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Abstract: This report proposes two electricity pricing strategies for the Major of an American town of 16,000 people. An implementation with the AIMMS software is presented along with an interface that was built to be used by people not familiar with optimization. This work was awarded second place in the 6$^{\text{th}}$ AIMMS-MOPTA Optimization Modeling Competition in 2014.

Résumé : Ce rapport propose deux stratégies de fixation des prix de l'électricité pour un maire d'une petite ville américaine de 16 000 habitants. Une implémentation avec le logiciel AIMMS est présentée, ainsi qu'une interface graphique simple d'utilisation à l'usage de personnes non expertes en optimisation. Ce travail a remporté la deuxième place à la 6$^{\text{e}}$ compétition d'optimisation et de modélisation AIMMS-MOPTA en 2014.

1 Problem 1: Determining price based on available data

This section will examine our approach for the problem presented in Section 1.1 of the AIMMS Competition Outline. The objective of this problem is to determine a pricing strategy for the Major to use that satisfies the constraints he has laid out. The basic idea of our approach is to estimate the price-consumption relationship with a piecewise linear function for each time period and each class. The piecewise linear interpolation is calculated in AIMMS for each class of customer and each hour of the day using an optimization problem. After that, the consumption for each customer is generated with a normal distribution from the approximate consumption of the class it belongs to. The result is the price that the Major should charge each customer for each time of the day and the estimated consumption level (also for each customer and each time of the day) that he can expect.

The remaining of this section is divided as follows: Section 1.1 will describe the assumptions and modeling decisions we made; Section 1.2 describes the process we use to solve the problem including the two types of optimization models solved; Section 1.3 presents the solution we found and Section 1.4 presents an analysis of both our process and solutions.

1.1 Modeling decisions and assumptions

Certain assumptions and modeling decisions have been made to the model presented in the AIMMS Competition Outline. These assumptions/changes are: varying individual customer consumption around the class consumption average, constructing a piecewise linear relationship between consumption and price, bounding price and discretizing price. These modeling decisions are discussed in depth in the sections that follow.

1.1.1 Customer classes and individual consumptions

Five classes of customers have been defined by the Major. Let Φ be a mapping function which returns the class $\Phi(i)$ of each customer i. In addition, we assume that all customers within a customer class follow a class consumption profile and we assume that the consumption of a customer in a class is a random variable following a normal distribution around this class consumption profile. Thus, the consumption of the customer i belonging to the class $\Phi(i)$ at the time period t_j can be decomposed on the following form:

$$\widetilde{C}^p_{i,t_j} = \widehat{C}^p_{\Phi(i),t_j} + X_{i,t_j}$$

where p is the price, $\widetilde{C}^p_{i,t_j}$ is an estimation of the consumption of a customer i at the time period t_j when the price is p, $\widehat{C}^p_{\Phi(i),t_j}$ is the approximation of the consumption for the class $\Phi(i)$ at the time period t_j when the price is p, and X_{i,t_j} is a random variable which represents the deviation in kWh from the class consumption average. This random variable X_{i,t_j} follows a truncated normal distribution $\mathcal{N}(0, \sigma(\Phi(i), j)))$ such that $\widehat{C}^p_{\Phi(i),t_j} + X_{i,t_j} \geq 0$. The standard deviation $\sigma(c, j)$ is equal to 30% of the consumption at the price of $\$0.20/kWh$. This value of 0.30% is the one observed for small residence customer in [4]. As we did not find reliable informations about the other classes we took the same value for each class, but it could be improved.

In the preprocessing done prior to solving the model we define the consumption $\widehat{C}^p_{\Phi(i),t_j}$ of each class at each time period and then add the random part which will individualize each customer.

At each time period t_j the price (p_{c,t_j}) is the same for every customer in a class c. We denote $\mathcal{S}$ as the set of customer classes ($|\mathcal{S}| = 5$). The following implications result from the assumption that the consumption of each customer is under the form $\widehat{C}^p_{\Phi(i),t_j} + X_{i,t_j}$ and that the prices p_{i,t_j} are equal for all customers within a class:

- The variable $\widehat{C}^p_{\Phi(i),t_j}$ and p_{i,t_j} only need to be included once for each customer class.
- As opposed to summing over all customers in the objective function we can sum over each customer class and multiply this by the number of customers in that class. Let N be the total number of customers

in all classes and $\mathcal{I}^c := \{i \mid \Phi(i) = c\}$ be the subset of customers that are in class c. Hence, $|\mathcal{I}^c|$ is the number of customers in class c, and we then have:

$$\sum_{i=1}^{N}\sum_{j=1}^{m}\widetilde{C}^{p}_{i,t_j}p_{\Phi(i),t_j} = \left(\sum_{c\in\mathcal{S}}\sum_{j=1}^{m}|\mathcal{I}^c|\widehat{C}^{p}_{c,t_j}p_{c,t_j}\right) + \left(\sum_{i=1}^{N}\sum_{j=1}^{m}X_{i,t_j}p_{\Phi(i),t_j}\right)$$

Figure 1 shows an example of how the consumption of small residence customers is distributed at 6:00 a.m. and 8:00 p.m. We randomly generated the deviation using the process described above and the normal function in AIMMS. The bars show the number of customers that have a consumption within each range.

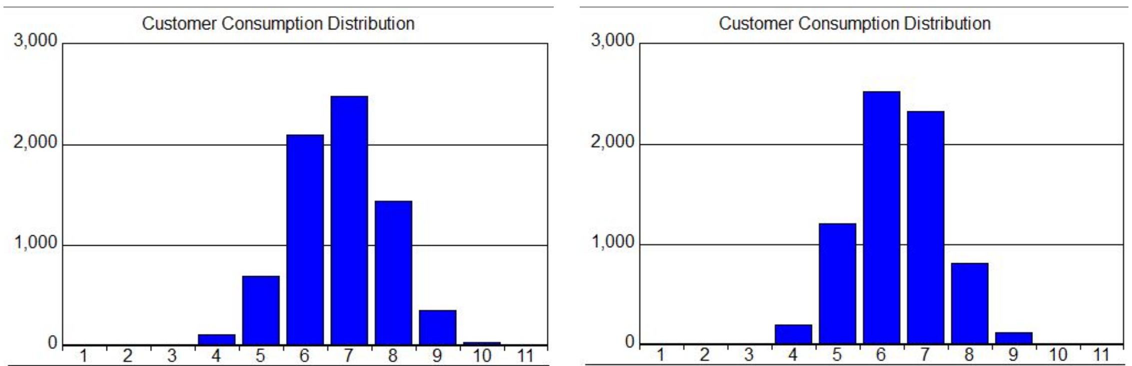

Figure 1: Example of individual small residence consumption at 6:00 a.m. and 8:00 p.m. for problem 1

1.1.2 Consumption-price relation

The relationship between consumption and price is complicated. Our decision was to model the relationship of consumption vs price with piecewise linear functions. The consumption-price data was provided in five spreadsheets. Figure 2 shows the consumption at each price for a standard small residential customer with each curve represents one of the 24 time periods.

