights, *i.e.*  $\gamma_i = 1$ . In this case, all capacity  $T$  would be used and the market would be competitive following Definition 3. Also, for

$$T > \frac{AB}{B^2 + D_i D_j}$$

a duopoly regime - where  $\gamma_i^* < 1$  for any  $i$  - exists, and as  $T$  increases, its share rises. This could be explain by the comparison between duopoly and monopoly. It is well-known that the output for a duopolistic market is greater in quantity than for a monopolistic one. And so, it is easier to get bounded in a constrained framework. Hence, as capacity rises such bounds get away,  $\sigma_i^D$  decreases with  $T$  for all  $i$ , and extends the array of rights allocation which sets duopolistic market regime.

<sup>7</sup>In monopoly regime, other player's strategy is constrained, hence its allocation can be considered as a part of the competitive fringe.

<sup>8</sup>For results in each regime, see the Appendix A.2.

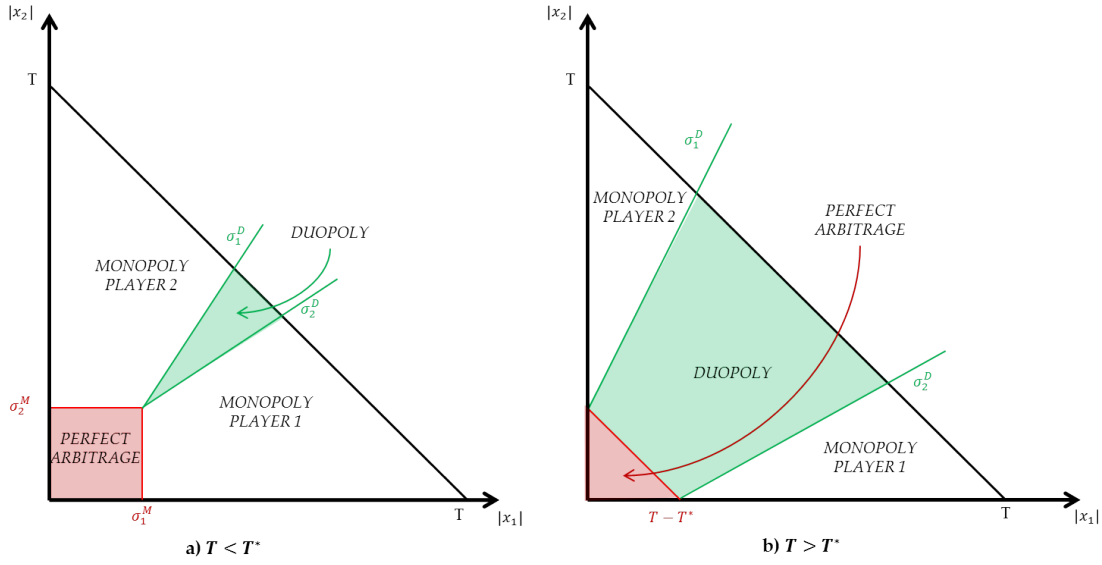


Figure 2: Market regime space

Existence of price-making behaviour does not necessarily mean withholding of physical transmission rights. Whether rights' withholding by a strategic producer is an equilibrium strategy depends on three factors:

1. The relative fundamentals of each market which are summarized by  $A$ ,  $D_1$  and  $D_2$ .  $A$  represents the difference of reserve price between node 2 and 1, each of them weighted by the supply quantity sensitivity.  $D_i$  gives information about the sensitivity to quantities of market  $i$ . The higher is  $|A|$ , the higher will be the price and more quantities will be supplied by a strategic exporter. If  $D_i$  is high, meaning that the market price  $p_i$  is sensitive to quantity changes, then a strategic exporter from node  $j$  will have incentive to withhold its output in order to increase the price.
2. The size of the interconnection  $T$  which will set the size of the market, as we assumed that all capacity is distributed through PTRs. Hence an important value of  $T$  (relatively to  $|A|$ ) is a necessary condition to have a competitive interconnection market. But it is not sufficient: if  $T$  is large but no PTR is allocated to the competitive fringe of traders, then the market will be at best a duopoly.
3. The allocation  $\mathbf{x}$  of PTRs defines the degree of competition in the interconnection market, from perfect competition to monopoly. It is remarkable to note that for relatively low value of rights, strategic generators play perfect arbitrage as a dominant strategy. Indeed, knowing the strategy of other players, they define their own strategy as a duopolist or a monopolist depending on the allocation of the other players. If unconstrained ( $|x_i|$  very large in the arbitrage direction), their monopolistic or Cournot-Nash output is not constrained, and a withholding strategy emerges. But if  $|x_i|$  is lower to that unconstrained strategy level  $\sigma_i^D$  or  $\sigma_i^M$  (see the proof of Theorem 3), then  $\gamma_i$  mechanically increases to compensate such reduction up to the point where  $\gamma_i$  is constrained above by 1.

Finally, we can define, thanks to this characterisation, a set of rights allocation  $S^P$  such that the equilibrium will be of perfect arbitrage.

**Definition 5** Let  $S^P$  be the set of perfect arbitrage equilibrium  $S^P = \mathcal{V} \setminus \{S_i^M \cup S_j^M \cup S^D\}$

In this set, allocation of rights to strategic players may be not null. The next section goes further in the analysis of strategies and their associated payoff.

