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Credit risk in corporate spreads during the financial crisis of 2008: A regime-switching approach

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Abstract: Credit spreads and CDS premiums are investigated before, during and after the financial crisis with a flexible credit risk model. The latter is designed to capture empirical facts: a regime-switching framework adjusts its behaviour to the financial cycles and the negative relationship between recovery rates and default probabilities appears endogenously.

Using a firm-by-firm estimation of 225 companies, notorious empirical questions are revisited, including the famous credit spread puzzle. The proportion of the spread explained by credit risk decreases during the crisis. Liquidity plays a significant role in explaining this gap throughout the financial turmoil and persists thereafter.

Key Words: Credit spread, credit default swap, crisis, credit risk, hybrid model, liquidity risk.

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1 Introduction

The so-called Global Financial Crisis of 2008 is considered by various economists as the worst crisis since the Great Depression. It is now clear that the 2008 crisis was primarily caused, among other things, by a decline in the United States' housing market (see Acharya et al. (2009)). This led to increased levels of mortgage defaults and induced a steep decrease in the value of mortgage-backed securities. At some point, investors began avoiding all risks, liquidity dried up and turmoil hit financial markets.

Worldwide, 101 corporate issuers rated by Moody's defaulted on a total of \$281.2 billion of debt in 2008 (Moody's, 2009). For the sake of comparison, only 18 issuers defaulted in 2007 on a total of \$6.7 billion of debt. The year 2008 witnessed the largest defaulter in history: Lehman Brothers Holdings Inc. Other large financial institutions, including Washington Mutual and three large Icelandic banks (Landsbanki, Kaupthing and Glitnir) also defaulted. The situation was so dire in the United States that the federal government implemented a program (Troubled Asset Relief Program, or TARP) in October 2008 to purchase assets and equity from financial institutions to strengthen the financial sector. Unsurprisingly, the panic that struck the marketplace had a major impact on the credit market. Therefore, corporate credit spreads and credit default swap (CDS) premiums rose significantly. However, it is not clear whether these increases were caused by increased risk of default or other factors. For instance, recovery rate risk, liquidity risk, risk aversion, market risk and macroeconomic risk could also have an impact on credit spreads and CDS premiums.

Understanding what proportion of credits spreads can be explained by credit risk and finding other factors of interest are two main research concerns found in the recent financial literature. These questions have been addressed by many: Elton et al. (2001), Collin-Dufresne et al. (2001), Lando and Skødeberg (2002), Driessen (2005), Chen et al. (2009), Dionne et al. (2010) and Huang and Huang (2012) among others. Two general conclusions arise from these studies. First, the proportion of the credit spreads explained by credit risk depends on the model, the estimation methodology and the dataset used. Second, the other risks mentioned above could be priced in corporate bonds, and again, the findings depend on the model, the estimation procedure and the data.

The determinants of credit default swap premiums are somewhat less exhaustive than those of corporate bond spreads. For instance, according to Ericsson et al. (2009), leverage, asset volatility and the risk-free rate are important determinants of credit default swap premiums, as predicted by theory. Moreover, applying a principal component analysis on the residuals, the authors find only weak evidence for a residual common factor.

In this paper, the main objective is to analyze the determinants of credit spreads and CDS premiums. The focus is placed on three different periods: pre-crisis era, crisis era, and post-crisis era. According to the literature, conclusions are sensitive to the characteristics of the model, which needs to be constructed carefully to capture the desired empirical facts. Our model includes a regime-switching variable that accommodates behavioural changes during financial turmoils. As in Boudreault et al. (2013), the negative relationship between credit ratings and recovery rates is modelled with an endogenous random recovery rate that depends on a firm's financial health; this empirical observation was highlighted by Altman et al. (2004) and Altman (2006), among others.

The estimation method must be tailored for the issue at hand: two latent variables shall be filtered simultaneously, i.e. the hidden regime and the firm's leverage, in addition to the model's parameters, which need to be estimated. The proposed filter has numerous advantages. First, it allows for multiple data sources that help disentangle some effects¹ that cannot be separated using a single product. Second, using time series of derivatives captures the dynamics under both objective and risk-neutral probability measures simultaneously. Third, the estimation of parameters is performed through the quasi-maximum likelihood function. Finally, the numerical implementation is fast and efficient. The model is estimated using different tenors from the term structure of CDS premiums for more than 200 firms on a firm-by-firm basis.

One in-sample and two out-of-sample analyses show that the model outperforms three others benchmarks. The relative pricing errors are also analyzed: the errors are regressed against different market-wide, interest rate, liquidity and firm-specific factors. The coefficient of determination of such regressions are rather small,

¹For instance, liquidity issues in some tenors, recovery rate uncertainty, default risk, etc.

ranging between 2.6% and 13.2%, implying that the model captures most of the variation in the CDS premiums.

The effects of stochastic recovery and the presence of regimes on the yield spread curve are investigated. Recovery uncertainty has a major impact on mid- and long-term credit spreads. While the presence of regimes has a second-order effect on average credit spread curves, it affects the long-term credit spread of high rated firms in the pre- and post-crisis periods. During the financial crisis, the short-term shape of the average credit spread curves is modified in the presence of regimes, especially for well-rated firms.

Theoretical spot spreads obtained through the credit risk model are compared with corporate bond benchmark spot spreads. In the pre-crisis period, the proportions of credit spread explained by credit risk found by our computations are much larger than those obtained by Huang and Huang (2012) and Elton et al. (2001), among others. However, the crisis seems to have had an impact on these figures: for the proposed model, the explained proportion of 15-year BBB spot spreads decreases from 67% in the pre-crisis era to 42% during the crisis, on average. Recovery rate risk and its dependence with default risk partly accounts for the high proportion of the empirical spreads explained by credit risk. For instance, applying a constant exogenous recovery rate decreases the proportion of credit spread explained by the model for 15-year spreads by an average of 14.2%.

Since credit risk does not fully explain the credit spreads, liquidity risk is investigated in the bond market. In the spirit of Dick-Nielsen et al. (2012), a liquidity proxy is computed. To control for credit risk, bond yield-to-maturity spreads implied by the regime-switching model are employed. Using regression analyses, the liquidity proxy terms are statistically different from zero for most ratings and periods. Moreover, the liquidity component generally tends to increase during the crisis; for B-rated firms, it is about 37.8 basis points (bps) during the pre-crisis era, 118.6 bps during the crisis, and 83.1 bps during the post-crisis era. The spread fraction also rises during the crisis: for B-rated firms, the fraction increases by 7.7%.

Last but not least, other regressors are added to previous liquidity regressions: two market-wide factors, one variable related to the interest rate market, and two firm-specific factors. In the three periods, the theoretical spreads explain a significant part of the observed credit spread's variation with an R -squared of 65%. When liquidity is taken into account, the R -squared reaches 67%. All additional variables brings the R -squared to 72%, implying that most of the variation presents in the observed CDS premiums are mainly explained by the model and the liquidity proxies.

The rest of this paper is organized as follows. The joint default and loss model is presented in Section 2. Section 3 explains the model estimation approach and discusses the estimation results. In Section 4, the impact of two important features of the model (i.e. endogenous random recovery and regime-switching risks) are analyzed in a comparison study. An assessment of the proportion of the corporate credit spread explained by credit risk is described in Section 5. Section 6 discusses the issue of liquidity in the corporate bond market. Other factors are analyzed in Section 7. Finally, Section 8 concludes.

2 Joint default and loss model

According to Ericsson et al. (2009), the three main determinants of default are leverage, asset volatility and interest rates. By considering Markov-switching dynamics for the firm's assets and liabilities, the proposed model can handle, at least partially, two of the three main determinants of default. However, even when accounting for these three variables, a large proportion of spreads remain unexplained (Ericsson et al., 2009); in the empirical literature, this issue is referred to as the credit spread puzzle. In addition, continuous-path structural models have failed to appropriately represent short term credit spreads, mainly because the default is a predictable stopping time. To solve these issues, structural models with incomplete information were proposed; the presence of a surprise element that adds randomness to the default trigger is an important feature of these models. They are part of the so-called hybrid approach in credit risk modelling. For instance, Duffie and Lando (2001) suppose that bond investors cannot observe the issuer's assets directly and receive periodic and imperfect accounting reports instead. Jarrow and Protter (2004) show that the incomplete knowledge of the firm's assets and liabilities leads to an inaccessible default time.

Other frameworks combine ideas from both structural and reduced-form approaches to obtain hybrid models: firms' liabilities and assets are modelled as stochastic processes and the default time is given by the first jump of a Cox process for which the intensity depends on the firm's fundamentals (e.g. Madan and Unal (2000), Bakshi et al. (2006b), and Boudreault et al. (2013)).

Moreover, as documented by various authors, CDS premium dynamics and credit spreads change during financial crises. Indeed, Dionne et al. (2011) estimate the default risk component of corporate yield spreads using a reduced-form model based on observed macroeconomic factors (consumption and inflation) with the possibility of regime changes. Their factors are linked to sharp increases in default spreads in two out of three NBER economic recessions over the 1987–2008 period.

During the last financial crisis, the dynamics of credit default swap premiums is investigated by Huang and Hu (2012). Applying a smooth transition autoregressive model, the authors provide clear evidence for transitions between low-price and high-price regimes in CDS spreads of 28 firms from 2007 to 2009. Maalaoui Chun et al. (2013) apply a regime-detection technique distinguishing between level and volatility regimes in credit spreads. They show that most breakpoints occur around economic downturns, thus linking the statistical regimes to the financial crises.

In this paper, the shell approach of Boudreault et al. (2013) is extended to capture the time-varying nature of the volatility.² The shell is essentially an intensity process that depends on the firm's leverage, which is modelled by Markov-switching dynamics. This flexible approach allows for an endogenous recovery rate that is both stochastic and negatively correlated with the firm's probability of default. The negative correlation between these two quantities is of paramount importance: as the firm's financial health becomes precarious (i.e. the firm's leverage raises), its default probability increases, but the recovery rate decreases accordingly.

According to Altman et al. (2005) and Acharya et al. (2007), a negative correlation between default probabilities and recovery rates exists, and both variables seem to be driven by the same factor. Using data from 1982 to 2002, Altman et al. (2005) show that losses are vastly understated if one assumes that default probabilities and recovery rates are uncorrelated.

Other contributions show the importance of the negative relationship between default probabilities and recovery rates. Using BBB-rated corporate bonds, regression analyses, and the information on all companies that have defaulted between 1981 and 1999, Bakshi et al. (2006a) find that, on average, a 4% increase in the risk-neutral hazard rate is associated with a 1% decline in risk-neutral recovery rates. Gaspar and Slinko (2008) explain empirically observed features, such as the negative correlation between the default probabilities and recovery rates through a reduced-form approach. An econometric model is developed by Bruche and González-Aguado (2010) which tries to assess by how much one underestimates credit risk if ignoring the negative relationship between default probabilities and recovery rates. Using a simple structural model, Bade et al. (2011) find that default and recovery are highly negatively correlated; the recovery is modelled as a stochastic quantity that depends on observable risk factors and a systematic random variable. Das and Hanouna (2009) use stock prices and CDS premiums to calibrate a hybrid model for default probabilities and recovery rates using 3,130 different firms from 2000 to 2002. The use of both markets helps identify the recovery rates.

2.1 Model

Let L_t and A_t be the time t market value of the firm's liabilities and assets respectively. The leverage ratio is defined as the quotient of these two values.³ Because the dataset overlaps the financial crisis, the model should be flexible enough to capture the changes in the state variable dynamics before, during, and after the financial turmoil. Hence, the market value of the firm's log-leverage is characterized by the following

²Zhang et al. (2009) show that volatility risk predicts 48% of the variation in the CDS spreads using a Merton-type structural model and a calibration approach.

³The leverage ratio L_t/A_t is not constrained to lie within the unit interval since L_t is the liabilities value and not the risky debt value. The liabilities value L_t could thus be larger than A_t .

regime-switching dynamics:

$$\log\left(\frac{L_t}{A_t}\right) = x_t = x_{t-1} + \left(\mu_{h_t}^{\mathbb{P}} - \frac{1}{2}\sigma_{h_t}^2\right)\Delta_t + \sigma_{h_t}\sqrt{\Delta_t}\epsilon_t^{\mathbb{P}}, \quad (1)$$

where h_t is the regime prevailing during the time interval $((t-1)\Delta_t, t\Delta_t]$, Δ_t is the time step between two consecutive observations, $\{\epsilon_t^{\mathbb{P}}\}_{t=1}^{\infty}$ is a sequence of independent standardized Gaussian random variables under the statistical probability measure \mathbb{P} , and $\mu_1^{\mathbb{P}}, \dots, \mu_K^{\mathbb{P}}, \sigma_1, \dots, \sigma_K$ are parameters that drive the leverage dynamics under the K possible regimes. The information structure is captured with the filtration $\{\mathcal{G}_t\}_{t=0}^{\infty}$ generated by the noise series $\{\epsilon_t^{\mathbb{P}}\}_{t=1}^{\infty}$ and the regimes $\{h_t\}_{t=0}^{\infty}$.

Because the leverage ratio is one of the potential drivers of default, it is incorporated in an intensity process $\{I_t\}_{t=0}^{\infty}$ that characterizes the potential default:

$$I_t = \beta + \left(\frac{L_t/A_t}{\theta}\right)^{\alpha} \quad (2)$$

where α , β , and θ are positive constants. The default intensity increases with the leverage ratio, making the default more likely to happen. As usual in intensity-based models, the default time arises as soon as the intensity accumulation reaches a random level determined by an exponentially distributed random variable E_1 of mean 1 independent of $\{\mathcal{G}_t\}_{t=0}^{\infty}$:

$$\tau = \inf\left\{t \in \{1, 2, \dots\} : \sum_{u=0}^{t-1} I_u \Delta_t > E_1\right\}. \quad (3)$$

The recovery rate at default time is proxied by

$$R_{\tau} = \min\left(\left(1 - \kappa\right)\frac{A_{\tau}}{L_{\tau}}; 1\right) = \min\left(\left(1 - \kappa\right)e^{-x_{\tau}}; 1\right), \quad (4)$$

where κ represents the legal and restructuring fees, expressed as a proportion of the asset value at default time. Consequently, an endogenous random recovery rate negatively correlated to default probabilities is implied by Equation (4).

The unobserved regime is modelled as a time-homogenous Markov chain with transition probabilities

$$p_{ij}^{\mathbb{P}} = \mathbb{P}(h_t = j | h_{t-1} = i), \quad i, j \in \{1, 2, \dots, K\}. \quad (5)$$

The market model is incomplete, implying that there are an infinite number of pricing measures. Among these measures, we restrict the choices to those preserving the model structure:

$$x_t = x_{t-1} + \left(\mu_{h_t}^{\mathbb{Q}} - \frac{1}{2}\sigma_{h_t}^2\right)\Delta_t + \sigma_{h_t}\sqrt{\Delta_t}\epsilon_t^{\mathbb{Q}} \quad (6)$$

and

$$p_{ij}^{\mathbb{Q}} = \mathbb{Q}(h_t = j | h_{t-1} = i), \quad i, j \in \{1, 2, \dots, K\} \quad (7)$$

where $\{\epsilon_t^{\mathbb{Q}}\}_{t=1}^{\infty}$ is a sequence of independent standardized Gaussian random variables under \mathbb{Q} .

2.2 Bond and credit default swap pricing

The derivative securities priced within this model take into consideration default, recovery, and regime risks. However, stochastic recovery rates and regime-switching dynamics prevent us from using closed-form solutions for CDS premiums and corporate coupon bond prices. An extension of Yuen and Yang (2010)'s trinomial lattice is used in this paper. Details about the pricing scheme are provided in Appendix A.

3 Estimation

The model is estimated on a firm-by-firm basis by filtering out the unobservable firm leverages, the hidden regime probabilities and all \mathbb{P} - and \mathbb{Q} -parameters from time series of CDS premiums of various tenors.

The question of whether CDS premiums include liquidity effects or counterparty risk is controversial. Based on CDS premiums from March 2008 to January 2009 on firms of widely followed CDX indices and regression analyses, Arora et al. (2012) show that even though counterparty credit risk is significantly priced in the CDS market, the magnitude of the effect is relatively modest.⁴ Based on CDS data from 2001 to 2007 (when the CDS market was still young), Bühler and Trapp (2009) find that the default component accounts for 95% of the CDS premiums on average. The nature of this dataset is nonetheless very different from ours.⁵

Using pre-crisis data and regression analyses, Tang and Yan (2007) show that liquidity could have a mild positive impact on CDS spreads, mainly due to search frictions and adverse selection. Using a number of quote providers as a measure of CDS market depth and a similar dataset to ours, Qiu and Yu (2012) explore the behaviour of CDS liquidity across 732 firms over 2001–2008. Large firms and firms near the “IG/HY boundary” tend to be the most liquid ones.

