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Abstract

In this paper, we empirically compare open-loop and closed-loop investment strategies in production capacities in the three-player Finnish electricity industry. The closed-loop equilibrium is approximated using a moving-horizon approach. Investment levels are compared first in a deterministic setting and next in a stochastic environment. The results show that the choice of information structure does not really matter when the dynamic game is deterministic, whereas it significantly affects the equilibrium strategies when the demand is uncertain.

Key Words: Investment Dynamics, Electricity Market, Oligopoly, Dynamic Games, Open-Loop Nash Equilibrium, Sequential Closed-Loop Nash Equilibrium, Moving Horizon Control.

Résumé

Dans cet article, en utilisant les données finlandaises, les stratégies d'investissements obtenues sous la structure de l'information en boucle ouverte sont comparées avec celles obtenues sous la structure d'information en boucle fermée. L'équilibre en boucle fermée est approximé en utilisant la méthode d'horizon mobile. La comparaison a été réalisée à la fois dans un cas déterministe et dans un environnement stochastique. Les résultats montrent que le choix de la structure de l'information n'est pas déterminant dans un cadre de jeu dynamique déterministe. Par ailleurs, il joue un rôle de premier plan quand les joueurs font face à une demande incertaine.

Mots clés : Investissements; Marchés de l'électricité; Oligopole; Jeux dynamiques; Équilibres de Nash en boucle ouverte; Équilibres de Nash en boucle fermée; Horizon mobile.

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1 Introduction

For almost a century, the business model of the electricity industry was one of a vertically integrated utility with the mandate of producing and delivering power to its customers at the lowest cost. Investment decisions in production capacities were mainly based on long-term demand forecasts that also took into account a desired reliability level. Since deregulation, the industry in many countries (states or regions) now involves a handful of firms competing for profits. Investment decisions are no longer made by solving a dynamic cost-minimization problem, but as an equilibrium of a noncooperative dynamic game. It is well-known, see., e.g., Basar and Olsder (1995) or Dockner et al. (2000) that the equilibrium levels (output, investment, etc.) and outcomes of a dynamic game critically depend on the information used by the players to make their decisions. In an *open-loop* information structure, each player's information set contains the current calendar date and initial values of the state variables (e.g., production capacity). In a *closed-loop* information structure, each player's information set contains the current calendar date and the current and the history of the states. And lastly, in a feedback (or Markovian) information structure, the information set contains the current calendar date and current states. Determining open-loop equilibrium strategies is much easier than computing their closed-loop or feedback counterparts which are, for many real, and even realistic problems, practically out of reach. Because an open-loop strategy chooses the control actions as a function of time only, the underlying problem is a one-stage (static) game. In such a case, one can easily write the first-order equilibrium conditions for a Nash equilibrium as a mixed complementarity problem for which efficient algorithms exist. This explains the popularity of open-loop approach in the literature in general. The drawback is that the corresponding equilibrium is not subgame perfect, and, therefore, is conceptually less appealing than the closed-loop and feedback equilibria. In fact, an open-loop strategy implies that the firm has perfect foresight that allows it to commit at time zero to a fixed time path for its control actions. Perfect foresight, however, is a strong assumption in practice, especially in the new deregulated electricity markets.

The literature on capacity expansion in the electricity sector, to which this paper naturally belongs, is of relatively recent vintage and includes, e.g., Chuang et al. (2001), Ventosa et al. (2002), Pineau and Murto (2003), Murphy and Smeers (2005), Ehrenmann and Smeers (2006), Bushnell and Ishii (2007), Genc et al. (2007), Genc and Sen (2008), Lise and Krusemann (2008), Pineau et al. (2009, 2010). These studies have in common an oligopoly model of the power-generation sector and they are all interested in analyzing the strategic behavior of the players (generators).