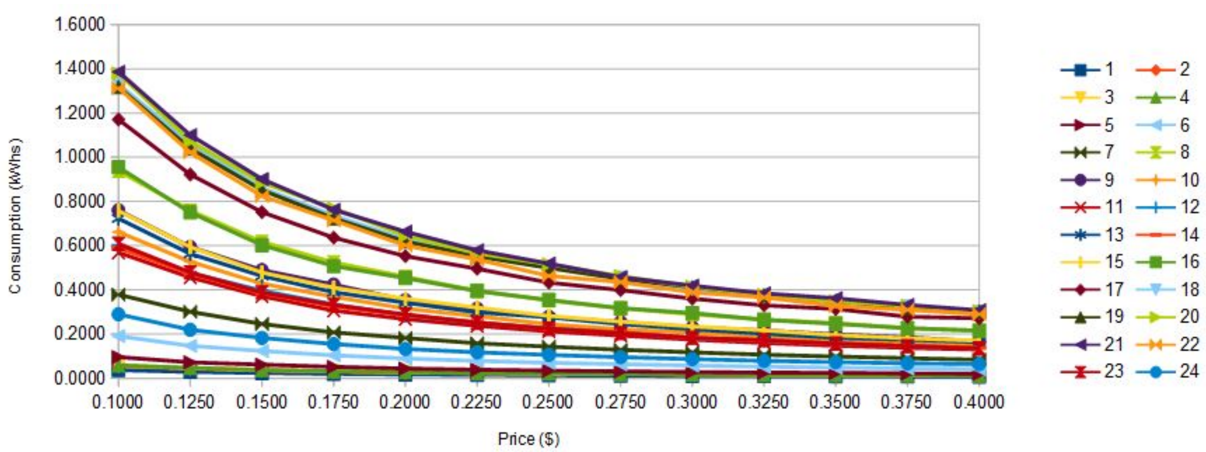

Figure 2: Relationship between price and consumption for small residents

Examining the data provided we observed one half of a near quadratic relationship between consumption and price, however, AIMMS was unable to find feasible solutions to this non-linear problem (specifically the program status was 'locally infeasible'). As a result we further simplified the model and assumed a piecewise linear relationship. We then decided to approximate the consumption-price curves by using a piecewise linear function. We decide that each piecewise function will have 4 pieces meaning that for each customer class at each time period we are approximating the consumption-price data by 4 linear functions. The 4 pieces are defined by the ranges: $p \in [0.1, 0.15]$; $p \in [0.15, 0.225]$; $p \in [0.225, 0.325]$ and $p \in [0.325, 0.4]$.

Figure 3 shows two specific curves (price-consumption data from small residence at 6:00 a.m. and 8:00 p.m.) and their respective piecewise approximation.

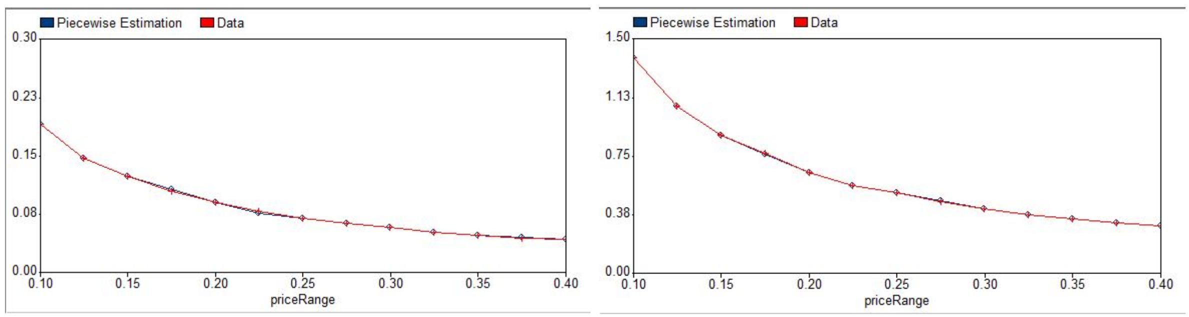

Figure 3: Piecewise estimation for small residence at 6:00 a.m. ($j = 6$) and 8:00 p.m. ($j = 20$)

Note that the curves for all customer classes in $\mathcal{S}$ and all time periods t_j, $\forall j = 1, \dots, m$ follow a similar pattern, therefore we apply the piecewise linear relationship to all 5*24=120 curves. The piecewise linear relationship is a linear relationship on each portion b, and is modeled as follows:

$$\text{If } p_{i,t_j} \in \mathcal{P}^b, \text{ then } \alpha^b_{i,t_j} p_{i,t_j} + \beta^{i,b}_{t_j} = \widehat{C}^{p_{\Phi(i),t_j}}_{\Phi(i),t_j} \;\; \forall c \in \mathcal{S}, \forall j = 1, \dots, m$$

where $p_{\Phi(i),t_j}$ is the price for the class $\Phi(i)$ at time t_j. Appropriate values of α^b_{i,t_j} and $\beta^{i,b}_{t_j}$ are found by solving an optimization model that minimizes the total absolute error between the piecewise function and the real data. The details are discussed in Section 1.2.1.

1.1.3 Price bounds

Bounds were added to the price because the further the price deviates from the range used in the provide data the less valid the piecewise approximation becomes. Specifically since the data provided only varied in price from \$0.10 to \$0.40 our piecewise functions are only valid within that range. Outside this range we have no data to describe how price and consumption relate. For example when the price approaches 0 there would likely be very strange behaviour in the actual level of consumption the Major could expect. In addition, since price and consumption are inversely related at some point the piecewise function would conclude that consumption becomes 0. This is also not practical as energy consumption is in integral part of our society at the moment and unlikely to disappear.

1.1.4 Discretization of price

As opposed to allowing the price that the Major will charge to be continuous we consider a finite number of potential prices. We discrete price between the minimum and maximum price provided with a step size of \$.005/kWh. We chose these values to balance the accuracy of the model with computational time. Increasing the number of potential prices increases the computational complexity (and therefore the run time), however having too few price options limits the accuracy of the result and may limit the Majors' revenue. In addition, this discretization will be easy for the customers to understand. Having a price with many decimals will be inconvenient during the billing process. We believe 3 decimals is much more manageable, especially for the residential consumers. It may be possible to change the step size (possibly to \$.001/kWh) as software improves or if the Major is willing to invest more computational time.

1.2 Process

This section will describe the algorithm we used to solve the problem. The basic idea of our method is to approximate the relationship between consumption and price with a piecewise regression model and then use this as parameters within the main model. The following outlines the steps within the process:

1. Define the relationship between price and consumption for each customer class and for each time period by solving an optimization problem. We assume a piecewise linear relationship See Section 1.1.2 for details.
2. Discretize price and for each price find the approximate consumption using the piecewise functions found above. Approximate consumption is found for each customer class at each time period for each price. See Section 1.1.2 for details.
3. The consumptions found in the previous step are considered averages as they are approximations of a representative customer. For each customer we estimate their consumption deviating around this average. See Section 1.1.1 for details.
4. Using an optimization model we select the price that each customer class should be charged to maximize revenue. Section 1.2.2 presents the optimization model and describes how binary variables are used.

1.2.1 Piecewise estimation model

Within this section we describe the linear piecewise model that is used to approximate the relationship between consumption and price. We begin by defining the notation used:

Parameters:

$\mathcal{P} :=$ $\{0.1, 0.125, 0.15, 0.2, 0.225, 0.25, 0.275, 0.3, 0.325, 0.35, 0.375, 0.4\}$ is the set of prices considered.

$\bar{C}^{p}_{c,t_j} :=$ the consumption for customer class c, at time t_j and price p. This is the exact information provided in the spreadsheets provided with the AIMMS Competition Report.

$\mathcal{P}^b :=$ is the range of prices over which the b$^{\text{th}}$ piece of the piecewise function is used to estimate the consumption. (Note $\mathcal{P}^b \subseteq \mathcal{P}\ \forall b = 1, \ldots, 4$).

