

**Short-Run Stick and Long-Run  
Carrot Policy: The Role of  
Initial Conditions**

D. Claude, C. Figuières,  
M. Tidball

G-2008-19

March 2008

Les textes publiés dans la série des rapports de recherche HEC n'engagent que la responsabilité de leurs auteurs. La publication de ces rapports de recherche bénéficie d'une subvention du Fonds québécois de la recherche sur la nature et les technologies.



# Short-Run Stick and Long-Run Carrot Policy: The Role of Initial Conditions

**Denis Claude**

*GERAD and HEC Montréal  
3000, chemin de la Côte-Sainte-Catherine  
Montréal (Québec) Canada, H3T 2A7  
denis.claude@gerad.ca*

**Charles Figuières  
Mabel Tidball**

*INRA, LAMETA  
2 Place Viala  
34060 Montpellier cedex 1, France  
figuiere@ensam.inra.fr, tidball@supagro.inra.fr*

March 2008

*Les Cahiers du GERAD*

G-2008-19

Copyright © 2008 GERAD



## Abstract

This paper explores the dynamic properties of price-based policies in a model of competition between two jurisdictions. Jurisdictions invest over time in infrastructure to increase the quality of the environment, a global public good. They are identical in all respects but one: initial stocks of infrastructure. This is a dynamic type of heterogeneity that disappears in the long run. Therefore, at the steady state, usual intuitions from static settings apply: identical jurisdictions inefficiently under-invest, calling for public subsidies. In the short run, however, counterintuitive properties are established: i) the evolution of capital stocks can be non-monotonic, ii) one jurisdiction can be temporarily taxed, even though it should increase its investment, whereas the other is subsidized. It is shown how these phenomena are related to initial conditions and the kind of interactions between infrastructure capitals, complementarity or substitutability.

## Résumé

L'article étudie les propriétés dynamiques des instruments en prix dans un modèle de concurrence en infrastructures. Il suppose l'existence de deux juridictions adjacentes. Chaque juridiction investit dans ses infrastructures « vertes » de façon à fournir à ses résidents un bien public : un environnement de meilleure qualité. Les deux juridictions sont supposées identiques en tout sauf leur stock initial d'infrastructures ; une hétérogénéité qui s'effacera dans le long terme. Ainsi, à l'état stationnaire, nos intuitions usuelles s'appliquent. Les deux juridictions sous-investissent, de sorte qu'une politique de subvention à l'investissement est requise pour parvenir à l'optimum social. Cependant, dans le cours terme, la propriété contre-intuitive suivante est établie : la politique en prix optimale peut impliquer que le planificateur social taxe une juridiction et subventionne sa concurrente. L'article relie cette propriété à l'écart entre les dotations initiales en infrastructures « vertes » et à la substituabilité ou complémentarité des deux stocks d'infrastructures dans le processus de production de la qualité de l'environnement.

**Acknowledgments:** We are grateful for financial assistance from NSERC Canada. We thank Alain Jean-Marie, Erik Ansik, Ngo Van Long and Georges Zaccour for helpful comments. Also, we thank participants at the 15th annual conference of the EAERE and 7th Meeting on Game Theory and Practice for helpful discussions. The usual disclaimer applies.



## 1 Introduction

In recent years the theoretical analysis of price-based policies for the control of environmental externalities under imperfect competition has received a renewed attention. Previously, most of the literature focused on the design of optimal tax or subsidy policies in a static setting where several instantaneous effects of these instruments should be balanced; e.g., with respect to taxation, the gain in terms of social welfare arising from the reduction of pollution emissions against the loss from output restriction.

However, little is known about how intertemporal externalities affect the design and the dynamic properties of price-based policies. Introducing the time dimension opens the possibility to raise the critical issue of credibility of public policies, namely how regulations should be framed to ensure that they remain optimal in their ability to achieve or increase social efficiency as circumstances change over time. Such an explicit consideration of credibility requirements may qualify substantially the intuitions about price-based regulations gained in a static setting and provide interesting and sometimes counter-intuitive policy advices.

An important contribution to this literature is Benckroun and Van Long (1998). They consider efficiency inducing taxation<sup>1</sup> for the regulation of an oligopolistic industry which is responsible for releasing a stock pollutant – one for which pollution accumulation generates present as well as long-term environmental damages. They formulate a differential game of pollution control in which the environmental regulator imposes a taxation rule in a symmetric oligopolistic industry. In this game, the state variable is the pollution stock, the tax policy is the control of the environmental regulator and output decisions are the controls of the firms which are assumed to use either open-loop or markov strategies.

Benckroun and Van Long (1998) analyze a Markovian tax policy whereby the output tax rate faced by a firm at any given time depends solely on the current pollution stock. By construction, such a linear markovian tax rule is credible, in the sense that it is time consistent and subgame perfect. The authors provide a characterization of the optimal tax rule that is shown to be increasing in the pollution stock and to 'decentralize' the socially optimal time-path of production. As for the dynamic properties of the tax, they obtain a surprising result. In an initial time interval where the stock of pollution is low, the tax rate may be negative implying a subsidy. Paradoxically, this subsidy induces firms to produce less than they would have if the industry had not been regulated. Upon reflection, the explanation for this result is simple. Since the tax rate at any given time depends solely on the pollution stock, firms anticipate that an increase in their production will lead to reduced subsidies in the future and eventually precipitate the turn of the subsidy into a tax. As noted by Benckroun and Van Long (1998), this is an instance of 'carrot and stick' policy.

Motivated by a long standing concern for infrastructure competition, this paper elaborates on Benckroun and Van Long's seminal contribution by studying how differences in initial stocks of infrastructure alter the dynamic properties of the optimal and credible tax or subsidy policy. We consider a stylized dynamic extension of the model of interjurisdictional spillovers introduced by Wildasin (1991). The focus of our attention are two jurisdictions that are located in the same watershed or airshed.<sup>2</sup> Each jurisdiction invest in public (or green)

---

<sup>1</sup>On efficiency inducing taxation, see also Bergstrom et al. (1981), Karp and Livernois (1992) and Karp and Livernois (1994).

<sup>2</sup>Here, we depart from the original meaning of the terms 'watershed' and 'airshed' and adopt the North-American usage in which they have come to describe the geographical boundary for water and air quality standards.

infrastructure<sup>3</sup> in order to provide a public good to its own residents. However, the public good produced by one jurisdiction benefits also the residents in the other jurisdiction who cannot be excluded from ‘its’ consumption once it is provided. In the absence of any regulation initiative, the presence of positive interjurisdictional spillovers will result in the underprovision of public infrastructure. Indeed, local jurisdictions will not take into account the positive spillovers that benefit the non-residents when setting their investment policies. A first remedy is to elevate the decision making process to a higher level of jurisdiction so that external benefits of public infrastructure become internal to the jurisdiction which funds them. A drawback of this solution is that it alienates local residents from the control they have over issues that impact their local community and daily life. A second remedy is for the higher level of jurisdiction to implement an infrastructure capital subsidization policy that will help coordinate local investment decisions while preserving subsidiarity. This is the route we travel by in this paper.

The logic of regulation in our model is similar to that of Bencheikroun and Van Long (1998). A benevolent authority sets the capital infrastructure subsidization scheme and local jurisdictions decides upon their expenditures in public infrastructure taking the subsidization rule as given. A key difference with Bencheikroun and Van Long (1998) lies in the state of the system which is not scalar. Instead it is a two-dimensional vector describing the stocks of infrastructure of each jurisdiction at any given time.

Consistently with the purposes of our paper we expunge the model of any asymmetry across jurisdictions except regarding their initial stock of infrastructure in order to highlight how this particular asymmetry affects the dynamic properties of the subsidy.<sup>4</sup> This is a dynamic type of heterogeneity that vanishes in the long-run. Consequently, at the steady state, usual intuitions from static symmetric settings apply. Due to the presence of positive interjurisdictional spillovers, both jurisdictions will inefficiently under-invest, which calls for the implementation of a green capital subsidization scheme. In the short run, however, a counterintuitive property appears: It is shown that the optimal scheme *may* require to simultaneously tax one jurisdiction and subsidize the other for an initial period of time. And which jurisdiction should be initially taxed depends on whether infrastructure stocks are substitute or complement. When they are substitute (respectively complement), the jurisdiction with the lower (resp. larger) initial stock is first taxed.

The remainder of this paper is organized as follows. Section 2 presents the basic model. In Section 3, the utilitarian social optimum is characterized. Then, in Section 4, we derive the optimal infrastructure capital subsidization scheme and we discuss its dynamic properties in Section 5. Section 6 provides examples and discusses intuitions. Finally, in Section 7, we conclude.

## 2 A dynamic framework for infrastructure competition

We consider a dynamic extension of the model of interjurisdictional spillovers introduced by Wildasin (1991). Two jurisdictions indexed by  $i = 1, 2$  are located in the same watershed or airshed. Each jurisdiction is inhabited by identical households who are assumed to be immobile and infinitely lived. These households can be treated as a representative consumer whose preferences are defined over a composite private commodity, denoted by  $x_i$ , and an

---

<sup>3</sup>Throughout this article, we shall use the terms ‘public’ and ‘green’, interchangeably.