Chuang et al. (2001) is one of the first attempts to understand the dynamics of capacity expansion in a competitive electricity industry. The authors assume a Cournot-based expansion model and compute the equilibrium using an iterative-search procedure. Ventosa et al. (2002) present two different approaches to model expansion planning in electricity markets under imperfect competitive conditions. The first model is à la Cournot and is formulated as a Mixed Complementarity Problem (MCP), wherein each firm not only decides its output but also its new generating capacity. The second model is à la Stackelberg and is formulated as a Mathematical Program with Equilibrium Constraints (MPEC), in which the leader firm decides its capacity expansion, subject to a complementarity problem that defines the market equilibrium in terms of prices and quantities. Using a deterministic framework, Ventosa et al. (2002) find minor differences between the two approaches. Pineau and Murto (2003) propose a stochastic dynamic oligopolistic model for the Finnish electricity market. They assume a Cournot-based model and use a variational-inequality approach to compute the equilibrium evolving along a sample-path-adapted, open-loop information structure. Murphy and Smeers (2005) consider two capacity-expansion games à la Cournot under an open-loop and a closed-loop information structure, respectively. In the open-loop game, the players make both capacity and quantity decisions simultaneously, whereas in the closed-loop game, a two-stage structure is assumed. The players choose their capacity levels in the first stage and compete in quantity in the second stage. As would be expected, the corresponding equilibrium results differ, but, interestingly, open-loop and closed-loop equilibria share some common features. Additionally, it is shown that the closed-loop prices and quantities are lower than their open-loop counterparts. Ehrenmann and Smeers (2006) pursue the comparison of one-stage and two-stage games in the same context of capacity expansion in the electricity industry. The main takeaway point from this paper is that, even if one assumes a deterministic demand, the existence of old power plants prevents the one and two-stage models from being equivalent. Bushnell and Ishii (2007) propose a framework for modeling investments in restructured electricity markets, in which firms make lumpy investment decisions. They adopt a model with two components–a detailed model of short-run, or spot-market competition in electricity markets, and a dynamic long-run equilibrium model of investment decisions of firms– and compute the Markov-perfect equilibrium investment levels in an oligopoly. The investment choices made by the firms will be driven by the underlying profits implied by the short-term markets under different investment paths. Firms will choose the investment paths that lead them to more profitable short-term states. In the same vein, Lise and Krusemann (2008) distinguish between two separate sets of decisions, in order to assess the effect on price and on the environment, of endogenous investment decisions in a liberalized electricity market. While the first set of decisions relates to the supply of electricity in the short run, the second set of decisions is associated to the investment in electricity production capacity. Investment decisions are based on the feedback information structure. This means that, in each period, electricity producers make their investment decisions according to the most recent information. Lise and Krusemann (2008) then extend the time horizon of the firm by one period and calculate the investments in capacity required to meet the expected next-period demand. In the next period, installed capacity is adjusted and where new capacity is added, depreciated capacity is subtracted from the amount of installed capacity in the previous period– and the two-period model is run again. The approach in Lise and Krusemann (2008) is the used again by Lise and Hobbs (2008, 2009) to study the liberalized European natural-gas market. Genc et al. (2007) and Genc and Sen (2008) use the tools of stochastic programming to study capital-investment dynamics and the pricing behavior of suppliers as uncertain electricity demand evolves over time in the Ontario (Canada) electricity market. In these papers, the *S-adapted open-loop Nash equilibrium* (see Haurie et al. (1990) and Haurie and Zaccour (2004)) is computed using Games with Probabilistic Scenarios (GPS) where players make production and investment decisions based on a collection of probabilistic scenarios. Pineau et al. (2009) propose a deterministic, discrete-time, finite-horizon oligopoly model to investigate investment and production equilibrium strategies, in a setting where demand evolves over time and the two market-segment loads (peak and base loads) are interdependent. They use data from Ontario to calibrate the model and assess the impact on the equilibrium strategies of a generation sector with more market power than the actual situation. Pineau et al. (2010) explore the impact of various modeling features on production and investment equilibria in an oligopolistic electricity market, using as a reference model the one in Pineau and Murto (2003). Pineau et al. (2010) obtain that depreciation rate, demand elasticity and time horizon have a clear impact on the results, but that using two dependent or independent market segments does not much alter the model's outcomes.

Outside the area of the capacity-expansion problem in deregulated electricity markets, there is a significant literature comparing closed-loop/feedback and open-loop strategies. In capital-accumulation games, where the focus is on investment-commitment values, the literature includes, e.g., Spence (1979), Dixit (1980), Fershtman and Muller (1984), Reynolds (1987), Cellini and Lambertini (1998), Dockner et al. (2000), Figuières (2002, 2009) and Genc and Zaccour (2009). Kossioris et al. (2008) and Long et al. (1999) make this comparison in environmental games and resource-economics games (see also Jørgensen et al. (2010), and Piga (1998), Erickson (2003) and Breton et al. (2006) for advertising games). Noticeable common denominators of a large majority of these contributions are the linear-quadratic framework,[1] and the limitation to two symmetric firms. As (theoretically is) expected, the authors unanimously find that open-loop and feedback equilibria do not coincide. However, their results are different in terms of investment ranking. For instance, Reynolds (1987) finds, in an infinite-horizon differential game, that Markov strategies increase competition, i.e., Markov-perfect equilibrium investments exceed the open-loop ones; whereas, Figuières (2002) and Genc and Zaccour (2009) obtain that Markov behavior softens competition. More specifically to our context, Murphy and Smeers (2005) find that the closed-loop game may not have an equilibrium. When it does, the equilibrium is unique and the total capacity in the closed-loop equilibrium is at least as large as the total capacity in the open-loop equilibrium. These results are confirmed by Ehrenmann and Smeers (2006), in the context of deterministic demand and the existence of old power plants. Genc and Thille (2009)

[1] In a linear-quadratic dynamic game, the objective function is quadratic and the dynamics are linear. An interesting feature of these games is that the player's value function is quadratic and the strategies are linear in the state.

characterize both Markov-perfect and open-loop equilibria when analyzing competition between hydro- and thermal-power generators under demand uncertainty. In the Markov-perfect equilibrium, they find that investment is discontinuous in initial capacity and higher than in the open-loop equilibrium. A common feature of this literature is that the comparison is done considering two-stage models, an assumption made to ease the computation burden.

The aim of this paper is to empirically compare open- and closed-loop investment decisions using data on the Finnish market. Our contribution to the literature on dynamic games and on electricity markets is fourfold. First, the model is nonlinear (and is not linear-quadratic) which is a notable deviation from what has often been assumed. Second, the comparison of the two equilibrium results is carried out in both a deterministic and a stochastic framework. This will enable us to isolate, *ceteris paribus*, the impact on the results, of using uncertainty. Third, the model is a truly multi-period model, allowing for many instances of investment decisions rather than one as in some two-stage models. Finally, for the first time in this area, to the best of our knowledge, we approximate the closed-loop equilibrium of the dynamic oligopolistic electricity-market model using a moving horizon (MH) approach. The later consists of solving, at each point in time, an open-loop Nash equilibrium (OLNE), in which the players determine their optimal strategies for the next T periods, but only implement the initial control actions. The use of this approach is discussed in van den Broek (2002), with special attention to the class of linear-quadratic (LQ) games. An interesting empirical application is found in Yang (2003) where the MH-solution concept is used to study the issues of reevaluation and renegotiation of climate-change coalitions.