On a different note, Longstaff et al. (2005) argue that CDSs are of a contractual nature that affords relative ease of transacting large notional amounts compared to the corporate bond market. Moreover, an investor can liquidate a position by entering into a new swap in the opposite direction instead of selling his current position. Therefore, liquidity is less relevant given the ability to replicate swap cash flows using another CDS. Thus, CDS premiums may not be significantly affected by liquidity and reflect, in a way, pure measures of credit risk. This assumption is commonly used in the recent literature. For instance, Mahanti et al. (2008), Han and Zhou (2008), Dionne and Maalaoui Chun (2013), and Guarin et al. (2014) used CDS premiums as pure measures of credit risk.

In light of these investigations, it is assumed that CDS premiums are mainly driven by credit risk. It is believed that even if credit risk is not the only risk involved in CDS, it is the main one and, ergo, a good proxy to understand a firm’s credit risk. Moreover, the filtering approach adopted in this paper allows for potential lack of liquidity in specific tenors to be absorbed by the noise terms. Thus, the selected methodology would help to reduce the impact of illiquidity in our dataset, if ever there is any. To minimize the impact of such risk, the study also focuses on firms that are part of the widely-followed CDX indices.

3.1 Data

The investigations were performed on 225 firms of the CDX North American IG and HY indices (CDS.NA.IG.21.V1 and CDS.NA.HY.21.V1) provided by the Markit Group on September 20, 2013. The indices span multiple credit ratings and sectors. The weekly⁶ term structure of CDS premiums from January 5, 2005, to December 25, 2013, is also provided by Markit for a maximum of 469 weeks. CDS premiums up to the end of 2012 are used in the estimation; the last year of data (i.e. 2013) is kept for an out-of-sample analysis. We use Wednesday CDS premiums.⁷ Prices for maturities of 1, 2, 3, 5, 7 and 10 years are available for most firms (125 IG and 100 HY). However, 15 firms (4 IG and 11 HY) were removed from the sample since there were not enough observations for the estimation procedure (i.e. less than 100 observations per maturity). This yields a grand total of 487,796 observations in our final sample of CDS premiums. Markit acquires closing premiums from dealers’ books. After filtering out stale premiums and outliers, they use the adequate pricing information from the contributors to compute a daily composite term structure of CDS premiums for each reference entity. The “No Restructuring” clause is selected to capture only the credit risk of the firm. By using other clauses, premiums would include margins for possible restructuring event (that are not related to default). According to Markit (2013), this clause is mainly traded in North America.

⁴The credit spread of CDS dealer would have to increase by nearly 645 basis points to result in a one basis point decline in the price of credit protection.

⁵They use CDS bid and ask premiums; this study’s CDS dataset undergoes a rigorous cleaning process which is achieved by the data provider.

⁶Weekly credit default swap observations means $\Delta_t = 1/52$.

⁷We focus on Wednesday CDSs because it is the least likely day to be a holiday and it is least likely to be affected by weekend effects. For more details on the advantages of using Wednesday data, see Dumas et al. (1998).

The data provider also argues that its “selection methodology ensures that the indices represent the most liquid segment of the market” (Markit, 2013). Hence, liquidity issues shall be somewhat reduced when using single-name CDS present in CDX.NA.IG and CDX.NA.HY indices.

To illustrate how CDS premiums move over time, the first panel of Figure 1 shows the evolution of the mean 5-year CDS premium taken across firms for both IG and HY portfolios. Premiums were more or less stable during the pre-crisis era. They increased during the financial crisis: the mean 5-year premiums jumped to a high of 321 bps for IG firms and 1,422 bps for HY firms. In the post-crisis era, the premiums decreased, but did not reach their pre-crisis levels.

The second panel of Figure 1 shows the evolution of the difference between mean 10-year CDS premiums and 1-year CDS premiums taken across firms for both IG and HY portfolios. The slope of the CDS premiums’ term structure became negative during the crisis for IG and HY firms on average. According to Figure 1, both the level and the slope of the CDS premiums change during the financial crisis and the aftermath of the crisis. The regime-switching component of the proposed framework is required to capture these important changes in behaviour.

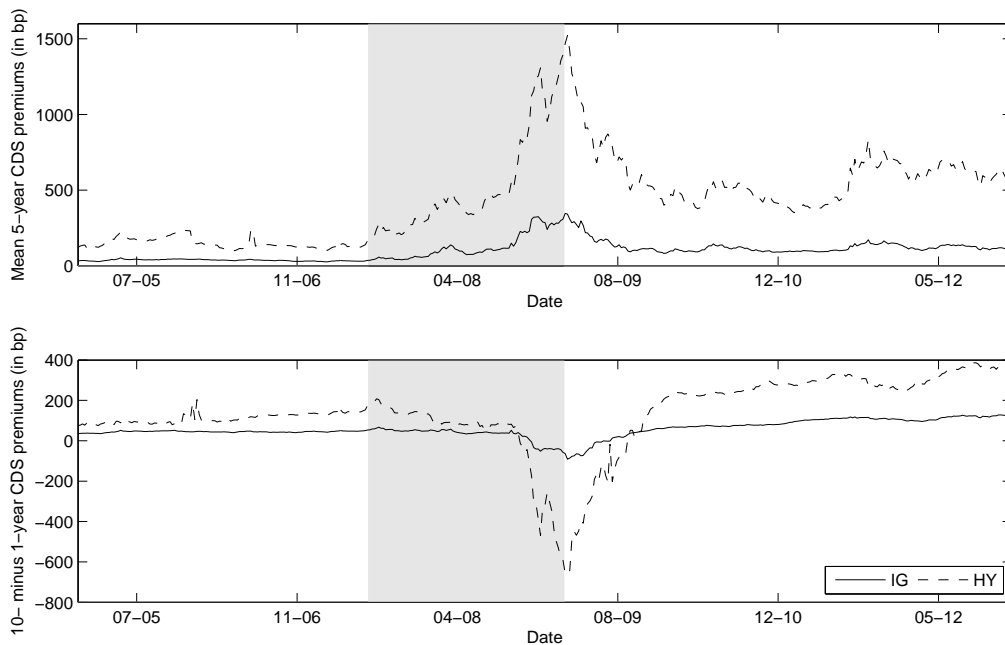


Figure 1: Evolution of the mean 5-year CDS premiums (across firms) and of the difference between mean 10-year CDS premiums and 1-year CDS premiums (across firms).

The CDS premiums were taken from both CDX.NA.IG.21.V1 and CDS.NA.HY.21.V1 portfolios, between January 2005 and December 2012. The grey surface corresponds to the financial crisis (July 2007 to March 2009).

In all experiments and unless stated otherwise, the three-month Treasury constant maturity rate serves as a good proxy for the short rate.⁸

Saunders and Allen (2010) decompose the recent financial crisis in three periods. The first period corresponds to the credit crisis in the mortgage market (June 2006 to June 2007), the second one covers the period of the liquidity crisis (July 2007 to August 2008), and the third period covers the default crisis period (September 2008 to March 2009). This study focuses on the second and the third periods; thus, the financial crisis started in July 2007 and finished in March 2009 throughout this paper.

⁸These rates are provided by the Federal Reserve of St. Louis website via FRED (Federal Reserve Economic Data). The Treasury constant maturity rate series ID is DGS3MO.

3.2 Estimation results

The log-leverage model used hereafter is a simplified version of (1); we only consider two regimes. Considering more regimes is theoretically feasible; however, the number of parameters increase drastically and the numerical optimization of the log-likelihood function becomes unmanageable. We also consider the same drift parameter across both regimes ($\mu^{\mathbb{P}} = \mu_1^{\mathbb{P}} = \mu_2^{\mathbb{P}}$ and $\mu^{\mathbb{Q}} = \mu_1^{\mathbb{Q}} = \mu_2^{\mathbb{Q}}$). Indeed, the drift parameter estimators of the latent variable is rather inaccurate and create numerical instability due to the short span of the time series used.⁹ Besides this caveat, the regime-switching framework still captures the variation of volatility across the different regimes. The estimation is performed using the detection-estimation algorithm of Tugnait (1982) coupled with the unscented Kalman filter (see Appendices B and C).

Table 1 shows descriptive statistics on estimated parameters. All the firms have a positive intensity process since β is always positive. The parameter α is always greater than 1.59, confirming the convex relationship between the intensity process and the leverage ratio as in Boudreault et al. (2013). The volatility parameters σ_1 and σ_2 are very different one from another. The low-volatility regimes correspond normally with the end of the pre-crisis era and the post-crisis period; its average value is around 12%. For most of the firms, the high-volatility regimes correspond to the financial crisis. Its volatility is roughly 36% on average. The regimes are persistent since both \mathbb{P} - and \mathbb{Q} -transition probability matrices are concentrated over the main diagonal.

Noise term standard errors for mid- and long-term tenors are small on average: for instance, around 13% for 2-year maturity and 3.5% for 5-year. It is the highest for the 1-year CDS on average. Two reasons can explain this result: elements not necessarily related to the entity's true default and recovery risks, and fitting error due to model misspecification. Credit default swaps with a maturity of 5 years are the ones with the smallest noise standard errors on average. This makes sense empirically: 5-year is known to be the most liquid maturity for CDS.

The regime-switching hybrid credit risk model is compared to three benchmarks: the “one-regime” equivalent of our model (i.e. the one presented in Boudreault et al. (2013)), a regime-switching structural version¹⁰ of the proposed framework, and a regime-switching reduced-form model.¹¹ A comparison of the models' performances is presented in Appendix D. In summary, the results of these in- and out-of-sample tests show that the full model outperforms the benchmarks.

3.3 Analysis of pricing errors

To verify if the credit risk model captures most of the information present in the CDS premiums, relative pricing errors are regressed against one liquidity proxy, two market-wide factors, one variable related to the risk-free interest rate market, and two firm-specific factors.

For each tenor, the following dummy variable regression is estimated:

$$\begin{aligned} \nu_{jt}^{*(i)} = & \gamma_0 + \gamma_1(\text{Observed CDS premium})_{jt}^{(i)} + \gamma_2(\text{Cont})_{jt} + \gamma_3(\text{VIX})_t + \gamma_4(\text{S\&P})_t + \\ & \gamma_5(\text{Slope})_t + \gamma_6 R_{jt} + \gamma_7 \sigma_{jt}^R + \gamma_8 \mathbb{I}_{\text{Crisis}}(t) + \gamma_9 \mathbb{I}_{\text{Post}}(t) + \gamma_{10} \mathbb{I}_{\text{HY}}(j) + \epsilon_{jt}^{(i)}, \end{aligned} \quad (8)$$

where $\nu_{jt}^{*(i)}$ is the relative pricing error¹² on the i -year CDS premiums at time t of the j th reference entity and $(\text{Observed CDS premium})_{jt}^{(i)}$ is the time t observed i -year CDS premium of the j th reference entity.

As in Jacoby et al. (2009) and Qiu and Yu (2012), the number of distinct contributors to generate the 5-year CDS spread, $(\text{Cont})_{jt}$, serves as a proxy for the liquidity in the credit default swap market. It shall

⁹Even in a “one-regime” framework where the log-leverage is assumed to be observed, the precision of the drift parameter estimate is proportional to the square root of the sampling period length. Hence, long time series are required to pin down these parameters.

¹⁰It is accomplished by using $\beta = 0$ and $\alpha \rightarrow \infty$. The intensity process jumps from 0 to infinity inducing the default as soon as the leverage ratio reaches 1.

¹¹The intensity is a regime-switching geometric Brownian motion which is totally independent of the leverage ratio. The recovery rate is an estimated parameter that remains constant in this framework.

¹²Observed premium minus the theoretical one divided by the observed premium.

Table 1: Descriptive statistics on the distribution of parameters and noise terms across the portfolio of firms of the CDX indices.

Panel A: Descriptive statistics on leverage dynamics parameters (under \mathbb{P}) and fees κ.							
	$\mu^{\mathbb{P}}$	$p_{12}^{\mathbb{P}}$	$p_{21}^{\mathbb{P}}$	σ_1	σ_2	κ	Obs.
Mean	0.0200	0.0127	0.0223	0.1167	0.3570	0.6153	210
SD	0.0499	0.0182	0.0701	0.0392	0.0791	0.1713	
10%	-0.0074	0.0033	0.0058	0.0752	0.2611	0.4096	
25%	-0.0005	0.0041	0.0080	0.0942	0.3175	0.5002	
50%	0.0004	0.0089	0.0121	0.1123	0.3542	0.6014	
75%	0.0217	0.0138	0.0209	0.1360	0.3747	0.7007	
90%	0.0756	0.0265	0.0336	0.1634	0.4441	0.8609	
IG	0.0131	0.0122	0.0253	0.1111	0.3738	0.5949	121
HY	0.0292	0.0134	0.0184	0.1244	0.3341	0.6431	89
Panel B: Descriptive statistics on leverage dynamics parameters (under \mathbb{Q}) and intensity.							
	$\mu^{\mathbb{Q}}$	$p_{12}^{\mathbb{Q}}$	$p_{21}^{\mathbb{Q}}$	α	θ	β	Obs.
Mean	0.0163	0.0059	0.0381	11.3930	1.4354	0.0086	210
SD	0.0297	0.0060	0.0257	8.3380	0.3268	0.0235	
10%	-0.0058	0.0014	0.0093	5.6210	1.0525	0.0002	
25%	0.0000	0.0026	0.0150	7.6341	1.2962	0.0008	
50%	0.0099	0.0046	0.0370	9.8976	1.4695	0.0026	
75%	0.0262	0.0074	0.0560	11.7433	1.5111	0.0086	
90%	0.0439	0.0124	0.0692	15.4900	1.7424	0.0196	
IG	0.0033	0.0059	0.0517	12.2205	1.3551	0.0064	121
HY	0.0340	0.0061	0.0197	10.2679	1.5445	0.0117	89
Panel C: Descriptive statistics on error terms.							
	$\delta^{(1)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(5)}$	$\delta^{(7)}$	$\delta^{(10)}$	Obs.
Mean	0.2488	0.1320	0.0807	0.0345	0.0479	0.0781	210
SD	0.0647	0.0371	0.0298	0.0266	0.0408	0.0475	
10%	0.1739	0.0853	0.0404	0.0004	0.0001	0.0292	
25%	0.2007	0.1058	0.0669	0.0136	0.0101	0.0380	
50%	0.2416	0.1328	0.0823	0.0354	0.0462	0.0694	
75%	0.2971	0.1572	0.1013	0.0465	0.0675	0.1015	
90%	0.3410	0.1786	0.1163	0.0729	0.1068	0.1455	
IG	0.2299	0.1254	0.0767	0.0339	0.0663	0.0982	121
HY	0.2745	0.1409	0.0861	0.0353	0.0228	0.0507	89

[1] For each of the 210 firms, the parameters of the model are estimated using weekly CDS premiums of maturities 1-, 2-, 3-, 5-, 7-, and 10-year, using the DEA-UKF filtering technique. The mean, standard deviation (SD) and quantiles are computed across firms. The last two rows compute the mean across firms of CDX.NA.IG.21.V1 and CDX.NA.HY.21.V1 portfolios.

[2] The δ 's represent the standard deviation of the noise terms present in the observation equation of the filter.

explain the depth of the CDS market and is available for every reference entity and every day in our sample through the Markit dataset.

The first market-wide factor is the CBOE Volatility Index VIX (denoted by $(VIX)_t$) which is a popular measure of the implied volatility of S&P 500 index options. It is extracted from the Federal Reserve of St. Louis website via FRED.¹³ Weekly averages are considered from January 2005 to December 2012 to be consistent with the CDS dataset. The Standard and Poor's 500 market index return, $(S\&P)_t$, is the second

¹³The series ID is VIXCLS.

market-wide variable. It is provided by FRED¹⁴ from January 2005 to December 2012. The returns are summed over each week.

The slope of the term structure¹⁵ (denoted by $(\text{Slope})_t$) is included in the regression. This time series is also acquired via FRED.¹⁶ Other studies generally include the three-month constant maturity Treasury rate as a regressor. However, a negative correlation of -88% is found between the three-month Treasury rate and the slope of the term structure during the considered sampling period. Hence, to diminish possible collinearity effects between the two variables, only the slope of the term structure is considered.

Stock returns R_{jt} ¹⁷ are obtained via the Center for Research in Security Prices (CRSP) available in WRDS. The firm's tickers are manually matched to obtain the proper returns' time series. These returns are aggregated to obtain weekly values: we sum the daily returns over each week from 2005 to 2012. A proxy for the stock returns' volatility (denoted by σ_{jt}^R) is also computed: the standard deviations on daily returns using one-year rolling windows are calculated (as in Doshi et al. (2013)). Collinearity could be a concern for these time series; however, the sample correlation between each couple of time series is rather small (an average of -3%). The rolling window methodology could explain this conclusion. Hence, both time series are added to the regression analyses.

To take into account fixed effects related to the risk groups (IG and HY) and the periods (pre-crisis, crisis, and post-crisis), three indicator functions are added to the regression equation, as it is common in dummy variable regression analyses.

Note that we winsorize the 0.5% highest and lowest values of all pricing errors and observed CDS premiums for each tenor. Regression coefficient variances are estimated using Thompson (2011) allowing for estimators that are robust to simultaneous correlation along two dimensions (i.e. firm and time).