### 4.3 Comparative statics

To each of the above mentioned regime correspond a different payoff function for each producer, and different local welfare functions at nodes. By doing comparative statics w.r.t. the portfolio of transmission rights, we analyse how players evaluate them. We proceed in a top-down approach, starting with local welfare, which is the objective of the regulator, and then disaggregate it in monopolist's and representative consumer's objectives.

When analysing local welfare at nodes, it is interesting to note that, according to this benchmark, most of the rights allocation results in a Pareto-dominated equilibrium by the perfect arbitrage solution. Hence, it would be profitable for the system operators at both nodes to coordinate in most of the cases, in order to prevent the strategic behaviour of their respective monopolist. But they may not be encouraged to do so if their goal is to maximise local welfare. Indeed, we could show that if monopolist at node  $i$  owns a sufficient amount of rights in the market, then this equilibrium would maximise local welfare at this node.<sup>9</sup>

To state that result, we first show that  $\gamma_i$  is continuous in  $x_i$  for any  $i$ . Then we define the local welfare function, and compare it at equilibrium with the same function assuming every players always act competitively. We do so for every market regime, and using continuity of  $\gamma_i$ , we demonstrate the above mentioned general result.

But, before this, it is noteworthy to mention that since the beginning  $\gamma_i$  and  $x_i$  often behave together. For notation simplicity, it is convenient to define  $z_i^*(\mathbf{x}) = \gamma_i^*(\mathbf{x})x_i$ . This makes  $z_i$  convenient in term of interpretation, as  $z_i$  is the power that producer  $i$  exports to the other node.

Also it is useful at this stage to normalize variable  $x$  in order to analyse more efficiently solutions.

**Definition 6** Let  $X = (\chi_1, \chi_2, \chi_f)$  be the normalized version of  $\mathbf{x}$ :

$$X = (\chi_1, \chi_2, \chi_f) = (x_1, -x_2, x_f)$$

Hence  $X$  may be interpreted as the allocation of rights to export from node 1 to 2, negative value of  $X$  being imports from 2 to 1. Such normalization is very useful to simplify mathematical demonstration. Hence we can set  $z_i^*(X) = \gamma_i^*(X)\chi_i$ .

Because everything is normalized we could assume without loss of generality that  $A > 0$ , which implies  $X > 0$ ,  $\Delta > 0$  and  $\Theta > 0$ . As we analyse the result of the interconnection game for any player  $i$ , we still analyse behaviour of importer as well as exporter. And this permit us to set the relationship:

$$\sum_{i=1,2,f} \chi_i = T \quad (35)$$

**Lemma 1** If  $X \in \mathcal{V}$ , then  $\gamma_i(X)$  is continuous  $\forall X$ .

**Proof.** Within each strategy regime defined in Proposition 3, the function  $\gamma_i(X)$  is continuous in  $(X)$ . Thus, continuity need to be checked at the limit of each transition point. From interior (monopolistic and duopolistic) to constrained solution, the function is, by definition of the thresholds  $\sigma_i^D$  and  $\sigma_i^M$ , continuous in  $\chi_i$ . Continuity at the transition point between monopolistic and duopolistic strategy remains to be checked. We know:

$$\begin{aligned} X \in S_i^M &\iff \chi_i > \sigma_i^M \text{ and } \chi_j \leq \sigma_j^D \\ X \in S^D &\iff \chi_i > \sigma_i^D \text{ and } \chi_j > \sigma_j^D \\ \chi_j = \sigma_j^D &\iff \sigma_i^M = \sigma_i^D \end{aligned}$$

<sup>9</sup>To note, a Regulator who act as a social planner, setting  $y_i$ ,  $\gamma_i$  and not just  $p_i$ ,  $i = 1, 2$  as here, would not have this problem. A Social planner would act on the interconnection market as well as the local one. In this position, he would not suffer from externalities under the monopolistic regime. See Appendix A.3 for the social planner's program.

As  $S^D$  is an open set in the space  $(\chi_i; \chi_j)$ , we show that  $\gamma_i^D$  ( $\gamma_i^*$  iff  $X \in S^D$ ) converge to  $\gamma_i^M$  ( $\gamma_i^*$  iff  $X \in S_i^M$ ) for  $\chi_j = \sigma_j^D$ . This is sufficient since at this point  $\chi_i > \sigma_i^M \iff \chi_i > \sigma_i^D$ , and  $S_i^M$  is bounded in the space  $(\chi_i; \chi_j)$ . We calculate:

$$\lim_{\chi_j = \sigma_j^D} \gamma_i^D = \frac{D_i(|A| - B(T - \chi_i - \sigma_i^D))}{\chi_i(B^2 + D_i D_j)}$$

We finally found the researched equality:  $\lim_{\chi_j = \sigma_j^D} \gamma_i^D = \gamma_i^M \forall \chi_i$ .  $\square$

As  $\gamma_i^*(X)$  is continuous despite change of regimes, it comes that any function which is continuous in  $\gamma_i$  is also continuous in  $X$  at equilibrium. This is true for profit function as well as consumers' utility function.

We now define the local welfare function at node  $i$ .

