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Incentive strategies for an optimal recovery program in a closed-loop supply chain

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Abstract: We consider a dynamic closed-loop supply chain made up of one manufacturer and one retailer, with both players investing in a product recovery program to increase the rate of return of previously purchased products. Product returns have two impacts. First, they lead to a decrease in the production cost, as manufacturing with used parts is cheaper than using virgin materials. Second, returns boost sales through replacement.

We show that the coordinated solution can be implemented by using so-called incentive strategies, which have the property of being best-reply strategies if each player assumes that the other is also implementing her incentive strategies. A numerical example illustrates the theoretical results.

Key Words: Closed-loop supply chain, coordination; incentive strategies, pricing, product recovery programs.

Résumé : Nous considérons une chaîne d'approvisionnement fermée composée d'un fabricant et d'un détaillant dans un contexte dynamique. Les joueurs investissent dans un programme de récupération de produits pour augmenter le taux de retour des unités précédemment achetées. Ces retours ont deux effets. Premièrement, ils permettent une économie de coût dans la mesure où la fabrication avec des pièces recyclées est moins chère que d'utiliser de nouveaux matériaux. Deuxièmement, ces retours induisent une augmentation de la demande de remplacement.

Nous montrons que la solution coordonnée peut être mise en œuvre en utilisant des stratégies incitatives qui ont la propriété d'être les meilleures réponses quand l'autre joueur implémente sa stratégie incitative. Nous illustrons nos résultats théoriques avec un exemple numérique.

Mots clés : Chaîne d'approvisionnement fermée, coordination, stratégies incitatives, tarification, programme de récupération de produits.

1 Introduction

It is by now well documented in many industries that using parts from returned products leads to substantial cost savings comparatively to using new materials. For instance, returns of car engines lead to a 70% cost savings at Volkswagen (Volkswagen (2011)), and using returned, previously purchased cameras allows a 40 to 60% cost saving at Kodak (Savaskan et al. (2004)). Similarly, Xerox saves 40 to 50% on its manufacturing costs by reusing parts, components and materials from returned products (Savaskan et al. (2004)). On a more macro scale, the remanufacturing sector in the US domestic steel industry reaches annual sales in excess of $53 billion (Lund (1996)), and Americans buy approximately 60 million remanufactured automotive products annually (Seitz and Peattie (2004)). Remanufacturing is only one of the many value-added reverse activities that a closed-loop supply chain (CLSC) undertakes. Other examples include, e.g., product acquisition, reverse logistics, recycling and re-marketing. For a comprehensive list and discussion of such activities, see Fleischmann et al. (2000), Guide and Van Wassenhove (2002) and Flapper et al. (2005).

When consumers recycle used products, they are likely to buy new ones. Based on data provided by a report of the Environmental Protection Agency (EPA (2011)), we obtain a correlation between returns and sales of over 90% for several electronic products (see Table 1). This return/repurchase relationship also applies to many other product categories such as printer cartridges, tires and toner to name a few. Of course, it may well be the case that that causality works in the opposite direction, i.e., that consumers are returning the used product because they need a new one. In any event, the point here is that if the rebuy rate is high, then the firm could engage in marketing activities and provide incentives to consumers to return products. One often-cited reason for returning used products is to repurchase greener products; see, e.g., Han et al. (2009), Ko et al. (2013) and Chen (2013). In such a context, there is an easy case to be made for marketing to invite consumers to even anticipate their rebuys.

Table 1: Correlation between purchases and returns in 2006–2010.

Product category	**Pearson Correlation**
Computers	0.94
Computer monitors	0.94
Hard-copy devices	0.93
Keyboards and mice	0.95
TVs	0.92
Mobile devices	0.97

Although situations where consumers who return/recycle products also buy new ones have been empirically observed (think of soda cans), the marketing aspects of recycling and remanufacturing have been overlooked in the CLSC literature. Indeed, research has focused on using operational levers (leverage?) to improve economic performance, more specifically by cutting costs through the remanufacturing of returns instead of using new raw materials, e.g., Savaskan et al. (2004), Savaskan and van Wassenhove (2006). Put differently, the implicit assumption is that neither the return rate nor the advantages of remanufacturing exert an impact on demand, which has mainly been modeled as a function of price only in, e.g., Debo et al. (2005), Guide et al. (2003), Savaskan et al. (2004), Ferguson and Tokay (2006), Atasu et al. (2005), Robotis et al. (2005), Savaskan and van Wassenhove (2005), Bakal and Akcali (2006), Ferrer and Swaminathan (2006), Hammond and Beullens (2007), De Giovanni and Zaccour (2013, 2014). A first contribution of this paper is in accounting for both the operational (cost-saving) and marketing (market-expansion) potential benefits of remanufacturing in a dynamic model of a CLSC. We assume that both members of the CLSC, i.e., the manufacturer and the retailer, invest in a product recovery program that aims at increasing returns by consumers.

When the main purpose of product recovery is reducing production costs, then the direct benefits go to the manufacturer, and the retailer has no direct incentive in contributing to a product recovery program. Of course, one can argue that if the production cost is lower, then this will be reflected in a lower wholesale price, resulting in the retailer also benefiting from product recovery. In any event, when returns boost demand in the retailer's outlet, then it becomes crystal clear to this player that it is in her best interest to participate in a product recovery program (PRP) to increase the return rate. Now, although both members of the CLSC are interested in the PRP, this does not imply that the manufacturer and the retailer will choose the optimal investment levels that would maximize the total chain profits. To increase the players' contributions in a decentralized supply chain, one needs to implement some incentive schemes. Here, the literature has focused on per-return incentives, that is, where the collector (manufacturer, retailer or third party) receives a per-return amount, which is often an exogenous parameter (e.g., Corbett and DeCroix (2001), Fleischmann et al. (2002), Savaskan et al. (2004), Ray et al. (2005), Bakal and Akcali (2006), Savaskan and van Wassenhove (2006), De Giovanni and Zaccour (2013, 2014)).

Considering a CLSC with one manufacturer and one retailer, we show that so-called incentive strategies lead to the implementation of the joint optimal solution in a dynamic game where the players invest in a PRP. The manufacturer chooses the wholesale price and her investment in the PRP, and the retailer selects the price to the consumer and also invests in the PRP. The idea of implementing some form of incentive strategy in dynamic supply chains (or marketing channels) has previously been pursued by Jørgensen and Zaccour (2003) and Jørgensen et al. (2006). In both papers, the (dynamic) pricing and remanufacturing were not an issue, and in Jørgensen et al. (2006) the incentive game was one-sided, that is, there was a leader in charge of designing the incentive. In our setting, we have a two-sided incentive problem. The implementation of incentive strategies in a CLSC and the optimality result constitute our second main contribution to this literature.

The rest of the paper is organized as follows. Section 2 introduces the models and notations. Section 3 characterizes the equilibria in those models and presents some results. Section 4 presents a numerical simulation to compare strategies and profits, and discusses some managerial implications. Section 5 provides some concluding remarks and suggestions for future research.

2 Model

We consider a closed-loop supply chain composed of one (re)manufacturer, player M; and a retailer, player R. Time t is continuous and the planning horizon is $[0,\infty)$. The manufacturer chooses the wholesale price, $\omega(t)$, and the retailer fixes the retail price to consumers, $p(t)$. Both players are actively engaged in product recovery programs (PRP) whose objective is to induce consumers to return past-purchased products (Guide and van Wassenhove (2009)). The rationale of the PRP is twofold. First, investments in PRP have a marketing purpose, since consumers who return the product are likely to replace it by buying a new one, and that boosts total demand, benefiting both members of the supply chain. Second, returns contribute to reducing the production cost.

Denote by $G_i(t)$ the investment level in PRP undertaken by player $i \in \{M,R\}$ at time $t \in [0,\infty)$. Examples of PRP activities include monetary and symbolic incentives to consumers to return used products, advertising and communications campaigns about recycling policies, logistics services, employee-training programs, etc. For a discussion of PRP activities, see, e.g., Fleischmann et al. (2001), Corbet and Savaskan (2003), Guide and Van Wassenhove (2009), Ko et al. (2009) and De Giovanni (2011). These investments influence the consumers' willingness to return products, represented by the return rate $r(t)$. The latter consists of the fraction of consumers who are willing to return past-purchased products directly to the manufacturer or to specific drop-off locations. For instance, Samsung has undertaken some collection programs (e.g., S.T.A.R.—Samsung Takeback and Recycle Program—www.samsung.com) by which consumers can return end-of-use toner cartridges through reusing the shipping box from new Samsung toner cartridges: customers simply print and attach a label to the box and drop it off at any U.S. postal office, and Samsung takes care of the shipping cost. Samsung invests in such programs to enhance consumers' willingness to return end-of-use toner cartridges, as a demonstration of their commitment to their sustainability policies. Therefore, the return rate $r(t)$ also represents a proxy for environmental performance, as it provides information about the percentage of the products that have been returned to manufacturers rather than dispersed into the environment.