Table 2 shows the results of the regression for each tenor. Even if many coefficients are statistically significant, it is mainly a consequence of the sample's large size: any small departure from zero makes the coefficients significant. Hence, the focus should be put on R -squareds that are very low, ranging from 2.6% to 13.2%, indicating that all these factors do not explain linearly the variations in the pricing errors.

Based on R -squared estimates for different specifications of Equation (8)¹⁸ (for which some γ_i are set to zero), it is possible to determine that the liquidity proxy used do not linearly explain the relative pricing error on CDS premiums. Furthermore, when considering only the slope of the term structure in the regression (in addition to the dummy variables), the R -squareds are almost equivalent to the one given in Table 2, meaning that the slope of the term structure accounts for most of the pricing error.

As a robustness test, we also used innovations of the VIX and of the equity volatility as regressors (instead of the levels). Results were similar to what is obtained in this subsection.

3.4 Most likely regimes through time

The approach of Viterbi (1967) is adapted to the context of a hidden regime and a latent variable to extract the most probable regime path. The regime path that maximizes the likelihood function given the estimated parameters is constructed recursively.

Figure 2 shows the proportion of firms considered in the high-volatility regime for each week and for each risk class. In our context, being in the second (high-volatility) regime is synonymous with more uncertainty in the firm's leverage. The fast transition from the first (low-volatility) regime to the second (high-volatility) regime during 2007 and 2008 is obviously very natural due to the financial conditions at that time. At the beginning of the sample, many HY firms are in the high-volatility regime. This means that the filtered leverage is more uncertain at the beginning of the sample. To understand this effect, we compare the slope of the term structure of CDS premiums (i.e. 10-year minus 1-year CDS premiums) during the pre-crisis era.¹⁹

¹⁴The S&P 500 series ID is SP500.

¹⁵The difference between the 10-year and three-month constant maturity Treasury rate.

¹⁶This series ID is T10Y3M respectively.

¹⁷ R_{jt} is the weekly return of the stock associated to the j th reference entity at time t .

¹⁸Not reported here, but available on request.

¹⁹Not reported here, but available on request.

Table 2: Dummy variable regression results on relative pricing errors.

	1-year	2-year	3-year	5-year	7-year	10-year
Intercept	-0.103	-0.050	-0.050	0.017	0.024	0.026
Obs. CDS premium	-0.005	0.001	0.003	-0.003	0.023	0.055
Number of contributors	0.002	0.001	0.001	0.000	-0.001	-0.001
VIX	0.003	0.001	0.000	-0.001	-0.001	-0.001
S&P 500 returns	0.414	0.274	0.160	-0.042	-0.174	-0.275
Slope	0.053	0.020	0.013	0.004	-0.022	-0.030
Equity return	0.013	0.001	0.012	0.005	-0.014	-0.020
Equity volatility	0.014	-0.005	-0.001	-0.006	0.006	0.004
Crisis dummy	-0.043	-0.047	-0.007	0.001	0.009	0.014
Post-crisis dummy	-0.102	-0.061	-0.036	-0.003	0.036	0.054
HY dummy	-0.017	-0.012	-0.010	0.011	-0.001	-0.014
R^2	0.065	0.026	0.035	0.037	0.132	0.107
Number of observations	75,628	75,628	75,628	75,628	75,628	75,628

[1] For each of the 210 firms, the regression

$$\nu_{jt}^{*(i)} = \gamma_0 + \gamma_1(\text{Observed CDS premium})_{jt}^{(i)} + \gamma_2(\text{Cont})_{jt} + \gamma_3(\text{VIX})_t + \gamma_4(\text{S\&P})_t + \gamma_5(\text{Slope})_t + \gamma_6 R_{jt} + \gamma_7 \sigma_{jt}^R + \gamma_8 \mathbb{I}_{\text{Crisis}}(t) + \gamma_9 \mathbb{I}_{\text{Post}}(t) + \gamma_{10} \mathbb{I}_{\text{HY}}(j) + \epsilon_{jt}^{(i)},$$

is estimated for each tenor.

[2] The conclusions of the statistical test $H_0 : \gamma_i = 0$ against $H_1 : \gamma_i \neq 0$, $i = 0, \dots, 10$, are reported. Estimates in bold are significant at a confidence level of 95%.

[3] Obs. CDS premium means the observed CDS premium times 1,000.

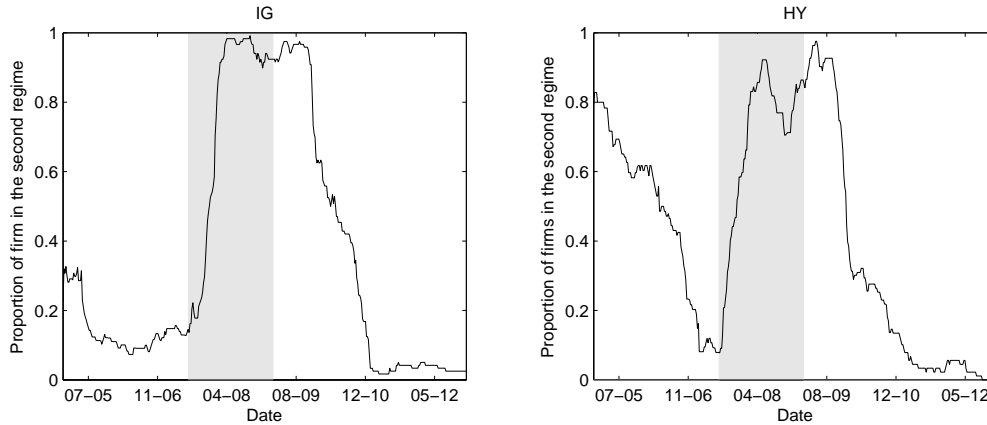


Figure 2: Proportion of firms in the high-volatility regime.

Each week, the proportion of firms considered in the second regime is computed for the 121 IG and the 89 HY ones. The grey surface corresponds to the financial crisis (July 2007 to March 2009).

In a non-reported quantitative analysis, the second regime is strongly associated with smaller slopes, which is the case of many firms in the early stage of our sample.

For HY firms, the moment of transition between regime 1 and 2 corresponds to the beginning of the financial crisis (i.e. July 2007). During the first six months of the crisis, the proportion of firms in the high-volatility regime goes from 8% to almost 60%. For IG firms, the transition happens a little later and is consistent with the beginning of the NBER economic recession in the United States (i.e. December 2007). At the end of the financial crisis (or the recession), both IG and HY firms remain in the high-volatility regime for several weeks. These results are in line with the recent literature. Indeed, using a different approach, Maalaoui Chun et al. (2013) detect some persistence in the volatility regimes of credit spreads. These observations are also consistent with those of Garzarelli (2009) and Mueller (2008). These authors

document that credit spread levels increase before the onset of the NBER recession and somehow persist until long after the recession is over.

4 Recovery rate versus regime-switching: term structure comparison

This section assesses the relative importance of recovery rate and regime-switching risks in credit spread curves. Essentially, we compare the difference between risky and riskless zero-coupon yields for various model specifications.

The time t value of risky zero-coupon bond (see Appendix A.3) is given by $V(t, T, \hat{x}_t, \hat{h}_t; R_\tau, 0)$ where \hat{x}_t is the time t filtered log-leverage, and \hat{h}_t is the most probable regime²⁰ at time t .

To compare the relative importance of recovery-rate risk and regime-switching risk, four different formulations of the model are estimated. The first one (RS-ENDR) is described in Section 2. The second model (RS-EXOR) is a variation of the full model: instead of using endogenous recovery rates, a constant exogenous recovery rate $R_t = R \in [0, 1]$ is incorporated into the set of parameters to be estimated. The third model is the “one-regime” equivalent with endogenous recovery rates (1R-ENDR) presented in Boudreault et al. (2013). The last model (1R-EXOR) is a “one-regime” equivalent of the full model with a constant exogenous recovery rate to be estimated.

In this and the following sections, some of the results are broken down by credit rating. These ratings are attributed by Standard and Poor’s between January 31, 2005, and December 31, 2012, and are available from Compustat in WRDS. The rating used is identified as “S&P Domestic Long Term Issuer Credit Rating” (SPLTICRM) in the database. The firm’s ticker symbol is matched to the data in Compustat. For 2 firms, no rating is available, leaving a sample of 208 firms (121 IG and 87 HY) whenever results are presented by credit rating.

Table 3 and Figure 3 show average credit spreads across credit ratings and models. Curves obtained from AAA- and AA-rated firms are not shown in the figure since the number of firms with such rating is too small to be representative.

There has been an important rise in average credit spreads during the financial turmoil that affects mainly the short and mid terms. According to any of the tested models, the 2-year average credit spread of A-rated firms is about 10 times larger during the crisis period than before. For riskier firms rated BBB, BB and B, the average credit spread is about 4 times larger. When considering the 5-year credit spread, it is 4 times larger for the A-rated firms and about 2.5 times larger for the credit ratings BBB, BB and B. There is a reduction of the credit spread in the post-crisis period, yet it never reaches the pre-crisis levels.

Recovery uncertainty and its negative relationship with default probability have a major impact on mid- and long-term credit spreads. Indeed, in a regime-switching environment, the 5-year credit spread increases by a factor of 13% to 31% with the endogenous recovery assumption when compared to an equivalent model with a constant recovery rate during the crisis. On a longer horizon, the effect is even more significant, ranging between 12% and 73% depending on the credit rating or the period considered. The dependence between default probabilities and recovery rates have been documented empirically (i.e. lower-rated firms have lower recovery rates), but most modelling approaches neglect the accrued risk associated with this negative relation. Thus, assuming constant recovery rates seriously impacts credit spread curves, especially over the long run.

The presence of regimes has a second-order effect on average credit spread curves and is mainly attributed to how these averages are constructed. The parameters of the “one-regime” model capture the behaviour during good and bad times. Unlike the previous approach, the two-regime model allows for a distinct set of parameters for each state. However, the curves presented in Figure 3 are constructed by averaging all

²⁰Normally, we would need to take a weighted average of the different prices, given the regime at t ; the weights correspond to $\mathbb{Q}(h_t = h | \mathcal{F}_t)$, the risk-neutral probability of being in regime h at time t conditional on \mathcal{F}_t . Though, these probabilities cannot be computed readily, and this is why we use the most probable regime at time t as a proxy. We could also use the probability under the physical measure as a proxy; however, this shall lead to the same estimate since the probability of being in a regime is most of the time in the neighbourhood of 1 or 0.

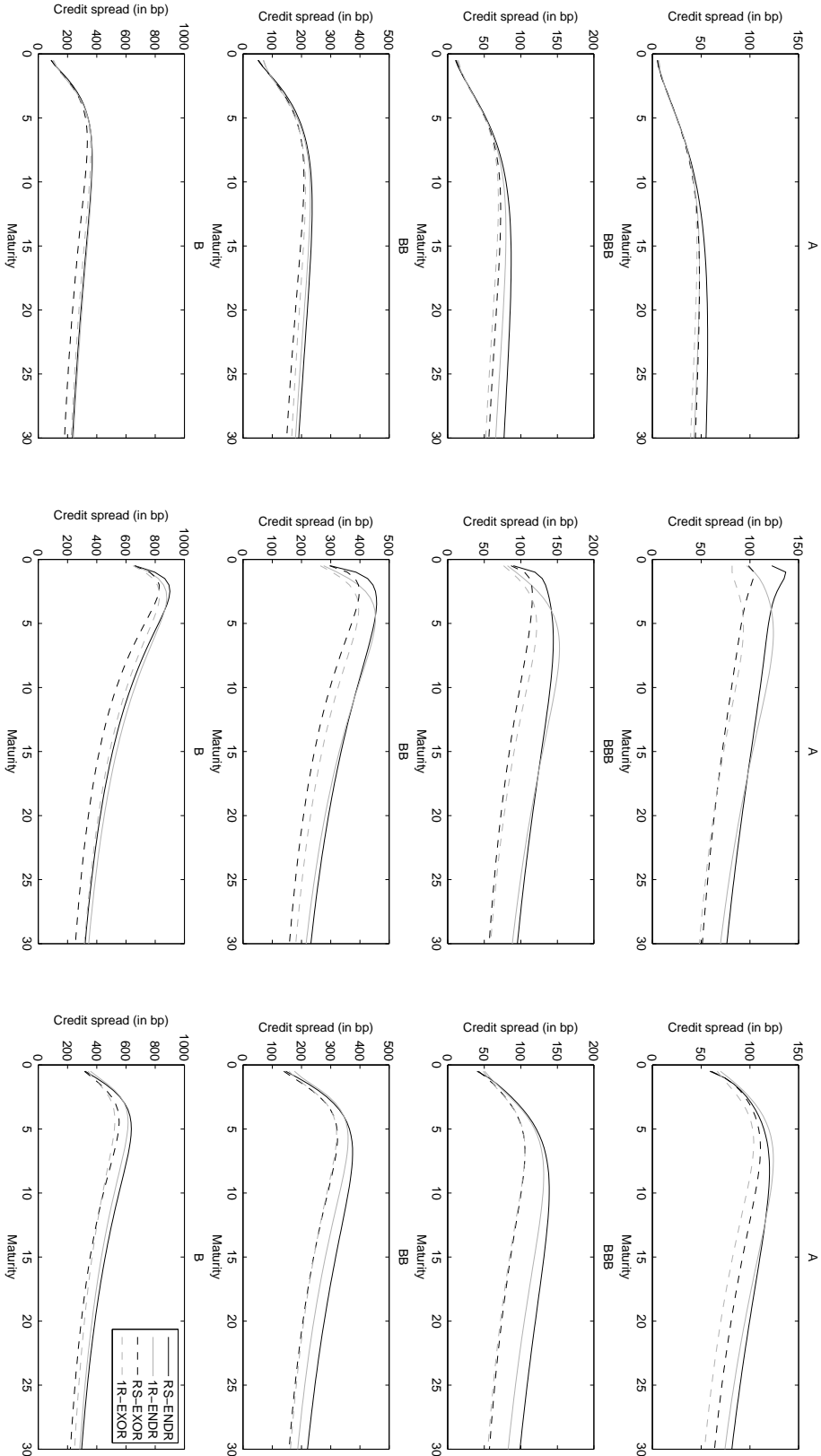


Figure 3: Credit spread as a function of maturities, credit ratings and eras. For each of the 208 firms, the parameters of the model have been estimated using weekly CDS and the DEA-UKF technique. The data is available for January 2005 to December 2012. The term structures are evaluated each week. Then, we average the spreads across the time periods (i.e. pre-crisis, crisis, post-crisis era). Each graph shows average term structures for four different models. Two versions of our regime-switching model are used: RS-ENDR and RS-EXOR. Moreover, two versions of the one-regime equivalent are used: 1R-ENDR and 1R-EXOR.

Table 3: Average credit spreads (in basis points) across credit ratings and models.

		Pre-crisis					Crisis					Post-crisis				
		2-y	5-y	15-y	30-y	Obs.	2-y	5-y	15-y	30-y	Obs.	2-y	5-y	15-y	30-y	Obs.
AAA	RS-ENDR	5	12	29	33	3	133	140	111	79	3	-	-	-	-	0
	1R-ENDR	6	12	25	23		120	140	108	70		-	-	-	-	
	RS-EXOR	5	12	25	21		101	75	43	25		-	-	-	-	
	1R-EXOR	5	11	24	24		112	112	83	55		-	-	-	-	
AA	RS-ENDR	10	20	39	40	4	208	174	119	82	2	88	121	121	89	3
	1R-ENDR	11	20	33	28		168	176	127	79		87	122	107	71	
	RS-EXOR	9	20	39	41		175	135	97	73		75	85	48	27	
	1R-EXOR	10	20	32	25		137	115	75	47		85	109	88	58	
A	RS-ENDR	10	25	53	55	55	131	119	101	76	48	91	114	110	81	35
	1R-ENDR	10	25	48	43		114	124	103	70		93	119	108	74	
	RS-EXOR	10	25	48	44		101	91	72	51		90	109	92	64	
	1R-EXOR	10	25	46	39		86	93	73	48		83	103	82	53	
BBB	RS-ENDR	23	52	87	77	100	134	144	128	95	88	78	121	132	99	87
	1R-ENDR	23	51	79	66		114	149	130	88		76	119	119	83	
	RS-EXOR	23	51	71	56		115	113	84	57		71	102	86	57	
	1R-EXOR	24	51	67	52		104	122	91	59		70	102	85	55	
BB	RS-ENDR	106	189	233	191	40	445	448	336	232	47	258	364	317	221	48
	1R-ENDR	102	185	225	180		395	452	329	216		266	357	282	187	
	RS-EXOR	103	180	197	149		395	371	244	159		236	322	242	157	
	1R-EXOR	102	178	207	166		364	390	270	180		238	319	242	162	
B	RS-ENDR	203	340	329	234	16	894	823	508	320	29	514	636	458	296	36
	1R-ENDR	191	340	335	240		857	834	536	345		524	612	432	284	
	RS-EXOR	196	323	274	179		827	728	417	254		466	550	355	220	
	1R-EXOR	190	335	314	225		814	777	489	315		455	521	368	248	

[1] For each of the 208 firms, the parameters of the model are estimated using weekly CDS premiums of maturities 1-, 2-, 3-, 5-, 7- and 10-year, using the DEA-UKF filtering technique. The data is available for January 2005 to December 2012.