**Definition 7** *The local welfare function at node  $i$  is defined:*

$$W_i(\cdot) = U_i(\cdot) - C_i(\cdot) + \gamma_i x_i U_j'(\cdot) + (\theta_i(\cdot) - \gamma_i x_i) p_i$$

After some calculus, the normalized equilibrium local welfare functions is described:

$$W_i^* = \frac{a_i^2 + c_i b_i (\Theta^*(X))^2}{2(b_i + c_i)} + z_i(X) \Delta^*(X)$$

where  $\chi_1 = x_1$  and  $\chi_2 = -x_2$  are the normalized value of  $x_i$ , and  $X$  is the vector  $(x_f, \chi_1, \chi_2)$ . The first term, which is a monotone convex-increasing function of  $\Theta$ , represents welfare from local production and consumption. The second term is the plus-value earned on the interconnection market. We compare this function with the same function assuming perfect arbitrage:

$$W_i^P = \frac{a_i^2 + c_i b_i (\Theta^P)^2}{2(b_i + c_i)} + \chi_i \Delta^P$$

where

$$|\Theta^P| = \min\{T, T^*\} \implies |\Delta^P| = \max\left\{\frac{|A| - BT}{(b_i + c_i)(b_j + c_j)}; 0\right\}$$

As  $\Theta^P$  and  $\Delta^P$  are constant, such function is linear in  $X$ , and even constant if  $\Delta^P = 0$ . So by analysing the difference between the two functions, we reach two goals at the same time. Firstly, we compare the two situations. Secondly, we proceed to the analysis of an affine transformation of the local welfare function, which is equivalent to the initial function. To this end, we set the difference of the two functions as another function:

$$F_i(X) = W_i^* - W_i^P = \frac{b_i c_i (\Theta^*(X))^2 - (\Theta^P)^2}{2(b_i + c_i)} + z_i^*(X) \Delta^*(X) - \chi_i \Delta^P \quad (36)$$

The graphic below shows an example of  $F_i(X)$ : As we have seen before, the interconnection market follows four regimes. Each one of them triggers different strategies of players, and consequently different payoffs. The main element to emphasize is that the function is not monotonic in  $\chi_i$  in the monopoly regime  $i$ , and the function is negative in most of the cases. The goal of this section is to provide an explanation of this phenomenon. Let define the gradient of  $F_i$ ,  $\nabla F_i$ :

$$\nabla F_i = \begin{Bmatrix} \frac{\partial F_i}{\partial \chi_i} \\ \frac{\partial F_i}{\partial \chi_j} \end{Bmatrix} \quad (37)$$

From this definition,  $\chi_f$  is the residual part of  $\chi_i$  and  $\chi_j$ . Hence, every incremental add or removal of  $\chi_i$  or  $\chi_j$  is reversely removed or added to  $\chi_f$ . Similarly we could define  $\nabla_{z_i^*}$ ,  $\nabla_{\Theta^*}$  and  $\nabla_{\Delta^*}$ .

Hence,  $F_i$  has the following gradient:

$$\nabla F_i = \frac{b_i c_i \Theta^*(X)}{b_i + c_i} \nabla_{\Theta^*} + \Delta^*(X) \nabla_{z_i^*} + z_i^*(X) \nabla_{\Delta^*} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta^P$$

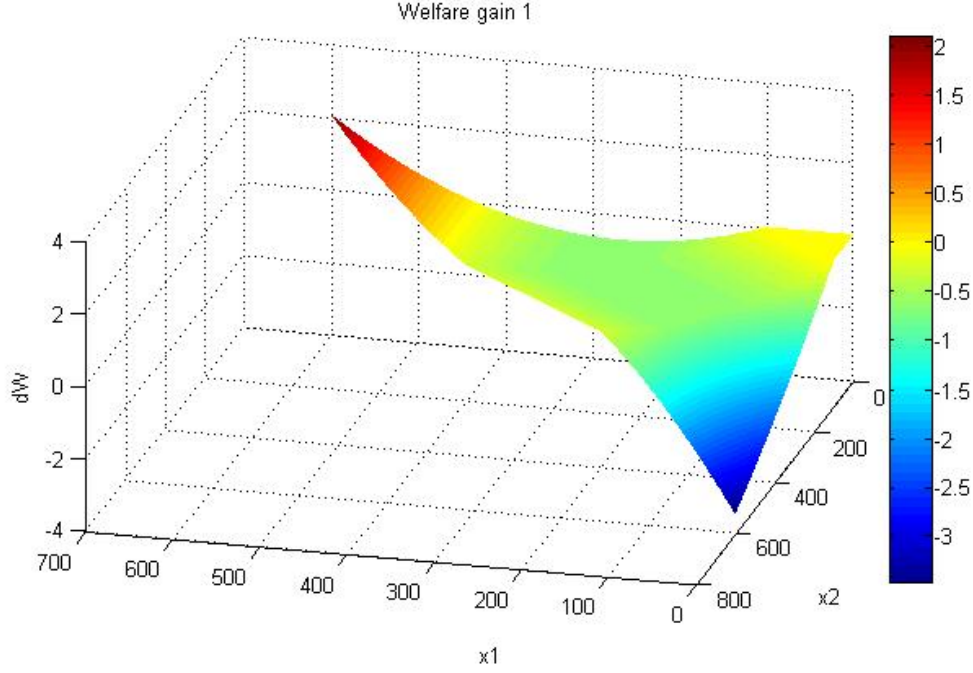


Figure 3: Local welfare: Equilibrium vs. perfect arbitrage

and from systems (38)–(40),  $\Theta^*(X)$ ,  $\Delta^*(X)$  and  $z_i^*(X)$  are linear in  $X$ , the second derivative gives:

$$\frac{\partial^2 F_i(X)}{\partial \chi_i^2} = \frac{b_i c_i}{b_i + c_i} \left( \frac{\partial \Theta^*(X)}{\partial \chi_i} \right)^2 + 2 \frac{\partial z_i^*(X)}{\partial \chi_i} \frac{\partial \Delta^*(X)}{\partial \chi_i}$$

First of all, let remark that in case that the equilibrium is perfectly arbitrated, then the above defined functions coincide.

