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Abstract

This paper deals with the class of continuous-time singular uncertain linear systems with time-varying delay in the state vector. The uncertainties we are considering are of norm bounded type. Delay-dependent sufficient conditions on robust stability and robust stabilizability are developed. A design algorithm for a memoryless state feedback controller which guarantees that the closed-loop dynamics will be regular, impulse-free and robust stable is proposed in terms of the solutions to linear matrix inequalities (LMIs).

Key Words: Singular systems, Continuous-time uncertain linear systems, Norm bounded type uncertainties, Linear matrix inequality, Robust stability, Robust stabilizability, Memoryless state feedback.

Résumé

Cet article traite de la classe des systèmes incertains singuliers continus avec retard variant dans le temps. Les incertitudes sont bornées en norme. Des conditions suffisantes dépendantes du délai sur la stabilité et la stabilisabilité sont établies. Un algorithme de design d'un correcteur par retour d'état assurant que le système en boucle fermée soit régulier, sans impulsion et stable, est proposé sous forme d'inégalités matricielles.

1 Introduction

During the last decades, we witnessed an increasing interest to the class of singular continuous-time linear systems. Researchers from mathematics and control communities have contributed widely to the development of the control of this class of systems which is also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems (see [1, 2]). Singular systems are more appropriate to describe the behavior of some practical systems in different fields (see [1, 3–5]). Many problems for the class of continuous-time and discrete-time singular linear systems have been tackled and interesting results have been reported in the literature. Among these contributions we quote those of [6–17], and the references therein.

Some practical systems that can be adequately modelled by the class of singular systems that we are considering here may have time-delay and uncertainties in their dynamics which may be the cause of instability and performance degradation of such systems (see [18]). Therefore, more attention should be paid to this class of systems. To the best of our knowledge, the class of continuous-time uncertain linear singular systems with time delays has not yet been fully investigated. Particularly delay-dependent sufficient conditions are few even not existing in the literature.

When the system is nonsingular and has time-delay either time-varying or constant delay, the problem of stability or the one of stabilizability have been tackled by many researchers and many results in the LMI setting have been reported in the literature. These results can be divided in two categories, delay-independent and delay-dependent. The delay-independent conditions are in general more conservative than the delay-dependent ones. But even for the delay-dependent conditions the conservatism will dependent on the chosen Lyapunov functional to establish the results. For more details on this matter, we refer the reader to [18–26] and the references therein. In these references we witnessed interesting results with less conservatism. A good review of the literature on all the recent advances on the class of time-delay systems up to 2003 can be find in [24].

This paper deals with the problems of robust stability analysis and robust stabilization for singular continuous-time linear systems with time delays. Firstly, we develop a delay-dependent sufficient condition, which guarantees that the system is regular, impulse-free and stable for all admissible uncertainties. Based on this, a delay-dependent sufficient condition for the existence of a state feedback controller guaranteeing that the closed-loop dynamics is regular, impulse-free and stable is proposed. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed results. All the developed results are in the LMI framework which makes them more interesting since the solutions are easily obtained using existing powerful tools like the LMI toolbox of Matlab or any equivalent tool.

The rest of this paper is organized as follows. In Section 2, the problem is formulated and the goal of the paper is stated. In Section 3, the main results are given and they include results on robust stability, and robust stabilizability. A memoryless controller is used in this paper and a design algorithm in terms of the solutions to linear matrix inequalities is proposed to synthesize the controller gains we are using.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$^\top$" denotes matrix transposition and the notation $X \geq Y$(respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\lambda_{min}(A)$ and $\lambda_{max}(A)$ are minimal and maximal eigenvalues of the matrix A. $\mathbb{I}$ is the identity matrices with compatible dimensions. $\mathscr{L}_2$ is the space of integral vector over $[0, \infty)$. $\| \cdot \|$ will refer to the Euclidean vector norm whereas $\|\cdot\|$ denotes the $\mathscr{L}_2$-norm over $[0,\infty)$ defined as $\|f\|^2 = \int_0^\infty f^T(t)f(t)\ dt$.

2 Problem Statement

Let $x(t) \in \mathbb{R}^n$ be the physical state of the studied system, which is assumed to satisfy the following dynamics:

$$\begin{cases} E\dot{x}(t) = \left[A + D_A F_A E_A\right] x(t) + \left[A_1 + D_{A_1} F_{A_1} E_{A_1}\right] x(t-h(t)) + \left[B + D_B F_B E_B\right] u(t), \\ x(s) = \phi(s), s \in [-\bar{h}, 0] \end{cases} \quad (1)$$

where $u(t) \in \mathbb{R}^m$ is the control input system, A, A_1, B, D_A, E_A, D_{A_1}, E_{A_1}, D_B, and E_B are known real matrices with appropriate dimensions, the matrix E may be singular, and we assume that $rank(E) = n_E \leq n$, $h(t)$ represents the system delay that we assume to satisfy $0 \leq h(t) \leq \bar{h}$, with $\bar{h}$ is a known constant, and $\phi(t)$ is a smooth vector-valued initial function in $[-\bar{h}, 0]$ representing the initial condition of the system such that $x(s) = \phi(s) \in \mathscr{L}_2[-h, 0] \triangleq \{f(\cdot) | \int_0^\infty f^\top(t)f(t)dt < \infty\}$.

The uncertainties F_A, F_{A_1} and F_B are assumed to satisfy the following conditions:

$$\begin{cases} F_A^\top F_A \leq \mathbb{I} \\ F_{A_1}^\top F_{A_1} \leq \mathbb{I} \\ F_B^\top F_B \leq \mathbb{I} \end{cases}$$

The uncertainties that satisfy these conditions are referred to as admissible.