[2] The credit spreads (in basis points) are averaged across the models, the credit ratings, and the periods.

[3] In the table, Obs. means the average number of observations (firms) per rating-period, 2-y stands for 2-year, 5-y corresponds to 5-year and so on.

weekly curves on a given period and correspond to a mix of various firms and regime weights. Even then, the presence of regimes affects the long-term credit spread of highly rated firms in the pre- and post-crisis periods. During the financial turmoil, the presence of regimes modifies the short-term shape of the average credit spread curves, especially for highly rated firms.

To further assess the importance of the negative correlation between default probabilities and recovery rates, additional tests are performed. Figure 4 exhibits the implied probability of default (under the objective measure) and the expected recovery rate given default for each week considered in our sample, and for three different time horizons: 1, 5, and 10 years. The two variables seem to be highly negatively correlated: as the implied default probability increases, the expected recovery rate given default decreases. Indeed, the average correlation²¹ between the two variables is -77.5% for a time horizon of 1 year, -87.8% for 5 years and -88.6% for 10 years.

²¹The correlation is computed on a firm-by-firm basis, and then averaged across firms.

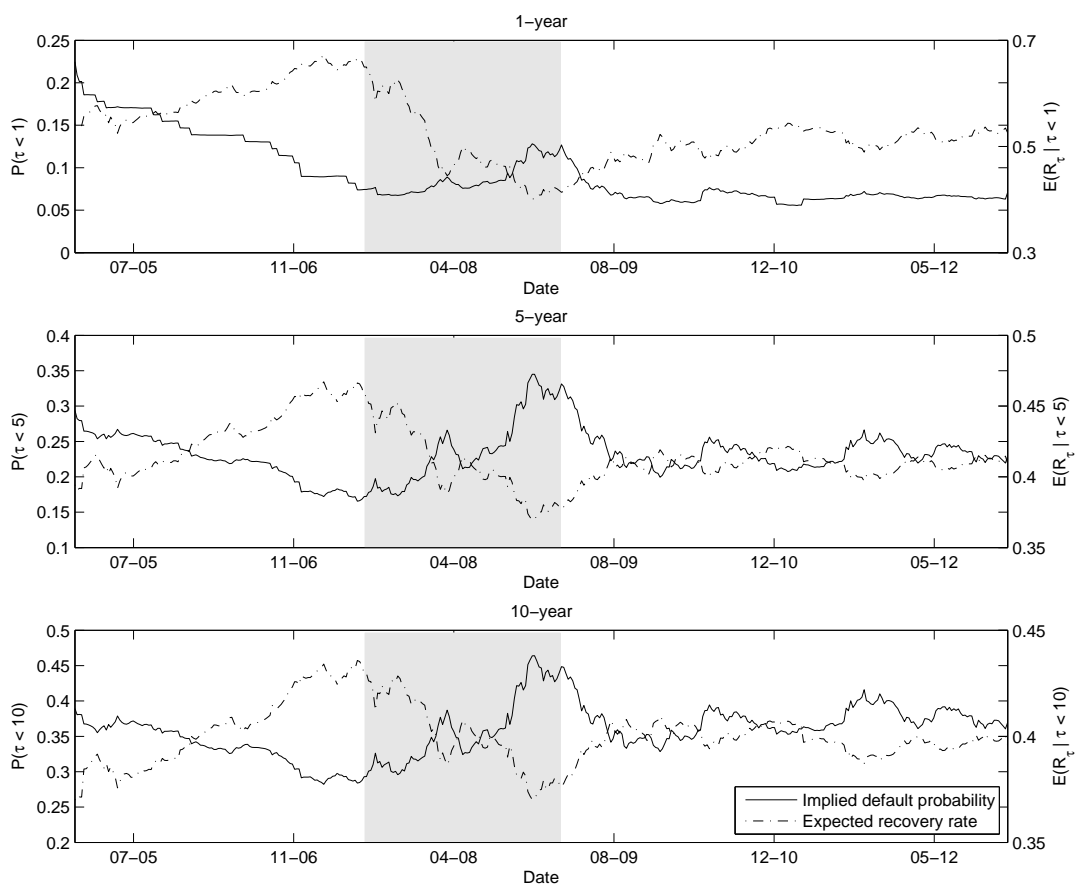


Figure 4: Implied default probability and expected recovery as a function of the date.

For each of the 208 firms and using the regime-switching model with endogenous recovery, the parameters of the model have been estimated using weekly CDS and the DEA-UKF technique. The data is available for January 2005 to December 2012. The left axis corresponds to the implied probability of default (under the real measure) and the right axis shows the expected recovery rate given default, both computed each week using three different time horizons: 1, 5, and 10 years. Then, the two quantities are averaged across firms.

5 How much of the credit spread is explained by the model?

Many have tried over the past fifteen years to provide insights on the contribution of credit risk in corporate bond spreads. This question is obviously important to credit risk modellers. Moreover, the puzzle's resolution could help us grasp the factors contributing to the so-called yield spreads.

Some studies measure the empirical zero-coupon bond yield spreads using a Nelson and Siegel (1987) approach on Warga (1998)'s dataset.²² The theoretical zero-coupon yield spreads are computed by some credit risk models, and these studies differ in the choice of this model and by the estimation method used.

For instance, Elton et al. (2001) claim that only a small fraction of corporate spreads can be attributed to expected default loss. Using a reduced-form model calibrated on Moody's credit rating transition matrix and using a constant recovery rate available from other empirical studies, the authors conclude that no more than 25% of the credit spread can be explained by default risk. Dionne et al. (2010) show that the proportion of empirical spreads explained by default risk is responsive to the data used. Indeed, the sampling period, the estimation approach and the data filtering procedure impact significantly on theoretical credit spread measures. According to this study, the proportion of credit spread attributed to default risk varies from 12% to 49%; when variations in recovery rate are considered, this proportion reaches 54%.

²²This dataset, also called Lehman Brothers via the Fixed Income Database, contains information on all investment-grade corporate and government bonds before March 1998; they also eliminate all bond with special features.

Using linear regression analyses, Collin-Dufresne et al. (2001) show that variables that should in theory determine credit spread changes have limited explanatory power: about 25% of the variation is explained by factors suggested by traditional models. The authors use equity returns, Treasury rate levels, changes in the slope of the yield curve, firm leverage changes, VIX index changes, and changes in the slope of implied volatilities of options on S&P 500 futures.

Using structural models,²³ Huang and Huang (2012) find that credit risk accounts for only a small portion of observed spreads on IG bonds over the period 1973–1998. More precisely, the credit component is about 30% for 10-year BBB-rated bonds and about 20% for riskier bonds. The parameters are calibrated using proxies for each credit rating. Eom et al. (2004) show that it is possible to obtain higher spreads than those seen in the bond market using several structural models.²⁴ One of their conclusions is that more sophisticated models tend to overestimate the spreads. This conclusion is obviously dependent on the parameters and proxies used by the authors. Cremers et al. (2008) develop a structural model where the firm value is exposed to correlated diffusion shocks, common jumps and firm-specific jumps. The authors allow for risk premiums on both diffusion and common jumps. According to them, this model could explain a significant part of observed credit spread levels and jump risk embedded in corporate bonds and in equity options that are close to each other.

5.1 Data and methodology

In the spirit of Dionne et al. (2010) and Elton et al. (2001), we compare spot spreads obtained by the credit risk model with corporate bond benchmark spot spreads. This exercise is performed “out-of-sample” since no corporate bonds were used in the estimation step. The empirical spot curves are proxied by the Bank of America (BofA) Merrill Lynch bond indices. The option-adjusted spreads (OAS) are selected for A-, BBB-, BB- and B-rated bonds. These spreads are also available for different maturities (e.g. 1–3 years, 3–5 years, 5–7 years, 7–10 years, 10–15 years and 15+ years). The benchmarks are available through the Bloomberg Terminal in BofA Merrill Lynch Bond Indices page. These empirical curves are compared to the yield curves derived from the proposed model. Each week, we only consider A-, BBB-, BB- and B-rated firms. For many weeks during the considered period, no AAA- and AA-rated firms are available in our sample to construct the model-implied yield curve.

5.2 Results

Figure 5 shows the evolution of 15-year theoretical and observed spreads. Theoretical spreads are computed for each week using the parameters estimated in Section 3 on a firm-by-firm basis and data from January 2005 to December 2012. Then, the weekly spreads across credit ratings are averaged to obtain one time series of spreads per rating. Observed spreads are computed using the benchmarks: using daily spread, we compute a weekly average for each rating category. For each week, the ratio of theoretical to observed weekly spreads is also computed and reported on Figure 5.

In the pre-crisis era, the observed spreads are similar to the one given by the model on average, except for B-rated firms. For the latter firms, the theoretical spreads are sometimes higher than the observed spreads. One reason comes to mind: the firms in our subsample might be less risky B-rated firms than the average B-rated firms in the economy.

During the crisis, the observed spreads are much more important than those given by the model: this is true for every rating considered in this study. In the post-crisis period, the 15-year spreads remain higher than those given by the model. However, for shorter maturities, the model’s post-crisis spreads are in accordance with the observed spreads.

Figure 6 shows the average proportions of credit spread explained by the full model and a variation of the latter. The first model (RS-EXOR) is a regime-switching model with a constant exogenous recovery rate

²³Huang and Huang (2012) use Longstaff and Schwartz (1995), Leland and Toft (1996), Anderson et al. (1996), Anderson and Sundaresan (2000), Mella-Barral and Perraudin (1997) and Collin-Dufresne and Goldstein (2001).

²⁴They use those of Merton (1974), Geske (1977), Longstaff and Schwartz (1995) Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001).

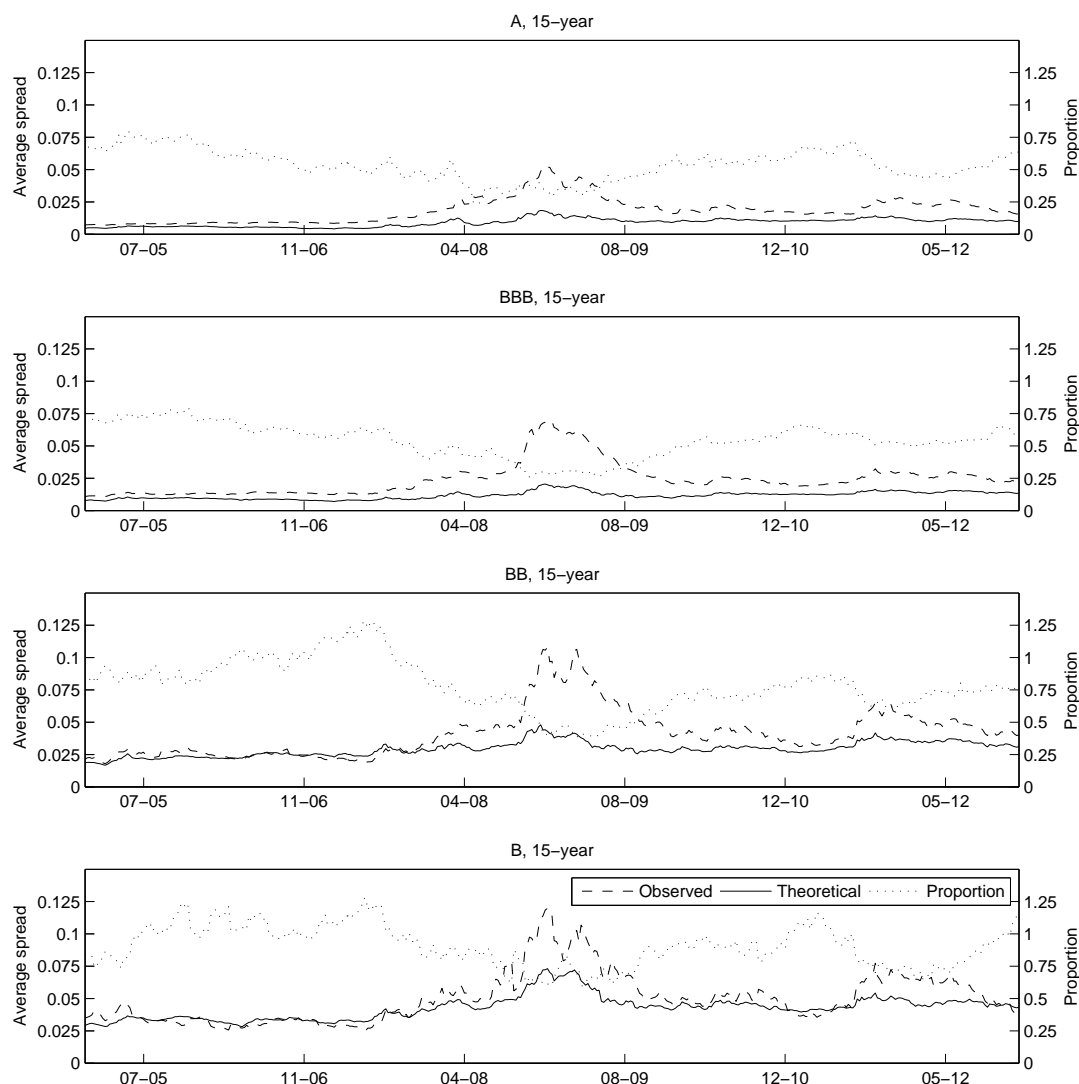


Figure 5: Evolution of the 15-year theoretical spreads, observed spreads, and quotient of observed and theoretical spreads (so-called proportion).

Every week, we average 15-year spreads implied by the model for each rating. The observed spreads are assumed to be the “10–15 years” BofA Merrill Lynch benchmark option-adjusted spread also available by rating. Averages of the “10–15 years” BofA Merrill Lynch benchmark option-adjusted spreads are calculated to obtain one spread per week. The left axis corresponds to the average spread and the right axis shows the proportion of observed to theoretical spreads.

as exposed in the previous section; the second model (RS-ENDR) is the full model described in Section 2. The average of these ratios is reported by credit rating and for different eras. In general, the proportion of the empirical spreads explained by credit risk for the full model is high. It is virtually more than 50% across ratings and periods. Recovery rate risk and its dependence with default risk could partly explain these high figures. When considering a constant exogenous recovery rate, the proportion of credit spread explained by the model decreases by 14.2% on average. For instance, the proportion decreases by 8.4% for A-rated bonds and 17.4% for B-rated bonds.

For all ratings, a decrease in the proportion of the spread explained by credit risk is observed during the financial crisis, going from around 83% to 60% for 15-year spreads when using the full model. Specifically, the explained proportion of 15-year BBB spot spreads decreases from 67% in the pre-crisis era to 42% during the crisis on average. These proportions go up during the post-crisis period (i.e. 52%), but do not reach pre-crisis levels in general. Some other risks could explain this decrease. For instance, more liquidity risk in the corporate bond market is a plausible idea: the next section will be devoted to this thought.

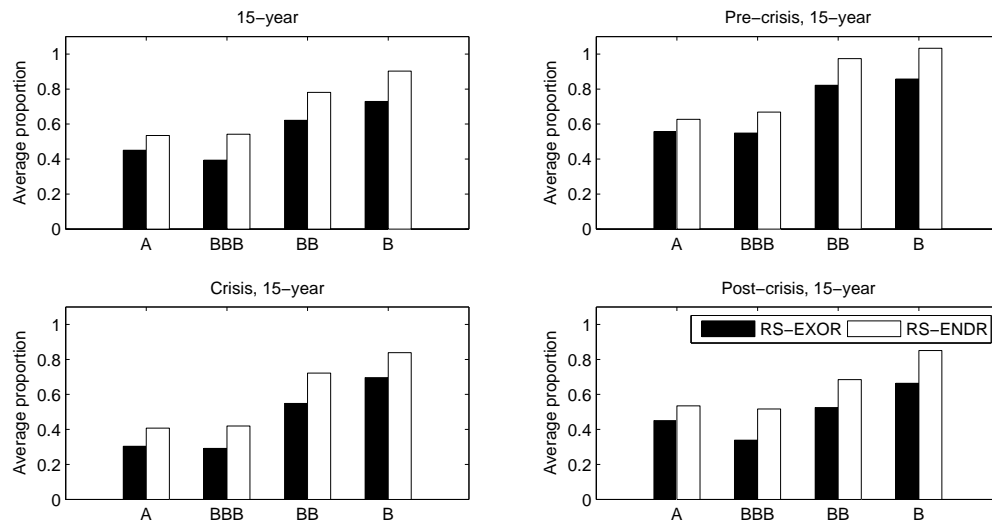


Figure 6: Average proportions of credit spread explained by two regime-switching models for 15-year spreads. The first model (RS-EXOR) is a regime-switching model with a constant exogenous recovery rate; the second model (RS-ENDR) is the full model described in Section 2. Average spreads implied by the model are computed for each rating and each week. Then, averages of the “10–15 years” BofA Merrill Lynch benchmark option-adjusted spreads are also calculated to obtain one spread per week. Both quantities are divided, averaged, and reported in the bar charts. Results are divided in four periods: whole sample, pre-crisis era, crisis era, and post-crisis era.

