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Abstract: Generation Expansion Planning (GEP) with load uncertainty is modeled in this paper using chance constrained programming, and new iterative solution algorithms identifying the stressed and non-stressed buses in the system are developed for solving this model. The algorithms use a risk-based approach to dynamically allocate risk at each bus depending on the response of the system to the load behaviour. Computational results for the IEEE 30- and 118- bus systems are presented and compared with previous approaches. It is observed that expansion solutions meeting the required reliability at lower cost are obtained using this approach.

Key Words: Generation Expansion Planning, Chance Constrained Programming, Risk Allocation.

1 Introduction

Generation Expansion Planning (GEP) is a well-defined problem in power systems. Both regulated and deregulated power systems are exposed to uncertainty due to several sources of random events. The many sources of uncertainty, including renewable sources integration, demand participation, generation and transmission availabilities and the cost of fuel, make the problem more challenging. The inability to predict random events introduces risk into the power system economics and operation. This motivates the use of stochastic optimization approaches to incorporate uncertainty in GEP models.

Utility planners always seek to provide reliable power to the consumers at the lowest possible cost. The increasing power demand makes this more difficult to achieve. The investment cost and the time taken to build a generation unit is significant [1] and hence decisions have to be taken well ahead of time. In the GEP, one seeks to find out the number and size of new generation units needed to satisfy the forecasted demand growth with the minimal investment and operation cost [2, 3].

Optimal solutions to decision support models like those used for the GEP are often sensitive to uncertainty in the model parameters. Neglecting the uncertainty may lead to unrealistic solutions. For the GEP, a survey of all the optimization models used in power system planning is discussed in [4]. A deterministic multi-period and multi-objective GEP is solved in [5]. A GEP model with uncertain demand was formulated in [6] using a Markov chain, and was solved using stochastic dynamic programming in [7]. Different applications have given rise to different types of stochastic optimization models. One is the recourse-based model in [8] in which optimal decisions are taken in a first step, and subsequently there is a recourse to re-optimize after some of the uncertainty is resolved. Another is the expected value model (E-model) that minimizes the expected value of the cost subject to the expected values of constraints [8, 9]. Yet another way to handle the uncertainty as probabilistic measures is chance constrained programming (CCP).

The contribution of this paper is an iterative algorithm to solve the the GEP with load uncertainty for a vertically integrated power system using CCP. The proposed iterative algorithm distinguishes between the stressed and non-stressed buses in the system and thus enables dynamic risk allocation among the chance constraints. This work is different from other similar algorithms such as in [10] and [11] because the risk is allocated dynamically. It improves on the work in [12] that only allocates risk among the stressed bus becasue in this work we consider the information from both stressed and non-stressed buses.

The rest of the paper is structured as follows. In Section 2, CCP and its application to power systems is briefly described. In Section 3, the formulation of the GEP with load uncertainty using CCP is presented. In Section 4, previous iterative solution procedures to solve the CCP formulation are discussed. In Section 5, the improved algorithms proposed in this paper are explained and computational results demonstrating their benefits are reported. Section 6 concludes the paper.

2 Literature on CCP in Power Systems

The idea behind chance-constrained optimization (CCO) is to require that one or more constraints be satisfied with a given probability. It was first introduced by [13] and has been applied extensively to a wide range of engineering, financial and management applications. CCP was first used as an analytical tool for planning problems as it incorporates risk explicitly.

CCP has been applied to planning and operation problems in power systems. A generation planning model was introduced in [14] where a probabilistic reliability criterion was considered for both discrete and continuous random generation. In [15], CCP-based transmission expansion planning was solved using a genetic algorithm. The effect of wind uncertainty in transmission expansion planning was studied in [16]. A probability density function was modeled for wind uncertainty and the resultant CCP-based problem was solved using genetic algorithm. A generation and transmission expansion problem was modeled in [17] using two-stage stochastic programming where a risk factor is introduced in the objective function. The solution algorithm was based on the minimum variance approach [18] which minimizes the risk in an investment project. A market-based generation and transmission expansion planning model was solved in [19] using scenario-based formulation and Monte Carlo Simulation (MCS). A reduction technique has been applied to reduce the number of scenarios considered. A GEP problem for vertically integrated systems with load uncertainty is modeled using CCP and solved with a modified iterative algorithm in [11] and has proven to have less iterations.