We suppose that the return rate $r(t)$ depends on the whole history, and not only on the current level of green activities. A common hypothesis in such a context is to assume that $r(t)$ corresponds to a continuous and weighted average of past green activities with an exponentially decaying weighting function. This assumption is intuitive because the return rate is related to environmental awareness, which is a "mental state" that consumers acquire over time, not overnight. This process is captured by defining $r(t)$ as a state variable whose evolution is governed by the linear-differential equation

$$\dot{r}(t) = b_M G_M + b_R G_R - \delta r(t), \quad r(0) = r_0 \geq 0, \tag{1}$$

where $\delta > 0$ is the decay rate and represents the proportion of consumers who forget to return end-of-use products; r_0 is the initial rate of return and consists of the initial fraction of consumers willing to voluntarily return products, i.e., independently of the firm's PRP investments; $b_i > 0$, $i \in \{M, R\}$ is the PRP effectiveness and measures the marginal impact of firms' PRP on the return rate.

We assume that the cost of PRP activities is given by the following increasing convex function:

$$C(G_i(t)) = \frac{\kappa_i (G_i(t))^2}{2}, \quad i \in \{M, R\}, \tag{2}$$

where $\kappa_i > 0$ is a scaling parameter. To save on parameters, we let $\kappa_i = 1$, $i \in \{M, R\}$. This normalization is by no means severe as the players are still asymmetric with respect to their green activities through the b parameters.

The demand for the product depends on the retail price and on the return rate, that is, $d(p, r)$, with $\frac{\partial d}{\partial p} < 0$ and $\frac{\partial d}{\partial r} \geq 0$. We adopt the following functional form:

$$d(p(t), r(t)) = (1 + \mu r(t))(\alpha - \beta p(t)), \tag{3}$$

where α and β are strictly positive parameters representing the market potential and consumers' sensitivity to price, respectively. Note that sales are reinforced by the return rate according to $\mu \geq 0$, which shows the capacity for market expansion. When $\mu \geq 1$, all consumers returning end-of-use products purchase one or more new goods. When $\mu \in (0, 1)$ only a fraction of consumers show some intention to repurchase. The case $\mu = 0$ corresponds to a setting where returns do not influence the current demand, and the demand function assumes the classical form that has been extensively used in CLSC research (e.g., Ferrer and Swaminathan (2006)), in which the outcomes of closing the loop have no effect on demand.

The second purpose of implementing a successful return policy relates to the traditional operational benefits that the manufacturer gains in production when using returns. The classical rationale behind the implementation of a CLSC is that making new products by means of returns is always cheaper than using virgin material (see, e.g., Majumder and Groenevelt (2001)). The related cost savings are a function of the return rate, as the operational advantages of remanufacturing depend on the number of past-sold products that are returned. Following Savaskan et al. (2003), Savaskan and van Wassenhove (2005), and De Giovanni and Zaccour (2013), we suppose that the unit-production cost is given by the downward sloping function

$$C(r(t)) = c_0 - c_r r(t), \tag{4}$$

where $c_0 > 0$ is the cost of producing one unit with only new raw materials, and $c_0 - c_r > 0$ is the unit cost when producing with only used materials, i.e., when $r(t) = 1$. We assume that returns can be remanufactured an infinite number of times; thus the cost savings do not depend on the number of times that returns have already been remanufactured. Remanufacturing activities, in fact, allow returns to be sold "as new" in the market, regardless of their condition. For instance, HP sells remanufactured products "as new" and uses the advertising slogan: "Sitting side-by-side, you can't tell the difference between a new HP product and a Renew product other than the serial number" (HP (2014)). The operational problem linked to remanufacturing is not about the number of times returns can be remanufactured but rather the remanufacturing operations required to bring the products' functionality to an "as new" state. This fact is linked to the type of returns that manufacturers get back. According to Guide and van Wassenhove (2009), returns can be be broken down into the following categories:

- Commercial returns are products that have barely been used and consumed, and are best reintroduced to the market as quickly as possible. The majority of these returns require only light repair operations (e.g., cleaning).

- End-of-use returns are products that have been intensively used over a period of time and may therefore require more extensive remanufacturing activities.
- End-of-life returns are predominantly technologically obsolete products and are often worn out. This makes parts recovery and recycling the only practical recovery alternatives.

According to the type of return, manufacturers decide on the number of recovery programs, remanufacturing operations and a commercial policy. Therefore, the assumption of an infinite number of remanufacturing operations as well as the findings of this research can be easily applied to commercial and end-of-use returns, whose "as new" functionality can be restored through some remanufacturing operations, while the operational efficiency depends on the type of return (e.g., commercial returns are cheaper to remanufacture than end-of-use returns). In our model, the operational efficiency is captured by $c_o - c_r$: high vs. low recovery values indicate remanufacturing of commercial vs. end-of-use returns.

Assuming profit maximization behavior, the objective functionals of the manufacturer and the retailer are as follows:

$$J_M = \int_0^{+\infty} e^{-\rho t} \left(\left((1 + \mu r(t)) (\alpha - \beta p(t)) \right) (\omega(t) - c_0 + c_r r(t)) - \frac{(G_M(t))^2}{2} \right) dt, \tag{5}$$

$$J_R = \int_0^{+\infty} e^{-\rho t} \left(\left((1 + \mu r(t)) (\alpha - \beta p(t)) \right) (p(t) - \omega(t)) - \frac{(G_R(t))^2}{2} \right) dt, \tag{6}$$

where $\rho > 0$ is the common discount rate. To wrap up, by (1) and (5)–(6) we have defined a two-player infinite-horizon differential game having one state variable (r) and four control variables (ω and G_M for the manufacturer, and p and G_R for the retailer). Note that the case were returns only affect the unit cost (no demand expansion) is obtained by setting $\mu = 0$. Similarly, if returns do not contribute to lowering the unit cost, then it suffices to take $c_r = 0$. From now on, we shall omit the time argument when no ambiguity may arise.

In the following section, we characterize the cooperative solution.

3 Vertical integration

Suppose that the two players of the supply chain are vertically integrated. By this, we mean that the pricing and green activities are determined such that they maximize the joint optimization objective of the two players, given by

$$J = J_M + J_R = \int_0^{+\infty} e^{-\rho t} \left((1 + \mu r) (\alpha - \beta p) (p - c_0 + c_r r) - \frac{G_M^2}{2} - \frac{G_R^2}{2} \right) dt.$$

Observe that the wholesale price has been cancelled out in J and therefore has to be determined ex post. We will come back to this issue later on.

The current-value Hamiltonian of this optimal-control problem reads as follows:

$$H^C = (1 + \mu r) (\alpha - \beta p) (p - c_0 + c_r r) - \frac{G_M^2}{2} - \frac{G_R^2}{2} + \lambda (b_M G_M + b_R G_R - \delta r),$$

where λ is the adjoint variable appended to the state dynamics in (1), and the superscript C stands for cooperation.

Assuming an interior solution, the first-order optimality conditions include

$$H_p^C = \alpha - \beta (2p - c_0 + c_r r) = 0, \tag{7}$$

$$H_{G_M}^C = -G_M + \lambda b_M, \tag{8}$$

$$H_{G_R}^C = -G_R + \lambda b_R, \tag{9}$$

$$\begin{aligned}
\dot{\lambda} &= (\rho+\delta)\lambda - (\alpha-\beta p)(\mu(p-c_0+2c_r r)+c_r), \quad (10)\\
\lim_{t\to\infty} e^{-\rho t}\lambda(t) &= 0, \quad (11)\\
\dot{r} &= b_M G_M + b_R G_R - \delta r, \quad r(0)=r_0. \quad (12)
\end{aligned}$$

The first three conditions are equivalent to

$$p = \frac{\alpha+\beta(c_0-c_r r)}{2\beta}, \quad G_M = \lambda b_M, \quad G_R = \lambda b_R. \quad (13)$$

Substituting for p, G_M and G_R in (10)–(12) leads to the following pair of differential equations:

$$\begin{aligned}
\dot{r}^C &= (b_M+b_R)\lambda^C - \delta r^C, \quad (14)\\
\dot{\lambda}^C &= (\rho+\delta)\lambda^C - \left(\frac{\alpha-\beta\left(c_0-c_r r^C\right)}{2}\right)\left(\frac{\mu\left(\alpha-\beta\left(c_0-3c_r r^C\right)\right)}{2\beta}+c_r\right), \quad (15)
\end{aligned}$$

with the boundary conditions $\lim_{t\to\infty} e^{-\rho t}\lambda(t)=0$ and $r(0)=r_0$. Inserting back λ^C and r^C in (13), we get the optimal retail price p^C and green activities G_M^C and G_R^C.