Remark 2.1 *When the matrix $E = \mathbb{I}$ the system will be called nonsingular and otherwise it will be referred to as singular as it is the case in this paper. Most of the results on singular systems will be in theorems and the ones related to nonsingular systems are given in form of corollaries.*

The following definitions will be used in the rest of this paper. For more details on the singular systems properties, we refer the reader to [1, 13] and the references therein.

Definition 2.1 *[13]*

i. System (1) is said to be regular if the characteristic polynomial, $\det(sE - A)$ *is not identically zero.*

ii. System (1) is said to be impulse-free, i.e. $\deg(\det(sE - A)) = rank(E)$*.*

For more details on other properties and the existence of the solution of system (1), we refer the reader to [13], and the references therein. In general, the regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

This paper studies the robust stability and the robust stabilizability of the class of systems (1). Our main objective in this paper is to design a state feedback controller guaranteeing that the closed-loop is regular, impulse-free and robust stable. In the rest of this paper, we will assume the complete access to the system state. Our methodology in this paper will be mainly based on the Lyapunov theory and some algebraic results. The conditions we will develop here will be in terms of the solutions to linear matrix inequalities that can be easily obtained using LMI control toolbox. These conditions are delay-dependent, which makes them less conservative compared to delay-independent conditions. The stabilizing controller that we would like to design has the following form:

$$u(t) = Kx(t) \tag{2}$$

where K is a design parameter that has to be determined.

Before closing this section, let us give some lemmas that we will use in our development. The proofs of these lemmas can be found in the cited references.

Lemma 2.1 *[18] Let H, F and G be real matrices of appropriate dimensions then, for any scalar $\varepsilon > 0$ for all matrices F satisfying $F^\top F \leq \mathbb{I}$, we have:*

$$HFG + G^\top F^\top H^\top \leq \varepsilon H H^\top + \varepsilon^{-1} G^\top G \tag{3}$$

Lemma 2.2 *[18] The linear matrix inequality*

$$\begin{bmatrix} H & S^\top \\ S & R \end{bmatrix} > 0$$

is equivalent to

$$R > 0, H - S^\top R^{-1} S > 0$$

where $H = H^\top$, $R = R^\top$ and S is a constant matrix.

3 Main Results

In this section, we will firstly develop results that assure that the free system (i.e. $u(t) = 0$ for all $t \geq 0$) is regular, impulse-free and robust stable. Then using these results, we will design a state feedback controller of the form (2) that guarantees the same goal. Delay-dependent sufficient conditions are developed in the LMI setting.

Let us now consider the free nominal system (all the uncertainties are equal to zero) and see under which conditions the corresponding dynamics will be regular, impulse-free and stable. The following theorem gives such results.

Theorem 3.1 *The free nominal singular linear system (1) is regular, impulse-free and stable if there exist a nonsingular matrix P and symmetric and positive-definite matrices $Q > 0$, and $R > 0$, such that the following set of LMIs holds:*

$$E^\top P = P^\top E \geq 0 \tag{4}$$

$$\begin{bmatrix} R & P^\top \\ P & \mathbb{I} \end{bmatrix} > 0 \tag{5}$$

$$\begin{bmatrix} J & P^\top A_1 - P^\top E & \bar{h} A^\top \\ A_1^\top P - E^\top P & -Q & \bar{h} A_1^\top \\ \bar{h} A & \bar{h} A_1 & -\bar{h}\mathbb{I} \end{bmatrix} < 0. \tag{6}$$

where $J = P^\top A + A^\top P + Q + P^\top E + E^\top P + \bar{h} R$.

Proof. The proof of this theorem is divided into two parts. The first one deals with the regularity and impulse-free properties and the second one treats the stability property of the studied class of systems. Let us first of all show that the free system (1) is regular and impulse-free, which is equivalent to say that $(E, A + A_1)$ and (E, A) are regular and impulse-free for any time-delay $h(t) \geq 0$. Using (6), it is easy to see that the following holds:

$$\begin{bmatrix} J & P^\top A_1 - P^\top E \\ A_1^\top P - E^\top P & -Q \end{bmatrix} < 0$$

which gives:

$$J + \left(P^\top A_1 - P^\top E\right) Q^{-1} \left(A_1^\top P - E^\top P\right) < 0$$

Using the fact that:

$$\left(P^\top A_1 - P^\top E\right) + \left(A_1^\top P - E^\top P\right) \leq Q + \left(P^\top A_1 - P^\top E\right) Q^{-1} \left(A_1^\top P - E^\top P\right)$$

and the fact that $Q > 0$, $R > 0$ and $\bar{h} > 0$, we get:

$$(A + A_1)^\top P + P^\top (A + A_1) < 0$$

which implies that $(E, A + A_1)$ is regular and impulse-free.

Similarly using (6), it is easy to see that the following holds:

$$P^\top A + A^\top P < 0 \tag{7}$$

Now, choose two nonsingular matrices $\widehat{M}$ and $\widehat{N}$ such that

$$\widehat{M} E \widehat{N} = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{bmatrix}$$

and write

$$\widehat{M} A \widehat{N} = \begin{bmatrix} \widehat{A}_1 & \widehat{A}_2 \\ \widehat{A}_3 & \widehat{A}_4 \end{bmatrix}, \quad \widehat{M}^{-\top} P \widehat{N} = \begin{bmatrix} \widehat{P}_1 & \widehat{P}_2 \\ \widehat{P}_3 & \widehat{P}_4 \end{bmatrix}.$$

Then, using (4), it can be shown that $\widehat{P}_2 = 0$. Pre- and post-multiplying (7) by $\widehat{N}^\top$ and $\widehat{N}$, respectively, we get:

$$\begin{bmatrix} \star & \star \\ \star & \widehat{A}_4^\top \widehat{P}_4 + \widehat{P}_4^\top \widehat{A}_4 \end{bmatrix} < 0,$$

where $\star$ will not be used in the following development. Then, from this, we get:

$$\widehat{A}_4^\top \widehat{P}_4 + \widehat{P}_4^\top \widehat{A}_4 < 0$$

which implies that $\widehat{A}_4$ is nonsingular. Therefore, system (1) is regular and impulse-free.