CCP has been also applied to operation and stability problems. The unit commitment problem can be modeled using CCP [20]. A reactive power planning for deregulated electric industry is modeled using CCP under the assumption that the generation output and load demand have a specific distribution function [21], and the problem is again solved using MCS and a genetic algorithm. Evaluation of available transfer capacity for interconnected power networks is carried out in [22] using both recourse-based stochastic programming and CCP. A mathematical model with objective as minimizing spinning reserve procurement cost with system security as a chance constraint is modeled [23], and a MCS-based genetic algorithm solved the problem. CCP and risk allocation was applied to dynamic systems [24]. Its advantages and effectiveness are discussed in [25].

There are two drawbacks for the previous applications of CCP to power system problems: separate chance constraints were used [14, 21] where only one probabilistic constraint was of interest, and the solution approach for the joint CCP was computationally costly. Problems based on joint CCP are difficult to solve and hence they are usually transformed into equivalent deterministic approximations as proposed in [26].

3 Generation Expansion Planning under Uncertainty

The main objective in GEP is cost minimization subject to the operational constraints of the system. Mathematically this can be expressed using a modified version of the formulation presented in [14]:

$$\min \sum_{i=1}^{nb} w_i C_{bn} P_{ngi}^{max} + \sum_{i=1}^{nb} C_{pni} p_{ngi} + \sum_{i=1}^{nb} C_{pei} p_{egi} \tag{1a}$$

$$p_{ngi} + p_{egi} - p_{si} = p_{li} \quad i = 1...nb \tag{1b}$$

$$p_{si} = \sum_j -b_{ij}(\delta_i - \delta_j) \quad i = 1...nb \tag{1c}$$

$$p_{egmin} \le p_{egi} \le p_{egmax} \quad i = 1...nb \tag{1d}$$

$$w_i p_{ngmin} \le p_{ngi} \le w_i p_{ngmax} \quad i = 1...nb \tag{1e}$$

$$w_i = \{0, 1\} \tag{1f}$$

where
C_{bn}, C_{pn} are the investment and production cost of new units
C_{pe} are the production costs of existing units
p_{ng}, p_{eg} are the active power levels of new and existing units
p_{li} is the load connected to bus i
p_{si} is the net power flow in all the lines connected to bus i
δ_i is the voltage angle at bus i
b_{ij} is the susceptance of the line
nb is the number of buses in the system
w_i is the binary decision variable for new generation at bus i.

The objective function (1a) is the total of investment and operation costs. The constraint (1b) is the real power balance equation, and (1c) is the summation of the line flows over all the lines connected to bus i. Constraints (1d) and (1e) are the operational limits on the generating units. Finally constraint (1f) represents the binary nature of the decision variables w_i.

The above formulation ignores load uncertainty. The load represented here is the average forecasted load. To include uncertainty using probabilistic terms, the constraint (1b) is changed to:

$$Pr\left(\bigcap_{i=1}^{nb}(p_{ngi} + p_{egi} - p_{si} \ge p_{li})\right) \ge \alpha \tag{2}$$

where α represents a user-defined confidence level (or reliability level). The left-hand side of (2) is a joint probability, i.e., it is the probability of nb events occurring simultaneously; hence constraint (2) is a Joint Chance Constraint [27]. When the probability distribution of the random vector p_{li} is known and continuous, then the left-hand side of (2) can be computed directly using numerical integration techniques. However this computation involves an nb-dimensional integral [27] and is only possible for small problems in practice.