<sup>4</sup>We also assume away any informational obstacles to regulation. For a recent review on such issues, see for instance Lewis (1996).



index of environmental quality denoted by  $s_i$ . These preferences can be represented by the utility function

$$u_i(x_i, s_i) = x_i + s_i, \quad \forall i = 1, 2. \quad (1)$$

Local stocks of infrastructure are the inputs in the production process of environmental quality. This relationship can be expressed by a quadratic production function  $s_i = P_i(K_i, K_j)$ .

We assume that jurisdictions compete in public infrastructure over an infinite time period. Let  $e_i(t)$  denote Jurisdiction  $i$ 's expenditure on its public infrastructure at time  $t \in [0, \infty[$  and  $K_i(t)$  denote its stock of green infrastructure. Each jurisdiction is endowed with an initial stock of green infrastructure equal to  $K_i(0) = K_i^0$ . Investment is a flow that allows jurisdictions to adjust their stocks of public infrastructure. Jurisdiction  $i$ 's public expenditure  $e_i(t)$  modifies its current stock of infrastructure according to the following law of motion

$$\dot{K}_i(t) = e_i(t) - \delta K_i(t), \quad \forall i = 1, 2, \quad (2)$$

where  $\delta$  is the constant rate of depreciation. In this paper, we assume that investment is reversible and resale of infrastructure capital is impossible. In other words,  $e_i(t)$  is restricted to be non-negative and  $\delta$  is strictly positive.

Investment in infrastructure capital is costly. Let  $C_i(e_i)$  denotes Jurisdiction  $i$ 's cost of infrastructure capital adjustment. We assume that  $C_i(0) = 0$  and  $C_i'(e_i) > 0$ ,  $C_i''(e_i) > 0$ . In other words, Jurisdiction  $i$ 's cost of altering its infrastructure stock is an increasing and convex function of the rate of investment. In our model, where investment is reversible ( $\delta > 0$ ), this assumption implies that instantaneous adjustments of capital stocks are ruled out.

We assume that each jurisdiction is endowed with an exogenous revenue  $y_i$  which can be used to finance public expenditures and consumption good expenses. Accordingly, Jurisdiction  $i$ 's budget constraint is

$$x_i + C_i(e_i) = y_i. \quad (3)$$

Plugging the budget constraint (3) into the utility function (1) yields the reduced-form utility for each jurisdiction:

$$W_i(K_i, K_j, e_i) = y_i + P_i(K_i, K_j) - C_i(e_i). \quad (4)$$

Each local jurisdiction is assumed to choose the time-path of public expenditure in infrastructure capital that maximizes the integral of its discounted stream of net social benefits. Denoting by  $r \in ]0, 1[$  the common discount factor of both jurisdictions, the objective of Jurisdiction  $i$  is to maximize

$$J_i = \int_0^{\infty} [y_i + P_i(K_i(t), K_j(t)) - C_i(e_i(t))] e^{-rt} dt, \quad \forall i = 1, 2, \quad (5)$$

with respect to the state equations (2) and the non-negativity constraint  $e_i(t) \geq 0$ . To complete the description of each jurisdiction's problem, the information structure must be specified. In this paper, it is assumed that each jurisdiction is able to observe the state of the game at any given time and make investment decisions based on this information. Namely, we assume that both jurisdictions use markov strategies; i.e., decision rules of the form  $e_i(t) = \phi_i(K_i(t), K_j(t))$ .

The above elements define a game of competition in infrastructure which belongs to the extensively studied class of capital accumulation games.<sup>5</sup> In analyzing the outcome of inter-jurisdictional competition, the relevant solution concept is then the markov perfect Nash equilibrium : a pair of markov perfect strategies that are mutual best-responses. In our model where green infrastructures generate positive spillovers across the boundaries of jurisdictions, the Nash equilibrium outcome predicts that both jurisdictions will underinvest. This underinvestment conclusion has been a major argument in favour of transferring decision making about public infrastructure to a higher level of jurisdiction that encompasses all the spillovers.

In this paper, we consider a different remedy. We assume that the higher level jurisdiction wishes to coordinate local jurisdictions investment decisions and 'decentralize' the social optimum by means of a capital investment subsidization scheme. Specifically, we assume that the social regulator implements a linear markovian subsidization policy to support local expenditures in green capital. Under this tax scheme, each Jurisdiction  $i$  is granted an amount  $\tau_i(K_i, K_j)$  per unit of investment in public infrastructure capital  $e_i$ . It is important to note here that the unit rate of subsidization depends exclusively on the two jurisdictions stocks of infrastructure at any given time  $t$ .

In the remainder of this paper, we restrict our attention to the qualitative implications of initial differences in public infrastructure for the dynamic properties of the optimal subsidy policy. With this purpose in mind, we assume that the two jurisdictions are identical in all respects, except (perhaps) their initial stocks of public infrastructure. In other words, we assume identical cost functions,  $C_1(e) = C_2(e) = C(e)$ ,  $\forall e \in \mathfrak{R}_+$  and symmetric environmental quality indexes  $P_2(K_2, K_1) = P_1(K_2, K_1)$ ,  $\forall (K_1, K_2) \in \mathfrak{R}_+^2$ . However, we allow for different initial stocks of infrastructure by assuming that  $K_1^0 \geq K_2^0$ .

Furthermore, in order to actually solve for the optimal capital investment subsidization scheme we make specific assumptions about functional forms. Environmental quality indexes are assumed to be quadratic and given by

$$P_i(K_i, K_j) = p_0 + p_1 K_i + p_2 K_j + \frac{p_3}{2} K_i^2 + p_4 K_i K_j + \frac{p_5}{2} K_j^2, \quad \forall i = 1, 2,$$

with  $p_1, p_2 > 0, p_3 < 0$  and  $p_5 < 0$ . Furthermore we assume that  $p_1$  and  $p_2$  are sufficiently large to ensure that  $\frac{\partial P_i}{\partial K_i} > 0$  and  $\frac{\partial P_i}{\partial K_j} > 0$ ; i.e., the quality of the environment in Jurisdiction  $i$  is increasing in its own stock of infrastructure and interjurisdictional externalities are positive. These assumptions imply that environmental quality is produced through a technology that features decreasing returns to scale. Parameter  $p_4 \neq 0$  is not restricted in sign and will play an important part in our investigations. We introduce the following terminology due to Figuières (2004). Capital stocks are said to be *complements* when an increase in the stock of capital accumulated by one jurisdiction enhances the marginal productivity of its rival's own stock of capital. Conversely, when an increase in the capital stock of one jurisdiction lowers the marginal productivity of its rival's own stock of capital, they are said to be *substitutes*. Correspondingly, stocks are complements if  $p_4$  is positive, whereas they are substitutes if  $p_4$  is negative.

Lastly, we assume that the capital adjustment cost function is quadratic and given by

$$C(e) = c_1 e + \frac{c_2}{2} e^2 \tag{6}$$

---

<sup>5</sup>See Dockner et al. (2000, chapt. 9) for an introduction. Also, see Driskill and McCafferty (1989); Fershtman and Muller (1984, 1986); Figuières (2002, 2004); Figuières, Gardères and Rychen (2002); Reynolds (1991).

where  $c_1 \geq 0, c_2 > 0$  are cost parameters. The marginal cost of investment is thus increasing in  $e_i$ . Observe that this specification of the two jurisdictions cost functions as a strictly convex function of their investments has important consequences for the analysis: it induces firms to adjust their stocks of infrastructure gradually.

### 3 The utilitarian social optimum

Let us assume that the responsibility for infrastructure financing has been transferred to a higher level of government that encompasses both local jurisdictions; e.g., an intercommunal or interregional association. As a consequence of this delegation, interjurisdictional spillovers are now internalized into the decision making of a single economic agent. Then, the problem faced by the social planner is to find the time-paths of investment  $(e_1(\cdot), e_2(\cdot))$  that solve

$$\max_{(e_1(\cdot), e_2(\cdot))} J^1 + J^2 \quad (7)$$

subject to (2) and  $e_i(t) \geq 0, \forall t \in ]0, \infty[$ . This amounts to solving a two-state variable optimal control problem. We will refer to the solution to this problem as the *utilitarian social optimum* and use it as a benchmark for the remainder of the analysis.