As a laboratory for our computational experiments, we use a deterministic (resp. stochastic), dynamic, discrete-time, finite-horizon model of the Finnish market. In the case of stochastic demand, an event tree is used to describe the (uncontrolled) uncertainty affecting the market demand. The resulting closed-loop equilibrium-approximation strategies are then contrasted with the open-loop equilibrium ones in order to establish their similarities and differences. Our main results are:

1. When demand is deterministic, OL and CL strategies lead to very similar investment levels for the first periods. This means that in the "short" run, it does not matter much, investment-wise, which information structure the players are using.
2. When demand uncertainty is introduced, we do find differences between the two equilibria, with the result that closed-loop strategies soften competition, i.e., investment levels under an open-loop information structure exceed their closed-loop counterparts.

The rest of the paper is organized as follows. In Section 2, we present the deterministic dynamic model. A short discussion on the solution concept is provided in Section 3. We present a numerical example in Section 4, and briefly conclude in Section 5.

2 Dynamic Model

In this section, we introduce, in relatively general terms, a deterministic dynamic model for a deregulated electricity market. Denote by $M = \{1, ..., m\}$ the set of players (producers), by J the set of available technologies (hydro, nuclear, coal, gas-fired turbine, etc.) and by L the set of load periods. In principle one may consider a larger number of load periods and technologies. We follow the literature and focus on only two periods, namely, the base-load period (b) and peak-load period (p); and on two technologies, i.e., base-load technology (b), such as hydro or nuclear, and peak-load technology (p), e.g., natural gas or renewable technologies. Therefore, we have $J = \{b, p\}$ and $L = \{b, p\}$. Time t is discrete, $t = 0, ..., T$, where T is the planning horizon.

Denote by $q_{ijl}(t)$ the quantity of energy (in MWh) produced by player i using technology j during load period l at period t. Let $q_{ij}(t) = \sum_l q_{ijl}(t)$ be the total quantity produced using technology j at period t and $Q_l(t) = \sum_i \sum_l q_{ijl}(t)$ be the total available quantity in load period l at period t. We suppose that the price during load period l depends linearly on the total quantity put on the market during that load period, i.e.,

$$P_l\left(Q_l\left(t\right)\right) = A_l\left(t\right) - B_l Q_l\left(t\right), \quad t = 0, ..., T-1, \tag{1}$$

where $A_l\left(t\right)$ and B_l are positive parameters. We assume that $A_l\left(t\right)$ evolves over time according to

$$A_l\left(t\right) = \left(1+g\right)^t A_l\left(0\right),$$

where g is a given positive parameter.

Denote by $I_{ij}\left(t\right)$ the investment in technology j (expressed in MW) made by player i at period t, and by $k_{ij}\left(t\right)$ the installed capacity. Assuming a one-period lag before an investment becomes productive, the evolution of the production capacity of player i of type j is then described by the following state equation:

$$k_{ij}\left(t\right) = (1-\delta_j)k_{ij}\left(t-1\right) + I_{ij}\left(t-1\right), \quad k_{ij}\left(0\right) = k_{ij}^0, \tag{2}$$

where k_{ij}^0 denotes the initial capacity, and δ_j, $0 \leq \delta_j < 1$, is the capacity-depreciation rate. The energy production during load period l is subject to available capacity, i.e.,

$$q_{ij}\left(t\right) \leq h_l k_{ij}\left(t\right), \quad l \in \left\{b,p\right\}, \quad t = 0, ..., T-1, \tag{3}$$

where h_l is the number of operating hours during load period l.

As in Andersson and Bergman (1995), we adopt the following functional forms for the production costs:

$$G_{ibl}\left(q_{ibl}\left(t\right), k_{ib}\left(t\right)\right) \triangleq \alpha_b q_{ibl}\left(t\right) \tag{4}$$

$$G_{ipl}\left(q_{ipl}\left(t\right), k_{ip}\left(t\right)\right) = \alpha_p q_{ipl}\left(t\right) + \nu_p\left(\frac{h_l k_{ip}\left(t\right)}{\phi+1}\right)\left(\frac{q_{ipl}\left(t\right)}{h_l k_{ip}\left(t\right)}\right)^{\phi+1} \tag{5}$$

where α_b, α_p and ν_p are positive parameters and $\phi > 1$. In (4), we assume that the total cost is independent of the total installed capacity, and that the marginal production cost is constant. The production cost function in (5) is jointly convex in both its arguments and assumes that the marginal cost is increasing with production and decreasing with capacity, i.e., the higher the available capacity, the lower the cost for producing a given quantity. Hence, a thermal operator needs to commit, as demand grows, to higher-cost production units to satisfy it. Further, an implicit assumption is that the output of the two technologies (base- and peak-load capacities) is homogenous, that is, the total power produced during the load period l is given by

$$Q_l = \sum_j \sum_s q_{jsl}.$$

In addition, we suppose that the total production cost of generator i at load period l is additively separable in technologies. That is,

$$G_{il}\left(q_{ib}\left(t\right), q_{ip}\left(t\right), k_{ib}\left(t\right), k_{ip}\left(t\right)\right) = G_{ibl}\left(q_{ibl}\left(t\right), k_{ib}\left(t\right)\right) + G_{ipl}\left(q_{ipl}\left(t\right), k_{ip}\left(t\right)\right)$$