3.1 Optimal solution

Recall here that vertical integration results in an infinite-horizon, discounted, autonomous, one-state optimal-control problem. The optimality conditions are incomplete (see Section 3.7 in Grass et al. (2008)), as the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda^C(t)=0$ still does not help to single out an optimal solution. However, the optimal solution can be obtained by a phase-portrait analysis (see appendix) of the optimal system (14–15). In the appendix, we show that the above optimal vector field displays different qualitative behaviors, also called bifurcations, depending upon the parameter values. Firstly, we have the following remark, which rules out the possibility of a cycle as an optimal solution.

Remark 1 *It is well known that in infinite-horizon, discounted, autonomous, one-state optimal-control problems with $\rho>0$, cycles are ruled out as optimal solutions. See Proposition 3.83 in Grass et al. (2008).*

Next, we make the following assumption on the parameters and nature of the optimal solution of the system (14–15).

Assumption 1 *The state variable $r^C(t)$ takes values in $[0\ 1]$ for all t. Here, we seek optimal candidates where the state trajectory $r^C(t)$ starts in the set $[0\ 1]$ and stays in this set for all times.*

Remark 2 *If $\alpha-\beta c_0>0$, then the optimal demand is strictly positive at all times t. Indeed, at retail price*

$$p^C = \frac{\alpha+\beta\left(c_0-c_r r^C\right)}{2\beta},$$

we have

$$d\left(p^C(t), r^C(t)\right) = \left(1+\mu r^C(t)\right)\left(\alpha-\beta p^C(t)\right) = \frac{1}{2}\left(1+\mu r^C(t)\right)\left(\frac{\alpha-\beta c_0+\beta c_r r^C(t)}{2}\right),$$

which is strictly positive for $\alpha-\beta c_0>0$. This means that demand is positive if the price is equal to the initial (highest) marginal cost, which is a typical assumption in economics.

Under Assumption 1, the above remarks, and from the detailed bifurcation analysis of the optimal vector field, it is obvious that saddle points and their stable manifolds are identified as the most important candidates for optimal solutions. We have the following proposition regarding the solution of the optimal system (14–15). The proposition follows from the detailed bifurcation analysis of the system (14–15), which is provided in the appendix.

Proposition 1 *Let the following conditions on the parameters hold true:*

$$\Delta = \left(\frac{2\delta(\rho+\delta)}{3(b_M+b_R)\mu\beta c_r^2} + \frac{1}{3}\left[\frac{\alpha-\beta c_0}{\beta c_r} - \frac{1}{\mu}\right]\right)^2 - \frac{4\delta(\rho+\delta)}{9(b_M+b_R)\mu\beta c_r^2}\left(\frac{2}{\mu} + \left[\frac{\alpha-\beta c_0}{\beta c_r}\right]\right) > \frac{1}{4} \quad (16)$$

$$0 \leq \frac{2\delta(\rho+\delta)}{3(b_M+b_R)\mu\beta c_r^2} - \frac{1}{3}\left(\frac{1}{\mu} + 2\left[\frac{\alpha-\beta c_0}{\beta c_r}\right]\right) - \sqrt{\Delta} \leq 1. \quad (17)$$

Then, Assumption 1 holds true. Further, the optimal solution satisfies $(r^C, \lambda^C) \in W^s(E_1)$*, where* $W^s(E_1)$ *is the stable manifold of the saddle node* E_1*.*

Proof. From (34) we have $\Delta = n + \left(\frac{\delta(\rho+\delta)}{2ps}\right)^2 - \frac{\delta(\rho+\delta)m}{ps}$. If $\Delta > 0$ then there exists two equilibrium points for the optimal system (14–15). Further, the equilibrium points are separated by a distance $2\sqrt{\Delta}$ (see appendix) in the r^C axis. So, condition (16), i.e., $\Delta > \frac{1}{4}$, ensures the existence of two equilibrium points, which are separated by a distance equal to 1. Next, condition (17) ensures that the r^C coordinate of the first equilibrium point, which is a saddle node, lies in the interval [0 1] and the r^C coordinate of second equilibrium point, which is either a source node or source spiral, lies outside the interval [0 1]. This condition also ensures that the stable manifold $W^s(E_1)$ (see Figures 3(c) and 3(d) in the appendix) is well defined for all the initial points $r^C(0) \in [0\ 1]$. So, every trajectory $(r^C(t), \lambda^C(t))$ lying on the stable manifold $W^s(E_1)$ starting in the interval [0 1] will converge to the equilibrium point E_1, and as a result, the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda^C(t) = 0$ also holds. So, clearly Assumption 1 holds true. □

4 Incentive strategies

As the cooperative solution is not (in general) an equilibrium, it makes sense to argue that each player would like to be assured that if she implements her part of this solution, her partner will also. One way of realizing the desired cooperative outcome is to apply the concept of an incentive equilibrium. This concept was used in resource games by Ehtamo and Hämäläinen (1993, 1995), and by Jørgensen and Zaccour (2003a, 2003b) in marketing channels and retail promotions, respectively. The origin of the incentive equilibrium is in Stackelberg games, where the leader designs an incentive for the follower in a way that is desirable to the leader. An incentive equilibrium may be seen as a two-sided "Stackelberg incentive" problem: when one player implements her incentive strategy, the other player can do no better than to act in accordance with the agreement.

Define by

$$u_M = (\omega, G_M), \quad u_R = (p, G_R),$$

the control vectors of the two players, and by

$$\begin{aligned} \gamma_M(u_R) &= \left(\gamma_R^1(\omega), \gamma_R^2(G_M)\right), \\ \gamma_R(u_M) &= \left(\gamma_M^1(p), \gamma_M^2(G_R)\right), \end{aligned}$$

the manufacturer's and retailer's incentive strategies. Denote by U_M and U_R the set of feasible controls for the manufacturer and the retailer, respectively. (Feasibility in our setting is simply non-negativity of the control variables ω, p, G_M and G_R). Denote by Γ_M and Γ_R the set of feasible strategies for the manufacturer and the retailer, respectively, that is,

$$\Gamma_M = \{\gamma_M | \gamma_M : U_R \to U_M\}, \quad \Gamma_R = \{\gamma_R | \gamma_R : U_M \to U_R\}.$$

It is readily seen that each player's strategy is a function of the other player's control variables. We now define the incentive equilibrium at cooperative solution $\left(u_M^C, u_R^C\right)$, with $u_M^C = \left(\omega^C, G_M^C\right)$ and $u_R^C = \left(p^C, G_R^C\right)$.

Definition 1 *The strategy pair* $\gamma_M \in \Gamma_M, \gamma_R \in \Gamma_R$ *is said to be an incentive equilibrium at* $\left(u_M^C, u_R^C\right)$ *if*

$$\begin{aligned} J_M\left(u_M^C, u_R^C\right) &\geq J_M\left(u_M, \gamma_R(u_M)\right), \forall u_M \in U_M, \\ J_R\left(u_M^C, u_R^C\right) &\geq J_R\left(\gamma_M(u_M), u_R\right), \forall u_R \in U_R, \\ u_M^C &= \gamma_M\left(u_R^C\right), \quad u_R^C = \gamma_R\left(u_M^C\right). \end{aligned}$$

The above definition states that when the incentive strategies are jointly carried out, then the cooperative outcome is realized. Observe that the above definition could be written for any desired point, and not only for the cooperative controls. The two inequalities in the definition mean that when one player implements her incentive strategy, the best choice for the other player is then to implement her part of the cooperative solution.