In this section we show that there exists a unique optimal path of investment in public infrastructure. To solve for the social optimum we make use of Pontryagin's maximum principle. The current value Hamiltonian of the centralized problem (7) is defined as<sup>6</sup>

$$H(e_1, e_2, K_1, K_2, \lambda_1, \lambda_2) = \sum_{i=1}^2 (P_i(K_i, K_j) - C(e_i)) + \sum_{i=1}^2 \lambda_i (e_i - \delta K_i), \quad (8)$$

where  $\lambda_1$  and  $\lambda_2$  are the co-state variables associated with  $\dot{K}_1$  and  $\dot{K}_2$ , respectively. Assuming interior solutions, Pontryagin's maximum principle implies the following necessary conditions for optimality ( $\partial H / \partial e_i = 0, \dot{\lambda}_i = r\lambda_i - \partial H / \partial K_i$ ):

$$\lambda_i = c_1 + c_2 e_i, \quad \forall i = 1, 2, \quad (9)$$

$$\dot{\lambda}_i = (r + \delta) \lambda_i - (p_1 + p_2) - (p_3 + p_5) K_1 - 2p_4 K_2, \quad \forall i (i \neq j) = 1, 2, \quad (10)$$

along with the dynamic process of capital accumulation (2); the transversality condition at infinity is

$$\lim_{t \rightarrow \infty} [\lambda_1(t) (K_1(t) - K_1^c(t)) + \lambda_2(t) (K_2(t) - K_2^c(t))] e^{-rt} = 0, \quad (11)$$

where  $K_i^c(\cdot)$  denotes a candidate for optimization and  $K_i(\cdot)$  is any other path. Using Equation (9) to eliminate  $e_i$  from (2), optimality conditions can be summarized as

$$\begin{pmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{K}_1 \\ \dot{K}_2 \end{pmatrix} = \begin{pmatrix} (r + \delta) & 0 & -(p_3 + p_5) & -2p_4 \\ 0 & (r + \delta) & -2p_4 & -(p_3 + p_5) \\ 1/c_2 & 0 & -\delta & 0 \\ 0 & 1/c_2 & 0 & -\delta \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ K_1 \\ K_2 \end{pmatrix} - \begin{pmatrix} (p_1 + p_2) \\ (p_1 + p_2) \\ c_1/c_2 \\ c_1/c_2 \end{pmatrix} \quad (12)$$

along with the initial conditions  $(K_1(0) = K_1^0, K_2(0) = K_2^0)$  and the transversality condition (11). The above system of differential equations (12) can be rewritten more compactly as

<sup>6</sup>We have not incorporated explicitly the constraints  $e_i(t), e_j(t) \geq 0$  at the formulation stage of the problem. We preferred to solve it and check afterward that those constraints are verified. The same remark applies to the study of decentralized behaviors in the next section.

$\dot{x} = Ax - b$ . A steady-state solution  $(K_1^\infty, K_2^\infty)$  is defined as a constant trajectory that solves (12); i.e.,  $\dot{x} = 0$ . Setting time derivatives equal to zero, we obtain a system of algebraic equations which can be solved for  $(K_1^\infty, K_2^\infty, \lambda_1^\infty, \lambda_2^\infty)$  to yield:

$$e_1^\infty = e_2^\infty = \delta K^\infty, \quad (13)$$

$$K_1^\infty = K_2^\infty = K^\infty = \frac{(r + \delta) c_1 - (p_1 + p_2)}{(p_3 + p_5) + 2p_4 - \delta(r + \delta) c_2}. \quad (14)$$

The stability properties of the steady-state can be determined by examining the eigenvalues of the coefficient matrix  $A$ . Solving the characteristic equation

$$\det(\rho I - A) = \frac{[(p_3 + p_5) - (r + \delta - \rho)(\delta + \rho) c_2]^2 - 4p_4^2}{c_2^2} = 0, \quad (15)$$

where  $I$  is the identity matrix, yields four real and distinct eigenvalues, two of which are positive and two of which are negative, confirming a saddle-point solution. The following proposition provides a characterization of the social optimum.

**Proposition 1** *The socially optimal time-paths of investment in public infrastructure are*

$$e_1(t) = \frac{1}{2}(\delta + \rho_1)(K_1^0 - K_2^0)e^{\rho_1 t} + \frac{1}{2}(\delta + \rho_2)(K_1^0 + K_2^0 - 2K^\infty)e^{\rho_2 t} + \delta K^\infty, \quad (16)$$

$$e_2(t) = \frac{1}{2}(\delta + \rho_1)(K_2^0 - K_1^0)e^{\rho_1 t} + \frac{1}{2}(\delta + \rho_2)(K_1^0 + K_2^0 - 2K^\infty)e^{\rho_2 t} + \delta K^\infty, \quad (17)$$

where  $(\rho_1, \rho_2)$  are the negative roots of the coefficient matrix  $A$ :

$$\rho_1 = \frac{1}{2} \left[ r - \frac{\sqrt{(r + 2\delta)^2 c_2 - 4(p_3 + p_5 - 2p_4)}}{\sqrt{c_2}} \right], \quad (18)$$

$$\rho_2 = \frac{1}{2} \left[ r - \frac{\sqrt{(r + 2\delta)^2 c_2 - 4(p_3 + p_5 + 2p_4)}}{\sqrt{c_2}} \right]. \quad (19)$$

The stocks of green infrastructure  $(K_1(t), K_2(t))$  evolve along the following trajectories

$$K_1(t) = \frac{1}{2}(K_1^0 - K_2^0)e^{\rho_1 t} + \frac{1}{2}(K_1^0 + K_2^0 - 2K^\infty)e^{\rho_2 t} + K^\infty, \quad (20)$$

$$K_2(t) = \frac{1}{2}(K_2^0 - K_1^0)e^{\rho_1 t} + \frac{1}{2}(K_1^0 + K_2^0 - 2K^\infty)e^{\rho_2 t} + K^\infty, \quad (21)$$

which converge to the unique steady state  $K^\infty$ .

**Proof.** See Appendix A. □

Finally, note that the two jurisdictions' optimal rates of investment at any time  $t$  can be written as functions of the state vector  $(K_1(t), K_2(t))$ . The so-called feedback representations of the optimal controls then read as

$$\hat{e}_1(K_1, K_2) = \frac{1}{2} [2\delta + \rho_1 + \rho_2] K_1 + \frac{1}{2} (\rho_2 - \rho_1) K_2 - \rho_2 K^\infty, \quad (22)$$

$$\hat{e}_2(K_2, K_1) = \frac{1}{2} [2\delta + \rho_1 + \rho_2] K_2 + \frac{1}{2} (\rho_2 - \rho_1) K_1 - \rho_2 K^\infty. \quad (23)$$

Having characterized the utilitarian optimal solution, we now turn to the analysis of the decentralized scenario.

## 4 A pigovian remedy to infrastructure competition

Now we assume that the higher level jurisdiction seeks to implement the social optimum through the choice of a capital investment subsidy scheme. With the aim of coordinating local expenditures, the social regulator announces a linear-markovian subsidy scheme  $T^* = \{\tau_1^*(K_1, K_2), \tau_2^*(K_1, K_2)\}$  before jurisdictions take their decisions.<sup>7</sup> Under this scheme, each Jurisdiction  $i$  is granted a subsidy  $\tau_i(K_i(t), K_j(t))$  per unit of expenditure in public infrastructure at time  $t$ , where the unit subsidy rate depends only on the two jurisdictions current period stocks of green infrastructure capital,  $K_i(t)$  and  $K_j(t)$ . In the remainder of this paper, it is assumed that  $\tau_i(K_i, K_j)$  is not restricted in sign. Indeed, it may be negative implying a tax. It is shown that this pigouvian scheme  $T^*$  decentralizes the social utilitarian optimum as a markov perfect Nash equilibrium. In other words, the equilibrium that the economy will reach when each Jurisdiction  $i$  determines its preferred investment rule – taking as given both Jurisdiction  $j$ 's investment rule  $e_j(K_j, K_i)$  and the subsidy rule  $\tau_i^*(K_i, K_j)$  – coincide with the social optimum.

To begin with, let us consider how a linear-markovian subsidy affects local jurisdictions' incentives to invest in green infrastructure. In the presence of subsidization, at a markov perfect Nash equilibrium Jurisdiction  $i$  solves the dynamic optimization problem

$$\max_{e_i} \quad J_i = \int_0^{+\infty} e^{-rt} [P_i(K_i, K_j) - C(e_i) + \tau_i(K_i, K_j)e_i] dt \quad (24)$$

$$s.t. \quad \dot{K}_i = e_i - \delta K_i, \quad K_i(0) = K_i^0, \quad (25)$$

$$\dot{K}_j = e_j(K_j, K_i) - \delta K_j, \quad K_j(0) = K_j^0. \quad (26)$$

The current value Hamiltonian for this problem is defined as

$$H_i = P_i(K_i, K_j) - C(e_i) + \tau_i(K_i, K_j)e_i + \mu_i(e_i - \delta K_i) + \sigma_i(e_j(K_j, K_i) - \delta K_j), \quad (27)$$

where  $\mu_i$  and  $\sigma_i$  are the costate variables associated with  $\dot{K}_i$  and  $\dot{K}_j$ , respectively. Let us recall that the optimal strategies of Jurisdiction  $i$ 's opponent are necessarily of form  $e_j(K_j, K_i) = \phi_1 + \phi_2 K_j + \phi_3 K_i$ , given the linear-quadratic structure of the game. Assuming interior solutions, Pontryagin's maximum principle then implies that the following conditions

$$\mu_i = c_1 + c_2 e_i - m - n K_i - q K_j, \quad (28)$$

$$\dot{\mu}_i = (r + \delta) \mu_i - (p_1 + p_3 K_i + p_4 K_j) - n e_i - \sigma_i \phi_3, \quad (29)$$

$$\dot{\sigma}_i = (r + \delta - \phi_2) \sigma_i - (p_2 + p_4 K_i + p_5 K_j) - q e_i. \quad (30)$$

hold along Jurisdiction  $i$ 's optimal trajectory of investment (where the transversality condition has been omitted for sake of brevity).

The optimality conditions provide the higher level jurisdiction with the information needed to foresee how a subsidy policy will alter local jurisdictions' incentives to invest in public infrastructure. On the basis of this information, the regulator will select the infrastructure capital subsidization scheme  $T^*$  so as to decentralize the social optimum. Formally, this amounts to choosing  $T^*$  in such a way that the optimality conditions (28)-(30) match the conditions for a social optimum (9) and (10). The following proposition characterizes the optimal tax rule:

<sup>7</sup>Benckroun and Van Long (1998) more precisely state that the scheme is announced at date  $t = 0$  before economic agents take their decision. In a linear quadratic infinite horizon model, such a scheme is subgame perfect. Thus an alternative timing, more in line with the dynamic spirit of our analysis, is for the regulator to revise and announce the tax or subsidy rate at each date before jurisdictions decide upon their investment.