Let us now show the stability. For this purpose, since the free system (1) is regular and impulse-free, we can choose two nonsingular matrices $\widetilde{M}$ and $\widetilde{N}$ such that

$$\widetilde{M}E\widetilde{N} = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{bmatrix}, \quad \widetilde{M}A\widetilde{N} = \begin{bmatrix} \widetilde{A} & 0 \\ 0 & \mathbb{I} \end{bmatrix}.$$

Then, by defining:

$$\widetilde{P} = \widetilde{M}^{-\top}P\widetilde{N} = \begin{bmatrix} \widetilde{P}_1 & \widetilde{P}_2 \\ \widetilde{P}_3 & \widetilde{P}_4 \end{bmatrix}, \widetilde{Q} = \widetilde{N}^\top Q\widetilde{N} = \begin{bmatrix} \widetilde{Q}_1 & \widetilde{Q}_2 \\ \widetilde{Q}_2^\top & \widetilde{Q}_4 \end{bmatrix},$$

$$\widetilde{M}A_1\widetilde{N} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{13} & \widetilde{A}_{14} \end{bmatrix}, \widetilde{R} = \widetilde{N}^\top R\widetilde{N} = \begin{bmatrix} \widetilde{R}_1 & \widetilde{R}_2 \\ \widetilde{R}_2^\top & \widetilde{R}_4 \end{bmatrix},$$

then, system (1) becomes equivalent to the following one:

$$\begin{aligned} \dot{\xi}_1(t) &= A\xi_1(t) + \widetilde{A}_{11}\xi_1(t-h(t)) + \widetilde{A}_{12}\xi_2(t-h(t)), \\ 0 &= \xi_2(t) + \widetilde{A}_{13}\xi_1(t-h(t)) + \widetilde{A}_{14}\xi_2(t-h(t)). \end{aligned}$$

where

$$\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \widetilde{N}^{-1}x(t).$$

Using (6), it is easy to see that the following holds:

$$\begin{bmatrix} J & P^\top A_1 - P^\top E \\ A_1^\top P - E^\top P & -Q \end{bmatrix} < 0 \tag{8}$$

Now, pre- and post-multiplying (8) by $\mathrm{diag}\left(\widetilde{N}^\top, \widetilde{N}^\top\right)$ and its transpose, and using the fact that $\widetilde{P}_2 = 0$, we get:

$$\begin{bmatrix} \star & \star & \star & \star \\ \star & \widetilde{P}_4 + \widetilde{P}_4^\top + \widetilde{Q}_4 + \bar{h}\widetilde{R}_4 & \star & \widetilde{P}_4^\top \widetilde{A}_{14} \\ \star & \star & \star & \star \\ \star & \widetilde{A}_{14}^\top \widetilde{P}_4 & \star & -\widetilde{Q}_4 \end{bmatrix} < 0$$

which implies that the following holds:

$$\begin{bmatrix} \widetilde{P}_4 + \widetilde{P}_4^\top + \widetilde{Q}_4 + \bar{h}\widetilde{R}_4 & \widetilde{P}_4^\top \widetilde{A}_{14} \\ \widetilde{A}_{14}^\top P_4 & -\widetilde{Q}_4 \end{bmatrix} < 0.$$

Using the fact that $\widetilde{R} > 0$ and pre- and post-multiply this by $\begin{bmatrix} -\widetilde{A}_{14}^\top & \mathbb{I} \end{bmatrix}$ and $\begin{bmatrix} -\widetilde{A}_{14} \\ \mathbb{I} \end{bmatrix}$ respectively, we get: $\widetilde{A}_{14}^\top \widetilde{Q}_4 \widetilde{A}_{14} - \widetilde{Q}_4 < 0$. Therefore, it results that the following holds:

$$\rho\left(\widetilde{A}_{14}\right) < 1. \tag{9}$$

where $\rho(\widetilde{A}_{14})$ is the spectral radius of the matrix $\widetilde{A}_{14}$.

Now, let us choose the following Lyapunov functional:

$$\begin{aligned}V(x(t),r_t) &= x^\top(t)E^\top Px(t) + \int_{-\bar{h}}^{0}\mathcal{Q}(t,s)ds + \int_{-\bar{h}}^{0}\int_{t+v}^{t}(E\dot{x}(s))^\top \mathbb{I}E\dot{x}(s)dsdv \\ &+ \int_0^t\int_{v-h(v)}^{v}\begin{bmatrix} x^\top(v) & (E\dot{x}(s))^\top\end{bmatrix}\begin{bmatrix} R & P^\top \\ P & \mathbb{I}\end{bmatrix}\begin{bmatrix} x(v) \\ E\dot{x}(s)\end{bmatrix}dsdv,\end{aligned}$$

with $\mathcal{Q}(t,s) = \sup_{s\le v\le 0} x^\top(t+v)Qx(t+v)$.