Following the literature (see, e.g., Ventosa et al. (2002), Murphy and Smeers (2005), Ehrenmann and Smeers (2006), Lise and Krusemann (2008)), we assume that the investment cost function $\Gamma_{ij}\left(I_{ij}\left(t\right)\right)$ is linear and given by

$$\Gamma_{ij}\left(I_{ij}\left(t\right)\right) = \theta_j . I_{ij}\left(t\right) \quad j \in \left\{b,p\right\},\ t = 0, ..., T-1,$$

where $\theta_j > 0$.

Denote by $\beta_i, 0 \leq \beta_i < 1$, the discount factor of player i. Assuming profit-maximization behavior, player i the faces the following optimization problem,

$$\max \Pi_i = \sum_{t=0}^{T-1} \beta_i^t \sum_{j \in J}\left(\sum_{l \in \left\{b,p\right\}}\left[q_{ijl}\left(t\right) P_l\left(\cdot\right) - G_{ijl}\left(\cdot,\cdot\right)\right] - \Gamma_{ij}\left(\cdot\right)\right) \tag{6}$$

s.t. (2), (3), and

$$\text{Non-negativity constraints} \quad : \quad I_{ij}\left(t\right) \geq 0,\ q_{ijl}\left(t\right) \geq 0, j \in J, l \in \left\{b,p\right\}, t = 0, ..., T-1,$$

where the inverse-demand laws are given by (1).

3 Solution Concepts

Our objective is to compute and contrast open-loop and closed-loop Nash equilibria for representative instances of capacity-expansion games in the electricity industry. The closed-loop equilibrium is approximated using a moving-horizon approach. The principle is simple: At each period t, the players determine the open-loop Nash-equilibrium strategies for a given T -period planning horizon. However, only the initial control action is implemented. At period $t+1$, the players again compute the equilibrium strategies for the next T periods, implement the (new) initial action, and so on. The resulting moving-horizon equilibrium trajectories constitute an approximation of the closed-loop equilibrium trajectories that would have been obtained at the outset of the dynamic game.

Denote by u_i the vector of decision variables of player i, that is,

$$u_i = \left[\left(q_{ijl}\left(t\right), \forall j, l \in \{b, p\}, t = 0, ..., T\right); \left(I_{ij}\left(t\right), \forall j \in \{b, p\}, t = 0, ..., T-1\right)\right],$$

and by U_i the set of feasible actions of player i. Let $U = \Pi_{i \in M} U_i$, be the feasible set of all players. The vector $u^* = (u_1^*, \ldots, u_m^*)$ is an open-loop Nash equilibrium if

$$\Pi_i\left(u^*\right) \geq \Pi_i\left(u_1^*, \ldots, u_{i-1}^*, u_i, u_{i+1}^*, \ldots, u_m^*\right), \forall u_i \in U_i, \forall i \in M.$$

Given that the payoff function of each player is concave in its actions and that the feasible set is convex, there exists at least one open-loop Nash equilibrium. Further, the first-order equilibrium conditions for a Nash equilibrium constitute a mixed-complementarity problem (MCP) for which efficient algorithms exist. In particular, we will use the PATH solver for the case study.

The closed-loop strategy profile is the following triplet:

$$S_i\left(\left\{u_i\left(t\right) | \Omega\left(\tau \leq t\right)\right\}, \Pi_i, S_{-i}\right), \ \forall \iota \in M,$$

where Ω is the information set at time t and subscript $-i$ represents all agents but i. The set Ω includes the current states involving demand and capacity states, the distribution of future demand, and the history of the states. Note that, unlike the pre-committed open-loop strategies, the closed-loop strategies are subgame perfect where players are allowed to condition their decisions at time t based on the history of the states up to time t.

Following Yang (2003), we denote the sets of strategy profiles (trajectories of all state and control variables) of open- and closed-loop solutions started at $t = 0$, in semi-closed intervals $[a, b)$ as $SP_0^O\left\{[a, b)\right\}$ and $SP_0^C\left\{[a, b)\right\}$, respectively. Hence, we can decompose the entire trajectories of open- and closed-loop strategies into the unions of piecemeal sections. That is,

$$\begin{aligned} SP_0^O\left\{[0, \infty)\right\} &= SP_0^O\left\{[0, 1)\right\} \cup SP_0^O\left\{[1, 2)\right\} \cup SP_0^O\left\{[2, 3)\right\} \\ &\quad \cup ... \cup SP_0^O\left\{[t, t+1)\right\} \cup ... \end{aligned}$$

and,

$$\begin{aligned} SP_0^C\left\{[0, \infty)\right\} &= SP_0^C\left\{[0, 1)\right\} \cup SP_0^C\left\{[1, 2)\right\} \cup SP_0^C\left\{[2, 3)\right\} \\ &\quad \cup ... \cup SP_0^C\left\{[t, t+1)\right\} \cup .... \end{aligned}$$