**Proposition 2** *The optimal subsidization scheme that decentralizes the socially optimal time-path of expenditure in public infrastructure capital as a markov perfect Nash Equilibrium is*

$$\tau_i^*(K_i, K_j) = m^* + n^* K_i + q^* K_j, \quad \forall i(i \neq j) = 1, 2, \quad (31)$$

where

$$q^* = \frac{-1}{4(r - \rho_1 - \rho_2)(-2(r + \delta) + \rho_1 + \rho_2)} \left\{ 8p_4(-r + \rho_1 + \rho_2) + \left[ 4p_5 - c_2(2r - \rho_1 - 3\rho_2)(2r - 3\rho_1 - \rho_2) \right] (\rho_1 - \rho_2) \right\}, \quad (32)$$

and

$$n^* = \frac{2(p_4 - p_5)(r - 2\rho_1)}{(r + 2\delta)(-2r + 3\rho_1 + \rho_2)} + q^* \left[ 1 + \frac{2(r - 2\rho_1)(\delta + \rho_1)}{(r + 2\delta)(-2r + 3\rho_1 + \rho_2)} \right], \quad (33)$$

and

$$m^* = -\frac{1}{(r + \delta)} \left\{ \left[ p_1 - (r + \delta)c_1 \right] + \frac{p_2(\rho_1 - \rho_2)}{(-2r + \rho_1 + \rho_2)} \right\} + \frac{K^\infty}{(r + \delta)} \left\{ \left[ \delta(r + \delta)c_2 - p_3 + p_5 \right] + \frac{2(p_4 + p_5)(r - \rho_1)}{(-2r + \rho_1 + \rho_2)} \right\} + \frac{q^*}{(r + \delta)} \left[ -r + \frac{2\delta(r - \rho_1)}{(-2r + \rho_1 + \rho_2)} \right] K^\infty + n^* \left[ -2 + \frac{r}{(r + \delta)} \right] K^\infty. \quad (34)$$

**Proof.** See Appendix B. □

## 5 Complementarity, substitutability and the role of initial conditions

In the next section, we rely on selected numerical examples and illustrations to provide important insights about the qualitative properties of the optimal markovian scheme. Most notably, on the basis of these examples it will be shown that for an initial period of time the optimal pigovian rule may require to simultaneously tax one jurisdiction and subsidize the other. This property is to be linked to the gap between initial endowments of public infrastructure and the nature of the technical relationship between the two stocks. The following qualitative results add important information about how the technical relationship among the stocks alters the strategic features of the game. Moreover, it will provide additional guidance in the choice of our numerical examples.

First of all, let us state a result which shows the connection between the technical relationship among stocks and best reply functions under optimal regulation.

**Proposition 3** *Let the stocks be complement,  $p_4 > 0$  (resp. substitute,  $p_4 < 0$ ). Then:*

- *the pigovian scheme is an increasing (resp. decreasing) function of the rival stock, i.e.  $q^* > 0$  (resp.  $q^* < 0$ ).*

- at the optimally regulated markov perfect equilibrium, Jurisdiction  $i$ 's decision rule is an increasing (resp. a decreasing) function of the rival stock.

**Proof.** Appendix C1. □

**Assumption 1**  $p_4 > 0$  and  $\frac{-2p_4+p_3+p_5}{(r+\delta-\rho_1)c_2} = \delta + \rho_1 \simeq 0$ .

Assumption 1 captures a family of investments problems where capital stocks are complements ( $p_4 > 0$ ) and costs are relatively large ( $c_2 \gg 0$ ).

**Lemma 1** Let Assumption 1 hold. Then  $K_i^*(t) > K_j^*(t) \iff \tau_i^*(t) < \tau_j^*(t)$  .

**Proof.** Appendix C2. □

Therefore one can deduce:

**Proposition 4** Let Assumption 1 hold. Assume also that Jurisdiction  $j$  is initially taxed whereas Jurisdiction  $i$  is subsidized,  $\tau_j^*(0) < 0$  and  $\tau_i^*(0) > 0$ . Then  $K_j^*(0) > K_i^*(0)$ .

This proposition means in particular that, under complementarity and large variable costs, when there is an initial taxation, then it applies to the jurisdiction with the largest initial capital stock.

**Assumption 2**  $p_4 < 0$  and  $p_4 < p_5$  and  $p_4 < (p_3 + p_5)/2$ .

Assumption 2 captures a family of investments problems where capital stocks are substitutes ( $p_4 < 0$ ) and the degree of substitutability is strong enough ( $p_4 < p_5$ ).

**Lemma 2** Let Assumption 2 hold. Then  $K_i^*(t) > K_j^*(t) \iff \tau_i^*(t) > \tau_j^*(t)$  .

**Proof.** Appendix C3. □

Thus, the following reversed property can now be established:

**Proposition 5** Let Assumption 2 hold. Assume also that Jurisdiction  $j$  is initially taxed whereas Jurisdiction  $i$  is subsidized,  $\tau_j^*(0) < 0$  and  $\tau_i^*(0) > 0$ . Then  $K_j^*(0) < K_i^*(0)$ .

This proposition indicates that, under "strong" substitutability, when there is an initial taxation, then it applies to the jurisdiction with the lowest initial capital stock.

The next section uses numerical examples to illustrate those results and to discuss the corresponding intuitions.

## 6 Illustrations

First of all, it is useful to highlight in what respects static and dynamic Pigovian instruments differ. In a static setting the subsidization rate faced by Jurisdiction  $i$  is constant so that an increase in its level of investment translates directly into an increase of the subsidy it receives. In addition to this quantity effect, in a dynamic setting, jurisdictions have to take into account the intertemporal 'price' effects of their decisions. First, since the rate of subsidization of each

jurisdiction depends on both stocks of infrastructure, a change in the rate of investment of any jurisdiction directly alters the subsidy rate enjoyed by both jurisdictions. Second, at a MPE Jurisdiction  $i$  correctly anticipates the decision rule of its rival,  $e_j(K_j, K_i)$  and so the indirect effect a stock increase will have on  $K_j$  and  $\tau_i^*(K_i, K_j)$ . Namely, Jurisdiction  $i$  anticipates that Jurisdiction  $j$  will reply to a change in  $K_i$  by adjusting its capital stock  $K_j$  and that this strategic move will affect  $\tau_i^*(K_i, K_j)$ .

Second, social efficiency requires that *i*) the gap between the two stocks remains optimal all along the transition phase and *ii*) eventually closes when the socially optimal steady-state is reached. It should be noted that while  $K^\infty$  (see expression 14) is independent of the initial conditions, the optimal evolution of the gap depends on the technological relationship that links capital stocks. Indeed, cost efficiency requires that the substitutability (or complementarity) property of the stocks be used to minimize the cost of the transition to the steady-state. Hence, it should come as no surprise that both the optimal evolution of the stocks (equations 16 and 17) and the optimal evolution of the corrective instrument designed to decentralize the social optimum (propositions 4 and 5) depend on the initial conditions.

The challenge is to come to grips with these uncommon features. With this purpose in mind, we now go through four numerical examples in which stocks are assumed to be complements and all parameters except initial capital stocks remain unchanged. A fifth example will be used to illustrate a situation in which stocks are substitute.

## 6.1 An example with technical complementarity

For the selected numerical parameters, the optimal tax/subsidy policy (2) is given by:

$$\tau_i^*(K_i, K_j) = 158.564 - 1.89455 K_i + 1.8511 K_j. \quad (35)$$

Note that  $\tau_i^*(K_i, K_j)$  is decreasing in its first argument and increasing in its second (see Proposition 3). Furthermore, observe that the optimal policy rule may not offer a subsidy to both jurisdictions. For example, if Jurisdiction  $i$  is initially endowed with a large stock of green infrastructure and competes with a rival which lacks infrastructure, then  $\tau_i^*(K_i, K_j)$  may be negative for an initial period of time – implying that Jurisdiction  $i$  is temporarily taxed. The optimal corrective instrument (35) induces Jurisdiction  $i$  to adopt the following equilibrium Markov strategy<sup>8</sup>

$$e_i(K_i, K_j) = 26.1367 - 0.262795 K_i + 0.255191 K_j \quad (36)$$

which coincide with the socially optimal time-path of investment in green infrastructure. Note that this strategy is increasing in the rival stock. A property that could have been anticipated from Proposition 4 since the selected parameter values satisfy Assumption 1.

### 6.1.1 Identical initial endowments

To begin with, assume that both jurisdictions are initially endowed with identical stocks of green infrastructure. Naturally, this case implies that both jurisdictions are identical and will follow the same time-path of investment in public infrastructure,  $K_1(t) = K_2(t) = K(t)$ ,  $\forall t$ . Consequently, the optimal tax/subsidy policy rewrites as:

$$\tau_i^*(K) = m^* + (n^* + q^*) K$$

<sup>8</sup>Details on the computation of the unregulated markov-perfect Nash-equilibrium of the game among jurisdictions are omitted here as they can be found in Figuières (2002).