Although different functional forms can be envisioned for incentive strategies, we keep things as simple as possible and retain the following linear specifications:

$$\begin{aligned} \gamma_M^1(p) &= \omega^C + \eta_M\left(p - p^C\right), && (18)\\ \gamma_M^2(G_R) &= G_M^C + \psi_M\left(G_R - G_R^C\right), && (19)\\ \gamma_R^1(\omega) &= p^C + \eta_R\left(\omega - \omega^C\right), && (20)\\ \gamma_R^2\left(G_M^C\right) &= G_R^C + \psi_R\left(G_M - G_M^C\right), && (21)\end{aligned}$$

where the incentive (penalty) coefficients η_M, ψ_M, η_R and ψ_R are positive time functions. The idea behind the above formulation is simple. Indeed, each player is letting her choice depend on the other player's controls. The manufacturer's strategies described in (18) and (19) state that if the retailer implements the cooperative retail price and PRP investment, then the manufacturer plays the cooperative wholesale price (yet to be defined) and PRP investment. The retailer's strategies in (20) and (21) have a similar interpretation. Note that the assumption here is that if the manufacturer cheats on the cooperative solution, then she will increase the wholesale price and decrease her investment in PRP, that is, $\omega > \omega^C$ and $G_M < G_M^C$, causing an increase in the retailer's price and a decrease in advertising. Similarly, if the retailer deviates from the desired strategy, then she will select a higher retail price and a lower advertising effort than under cooperation $\left(p > p^C, G_M < G_M^C\right)$. The manufacturer reacts by increasing the wholesale price and decreasing her advertising effort.

To determine the incentive coefficients, we have to solve an optimal-control problem for each player, assuming that the other player will indeed implement her part of the cooperative solution. Put differently, we characterize for each player her best response to the incentive strategies of the other player. The current-value Hamiltonians of the manufacturer and the retailer are given by

$$\begin{aligned} H_M^I &= (1+\mu r)\left(\alpha - \beta\left(p^C + \eta_R\left(\omega - \omega^C\right)\right)\right)\left(\omega - c_0 + c_r r\right) - \frac{1}{2}G_M^2\\ &\quad + \lambda_M\left(b_M G_M + b_R\left(G_R^C + \psi_R\left(G_M - G_M^C\right)\right) - \delta r\right),\\ H_R^I &= (1+\mu r)(\alpha - \beta p)\left(p - \left(\omega^C + \eta_M\left(p - p^C\right)\right)\right) - \frac{1}{2}G_R^2\\ &\quad + \lambda_R\left(b_M\left(G_M^C + \psi_M\left(G_R - G_R^C\right)\right) + b_R G_R - \delta r\right),\end{aligned}$$

where λ_M and λ_R are the adjoint (co-state) variables appended by the players to the state dynamics. Observe that in each player's Hamiltonian, we have substituted the incentive strategies for the other player's controls from (18)–(21).

Assuming an interior solution, the first-order optimality conditions for the manufacturer are given by

$$\begin{aligned} \frac{\partial H_M^I}{\partial \omega} &= \alpha - \beta\left(p^C + \eta_R\left(2\omega - \omega^C\right)\right) + \beta\eta_R\left(c_0 - c_r r\right) = 0, && (22)\\ \frac{\partial H_M^I}{\partial G_M} &= -G_M + \lambda_M\left(b_M + b_R\psi_R\right) = 0, && (23)\\ \dot{\lambda}_M &= (\rho+\delta)\lambda_M - \left(\alpha - \beta\left(p^C + \eta_R\left(\omega - \omega^C\right)\right)\right)\left(\mu\left(\omega - c_0 + c_r r\right) + c_r(1+\mu r)\right), && (24)\\ \dot{r} &= b_M G_M + b_R\left(G_R^C + \psi_R\left(G_M - G_M^C\right)\right) - \delta r, && (25)\end{aligned}$$

and those for the retailer by

$$\frac{\partial H_R^I}{\partial p} = \alpha(1-\eta_M) - \beta\left(2p - \left(\omega^C + \eta_M\left(2p - p^C\right)\right)\right) = 0, \quad (26)$$

$$\frac{\partial H_R^I}{\partial G_R} = -G_R + \lambda_R\left(b_R + b_M\psi_M\right) = 0, \quad (27)$$

$$\dot{\lambda}_R = (\rho+\delta)\lambda_R - \mu(\alpha - \beta p)\left(p - \left(\omega^C + \eta_M\left(p - p^C\right)\right)\right), \quad (28)$$

$$\dot{r} = b_R G_R + b_M\left(G_M^C + \psi_M\left(G_R - G_R^C\right)\right) - \delta r. \quad (29)$$

From (22)–(23) and (26)–(27), we get (superscript I stands for incentive)

$$\begin{aligned}
\omega^I &= \frac{\alpha - \beta p^C + \beta\eta_R\left(\omega^C + c_0 - rc_r\right)}{2\beta\eta_R}, \\
G_M^I &= \lambda_M^I\left(b_M + b_R\psi_R\right), \\
p^I &= \frac{\left(\alpha(1-\eta_M) + \beta\left(\omega^C - p^C\eta_M\right)\right)}{2\beta(1-\eta_M)}, \\
G_R^I &= \lambda_R^I\left(b_R + b_M\psi_M\right).
\end{aligned}$$

To determine the penalty coefficients, we equate the incentive strategies and optimal (cooperative) policies, that is,

$$\begin{aligned}
\omega^C &= \omega^I \Leftrightarrow \eta_R = \frac{\alpha - \beta\left(c_0 - c_r r^C\right)}{2\beta\left(\omega^C - \left(c_0 - c_r r^C\right)\right)}, \\
p^C &= p^I \Leftrightarrow \eta_M = \frac{2\beta\left(\omega^C - \left(c_0 - c_r r^C\right)\right)}{\alpha - \beta\left(c_0 - c_r r^C\right)}, \\
G_M^C &= G_M^I \Leftrightarrow \psi_R = \frac{b_M}{b_R}\frac{\left(\lambda^C - \lambda_M^I\right)}{\lambda_M^I}, \\
G_R^C &= G_R^I \Leftrightarrow \psi_M = \frac{b_R}{b_M}\frac{\left(\lambda^C - \lambda_R^I\right)}{\lambda_R^I}.
\end{aligned}$$

Observe that the penalty coefficients are time functions, and not constants that are determined once for the whole game. The interpretation of the PRP penalty coefficients ψ_R and ψ_M is straightforward. Indeed, each involves the ratio of efficiency coefficients of PRP activities in raising the return rate times the relative difference in the shadow prices of the returns, that is, $\frac{\lambda^C - \lambda_j^I}{\lambda_j^I}, j = M, R$. To interpret the price-penalty coefficients, we first observe that η_M can be rewritten equivalently as follows:

$$\eta_M = \frac{\omega^C - \left(c_0 - c_r r^C\right)}{\left(p^C - \omega^C\right) + \left(\omega^C - \left(c_0 - c_r r^C\right)\right)}.$$

Then, η_M is the ratio of the manufacturer's margin to the sum of retailer's and manufacturer's margins in the cooperative solution, that is, the manufacturer's margin share. Second, we note $\eta_R = \frac{1}{\eta_M}$, that is, the total margin divided by the manufacturer's margin share.

Substituting for the incentive strategies in the adjoint variables

$$\dot{\lambda}_M^I = (\rho+\delta)\lambda_M^I - \left(\alpha - \beta\left(p^C + \eta_R\left(\omega - \omega^C\right)\right)\right)\left(\mu\left(\omega - c_0 + c_r r\right) + c_r(1+\mu r)\right) \quad (30)$$

$$\dot{\lambda}_R^I = (\rho+\delta)\lambda_R^I - \mu(\alpha - \beta p)\left(p - \left(\omega^C + \eta_M\left(p - p^C\right)\right)\right), \quad (31)$$

we get

$$\begin{aligned}
\dot{\lambda}_M^I &= (\rho+\delta)\lambda_M^I - \left(\alpha - \beta p^C\right)\left(\mu\left(\omega^C - c_0\right) + c_r\right) \\
\dot{\lambda}_R^I &= (\rho+\delta)\lambda_R^I - \mu\left(\alpha - \beta p^C\right)\left(p^C - \omega^C\right).
\end{aligned}$$

Substituting for the cooperative price

$$p^C = \frac{\alpha + \beta\left(c_0 - c_r r^C\right)}{2\beta}$$

yields the following pair of differential equations:

$$\begin{aligned}
\dot{\lambda}_M^I &= (\rho+\delta)\,\lambda_M^I - \left(\frac{\alpha - \beta\left(c_0 - c_r r^C\right)}{2}\right)\left(\mu\left(\omega^C - c_0\right) + c_r\right),\\
\dot{\lambda}_R^I &= (\rho+\delta)\,\lambda_R^I - \mu\left(\frac{\alpha - \beta\left(c_0 - c_r r^C\right)}{2}\right)\left(\frac{\alpha + \beta\left(c_0 - c_r r^C\right)}{2\beta} - \omega^C\right).
\end{aligned}$$

Here, the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\lambda_M^I(t) = 0$ and $\lim_{t\to\infty} e^{-\rho t}\lambda_R^I(t) = 0$ should hold true as a part of the manufacturer's and retailer's optimization problems. We have the following assumption along with Assumption 1.