Then, we have

$$\begin{aligned}\dot{V}(x(t)) &\le x^\top(t)\left[A^\top P + P^\top A + Q + P^\top E + E^\top P + \bar{h}R\right]x(t) \\ &+x^\top(t)\bar{h}A^\top\mathbb{I}Ax(t) + x^\top(t)\left[P^\top A_1 + \bar{h}A^\top\mathbb{I}A_1 - P^\top E\right]x(t-h(t)) \\ &+x^\top(t-h(t))\left[A_1^\top P + \bar{h}A_1^\top\mathbb{I}A - E^\top P\right]x(t) \\ &+x^\top(t-h(t))\left[-Q + \bar{h}A_1^\top\mathbb{I}A_1\right]x(t-h(t)) \\ &\triangleq \chi^\top(t)\left[\Psi + \bar{h}\begin{bmatrix} A^\top \\ A_1^\top\end{bmatrix}\mathbb{I}\begin{bmatrix} A & A_1\end{bmatrix}\right]\chi(t) = \chi^\top(t)\Gamma\chi(t)\end{aligned} \tag{10}$$

where

$$\begin{aligned}\chi(t) &= \begin{bmatrix} x^\top(t) & x^\top(t-h(t))\end{bmatrix}^\top, \\ \Psi &= \begin{bmatrix} J & P^\top A_1 - P^\top E \\ A_1^\top P - E^\top P & -Q\end{bmatrix}.\end{aligned}$$

Using now (6), (9), (10), we conclude that $\dot{V}(x(t)) < 0$. Using now this fact and the change of variable, $\xi(t) = \widetilde{N}^{-1}x(t)$, we get:

$$\begin{aligned}\alpha\|\xi_1(t)\|^2 - V(x(0)) &\le x^\top(t)E^\top Px(t) - V(x(0)) \\ &\le V(x(t) - V(x(0))) \\ &\triangleq \int_0^t \dot{V}(\widetilde{N}\xi(v))dv \\ &\le -\beta\int_0^t \|\xi(v)\|^2 dv \\ &\le -\beta\int_0^t \|\xi_1(v)\|^2 dv < 0\end{aligned}$$

with $\alpha = \lambda_{min}(\widehat{P}_1) > 0$ and $\beta = -\lambda_{max}(\widetilde{N}^\top\Gamma\widetilde{N}) > 0$.

From this we conclude that $\alpha\|\xi_1(t)\|^2 + \beta\int_0^t \|\xi_1(v)\|^2 dv \leq V(x(0))$, which implies that $\|\xi_1(t)\|$ and $\int_0^t \|\xi_1(v)\|^2 dv$ are bounded, i.e.:

$$\begin{aligned} 0 \leq \|\xi_1(t)\|^2 &\leq \alpha^{-1} V(x(0)) \\ 0 \leq \int_0^t \|\xi_1(v)\|^2 dv &\leq \beta^{-1} V(x(0)) \end{aligned}$$

which implies in turn that $\|\xi_2(t)\|$ is bounded too from the equivalent dynamics after the variable change. In consequence, $\|\dot{\xi}(t)\|$ is bounded and uniformly continuous and by using the well known Barbalat's results we conclude that $\xi_1(t)$ goes to zero as t goes to infinity. From the other side, using the fact that: $0 = \xi_2(t) + \widetilde{A}_{13}\xi_1(t-h(t)) + \widetilde{A}_{14}\xi_2(t-h(t))$, we conclude also that $\xi_2(t)$ will go to zero when t goes to infinity. This implies that system (1) is stable. This completes the proof. □

Remark 3.1 *The condition we developed in this theorem gives a delay-dependent sufficient condition which once it is satisfied will guarantee that the considered system is regular, impulse-free and stable. The results of this theorem can be easily extended to handle the case of multiple time-varying delays with the classical appropriate assumptions.*

Let us now see how we can design a state feedback controller that guarantees that the closed-loop dynamics of the nominal system will be regular, impulse-free and stable. Combining the dynamics of the nominal system and the controller expression give:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_1x(t-h(t)) + BKx(t) = [A + BK]\,x(t) + A_1x(t-h(t)) \\ &= A_{cl}x(t) + A_1x(t-h(t)) \\ x(s) &= \phi(s), s \in [-\bar{h}, 0] \end{aligned}$$

with $A_{cl} = A + BK$.

Based on the results of Theorem 3.1, this dynamics will be regular, impulse-free and stable if there exist a nonsingular matrix P and symmetric and positive-definite matrices $Q > 0$, and $R > 0$ such the following set of LMIs holds:

$$\begin{aligned} & E^\top P = P^\top E \geq 0 \\ & \begin{bmatrix} R & P^\top \\ P & \mathbb{I} \end{bmatrix} > 0 \\ & \begin{bmatrix} J & P^\top A_1 - P^\top E & \bar{h}\,[A + BK]^\top \\ A_1^\top P - E^\top P & -Q & \bar{h}A_1^\top \\ \bar{h}\,[A + BK] & \bar{h}A_1 & -\bar{h}\mathbb{I} \end{bmatrix} < 0, \end{aligned}$$

where $J = P^\top [A + BK] + [A + BK]^\top P + Q + P^\top E + E^\top P + \bar{h}R$.

Pre- and post-multiplying the last matrix inequality respectively by $\mathrm{diag}\left(P^{-\top}, P^{-\top}, \mathbb{I}\right)$ and its transpose, give:

$$\begin{bmatrix} J_P & A_1P^{-1} - EP^{-1} & \bar{h}\left[AP^{-1} + BKP^{-1}\right]^\top \\ P^{-\top}A_1^\top - P^{-\top}E^\top & -P^{-\top}QP^{-1} & \bar{h}P^{-\top}A_1^\top \\ \bar{h}\left[AP^{-1} + BKP^{-1}\right] & \bar{h}A_1P^{-1} & -\bar{h}\mathbb{I} \end{bmatrix} < 0,$$

where $J_P = \left[AP^{-1} + BKP^{-1}\right] + \left[AP^{-1} + BKP^{-1}\right]^\top + P^{-\top}QP^{-1} + EP^{-1} + P^{-\top}E^\top + \bar{h}P^{-\top}RP^{-1}$.