Variables:

$\Delta_p :=$ the error between the real value and our approximation at price p

$\Delta^+_p :=$ the absolute value of the error between the real value and our approximation at price p

$\alpha^b_{c,t_j}, \beta^b_{c,t_j} :=$ coefficients of the b$^{\text{th}}$ piece of the piecewise linear estimation of consumption for customer class c at time t_j

The following model finds the linear regression line ($\alpha^b_{c,t_j} p + \beta^b_{c,t_j}$) that minimizes the error from the given value of consumption ($\bar{C}^p_{c,t_j}$). The model is solved for each customer class and each time period and the optimal solution ($\alpha^b_{c,t_j}, \beta^b_{c,t_j}$) defines the linear equation of the b$^{\text{th}}$ piece in the piecewise function. Note that the last two constraints and the minimization imply that $\Delta^+_p = |\Delta_p|$.

$$
\begin{aligned}
(\alpha^b_{c,t_j}, \beta^b_{c,t_j}) = \min\ & \sum_{p \in \mathcal{P}} \Delta^+_p \\
\text{s.t. }\ & \alpha^b_{c,t_j} p + \beta^b_{c,t_j} - \bar{C}^p_{c,t_j} = \Delta_p \qquad \forall p \in \mathcal{P}^b, \forall b = 1, \ldots, 4 \\
& \Delta^+_p \geq \Delta_p \qquad \forall p \in \mathcal{P} \\
& \Delta^+_p \geq -\Delta_p \qquad \forall p \in \mathcal{P}
\end{aligned}
$$

1.2.2 Main model

This section will define the optimization model we use to find the prices and describes the simplifications and assumptions that have been made from the non-linear model presented in Section 1.1 of the AIMMS Competition Outline. We begin by presenting the notation used within the model:

Parameters:

$N :=$ the total number of customers ($N = 16,000$)

$\mathcal{I} :=$ the set of customer ($|\mathcal{I}| = N$)

$\mathcal{P} := \{.1 : .005 : .4\}$ is the set of prices that will be considered.

$\mathcal{S} :=$ the set of customer classes ($|\mathcal{S}| = 5$)

$\Phi(i) :=$ the class of customer i. Therefore $\Phi(i) \subset \mathcal{S}$ and $|\Phi(i)| = 1$ (since each customer only belongs to one class).

$\alpha^b_{c,t_j}, \beta^b_{c,t_j} :=$ coefficients of the b$^{\text{th}}$ piecewise model that defines the linear relationship between price and consumption at time t_j for customer class c

$\widetilde{C}^p_{i,t_j} :=$ the estimated consumption of customer i at time t_j when the price is p.

$\bar{C}^{p^a}_{c,t_j} :=$ the consumption for customer class c, at time t_j and the current price p^a. This is the exact information provided in the spreadsheet provided with the AIMMS Competition Report.

The following parameters are defined as in the competition report: $m, t_j, \mu^+, \mu^-, W_i, \gamma, \mathcal{S}_1, \mathcal{S}_2$.

$\mathcal{P}^b\ \forall b = 1, \ldots, 4$ is defined as in the previous section, however note that $\mathcal{P}$ is defined differently. Recall that α^b_{c,t_j} and β^b_{c,t_j} are parameters in the main model, however they are variables in the piecewise linear problems (see Section 1.2.1).

Variables:

$$v^p_{c,t_j} := \begin{cases} 1 & \text{if all customers in class } c \text{ are charged price } p \text{ at time } t_j \\ 0 & \text{otherwise} \end{cases} \qquad \forall c \in \mathcal{S}, \forall j = 1, \ldots, m, \forall p \in \mathcal{P}$$

The main mathematical model is as follows:

$$\max \sum_{i=1}^{N} \sum_{j=1}^{m} \sum_{p \in \mathcal{P}} (t_j - t_{j-1})(v^p_{\Phi(i),t_j})(p)(\widetilde{C}^p_{i,t_j})$$

$$\text{s.t.} \quad \mu^- W_i \leq \sum_{j=1}^{m} \left((t_j - t_{j-1}) v^p_{\Phi(i),t_j} \widetilde{C}^p_{i,t_j} \right) \leq \mu^+ W_i \qquad \forall i \in \mathcal{I}$$

$$\sum_{i \in \mathcal{I}_1} \frac{1}{|\mathcal{I}_1|} \sum_{p \in \mathcal{P}} \left(v^p_{\Phi(i),t_j} p \right) \leq \gamma \sum_{i \in \mathcal{I}_2} \frac{1}{|\mathcal{I}_2|} \sum_{p \in \mathcal{P}} \left(v^p_{\Phi(i),t_j} p \right) \qquad \forall j = 1, \ldots, m$$

$$\sum_{p \in \mathcal{P}} v^p_{c,t_j} = 1 \qquad \forall c \in \mathcal{S}, j = 1, \ldots, m$$

$$v^p_{c,t_j} \in \{0, 1\} \qquad \forall c \in \mathcal{S}, j = 1, \ldots, m, p \in \mathcal{P}$$

$$\text{where } W_i = \left(\sum_{j=1}^{m} (t_j - t_{j-1}) \bar{C}^{p^a}_{\Phi(i),t_j} \right)$$

The last two constraints ensure that only one v^p_{i,t_j} variable will be nonzero for each customer class and each time period. Therefore the revenue of each class at each time period will equal $\sum_{p \in \mathcal{P}} (v^p_{\Phi(i),t_j})(p)(\widetilde{C}^p_{i,t_j})$. The objective function sums these revenues to get the total revenue. The first constraint models the consumption deviation relations from the model presented in Section 1.1 of the AIMMS Competition Report. The difference is that as opposed to using a variable consumption we use the parameter for estimated consumptions. Similar to the objective function $v^p_{\Phi(i),t_j}$ is used so that only the estimated consumption associated with the selected price will be nonzero. Similarly models the price equity.

1.3 Solution

The process presented in Section 1.2 is coded into AIMMS version 3.13. Figure 4 shows two of the page pages for problem 1.

The Major runs the aforementioned process on the page on the left. Following the steps on the left of this page the Major should begin by clicking 'Load Basic Data' to initialize many of the parameters. Note that

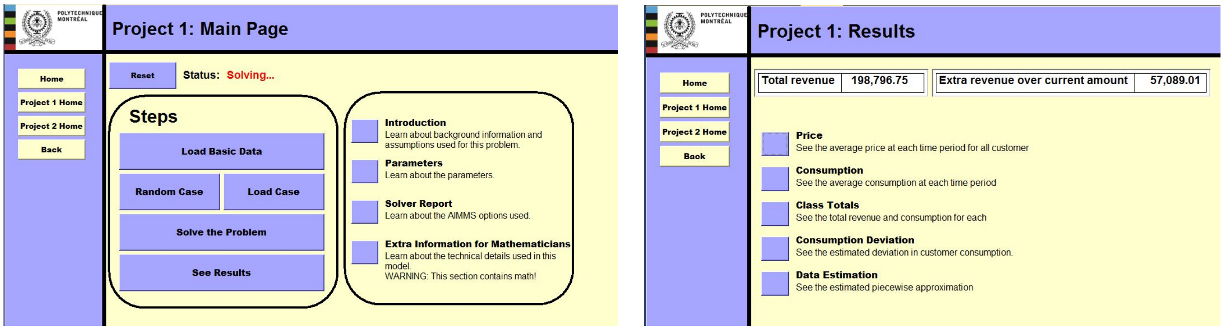

Figure 4: User interface: The main page for problem 1

this step also runs some pre-processing which includes the piecewise regression models that calculate α^b_{i,t_j} and β^b_{i,t_j}. The Major should then click either ‘Random Case’ to generate the random portion of the data related to the deviation of customer consumption or ’Load Case’ to load a pre-existing random case. Clicking ‘Solve the Problem’ runs the main optimization (and runs some post-processing to calculate values needed for the charts). Finally clicking ‘See Results’ takes the Major to a new page that summarizes the results. The ’Status’ of this page is currently solving the main optimization model. This status identifier will change between ’Processing...’, ‘Solving...’ and ‘Completed!” depending on the status. The page on the right is the main results page. It shows the total revenue (objective value of the main model) and the change in revenue the Major can expect. Clicking the links takes the Major to specific results.