Assumption 2 *The co-state trajectories $\lambda_M^I(t)$ and $\lambda_R^I(t)$ reach their steady-state values as t approaches infinity.*

Considering the pair of differential equations in (14)–(15) and the above equations, the steady-state values of the co-state variables are given by the following expressions:

$$\begin{aligned}
\bar{\lambda}_M^I &= \left(\frac{\alpha - \beta\left(c_0 - c_r \bar{r}^C\right)}{2(\rho+\delta)}\right)\left(\mu\left(\omega^C - c_0\right) + c_r\right),\\
\bar{\lambda}_R^I &= \mu\left(\frac{\alpha - \beta\left(c_0 - c_r \bar{r}^C\right)}{2(\rho+\delta)}\right)\left(\frac{\alpha + \beta\left(c_0 - c_r \bar{r}^C\right)}{2\beta} - \omega^C\right),\\
\bar{\lambda}^C &= \left(\frac{\alpha - \beta\left(c_0 - c_r \bar{r}^C\right)}{2(\rho+\delta)}\right)\left(\frac{\mu\left(\alpha - \beta\left(c_0 - 3c_r \bar{r}^C\right)\right)}{2\beta} + c_r\right),
\end{aligned}$$

where $\bar{r}^C$ is the steady-state value of the returns $\bar{r}^C$, which is obtained by solving the second-degree polynomial

$$\bar{r}^C - \frac{(b_M + b_R)}{\delta}\left(\frac{\alpha - \beta\left(c_0 - c_r \bar{r}^C\right)}{2(\rho+\delta)}\right)\left(\frac{\mu\left(\alpha - \beta\left(c_0 - 3c_r \bar{r}^C\right)\right)}{2\beta} + c_r\right) = 0.$$

Remark 3 *We make the following observations regarding two special cases:*

Only cost benefits ($\mu = 0$): *When returns involve only operational benefits, then the system of differential equations to be solved $\left(\dot{\lambda}^C, \dot{r}^C\right)$ becomes linear, and its steady state is given by*

$$\begin{aligned}
\bar{r}^C &= \frac{c_r(b_M + b_R)(\alpha - \beta c_0)}{2\delta(\delta+\rho) - \beta c_r^2 (b_M + b_R)},\\
\bar{\lambda}^C &= \frac{c_r \delta(\alpha - \beta c_0)}{2\delta(\delta+\rho) - \beta c_r^2 (b_M + b_R)}.
\end{aligned}$$

The steady-state values for the adjoint variables in the incentive-strategies game are given by

$$\begin{aligned}
\bar{\lambda}_M^I &= c_r \frac{\alpha - \beta(c_0 - c_r \bar{r}^C)}{2(\rho+\delta)},\\
\bar{\lambda}_R^I &= 0.
\end{aligned}$$

The reason for having $\bar{\lambda}_R^I = 0$ is that when $\mu = 0$, the retailer's payoff function is independent of the return rate.

Only market expansion $(c_r = 0)$: *When returns involve only marketing benefits, then the system of differential equations to be solved* $\left(\dot{\lambda}^C, \dot{r}^C\right)$ *becomes linear, with its steady state given by*

$$\bar{r}^C = \frac{\mu(b_M + b_R)\left(\alpha - \beta c_0\right)^2}{4\delta\beta(\rho+\delta)},$$
$$\bar{\lambda}^C = \frac{\mu\left(\alpha - \beta c_0\right)^2}{4\beta(\rho+\delta)},$$

The steady-state values for the adjoint variables in the incentive-strategies game are given by the simple expressions

$$\bar{\lambda}^I_M = \mu\left(\frac{\alpha - \beta c_0}{2(\rho+\delta)}\right)\left(\omega^C - c_0\right),$$
$$\bar{\lambda}^I_R = \mu\left(\frac{\alpha - \beta c_0}{2(\rho+\delta)}\right)\left(\frac{\alpha + \beta c_0}{2\beta} - \omega^C\right).$$

5 Numerical illustration

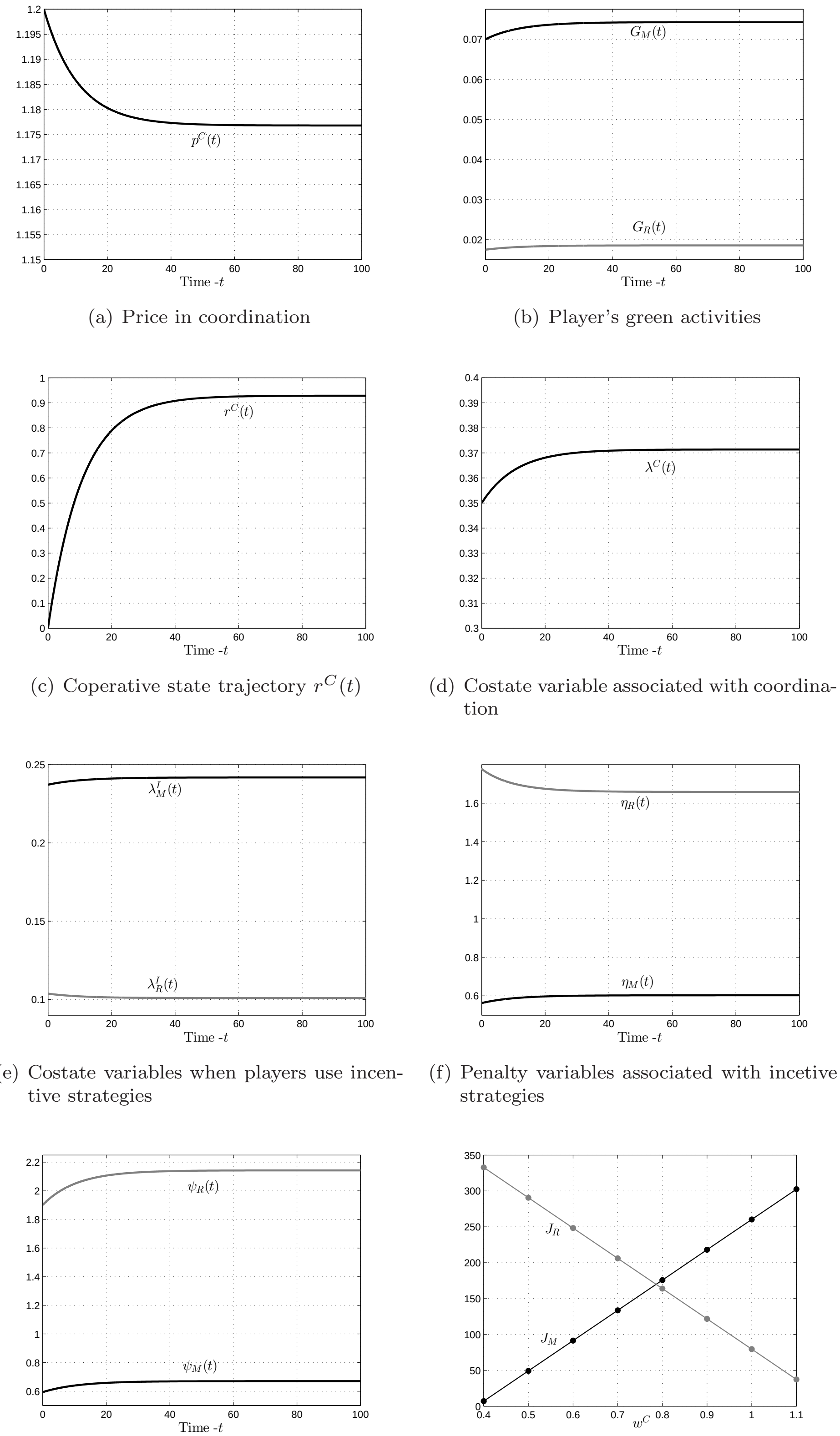
To illustrate the theory presented above, we provide a numerical example. We choose the following parameter values:

Demand parameters	:	$\alpha = 1,\ \beta = 0.5, \mu = 0.15,$
Cost parameters	:	$c_0 = 0.4,\ c_r = 0.5,$
PRP efficiency parameters	:	$b_M = 0.2$ (or 0.05), $b_R = 0.05$ (or 0.20),
Decay and discount parameters	:	$\delta = 0.1,\ \rho = 0.1,$
Cooperative wholesale price	:	$w^C = 0.85.$

For the PRP efficiency parameters, we consider two cases, namely, a case where the manufacturer is more efficient than the retailer ($b_M = 0.2,\ b_R = 0.05$), and the mirror case where the retailer is more efficient ($b_M = 0.05,\ b_R = 0.20$). It is easy to verify that the above parameters satisfy Assumption 1 and the conditions stated in Proposition 1. The cooperative solution is obtained by computing the stable manifold of the saddle node E_1 associated with the optimal system (14–15). Next, the incentive strategies are obtained by augmenting the costate variables (30) and (31) with the above optimal system. Further, as alluded to before, the wholesale price, which is a decision variable in the incentive-equilibrium problem, cancels out in the vertically integrated (or cooperative) solution, and therefore w^C has to be chosen in some way, e.g., by negotiation, or by requiring a property to be satisfied, such as a fair division of total revenues. Although the selection of a particular value for w^C is not a focal point in this paper, we shall vary w^C and see its impact on the players' profits.

The results call for the following observations:

1. Figures 1(a) and 2(a) show that the price is monotonically decreasing over time. This follow-the-cost pricing scheme has also been obtained in other dynamic-pricing models where the cost declines over time because of, e.g., learning-by-doing in the production process. In such a case, the unit-production cost decreases with cumulative production (state variable in learning-by-doing literature), whereas here, it decreases with the returns of previously purchased products. The price reaches the steady state according to the manufacturer's production benefits, $c_0 - c_r r^C$. When the difference is small (large), the price reaches its steady state earlier (later) over the planning horizon. In addition, its shape is governed by the return rate. When the CLSC achieves all the environmental performance, i.e., $r^C \simeq 1$, the pricing strategies quickly reach the steady state. For a discussion of pricing in a dynamic oligopoly (horizontal competition) and a dynamic vertical structure (supply chain or marketing channel), see, e.g., Jørgensen and Zaccour (2004). Finally, the CLSC seeks to promptly adjust the price at the steady state to fully take advantage of a higher demand and market expansion.



(a) Price in coordination

(b) Player's green activities

(c) Coperative state trajectory $r^C(t)$

(d) Costate variable associated with coordination

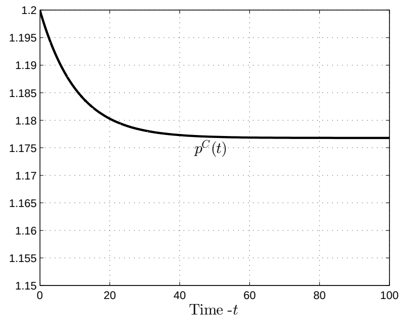
(e) Costate variables when players use incentive strategies

(f) Penalty variables associated with incetive strategies

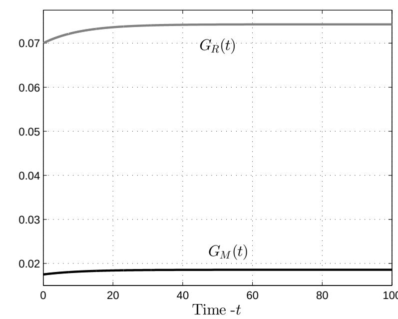
(g) Penalty variables associated with incetive strategies

(h) Variation of profits with wholesale price

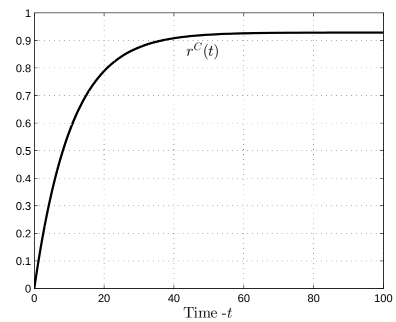
Figure 1: **Efficient Manufacturer** with parameter values $b_M = 0.2$, $b_R = 0.05$, $\delta = 0.1$, $\rho = 0.1$, $\alpha = 1$, $\beta = 0.5$, $c_0 = 0.4$, $c_r = 0.05$, $\mu = 0.15$. For Figures 1(a)–1(g), we use $w^C = 0.85$.



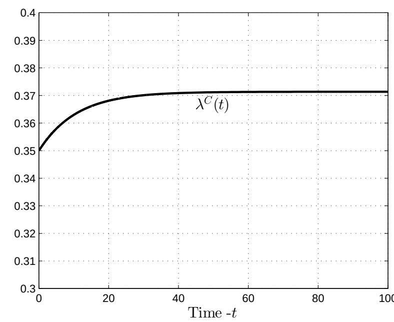
(a) Price in coordination



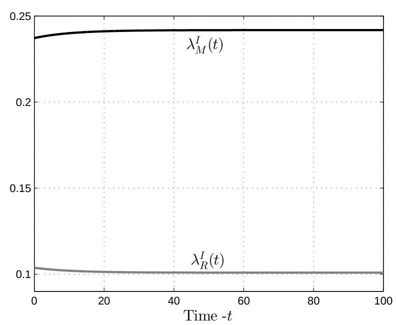
(b) Player's green activities



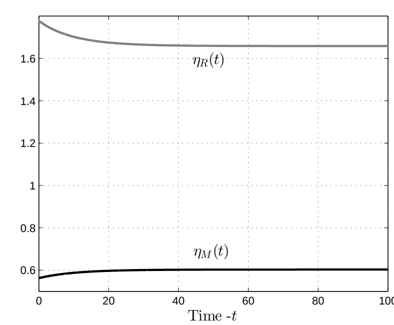
(c) Coperative state trajectory $r^C(t)$



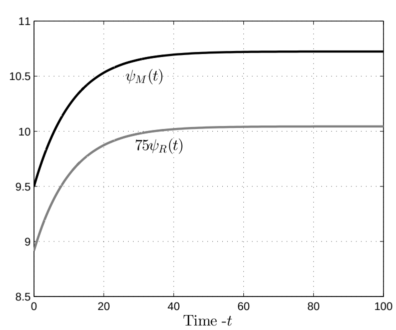
(d) Costate variable associated with coordination



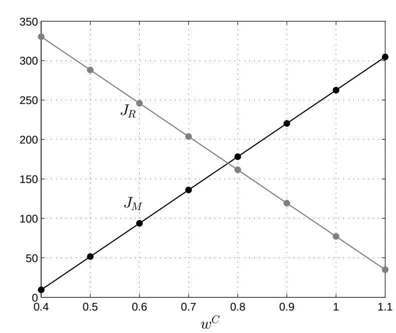
(e) Costate variables when players use incentive strategies



(f) Penalty variables associated with incetive strategies



(g) Penalty variables associated with incetive strategies



(h) Variation of profits with wholesale price

Figure 2: **Efficient Retailer** with parameter values $b_M = 0.05$, $b_R = 0.2$, $\delta = 0.1$, $\rho = 0.1$, $\alpha = 1$, $\beta = 0.5$, $c_0 = 0.4$, $c_r = 0.05$, $\mu = 0.15$. For Figures 2(a)–2(g), we use $w^C = 0.85$.