By the optimality principle (see, e.g., Beevis and Dobbs (1990)), we clearly have

$$SP_0^C\left\{[0, 1)\right\} = SP_0^O\left\{[0, 1)\right\}. \tag{7}$$

At the beginning of the planning horizon, the open- and closed-loop equilibrium problems share identical initial conditions. The information sets of the two strategies at $t = 0$ are also identical. Based on (7), Yang

(2003) showed that a closed-loop strategy in $[0, \infty)$ can be expressed as a union of the subsets of distinctive open-loop strategies in a discrete time case. Formally,

$$\begin{aligned} SP_0^C \{[0,\infty)\} &= SP_0^O \{[0,1)\} \cup SP_0^O \{[1,2)\} \cup SP_0^O \{[2,3)\} \\ &\quad \cup ... \cup SP_0^O \{[t,t+1)\} \cup .... \end{aligned} \tag{8}$$

To implement (8) we use the following algorithm.[2]

Step 1: Solve simultaneously the optimization problem (6) for all players, which starts at $t = 0$ and ends at $t = T$. Only the decisions that affect the actions in $t = 1$ are placed in the set of closed-loop strategies, i.e., the actions in $t = 0$.

Step 2: When time moves to $t = 1$, firms will re-evaluate their decisions made at $t = 0$ according to the new available information. In that case, they will restart new open-loop decision processes from $t = 1$ to $t = T + 1$ with the new information. In the strategy profile of this new open-loop solution, only the actions in $t = 1$ are incorporated into the closed-loop solution. Similar decision processes repeat at $t = 2, 3, ..., T$. The first periods in each sequential open-loop decision make up the closed-loop strategic profiles.

4 Empirical Analysis

We are interested in empirically assessing the extent to which information structure affects electricity producers' investment decisions. To do so, we apply the model in Section 2 to the Finnish industry, where competition was introduced in 1995 (see, e.g., Pineau and Hämäläinen (2000) for a description of the deregulation of the Finnish electricity market). The comparison between open-loop and closed-loop investment levels will be carried out under different experimental conditions related to planning horizon, demand elasticity, demand nature (deterministic and stochastic) and discount rate. We focus on comparing the investment-equilibrium levels of the first six five-year periods. We believe that this is more than enough to characterize qualitatively and quantitatively the differences between the two information structures. Actually, what really matters is how open- and closed loop approaches affect "today's" decisions.

4.1 Base Case

In 1996, the main player in Finnish electricity was the state-owned company Fortum (known at the time as Imatran Voima Oy, or IVO), which supplied approximately 30% of Finnish electricity consumption. The other players were industrial and municipally owned energy firms that produced electricity mainly for their own usage. A large number of these industrial firms were grouping their production under a common organization, Pohjolan Voima Oy (PVO), that supplied approximately 20% of the total consumption. In the sequel, we shall refer to the remaining producers as Player 3. Currently, the Finnish industry still has the same structure, but is more integrated with its Nordic neighbors (Sweden and Norway), and is engaged in developing its interconnection lines with the Baltic countries (Estonia and Latvia) and Russia.[3] Between 1996 and 2006, the total installed capacity increased by 10%, whereas the Finnish production grew by 18% and consumption by 29% (Nordel, 1996 and 2006). The average spot price of electricity increased from about 20 euro/MWh in 1996 to 48 euro/MWh in 2006.[4] These numbers tend to say that this decade of open market has clearly been good for electricity producers, with a more intense use of their production capacity and higher prices.

[2] This algorithm of sequential open-loop equilibria is valid and robust (Yang (2003)).

[3] "In April 2003, power companies from Estonia, Latvia and Finland signed an agreement for the laying of a 315-MW underwater electricity cable linking Finland and Estonia. The "Estlink" line is part of the "Baltic Ring" project and became operative in the beginning of 2007. It enables the Baltic States to be integrated into the Inter-Nordic electricity market. (...) In addition, Finland is also planning the laying of an underwater cable to connect the Leningrad NPP west of St. Petersburg to Finland. The project, which will cost about $350 million, was approved in 2004 by the Finnish foreign and trade and industry ministries, and by the Russian government." Foratom (2009).

[4] It is worthwhile to note that, in Finland, prices were affected by such exogenous factors as the level of water reservoirs and the related impact on electricity supplied by hydro-power generators, as well as the price of emissions rights. For example, due to abundant hydro power and cheap emission rights in 2007, the average day-ahead area price for Finland in 2007 was 30.01 euros per MWh compared with 48.57 euros in 2006 (EMA, 2008).

As a base case, we adopt the parameter values in Pineau and Murto (2003) and Pineau et al. (2010). These values are provided in Table 1. Note, in particular, that the planning horizon in the base case is forty years (8 five-year periods), and that all players face the same cost- and demand-parameter values. This symmetry assumption is fully justified here as our objective is to focus on differences that are due, and only due, to varying the information structure adopted by the players when determining their investments. The only asymmetric feature of the model lies in the endowment of initial capacities. We shall assess later on the impact on investment strategies of varying some of the parameter values.

Table 2 provides the predicted-equilibrium investment levels computed under the two information structures. One striking observation is that little difference (less than 1%) is observed between the OL- and MH-investment levels during the first half of the planning horizon (the first four five-year periods). In contrast, we do observe, not only significant quantitative differences, but also qualitative ones during period 5 and 6. Indeed: (i) The players invest more in a closed-loop equilibrium; (ii) while no investment is made in base-load technology, in period 6 of the open-loop solution, the three players in the closed-loop equilibrium do invest in such technology. One interpretation of these results is that firms are less farsighted (or more sensitive to the "finite-horizon"effect) when playing open-loop than when adopting an MH strategy. Actually, in the later case, the players behave as if the planning horizon were infinite.