The entire process takes approximately 15 minutes. It takes just over a minute to solve all of the piecewise linear models and 10.2 seconds to solve the main optimization model. The majority of the time is spent dealing with the large quantity of random data associated with the deviation in customer consumption.

1.3.1 Optimality

The model is a binary optimization model with linear constraints and solves to find an optimal solution (AIMMS:Program Status is ‘optimal’). This means that an integer solution was found and that it was the (or one of the) feasible integer solutions with the best objective value. It is important to note that the solution provided is optimal for the model we solved and not for the exact model provided in the AIMMS Competition Outline. We are not able to state if the actual optimal value from the model presented in the competition outline would be larger or smaller than our solution since our model is neither a relaxation (would result in a ‘better’ objective value) or a tightening (would result in a ‘worse’ objective value). Our model is an approximation as we approximate the relationship between consumption and price.

1.3.2 Parameter choice

The model has 5 parameters: γ, μ^-, μ^+ as defined in the AIMMS Competition Outline and $\underline{p}$ and $\bar{p}$ as the minimum and maximum value of the price. The values of the parameters are as follows:

$\gamma := 0.80$	$\mu^- := 0.80$	$\mu^+ := 4$	$\underline{p} := 0.10$	$\bar{p} := 0.40$

The choice of γ is such that the average price of small residential, large residential and office building and commercial customers will be at most 80% of the price paid by shift and no-shift industrial customers. The choice of μ^- is such that the consumption will not decrease too much from the current consumption. We found μ^+ had quite an impact on the model and in the end was selected so that the model was feasible. Recall that customer consumption is explicitly found for each potential price before the main optimization model is solved. This means that individual customer consumption is already bounded from deviating too much from an unrealistic value (specially it does not deviate much from the representative customer consumption at each time period). If the price drops one would expect consumption to increase and if the price drastically drops then customers could choose to significantly increase their consumption (for example they may buy a dishwasher or do laundry more often). The values we chose are the minimum and maximum price for which data is provided. Section 1.1.3 describes our reasoning in full.

1.3.3 Results

Using the parameters outlined in the previous section the total revenue (i.e. optimal objective value) of our approach is \$198,796.75. Comparing this with the current total price \$141,707.74 gives the Major a total gain of \$57,089.01. The exact prices and consumption for each customer class and each time period are shown in the Appendix (see Tables 1 and 2 respectively). Figure 5 shows the price and consumption for small residential customers. The results for the other 4 classes can be found under the 'Price' and 'Consumption' link of the interface respectively but have been omitted from this report due to space constraints.

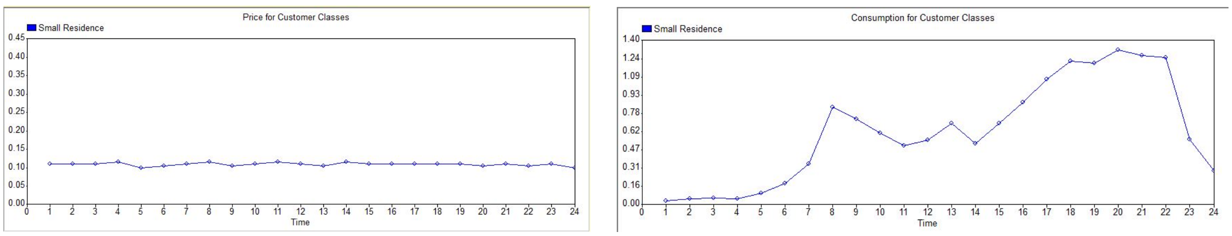

Figure 5: Small residence price and consumption results for problem 1 at each time period

1.4 Analysis of model and solution

1.4.1 Advantages and limits of the model

Advantages of the model:

- The range of prices within each customer class of the course of the 24 hours is small.
- Our model has linear constraints and a linear objective function.
- Our solution methodology uses an approximation of the relationship between consumption and price. However the piecewise approximate is a very close approximation of the real data. having all linear constraints has advantages.

Limits of the model:

- The price is different for each hour of the time period and for each customer class. This structure is not practical for the Major or for consumers, especially since the current system uses a single price of \$0.20/kWh. The Borough does not have the ability to track each customers' consumption, therefore it is fair to assume that tracking different prices for each hour would also be logistically difficult (if not impossible). In addition there are many concerns with how using different hourly prices would affect the customers. For example customers likely would not understand how these prices were calculated or why prices were higher at certain times and reporting the prices would be difficult in a single bill.
- The generation of the deviation consumption for each customer can be criticized. Indeed, we choose to generate a deviation different at each hour for each customer, whereas it could also be possible to generate a relative deviation for each customer for the entire day. However, this second approach assume that the deviation behaviour of a customer is similar during the entire day. This is a strong assumption, because it is possible that a customer who consumes more at one time of the day will use less electricity at a other time periods. Both approaches have advantages and limits and perhaps a discussion with the Major will help to make the choice he would prefer.
- After the random generation of consumption deviation the distribution of customer consumption does not always follow a normal distribution. An example is shown in Figure 6. This happens because some of the customer classes are very small. This only happens in the small sized classes: office building and commercial, shift industrial and no-shift industrial.

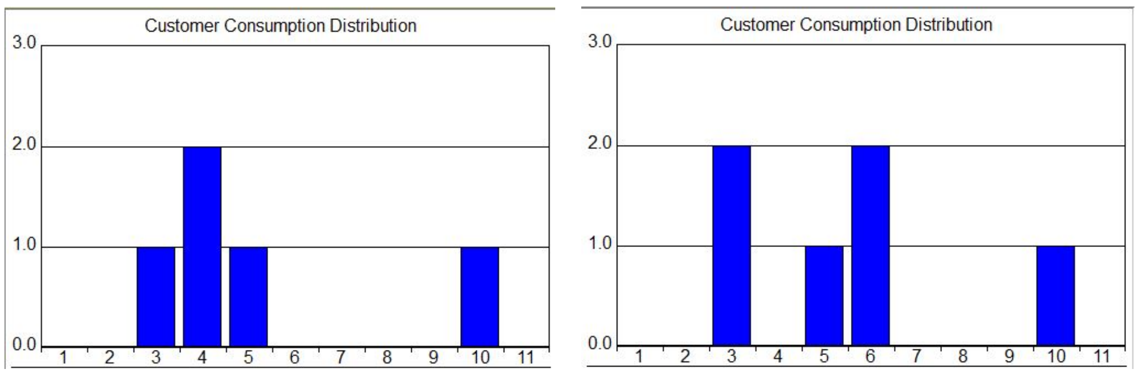

Figure 6: Example of individual consumption that does not follow a normal distribution

2 Problem 2: Determining price based on the monetary utility of the customers

This section will present our work for the problem from Section 1.2 of the AIMMS Competition Outline. A bilevel optimization problem is proposed based on the monetary utility of the customer. The basis of our approach is to decouple the inner problem from the outer problem. The inner problem is used to relate consumption to price and the newly reformulated outer problem selects the best pricing strategy. Section 2.1 presents the assumptions and modeling decisions we have made and includes some background information about electricity markets, usual prices and electricity meters (see Section 2.1.3). The process used and the newly proposed optimization model that simplifies the bilevel problem is discussed in Section 2.2. Finally, the solution we found and an analysis of both our process and solutions are in Sections 2.3 and 2.4 respectively. We begin with the following definitions that are essential to our approach:

- A **pricing policy** is a set of prices for each hour of the day. The π^{th} pricing policy is denoted as the vector $[p^{\pi}_{t_j}]_{\forall j}$.
- **Price-consumption strategy** defines the relationship between price and consumption based on customer class. It is a pricing policy and the consumption that would be expected.