2. Figure 1(b) and 2(b) show that the investment levels of both players in the product recovery program are slightly increasing in the beginning till they reach their steady-state values. This suggests that firms within a CLSC should substantially invest at the beginning to exploit the market expansion and increase profits. Plus, manufacturers fully benefit from increasing PRP rates from an operational perspective. The environmental performance drives firms' PRP strategies. In fact, when the CLSC's environmental performance is near perfect ($r^C \simeq 1$), investments in PRP aim at maintaining a good consumer portfolio that would otherwise naturally decrease. While firms' PRP investments assume the same shape, the difference in the firms' operational efficiency, b_i, determines the different investment levels at each instant of time (see Figures 1(b) and 2(b)). When the manufacturer's (retailer's) operational efficiency is higher than that of the other firm, she invests a higher amount. Interestingly, an efficient manufacturer reaches its steady state with the same speed as an efficient retailer, although firms have misaligned interests for closing the loop: the manufacturer has both operational and marketing motivations to invest in PRP, while the retailer only exploits the market-expansion potentiality. This result depends on the construction of credible incentives where when one player sets an incentive strategy, it is in the best interest of the opponent to act in accordance with the agreement. If the latter cheats on the cooperative solution, the former deviates from the cooperative solution as well.
3. The return rate increases over time and reaches a steady-state value (of approximately 0.92) really early in the planning horizon, independent of the firms' operational efficiency (see Figures 1(c) and 2(c)). The behavior of the return-rate trajectories is the product of an increase in PRP spending over time, market expansion and a decline in production cost (and consequently a lower price). Firms within a CLSC are aware of the operational and marketing benefits that the return rate supplies; thus reaching the steady-state value represents a major target. In fact, while $r^C(0) = 0$, the CLSC performs $r^C(20) = 0,8$, that is, incentives drive firms through the implementation of the cooperative solution, which represents the best deal to perform the return rate. Plus, firms in CLSCs should be aware of the fact that the effectiveness of the incentives already provides environmental and economic advantages at the beginning of the planning horizon.
4. Looking at the shadow prices under cooperation (Figures 1(d) and 2(d)) and under incentive strategies (Figures 1(e) and 2(e)), we observe that $\lambda^C(t)$ is larger than the sum of $\lambda_M^I(t)$ and $\lambda_R^I(t)$, independently of the difference in the firms' operational efficiency. This result can be interpreted as follows: in the absence of cooperation, each firm values the returns only in terms of its own payoff while disregarding the positive impact they have on the other player's profit. It is a well-known result that in a noncooperative game, the provision of public good (here the investment in PRP) is lower than its level in a cooperative solution. By implementing incentive strategies, the two CLSC partners eliminate the temptation of deviating from the cooperative solution, which would otherwise signify under-investments in PRP. This is achieved through imposing some penalty terms on any deviation from the cooperative solution. By design, these penalties imply that each player is economically better off implementing the cooperative solution rather than deviating from it, conditional on the fact that the other player also chooses to cooperate. Interestingly, $\lambda_M^I(t)$ and $\lambda_R^I(t)$ reach their steady state through opposite trajectories: while $\lambda_M^I(t)$ shows an increasing path before reaching the steady state, $\lambda_R^I(t)$ decreases in the beginning of the planning horizon. This is probably due to the fact that the retailer's price incentive is constructed through ω^C, which is exogenous; therefore, the rate of improvement in the retailer's objective function decreases in the short term and then reaches its steady-state value as the benchmark value $\omega^C = 0.85$, which is not optimal.
5. Figures 1(h) and 2(h) display the total individual profits at the steady state as a function of the wholesale price ω^C. As one would expect, the manufacturer's payoff is increasing in ω^C, and the retailer's profit is decreasing in ω^C. Independent of the firms' operational efficiency, there exists a $\omega^C = \omega^{C*}$through which a CLSC equally shares the total cooperative value. As Figures 1(h) and 2(h) show, $\omega^{C*} = 0.8$. Finally, as mentioned earlier, the ultimate choice of ω^C depends on several exogenous elements such as firms' decisional power, the implementation of specific contracts, and the presence of competition. Thus, CLSC parties should identify and negotiate its amount according to the most appropriate profit splitting rule.

6. The trajectories of the price-penalty terms $\eta_i(t)$ are exhibited in Figures 1(f) and 2(f). These trajectories do not depend on the firms' operational efficiency while it always results that $\eta_R(t) > \eta_M(t)$. Therefore, the manufacturer's (retailer's) strategy has a high (low) weight on the retailer's (manufacturer's) pricing strategy. This result derives from the fact that the manufacturer has a dual motivation (operational and marketing) to close the loop; therefore the low incentive $\eta_M(t)$ signifies that she already sets her pricing strategies to do more. Her move to the cooperative solution is thus simple. In contrast, the retailer has only marketing motivations to contribute to the return rate; therefore the cooperative solution is a substantial challenge and the high incentive $\eta_R(t)$ translates into a high dependency on ω. Notice that the trajectories $\eta_i(t)$ (as for $\lambda_i(t)$) have opposite directions. The manufacturer's price incentive to set the wholesale price at the cooperative solution increases over time and reaches the steady-state value quickly. In contrast, the retailer's price incentive decreases over time and the steady state is reached later than the manufacturer's price incentive. This difference is again due to the effect of ω^C, which plays an opposite role inside $\eta_M(t)$ and $\eta_R(t)$. High values for ω^C lead to a lower impact of the manufacturer's pressure on the retailer's pricing strategy and thus, a lower willingness to perform the cooperative solution.
7. The trajectories of PRP-penalty terms are exhibited in Figures 1(g) and 2(g). Note that in Figure 2(g), as $\psi_R(t)$ is very small, its value has been multiplied by 75 for readability purposes. All trajectories are always increasing over time, independent of the firms' operational efficiency. This result demonstrates that both firms wish to set their PRP efforts at the cooperative solution because it is in their best interest to invest in PRP to increase the return rate, and this is the best decision for the opponent as well. Interestingly, the difference in the firms' efficiency b_i plays a role. In fact, the PRP incentive will be lower for the most efficient firm. This result is intuitive: when a firm is efficient, she already invests more in PRP (e.g., see Figures 1(b) and 2(b)) and the move to the cooperative solution will be much easier.

6 Concluding remarks

In this paper, we showed how to implement a cooperative (joint-maximization) solution in supply chain by using incentive strategies. We recall that in a vertical structure, coordination has three important implications. First, it leads to higher total payoffs than does decentralization. Second, it leads to lower prices and higher investments in product recovery programs. Finally, coordination allows a higher environmental performance. These results are not surprising as it has been repeatedly shown in both supply-chain and marketing-channel literature that coordination is better that decentralization for all parties involved. What is new, and therein lies our main contribution, are (i) the use of incentive strategies to reach the coordinated solution in a dynamic CLSC; and (ii) the development of a model that combines both operational and marketing aspects in a dynamic CLSC. The marketing-operations interface has clearly been overlooked in the literature up to now.

In any modeling effort, some hypotheses are needed either for mathematical tractability or to enable the authors to focus on some particular points in a parsimonious way. Our study does not escape this rule, and consequently, we believe some extensions are worth conducting. First, the assumption of infinite reuse for returned products should be relaxed and should instead suppose a finite number of reuses. In the same vein, we could analyze the deteriorating quality of reused products with the number of times they enter the manufacturing process. Second, the assumption that the return rate is a function of PRP activities may be a strong one in some industries. Here, monetary incentives to the consumer can be explicitly added to boost the returns. Also, we can analyze the case where the returns also depend on some stochastic parameter. Finally, the case where competition is present in the returns market is clearly of empirical interest.

A Bifurcation analysis of the optimal vector field (14–15)

In this section, we analyze the optimal vector field (14–15) in more detail. To simplify calculations, we denote $p = b_M + b_R, s = \frac{3\mu\beta c_r^2}{4},\ n = \left(\frac{1}{3}\left[\frac{\alpha-\beta c_0}{\beta c_r} - \frac{1}{\mu}\right]\right)^2$ and $m = \frac{1}{3}\left(\frac{1}{\mu} + 2\left[\frac{\alpha-\beta c_0}{\beta c_r}\right]\right)$. The optimal system (14–15) in state-costate coordinates is given by

$$\dot{r}^C = -\delta r^C + p\lambda^C \tag{32}$$
$$\dot{\lambda}^C = (\rho+\delta)\lambda^C - s\left((r^C+m)^2 - n\right). \tag{33}$$

The equilibrium points $(\bar{r}^C, \bar{\lambda}^C)$ of the above system are obtained by solving

$$(\bar{r}^C+m)^2 - n - \frac{\delta(\rho+\delta)}{ps}\bar{r}^C = 0$$
$$\left(\bar{r}^C + m - \frac{\delta(\rho+\delta)}{2ps}\right)^2 = \Delta = n + \left(\frac{\delta(\rho+\delta)}{2ps}\right)^2 - \frac{\delta(\rho+\delta)m}{ps}. \tag{34}$$

Next, the Jacobian matrix, to be analyzed at the equilibrium points for stability, is given by

$$J = \begin{bmatrix} -\delta & p \\ -2s(\bar{r}^C+m) & (\rho+\delta) \end{bmatrix}.$$