Table 1: Parameter Calibration

Category	**Parameter**	**Description**	**Value**
Players[5]	m	Number of players	3
	$k_{1b}\left(s^0\right)$	Fortum's initial installed base-load capacity	3,250 MW
	$k_{1p}\left(s^0\right)$	Fortum's initial installed peak-load capacity	4,000 MW
	$k_{2b}\left(s^0\right)$	PVO's initial installed base-load capacity	1,950 MW
	$k_{2p}\left(s^0\right)$	PVO's initial installed peak-load capacity	2,800 MW
	$k_{3b}\left(s^0\right)$	Player3's initial installed base-load capacity	90 MW
	$k_{3p}\left(s^0\right)$	Player3's initial installed peak-load capacity	3,710 MW
Demand	ε_b	Base-load price elasticity	-0.6
	ε_p	Peak-load price elasticity	-0.9
	$P_b\left(s^0\right)$	Base-load price at $t=0$	16.82 €/MWh
	$P_p\left(s^0\right)$	Peak-load price at $t=0$	33.64 €/MWh
	g	(Yearly) Demand growth rate	1.5%
Production Cost	α_b	Base load marginal production cost	4.20 €/MWh
	α_p	Low-peak-load marginal production cost	15.14 €/MWh
	$\alpha_p+\nu_p$	High-peak-load marginal production cost	40.36 €/MWh
	ϕ	Convexity parameter	2
Investment Cost[6]	θ_b	Base-load investment cost	1,700,000 €/MWh
	θ_p	Peak-load investment cost	340,000 €/MWh
Others	h_b	Number of hours of base-load period	7,008 hours
	h_p	Number of hours of peak-load period	1,752 hours
	β	Discount factor	0.95
	T	Time length	8 periods of 5 years
	δ_b	Base-load capacity depreciation rate	0.025
	δ_p	Peak-load capacity depreciation rate	0.05

[5] We refer here to the base case in Pineau and Murto (2003). In this case, it is assumed that some of the fringe capacity is divided between Fortum and PVO, and also, that all the nuclear and hydro capacity comes under their control. The rest of the fringe becomes a third strategic player and gets one-third of the thermal capacity. The market's merger and acquisition pressures justify this scenario.

[6] These values are used to calibrate the non-linear investment-cost parameters by using the same approach as in Appendix B of Pineau et al. 2009.

Table 2: Base Case - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL	Fortum	Base			233/235	694/696	653/692	0/573
		Peak						955/644
	PVO	Base		458/463	719/717	694/696	653/692	0/573
		Peak					314/266	970/696
	Player 3	Base	1,335/1,335	726/731	719/717	694/696	653/692	0/573
		Peak					283/235	970/696

4.2 Sensitivity Analysis

To verify whether the result obtained above is robust, we vary the values of some of the model's key parameters. These changes involve: (i) the length of the planning horizon T; (ii) the depreciation rates of base- and peak-load production capacities; and (iii) the demand coefficients. To save on space, we only include the results for a few modifications of the base case. Repeating the same exercise for different parameter values and for different parameters, did not qualitatively change the conclusions.[7]

Impact of the planning horizon: Increasing the number of periods from 8 to 10 (i.e., lengthening the planning horizon from 40 to 50 years) leads to the results in Table 3. As in the base case, we obtain that the OL and MH strategies only differ toward the end of the planning horizon. Note that less than 10 MW of capacity separate the two strategies throughout the planning horizon. Comparing Tables 2 and 3 shows that investment decisions based on moving-horizon strategies are horizon-independent. This is a by-product of the fact that the algorithm behind the MH strategy is implicitly based on an infinite-horizon planning horizon.

Table 3: Ten-Period Planning Horizon - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL	Fortum	Base			233/235	694/696	690/692	580/573
		Peak						645/644
	PVO	Base		458/463	719/717	694/696	690/692	580/573
		Peak					277/266	689/696
	Player 3	Base	1,335/1,335	726/731	719/717	694/696	690/692	580/573
		Peak					246/235	689/696

Impact of the capacity-depreciation rates: Recall that, in the base case, the depreciation rates of base- and peak-load production capacities are fixed at $\delta_b = 0.025$ and $\delta_p = 0.05$. These rates correspond to a life duration of 40 and 20 years, respectively. Table 4 shows the results for zero-depreciation rates, that is, when both peak- and base-load capacities have an infinite life duration. Table 5 provides the investment levels for depreciation rates equal to half of those retained in the base case, i.e., $\delta_b = 0.0125$ and $\delta_p = 0.025$. As replacing ageing capacity becomes less a requirement, we obtain the fully expected result that the players invest less in the two scenarios being considered than in the base case. In terms of difference between the OL and CL investments, we again observe that the two information structures yield the same investment levels in the early periods, but differ as they near the end of the planning horizon. Comparing the results of Table 4 and 5 shows that the lower the depreciation rates, the earlier OL and CL start differing significantly.

Impact of demand parameters: The inverse demand law is given by

$$P_l\left(Q_l\left(t\right)\right) = \left(1+g\right)^t A_l\left(0\right) - B_l\left(t\right) Q_l\left(t\right),\ t = 0, ..., T-1,$$

[7] Numerical results are available from the authors upon request.