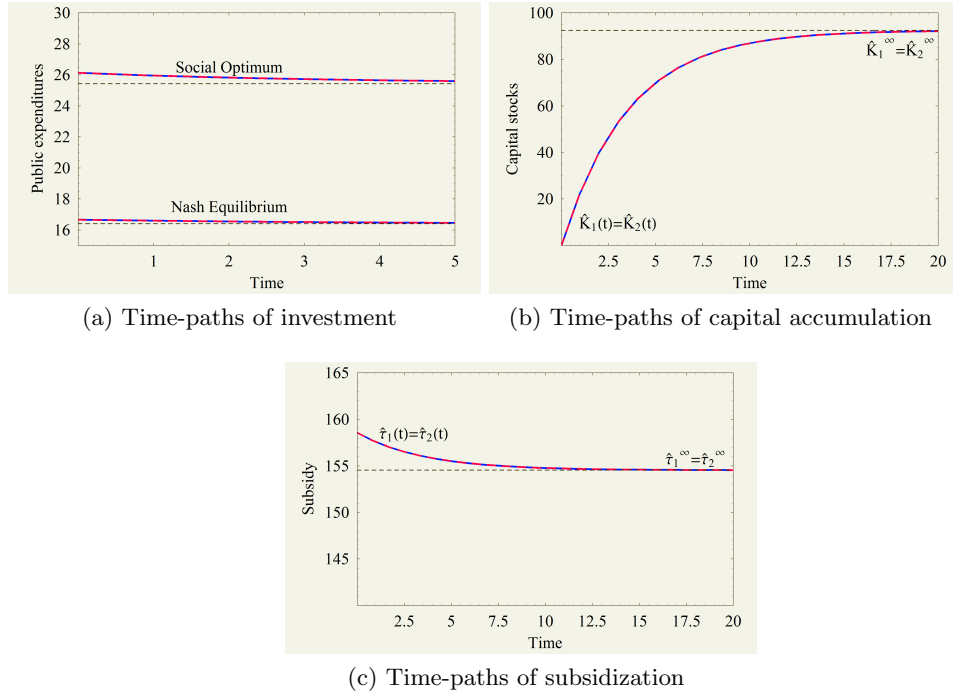


Figure 1: Socially optimal trajectories when jurisdictions are initially endowed with identical stocks of green infrastructure (parameter values:  $p_0 = 0, p_1 = 50, p_2 = 50, p_3 = -1.525, p_4 = 1.5, p_5 = -1.525, r = 1/10, c_1 = 0, c_2 = 10, \delta = 0.275, K_{10} = K_{20} = 0$ )

Socially optimal time-paths of investment in green infrastructure, capital accumulation and subsidization are depicted in Figure 1. Note that  $n^* + q^* < 0$  so that  $\tau_i^*(K)$  is a decreasing function of  $K$ . In other words, the optimal rate of subsidization monotonically decreases over time towards its steady-state level  $\hat{\tau}^\infty$ . This pattern of evolution should come as no surprise. Indeed, comparing the Feedback-Nash investment trajectory with the optimal one (see Figure 1a), it appears clearly that the need to reinforce incentives to invest is greater at earlier dates.

### 6.1.2 Difference in initial endowments

Now we assume that Jurisdiction 1 is initially endowed with a larger stock of green infrastructure than Jurisdiction 2. This implies that the two jurisdictions will follow separate trajectories of investment (and thus capital accumulation). However, such an asymmetry will disappear in the long-run since both trajectories converge to the same steady-state. The graphical comparison of unregulated and regulated investment levels reveals that Jurisdiction 2 should initially commit to larger investments than Jurisdiction 1, although this demand is smoothly lifted over time.

Figure 2 allows us to visualize how the required incentives are provided by the optimal markovian scheme. Initially, Jurisdiction 2 receives a higher subsidy for investing in green infrastructure than Jurisdiction 1. However as Jurisdiction 2's stock of infrastructure capital catches up with that of its rival this preferential treatment is smoothly removed by simultaneously reducing  $\tau_2^*(K)$  and increasing  $\tau_1^*(K)$  and eventually vanishes. Graphically, one sees

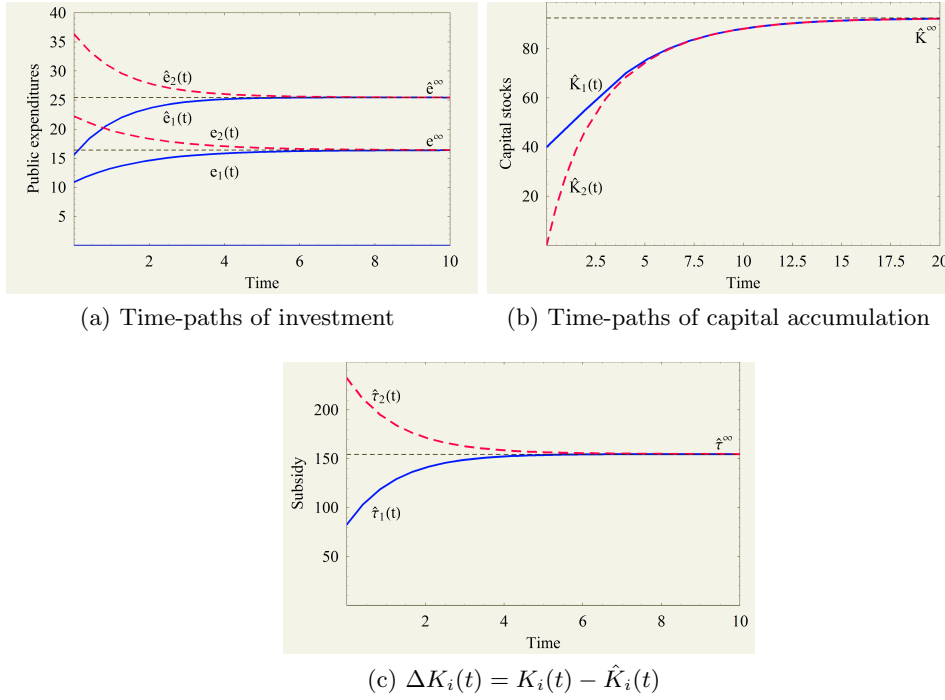


Figure 2: Socially optimal trajectories when Jurisdiction 1 initially is endowed with slightly more capital stock (1) (parameter values:  $p_0 = 0$ ,  $p_1 = 50$ ,  $p_2 = 50$ ,  $p_3 = -1.525$ ,  $p_4 = 1.5$ ,  $p_5 = -1.525$ ,  $r = 1/10$ ,  $c_1 = 0$ ,  $c_2 = 10$ ,  $\delta = 0.275$ ,  $K_{10} = 40$ ,  $K_{20} = 0$ )

that granted subsidies converge to the same steady-state value. However,  $\tau_2^*(K)$  approaches the steady-state from above whereas  $\tau_2^*(K)$  approaches  $\tau_2^\infty$  from below.

### 6.1.3 Carrot and stick policy with complementarity

In this example the gap between initial stocks of infrastructure has been increased further compared to the previous case. Figures 3 and 4 allows for a comparison between unregulated and socially optimal outcomes. The comparison reveals that the optimal tax/subsidy policy should alter jurisdictions' investment incentives in such a way that:

- Jurisdiction 2 is lead to increase its investment at all dates, although to a lesser extent as time passes by,
- Jurisdiction 1 is driven to reduce its level of investment for an initial time period before resuming its investment effort and eventually investing more than it would do in the laissez-faire scenario for the remainder of the planning horizon (Figures 3(a) and 3(b)),
- the gap between green capital stocks is first reduced so that the complementarity of the stocks is better exploited before allowing both capital stocks to increase well above their level in the unregulated scenario (Figures 4(a) and 4(b)).

The optimal Pigovian scheme (35) meets the above requirements. To see this, let us first consider the incentives it provides to the second jurisdiction. Observe that an increase in  $e_2$  augments  $K_2$  which in turn changes the subsidy rate through two channels. First, since  $\tau_2^*(K_2, K_1)$  is a decreasing function of  $K_2$ , an increase in  $K_2$  leads to a decrease in

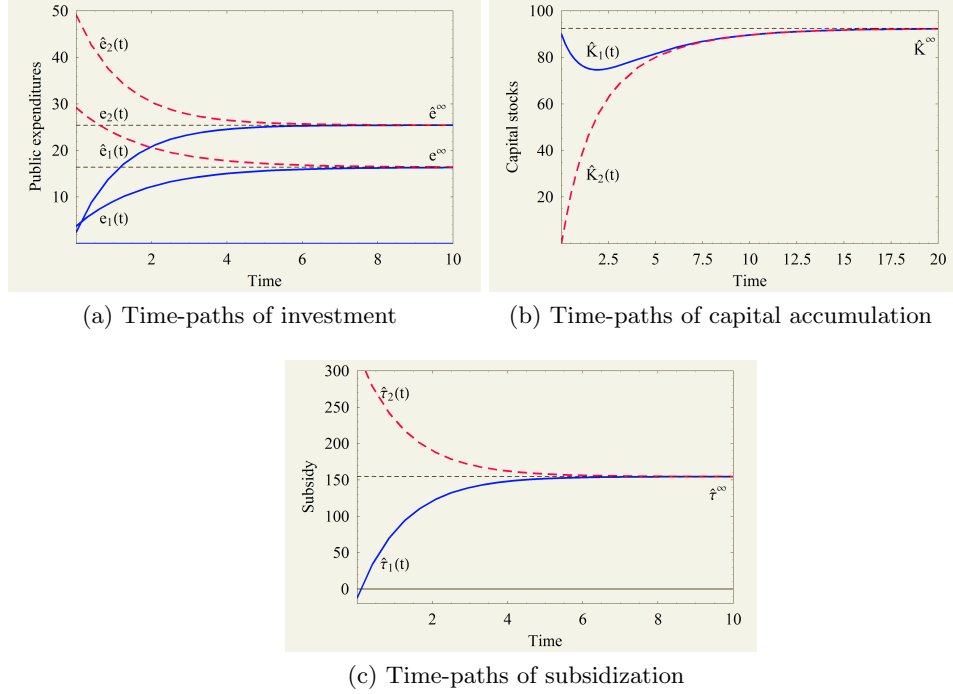


Figure 3: Carrot and stick policy with complementarity (parameter values:  $p_0 = 0, p_1 = 50, p_2 = 50, p_3 = -1.525, p_4 = 1.5, p_5 = -1.525, r = 1/10, c_1 = 0, c_2 = 10, \delta = 0.275, K_{10} = 90 = K_1^\infty, K_{20} = 0$ )