**Remark 3** If  $X \in S^P$ , then  $W_i^* = W_i^P$ ,  $i = 1, 2$ ,  $i \neq j$

Then, if the market is in monopoly  $j$  regime, then the local welfare at node  $i$  at equilibrium is strictly dominated by the perfect arbitrage solution.

**Lemma 2** For any  $X \in S_j^M$ ,  $W_i^* < W_i^P$ ,  $i = 1, 2$ ,  $i \neq j$

**Proof.** First, remark that for  $\chi_i = 0$ , we obtain by comparing  $\theta^*$  and  $\theta^P$  in the system of equation (39), that  $F_i(X) < 0$ . Using systems (38)–(40) we could express gradients for each function of  $X$

$$\nabla z_i^* = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \nabla \Theta^* = \begin{bmatrix} 0 \\ -\frac{D_i}{B+D_i} < 0 \end{bmatrix} \quad \nabla \Delta^* = \begin{bmatrix} 0 \\ \frac{BD_i}{(B+D_i)(b_i+c_i)(b_j+c_j)} > 0 \end{bmatrix}$$

Hence it comes that

$$\nabla F_i = \begin{bmatrix} \Delta^* - \Delta^P \\ -\frac{D_i}{B+D_i} \frac{b_i c_i \Theta^*(X)}{b_i + c_i} + \frac{BD_i z_i^*(X)}{(B+D_i)(b_i+c_i)(b_j+c_j)} \end{bmatrix}$$

We know that  $\Delta^* > \Delta^P$ , so the more  $\chi_i$ , or equivalently the fewer  $\chi_f$ , the higher  $F_i$ . Set  $\chi_f = 0$  and  $\chi_i = \sigma_i^P = AD_i/(B^2 + D_i D_j)$ . This correspond to the highest  $F_i(X \in S_j^M)$ . Replacing  $T - \chi_j$  by  $\chi_i$ , we obtain after some calculus a second degree polynomial:

$$F_i = \frac{-\left(\Theta^P - \frac{AB}{B^2 + D_i D_j}\right)^2}{2(b_i + c_i)(b_j + c_j)}$$

which one is always negative. □

But, even if node  $i$ 's generator is in a monopolistic regime, there exists an allocation of PTRs in this subset such that the equilibrium strategy is strictly dominated by the perfect arbitrage solution.

**Lemma 3** *There exists  $X \in S_i^M$  such that  $W_i^* < W_i^p$ ,  $i = 1, 2$*

**Proof.** Using systems (38)–(40) we could express gradients for each function of  $X$

$$\nabla_{z_i^*} = \begin{bmatrix} \frac{B}{B+D_j} > 0 \\ 0 \end{bmatrix} \quad \nabla_{\Theta^*} = \begin{bmatrix} -\frac{D_j}{B+D_j} < 0 \\ 0 \end{bmatrix} \quad \nabla_{\Delta^*} = \begin{bmatrix} \frac{BD_j}{(B+D_j)(b_i+c_i)(b_j+c_j)} > 0 \\ 0 \end{bmatrix}$$

The derivative with respect to  $\chi_j$  is null, and the one with respect to  $\chi_i$  is not monotonic. Furthermore,  $(F_i)'' > 0$ , such that this function admit a unique minimum  $\underline{\chi}_i$ . It is then sufficient to find necessary conditions for  $F_i(\underline{\chi}_i) \leq 0$ . This minimum is reached for  $(F_i)' = 0$ , i.e.

$$\underline{\chi}_i = \begin{cases} \frac{B}{D_j} \cdot \frac{A-T(B+D_j)}{B+D_i} + T & \text{if } T < \frac{A}{B} \\ T - \frac{A}{B+D_i} & \text{if } T \geq \frac{A}{B} \end{cases}$$

We let the careful reader check that these values are greater than  $\sigma_i^M$ . Calculating the value of  $F_i(\underline{\chi}_i)$ , we obtain a second degree polynomial which state that:

$$F_i(\underline{\chi}_i) \leq 0 \iff -(A - (B + D_j)T)^2$$

which is negative. □

It is striking that this result appears for node  $i$  in its own monopoly regime. Indeed, one would think here that the system operator maximises welfare over quantities through a local monopolist which is regulated and foreign players who own too little rights to behave strategically. However this effect could be explain because the market operator is not present in the first stage of the game. It takes the outcome of this stage as given, confident in the fact that the monopolist at node  $i$  does not mark-up local price setting, by over exporting for example. But it does not take in account the marginal revenue at other node in its own price setting. Comparing generators' strategies in equation (25), and in Appendix A.3, they tend to overestimate competition on the first case. Hence, the market operator does not provide the good incentives to the monopolist.

Nevertheless, by construction, for  $\chi_i = T$ , the value of  $W_i^*$  correspond to the maximum of local welfare, since at this point, local marginal utility is equal to marginal cost, which is equal to the marginal revenue at the foreign node. Hence, it is also true that there exists positive  $F_i$  for  $X \in S_i^M$ .