A practical means to address the difficulty introduced by replacing the constraints (1b) with the single constraint (2) is to approximate the latter constraint with a set of chance constraints on the individual loads. First observe that (2) can be reformulated as (3):

$$Pr\left\{\bigcup_{i=1}^{nb}(p_{ngi}+p_{egi}-p_{si}\geq p_{li})^{c}\right\}\leq 1-\alpha. \tag{3}$$

This reformulation follows from the fact that the sum of the probabilities of an event and of its complementary event equals 1. Now we can write a set of sufficient conditions for equation (3) to hold:

$$Pr\left\{(p_{ngi}+p_{egi}-p_{si}\geq p_{li})^{c}\right\}\leq \frac{1-\alpha}{nb},\quad i=1,\ldots,nb. \tag{4}$$

Therefore the probabilistic real power flow equation (2) can be approximated as:

$$Pr\left\{p_{ngi}+p_{egi}-p_{si}\geq p_{li}\right\}\geq 1-\frac{1-\alpha}{nb},\quad i=1,\ldots,nb. \tag{5}$$

In short, the constraint (2) can be replaced with the stronger set of constraints (5).

If the probability distribution of the load is known, each of the constraints in (5) can be replaced by its deterministic equivalent. For the GEP, it is reasonable to assume that the load at bus i follows a normal distribution [28]. Let μ_{li} denote its mean and σ_{li} its variance. Then we can rewrite the constraints (5) as:

$$Pr\left\{\frac{p_{ngi}+p_{egi}-p_{si}-\mu_{li}}{\sigma_{li}}\geq\frac{p_{li}-\mu_{li}}{\sigma_{li}}\right\}\geq 1-\frac{1-\alpha}{nb},\quad i=1,\ldots,nb. \tag{6}$$

Because $\frac{p_{li}-\mu_{li}}{\sigma_{li}}$ is a standard normal random variable, (6) is equivalent to

$$\phi\left\{\frac{p_{ngi}+p_{egi}-p_{si}-\mu_{li}}{\sigma_{li}}\right\}\geq 1-\frac{1-\alpha}{nb} \tag{7}$$

where ϕ is the normal probability distribution function. Finally let Z_α be the inverse cumulative distribution of $1-\frac{1-\alpha}{nb}$, i.e., $Z_\alpha=\phi^{-1}\left\{1-\frac{1-\alpha}{nb}\right\}$. Then (7) can be rewritten as

$$\frac{p_{ngi}+p_{egi}-p_{si}-\mu_{li}}{\sigma_{li}}=Z_\alpha \tag{8}$$

or equivalently

$$p_{ngi}+p_{egi}-p_{si}=\mu_{li}+\sigma_{li}Z_\alpha \tag{9}$$

The remainder of this paper is concerned with improving the choice of Z at each bus so as to reduce the resulting GEP total cost. In particular, while the above assumed that the load uncertainty is the same at every bus, this is generally not the case. Therefore it makes sense to adjust the value of Z at each bus, and by doing so it is possible to reduce the cost to achieve conditions sufficient to attain the required reliability level.

4 Previously Proposed Z-Update Algorithms

4.1 Uniform Z-Update Algorithm

Given the desired probability level α, the general framework for solving the GEP problem using the deterministic formulation described above is as follows (see also Figure 1):

1. A set of new generations is computed by solving the MILP problem (1a -1f) with the constraint (1b) replaced by the approximate deterministic equivalent (9).
2. Sample load scenarios are randomly generated using MCS and an Optimal Power Flow (OPF) is solved with the solution from step 1. Because the new generation is fixed, w_i is not a variable and there is no investment cost in the objective function. The number of feasible cases divided by the total number of scenarios gives the estimate of the probability.
3. If the estimated probability is not satisfactory, then the Z value is updated and the process is repeated from step 1 until the target probability is achieved.

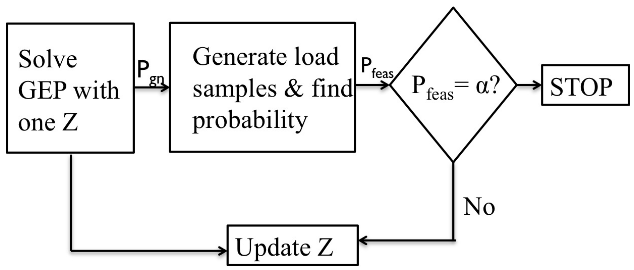

Figure 1: Overall Outline of the Iterative Algorithm

The update of the Z value implies a change in the load conditions p_{li} and is likely to have a significant impact on the optimal new generation decisions. The Z update method proposed in [10] is based on the interpolation of the univariate and multivariate variables and the false position method [29]. The steps in the algorithm are:

1. The GEP problem is initially solved twice with values Z_h and Z_l that are chosen to correspond to probabilities p_h and p_l respectively higher and lower than the desired probability α.
2. The probability values p_h and p_l are determined by solving OPFs for various normally distributed samples of load generated using MCS. These probabilities are converted to the corresponding univariate space Z-equivalents Z_1 and Z_2 using the probability distribution function of the random variable.
3. Based on these univariate Z and multivariate probability values, Z_α is updated for the next iteration using the formula:
$$Z_\alpha^{j+1} = Z_l + \left(\frac{Z_\alpha - Z_2}{Z_1 - Z_2} (Z_h - Z_l) \right). \tag{10}$$
The iterative algorithm basically tries to shrink the interval $[Z_h, Z_l]$. We refer to it as the uniform Z-update algorithm because the same Z value is used to update the constraint (9) for every bus.
4. The GEP problem is solved with the updated Z_α^i, then the OPF is solved to determine the probability P_{feas} of the new solution and the corresponding Z_{new} is calculated.
5. If $| P_{feas} - \alpha | \leq \triangle\alpha$ then the algorithm terminates. Here, $\triangle\alpha$ is a small tolerance allowed in the target probability. Whenever $P_{feas} > \alpha \pm \triangle\alpha$, i.e., the tolerance is not satisfied, the false position method [29] is used to choose new lower and higher values for Z:
   (a) If $Z_{new} < Z_\alpha^i$ then Z_l and Z_2 are replaced with the new Z_α^i and Z_{new} respectively.
   (b) If $Z_{new} > Z_\alpha^i$ then Z_h and Z_1 are replaced with the new Z_α^i and Z_{new} respectively.
6. The process is repeated until the target probability is reached.

Thus by updating the Z value, the loading conditions and the expansion decisions are modified to reach the target confidence level.

4.2 Risk-based Z Update - Increasing Z at stressed buses

In the uniform Z-update approach, the value of Z is updated uniformly for every bus. Hence the particular impact of each bus on the system reliability is not taken into consideration. This drawback in the algorithm was addressed in our earlier paper [12] by using a dynamic risk allocation approach. By focusing on the impact of the stressed buses, a reduction in cost was achieved.

The outline of this algorithm is similar to that of the uniform Z-update algorithm. The main difference is the identification of the stressed buses and the updating of the Z value independently for each bus. Given the desired probability level α, the steps of the algorithm are the same except for a modification in (10). The modified steps in this algorithm are:

1. After applying MCS, the success probability achieved with the current expansion solution is known. The OPF infeasible cases imply that there is not enough generation at one or more buses to meet the load. These buses are the stressed buses of the system. To identify them, we solve a different GEP formulation with a positive "slack" variable and a corresponding penalty introduced for each bus:

$$\min \sum_{i=1}^{nb} C_{pni} p_{ngi} + \sum_{i=1}^{nb} C_{pei} p_{egi} + \sum_{i=1}^{nb} C_{pen} s_i \tag{11a}$$

$$p_{ngi} + p_{egi} - p_{si} = p_{li} - s_i, \quad i = 1, \ldots, nb \tag{11b}$$

$$s_i \geq 0, \quad i = 1, \ldots, nb \tag{11c}$$

$$\text{Constraints}(1c) - (1e) \tag{11d}$$

   where C_{pen} is the penalty cost for the slack variables. Provided that C_{pen} is chosen sufficiently large, the slack variables will be non-zero only when there is no other way to make the OPF feasible, and only at those buses where reducing the load makes the OPF feasible.

2. The Z_α^i value for bus i is then updated as

$$Z_\alpha^i = Z_l + \left(\frac{Z_\alpha - Z_2}{Z_1 - Z_2} (Z_h - Z_l) \right) \left(\frac{\bar{s}_i}{\hat{s}} \right) \tag{12}$$

   where $\bar{s}_i$ is the sum of the slack variables for bus i over all the MCS scenarios, and $\hat{s} = \max_{i=1,\ldots,nb} \bar{s}_i$.