2.1 Decisions and assumptions

This section describes the assumptions and modeling decisions that we have made. We assume that each customer in a class has an average consumption that is similar to the average consumption of the class. In addition we assume that each customers' consumption deviates from this average and that this deviation follows a truncated normal distribution. This is identical to the previous problem (see Section 1.1.1). We also make the same assumptions about the bounds on the price. Our additional assumptions/decisions are: how the bilevel model is separated; the discretization of the price; and the use of 'peaks' and the patterns to describe the price. These are discussed in depth in the sections that follow.

2.1.1 Separating the bilevel problem

Recall that the monetary utility has the form:

$$U_{ij}(x) = 1 - e^{-a_{ij}x}$$

where a_{ij} is such that $U_{ij}(\bar{C}^i_{t_j,p^a}) = u_i$ and u_i is defined as in the AIMMS Competition Outline (namely $u_i = 0.9$ for small residential customers, $u_i = 0.85$ for large residential customers, $u_i = 0.9$ for office building and commercial customers, $u_i = 0.75$ for shift industrial customers and $u_i = 0.8$ for no-shit industrial customers).

The utility function is used within the inner problem of the bilevel model defined in the AIMMS Competition Outline. Bilevel problems are difficult to solve therefore we made the modeling decision to separate the two parts. The inner problem assumes price is fixed and determines a customers' consumption (based on total price and monetary utility). The outer problem finds the optimal price using the consumption that would be determined by the inner problem. We simplify the model by not considering all prices in the outer model. First we pick a finite combination of prices called pricing policies and find the expected consumption. Then we solve the outer problem to select one of these pricing policies. We describe what pricing policies are and how we select them in Section 2.2.1. Note that the consumption is still the result of the inner optimization problem, however the inner problem can be solved separately (making the bilevel model no longer bilevel) and we are only considering a finite subset of possible prices. The outer problem now is to select from the different vectors with price and the corresponding consumption the one which maximizes the revenue. The details of the inner problem and the revised outer problem are discussed in Section 2.2.2 and 2.2.3 respectively.

2.1.2 Discretization of price

The reasons for discretizing price are similar to the previous problem (see Section 1.1.4). However for this problem the potential prices are limited to 13 values which are bounded between 0.10/kWh and \$0.40/kWh in increments of \$0.025. The approach for this question is more complicate than in the previous one and it was necessary to discretized with a larger step size.

2.1.3 Electricity market and peaks

In addition to discretizing price we assume the prices during a day follow a pattern. This section presents some background informations about the electricity market prices and explain our decision to categorize the price throughout the day and to define patterns. A short overview of some big American electricity distributor websites (PG&E, BGE, Portland General Electric) shows that in most of the cases residential and commercial building customers have a maximum of 3 different prices through out a day. These are categorized as off-peak, partial-peak and on-peak. Figure 7 shows a typical way that a 24 hour period is divided into these categories. In fact, as said in [2], most of the 'Domestic variable-rate meters generally permit two to three prices ("peak","off-peak" and "shoulder")'. As a result of these findings we have decided to have no more than 3 prices during the day for each customer.

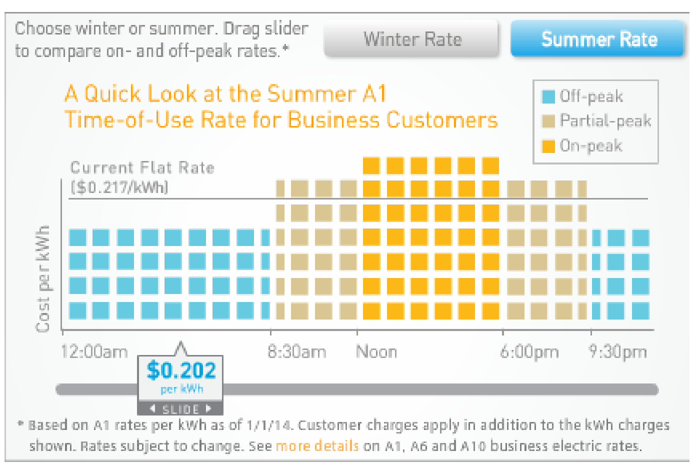

Figure 7: Different residential prices at PGE [1]

For industrial customer it seems to be more complicate, because the consumption is larger and having one price for every hour is common in practice. But it is also important to notice that applying more than 3 prices per day require specific equipment, which is very expensive, specially when amperage is high.

For small and big customers the cost of equipment and the installation of smart meter is huge. For example, in Montreal, Canada the company Hydro-Québec will spend \$845 CAD for the smart meter (including installation) for each residential customer [3]. The price for the equipment for high amperage (for industry customers) is even more expensive. This initial investment can not be ignored and reinforces our choice of three different prices per day for each class of customer.

As our model does not take into account the cost for the Major to buy electricity from producer (and the price is not the same at every hour) we will build our own period from the analysis of the consumption of the total of customers. The periods will be the same for every class of customers. Figure 8 shows the overall consumption at p^a as a function of the hour of the day. This overall consumption is computed from the provided data by calculating the total consumption for all of the 16,000 customers of the town.

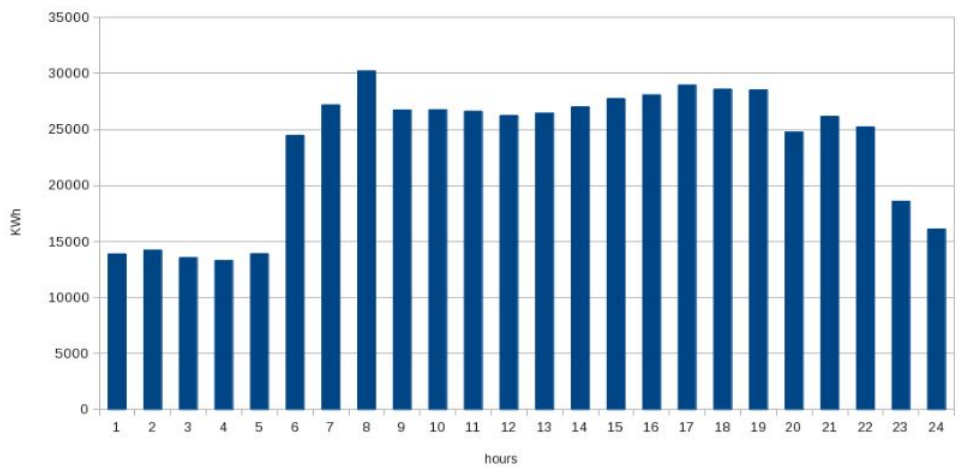

Figure 8: Overall Consumption

We have decided to have the following periods:

- On-Peak: from 6:00 a.m. to 8:00 a.m. and from 3:00 p.m. to 10:00 p.m.
- Partial-Peak: from 9:00 a.m. to 2:00 p.m.
- Off-Peak: from 10:00 p.m. to 5:00 a.m.