To sum up, for  $X \in S_i^M$ , node  $j$  welfare at equilibrium is always strictly dominated by perfect arbitrage solution. Furthermore, there always exists allocation of  $X \in S_i^M$  for which equilibrium welfare at node  $i$  is also strictly dominated at equilibrium. In those situations we thus have that a strictly Pareto dominated equilibrium. Market operators at both node would then have incentives to coordinate.

We end now with the result that if the interconnection market is in a duopolistic regime, then the equilibrium is always Pareto-Dominated.

**Lemma 4** *For any  $X \in S^D$ ,  $W_i^* \leq W_i^p$ ,  $i = 1, 2$*

**Proof.** Using systems (38)–(40) and equation (35) we could express gradients for each function w.r.t.  $\chi_f$

$$\frac{\partial z_i^*}{\partial \chi_f} = \frac{-BD_i}{B^2 + D_i D_j} \quad \frac{\partial \Theta^*}{\partial \chi_f} = \frac{D_i D_j}{B^2 + D_i D_j} \quad \frac{\partial \Delta^*}{\partial \chi_f} = \frac{-BD_i D_j}{B^2 + D_i D_j (b_i + c_i)(b_j + c_j)}$$

Hence it comes that  $F_i$  has a global minimum in  $\chi_f$ . Equalizing first derivative with zero, we obtain that

$$\underline{\chi}_f = \frac{AB}{(B + D_i)(B + D_j)} > 0$$



Hence, borders of the subset  $S^D$  constitute local maxima. We know that as  $X$  goes to the perfect arbitrage area,  $F_i \rightarrow 0^-$ . It remains to calculate for  $\chi_f = 0$ , such that  $T = \chi_i + \chi_j$ . In this case, output of players are constants, and consequently total volume and price also. Hence, it remains that:

$$\nabla_{F_i} = \begin{bmatrix} -\Delta^P \\ \Delta^P \end{bmatrix}$$

Hence the function is linear or constant depending on the size of the interconnection. We could conclude that  $F_i(\chi_i = \sigma_i^D) \geq F_i(\chi_j = \sigma_j^D)$ . However,  $F_i(\chi_i = \sigma_i^D) < 0$  since  $\chi_i = \sigma_i^D \implies X \in S_j^M$ . By continuity of  $\gamma_i(X)$  and monotonicity of  $F_i(X)$  in this region, we have that  $F_i(X) < 0 \forall X$ .  $\square$

Hence, the duopoly situation is always sub-optimal from the local welfare point of view.<sup>10</sup> We links all this lemmas with continuity of  $\Gamma$  in  $X$  to state the last result:

**Proposition 4** *There exists an allocation  $\mathbf{x} \in \mathcal{V}$ , such that the equilibrium is Pareto-Dominated by the perfect arbitrage solution from the local welfare perspective.*

**Proof.** Trivial.  $\square$

This result is of prime importance in the sense that the objective of regulation is to maximise welfare, but we find some situations where it is not the case, in a context where the system operator is not gamed. By comparing welfare function as defined in our utilities' problem (see Definition 7) and the social planner's one (see Appendix A.3), we can see that a local regulator does not take into account the marginal impact of other actions (everyone excepts the local monopolist) on the marginal utility of consumers at its node.

The associated interpretation would be the following. By putting weight on service of local consumption at a least cost, and so enforcing regulation at a local level, the regulated market operator fails in comparing marginal willingness to pay of exporter and marginal willingness to receive of its own consumers in exchange of this export. This is implicitly due to the fact that a physical market operator is quantity-taker of the interconnection game's outcome, *modulo* local monopolist does not game it. So the market operator cannot make such arbitrage efficiently. Hence we could see this effect as a non-internalized market externalities.

Consumer's surplus analysis is straightforward. Exporting is synonymous of lower consumption and higher price whereas importing has the opposite effect. Analysing profit, results are similar to one would expect according to [21]. An importer has more interest for imperfect arbitrage equilibrium, in order to let local price relatively high. He would prefer a situation where he owns all PTRs, but if an exporter get them, it is still better than a perfect arbitrage situation. For an exporter, he prefers a perfectly arbitrated interconnection in most cases. The worst case for him is that a strategic importer get all the rights. However, in the case where transmission capacity is large, as the two markets' reserve price become very different, it is more and more interesting for an exporter to own as much PTRs as possible. In such a case, exporter's monopoly results in an equilibrium which Pareto Dominates perfect arbitrage solution from generators point of view. In this case, producers have common interests and may collude.<sup>11</sup>

#### 4.4 Discussion on the PTRs allocation mode

The goal of this section is to discuss whether sub-optimal equilibrium might be a realistic output of the game, and discuss the incentives for system operators to coordinate in order to make an allocation process as transparent as possible. However, defining PTR' quantity and price settled for each participant goes beyond the scope of this article. Following the grid of analysis developed in Joskow and Tirole [21], the micro-structure of initial ownership of rights sets in large part the final result, but the link between system operators and generators also plays a role.

<sup>10</sup>This results has the flavour of prisoner's dilemma, but the acting players are the generators and not the market operators.

<sup>11</sup>See Appendix A.4 for mathematical details.

Overall, exporting generators have interest to buy PTRs and withhold them only for very large price differential – based on reserve prices difference – and an even larger interconnection capacity. In every other cases, an exporting generator prefers PTRs to be efficiently managed (i.e. by traders). The threshold between those two payoff regions is defined by the relative size of their local market demand. If it is important, then the rent earned on the other market by owning PTRs does not sufficiently compensate the increase of rent resulting from an increase in local price, assuming quadratic convex cost function and linear inverse demand function. For an importing generator, owning as much right as possible is always the best strategy, but they are fine as long as PTRs doesn't belong to the competitive fringe. For a system operator, assuming it could give some incentives – regulatory or monetary – to generators, it is best to have as much rights as possible, but this being a risky game as sub-optimal results exist if both SO play this strategy. Accordingly, their incentive to coordinate with other SO in order to develop transparent allocation process which foster perfect arbitrage equilibrium is rather poor in this model.