$\tau_2^*(K_2, K_1)$ . Second, due to feedback complementarity (see 36) an increase in  $K_2$  also provides Jurisdiction 1 with an incentive to increase its investment in order to augment  $K_1$  – But since  $\tau_2^*(K_2, K_1)$  is an increasing function of  $K_1$ , this strategic move leads to an increase in the subsidy rate. In the chosen example, the second positive effect dominates the first negative effect implying that Jurisdiction 2 is incited to invest more.

Incentives faced by Jurisdiction 1 are much more elaborated. Observe that an increase in its investment also has two opposite effects. Yet, which one prevails depends on the relative importance of the stocks, *i.e.* on the current state of the system. At a neighborhood of the initial conditions, with a large  $K_{10}$  and a small  $K_{20}$ , the negative effect prevails and gives Jurisdiction 1 the incentive to reduce its investment. Note that  $\tau_1^*(K_{10}, K_{20})$  is negative: this is a situation with complementarity ( $p_4 > 0$ ) which illustrates Proposition 4.

This leads to the required initial reduction in the gap between the two stocks. As the rival stock  $K_2$  increases, the positive effect soon dominates and  $\tau_1^*(K_1, K_2)$  turns into a subsidy which gives Jurisdiction 1 the incentive to invest more.

## 6.2 An example where stocks are substitute

Finally, Figures 5 and 6 illustrate a situation in which stocks are substitute ( $p_4 < 0$ ). For the chosen numerical values, the optimal pigovian scheme is:

$$\tau_i^*(K_i, K_j) = 104.926 - 1.3549 K_i - 1.7409 K_j. \tag{37}$$

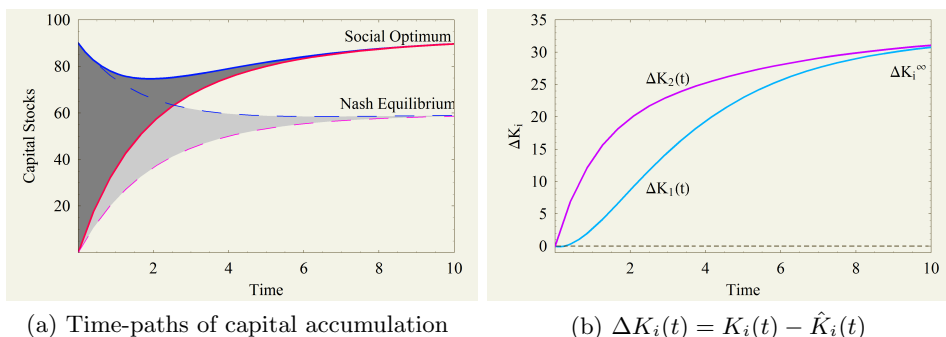


Figure 4: Differences in capital stocks with complementarity (parameter values: see Figure 3)

Observe that  $\tau_i^*(K_i, K_j)$  is now decreasing in both arguments. Furthermore, as in the previous cases, jurisdictions are not always subsidized. For sufficiently large stocks of green infrastructure (and especially a large rival stock) the optimal corrective instrument is negative, implying a tax on investment.

Optimal time-paths of investment now are

$$e_i(K_i, K_j) = 20.5688 - 0.181696 K_i - 0.342131 K_j, \quad (38)$$

and are decreasing functions of the rival stock. A property that could have been anticipated from Proposition 5 since the chosen parameter values satisfy Assumption 2.

The graphical comparison of regulated and unregulated time-paths of investment reveals that:

- Jurisdiction 1 underinvests for the whole planning horizon whereas Jurisdiction 2 should initially reduce its investment effort (roughly, until  $t = 24$ ) and resume its investment effort afterward (See Figures 5(a) and 6(b)).
- the gap between the stocks should initially be increased (Figure 6(a)) to fully exploit their substitutability which implies an increase of  $K_1$  and a reduction followed by an increase of  $K_2$ .

Let us now turn to the incentive properties of the optimal corrective instrument. Furthermore, let us focus our attention on the complex mix of incentives it should provide to Jurisdiction 2. At the beginning of the planning period,  $\tau_2^*(K_2, K_1)$  is negative which implies that Jurisdiction 2 faces a tax. This stems from the fact that Jurisdiction 2 should initially reduce its investment. *Ceteris paribus*, such a reduction would lead to a reduction of  $K_2$  which in turns would bring about a decrease of  $\tau_2^*(K_2, K_1)$ . Also the reduction of  $K_2$  increases  $e_1$  and  $K_1$  (because of strategic feedback substitutability) and decrease  $\tau_2^*(K_2, K_1)$ . At the initial state of the system, the second effect dominates; the scheme is a tax that reduces when  $e_2$  reduces. Yet, if this jurisdiction is taxed at early dates, it becomes subsidized long before  $t = 24$ . Still, despite this subsidization Jurisdiction 2 is correctly induced to lower its investments until  $t \simeq 24$ , because at the prevailing states of the system the subsidy is a decreasing function of  $e_2$ . As in Benchekroun and Van Long (1998) the sign of the instrument - negative for a tax, positive for a subsidy- in a dynamic context is relatively unrelated to the immediate goals of reducing or increasing the incentives. Instead, what matters is whether the scheme is a decreasing or an increasing function of the efforts.

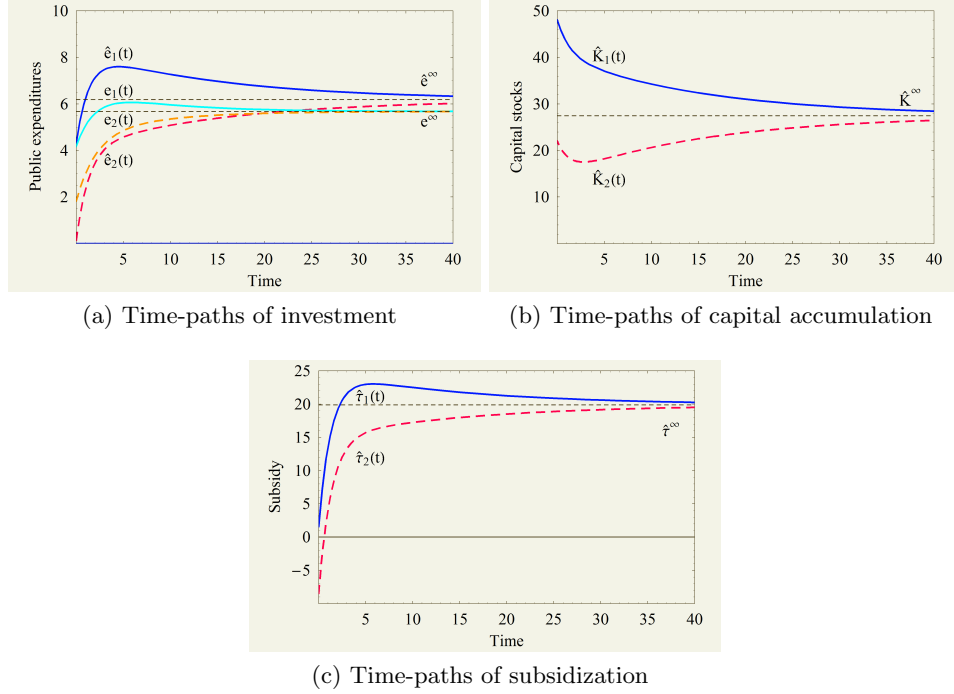


Figure 5: Carrot and stick policy with substitutability (parameter values:  $p_0 = 0, p_1 = 75, p_2 = 65, p_3 = -1, p_4 = -1.25, p_5 = -1, r = 1/10, c_1 = 1, c_2 = 8, \delta = 0.225, K_{10} = 48, K_{20} = 22$ )

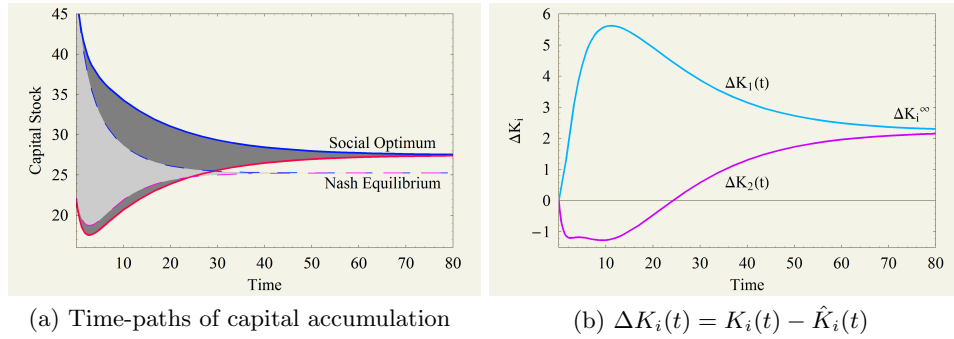


Figure 6: Differences in capital stocks with substitutability (parameter values: see Figure 5)

And the gap is correctly increased with the large initial stock jurisdiction being encouraged to invest more while the low initial stock jurisdiction being further induced to decrease its investment before being encouraged to invest.