The majority of market spreads before the crisis seems to be explained by credit risk. Two interesting things emerge from this observation. First, additional risks priced in corporate bonds during the crisis seem to be persistent in the aftermath of the crisis. Second, A-, BBB-, and BB-rated firms seem to be the ones for which credit risk explains a smaller proportion of the spread during and after the crisis.

6 Bond market liquidity risk

Since a proportion of credit spreads are unexplained by credit risk, it would be interesting to see what proportion could be attributed to liquidity risk. Using the model introduced above, theoretical yield spreads for corporate bonds can readily be obtained. Then, using these theoretical values and liquidity proxies, it is possible to regress these factors on observed spreads to assess their relative importance in the pricing of corporate bonds.

In opposition with the previous section, YTM spreads are utilized (i.e. the difference between a yield-to-maturity of the corporate bond and the linearly interpolated maturity-matched Treasury rate calculated on the same day). Indeed, since it is impossible to recover the spot rate for a given issue on a given day, we base our computations on YTM instead, as did Dick-Nielsen et al. (2012).

Many recent papers focus on liquidity issues in the bond market. Using a reduced-form approach, Longstaff et al. (2005) use credit default swap premiums to measure the size of the default component in corporate spreads. The authors find that the majority of the spread is due to default risk and that the non-default component is time varying and strongly related to measures of bond-specific illiquidity. Dick-Nielsen et al. (2012) analyze corporate bond spreads during 2005–2009 using a new robust illiquidity measure based on four different proxies. They show that the spread contribution from illiquidity increases dramatically with the onset of the subprime crisis. This effect is slow and persistent for investment-grade bonds while it is stronger but more short-lived for speculative-grade bonds. In He and Xiong (2012), the authors show that deterioration in debt market liquidity leads to an increase in not only liquidity premium of corporate bonds, but also credit risk. They argue that the latter effect is due to firms’ debt rollovers. According to De Jong and Driessen (2012), corporate bond returns have significant exposures to fluctuations in equity market and Treasury bond liquidity. This price factor seems to help explain the credit risk puzzle. The authors find that the total estimated liquidity risk premium is 0.6% for IG bonds, and 1.5% for HY bonds. Friewald et al.

(2012) use a unique dataset of 20,000 bonds traded between October 2004 and December 2008 to analyze whether liquidity is an important price factor in the US bond market. The authors employ a wide range of liquidity measures and find that liquidity can account for approximately 14% of the market-wide corporate yield spread changes. In addition, the economic impact of liquidity seems to be larger in periods of crisis and for HY bonds. Using a random non-parametric regime shift technique, Dionne and Maalaoui Chun (2013) identify two different regimes in the dynamics of credit spreads during 2002–2012: a liquidity regime and a default regime. The liquidity regime seems to explain the predictive power of credit risk on the 2007–2009 recession, whereas the default regime drives the persistence of credit spreads over the same recession. The authors use the non-parametric technique of Maalaoui Chun et al. (2013), which has the advantage of detecting possible break points in real time or when new data arrives.

Liquidity is usually vaguely defined as the degree to which an asset or security can be bought or sold in the market quickly without affecting the asset's price (Tang and Yan, 2007). However, it has multiple facets and cannot be defined efficiently by a single statistic. There are three oft-cited dimensions of the liquidity risk. The first one is tightness: if the bid-ask spread is small, it is assumed that the market is liquid. The second dimension is depth: it is related to the amount of security that can be traded without affecting the price. Finally, the third dimension is called resiliency: a market is liquid in this specific dimension if price recovers quickly after a demand or supply shock. By controlling for credit risk, the effect captured by the liquidity proxies are really related to liquidity issues (and not to residual credit risk in the spreads).

The trading data were acquired by the Trade Reporting And Compliance Engine (TRACE). The selected bonds are senior, non-callable, non-puttable bullet bonds with fixed coupon rates. In addition, they are active at the end of 2012; to be considered, the issues need to have at least 100 trades during the period considered (2005 to 2012). Moreover, bonds issued after April 2009 are discarded because bond issues should preferably be active in at least two of the three periods considered in this paper.

Since many firms issue only callable bonds, our final bond sub-sample contains 395 issues from 77 firms (52 IG and 25 HY), for a grand total of 2,420,193 observations. Dick-Nielsen (2009)'s algorithm is used to filter out the errors in TRACE data. Omitting this error filter step might result in high liquidity biases: if TRACE data are not cleaned up before use, the number of transactions will be too high. The filter is divided into three steps. First, true duplicates are deleted (i.e. intra-day trades with the same unique message sequence number). Then, reversals²⁵ are also deleted. Finally, same-day corrections are deleted. These are identified using the report's trade status.

To obtain spreads from our bond sample, Treasury constant maturity rate is used again. We use 1-, 2-, 3-, 5-, 7-, 10-, 20- and 30-year rates. We linearly interpolate the different rates to obtain the corresponding rate.²⁶ These rates are provided by the Federal Reserve of St. Louis website via FRED.

6.1 Regressions

To assess whether or not liquidity has an impact on corporate bond pricing, observed YTM spreads are regressed with respect to Dick-Nielsen et al. (2012)'s λ measure. It is a construction made of four different liquidity proxies: the Amihud (2002) measure, the Amihud risk, the imputed roundtrip cost (IRC), and the IRC risk. To be precise, each proxy is normalized and then summed to create the new λ measure. Note that the proxy is computed on a monthly basis.

To be certain that this measure is robust, seven different liquidity proxies used in Dick-Nielsen et al. (2012)²⁷ are computed and a principal component analysis is applied on these variables. The first component explains 73% of the variation and is strongly correlated with Dick-Nielsen et al. (2012)'s λ measure (i.e. 65%). Therefore, it seems fair to use this variable as a liquidity proxy.²⁸

²⁵A trade cancellation for a trade report that was originally submitted to TRACE on a previous date.

²⁶The rate that matches the product's maturity.

²⁷The Amihud (2002) measure, the Amihud risk, the imputed roundtrip cost (IRC), the IRC risk, the Roll (1984) measure, the turnover rate of a bond, and the proportion of zero trading days. For more details on these variables, see Dick-Nielsen et al. (2012).

²⁸As a robustness test, the analysis has been performed using other liquidity proxies and the results were about the same.

Bond yields-to-maturity are computed on clean prices from which the corresponding maturity-matched Treasury rate is removed. Then, the daily spreads are averaged over each month to be compared with the monthly liquidity proxy. Note that we winsorize the 0.5% highest and lowest values of every observed YTM spread. To control for credit risk, theoretical YTM spreads are obtained using the regime-switching hybrid credit risk model. One spread is computed for every week (since the time step of our estimation method was $\Delta_t = 1/52$) and the monthly theoretical spread is the average of the weekly ones. Like CDS premiums, theoretical bond prices are numerically computed using Yuen and Yang (2010)'s trinomial lattice with Schönbucher (2002)'s additional branch to account for possible default (see Appendix A).

We run a dummy variable regression using monthly observations. AAA- and AA-rated firms are removed since they correspond to a handful of firms²⁹ and no serious inference could be done for these credit ratings. We are thus able to have bond issues for four different ratings (A, BBB, BB and B). The liquidity regressions are based on the following equation:

$$\begin{aligned} & (\text{Observed YTM spread})_{it} \\ &= \gamma_0 + \gamma_1(\text{Theoretical YTM spread})_{it} + \gamma_2\lambda_{it} + \mathbb{I}_{\text{Crisis}}(t)(\gamma_3 + \gamma_4\lambda_{it}) + \\ & \quad \mathbb{I}_{\text{Post}}(t)(\gamma_5 + \gamma_6\lambda_{it}) + \mathbb{I}_{\text{BBB}}(i)(\gamma_7 + \gamma_8\lambda_{it}) + \\ & \quad \mathbb{I}_{\text{BB}}(i)(\gamma_9 + \gamma_{10}\lambda_{it}) + \mathbb{I}_{\text{B}}(i)(\gamma_{11} + \gamma_{12}\lambda_{it}) + \epsilon_{it}, \end{aligned} \quad (9)$$

where $(\text{Observed YTM spread})_{it}$ is the observed YTM spread on the market, the model YTM spread is given by $(\text{Theoretical YTM spread})_{it}$, and λ_{it} is the it th liquidity proxy. The 0.5% highest values of the liquidity proxy are winsorized. The dummy variables $\mathbb{I}_{\text{Crisis}}(t)$ and $\mathbb{I}_{\text{Post}}(t)$ account for the different periods in our sample, and $\mathbb{I}_{\text{BBB}}(t)$, $\mathbb{I}_{\text{BB}}(t)$ and $\mathbb{I}_{\text{B}}(t)$ for the different ratings.

Regression coefficient variances are again estimated using Thompson (2011) allowing for estimators that are robust to simultaneous correlation along two dimensions (i.e. bond issues and time). Although this method is an improvement when compared to OLS standard deviations, the inference surrounding the point estimators shall be handled with care.

6.2 Liquidity pricing

The composite λ measure has a zero mean by construction; however, its distribution seems highly asymmetrical. The first percentile is around -3.5 and the 99th percentile is at 12.9. The same asymmetric behaviour is true for the four constituents of the λ measure. Figure 7 shows the average value of the liquidity proxy and the average value of the theoretical spread through time. The dependence between both variables appears to be important.

Figure 8 shows the relationship between the theoretical and observed YTM spreads. The relation between both variables seems linear, with a great amount of noise.

Specific cases of Equation (9) have been estimated to measure how much of the observed yield-to-spread bond spreads are explained by the model. The first one (i.e. Regression (1) of Table 4) considers the theoretical YTM spread as the only regressor: this regression yields an R -squared of 65%. Another specific case of Equation (9) is estimated. In the latter, the liquidity components are ignored (i.e. Regression (2)):

$$\begin{aligned} (\text{Observed YTM spread})_{it} = & \gamma_0 + \gamma_1(\text{Theoretical YTM spread})_{it} + \mathbb{I}_{\text{Crisis}}(t)\gamma_3 + \\ & \mathbb{I}_{\text{Post}}(t)\gamma_5 + \mathbb{I}_{\text{BBB}}(i)\gamma_7 + \mathbb{I}_{\text{BB}}(i)\gamma_9 + \mathbb{I}_{\text{B}}(i)\gamma_{11} + \epsilon_{it}. \end{aligned}$$

The theoretical YTM spreads taken with dummies controlling for rating and period effects explain about 66.2% of the observed YTM spreads. Moreover, the theoretical YTM spread is statistically different from zero and economically significant. The coefficient associated with the theoretical YTM is not statistically different from 1: the model yields unbiased YTM spreads on average. In Regression (3), the liquidity proxy and the theoretical YTM spread are used as regressors. Both are statistically significant and the R -squared is about 67%, which is an increase of 2.3% from the base case. Table 4 also shows the coefficients when we perform the full regression. Again, the coefficient associated with the theoretical YTM is not statistically

²⁹This affected five firms in the pre-crisis era, five in the crisis period, and four during the post-crisis era.

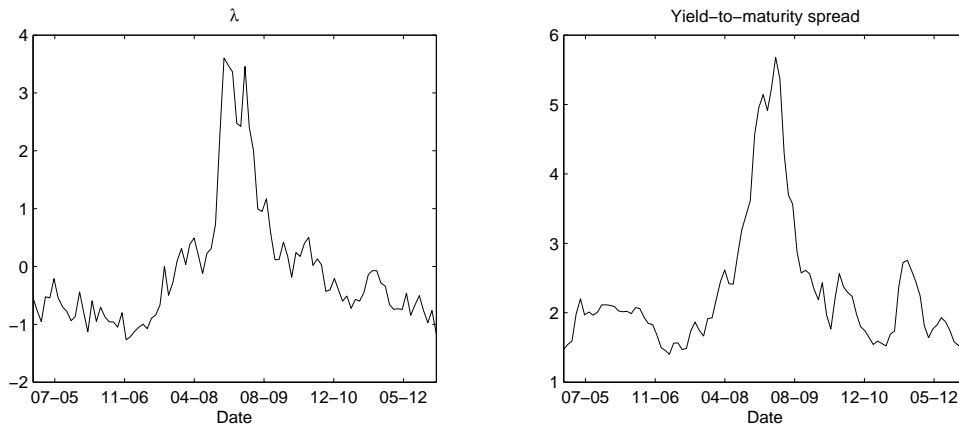


Figure 7: Time series of the average composite λ measure and the average YTM spread in percent. For each month, the average over each variable is computed across our dataset of 395 bond issues (paying no attention to the maturity of the bonds). The liquidity proxy is computed on a monthly basis from January 2005 to December 2012. Moreover, bond yields-to-maturity are computed on clean prices from which the corresponding maturity-matched Treasury rate is removed. The daily spreads are averaged over each month to be compared with the monthly liquidity proxy.

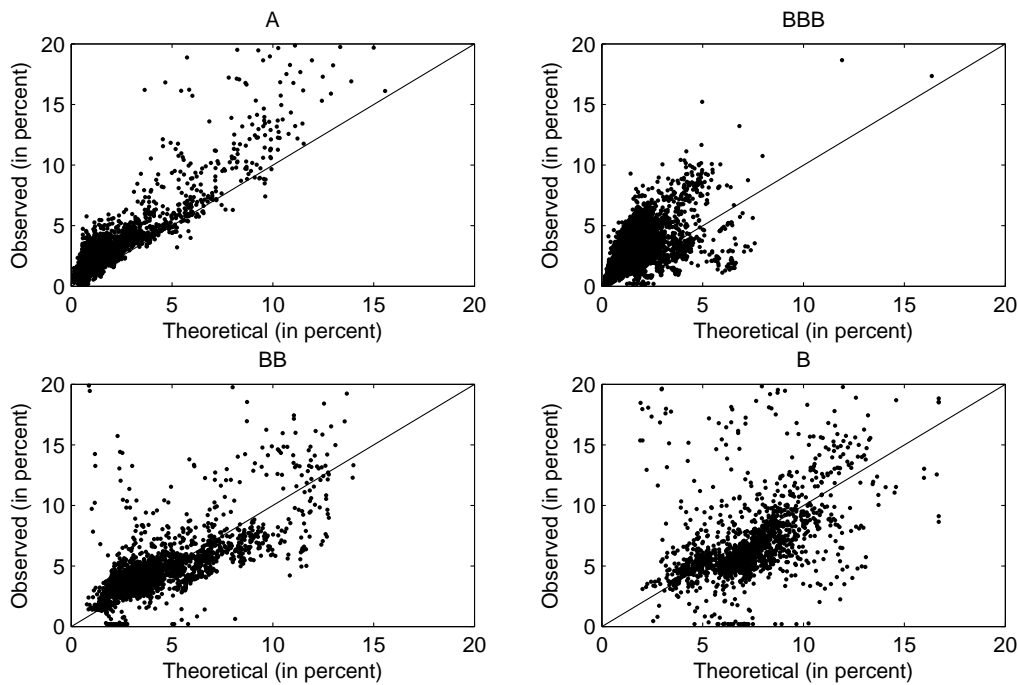


Figure 8: Observed versus theoretical yield-to-maturity spreads for each rating. The spreads are computed in percentage.

Observed bond YTM are computed on clean prices from which the corresponding maturity-matched Treasury rate is removed. Then, the daily spreads are averaged over each month to be compared with the monthly liquidity proxy. Theoretical YTM spreads are obtained using the regime-switching hybrid credit risk model on a weekly basis and are averaged over each month to be compared with the monthly liquidity proxy. The figure contains spreads from our dataset of 395 bond issues.

different from 1. The impact of the liquidity variable varies through time: moreover, the dummies related to liquidity in the crisis and post-crisis eras are statistically significant. The R -squared of this regression is 69.6%.

Table 4: Liquidity regressions' results.

	(1)	(2)	(3)	(4)
Intercept	0.625	-0.169	0.675	-0.097
Theoretical YTM spread	1.013	1.052	0.988	1.007
Liquidity proxy	-	-	0.141	-0.001
Crisis dummy	-	1.024	-	0.876
Crisis liquidity dummy	-	-	-	0.158
Post-crisis dummy	-	0.637	-	0.637
Post-crisis liquidity dummy	-	-	-	0.068
BBB dummy	-	0.493	-	0.480
BBB liquidity dummy	-	-	-	-0.007
BB dummy	-	-0.177	-	0.003
BB liquidity dummy	-	-	-	0.090
B dummy	-	-0.484	-	-0.225
B liquidity dummy	-	-	-	0.455
R^2	0.647	0.662	0.670	0.696
Number of observations	25,755	25,755	21,873	21,873

[1] Using our dataset of bond issues, the regression

$$\begin{aligned} & (\text{Observed YTM spread})_{it} \\ &= \gamma_0 + \gamma_1(\text{Theoretical YTM spread})_{it} + \gamma_2\lambda_{it} + \mathbb{I}_{\text{Crisis}}(t)(\gamma_3 + \gamma_4\lambda_{it}) + \\ & \quad \mathbb{I}_{\text{Post}}(t)(\gamma_5 + \gamma_6\lambda_{it}) + \mathbb{I}_{\text{BBB}}(i)(\gamma_7 + \gamma_8\lambda_{it}) + \\ & \quad \mathbb{I}_{\text{BB}}(i)(\gamma_9 + \gamma_{10}\lambda_{it}) + \mathbb{I}_{\text{B}}(i)(\gamma_{11} + \gamma_{12}\lambda_{it}) + \epsilon_{it}, \end{aligned}$$

is estimated. The conclusions of the statistical test $H_0 : \gamma_i = 0$ against $H_1 : \gamma_i \neq 0$, $i = 0, 1, \dots, 12$, are reported. Estimates in bold are significant at a confidence level of 95%.