Table 4: Zero Depreciation Rate - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL	Fortum	Base						0/41
		Peak						
	PVO	Base		102/105	271/269	71/299	0/326	0/342
		Peak						
	Player 3	Base	1,698/1,698	263/267	271/269	71/299	0/326	0/342
		Peak						

Table 5: Low Depreciation Rate - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL	Fortum	Base				248/250	408/546	0/533
		Peak						
	PVO	Base		268/272	523/521	532/533	408/546	0/533
		Peak						446/291
	Player 3	Base	1,485/1,485	513/517	523/521	532/533	408/546	0/533
		Peak						394/240

with $g = 1.5\%$ and $B_l(t) = B_l, \forall t$, in the base case.[8] Tables 6 and 7 give the investment levels for $g = 0$ and $g = 3\%$, respectively. The same observation applies as previously, namely, that the two strategies prescribe very similar investment levels in early periods but diverge later on. In the zero-growth scenario, OL investments are very low in period 5 and non-existent in period 6. In the high growth scenario, investments are high toward the end of the planning horizon, which is a direct consequence of the growing demand, with an emphasis on less-costly peak-load technology in the OL game. To interpret this result, recall that the investment cost is high in base-load technology, whereas the production cost is low, as compared with peak-load technology. In the OL equilibrium, as the planning horizon nears its end, peak-load technology is favoured because there is little time left to take advantage of the low production cost of base-load technology. However, the opposite happens in the MH strategy, where a longer planning horizon prescribes a greater investment in base-load technology.

Table 8 provides the results for a time-varying inverse-demand slope, that is,

$$P_l(Q_l(t)) = A_l(0) - B_l(t)\,Q_l(t)\,,\ t = 0, ..., T-1,$$

with

$$B_l(t+1) = \frac{B_l(t)}{(1+g)},$$

where $g = 1.5\%$. This amounts to assuming, as in Garcia and Shen (2010), that the slope of the inverse demand decreases over time to reflect the net effect of overall demand growth. Clearly, as before, OL and CL strategies are very close during the four first periods but then differ afterward.

[8] The parameters of the inverse-demand functions, i.e., A_l, B_l, $l \in \{b, p\}$, have been calibrated using the available information on prices, quantities and price elasticities computed at $t = 0$. The direct-price elasticity ε_l is given by:

$$\varepsilon_l = \frac{\partial Q_l}{\partial P_l} \cdot \frac{P_l}{Q_l},\ l = b, p.$$

Using the above definition, it can be easily shown that the parameters A_l, B_l, $l \in \{b, p\}$ can be expressed as follows:

$$A_l = P_l + B_l Q_l, \quad B_l = -\frac{P_l}{\varepsilon_l Q_l}$$

Table 6: No-Demand Growth Rate - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL)	Fortum	Base					0/149	0/283
		Peak						
	PVO	Base			262/262	357/357	39/320	0/283
		Peak						
	Player 3	Base	1,013/1,013	425/425	374/374	357/357	39/320	0/283
		Peak						

Table 7: High-Demand Growth Rate - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL	Fortum	Base		85/88	1,084/1,084	860/859	976/973	487/1,126
		Peak				1,014/1,007	1,667/1,661	2,571/2,100
	PVO	Base	36/36	1,063/1,065	1,084/1,084	860/859	976/973	487/1,126
		Peak			80/75	1,383/1,383	1,667/1,671	2,571/2,100
	Player 3	Base	1,675/1,675	1,063/1,065	1,084/1,084	860/859	976/973	487/1,126
		Peak			29/24	1,383/1,379	1,667/1,671	2,571/2,100

4.3 Stochastic Demand

Up to now, we have dealt with a deterministic setting. The assumption of absence of uncertainty is rather strong when the long term is involved. Therefore, we now relax this assumption and suppose, realistically, that demand growth forecasts are uncertain. One easy and appealing way to introduce (uncontrolled) demand uncertainty is through an event tree. Table 9 provides the ingredients for such a tree. As can readily be seen, we are assuming the simplest possible representation of an uncertain demand, namely, that each node has only two equally probable successors. This design is used purposely. Its rationale is that, if open-loop and closed-loop equilibria predict different investment levels in such a simple case, then it is safe to state that the two equilibrium strategies prescribe in general different investment paths in the class of stochastic dynamic games considered here.

The optimization problem of player i is given by

$$\begin{array}{rcl} \max \Pi_i & = & \displaystyle\sum_{t=0}^{T-1} \beta_i^t \sum_{s_n^t \in S^t} \pi\left(s_n^t\right) \sum_{j \in J} \left(\sum_{l \in \{b,p\}} \left[q_{ijl}\left(t, s_n^t\right) P_l\left(\cdot\right) - G_{ijl}\left(\cdot,\cdot\right) \right] - \Gamma_{ij}\left(\cdot\right) \right) \\ & & \text{subject to:} \\ \text{State equation} & : & k_{ij}\left(t, s_n^t\right) = (1-\delta_j) k_{ij}\left(t-1, a\left(s_n^t\right)\right) + I_{ij}\left(t-1, a\left(s_n^t\right)\right), \quad k_{js}\left(0, s^0\right) = k_{ij}^0, \\ \text{Production capacity} & : & q_{ijl}\left(t, s_n^t\right) \leq h_l k_{ij}\left(t, s_n^t\right), \ l \in \{b,p\}, \quad s_n^t \in S^t, \quad t = 0, ..., T-1, \\ \text{Non-negativity} & : & I_{ij}\left(t, s_n^t\right) \geq 0, \ q_{ijl}\left(t, s_n^t\right) \geq 0, \ j \in J, l \in \{b,p\}, \ s_n^t \in S^t, t = 0, ..., T-1. \end{array}$$