For relatively large level of interconnections ( $T > \frac{A}{B+D_i}$  for all  $i$ ), the impact of the micro structure of the allocation mechanism is similar to [21]. If PTRs' allocation is made through bilateral transactions, i.e. a single non-participating initial owner sells them through contracts, rights go to strategic producers, and equilibrium is of imperfect arbitrage. For multilateral agreements, e.g. many initial owners with no market power, participate in the market, as in [21], if conditional offers could be made, then strategic producers will be final owners of PTRs. If offers are unconditional, PTRs goes to the competitive fringe.

## 5 Concluding comments

We developed a model to analyse physical transmission rights withholding as a strategy to increase profit. Producers, who can't exert market power in their domestic node because of a tight regulation, are free to manipulate foreign market price as long as it is not part of a strategy to manipulate local market. This situation mirrors current regulation which focuses on local issues but does not coordinate with regulators from other control zones to insure efficient trade.

We first describe the model in a general way, assuming no specific functional forms. We define sufficient conditions for the model to converge. We also describe a general outcome of the model: an allocation of PTRs to strategic generators is a necessary condition for existence of imperfect arbitrage equilibrium, but not a sufficient one. Other factors, such as the relative size of the interconnection, or the allocation to the competitive fringe also play a role.

We then assume quadratic concave utility function and quadratic convex cost function. We fully characterize the equilibrium, and we find that withholding strategies could happen under quite smooth conditions. But, on the other hand, giving access to strategic generators to interconnection market does not always lead to imperfect arbitrage equilibrium, but it implies a non-null price difference between the two nodes. This situation relies heavily on the conditions that grounds the market: capacity of the interconnection, reservation price of demand, price sensitivity of local market, marginal cost parameters.

Strategic interactions are highlighted by analysing different interconnection's ownership structure and relative size with respect to reservation prices difference between the two nodes. The more different are reservation prices, the less probable is withholding strategy. When it comes to interconnection capacity, more is better only if, a certain amount of it is not allocated to strategic generators. To the concern of ownership allocation of PTRs to those generators, one could say that fewer is better, but this should be carefully studied for every condition.

When proceeding to comparative statics of local welfare with respect to ownership in a case of monopoly-fringe market structure, we find quite surprisingly that above some (relative) interconnection capacity, some Nash equilibrium are Pareto dominated by perfect arbitrage solution. This is quite counter-intuitive, given that regulator should set the price to maximise local welfare. This seems to be due to a market externalities: the regulator at node  $i$  does not take into account the reaction of the other node when it sets the local price. However, this result deserves more analysis.

This study may be the object of several extensions. First we could use the framework developed here to simulate numerically some real situations, and to infer profitability of interconnection projects. Second, one could develop a third stage where PTRs are allocated through an open market or an auction, or any multilateral agreement. Third, we may extend the analysis to  $n$  players at each node, and assume some form of local market power.

## A Appendices

### A.1 Absolute value

For example assume that  $A > 0$ , such that following arbitrage direction,  $x_f > 0$ ,  $x_1 > 0$  and  $x_2 < 0$ . Here the non-trivial operation is for  $\gamma_2$ :

$$\gamma_2 = \frac{- (|A| - B(\gamma_f |x_f| + \gamma_1 |x_1|))}{(B + D_1)(-|x_2|)} = \frac{|A| - B(\gamma_f |x_f| + \gamma_1 |x_1|)}{(B + D_1)|x_2|}$$

Reversely, assume that  $A < 0$ , such that  $x_f < 0$ ,  $x_1 < 0$  and  $x_2 > 0$ . Hence for  $\gamma_1$  and  $\gamma_2$ , we could express it:

$$\begin{aligned} \gamma_1 &= \frac{-|A| - B(-\gamma_f |x_f| - \gamma_2 |x_2|)}{(B + D_2)(-|x_1|)} = \frac{|A| - B(\gamma_f |x_f| + \gamma_2 |x_2|)}{(B + D_2)|x_1|} \\ \gamma_2 &= \frac{-(-|A| - B(-\gamma_f |x_f| - \gamma_1 |x_1|))}{(B + D_1)|x_2|} = \frac{|A| - B(\gamma_f |x_f| + \gamma_1 |x_1|)}{(B + D_1)|x_2|} \end{aligned}$$

### A.2 Normalized results

Normalization is defined such that positive value refers to export from node 1 to 2. Following this normalization we could state different results at equilibrium:

$$\begin{aligned} X \in S_i^M &\iff z_i^* = \frac{A - B(T - \chi_i)}{B + D_j} \\ X \in S^D &\iff z_i^* = \frac{D_i(A - B(T - \chi_i - \chi_j))}{B^2 + D_i D_j} \\ X \in S_j^M \cup |x_i| < \sigma_i^M &\iff z_i^* = x_i \\ x_f > T^* &\implies z_i^* = 0 \end{aligned} \tag{38}$$

Also, normalized notation is useful for stating result such as the total power  $\Theta^*$  which flows in the interconnection. Indeed, several interconnection market's regimes implies different output.