[2] The regressions called (1), (2) and (3) correspond to specific cases of the full regression equation in Equation (9) where some γ_i are set to zero.

[3] Regressions (3) and (4) have a lower number of observations since the composite liquidity measure could not be computed for some months in the sample.

6.3 Size of the liquidity component

To compute the impact of corporate bond illiquidity on observed spreads, we proceed as follows: using the previous regression results, a liquidity score is defined for a bond in a given month as $(\gamma_2 + \gamma_p + \gamma_r)\lambda_{it}$, where p and r are the appropriate indices.³⁰ Then, these scores are divided by ratings and periods (i.e. pre-crisis, crisis and post-crisis). The liquidity component of an average bond is the difference between the 50th percentile and the 5th percentile of each segment. Broadly speaking, this is the difference between the median liquidity contribution and the liquidity contribution of a very liquid bond.

Table 5 shows the liquidity component in basis points for the three periods. The results are given by ratings. In the pre-crisis era, liquidity contributions are rather small. For A- and BBB-rated bonds, the contribution is negative; however, the regression coefficients related to these values are not statistically different from zero at a confidence level of 95%. For all the ratings, the liquidity component increases during the crisis and this phenomenon is even more important for B-rated firms. Indeed, for A-rated bonds, the increase of the liquidity component during the crisis is 51 bps. For B-rated bonds, it goes from 38 to 119 bps. In the post-crisis era, the liquidity components decreased, but do not return to their original levels: there is some persistence in the liquidity component during the post-crisis era.

The fraction of the liquidity component to the total spread is also computed. For each bond, the liquidity component is defined for each month as $(\gamma_2 + \gamma_p + \gamma_r)(\lambda_{it} - \lambda_5)$, where λ_5 is the 5th percentile of the liquidity measure. The liquidity component is divided by the observed YTM spread, and then the median of this fraction is presented in Table 5 for each rating category during each period. According to our metric, liquidity explains 8.7% of the observed YTM spreads on average. During the crisis, this proportion reaches levels ranging from 10% to 21%, depending on the credit rating. Finally, in the post-crisis era, the proportion

³⁰The index p corresponds to the period and r to the rating. For instance, for a BB-bond during the crisis, $\gamma_p + \gamma_r = \gamma_4 + \gamma_8$.

Table 5: Liquidity component in basis point and liquidity component fraction of the YTM spread.

		Average	Pre-crisis	Crisis	Post-crisis
A	Liquidity component (in basis point)	21.747	-0.139	50.800	14.579
	Fraction of the spread (in percent)	9.566	-0.123	20.954	7.866
	Number of observations	4,529	1,010	1,058	2,461
BBB	Liquidity component (in basis point)	19.652	-1.441	46.532	13.864
	Fraction of the spread (in percent)	6.931	-0.811	15.297	6.308
	Number of observations	8,054	2,029	1,409	4,616
BB	Liquidity component (in basis point)	32.922	14.928	58.015	25.822
	Fraction of the spread (in percent)	6.629	3.936	10.251	5.699
	Number of observations	2,417	675	285	1,457
B	Liquidity component (in basis point)	79.847	37.775	118.622	83.145
	Fraction of the spread (in percent)	11.576	6.934	14.658	13.135
	Number of observations	1,644	280	424	940

[1] The liquidity component in each period is defined by $(\gamma_2 + \gamma_p + \gamma_r)(\lambda_{50} - \lambda_5)$ where λ_{50} is the median liquidity measure and λ_5 is the 5th percentile.

[2] The fraction of the total spread is defined as $(\gamma_2 + \gamma_p + \gamma_r)(\lambda_{it} - \lambda_5)/(\text{Observed YTM spread})_{it}$ for each month. Then, median for each credit rating is taken.

of observed YTM spreads explained by liquidity varies between 6% and 13%. These results contrast with those of Dick-Nielsen et al. (2012): their increases are much more severe. This difference can be attributed to the fact that both studies used a different sample of bond issues. Moreover, different methods to control credit risk were used. One could also argue that our selection of bonds is very liquid as CDS contracts are traded for firms issuing these bonds.

7 Other risks in the bond market

Since liquidity risk cannot entirely account for the unexplained part of the yield-to-maturity spreads, it could be interesting to see whether other factors would. Using a similar methodology to that introduced in Equation (9), interest rate, market-wide, and firm-specific variables are added to the regressions.

Collin-Dufresne et al. (2001) considers the slope of the risk-free term structure in their analyses. Since this variable could have some impact on the results (especially during the crisis), it is also included in this section's regressions even though they find that this factor is not very significant either statistically or economically.

According to Collin-Dufresne et al. (2001), the changes in the CBOE Volatility Index VIX are statistically significant. Ericsson et al. (2006) also use the VIX to explain residual bond spreads (i.e. the difference between the market spread and the level given by some structural models³¹). They find that VIX is statistically significant for residual bond spreads for Leland and Toft (1996) model.

The S&P 500 returns is another factor which tries to capture the market-wide risk in the regressions. This variable is justified by the work of Campbell and Taksler (2003) and Ericsson et al. (2006),³² among others. Campbell and Taksler (2003) show that the excess returns on an index is statistically significant when the bond yield spreads are regressed against this variable. They use data between 1995 and 1999. The same authors also consider the excess stock returns. Therefore, monthly stock returns are added to this section's regressions.

Finally, the annualized stock volatility is also considered. Under credit risk structural models, this variable makes considerable sense. It will be related to the volatility of the firm's assets, and is expected to explain credit spreads. Normally, we would expect a positive relationship between stock volatility and credit spreads.

³¹Three models are used in this study: Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000).

³²Ericsson et al. (2006) find that the S&P 500 returns are statistically significant in bond spread residuals in general.

The variable is considered by Chen et al. (2007) and Campbell and Taksler (2003), among others. It is again computed as the historical volatility based on daily returns using a one-year rolling window.

Using our dataset of bond issues, the following dummy variable regression is run:

$$\begin{aligned}
 (\text{Observed YTM spread})_{it} = & \\
 & \gamma_0 + \gamma_1(\text{Theoretical YTM spread})_{it} + \gamma_2\lambda_{it} + \gamma_3(\text{VIX})_t + \gamma_4(\text{S\&P})_t + \\
 & \gamma_5(\text{Slope})_t + \gamma_6R_{it} + \gamma_7\sigma_{it}^R + \gamma_8\mathbb{I}_{\text{Crisis}}(t) + \gamma_9\mathbb{I}_{\text{Post}}(t) + \\
 & \gamma_{10}\mathbb{I}_{\text{BBB}}(i) + \gamma_{11}\mathbb{I}_{\text{BB}}(i) + \gamma_{12}\mathbb{I}_{\text{B}}(i) + \epsilon_{it}.
 \end{aligned} \tag{10}$$

Note that we winsorize the 0.5% highest and lowest values of every observed and theoretical YTM spread. Also, the 0.5% highest values of Dick-Nielsen et al. (2012)s λ measure are winsorized.

Table 6 shows the regression results using different specifications. The liquidity proxy and the theoretical YTM spreads are statistically significant. The R -squared obtained with the full regression is 72%, about 4% more than Regression (2). Therefore, additional variables explain mildly the observed YTM spreads when theoretical YTM spreads and bond liquidity is accounted for. Based on the coefficients of determination of Table 6, firm-specific variables (and more specifically, equity volatility) are the variables that explain the most important part of the YTM spreads after the theoretical YTM spreads. The coefficient related to equity volatility is statistically significant and its sign is positive, meaning that an increase in the equity volatility will result in a rise in the spreads.

In the full regression, the post-crisis dummy is statistically significant and the coefficient is positive, meaning that YTM spreads are higher in the post-crisis period than in the pre-crisis era. During the crisis, the coefficient is positive but not statistically significant at a confidence level of 95%. However, this does not mean that the YTM spreads do not increase during the crisis: other variables positively related with the spreads have sharply increased during the crisis resulting in higher predicted spreads in the crisis era.

Overall, the theoretical YTM spreads explain an important part of the observed spreads. However, based on these regressions, the model does not capture every risk present in corporate bond spreads. Besides

Table 6: Regressions on other factors.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	-0.169	-0.053	-0.606	-0.070	-0.526	-0.526	-0.740
Theoretical YTM spread	1.052	1.021	0.961	1.016	0.813	0.799	0.783
Liquidity proxy	-	0.121	0.096	0.118	0.098	0.095	0.088
VIX	-	-	0.046	-	-	-	0.024
S&P 500 returns	-	-	2.265	-	-	-	-2.596
Slope	-	-	-	0.084	-	-	-0.223
Equity returns	-	-	-	-	-	1.760	2.364
Equity volatility	-	-	-	-	3.362	3.377	3.360
Crisis dummy	1.024	0.872	0.220	0.747	0.008	0.153	0.010
Post-crisis dummy	0.637	0.565	0.147	0.367	0.288	0.262	0.576
BBB dummy	0.493	0.488	0.469	0.486	0.427	0.413	0.410
BB dummy	-0.177	-0.048	0.140	-0.032	0.119	0.167	0.212
B dummy	-0.484	-0.328	0.046	-0.298	0.175	0.210	0.316
R^2	0.662	0.681	0.690	0.681	0.711	0.715	0.720
Number of observations	25,755	21,873	21,873	21,873	20,850	20,850	20,850

[1] Using our dataset of bond issues, the dummy variable regression

$$\begin{aligned}
 (\text{Observed YTM spread})_{it} = & \\
 & \gamma_0 + \gamma_1(\text{Theoretical YTM spread})_{it} + \gamma_2\lambda_{it} + \gamma_3(\text{VIX})_t + \gamma_4(\text{S\&P})_t + \\
 & \gamma_5(\text{Slope})_t + \gamma_6R_{it} + \gamma_7\sigma_{it}^R + \gamma_8\mathbb{I}_{\text{Crisis}}(t) + \gamma_9\mathbb{I}_{\text{Post}}(t) + \\
 & \gamma_{10}\mathbb{I}_{\text{BBB}}(i) + \gamma_{11}\mathbb{I}_{\text{BB}}(i) + \gamma_{12}\mathbb{I}_{\text{B}}(i) + \epsilon_{it},
 \end{aligned}$$

is estimated. The conclusion of the statistical test $H_0 : \gamma_i = 0$ against $H_1 : \gamma_i \neq 0$, $i = 0, 1, \dots, 12$, is reported. Estimates in bold are significant at a confidence level of 95%.

liquidity, the firm's equity volatility is also linearly related to the observed spreads. Even though the liquidity proxy is significant, its average contribution is moderate. Market-wide factors (and more specifically, VIX) do have an impact on the observed spreads during these periods.

As a robustness test, we also used innovations of the VIX and of the equity volatility as regressors (instead of the levels). Results were similar to what is obtained in this section.

8 Concluding remarks

The determinants of credit spreads and CDS premiums are investigated throughout the paper, paying particular attention to three different periods: before, during, and after the financial crisis of 2008. The fact that conclusions are sensitive to the model's characteristics prompted us to design a flexible credit risk model that captures the empirical evidence gathered in the last decade. A regime-switching variable is included to accommodate behavioural changes during the financial crisis. A negative dependence between endogenous recovery rates and the firm's default probability is addressed in Altman et al. (2005)'s empirical findings.

A firm-by-firm estimation procedure based on a filtering procedure deals with latent variables and noise. The in-sample performance of the model reveals that it is flexible enough to adjust to various firms and financial cycles. An out-of-sample study concludes that the model is reliable and outperforms other considered benchmarks.

Recovery uncertainty and its negative relationship with default probability have a major impact on mid- and long-term credit spreads. The presence of regimes affects the long-term credit spread of highly rated firms during the pre- and post-crisis periods. During the financial turmoil, the presence of regimes modifies the short-term shape of the average credit spread curves, especially for highly rated firms.

In the pre-crisis era, the observed spot spreads are almost fully explained by credit risk measured through the proposed framework. For instance, 67% of the BBB spot spreads are explained by the model. This is somewhat higher than figures obtained by older studies such as Elton et al. (2001) and Huang and Huang (2012), but consistent with recent literature. During the crisis, this proportion decreases to 42% and increases in the post-crisis period, but does not reach pre-crisis levels. The endogenous recovery rate and its dependence with default risk clarifies in part the high proportion of empirical spreads explained by the model.

Illiquidity in the bond market may explain part of these gaps. For all ratings, the liquidity component increases during the crisis and explains about 15.3% of the observed yield-to-maturity spreads. This is consistent with Dionne and Maalaoui Chun (2013)'s results. However, these results contrast with those of Dick-Nielsen et al. (2012): their increases are much sharper for every rating. This dichotomy could be attributed to the fact that both studies used a different sample of bond issues and different methods to control credit risk.

Since liquidity risk cannot entirely account for the unexplained part of the yield-to-maturity spreads, other factors such as interest rate, market-wide and firm-specific variables are added to the regressions. Overall, credit risk (proxied as the theoretical YTM spread) explains a large proportion of the observed YTM spreads. Firm-specific variables (and more specifically equity volatility) are statistically significant and increase the coefficient of determination from 68.1% to 71.5%. The VIX variable is also statistically significant.

A Derivative pricing

This appendix describes the numerical method inspired by Yuen and Yang (2010) used to price credit-sensitive derivatives.

A.1 Trinomial lattice approach

Yuen and Yang (2010) propose a trinomial lattice approach for Markov-switching dynamics. The authors use the same trinomial lattice for every regime, but change the risk-neutral weights if the regime state

changes so that the trinomial tree is a recombining one. Their method works for many kinds of options, but credit-sensitive derivatives require an adaptation of the algorithm.

A “up-across-down” branching structure is chosen with $x_u = xe^{\sigma\sqrt{\Delta t}}$, $x_m = x$, and $x_d = xe^{-\sigma\sqrt{\Delta t}}$ where x is the actual value of the log-leverage process at a typical node. Moreover, when the number of regimes is K , the value of σ is given by

$$\sigma = \max_{1 \leq i \leq K} \sigma_i + (\sqrt{1.5} - 1)\bar{\sigma} \tag{11}$$

where $\bar{\sigma}$ is the arithmetic mean of σ_i . This suggestion is based on the values used in the binomial and trinomial trees in the literature.

The weights for the i th regime are

$$\begin{aligned} \pi_m^i &= 1 - \frac{1}{\lambda_i^2}, \\ \pi_u^i &= \frac{e^{\mu_i^Q \Delta t} - e^{-\sigma\sqrt{\Delta t}} - (1-1/\lambda_i^2)(1-e^{-\sigma\sqrt{\Delta t}})}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}, \\ \pi_d^i &= \frac{e^{\sigma\sqrt{\Delta t}} - e^{\mu_i^Q \Delta t} - (1-1/\lambda_i^2)(e^{\sigma\sqrt{\Delta t}} - 1)}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}, \end{aligned}$$

where $\lambda_i = \frac{\sigma}{\sigma_i}$. These weights are different for each regime. The Schönbucher (2002) lattice that deals with credit-sensitive instruments is adapted in the trinomial lattice approach for Markov-switching dynamics by adding an “artificial” branch at each node. According to Equations (2) and (3), the default probability is

$$p = 1 - \exp\left(-\left(\beta + \left(\frac{e^x}{\theta}\right)^\alpha\right)\Delta t\right),$$

if the log-leverage value at this typical node is x .

Figure 9 shows the different branches to be considered to use this numerical scheme when $K = 2$. Note that even if the figure contains two different trees, these trees represent the same lattice: only the weights change across the different regimes.

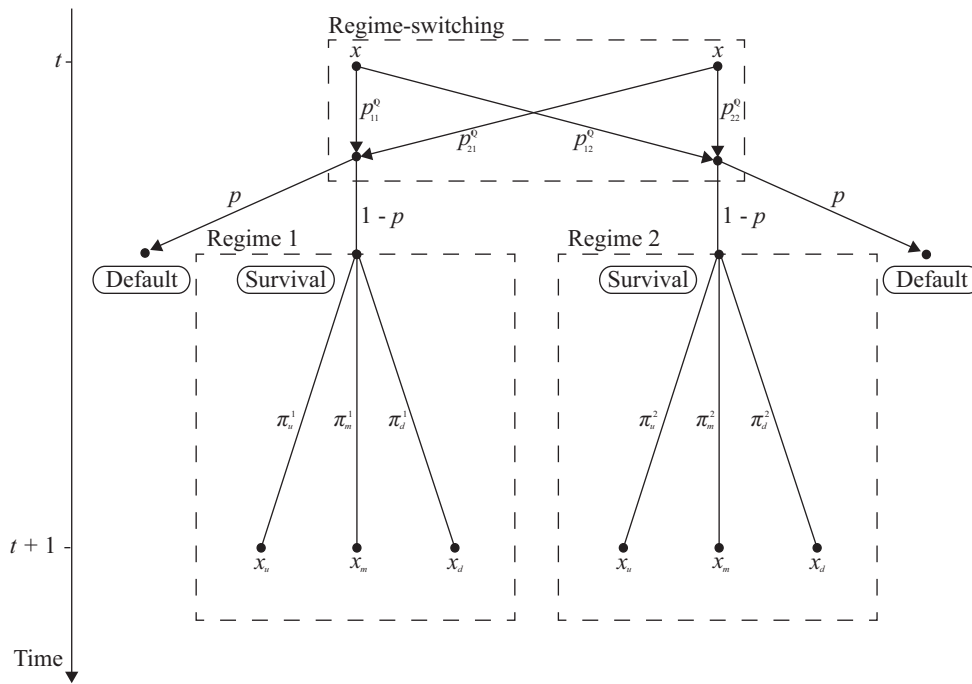


Figure 9: The branching to default at a typical node in the tree when the number of regimes is set to two.