where the contingent inverse-demand laws are given by

$$P_l\left(Q_l\left(t, s_n^t\right)\right) = A_l(t, s_n^t) - B_l Q_l\left(t, s_n^t\right) \quad \forall s_n^t \in S^t, \ t = 0, ..., T-1,$$

in which $A_l\left(t, s_n^t\right)$ and B_l are positive parameters scaling the level of demand. The total available quantity on the market in load period l at node n is given by $Q_l\left(n\right) = \sum_i \sum_j q_{ijl}\left(n\right)$. Note that the production- and investment-cost functions are the same as in the deterministic model.

With 8 periods and two successors per node, the event tree has 128 equally probable different paths. Comparing investment levels produced by OL and CL along these paths is feasible but not instructive. We shall carry out this comparison for the path that corresponds to the deterministic base case setting. This has the merit of avoiding repetition and, more importantly, allows us to point out possible differences induced by

Table 8: Increasing-Slope Inverse-Demand Function - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL	Fortum	Base			23/23	559/560	416/605	0/646
		Peak						
	PVO	Base		305/305	644/643	559/560	416/605	0/646
		Peak						
	Player 3	Base	$1,242/1,242$	654/654	644/643	559/560	416/605	0/646
		Peak						

Table 9: Description of the Event Tree

S	Set of nodes
S^t	Set of nodes at period t, $S^t = \{s_1^t, ..., s_{N_t}^t\} \subset S$ with $S^0 = \{s^0\}$
$a(s_n^t)$	Unique predecessor of s_n^t, $a(s_n^t) \in S^{t-1}$ and $s_n^t \in S^t$, $t = 0, ..., T$
$B(s_n^t)$	Set of successors of node s_n^t, $B(s_n^t) \subset S^{t+1}$ and $s_n^t \in S^t$, $t = 0, ..., T-1$
$\pi(s_n^t)$	Probability of a scenario to include node s_n^t [9]

randomness. Table 10 exhibits the investment levels for this particular path, namely, the path corresponding to having a demand growth rate $g = 1.5\%$ along the tree.

Table 10: High-Growth Case - Capacity Investment (MW) By Player

			1	2	3	4	5	6
OL/CL	Fortum	Base					0/114	0/77
		Peak			531/77	1,283/698	1,491/1,128	1,591/1,453
	PVO	Base				0/167		
		Peak		295/270	1,276/865	1,360/1,505	1,529/266	1,582/1,713
	Player 3	Base			0/64		0/692	
		Peak	106/106	1,646/1,057	1,558/1,564	1,574/1,744	1,717/235	1,728/1,871

The obvious message of Table 10 is that information structure does significantly matter in a stochastic context. Indeed, as early on as Period 2, we observe a huge difference in the investments made by Player 3 in the OL and CL equilibria (OL exceeds CL by more than 55%). In Period 3, the same player adds 64 MW to its base-load capacity in the CL equilibrium, and does not invest at all in the OL equilibrium.

Comparing the results in the deterministic base case (Table 2) to the results of the corresponding path in the stochastic game (Table 10) allows the following two observations. First, investments are systematically made in base-load technology in the deterministic game and mainly in peak-load technology in the stochastic game. As investment is sunk, the players opt for the lower-investment-per-MW option when uncertainty is present. Second, when accounting for demand-growth uncertainty, firms invest more in OL than in CL, but they do the opposite in the deterministic game. This result clearly shows the importance of uncertainty on the results in terms of strategic interactions between the players. When facing a stochastic environment, adopting the closed-loop information structure considerably softens competition.

5 Conclusion

To the best of our knowledge, this is the first study to empirically compare open-loop and closed-loop investment equilibrium strategies in a fully dynamic, deregulated electricity market. Although we knew beforehand that the two strategies generally produce different control-variables paths, still little is known on the short-term implications of using OL or CL in practice. Further, within the literature in each area

[9] It is the sum of probabilities of all scenarios that include s_n^t.

where such a comparison had been carried out (e.g., capital accumulation, resource- and environmental-economics, advertising, etc.) the results in terms of competition, welfare, etc. have seldom been clear cut. Therefore, it is of interest to attempt to link the differences in OL and CL predictions to some features of the generation-expansion-planning problem. Our investigation allows us to state two main conclusions, which have important methodological and managerial implications:

- When demand is deterministic, almost no difference is observed in the short term (first 20 years) between the OL and CL strategies. The two investment paths diverge later on, a result that can be safely attributed to the difference in the definition of remaining time-to-go. Indeed, an open-loop information structure assumes that the planning horizon is finite, whereas closed-loop investments, based on a moving-horizon approach, are done as if the game will be played forever. Interestingly, varying the values of key model parameters did not qualitatively alter this conclusion. The managerial implication here is that generators can compute and implement open-loop strategies for as long as they are considering a sufficiently long planning horizon.
- When demand is stochastic, the two strategies do not coincide anymore, not even in the short term. This indicates that in such a (realistic) setting, open-loop strategies cannot be trusted as a good approximation of the conceptually more appealing closed-loop strategies.

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