With this simplification the new problem is then to determine the prices for each period and each type of customer.

2.2 Process

Our model is based on generating price-consumption strategies solutions (which means price policies and the corresponding class consumptions). Specifically this means that the inner problem is rewritten as a set of potential solutions and then the outer problem selects the 'best' solution that also satisfies the other constraints. The set of potential solutions (called price-consumption strategies) is determined by solving the inner problem to find the consumption for varies prices (called pricing policies). The goal of the outer model is to select the best price-consumption strategy for each customer class i. The process we used for this problem is outlined below:

1. We determined an appropriate set Π of pricing policies to be considered. We examine the specifics of these pricing policies in Section 2.2.1.
2. The relationship between price and consumption is defined by solving the inner problem for each customer class $i \in \mathcal{S}$ and each pricing policies $p^{\pi}_{t_j}$ $\forall j = 1, \ldots, m$ to determine the expected consumption C^{π}_{c,t_j} $\forall j = 1, \ldots, m$ for customer class c and the π^{th} pricing policy. The combination of a pricing policy and related consumption is called a price-consumption strategy.
3. The main model is solved to determine which price-consumption strategy is used for each customer class $c \in \mathcal{S}$.

2.2.1 Determining pricing policies

The decision of which pricing policies to consider is critical since adding too many will make the problem computationally difficult yet adding too few will make the feasible region so small that the solution is very far from the true optimal and therefore not helpful for the Major to make accurate decisions.

We have decided that each pricing policies will have three different prices, one for each of the on-peak, partial-peak and off-peak periods in a day. The prices will vary between \$0.10/kWh and \$0.40/kWh in increments of \$0.025. Our strategy generates a total of $13*13*13 = 2197$ different pricing policies ($|\Pi| = 2197$), because we do not impose that the prices during on-peak are higher than during off-peak. As there is a finite number of vectors $p^{\pi}_{t_j}$ to test, we are able to compute each C^{π}_{i,t_j} by solving the inner problem.

2.2.2 Inner problem

The purpose of the inner problem is to define the relationship between price and consumption for each customer. Solving the inner problem for each individual customer would be too time consuming, instead we solve the inner problem for each class c and each pricing policy π to find an estimated 'average' consumption for class c when the prices are $p^{\pi}_{t_j}$ $\forall j = 1, \ldots, m$. We then deviate this consumption to find the expected individual customer consumption.

Parameters:

$p^{\pi}_{t_j}$:= the price at time t_j using the π^{th} pricing policy

The following parameters are defined as in the AIMMS Competition Outline: $m, t_j, \mu^+, W_i, \lambda_i, a_{ij}$.

Variables:

$\widehat{C}^{\pi}_{c,t_j}$:= the class consumption expected for the class c at time t_j if pricing policy $p^{\pi}_{t_j}$ is used.

The model (defined below) is solved for all $\pi \in \Pi$ and for all $c \in \mathcal{S}$.

$$\begin{aligned}
&\min \sum_{j=1}^{m} (t_j - t_{j-1}) \left(\widehat{C}^{\pi}_{c,t_j} p^{\pi}_{c,t_j} - \lambda_c \left(1 - e^{-a_{cj} \widehat{C}^{\pi}_{c,t_j}} \right) \right) \\
&\text{s.t.} \ \sum_{j=1}^{m} (t_j - t_{j-1}) \widehat{C}^{\pi}_{c,t_j} \leq \mu^+ W_c \\
&\text{where } W_c = \left(\sum_{j=1}^{m} (t_j - t_{j-1}) \bar{C}^{p}_{c,t_j} \right)
\end{aligned}$$

2.2.3 Main model

This section defines the main model that is used to select which price-consumption strategy the Major should pick for each customer class $c \in \mathcal{S}$. First we define the parameters and variables used within the model and then present the complete formulation.

Parameters:

C^{π}_{c,t_j} := the consumption expected for customer class c at time t_j if pricing policy $p^{\pi}_{t_j}$ is used.
X_{i,t_j} := is the deviation consumption for the customer i at the time period t_j.
$p^{\pi}_{t_j}$:= the price at time t_j for the π^{th} pricing policy
$\bar{C}^{p}_{c,t_j}$:= the consumption for customer class c, at time t_j at the current price p^a. This is the exact information provided in the spreadsheet provided with the AIMMS Competition Report.

The following parameters are defined as in the competition report: $m, t_j, \mu^+, \mu^-, W_i, \gamma, \mathcal{I}_1, \mathcal{I}_2$ and the following are defined as in Section 1.2.2: $\mathcal{S}, N$.

Variables:

$$v_c^\pi := \begin{cases} 1 & \text{if the } \pi^{\text{th}} \text{ pricing policy is used for class } c \\ 0 & \text{otherwise} \end{cases}$$

Note that since $\sum_{\pi\in\Pi} v_c^\pi = 1 \,\forall c \in \mathcal{S}$ and $v_c^\pi \in \{0,1\}\,\forall \pi \in \Pi,\, c \in \mathcal{S}$ then exactly one v_c^π equals 1 for each customer class and the rest equal 0. This corresponds to the one price-consumption strategy that is selected for each customer class.

We can then define the main problem as follows:

$$\begin{aligned} \max \; & \sum_{\pi\in\Pi}\sum_{j=1}^{m}\sum_{i\in\mathcal{I}} v^\pi_{\Phi(i)} \left(|\mathcal{I}^{\Phi(i)}| C^\pi_{\Phi(i),t_j} p^\pi_{t_j} + \sum_{i\in\mathcal{I}^c} X_{i,t_j} p^\pi_{t_j} \right) \\ \text{s.t.} \; & \sum_{\pi\in\Pi}\sum_{i\in\mathcal{I}_1} \frac{1}{|\mathcal{I}_1|} v^\pi_{\Phi(i)} p^\pi_{t_j} \le \gamma \sum_{\pi\in\Pi}\sum_{i\in\mathcal{I}_2} \frac{1}{|\mathcal{I}_2|} v^\pi_{\Phi(i)} p^\pi_{t_j} \qquad \forall j = 1,\ldots,m \\ & \sum_{\pi\in\Pi} v_c^\pi = 1 \qquad \forall c \in \mathcal{S} \\ & v_c^\pi \in \{0,1\} \qquad \forall c \in \mathcal{S},\, \forall \pi \in \Pi \end{aligned}$$

2.3 Solution

The process presented in Section 2.2 is coded into AIMMS version 3.13. Figure 9 shows two of the main pages for problem 2.