Letting $X = P^{-1}$, $Z = P^{-\top}QP^{-1}$, $W = P^{-\top}RP^{-1}$, and $Y = KX$, we get:

$$\begin{bmatrix} J_X & A_1X - EX & \bar{h}\left[AX + BY\right]^\top \\ X^\top A_1^\top - X^\top E^\top & -Z & \bar{h}X^\top A_1^\top \\ \bar{h}\left[AX + BY\right] & \bar{h}A_1X & -\bar{h}\mathbb{I} \end{bmatrix} < 0,$$

where $J_X = AX + BY + X^\top A^\top + Y^\top B^\top + Z + EX + X^\top E^\top + \bar{h}W$.

Noting that $\begin{bmatrix} R & P^\top \\ P & \mathbb{I} \end{bmatrix} > 0$ can be rewritten after pre- and post-multiplying the left hand side respectively by $\mathrm{diag}\left(P^{-\top}, \mathbb{I}\right)$ and its transpose as follows: $\begin{bmatrix} W & \mathbb{I} \\ \mathbb{I} & \mathbb{I} \end{bmatrix} > 0$. In a similar way the condition $E^\top P = P^\top E \geq 0$ can be rewritten as: $X^\top E^\top = EX \geq 0$.

Using now all these developments, we get the following results for the stabilization for our class of systems.

Theorem 3.2 *There exists a state feedback controller of the form (2) such that the closed-loop system (1) is regular, impulse-free and stable if there exist a nonsingular matrix X, a matrix Y, and symmetric and positive-definite matrices $Z > 0$, and $W > 0$, such that the following set of LMIs holds:*

$$X^\top E^\top = EX \geq 0 \tag{11}$$

$$\begin{bmatrix} W & \mathbb{I} \\ \mathbb{I} & \mathbb{I} \end{bmatrix} > 0 \tag{12}$$

$$\begin{bmatrix} J_X & A_1X - EX & \bar{h}\left[AX + BY\right]^\top \\ X^\top A_1^\top - X^\top E^\top & -Z & \bar{h}X^\top A_1^\top \\ \bar{h}\left[AX + BY\right] & \bar{h}A_1X & -\bar{h}\mathbb{I} \end{bmatrix} < 0, \tag{13}$$

where

$$J_X = AX + X^\top A^\top + BY + B^\top Y^\top + EX + X^\top E^\top + Z + \bar{h}W$$

The stabilizing controller gain is given by $K = YX^{-1}$.

Let us now consider the effect of the uncertainties and develop similar results on robust stability. For this purpose, let us consider the free uncertain system (1). Based on the results of Theorem 3.1, this dynamics will be regular, impulse-free and stable if there exist a matrix P and symmetric and positive-definite matrices $Q > 0$, and $R > 0$ such the following set of LMIs holds:

$$E^\top P = P^\top E \geq 0$$

$$\begin{bmatrix} R & P^\top \\ P & \mathbb{I} \end{bmatrix} > 0$$

$$\begin{bmatrix} J & P^\top\left[A_1 + D_{A_1}F_{A_1}E_{A_1}\right] - P^\top E & \bar{h}\left[A + D_AF_AE_A\right]^\top \\ \left[A_1 + D_{A_1}F_{A_1}E_{A_1}\right]^\top P - E^\top P & -Q & \bar{h}\left[A_1 + D_{A_1}F_{A_1}E_{A_1}\right]^\top \\ \bar{h}\left[A + D_AF_AE_A\right] & \bar{h}\left[A_1 + D_{A_1}F_{A_1}E_{A_1}\right] & -\bar{h}\mathbb{I} \end{bmatrix} < 0,$$

where $J = P^\top [A + D_A F_A E_A] + [A + D_A F_A E_A]^\top P + Q + P^\top E + E^\top P + \bar{h}R$.

The last LMI can be rewritten as follows:

$$\begin{bmatrix} J_0 & P^\top A_1 - P^\top E & \bar{h}A^\top \\ A_1^\top P - E^\top P & -Q & \bar{h}A_1^\top \\ \bar{h}A & \bar{h}A_1 & -\bar{h}\mathbb{I} \end{bmatrix} + \begin{bmatrix} P^\top D_A \\ 0 \\ \bar{h}D_A \end{bmatrix} F_A \begin{bmatrix} E_A & 0 & 0 \end{bmatrix}$$
$$+ \left[\begin{bmatrix} P^\top D_A \\ 0 \\ \bar{h}D_A \end{bmatrix} F_A \begin{bmatrix} E_A & 0 & 0 \end{bmatrix}\right]^\top + \begin{bmatrix} P^\top D_{A_1} \\ 0 \\ \bar{h}D_{A_1} \end{bmatrix} F_{A_1} \begin{bmatrix} 0 & E_{A_1} & 0 \end{bmatrix}$$
$$+ \left[\begin{bmatrix} P^\top D_{A_1} \\ 0 \\ \bar{h}D_{A_1} \end{bmatrix} F_{A_1} \begin{bmatrix} 0 & E_{A_1} & 0 \end{bmatrix}\right]^\top < 0,$$

where $J_0 = P^\top A + A^\top P + Q + P^\top E + E^\top P + \bar{h}R$.