   The idea is that the ratio $\frac{\bar{s}_i}{\hat{s}}$ is a measure of the relative failure rate of bus i. In parrticular, the Z value is incremented for the stressed buses according to the magnitude of the ratio, and it is maintained at Z_l for all the non-stressed buses.

3. The GEP is again solved using the new Z values, and the process is repeated until the target probability is reached.

The results in [12] show that this approach makes it possible to reduce the GEP total cost in comparison to the uniform Z-update. We now study further improvements to the choice of Z, and hence improved algorithms for solving the chance-constrained GEP.

5 Improved Z-Update Algorithms

5.1 Decreasing Z Approach at Non-Stressed Buses

The algorithm in Section 4.2 aims at increasing the Z value in the chance constraints corresponding to stressed buses. Nevertheless the minimum value of Z_l is maintained for every bus. It is plausible that for the buses in the system that are not stressed, i.e., the buses that least impact reliability under varying load, allowing the Z value to go below Z_l would be possible, and that the total cost could be further improved in this way.

In order to determine the buses that are least sensitive to the load fluctuations, slack variables are again used but in a different way. While the slack variables in Section 4.2 had the effect of reducing the load at a bus, these new slack variables attempt to increase the load at each bus. We therefore refer to them as "negative slack" variables. These negative slack variables are added to the real power balance constraint of the OPF-feasible scenarios. They play a role of increasing the load at certain buses such that the OPF remains feasible after the increase. In this way, it quantifies how much moer each bus can be loaded without affecting the reliability of the system. In other words, the higher the value of the negative slack variable at a particular bus, the more that bus is non-stressed.

The steps in this algorithm are:

1. The GEP is solved for a value of Z_l chosen in such a way that the probability of success with Z_l is much lower than the target probability.
2. After determining the probability of success using normally distributed samples of load, the feasible samples are selected and their loads are modified using the following optimization problem where the negative slack variables have been introduced:

$$\max \sum_{i=1}^{nb} m_i \tag{13a}$$

$$p_{ngi} + p_{egi} - p_{si} = p_{li} + m_i, \quad i = 1, \ldots, nb \tag{13b}$$

$$m_i \geq 0, \quad i = 1, \ldots, nb \tag{13c}$$

$$\text{Constraints } (1c) - (1e) \tag{13d}$$

   Note that the objective is to maximize the sum of the nagative slack variables, i.e., to determine the maximum loadibility point of each bus. A non-zero optimal vale of m_i indicates that bus i has the capacity to handle a higher load. Hence the negative slack values are a measure of the sensitivity of the buses to the loading conditions.
3. The non-stressed buses identified in the previous step are the candidate buses for decreasing Z. The Z values are thus updated as:

$$Z_\alpha^i = Z_l \left(1 - \frac{\bar{m}_i}{\hat{m}}\right) \tag{14}$$

   where $\bar{m}_i$ is the sum over all the scenarios of the negative slack variables at bus i, and $\hat{m} = \max_{i=1,\ldots,nb} \bar{m}_i$. Similar to the ratio $\frac{\bar{s}_i}{\hat{s}}$ in (12), the ratio $\frac{\bar{m}_i}{\hat{m}}$ is a meaure of the relative stress level of bus i. The buses that can take on the largest amount of additional load have a ratio of 1 while the buses unable to handle any additional load have a ratio of 0.
4. The expansion problem is again solved with the updated Z values, and the process is repeated until the target probability is achieved.

5.2 Computational Results for the Decreasing Z Approach

Computational results for the decreasing approach is presented for IEEE 30- and IEEE 118-bus test system [30]. The base case loading levels are the same as given in the test system. A reserve capacity of 5% is added to the load for both systems. For the IEEE 30-bus system, a load growth of 3% per year is assumed while for the 118-bus system, the load growth is assumed to be 40 % per year. The uncertainty in the load is modeled as a normal distribution satisfying the three standard deviation criterion $3\mu_{fi} = 0.25\sigma_{fi}$, where μ_{fi} and σ_{fi} are the mean and standard deviation of the normalized load growth.