A.2 Credit default swap premiums

A CDS is a credit derivative that compensates the buyer in the event of a default (or other credit events). In the most basic type of CDS, the protection seller provides a payment of par-minus-recovery (settled in cash) on default. This shall cover the loss incurred by a typical bondholder. In exchange, the protection buyer pays a periodic premium that ceases if a default occurs. Normally, these premiums are paid four times per year. The premium of such a contract is fixed by setting the expected present value of losses equals to the expected present value of the premiums.

The endogenous recovery rate defined in Subsection 2.1 introduces a negative relation between recovery and default risks.

Given that the CDS matures at time T and that the T -year risk-free rate r is constant, the protection leg (expected present value of the losses) is

$$\mathbb{E}^{\mathbb{Q}} \left[e^{-r(\tau-t)} (1 - R_{\tau}) \mathbb{I}_{\{t < \tau \leq T\}} \mid \mathcal{F}_t \right] = \mathbb{E}^{\mathbb{Q}} \left[\sum_{t \leq k < T} (1 - R_k) e^{-r(k-t)} \exp \left(- \sum_{t \leq u < k} I_u \Delta_t \right) (1 - \exp(-I_k \Delta_t)) \Bigg| \mathcal{G}_t \right] \mathbb{I}_{\{\tau > t\}}, \quad (12)$$

where the filtration $\{\mathcal{F}_t\}_{t=1}^{\infty}$ is defined as $\mathcal{F}_t = \sigma(\mathcal{G}_t, \mathcal{H}_t)$ and $\mathcal{H}_t = \sigma(\mathbb{I}_{\{\tau \leq s\}} : s \leq t)$.

To simplify the presentation, assume that a premium of 1 is paid. In this case, the premium leg (expected present value of the premiums) is given by

$$\mathbb{E}^{\mathbb{Q}} \left[\sum_{t_i^*} e^{-r(t_i^* - t)} \mathbb{I}_{\{t \leq t_i^* < \tau\}} \Bigg| \mathcal{F}_t \right] = \mathbb{E}^{\mathbb{Q}} \left[\sum_{t_i^*} \delta_i^* e^{-r(t_i^* - t)} \exp \left(- \sum_{t \leq u < t_i^*} I_u \Delta_t \right) \Bigg| \mathcal{G}_t \right] \mathbb{I}_{\{\tau > t\}} \quad (13)$$

where t_i^* are premium payment dates and $\delta_i^* = t_i^* - t_{i-1}^*$. The periodic premium is the ratio of (12) over (13).

The numerical scheme explained in Subsection A.1 shall come in handy to compute credit default swap premiums in the proposed framework. To price credit default swaps, a time step of 1 week (i.e. $\Delta_t = 1/52$) is used in the trinomial tree.

A.3 Coupon bond prices

To price coupon bonds on a single firm, recovery specifications are crucial. Again, the endogenous recovery of Subsection 2.1 is utilized. Bond investors will recover a fraction R_{τ} of an equivalent Treasury bond at default time τ .

Given a coupon rate of c , a T -year constant risk-free rate r , a maturity T , an initial log-leverage of x_t , an initial regime of h_t , and a face value of 1, the coupon bond price is given by

$$\begin{aligned} & V(t, T, x_t, h_t; R_{\tau}, c) \\ &= \mathbb{E}^{\mathbb{Q}} \left[\frac{c}{2} \sum_{t_i^*} e^{-r(t_i^* - t)} \mathbb{I}_{\{t \leq t_i^* < \tau\}} + e^{-r(T-t)} \mathbb{I}_{\{\tau > T\}} + e^{-r(\tau-t)} R_{\tau} P(\tau, T) \mathbb{I}_{\{t < \tau \leq T\}} \Bigg| \mathcal{F}_t \right] \\ &= \mathbb{E}^{\mathbb{Q}} \left[\frac{c}{2} \sum_{t_i^*} e^{-r(t_i^* - t)} \exp \left(- \sum_{t \leq u < t_i^*} I_u \Delta_t \right) \Bigg| \mathcal{G}_t \right] \mathbb{I}_{\{\tau > t\}} \\ &+ \mathbb{E}^{\mathbb{Q}} \left[e^{-r(T-t)} \exp \left(- \sum_{t \leq u < T} I_u \Delta_t \right) \Bigg| \mathcal{G}_t \right] \mathbb{I}_{\{\tau > t\}} \\ &+ \mathbb{E}^{\mathbb{Q}} \left[\sum_{t \leq k < T} R_k e^{-r(T-t)} \exp \left(- \sum_{t \leq u < k} I_u \Delta_t \right) (1 - \exp(-I_k \Delta_t)) \Bigg| \mathcal{G}_t \right] \mathbb{I}_{\{\tau > t\}} \end{aligned} \quad (14)$$

where $P(\tau, T)$ is the time τ value of a risk-free zero-coupon bond maturing at T and t_i^* are coupon payment dates. Note that a zero-coupon bond can be computed by replacing the coupon rate c by zero in Equation (14).

B Unscented Kalman filter-based method

Filtering techniques are useful when the state variables cannot be observed. In credit risk modelling, only a handful of individuals worked with the possibility of noise. Duan and Fulop (2009) propose a structural model estimation technique that allows for trading noise in observed equity prices. The estimation is carried out by using a particle filter and conducting a maximum likelihood estimation (MLE). It is argued that ignoring noise could non-trivially inflate one's estimate for the asset volatility. Huang and Yu (2010) introduce an alternative estimation method based on Markov-chain Monte Carlo methods for Merton (1974)'s model with independent and identically distributed noise. Using daily closing stock prices over 2003–2004 from emerging markets, they show that noises are positively correlated with firms' values. This method can be generalized to other structural models. Using the unscented Kalman filter, Boudreault et al. (2013) allow for noises in a hybrid credit risk model. Their estimation, like ours, is based on CDS premiums.

In the proposed model, leverage ratios and regimes are unobservable and should be inferred from observed quantities. Moreover, the filter must allow quasi-likelihood function computation in a somewhat direct manner.

A state space representation is commonly used to define a model for which the state is a Markov process and observed quantities (i.e. credit default swap premiums) are related to the state variable. The log-leverage dynamics is given by Equation (1). The dynamics under the martingale measure are similar to (1), except that the drift parameters would be $\mu_i^{\mathbb{Q}}$ instead of $\mu_i^{\mathbb{P}}$, and the transition probabilities would be $p_{ij}^{\mathbb{Q}}$ instead of $p_{ij}^{\mathbb{P}}$.

To reduce the number of parameters to be estimated, independence in the pricing error between each CDS has been assumed. Moreover, 1-, 2-, 3-, 5-, 7- and 10-year credit default swaps are used in the estimation. Thus, the measurement equation becomes

$$y_t^{(i)} = h^{(i)}(x_t, h_t) + \nu_t^{(i)} = \log \left(g^{(i)}(x_t, h_t) \right) + \nu_t^{(i)}, \quad i = 1, 2, 3, 5, 7, 10, \quad (15)$$

with $y_t^{(i)}$ the observed i -year credit default swap log-premium, $\nu_t^{(i)}$ being a Gaussian random variable (having a standard deviation $\delta^{(i)}$) independent across maturities and $g^{(i)}$ the theoretical premium of an i -year credit default swap (in basis points). Note that the CDS pricing formula (i.e. function $g^{(i)}$) is a nonlinear function of the log-leverage. To maintain positive premiums, a logarithmic transformation is applied; though, this tranformaton implies a small bias in CDS premiums. Note that the bias is negligible in most cases.

The short rate is not modelled explicitly, and once fixed, does not change: it remains at its initial level during the pricing for a given day. However, the rate changes over time in the physical measure (from one week to another). This assumption takes into account differences in the interest rate over time without having to explicitly model the variable.

A filtering method that simultaneously estimates both latent quantities is needed. An extension of Tugnait (1982)'s detection-estimation algorithm (DEA) is designed to filter both of our unobserved variables (see Appendix C for details on the DEA). To allow for our nonlinear state space representation, the DEA must be extended: instead of running M Kalman filters in parallel, we propose the use of M unscented Kalman filters in parallel³³ (UKF). To the best of our knowledge, this study is the first of using the DEA coupled with the UKF.

In the hybrid credit risk model, the state propagation equation is Gaussian and linear given the regime path, but the measurement equation is not. The UKF handles the nonlinearity and approximates the posterior state density using a set of deterministically chosen sample points (called sigma points). These points capture the mean and covariance of the Gaussian state variable, and when propagated through the measurement equation, it captures the posterior mean and covariance of the CDS premiums accurately up to

³³The parameters of the UKF technique have been assumed to be $\kappa_{\text{UKF}} = 2$, $\alpha_{\text{UKF}} = 0.1$, and $\beta_{\text{UKF}} = 0$.

the second order. According to Christoffersen et al. (2013), the unscented Kalman filter may prove to be a good approach for a variety of problems in fixed-income pricing. Moreover, for some applications in finance, it seems to significantly outperform the extended Kalman filter (EKF) and performs well when compared to the much more computationally intensive particle filter. The parameters are estimated using a quasi-maximum likelihood estimation³⁴ on a firm-by-firm basis.

It is important to note that the dynamics of the log-leverage ratio implied by the pricing of function $g^{(i)}$ have to be under the risk-neutral measure \mathbb{Q} while the transition of Equation (1) captures the parameters under the real measure \mathbb{P} .

The variance of the initial log-leverage³⁵ $P_{0|0}$ has been assumed to be around 0.05; different choices of initial variance do not seem to impact the filtering. Overall, the parameters to be estimated for each firm are $\mu^{\mathbb{Q}}$, $\mu^{\mathbb{P}}$, σ_1 , σ_2 , $p_{12}^{\mathbb{Q}}$, $p_{12}^{\mathbb{P}}$, $p_{12}^{\mathbb{Q}}$, $p_{12}^{\mathbb{P}}$, α , β , θ , κ , $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$, $\delta^{(5)}$, $\delta^{(7)}$, and $\delta^{(10)}$. Thus, a single set of parameters is used for each firm to explain the default risk and loss given default; this contrast with calibration techniques where the credit default swap term structure is fitted at every available period.

C Tugnait (1982)'s detection-estimation algorithm

Tugnait (1982) considers the problem of state estimation and system structure detection for discrete stochastic systems with parameters that may change among a finite set of values.

Assume the state space representation given by Equations (1) and (15):

$$\begin{aligned} x_t &= x_{t-1} + \left(\mu_{h_t}^{\mathbb{P}} - \frac{1}{2} \sigma_{h_t}^2 \right) \Delta_t + \sigma_{h_t} \sqrt{\Delta_t} \epsilon_t^{\mathbb{P}}, \\ y_t^{(i)} &= \log \left(g^{(i)}(x_t, h_t) \right) + \nu_t^{(i)}, \quad i = 1, 2, 3, 5, 7, 10, \end{aligned}$$

where $y_t^{(i)}$ the observed i -year credit default swap log-premium and $g^{(i)}$ the theoretical premium of an i -year credit default swap.

Tugnait (1982) consists of running $M = K^d$ Kalman (1960) filters in parallel at any stage t where K is the number of possible regimes in the model and d is determined by computation and storage capabilities. However, since the state space representation is nonlinear, the method cannot be applied directly. Thus, instead, the unscented Kalman filter is used.

Suppose that at stage $t - 1$, M/K regime paths have been chosen. At stage t , all possible extensions of these regime sequences shall be considered. By an extension $h_{0:t}^* \equiv \{h_0^*, h_1^*, \dots, h_t^*\}$ of $h_{0:t-1}^j$, one means

$$h_{0:t}^* = \{h_{0:t-1}^j, r(t)\}, \quad r(t) \in \{1, 2, \dots, K\}.$$

This shall create M new regime paths. Then, using Kalman filters, it is possible to compute the filtered value of x_t based on the first t observations and the regime path $h_{0:t}^*$:

$$x_{t|t}^* \equiv \mathbb{E}(x_t | y_{1:t}, h_{0:t}^*)$$

where $y_{1:t} \equiv \{y_1, y_2, \dots, y_t\}$ and $y_t = \{y_t^{(1)}, y_t^{(2)}, y_t^{(3)}, y_t^{(5)}, y_t^{(7)}, y_t^{(10)}\}$. Moreover, the *a posteriori* probability of observing this specific regime path based on the first t observations is

$$\mathbb{P}(h_{0:t}^j | y_{1:t}) = \frac{\mathbb{P}(y_{1:t} | h_{0:t}^j) \mathbb{P}(h_{0:t}^j)}{\sum_{i=1}^M \mathbb{P}(y_{1:t} | h_{0:t}^i) \mathbb{P}(h_{0:t}^i)}$$

³⁴Quasi-likelihood means here that the first two moments of the posterior distribution have a second-order precision and a posterior Gaussian distribution has to be assumed.

³⁵Initial leverages are approximated using their book values. More precisely, total liabilities divided by total assets is taken. Both quantities are acquired by Compustat, which is available from Wharton Research Data Services (WRDS). The fourth quarter of 2004's accounting data is selected to compute the proxy since Q4 of 2004 predates January 2005 (the beginning of our sampling period). In the database, the total liabilities is identified by LTQ and the total assets by ATQ. In addition, the firm's ticker symbol is matched to that of Markit's CDS premiums through the reference entity's name to ensure that the right information for each firm is used. For one firm, no data is available; this firm is thus removed from the sample.

where $\mathbb{P}(h_{0:t}^j)$ can be computed using $p_{ij}^{\mathbb{P}}$ of Equation (5). In addition,

$$\mathbb{P}(y_{1:t}|h_{0:t}^j) = \exp \left(-\frac{1}{2} \sum_{i=1}^t \log(\det V_i^j) - \frac{1}{2} \sum_{i=1}^t (e_i^j)^\top (V_i^j)^{-1} (e_i^j) - \frac{6t}{2} \log(2\pi) + \sum_{i=1}^t \log \left(p_{h_{i-1}^j, h_i^j}^{\mathbb{P}} \right) \right),$$

$e_i^j = y_i - y_{i|i-1}^j$ is the difference between the observations and the forecasted value of the observations at stage i , and $V_i^j = \mathbb{E} \left[(e_i^j)(e_i^j)^\top | h_{0:i}^j, y_{1:i-1} \right]$. The third term is multiplied by 6, as six tensors are used in this study.

This leads to the state estimate

$$x_{t|t} = \sum_{j=1}^M x_{t|t}^j \mathbb{P}(h_{0:t}^j | y_{1:t}).$$

Then, Tugnait (1982)'s idea is to collapse from M to M/K regime paths by keeping the paths that yield the most likely sequences in terms of *a posteriori* probabilities. This leads to minimum probability of error according to Van Trees (1968).

This method is similar to the one proposed by Kim (1994). The main difference between Kim (1994) and Tugnait (1982)'s DEA is the collapsing scheme. Kim (1994) proposes to take averages across the different regime paths to reduce the number of sequences instead of keeping the most likely ones.

The efficiency of the method is assessed by a simulation study (not reported here, but available on request). According to the study, the DEA-UKF technique yields virtually no bias on the parameter estimates for our application, in addition to being analytical and fast.³⁶

In this paper, the number of regimes is set to $K = 2$, and $d = 5$ is used. This choice appears to be a good compromise between accuracy and efficiency. Moreover, the results seem to be robust to different choices of d greater than 4.

D In- and out-of-sample performances

D.1 Benchmarks and model comparisons

The new regime-switching hybrid default model (RS) is compared to three other models: the “one-regime” (1R) equivalent of our model (i.e. the one presented in Boudreault et al. (2013)), a regime-switching structural version (SA) of the proposed framework, and a regime-switching reduced-form model (RFA).

The so-called 1R model is the one introduced in Boudreault et al. (2013); the main difference between this model and ours is that we use regime-switching dynamics to model the firm's leverage.

Letting $\beta = 0$ and $\alpha \rightarrow \infty$, the hybrid approach leads to a pure structural framework in which the log-leverage ratio is modelled by a regime-switching discretized version of a geometric Brownian motion. The structural model is nested in the hybrid framework; thus, its overall in-sample performance is expected to be weaker than the full model. However, it is possible that local performances dominate those of the full model.