Using Lemma 2.1 and Schur complement, we get the following results.

Theorem 3.3 *The free singular linear system (1) is regular, impulse-free and robust stable if there exist a nonsingular matrix P, and symmetric and positive-definite matrices $Q > 0$, and $R > 0$, and positive scalars ε_A and ε_{A_1} such that the following set of LMIs holds:*

$$E^\top P = P^\top E \geq 0 \tag{14}$$
$$\begin{bmatrix} R & P^\top \\ P & \mathbb{I} \end{bmatrix} > 0 \tag{15}$$
$$\begin{bmatrix} J & P^\top A_1 - P^\top E & \bar{h}A^\top & P^\top D_A & P^\top D_{A_1} \\ A_1^\top P - E^\top P & -Q + \varepsilon_{A_1} E_{A_1}^\top E_{A_1} & \bar{h}A_1^\top & 0 & 0 \\ \bar{h}A & \bar{h}A_1 & -\bar{h}\mathbb{I} & \bar{h}D_A & \bar{h}D_{A_1} \\ D_A^\top P & 0 & \bar{h}D_A^\top & -\varepsilon_A \mathbb{I} & 0 \\ D_{A_1}^\top P & 0 & \bar{h}D_{A_1}^\top & 0 & -\varepsilon_{A_1}\mathbb{I} \end{bmatrix} < 0, \tag{16}$$

where $J = P^\top A + A^\top P + Q + P^\top E + E^\top P + \bar{h}R + \varepsilon_A E_A^\top E_A$.

Remark 3.2 *Notice that our conditions are only sufficient and they should be satisfied to conclude on the robust stability of our systems. When the conditions are not satisfied no conclusion can be given. This means that we cannot conclude that the system is not robustly stable.*

When the matrix E is nonsingular, the results in this case can be obtained by setting E equal to $\mathbb{I}$ with the appropriate transformations. The corresponding results are given by the following corollary:

Corollary 3.1 *The free singular linear system (1) is regular, impulse-free and robust stable if there exist symmetric and positive-definite matrices P, $Q > 0$, and $R > 0$, and positive scalars ε_A and ε_{A_1} such that the following set of LMIs holds: (15) and (16) with $E = \mathbb{I}$.*

Let us now concentrate on the design of a state feedback controller of the form (2) which guarantees that the closed-loop will be regular, impulse-free and robust stable. Our objective is to establish LMI conditions that can determine the gain, K of the desired controller. To assure less conservatism a delay-dependent sufficient conditions are needed. For this purpose, we will use our previous results of Theorem 3.2. Plugging now, the controller (2) in the dynamics (1) gives:

$$\begin{cases} \dot{x}(t) = A_{cl}x(t) + [A_1 + D_{A_1}F_{A_1}E_{A_1}]\, x(t-h(t)) \\ x(s) = \phi(s), s \in [-\bar{h}, 0] \end{cases} \tag{17}$$

where $A_{cl} = A + D_A F_A E_A + BK + D_B F_B E_B K$.

Based on the results of Theorem 3.2, this dynamics will be regular, impulse-free and robust stable if there exist a nonsingular matrix X, a matrix Y, and symmetric and positive-definite matrices $Z > 0$, and $W > 0$, such that the following set of LMIs holds:

$$\begin{aligned} & X^\top E^\top = EX \geq 0, \\ & \begin{bmatrix} W & \mathbb{I} \\ \mathbb{I} & \mathbb{I} \end{bmatrix} > 0, \\ & \begin{bmatrix} \widehat{J}_X & U & T \\ U^\top & -Z & \bar{h}X^\top A_1^\top + h\,[D_{A_1}F_{A_1}E_{A_1}X]^\top \\ T^\top & \bar{h}A_1X + D_{A_1}F_{A_1}E_{A_1}X & -\bar{h}\mathbb{I} \end{bmatrix} < 0, \end{aligned}$$

where

$$\begin{aligned} T &= \bar{h}\,[AX + BY]^\top + h\,[D_A F_A E_A X]^\top + h\,[D_B F_B E_B Y]^\top \\ U &= A_1X - EX + D_{A_1}F_{A_1}E_{A_1}X \\ \widehat{J}_X &= AX + X^\top A^\top + BY + B^\top Y^\top + EX + X^\top E^\top + Z + \bar{h}W \\ &\quad + D_A F_A E_A X + X^\top E_A^\top F_A^\top D_A^\top + D_B F_B E_B Y + Y^\top E_B^\top F_B^\top D_B^\top \end{aligned}$$

Using the fact that:

$$\begin{bmatrix} D_A F_A E_A X & 0 & 0 \\ 0 & 0 & 0 \\ \bar{h}D_A F_A E_A X & 0 & 0 \end{bmatrix} = \begin{bmatrix} D_A \\ 0 \\ \bar{h}D_A \end{bmatrix} F_A \begin{bmatrix} E_A X & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} D_{A_1}F_{A_1}E_{A_1}X & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \bar{h}D_{A_1}F_{A_1}E_{A_1}X & 0 \end{bmatrix} = \begin{bmatrix} D_{A_1} \\ 0 \\ \bar{h}D_{A_1} \end{bmatrix} F_{A_1} \begin{bmatrix} 0 & E_{A_1}X & 0 \end{bmatrix}$$

$$\begin{bmatrix} D_B F_B E_B Y & 0 & 0 \\ 0 & 0 & 0 \\ \bar{h}D_B F_B E_B Y & 0 & 0 \end{bmatrix} = \begin{bmatrix} D_B \\ 0 \\ \bar{h}D_B \end{bmatrix} F_B \begin{bmatrix} E_B Y & 0 & 0 \end{bmatrix}$$