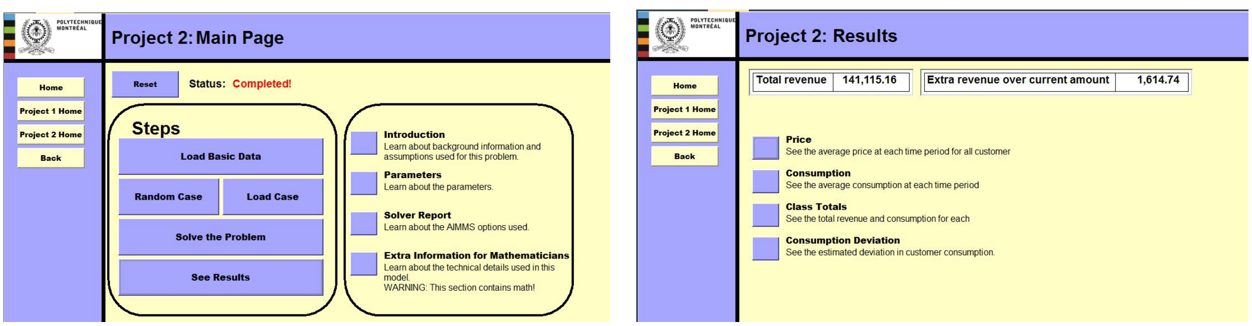

Figure 9: User interface: The main pages for problem 2

Similarly to problem 1, the Major follows the steps on the left of the left page to run problem 2. The Major should begin by clicking ‘Load Basic Data’ to initialize many of the parameters. Note that this step also runs some pre-processing which includes the generations of price policies and the inner optimization models that calculate price-consumption strategies. The Major should then click either ‘Random Case’ to generate the random portion of the data related to the deviation of customer consumption or ’Load Case’ to load a pre-existing random case. Clicking ‘Solve the Problem’ runs the outer optimization problem (and runs some post-processing to calculate values needed for the charts). Finally clicking ‘See Results’ takes the Major to a new page that summarizes the results. The ’Status’ of this page is currently not running anything and is waiting for the Major to select his next option. The page on the right is the main results page. Clicking the links takes the Major to specific results.

The entire process takes just over 5 minutes. It takes less than a minute to solve all the inner problems and 20.4 seconds to solve the outer optimization model. As in problem 1 the majority of the time is spent dealing with the large quantity of random data associated with the deviation in customer consumption.

2.3.1 Optimality

The model is a binary optimization model with linear constraints and solves to find a feasible integer solution (AIMMS: Program Status is ‘IntegerSolution’ and AIMMS:SolverStatus is ’ResourceInterrupt’). Since the integer solution is feasible but not guaranteed to be optimal we have a lower bound on the objective value of our outer optimization model. In addition, the process we used to simplify the model given in the AIMMS Competition Outline to get our outer optimization model is also a relaxation meaning that our solution is a lower bound on the actaul optimal value of the model in the competition outline. Note that our model is a relaxation since we are restricting the feasible region by only solving the inner problem for a finite number of prices. Combining these two facts means that the integer solution AIMMS found is a lower bound on the actual optimal value of the model in the competition outline.

2.3.2 Parameter choice

The model has 7 parameters: λ_i for each customer class, γ and μ^+. We chose the following values for these parameters:

$\lambda_{\mathrm{SR}} := 0.15$	$\lambda_{\mathrm{LR}} := 0.50$	$\lambda_{\mathrm{OB}} := 300$	$\lambda_{\mathrm{SI}} := 200$	$\lambda_{\mathrm{NSI}} := 2{,}000$	$\gamma := 0.80$	$\mu^+ := 1.2$

Let us define the policy π^a so that all prices are equal to p^a (i.e. $p^{\pi^a}_{t_j} = p^a \ \forall t_j \in \mathcal{T}$). We chose λ_i in such a way that for the policy π^a the consumption $C^{\pi^a}_{i,t_j}$ is approximately equal to the current consumption $\bar{C}^i_{t_j,p^a}$ (i.e. about the same order of magnitude).

2.3.3 Results

Using the parameters outlined in the previous section the total revenue (i.e. optimal objective value) of our approach is \$141,115.16. Comparing this with the current total price \$139,500.44 gives the Major a total gain of \$1,614.72. The exact prices and consumption for each customer class and each time period are shown in the Appendix (see Tables 3 and 4 respectively). Figure 10 shows the price and consumption for small residential customers. The results for the other 4 classes can be found under the 'Price' and 'Consumption' link respectively but have been omitted from this report due to space constraints.

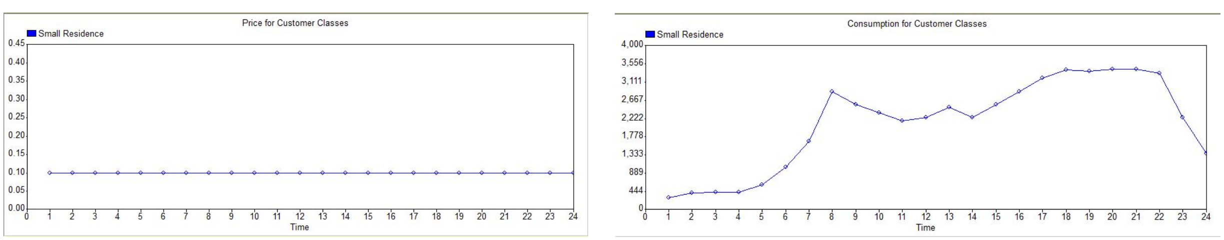

Figure 10: Small residence price and consumption results for problem 2 at each time period

2.4 Analysis of model and solution

2.4.1 Solution analysis

One can remark that the highest price in a day, for each customer, is not necessary during the on-peak time period. Although this is not realistic there is a mathematical explanation. Looking at the provided data we see that most of the time the consumption of a customer decreases when the price increases. However, the model of the monetary utility for the customers prevents the consumption from decreasing too much. Thus, the price can increase without resulting in a equivalent decrease in consumption (for example during the night for small residential customers). Since the total consumption is bounded within a range dependent on current consumption (deviation is no more than μ^+%) the total price is maximized by increasing price.

We can also observe that the price \$0.40/kWh appears very often (in more than 60% of the policy pricing). This results from the maximization of price (discussed above) being forced to the price upper bound. In addition with no lower bound on the consumption we observed that most of the time the consumption in the solution is lower than in the past (when the price is set at \$0.20/kWh).

Monetary utility (and the priority given to it) clearly affects the relationship between price and consumption. We examined what would happen if we changed this priority by changing the values of λ_i. If we decrease the value of λ_i then the prices of the optimal strategy decrease. This is because the weight given to the monetary utility decreases. However, consumption also decreases even for the current price \$0.20/kWh, this is not good as it does not reflect the data that is provided. Namely at the current price we know the current level on consumption.

2.4.2 Advantages and limits of the model

Advantages of the model:

- This model is more realistic than the one in question 1 because it takes into account the reaction of the customers.
- This approach is flexible for the Major. He has the possibility to test some other pricing policies and he does not necessary have to respect the periods. This would involving changes to the parameter used to define pricing policies in the AIMMS code.
- The strategies provided by this model are practical. Specifically, it is not necessary to adapt the equipment of the electricity network and easy to explain the changes to the customers.

Limits of the model:

- The Major is a distributor of electricity, not a producer. A more exact model would take into account the price of the energy that the Major must first buy. If this was done, the gains of the solution of question 2 would probably increase because (as described above) the consumption in the solution is less than in the reality, with a price of \$0.20/kWh. Moreover, he will then buy the electricity at different prices depending of the hours of the day. The objective function must take the cost of the electricity into account and most distributors vary their price by the time of day. The on-peak and off-peak will be more relevant if the price the Major has to pay also follows this pattern.
- The choice of the λ_i is very difficult to make and the choice of the model for the monetary utility can be questioned.
- As in 1.4.1, the way we generate the deviation can be improved.