The pure reduced-form version of the model (i.e. $\theta \rightarrow \infty$) is not the one used in this paper since this model would have constant intensity and this is obviously too restrictive. Instead, the intensity process is modelled by a regime-switching discretized version of a geometric Brownian motion:

$$I_t = \begin{cases} I_{t-1} + \left(\mu_S^{\mathbb{Q}} - \frac{1}{2} \sigma_{S,1}^2 \right) \Delta t + \sigma_{S,1} \sqrt{\Delta t} \epsilon_t^{\mathbb{Q}} & \text{if } h_t = 1 \\ I_{t-1} + \left(\mu_S^{\mathbb{Q}} - \frac{1}{2} \sigma_{S,2}^2 \right) \Delta t + \sigma_{S,2} \sqrt{\Delta t} \epsilon_t^{\mathbb{Q}} & \text{if } h_t = 2 \end{cases}$$

³⁶However, the DEA-UKF technique could have some problems with more general filtering applications. In that case, one could use the bias-free method of Fearnhead and Clifford (2003), even though this change could slow down the estimation procedure.

where $\mu_S^{\mathbb{Q}}$ is the drift parameter and $\sigma_{S,1}$ and $\sigma_{S,2}$ are volatility parameters for regime 1 and 2, respectively. Moreover, $\{\epsilon_t^{\mathbb{Q}}\}_{t=1}^{\infty}$ is a sequence of independent standardized Gaussian random variables under \mathbb{Q} . Since endogenous recoveries no longer make sense in the context of this model, we opt for a constant exogenous recovery rate $R_t = R \in [0, 1]$. This new parameter is estimated among all other parameters in the filtering procedure.

The models' performances are compared using the sum of squared errors:

$$\text{SSE} = \sum_{t=t_1}^{t_2} \sum_{i=1}^{N_t} (m_{t,i} - o_{t,i})^2 \quad (16)$$

where $m_{t,i}$ is the theoretical i -year CDS premium at time t , $o_{t,i}$ is the observed i -year CDS premium at time t , and N_t is the number of CDSs considered at time t . The parameters estimates are obtained using the entire sample and are kept fixed at any point in time. To standardize these fitting performances, the SSE is divided by the total variation of the observed CDS premiums:

$$\text{SST} = \sum_{t=t_1}^{t_2} \sum_{i=1}^{N_t} (o_{t,i} - \bar{o})^2 \quad \text{where } \bar{o} = \frac{1}{\sum_{t=t_1}^{t_2} N_t} \sum_{t=t_1}^{t_2} \sum_{i=1}^{N_t} o_{t,i}. \quad (17)$$

Consequently, the performance measure is the ratio SSE/SST. The closer it is to zero, the better the model is.

We also apply a very loose criterion to eliminate outliers: if the absolute difference between the model premium and the observed premium is more than 5 times the observed premium, it is considered an outlier:

$$|m_{t,i} - o_{t,i}| > 5o_{t,i}. \quad (18)$$

D.2 In-sample performance

The in-sample study is performed over 417 weeks, starting in January 2005 and ending in December 2012. Two hundred and ten firms were considered in this analysis.

Table 7 provides details about outliers. For the in-sample analysis, seven observations out of 489,796 observations were removed because the absolute value of the error on the premium was greater than five in at least one of the four models. Thus, we removed 0.0014% of our initial CDS sample.

Table 7: Number of outliers: in-sample study.

	RS	1R	SA	RFA	Total
Number of outliers	2	4	1	2	7

[1] The initial sample size is 489,796 CDS premiums.

Figure 10 shows the evolution of the SSE/SST ratio calculated by week. The measure used in these figures combines the six tenors. The in-sample performance of the full model is very good, with an average SSE/SST of 3.33%. It dominates the three benchmarks by a factor of 1.83 for the reduced-form approach, 2.05 for the structural model, and 3.45 for the "one-regime" equivalent. When considering IG firms only, the SSE/SST measure is about 1.52% for the full model on average; it outperforms the other models by factors of 1.97, 2.08, and 2.71 for RFA, SA and 1R, respectively. For HY firms, the average measures are higher: 4.28% for the full model. It is higher for the other benchmarks: the SSE/SSTs are 1.82 times larger for the reduced-form model, 2.06 for the structural approach, and 3.48 for Boudreault et al. (2013).

During the financial crisis of 2008, the SSE/SST ratio spikes when we consider the benchmark models. However, the full model seems to do well, even during these turbulent times. On September 17, 2008, the four models seem to yield large pricing errors on IG firms. This is mainly due to errors on AIG's credit

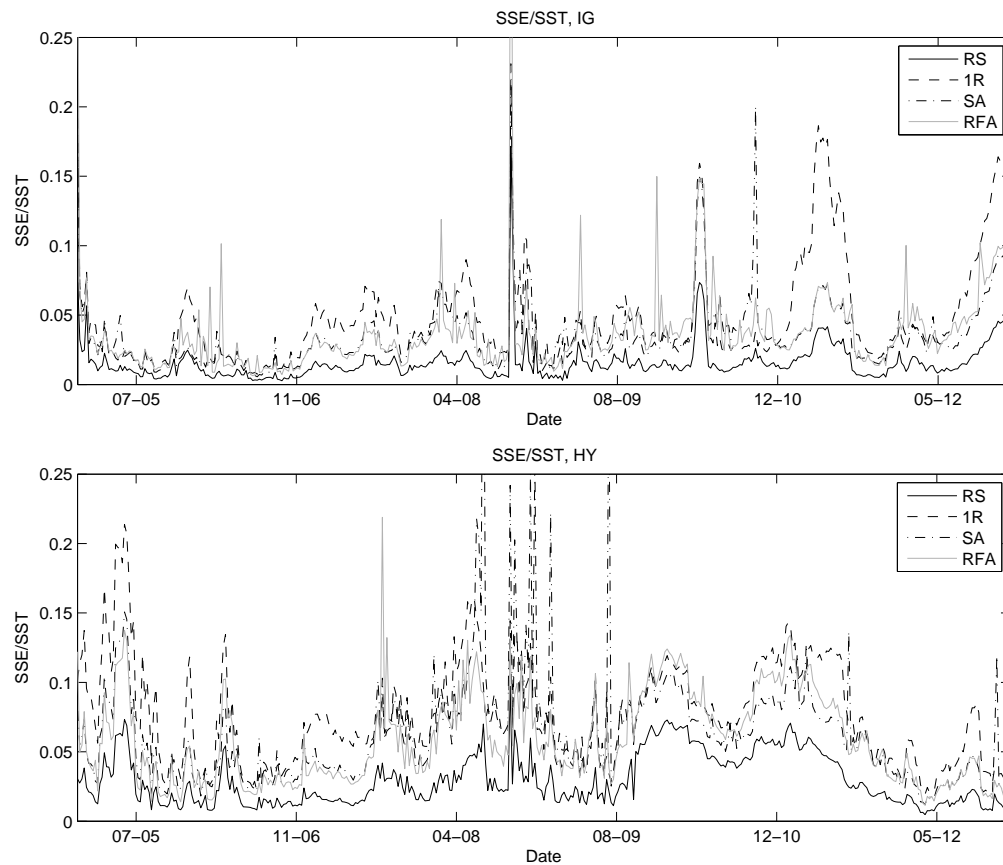


Figure 10: SSE/SST: evolution of the in-sample performance for IG and HY.

These figures show the evolution of the SSE/SST ratio calculated by week. For each firm, a single set of parameters is estimated using CDS premiums with maturities 1, 2, 3, 5, 7, and 10 years between January 2005 and December 2012.

default swap premiums. On September 15, 2008, AIG's credit rating was downgraded from AA- to A-; the downgrade had an important impact on CDS premiums.

Figure 11 shows the ratio calculated by maturity and by period. The performance of our model remains quite good, even during the crisis era (i.e. 2007–2009). Systematically, the full model outperforms the other ones. For IG firms, the performance is adequate, even for 1-year CDS premiums. For HY firms, the SSE/SST ratio seems to be larger for 1-year credit default swaps; however, SSE/SST ratios for the full model are always lower than the other models' ratios.

D.3 Out-of-sample performance: CDS subsample

In the first out-of-sample study, the parameters are estimated one more time. However, this time, we only use 2-, 5- and 10-year CDS premiums observed from January 2005 to December 2012. The model premiums are thus computed using different sets of parameters in comparison with those of the previous subsection. The out-of-sample measures are performed on 1-, 3- and 7-year CDS premiums.

Table 8 shows how many observations were removed from our analysis. In total, 75 observations were removed (0.015% of our initial sample).

Figure 12 shows the out-of-sample performance of the four models considered. For every maturity and every period, RS outperforms the three other models. For both IG and HY firms, the average SSE/SST is about 2.66% across the three unused tenors. This ratio is higher by factors of 2.06, 2.67 and 2.20 for reduced-form, structural, and “one-regime” models respectively. The “one-regime” equivalent and the structural model seem to have the worst out-of-sample performances according to this test; however, the performance

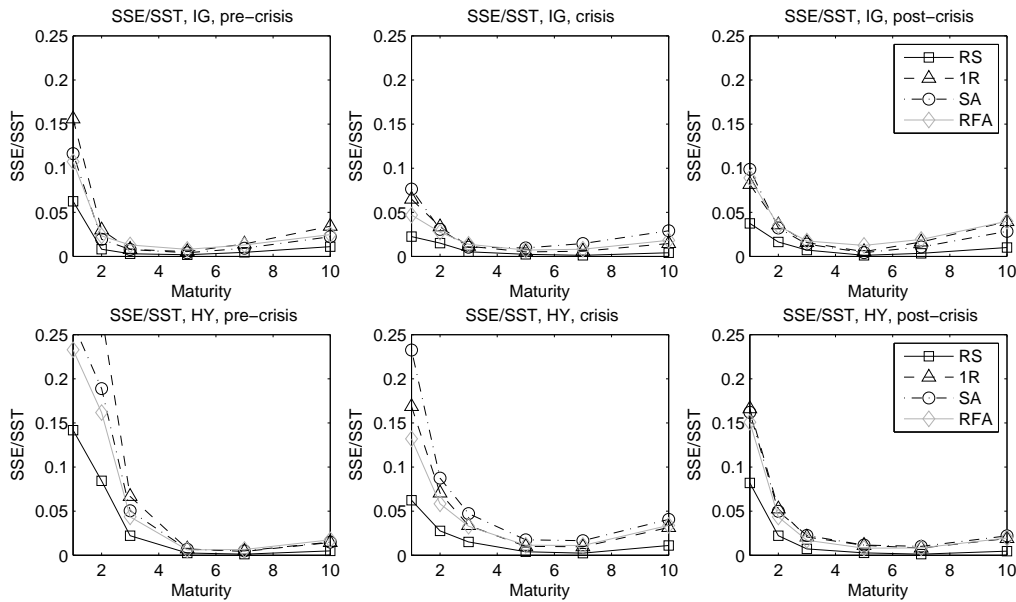


Figure 11: SSE/SST: in-sample performance for IG and HY.

These figures show the SSE/SST ratio calculated by maturity and period (i.e. pre-crisis, crisis, post-crisis). For each firm, a single set of parameters is estimated using CDS premiums with maturities of 1, 2, 3, 5, 7 and 10 years between January 2005 and December 2012.

Table 8: Number of outliers: first out-of-sample study.

	RS	1R	SA	RFA	Total
Number of outliers	8	33	47	6	75

[1] The initial sample size is 489,796 CDS premiums. There are 244,896 observations that are truly out-of-sample, and 244,900 observations used in the estimation.

of these models is not so dramatic. The reduced-form model performs very well in general. Performances seem to be somewhat constant within both risk classes.

Even for 1-year CDS premiums (which were not included in our estimation sample, and are known for being hard to price), the full model outperforms the other models and yields small SSE/SST ratios.

D.4 Forecasting 2013

In a second out-of-sample study, the parameters are estimated, again using 1-, 2-, 3-, 5-, 7- and 10-year CDS premiums from January 2005 to December 2012. Then, CDS premiums observed in 2013 are used to evaluate the out-of-sample fit of the model. The out-of-sample measures are calculated for every CDS premium observed in 2013.

As before, observations satisfying the rejection criterion of (18) are removed from the analysis. The number of these outliers is reported in Table 9. Overall, 41 out of 67,373 observations are discarded since they are considered to be outliers by at least one of the four models.

Figure 13 shows that our new model produces adequate one-week-ahead forecasts when compared to the benchmarks. The full model's curves seem to be lower than the other ones for almost every maturity and risk class, which is good.

For IG, SSE/SST ratios seem to be higher for short maturities. The main reason for this behaviour is that 1-year SST is much smaller than the other SSTs, even though the 1-year SSE is somewhat smaller than

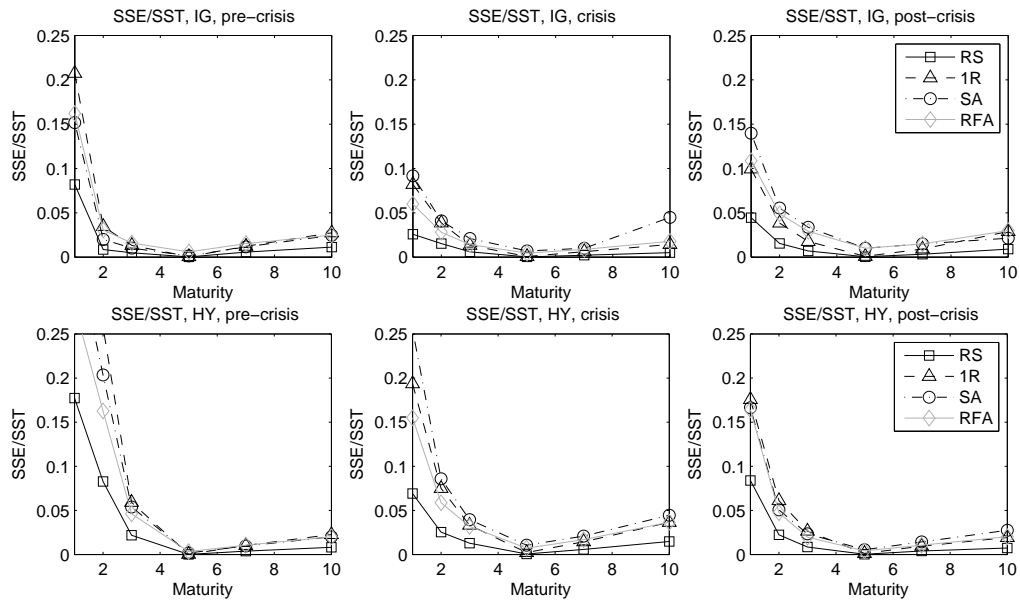


Figure 12: SSE/SST: out-of-sample performance for IG and HY. These figures show the SEE/SST calculated by maturity and by period (pre-crisis, crisis, and post-crisis). For each firm, a single set of parameters is estimated using CDS premiums with maturities 2, 5 and 10 years between January 2005 and December 2012.

Table 9: Number of outliers: second out-of-sample study.

	RS	1R	SA	RFA	Total
Number of outliers	4	13	29	16	41

[1] The initial sample size is 67,373 CDS premiums.

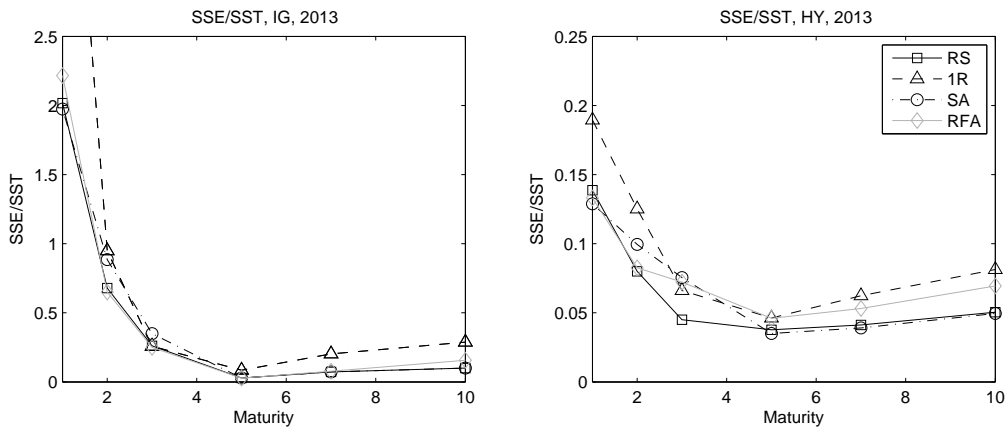


Figure 13: SSE/SST: out-of-sample performance for forecasted premiums in 2013. For each firm, a single set of parameters is estimated using CDS premiums with maturities 1, 2, 3, 5, 7 and 10 years between January 2005 and December 2012. Keeping the parameters constant, a one-week-ahead forecast of the CDS premium is compared to the realized premium for each firm, each maturity and each week of 2013.

those computed for other maturities. Therefore, it is natural to observe high SSE/SSTs for 1-year credit default swaps in 2013.

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