the third matrix inequality can be rewritten as follows:

$$\begin{bmatrix} \widehat{J}_X & A_1X - EX & \bar{h}\,[AX + BY]^\top \\ X^\top A_1^\top - X^\top E^\top & -Z & \bar{h}X^\top A_1^\top \\ \bar{h}\,[AX + BY] & \bar{h}A_1X & -\bar{h}\mathbb{I} \end{bmatrix}$$

$$+\begin{bmatrix} D_A \\ 0 \\ \bar{h}D_A \end{bmatrix} F_A \begin{bmatrix} E_A X & 0 & 0 \end{bmatrix} + \left[\begin{bmatrix} D_A \\ 0 \\ \bar{h}D_A \end{bmatrix} F_A \begin{bmatrix} E_A X & 0 & 0 \end{bmatrix}\right]^\top$$

$$+\begin{bmatrix} D_{A_1} \\ 0 \\ \bar{h}D_{A_1} \end{bmatrix} F_{A_1} \begin{bmatrix} 0 & E_{A_1} X & 0 \end{bmatrix} + \left[\begin{bmatrix} D_{A_1} \\ 0 \\ \bar{h}D_{A_1} \end{bmatrix} F_{A_1} \begin{bmatrix} 0 & E_{A_1} X & 0 \end{bmatrix}\right]^\top$$

$$+\begin{bmatrix} D_B \\ 0 \\ \bar{h}D_B \end{bmatrix} F_B \begin{bmatrix} E_B Y & 0 & 0 \end{bmatrix} + \left[\begin{bmatrix} D_B \\ 0 \\ \bar{h}D_B \end{bmatrix} F_B \begin{bmatrix} E_B Y & 0 & 0 \end{bmatrix}\right]^\top < 0,$$

where

$$\widehat{J}_X = AX + X^\top A^\top + BY + B^\top Y^\top + EX + X^\top E^\top + Z + \bar{h}W.$$

Using now Lemma 2.1, we get:

$$\begin{bmatrix} \widehat{J}_X & A_1 X - EX & \bar{h}\left[AX + BY\right]^\top \\ X^\top A_1^\top - X^\top E^\top & -Z & \bar{h}X^\top A_1^\top \\ \bar{h}\left[AX + BY\right] & \bar{h}A_1 X & -\bar{h}\mathbb{I} \end{bmatrix}$$

$$+\varepsilon_A \begin{bmatrix} D_A \\ 0 \\ \bar{h}D_A \end{bmatrix} \begin{bmatrix} D_A^\top & 0 & \bar{h}D_A^\top \end{bmatrix} + \varepsilon_A^{-1} \begin{bmatrix} X^\top E_A^\top \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_A X & 0 & 0 \end{bmatrix}$$

$$+\varepsilon_{A_1} \begin{bmatrix} D_{A_1} \\ 0 \\ \bar{h}D_{A_1} \end{bmatrix} \begin{bmatrix} D_{A_1}^\top & 0 & \bar{h}D_{A_1}^\top \end{bmatrix} + \varepsilon_{A_1}^{-1} \begin{bmatrix} 0 \\ X^\top E_{A_1}^\top \\ 0 \end{bmatrix} \begin{bmatrix} 0 & E_{A_1} X & 0 \end{bmatrix}$$

$$+\varepsilon_B \begin{bmatrix} D_B \\ 0 \\ \bar{h}D_B \end{bmatrix} \begin{bmatrix} D_B^\top & 0 & \bar{h}D_B^\top \end{bmatrix} + \varepsilon_B^{-1} \begin{bmatrix} Y^\top E_B^\top \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_B Y & 0 & 0 \end{bmatrix} < 0,$$

which can be rewritten as:

$$\begin{bmatrix} \widehat{J}_X & A_1 X - EX & \bar{h}\left[AX + BY\right]^\top \\ X^\top A_1^\top - X^\top E^\top & -Z & \bar{h}X^\top A_1^\top \\ \bar{h}\left[AX + BY\right] & \bar{h}A_1 X & -\bar{h}\mathbb{I} \end{bmatrix}$$

$$+\varepsilon_A \begin{bmatrix} D_A D_A^\top & 0 & \bar{h}D_A D_A^\top \\ 0 & 0 & 0 \\ \bar{h}D_A D_A^\top & 0 & \bar{h}^2 D_A D_A^\top \end{bmatrix} + \varepsilon_A^{-1} \begin{bmatrix} X^\top E_A^\top \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_A X & 0 & 0 \end{bmatrix}$$

$$+\varepsilon_{A_1} \begin{bmatrix} D_{A_1} D_{A_1}^\top & 0 & \bar{h}D_{A_1} D_{A_1}^\top \\ 0 & 0 & 0 \\ \bar{h}D_{A_1} D_{A_1}^\top & 0 & \bar{h}^2 D_{A_1} D_{A_1}^\top \end{bmatrix} + \varepsilon_{A_1}^{-1} \begin{bmatrix} 0 \\ X^\top E_{A_1}^\top \\ 0 \end{bmatrix} \begin{bmatrix} 0 & E_{A_1} X & 0 \end{bmatrix}$$

$$+\varepsilon_B \begin{bmatrix} D_B D_B^\top & 0 & \bar{h}D_B D_B^\top \\ 0 & 0 & 0 \\ \bar{h}D_B D_B^\top & 0 & \bar{h}^2 D_B D_B^\top \end{bmatrix} + \varepsilon_B^{-1} \begin{bmatrix} Y^\top E_B^\top \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_B Y & 0 & 0 \end{bmatrix} < 0,$$

Using now Schur complement 2.2, we get the results for the stabilization for our class of systems.