3 Further recommendations

The two problems presented in the AIMMS Competition Outline highlight a key difficulty in the energy field, namely the difficulty in modeling the relationship between price and consumption. The true relationship is too complex to model exactly and therefore was simplified. In Section 1 we approximated the relationship between price and consumption with a piecewise linear model. In Section 2 we simplified the bilevel nature of price and consumption by solving the inner problem for a subset of prices to define the corresponding consumption. Further analysis in the following areas could help strengthen the Majors understanding of energy markets and help improve the way we model the situations:

- We highly recommend to the Major to include the cost of energy since the Major is likely not a producer.
- If the Major opts for the second approach with the monetary utility, we suggest to make a deeper analysis based on a market study and an analysis of the behaviour of the consumer to determine the model of this utility and the coefficients λ_i used to define the trade-off.

4 Appendix

Table 1: Price of energy in \$/kWh found in problem 1 (i.e. p_{i,t_j} at optimality)

Time Period	Small Residential	Large Residential	Office Building	Shift Industrial	No-Shift Industrial
1	0.11	0.4	0.4	0.4	0.4
2	0.11	0.4	0.4	0.4	0.4
3	0.11	0.4	0.4	0.4	0.4
4	0.115	0.4	0.4	0.4	0.4
5	0.1	0.4	0.4	0.4	0.4
6	0.105	0.4	0.4	0.4	0.4
7	0.11	0.4	0.4	0.4	0.4
8	0.115	0.4	0.4	0.4	0.4
9	0.105	0.4	0.4	0.4	0.4
10	0.11	0.4	0.4	0.4	0.4
11	0.115	0.4	0.4	0.4	0.4
12	0.11	0.4	0.4	0.4	0.4
13	0.105	0.4	0.4	0.4	0.4
14	0.115	0.4	0.4	0.4	0.4
15	0.11	0.4	0.4	0.4	0.4
16	0.11	0.4	0.4	0.4	0.4
17	0.11	0.4	0.4	0.4	0.4
18	0.11	0.4	0.4	0.4	0.4
19	0.11	0.4	0.4	0.4	0.4
20	0.105	0.4	0.4	0.4	0.4
21	0.11	0.4	0.4	0.4	0.4
22	0.105	0.4	0.4	0.4	0.4
23	0.11	0.4	0.4	0.4	0.4
24	0.1	0.4	0.4	0.4	0.4

Table 2: Expected consumption in kWh found in problem 1 (i.e. C_{i,t_j} at optimality)

Time Period	Small Residential	Large Residential	Office Building	Shift Industrial	No-Shift Industrial
1	0.03466	0.5486	118.4	135.6761	69.078
2	0.05142	0.5693	116.4	136.6176	69.7371
3	0.05412	0.5052	114.9	136.8869	71.8921
4	0.05204	0.495	115.8	133.3988	70.7779
5	0.0964	0.4779	156.9	144.3262	139.5847
6	0.18264	0.8586	708.9	150.6897	1057.1356
7	0.34794	0.8689	878.6	161.3149	1417.4577
8	0.83054	0.8305	902.6	169.8122	2598.3004
9	0.7271	0.5857	919.3	170.1174	2510.0339
10	0.60828	0.6045	920.1	182.4144	2523.8352
11	0.50148	0.5907	918.4	188.3838	2794.4265
12	0.55294	0.5735	921.1	207.9914	1062.8092
13	0.69112	0.5983	918.3	190.1684	1527.9656
14	0.52044	0.5972	939.3	193.1985	2378.8882
15	0.69242	0.6101	955.2	189.5228	2446.8136
16	0.87302	0.635	948.3	189.2962	1784.0074
17	1.07134	0.7054	934.2	186.6539	1068.5694
18	1.22334	0.7437	790.9	182.9272	715.366
19	1.2066	0.8028	648.8	200.0025	222.5659
20	1.31794	0.8784	156.8	174.6471	152.1223
21	1.2721	1.0034	147.6	164.9931	83.0425
22	1.25486	1.002	141.9	158.231	82.0534
23	0.55676	0.6916	132.8	152.2748	82.0542
24	0.29	0.6496	127.2	136.3711	85.1212

Table 3: Price of energy in \$/kWh found in problem 2 (i.e. p_{i,t_j} at optimality)

Time Period	Small Residential	Large Residential	Office Building	Shift Industrial	No-Shift Industrial
1	0.1	0.1	0.1	0.375	0.225
2	0.1	0.1	0.1	0.375	0.225
3	0.1	0.1	0.1	0.375	0.225
4	0.1	0.1	0.1	0.375	0.225
5	0.1	0.1	0.1	0.375	0.225
6	0.1	0.1	0.1	0.375	0.275
7	0.1	0.1	0.1	0.375	0.275
8	0.1	0.1	0.1	0.375	0.275
9	0.1	0.1	0.1	0.325	0.275
10	0.1	0.1	0.1	0.325	0.275
11	0.1	0.1	0.1	0.325	0.275
12	0.1	0.1	0.1	0.325	0.275
13	0.1	0.1	0.1	0.325	0.275
14	0.1	0.1	0.1	0.325	0.275
15	0.1	0.1	0.1	0.375	0.275
16	0.1	0.1	0.1	0.375	0.275
17	0.1	0.1	0.1	0.375	0.275
18	0.1	0.1	0.1	0.375	0.275
19	0.1	0.1	0.1	0.375	0.275
20	0.1	0.1	0.1	0.375	0.275
21	0.1	0.1	0.1	0.375	0.275
22	0.1	0.1	0.1	0.375	0.275
23	0.1	0.1	0.1	0.375	0.225
24	0.1	0.1	0.1	0.375	0.225

Table 4: Expected consumption in kWh found in problem 2 (i.e. C_{i,t_j} at optimality)

Time Period	Small Residential	Large Residential	Office Building	Shift Industrial	No-Shift Industrial
1	297.581475	9144.986114	1152.19171	5982.557322	1439.651804
2	411.4682759	9295.706435	1207.381481	5970.38703	1586.940241
3	423.0412819	8756.160018	1188.630957	5838.935242	1723.834359
4	423.2090345	8596.627101	1289.094547	5974.677661	1475.02733
5	612.1551965	8451.936114	1551.262771	6102.936147	2705.333671
6	1032.433225	11700.06626	3440.033011	5935.749916	10115.93892
7	1672.613184	11845.02618	5245.234704	6192.971493	11854.30928
8	2886.650893	11565.95435	5963.206444	6020.441914	15377.33209
9	2560.115565	9651.863731	5048.982991	7277.959184	13259.21805
10	2371.69883	9633.071986	3585.20993	7062.908252	9741.971162
11	2173.593211	9474.273358	6178.24184	7369.153579	17993.76742
12	2243.823879	9356.462661	4041.245718	7256.621363	9240.006109
13	2492.157805	9476.768428	4797.410223	7770.047557	9114.384477
14	2252.329575	9605.317644	3808.745758	7205.745573	12550.68749
15	2559.201127	9762.617564	5300.918352	6443.099756	16634.91959
16	2880.597988	9931.440845	4130.028288	5735.280124	13830.99288
17	3217.252422	10529.50732	5285.76649	7009.677302	7781.220366
18	3411.90412	10794.39367	4414.480091	5557.707434	7778.648759
19	3379.842612	11255.32451	3365.415462	6486.870976	3673.07883
20	3421.449836	11941.77494	1460.405188	6257.872045	2919.766837
21	3420.816342	12717.14162	1537.759474	5532.892096	1762.583856
22	3333.24215	12608.60885	1527.678829	5932.846732	1719.710798
23	2244.826059	10508.90411	1447.703455	5917.997623	1910.766817
24	1373.38011	10048.72667	1425.640624	6214.119869	1753.198241

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