## 7 Conclusion

This paper complements earlier contributions on price-based policies in a dynamic setting by investigating how differences in initial conditions alter the dynamic properties of the optimal tax or subsidy policy. Specifically, we concentrate on a model of competition in public infrastructure between two jurisdictions. Each jurisdiction invests over time in green infras-

structure to provide environmental services to its own residents. However, once supplied, it is supposedly impossible to exclude the residents of the other jurisdiction from the consumption of these environmental services. We assume that the two jurisdictions are identical in all respects except (perhaps) their initial stocks of green infrastructure capital. As is well known, this is a dynamic type of heterogeneity that disappears in the long run. Consequently, at the steady state, usual intuitions from static settings apply. Due to the presence of positive interjurisdictional spillovers, both jurisdictions will inefficiently under-invest, which calls for the implementation of a green capital subsidization scheme. In the short run, however, counterintuitive properties are established:

- i)* the pigovian scheme is not necessarily a subsidy. This finding confirms that the sign of the instrument, negative for a tax, positive for a subsidy, in a dynamic context is relatively unrelated to the immediate goals of reducing or increasing the incentives. Intuitions gained from static settings cannot be transposed into dynamic frameworks without care; important qualifications are often required. For instance in situations where the goal is to encourage investments, to some extent it does not matter whether the incentive instrument is a tax rather than a subsidy, provided that the tax is a decreasing function of the investment.
- ii)* One jurisdiction can be temporarily taxed, even though at those taxation dates its investments should be increased, whereas the other is subsidized. It is shown how these phenomena are related to initial conditions and to the kind of technological link between stocks of infrastructure (complementarity or substitutability). Put differently, initial conditions can be important drivers for the qualifications alluded to above.

A follow-up research of the present analysis would be to investigate the pigouvian regulation of infrastructure competition when public capitals generate negative externalities. One may expect in this context that, despite the needs to discourage non cooperative investments, one jurisdiction might be subsidized at early dates.

## Appendices

### A Optimal time-paths of investment

In this appendix we characterize the socially optimal time-paths of investment in public infrastructure. From the theory of differential equations, solutions to the system of differential equations (12) are of the form

$$K_1 = \alpha_1 e^{\rho_1 t} + \beta_1 e^{\rho_2 t} + K^\infty, \quad (39)$$

$$K_2 = \alpha_2 e^{\rho_1 t} + \beta_2 e^{\rho_2 t} + K^\infty. \quad (40)$$

where the parameters  $\alpha_1, \alpha_2, \beta_1, \beta_2$  are constant coefficients to be determined. Differentiating (39) and (40) with respect to time yields

$$\dot{K}_1 = \alpha_1 \rho_1 e^{\rho_1 t} + \beta_1 \rho_2 e^{\rho_2 t}, \quad (41)$$

$$\dot{K}_2 = \alpha_2 \rho_1 e^{\rho_1 t} + \beta_2 \rho_2 e^{\rho_2 t}. \quad (42)$$

Also, we know that when optimal time-paths of public expenditure exist, they can be written in feedback form as

$$e_i(t) = \phi_1 + \phi_2 K_i(t) + \phi_3 K_j(t), \quad \forall i(i \neq j) = 1, 2. \quad (43)$$

Substituting these strategies into the Nerlove-Arrow equations yields an alternative (feedback) representation of optimal capital stock trajectories:

$$\dot{K}_1 = (\phi_2 - \delta)K_1 + \phi_3 K_2 + \phi_1, \quad (44)$$

$$\dot{K}_2 = \phi_3 K_1 + (\phi_2 - \delta)K_2 + \phi_1. \quad (45)$$

This system can be rewritten in matrix form as  $\dot{K} = BK + h$ . The coefficient matrix  $B$  admits two distinct real roots:

$$\rho_1 = \phi_2 - \delta - \phi_3, \quad (46)$$

$$\rho_2 = \phi_2 - \delta + \phi_3. \quad (47)$$

We are now in a position to determine the values of the coefficients  $\{\alpha_1, \alpha_2, \beta_1, \beta_2\}$  by identifying equations (39) and (40) with (44) and (45). From (44) we know that  $\phi_3 K_2 = \dot{K}_1 - (\phi_2 - \delta)K_1 - \phi_1$ . Plugging (39) and (41) into this expression, and rearranging terms yields:

$$\phi_3 K_2 = \alpha_1(\rho_1 - (\phi_2 - \delta))e^{\rho_1 t} + \beta_1(\rho_2 - (\phi_2 - \delta))e^{\rho_2 t} - (\phi_2 - \delta)K^\infty - \phi_1. \quad (48)$$

Now, from (46) and (47) we know that  $(\rho_1 - (\phi_2 - \delta)) = -\phi_3$  and  $(\rho_2 - (\phi_2 - \delta)) = \phi_3$ . Plugging this into (48) and rearranging terms yields

$$K_2 = -\alpha_1 e^{\rho_1 t} + \beta_1 e^{\rho_2 t} - \phi_3^{-1} ((\phi_2 - \delta)K^\infty + \phi_1), \quad (49)$$

By identification of (39) and (49), it comes that  $\alpha_1 = -\alpha_2 = \alpha$ ,  $\beta_2 = \beta_1 = \beta$  and  $K^\infty = -\phi_1/(\phi_3 + \phi_2 - \delta)$ . From (46) and (47), it comes that  $\phi_2 = (\rho_1 + \rho_2 + 2\delta)/2$  and  $\phi_3 = (\rho_2 - \rho_1)/2$ . Plugging this in  $K^\infty$  yields  $\phi_1 = -\rho_2 K^\infty$ . Now, let us denote  $\Delta K_i(t) = K_i(t) - K^\infty$ . Observe that  $K_1(0) = \alpha_1 + \beta_1 + K^\infty$  and  $K_2(0) = \alpha_2 + \beta_2 + K^\infty$  so that we have a system of equation

$$\alpha + \beta = \Delta K_1(0), \quad (50)$$

$$\beta - \alpha = \Delta K_2(0), \quad (51)$$

which can be solved to get the values of the coefficients  $\alpha$  and  $\beta$  :

$$\alpha = \frac{1}{2}(\Delta K_1(0) - \Delta K_2(0)) = \frac{1}{2}(K_1^0 - K_2^0), \quad (52)$$

$$\beta = \frac{1}{2}(\Delta K_1(0) + \Delta K_2(0)) = \frac{1}{2}(K_1^0 + K_2^0 - 2K^\infty). \quad (53)$$

Substituting  $\alpha$  for  $\alpha_1$ ,  $-\alpha$  for  $\alpha_2$  and  $\beta$  for  $\beta_1$  and  $\beta_2$  into (39) and (40) yields Equations (20) and (21). Finally, Equations (16) and (17) easily follow from Equation (2) by observing that  $e_i = \dot{K}_i + \delta K_i$  yields

$$e_1 = \alpha(\delta + \rho_1)e^{\rho_1 t} + \beta(\delta + \rho_2)e^{\rho_2 t} + \delta K^\infty, \quad (54)$$

$$e_2 = -\alpha(\delta + \rho_1)e^{\rho_1 t} + \beta(\delta + \rho_2)e^{\rho_2 t} + \delta K^\infty. \quad (55)$$

## B Optimal tax/subsidy policy

Let us recall that Jurisdiction  $i$ 's optimal time-path of investment in public infrastructure is given by (54). Plugging  $e_1^c$  into the short-run equilibrium condition (28) and rearranging terms yields

$$\mu_i = v_1 + v_2 \alpha e^{\rho_1 t} + v_3 \beta e^{\rho_2 t}, \quad (56)$$

where  $v_1 = -m + c_1 + (\delta c_2 - n - q) K^\infty$ ,  $v_2 = -n + q + c_2 (\delta + \rho_1)$  and  $v_3 = -(n + q) + c_2 (\delta + \rho_2)$ . Differentiating (56) with respect to time, we get

$$\dot{\mu}_i = v_2 \alpha \rho_1 e^{\rho_1 t} + v_3 \beta \rho_2 e^{\rho_2 t}. \quad (57)$$

Using Equation (57) to eliminate  $\dot{\mu}_i$  from Equation (29), solving for  $\sigma^i$  and rearranging terms yields

$$\sigma^i = w_1 + w_2 \alpha e^{\rho_1 t} + w_3 \beta e^{\rho_2 t}, \quad (58)$$

where

$$w_1 = -\frac{1}{\phi_3} (p_1 + K^\infty (n \delta + p_3 + p_4) - (r + \delta) v_1), \quad (59)$$

$$w_2 = \frac{1}{\phi_3} ((p_4 - p_3) + v_2 (r + \delta - \rho_1) - n (\delta + \rho_1)), \quad (60)$$

$$w_3 = -\frac{1}{\phi_3} ((p_3 + p_4) - v_3 (r + \delta - \rho_2) + n (\delta + \rho_2)). \quad (61)$$