$$\begin{aligned} \text{if } \mathbf{x} \in S_i^M &\implies \Theta^* = \frac{A + D_j(T - \chi_i)}{B + D_j} \\ \text{if } \mathbf{x} \in S^D &\implies \Theta^* = \frac{AB + D_i D_j(T - \chi_i - \chi_j)}{B^2 + D_i D_j} \\ \text{if } \mathbf{x} \in S^P &\implies \Theta^* = \min\{T; T^*\} \end{aligned} \tag{39}$$

Which induces the normalized price differential  $\Delta^*$ :

$$\begin{aligned} \text{if } \mathbf{x} \in S_i^M &\implies \Delta^* = \frac{D_j}{B + D_j} \frac{A - B(T - \chi_i)}{(b_i + c_i)(b_j + c_j)} \\ \text{if } \mathbf{x} \in S^D &\implies \Delta^* = \frac{D_i D_j}{B^2 + D_i D_j} \frac{A - B(T - \chi_i - \chi_j)}{(b_i + c_i)(b_j + c_j)} \\ \text{if } \mathbf{x} \in S^P &\implies \Delta^* = \max\left\{\frac{A - BT}{(b_i + c_i)(b_j + c_j)}; 0\right\} \end{aligned} \tag{40}$$

### A.3 Local social planner maximization

a Social planner who control price and quantity maximises the following local welfare function by backward induction, replacing  $q_i$  by  $y_i - \theta_i$ . For node  $i$ :

1. production at local node:

$$\max_{y_i} W_i^P = U_i(y_i - \theta_i) - C_i(y_i) \quad (41)$$

we obtain  $y_i(\theta_i)$ ,  $i = 1, 2$

2. export/import stage

$$\max_{\gamma_i} W_i(\gamma_i) = \hat{U}_i(\Gamma) - \hat{C}_i(\Gamma) + \gamma_i x_i \hat{U}'_j(\Gamma) + [\theta_i(\Gamma) - \gamma_i x_i] \hat{U}'_i(\Gamma) \quad (42)$$

by accounting that  $\partial \theta_i / \partial \gamma_i = x_i$ , it yields the FOC :

$$\frac{\partial W_i}{\partial \gamma_i} = 0 : \hat{U}'_i(\Gamma) - \hat{C}'_i + x_i \left[ \hat{U}'_j(\Gamma) + \gamma_i \frac{\partial \hat{U}'_j}{\partial \gamma_i} \right] + [\theta_i(\Gamma) - \gamma_i x_i] \frac{\partial \hat{U}'_i}{\partial \gamma_i} \quad (43)$$

In the example, this would be equal to

1. production at local node:

$$\max_{y_i} W_i^P = (y_i \mp \Theta) \left( a_i - \frac{b_i}{2} (y_i \mp \Theta) \right) - \frac{c_i}{2} y_i^2$$

which yields the FOC:

$$y_1(\Theta) = \frac{a_1 + b_1 \Theta}{b_1 + c_1} \quad y_2 = \frac{a_2 - b_2 \Theta}{b_2 + c_2}$$

2. the total welfare which depends on  $\Theta$

$$\begin{aligned} \max_{\gamma_1} W_1 &= \frac{a_1 - c_1 \Theta(\gamma_1)}{b_1 + c_1} \left[ a_1 - \frac{b_1}{2} \left( \frac{a_1 - c_1 \Theta(\gamma_1)}{b_1 + c_1} \right) \right] - \frac{c_1}{2} \left( \frac{a_1 + b_1 \Theta(\gamma_1)}{b_1 + c_1} \right)^2 \\ &+ \gamma_1 x_1 \left[ a_2 - b_2 \left( \frac{a_2 + c_2 \Theta(\gamma_1)}{b_2 + c_2} \right) \right] + (\gamma_f x_f - \gamma_2 x_2) \left[ a_1 - b_1 \left( \frac{a_1 - c_1 \Theta(\gamma_1)}{b_1 + c_1} \right) \right] \end{aligned}$$

$$\begin{aligned} \max_{\gamma_2} W_2 &= \frac{a_2 + c_2 \Theta(\gamma_2)}{b_2 + c_2} \left[ a_2 - \frac{b_2}{2} \left( \frac{a_2 + c_2 \Theta(\gamma_2)}{b_2 + c_2} \right) \right] - \frac{c_2}{2} \left( \frac{a_2 - b_2 \Theta(\gamma_2)}{b_2 + c_2} \right)^2 \\ &+ \gamma_2 x_2 \left[ a_1 - b_1 \left( \frac{a_1 - c_1 \Theta(\gamma_2)}{b_1 + c_1} \right) \right] - (\gamma_f x_f + \gamma_1 x_1) \left[ a_2 - b_2 \left( \frac{a_2 + c_2 \Theta(\gamma_2)}{b_2 + c_2} \right) \right] \end{aligned}$$

which yield the solution:

$$\gamma_1 x_1 = \frac{A - D_2(\gamma_f x_f - \gamma_2 x_2)}{B + D_2} \quad (44)$$

$$\gamma_2 x_2 = \frac{-A + D_1(\gamma_f x_f + \gamma_1 x_1)}{B + D_1} \quad (45)$$

$$\gamma_f x_f = \frac{A - B(\gamma_1 x_1 - \gamma_2 x_2)}{B} \quad (46)$$

### A.4 Profit sensitivity

Here, we don't proceed to the normalization because we refers to exporters and importers. Hence it is preferable for the demonstration that function are expressed form node  $i$  perspective. Set  $\gamma_i^*(\mathbf{x}) \chi_i = z_i^*(\mathbf{x})$ , and  $\delta_i$  the non-normalized price differential:

$$\begin{aligned} \delta_i &= \frac{a_j c_j}{b_j + c_j} - \frac{a_i c_i}{b_i + c_i} - \theta_i \left( \frac{b_j c_j}{b_j + c_j} + \frac{b_i c_i}{b_i + c_i} \right) \\ &= \frac{A_i - B \theta_i}{(b_i + c_i)(b_j + c_j)} \end{aligned}$$