Theorem 3.4 *There exists a state feedback controller of the form (2) such that the closed-loop system (1) is regular, impulse-free and stable if there exist a nonsingular matrix X, a matrix Y, and symmetric and positive-definite matrices $Z > 0$, and $W > 0$, such that the following set of LMIs holds:*

$$X^\top E^\top = EX \geq 0 \tag{18}$$

$$\begin{bmatrix} W & \mathbb{I} \\ \mathbb{I} & \mathbb{I} \end{bmatrix} > 0 \tag{19}$$

$$\begin{bmatrix} \widehat{J}_X + V & A_1X - EX & \bar{h}\left[AX + BY\right]^\top + \bar{h}V & X^\top E_A^\top & 0 & Y^\top E_B^\top \\ X^\top A_1^\top - X^\top E^\top & -Z & \bar{h}X^\top A_1^\top & 0 & X^\top E_{A_1}^\top & 0 \\ \bar{h}\left[AX + BY\right] + \bar{h}V & \bar{h}A_1X & -\bar{h}\mathbb{I} + \bar{h}^2V & 0 & 0 & 0 \\ E_AX & 0 & 0 & -\varepsilon_A\mathbb{I} & 0 & 0 \\ 0 & E_{A_1}X & 0 & 0 & -\varepsilon_{A_1}\mathbb{I} & 0 \\ E_BY & 0 & 0 & 0 & 0 & -\varepsilon_B\mathbb{I} \end{bmatrix} < 0, \tag{20}$$

where

$$\widehat{J}_X = AX + X^\top A^\top + BY + B^\top Y^\top + EX + X^\top E^\top + Z + \bar{h}W$$
$$V = \varepsilon_A D_A D_A^\top + \varepsilon_{A_1} D_{A_1} D_{A_1}^\top + \varepsilon_B D_B D_B^\top$$

The stabilizing controller gain is given by $K = YX^{-1}$.

In a similar way, we can get the following results for nonsingular systems with time-delay in the state vector.

Corollary 3.2 *There exists a state feedback controller of the form (2) such that the closed-loop system (1) is regular, impulse-free and stable if there exist a matrix Y, and symmetric and positive-definite matrices $X > 0$, $Z > 0$, and $W > 0$ such that the following set of LMIs holds: (19) and (20) with $E = \mathbb{I}$. The stabilizing controller gain is given by $K = YX^{-1}$.*

4 Numerical Example

To show the validness of our results, let us consider a numerical example of a singular system with state space in $\mathbb{R}^3$. The date of this system are as follow:

$$A = \begin{bmatrix} 1.0 & 1.5 & 1.0 \\ -0.2 & 1.0 & 2.0 \\ 0.0 & 0.0 & 0.1 \end{bmatrix}, A_1 = \begin{bmatrix} -1.5 & 1.0 & 0.0 \\ 0.2 & 0.0 & 0.5 \\ 1.3 & 0.5 & -1.6 \end{bmatrix}, B = \begin{bmatrix} 1.5 & 0.0 & 0.0 \\ 1.0 & -1.0 & 0.0 \\ -1 & 0.0 & -2.0 \end{bmatrix}$$
$$D_A = \text{diag}\left[0.1, 0.2, 0.1\right], E_A = \text{diag}\left[0.3, 0.1, 0.1\right], D_{A_1} = \text{diag}\left[0.2, 0.2, 0.3\right],$$
$$E_{A_1} = \text{diag}\left[0.1, 0.1, 0.2\right], D_B = \text{diag}\left[0.3, 0.1, 0.4\right], E_B = \text{diag}\left[0.1, 0.2, 0.2\right],$$

The singular matrix E is given by the following expression:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

Solving the LMIs (18)-(20) with $\bar{h} = 0.64$, $d = 0.8$ and $\varepsilon_A = \varepsilon_{A_1} = \varepsilon_B = 0.1$, we get:

$$Z = \begin{bmatrix} 0.2340 & 0.1320 & -0.0632 \\ 0.1320 & 0.1791 & -0.0135 \\ -0.0632 & -0.0135 & 0.3462 \end{bmatrix}, X = \begin{bmatrix} 0.0248 & 0.0174 & 0 \\ 0.0174 & 0.0236 & 0 \\ 0.0041 & -0.0259 & -0.0078 \end{bmatrix},$$

$$Y = \begin{bmatrix} -0.7660 & -0.1940 & 0.0797 \\ -0.4768 & 0.7475 & 0.0112 \\ 0.3227 & 0.0263 & 0.5986 \end{bmatrix}, W = \begin{bmatrix} 1.1706 & 0.0904 & -0.0449 \\ 0.0904 & 1.1290 & -0.0098 \\ -0.0449 & -0.0098 & 1.2704 \end{bmatrix}$$

which gives the following gains:

$$K = \begin{bmatrix} -0.0308 & -0.0290 & -0.0011 \\ 0.0033 & 0.0135 & -0.0005 \\ 0.0233 & -0.0282 & -0.0124 \end{bmatrix}.$$

For this system, we are able to find a feasible solution for the set of LMI for any $h \in [0, 0.74]$. This means that the maximum delay under which the system is stabilizable is 0.74.

5 Conclusion

This paper dealt with the class of continuous-time singular linear systems with time-delay in the state vector. Results on stability and its robustness, and stabilizability and its robustness are developed. The LMI framework is used to establish the different results on stability and stabilizability. The conditions we developed are delay-dependent. The results we developed can easily be solved using any LMI toolbox like the one of Matlab or the one of Scilab.

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