The line limits of the 30-bus system were modified as follows. The transfer capacity of the line connecting buses 5 and 7 was reduced to 0.45 from 0.7, and the capacity of the line between buses 6 and 8 was reduced to 0.28 from 0.32. The results for the 30-bus system were computed with these modified line limits. The 118 bus system has sufficient transmission line capacity and there were no modifications made to this system.

The simulation uses the system configuration, transmission line characteristics, generators capacity limits and transmission line flow limits as given in the test system. The installation cost for a new unit is $26,000/MW [31], and the cost of production for both existing and new generation units is 45$/MWh [31]. The MCS generates 1,000 samples of normally distributed demand per run. The iterative algorithm stops when P_{feas} is within ±0.5% of the target probability. The MILP problems were solved using the BONMIN [32] solver accessed via GAMS [33].

Figures 2 and 4 show the reductions in cost achieved using the increasing Z approach as well as the decreasing Z approach, for various levels of reliability. We see that the decreasing Z approach generally achieves lower costs than the increasing Z approach.

The Figures 3 and 5 show the final values of Z at each bus for the decreasing Z approach with a 92 % target probability in each of the test systems. We see that the Z value at the non-stressed buses decreases from Z_l whereas at other buses it is maintained at Z_l.

5.3 Combined Z-Update Approach

Since both the increasing and decreasing Z approaches are advantageous compared to uniformly updating Z, we now consider combining both approach into a single approach.

We proceed as follows. After running the initial MCS to compute the success probability, the infeasible and feasible scenarios are separated. The samples of load are modified using the respective slack variables and a set of optimization is carried out. From these results, both the stressed and non-stressed buses are determined. This classification is done after the first iteration. Once the classification is made, it is maintained for all other iterations to ensure that the Z Z value does not oscillate around Z_l. A bus that was neither stressed nor non-stressed in the first iteration can become either of them in the subsequent iterations. But if it was a stressed bus in the first iteration, it is not allowed to later become a non-stressed bus, and vice-versa. The Z updates at the stressed buses are done using the increasing Z approach while those at the non-stressed buses use the decreasing Z approach.

The steps in the combined Z-update approach are:

1. The GEP is solved for two initial values Z_l and Z_h.
2. The probability of success with the initial points Z_l and Z_h is determined by using the MCS and OPF. The probability values are made sure to be less and more than the target probabilities: p_l and p_h respectively.
3. The infeasible and feasible samples are grouped separately and the modifications are done as follows: For the infeasible samples, the optimization described by equations (11a) to (11d) is done whereas for the feasible samples, the optimization is according to equations (13a) to (13d). Thus the type of bus and its stress level are determined by the slack value and slack ratio.
4. From the previous step, the classification of buses is known and maintained throughout the process. There can be new addition of buses to this classification during the iteration process but no modifications to the existing classification.
   Based on this classification,
   if ($s_i \neq 0$)

   Z is updated using (12)

   elseif ($m_i \neq 0$)

   Z is updated using (14)
5. With the new values of Z at each bus, the GEP is solved again. Line search is used in the stressed buses update and hence swapping of bounds is done after every update. The iterative process is repeated till the target probability is reached.

5.4 Results for the Combined Z-Update Approach

Computational results are again shown for IEEE 30- and IEEE 118-bus test system. The details of the system are the same as in Section 5.2. The penalty C_{pen} is set to 0.4 M$. Figures 6 and 7 show the comparison of