Profit function at equilibrium is written:

$$\Pi_i^*(\mathbf{x}) = \frac{c_i(a_i + b_i\theta_i^*(\mathbf{x}))^2}{2(b_i + c_i)^2} + z_i^*(\mathbf{x})\delta_i^*(\mathbf{x}) \quad (47)$$

$z_i^*$ ,  $\theta_i^*$ ,  $\delta_i^*$  being linear functions, first and second order partial derivatives gives:

$$\begin{aligned} \frac{\partial \Pi_i^* \mathbf{x}}{\partial x_i} &= \frac{b_i c_i (a_i + b_i (\theta_i^*(\mathbf{x})))}{(b_i + c_i)^2} \frac{\partial \theta_i^*(\mathbf{x})}{\partial x_i} + z_i^*(\mathbf{x}) \frac{\partial \delta_i^*(\mathbf{x})}{\partial x_i} + \frac{\partial z_i^*(\mathbf{x})}{\partial x_i} \delta_i^*(\mathbf{x}) \\ \frac{\partial^2 \Pi_i^* \mathbf{x}}{\partial x_i^2} &= \frac{b_i^2 c_i}{(b_i + c_i)^2} \left( \frac{\partial \theta_i^*(\mathbf{x})}{\partial x_i} \right)^2 + 2 \frac{\partial z_i^*(\mathbf{x})}{\partial x_i} \frac{\partial \delta_i^*(\mathbf{x})}{\partial x_i} \end{aligned}$$

We know  $\forall \mathbf{x}$ :

$$\frac{\partial \theta_i(\mathbf{x})}{\partial x_i} \leq 0 \quad \frac{\partial \delta_i(\mathbf{x})}{\partial x_i} \geq 0 \quad \frac{\partial z_i(\mathbf{x})}{\partial x_i} \geq 0$$

For an importing generator:  $x_i \leq 0$ ,  $z_i(\cdot) \leq 0$ ,  $\delta_i(\cdot) \leq 0$ ,  $\theta_i(\cdot) \leq 0$  but  $a_i + b_i\theta_i(\cdot) > 0$  (otherwise price would be non-positive); this function is monotone non increasing in  $x_i$ . Considering  $\frac{\partial \Pi_i \mathbf{x}}{\partial x_j}$  gives the same results: an increase of  $x_j$  at the expense of  $x_f$  has a non-negative impact on price, and a non-positive effect on quantities. Hence an importing generator has interest that interconnection market be the less competitive.

For an exporting generator, this function is not monotonic and admit a minimum. For consumers of the exporting country it is easy to show that perfect arbitrage is their least preferred situation. Thus, since consumer surplus at equilibrium is greater or equal than consumer surplus in perfect arbitrage,  $W_i^* - W_i^P < 0 \implies \Pi_i^* < \Pi_i^P$ . So we have to show that there exists allocation  $\mathbf{x}$  for which  $\Pi_i^* > \Pi_i^P$ . for  $x_i = T$ :

$$\begin{aligned} Z_i &= \frac{A_i}{B + D_j} = \theta_i \quad \delta_i = \frac{A_i D_j}{(B + D_j)(b_i + c_i)(b_j + c_j)} \\ \Pi_i &= \frac{c_i(a_i(B + D_j) + A_i b_i)^2}{2(b_i + c_i)^2(B + D_j)^2} + \frac{A_i^2 D_j}{(B + D_j)^2(b_i + c_i)(b_j + c_j)} \end{aligned}$$

For  $T < T^*$ :

$$\Pi_i^P = \frac{c_i(a_i + b_i T)^2}{2(b_i + c_i)^2} + \frac{T(A - BT)}{(b_i + c_i)(b_j + c_j)}$$

For  $T \geq T^*$

$$\Pi_i^P = \frac{c_i(a_i B + b_i A_i)^2}{2B^2(b_i + c_i)^2}$$

We calculate:

$$\begin{aligned} \Pi_i^*(x_i = T) > \Pi_i^P(T < T^*) &\iff (B + D_j)^2 (T^2[(B + D_j)(b_i + c_i) + D_i c_i] - 2T[a_i D_i + A_i(b_i + c_i)]) \\ &\quad + A_i[(B + D_j)(A_i b_i + 2a_i D_i) + 2c_i D_j] > 0 \\ \Pi_i^*(x_i = T) > \Pi_i^P(T \geq T^*) &\iff A_i > a_i D_i \frac{2B^2(B + D_j)}{b_i D_j(B + D_j) + 2B^2 c_i} \end{aligned}$$

Hence if the difference between the two markets' reserve price is great ( $A_i$  large), then the exporter prefers to own as much rights as possible, rather than a perfect arbitrage equilibrium where local prices are higher, but potential profit on interconnection are greater. In this case, exporters and importers would have interest to collude in order to avoid perfect arbitrage equilibrium.

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