Finally, using Equation (58) to eliminate  $\sigma_i$  from (30) yields

$$\dot{\sigma}^i = z_1 + z_2 \alpha e^{\rho_1 t} + z_3 \beta e^{\rho_2 t} \quad (62)$$

with

$$z_1 = p_2 + K^\infty (q \delta + p_4 + p_5) - w_1 (r + \delta - \phi_2), \quad (63)$$

$$z_2 = (p_4 - p_5) + q (\delta + \rho_1) + w_2 (-r - \delta + \rho_1 + \phi_2), \quad (64)$$

$$z_3 = (p_4 + p_5) + q (\delta + \rho_2) + w_3 (-r - \delta + \rho_2 + \phi_2). \quad (65)$$

We now replace the coefficients  $w_1, w_2, w_3$  and  $v_1, v_2, v_3$  by their respective values into Equations (63)–(65) to get

$$z_1 = \frac{1}{\phi_3} [((r + \delta) (m - c_1) + p_1) (r + \delta - \phi_2) + p_2 \phi_3] + K^\infty (q \delta + p_4 + p_5) \quad (66)$$

$$+ \frac{K^\infty}{\phi_3} [((n + q) r + (2n + q) \delta - \delta (r + \delta) c_2 + p_3 + p_4) (r + \delta - \phi_2)] \quad (67)$$

$$z_2 = \frac{X}{\phi_3} (- (q (r + \delta)) + n (r + 2\delta) + p_3 - p_4 + q \rho_1 - c_2 (r + \delta - \rho_1) (\delta + \rho_1)) \quad (68)$$

$$+ [(p_4 - p_5) + q (\delta + \rho_1)], \quad (69)$$

$$z_3 = \frac{Y}{\phi_3} ((n + q) r + (2n + q) \delta + p_3 + p_4 - q \rho_2 - c_2 (r + \delta - \rho_2) (\delta + \rho_2)) \quad (70)$$

$$+ [(p_4 + p_5) + q (\delta + \rho_2)], \quad (71)$$

where

$$X = (r + \delta - \rho_1 - \phi_2), \quad (72)$$

$$Y = (r + \delta - \rho_2 - \phi_2). \quad (73)$$

Finally, we solve the algebraic system  $\{z_1 = 0, z_2 = 0, z_3 = 0\}$  for the parameters  $\{m, n, q\}$ . After tedious but straightforward manipulations, one obtains expressions (32)–(34).



## C Qualitative properties

### C.1 Analysis of $\partial\tau_i/\partial K_j$ and of Regulated best response functions

In order to establish the link between the sign of  $p_4$  and that of  $\partial\tau_i/\partial K_j$ , recall that

$$q^* = \frac{(4p_5 - c_2(2r - \rho_1 - 3\rho_2)(2r - 3\rho_1 - \rho_2))(\rho_1 - \rho_2) + 8p_4(-r + \rho_1 + \rho_2)}{4(r - \rho_1 - \rho_2)(2(r + \delta) - \rho_1 - \rho_2)}.$$

Note that the denominator is positive since both  $\rho_1$  and  $\rho_2$  are negative. Hence, the sign of  $q^*$  is the same as the sign of its numerator:

$$Num(q^*) = (4p_5 - c_2(2r - \rho_1 - 3\rho_2)(2r - 3\rho_1 - \rho_2))(\rho_1 - \rho_2) + 8p_4(-r + \rho_1 + \rho_2) \quad (74)$$

From the characteristic equation (15) and along with the fact that only its negative roots (18) and (19) are admissible, we have:

$$2p_4 = -(p_3 + p_5) + (r + \delta - \rho_1)(\delta + \rho_1)c_2, \quad (75)$$

and

$$-2p_4 = -(p_3 + p_5) + (r + \delta - \rho_2)(\delta + \rho_2)c_2. \quad (76)$$

Plugging (75) and (76) into equation (74) yields two alternative expressions for  $Num(q^*)$ . Adding these two expressions and rearranging terms, we obtain

$$Num(q^*) = -(\rho_1 - \rho_2)(c_2\rho_1^2 + 6c_2\rho_1\rho_2 - 4c_2r\rho_1 + c_2\rho_2^2 + 2c_2r^2 - 4p_5 - 4c_2r\rho_2) \quad (77)$$

Because  $\rho_1$  and  $\rho_2$  are negative the term between the second bracket is positive. Thus the sign of  $Num(q^*)$  depends only on the sign of  $\rho_1 - \rho_2$ . To complete the proof, from the expressions (18) and (19) observe that:

- $p_4 > 0$  implies  $(\rho_1 - \rho_2) < 0$  and then from (77)  $q > 0$ ,
- $p_4 < 0$  implies  $(\rho_1 - \rho_2) > 0$  and then from (77)  $q < 0$ .

Finally, from the last two equivalences along with expressions (22) and (23), one can easily observe that the second part of Proposition 3 holds; i.e.,  $sign(\partial\hat{e}_i/\partial K_j) = sign(p_4)$ .

### C.2 Proof of Lemma 1

Observe that:

$$\begin{aligned} \tau_i^*(t) - \tau_j^*(t) &= m^* + n^*K_i^*(t) + q^*K_j^*(t) - (m^* + n^*K_j^*(t) + q^*K_i^*(t)) \\ &= (n^* - q^*)(K_i^*(t) - K_j^*(t)). \end{aligned} \quad (78)$$

From (32) and (33), one obtains

$$n^* - q^* = \frac{2(p_4 - p_5)(r - 2\rho_1)}{(-2r + 3\rho_1 + \rho_2)(r + 2\delta)} + q^* \frac{2(r - 2\rho_1)(\delta + \rho_1)}{(-2r + 3\rho_1 + \rho_2)(r + 2\delta)}.$$

From the above expression it is straightforward to see that

$$sign(n^* - q^*) = -sign[(p_4 - p_5) + q^*(\delta + \rho_1)].$$

Now observe that  $sign(n^* - q^*) < 0$  if  $\delta + \rho_1 = 0$ . Using a continuity argument for cases where  $\delta + \rho_1$  is different from zero but small (Assumption A1) one also has  $sign(n^* - q^*) < 0$ . Hence, under Assumption 1 one has

$$K_i^*(t) > K_j^*(t) \iff \tau_i^*(t) < \tau_j^*(t).$$

### C.3 Proof of Lemma 2

From the proof of Lemma 1, recall that

$$\text{sign}(n^* - q^*) = -\text{sign}[(p_4 - p_5) + q^*(\delta + \rho_1)].$$

Note that under Assumption 2 we have  $q^* < 0$ ,  $(\delta + \rho_1) > 0$  and  $(p_4 - p_5) < 0$ . Hence we have  $\text{sign}(n^* - q^*) > 0$  so that

$$K_i^*(t) > K_j^*(t) \iff \tau_i^*(t) > \tau_j^*(t) .$$

## References

- Bencheikroun, H. and N. Van Long (1998), Efficiency inducing taxation for polluting oligopolists, *Journal of Public Economics* **70**, 325–342.
- Bergstrom, T., J. Cross and R. Porter (1981), Efficiency-inducing taxation for a monopolistically supplied depletable resource, *Journal of Public Economics* **15**, 23–32.
- Dockner, E., Van Long, N. Jorgensen, S. and G. Sorger (2000), *Differential games in economics and management science*, Cambridge University Press, Cambridge.
- Driskill, R.A. and S. McCafferty (1989), Dynamic duopoly with adjustment costs: A differential game approach, *Journal of Economic Theory* **49**, 324–338.
- Fershtman, C. and E. Muller (1984), Capital accumulation game of infinite duration, *Journal of Economic Theory*, **33**, 322–339.
- Fershtman, C. and E. Muller (1986), Turnpike properties of capital accumulation games, *Journal of Economic Theory*, **38**, 167–177.
- Figuières, C. (2002), Complementarity, Substituability and Strategic Accumulation of Capital, *International Game Theory Review* **4**, (4) 371–390.
- Figuières, C., Gardères, P. and F. Rychen (2002), Infrastructures publiques et politiques de développement décentralisées, *L'Actualité économique, Revue d'analyse économique* **78**, (4) 539–570.
- Figuières, C. (2004), Markov interactions in a class of dynamic games, *LAMETA Working Paper*, Montpellier.
- Lewis, T.R. (1996), Protecting the Environment When Costs and Benefits are Privately Known, *The Rand Journal of Economics* **4**, (27) 819–847.
- Karp, L. and J. Livernois (1992), On efficiency inducing taxation for a nonrenewable resource monopolist, *Journal of Public Economics* **49**, 219–239.
- Karp, L. and J. Livernois (1994), Using automatic tax changes to control pollution emissions, *Journal of Environmental Economics and Management* **27**, 38–48.
- Reynolds, S.S. (1991), Dynamic oligopoly with capacity adjustment costs, *Journal of Economic Dynamics and Control* **15**, (3), 491–514.
- Wildasin, D.E. (1991), Some rudimentary 'duopoly' theory, *Regional Science and Urban Economics* **27**, 38–48.