The eigenvalues of the Jacobian matrix at the equilibrium point $(\bar{r}^C,\ \bar{\lambda}^C)$ are then given by

$$z_1 = \frac{\rho}{2} - \sqrt{\frac{\rho^2}{4} - 2ps\left(\bar{r}^C + m - \frac{(\rho+\delta)\delta}{2ps}\right)}$$
$$z_2 = \frac{\rho}{2} + \sqrt{\frac{\rho^2}{4} - 2ps\left(\bar{r}^C + m - \frac{(\rho+\delta)\delta}{2ps}\right)}.$$

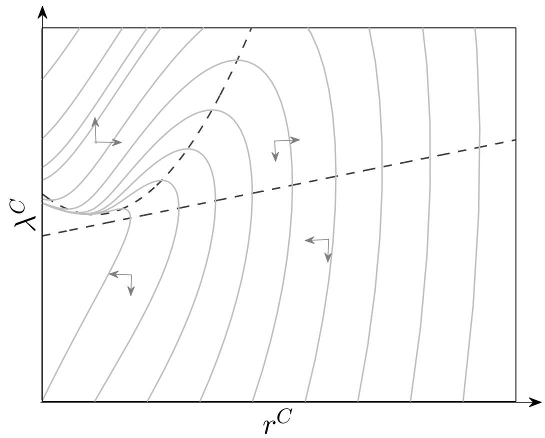
The system (32–33) displays bifurcations, that is, varying qualitative behaviors depending on the values of the parameters. We characterize these bifurcation scenarios as the following four possibilities:

1) When $\Delta < 0$, there exist no equilibrium points for the system (32–33). The phase diagram for this case is illustrated in 3(a). Here, the trajectories grow unbounded in magnitude with time.
2) When $\Delta = 0$, there exists one equilibrium point for the system (32–33). The eigenvalues associated with the Jacobian matrix are $\{0, \rho\}$. So this equilibrium point is not asymptotically stable. The phase diagram for this case is illustrated in 3(b).
3) When $\Delta > 0$, there exist two equilibrium points for the system (32–33) given by

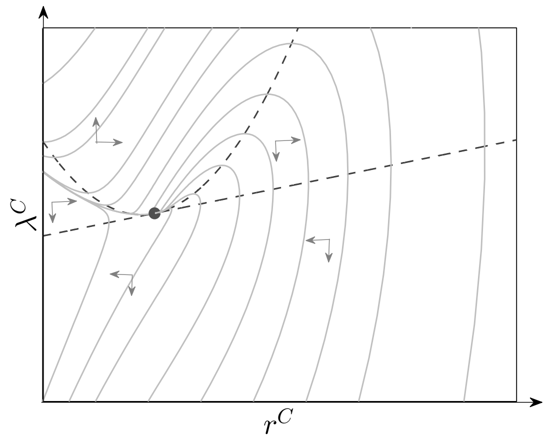
$$E_1 : (\bar{r}_1^C,\ \bar{\lambda}_1^C) = \left(\frac{\delta(\rho+\delta)}{2ps} - (m+\sqrt{\Delta}),\ \frac{\delta}{p}\left(\frac{\delta(\rho+\delta)}{2ps} - (m+\sqrt{\Delta})\right)\right)$$
$$E_2 : (\bar{r}_2^C,\ \bar{\lambda}_2^C) = \left(\frac{\delta(\rho+\delta)}{2ps} - (m-\sqrt{\Delta}),\ \frac{\delta}{p}\left(\frac{\delta(\rho+\delta)}{2ps} - (m-\sqrt{\Delta})\right)\right).$$

   Clearly we have $\bar{r}_2^C > \bar{r}_1^C$ and $\bar{\lambda}_2^C > \bar{\lambda}_1^C$. The eigenvalues of the Jacobian matrix associated with equilibrium point E_1 are $z_1 = \frac{\rho}{2} + \sqrt{\frac{\rho^2}{4} + 2ps\sqrt{\Delta}} > 0$ and $z_2 = \frac{\rho}{2} - \sqrt{\frac{\rho^2}{4} + 2ps\sqrt{\Delta}} < 0$. So, E_1 is a saddle node. Then we have the following two cases regarding the stability properties of the equilibrium point E_2.

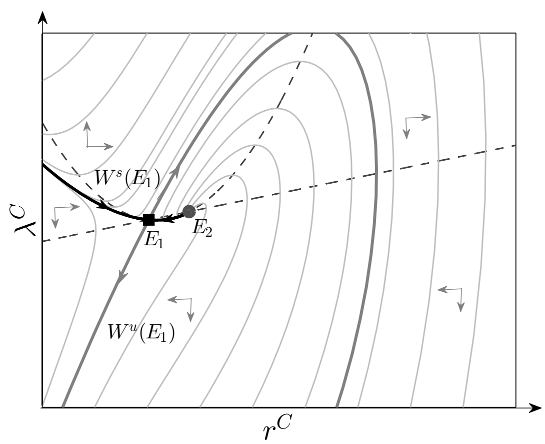
   a) When $\rho^2 > 8\sqrt{\Delta}ps$, the eigenvalues of the Jacobian matrix associated with the equilibrium point E_2 are $z_1 = \frac{\rho}{2} + \sqrt{\frac{\rho^2}{4} - 2ps\sqrt{\Delta}} > 0$ and $z_2 = \frac{\rho}{2} - \sqrt{\frac{\rho^2}{4} - 2ps\sqrt{\Delta}} > 0$. So, clearly E_2 is a source node. The phase diagram for this case is illustrated in Figure 3(c). Further, the stable manifold is not defined for the entire state space, i.e., only for $r^C(0) \in (-\infty, \bar{r}_2^C)$ do we have trajectories $(r^C(t), \lambda^C(t))$ converging to $(\bar{r}_1^C, \bar{\lambda}_1^C)$ along the stable manifold $W^s(E_1)$.
   b) When $\rho^2 < 8\sqrt{\Delta}ps$, the eigenvalues of the Jacobian matrix associated with the equilibrium point E_2 are $z_1 = \frac{\rho}{2} + i\sqrt{-(\frac{\rho^2}{4} - 2ps\sqrt{\Delta})}$ and $z_2 = \frac{\rho}{2} - i\sqrt{-(\frac{\rho^2}{4} - 2ps\sqrt{\Delta})}$. So, clearly, E_2 is a spiral source. The phase diagram for this case is illustrated in Figure 3(d). Again, the stable manifold is not defined for the entire state space. Here, there exists an r_u^C such that for $r^C(0) \in (-\infty, r_u^C)$ we have trajectories $(r^C(t), \lambda^C(t))$ converging to $(\bar{r}_1^C, \bar{\lambda}_1^C)$ along the stable manifold $W^s(E_1)$.



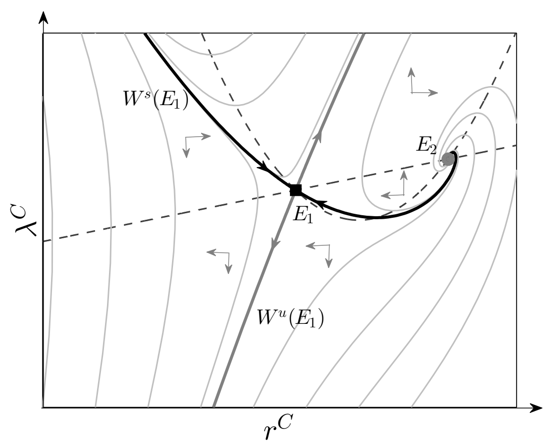
(a) With $\Delta < 0$.



(b) With $\Delta = 0$. The equilibrium point (denoted by a circle) is not asymptotically stable.



(c) With $\Delta > 0$, $\rho^2 - 8\sqrt{\Delta}ps > 0$. The equilibrium point E_1 is a saddle node and E_2 is a source node. $W^s(E_1)$ (thick dark line) and $W^u(E_1)$ (thick gray line) represent stable and unstable manifolds associated with E_1.



(d) With $\Delta > 0$, $\rho^2 - 8\sqrt{\Delta}ps < 0$. The equilibrium point E_1 is a saddle node and E_2 is a source spiral. $W^s(E_1)$ (thick dark line) and $W^u(E_1)$ (thick gray line) represent stable and unstable manifolds associated with E_1.

Figure 3: Bifurcation analysis of the optimal vector field (32–33). Dashed lines in the above figures represent $\dot{r}^C$- and $\dot{\lambda}^C$- isoclines, which divide the state-costate space into disjoint regions wherein the arrows reflect the direction of the vector field.

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