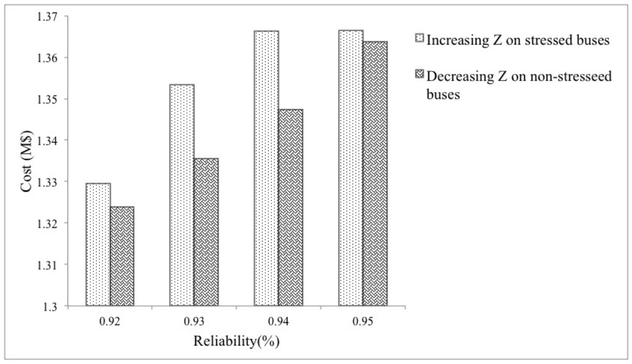

Figure 2: Costs for the 30-bus system

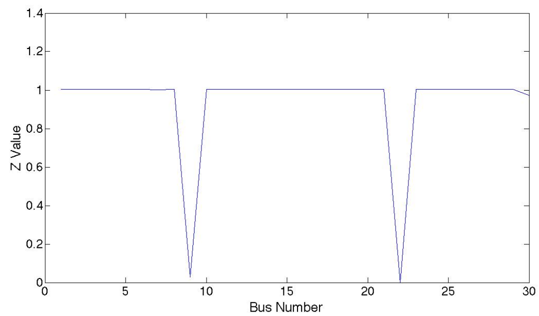

Figure 3: Final Z values for the decreasing Z approach for the 30-bus system with 92% target reliability

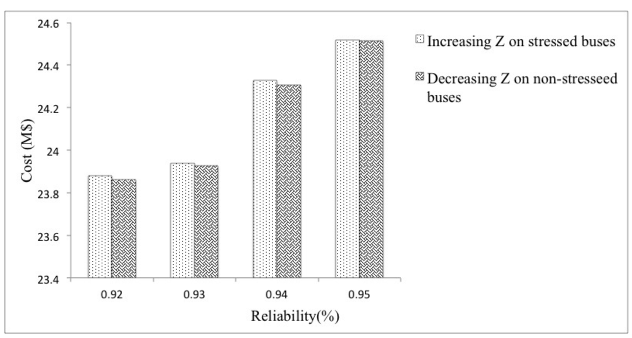

Figure 4: Costs for the 118-bus system

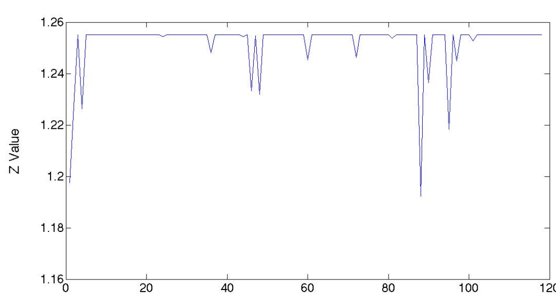

Figure 5: Final Z values for the decreasing Z approach for the 118-bus system with 92% target reliability

cost for all the Z-update methods presented. We see that the combined Z-update approach gives the most economic GEP solution.

The economic results are consistent with the results of the dynamic risk allocation. The Z value is increased only for those buses that are potentially problematic for reliability due to changes in load, and similarly it is reduced for those buses that are least affected to the changes in load.

Similarly, Figures 8 and 9 show the final value of Z using different approaches at each bus for IEEE 30- and IEEE 118- bus test system for a target probability of 92 %. It can be seen that the Z value is allowed to change in both directions around Z_l in the combined Z update approach thus identifying the stressed and non-stressed buses. The stressed buses are good candidates for new generation installation. It can also be seen that the bus identification using slack variables individually and in a combined way are very similar though not identical.

Tables 1 and 2 give the results using the combined Z approach for target probabilities between 92% and 95%. Because of the modified line limits, the 30-bus system faces congestion, and hence the existing units are not dispatched to their maximum capacity whereas in the118-bus system, the existing units are dispatched to their maximum. The cost increases for increasing reliability levels and thus CCP is able to give a relationship between profitability and reliability. It can also be seen that the achieved success probability is within the bounds allowed for the target probability.

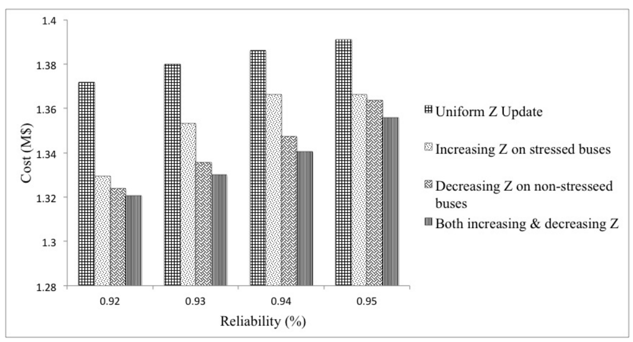

Figure 6: Costs for the 30-bus system

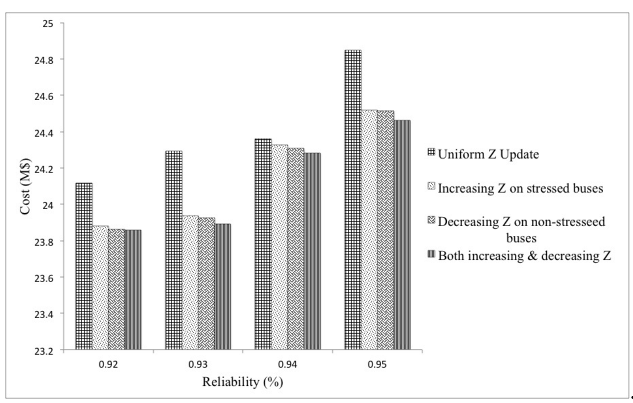

Figure 7: Costs for the 118-bus system

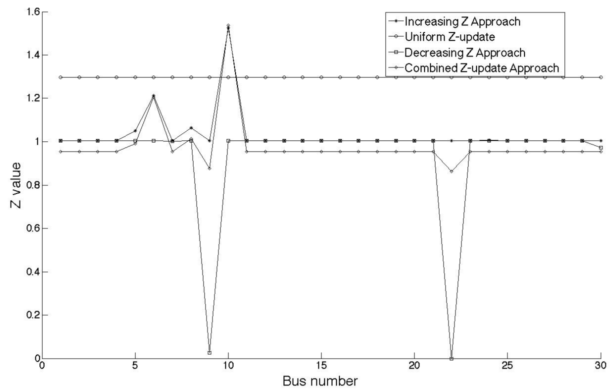

Figure 8: Final Z values for the 30-bus system with 92% target reliability

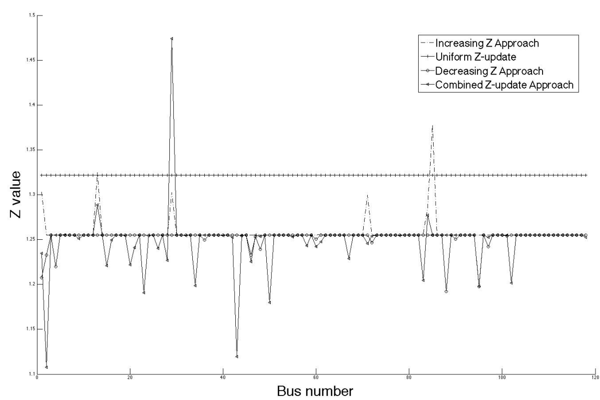

Figure 9: Final Z values for the 118-bus system with 92% target reliability

Table 1: Results for 30-bus system using combined Z-update

Target probability (α)	Demand	Existing Unit	New Unit	Cost (M$)	P_{feas}
0.92	3.3165	3.2646	0.0519	1.3208	0.9220
0.93	3.3307	3.2651	0.0655	1.3300	0.9300
0.94	3.3471	3.2657	0.0814	1.3406	0.9440
0.95	3.3701	3.2655	0.1045	1.3557	0.9510

Table 2: Results for 118-bus system using combined Z-update

Target probability (α)	Demand	Existing Unit	New Unit	Cost (M$)	P_{feas}
0.92	59.5391	58.04	1.4991	23.860	0.9190
0.93	59.5912	58.04	1.5512	23.894	0.9270
0.94	60.1788	58.04	2.1472	24.284	0.9360
0.95	60.5292	58.04	2.4892	24.508	0.9450

6 Conclusion

The GEP with load uncertainty was modeled using chance constrained programming, and new iterative solution algorithms identifying the stressed and non-stressed buses in the system were developed to solve this model. The algorithms dynamically allocate risk to each bus depending on the response of the system to the load behaviour. Computational results for the IEEE 30- and 118- bus systems are presented and compared with previous approaches. It is observed that expansion solutions meeting the required reliability at lower cost are obtained using this approach.

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