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An Analytical Study of the Modification Ability of Distribution Centers

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Abstract

This paper considers a two-level vendor managed inventory (VMI) system comprising a distribution center (DC) and a retailer. Both the DC's and the retailer's replenishment decisions follow the order-up-to-level policy and aim at maximizing the profit of the overall system. We critically examine the potential of the DC's ability to modify delivery decisions, identify and quantify the cost factors that influence the DC's modification ability, establish a relationship between the DC's location and its modification ability, and show the trade-off between the DC's modification ability and related costs. Our analysis provides a new insight into the role of the DC and reveals the full potential of the VMI system. Our findings and their practical implications, demonstrated with the aid of computational examples, are helpful for enhancing the practice of VMI at both strategic and operational levels.

Key Words: Vendor managed inventory system; distribution center; modification ability; order-up-to-level policy; location.

Résumé

Cet article considère un système à deux niveaux de stockage géré par le vendeur (SGV) comprenant un centre de distribution (CD) et un détaillant. Les décisions d'approvisionnement du CD et du détaillant suivent toutes deux une politique de commande jusqu'à un niveau donné et visent à maximiser le profit du système dans son ensemble. Nous procédons à un examen critique du potentiel du CD à modifier les décisions de stockage, identifions et quantifions les facteurs de coûts qui influencent l'habileté de modification du CD, établissons une relation entre la localisation du CD et ses habiletés de modification et montrons l'arbitrage entre l'habileté à modifier du CD et les coûts afférents. Notre analyse donne une nouvelle perspective sur le rôle du CD et révèle tout le potentiel du système SGV. Nos résultats et leurs implications pratiques, démontrés à l'aide d'exemples numériques, seront utiles pour favoriser la pratique du SGV tant au niveau stratégique qu'au niveau opérationnel.

Mots clés : système de stockage géré par le vendeur; centre de distribution; habileté à modifier; politique d'approvisionnement jusqu'à un niveau; localisation.

1 Introduction

There are normally three stages in a supply chain, namely *procurement*, *production* and *distribution*, each of which may consist of several facilities. In the distribution stage, the distribution center (DC) plays an important role. It is not only the confluence point for collecting and delivering products, but also the main vehicle used to meet customer demand through such activities as ordering, inventory management, transportation, transaction and information processing, etc. The role of the DC has been highlighted in the supply chain management (SCM) literature. For example, under a vendor managed inventory (VMI) system, the DC acts as a supplier that is committed to the mutual agreements between trading partners on inventory levels, fill rates and transaction costs, so trading partners can maximize their benefits by reducing inventories and stock-outs (Aviv and Federguen, 1998; Angulo et al., 2004).

In this paper we study a new role of the DC – its ability to modify replenishment decisions for retailers – by considering a VMI system that consists of a DC and a retailer. Both the DC’s and the retailer’s replenishment decisions follow the order-up-to-level (OUL) policy and aim at maximizing the profit of the overall system. We critically examine the potential of the DC’s ability to modify replenishment decisions for retailers, identify and quantify the cost factors that influence the DC’s modification ability, and shed new light on the relationship between the DC’s location decision and its modification ability. Our analysis provides a new insight into the role of the DC in the supply chain and reveals the full potential of the VMI system. Our findings are helpful for enhancing VMI and SCM practices at both strategic (i.e., the DC’s location decisions) and operational (i.e., the DC’s and the retailer’s optimal replenishment decisions) levels.

This paper is organized as follows. We present a literature review and discuss the problem formulation in Sections 2 and 3, respectively. In Section 4 we propose and compare two ordering and delivery strategies in order to examine the DC’s ability to modify delivery decisions. In Section 5 we classify the DC’s modification ability as positive or negative according to certain conditions, and identify and quantify the cost factors that influence it. We present in Section 6 computational examples to demonstrate the practical implications of the theoretical results. In Section 7 we conclude the study’s major findings and suggest further research directions.

2 Literature Review

Distribution centers are one kind of suppliers, responsible for the supply of products to their downstream customers, which include wholesalers, retailers, etc. Plentiful of research has been conducted on DCs’ location decisions (Eskigun et al., 2005; Snyder, 2006; Shen et al., 2003, etc.). These studies were usually concerned with setting up integrated models of the considered problems and proposing solution algorithms that aim at minimizing the expected cost of the system. In addition to studying location decisions, cooperation of the DC with its horizontal or vertical partners in the supply chain has also received considerable attention, which is significant in helping the supply chain to achieve such chain-wide objectives as shorter cycle time, lower inventory, lower cost and better customer service (Bordley et al., 1999; Neubert et al., 2004; Chen et al., 2005, etc.)

The increasing application of VMI in the practice of SCM in recent years has further heightened the importance of the DC’s function, since different decisions can be integrated under the VMI mode to achieve global optimal outcomes. In Bertazzi et al.(2005), under the assumption that the supplier takes care of the retailer’s replenishment decisions, two types of

VMI policy, namely the OUL policy and the fill-fill-dump (FFD) policy, were presented and compared, and an algorithm to solve the problem was proposed. With the consideration of fixed production cost and fixed cost of the delivery vehicles, the computational results showed that the FFD policy yields a lower average cost than the OUL policy. Jaruphongsa et al. (2004) studied a single-item, two-echelon system consisting of a warehouse and a distribution center. Studying the optimality properties of the problem, the authors presented a dynamic lot-sizing VMI model with delivery time windows and early shipment penalties, followed by a polynomial time algorithm for computing its solution.

In addition to treating the DC as the supplier, there are studies that considered other vendors such as manufacturers and developed the corresponding VMI strategies. These studies usually addressed a specific problem, analyzed various integration techniques in the form of information sharing, synchronized replenishment, and(or) collaborative product design and development, and derived the advantages of the proposed VMI strategies in terms of lower inventory, lower cost and better customer service, etc. Disney et al. (2003) investigated the impact of a VMI strategy on the transportation operations in a supply chain. It was shown that the holistic nature of inventory management within VMI enables batching to minimize transport demand without negatively impacting the overall dynamic performance of the supply chain. Gurbuz et al. (2007) studied the impact of coordinated replenishment and shipment in an inventory/distribution system. A new policy equivalent to the introduction of a warehouse with no inventory that is in charge of the ordering, allocation and distribution of inventory for the retailers was developed. They numerically compared the performance of the proposed policy with three other policies to identify the settings in which each policy would perform well.

Different from past studies on VMI that mainly focused on tackling the challenges of designing an integrated replenishment strategy for a complicated VMI system, we offer in this paper a new viewpoint of VMI by developing an analytical model to critically examine the role of the DC from the perspective of its ability to modify delivery decisions, i.e., the DC's ability to hold inventory and outsource supplies through its own ordering and delivery decisions. To focus on studying the modification ability of the DC, we confine our study to a simple VMI system comprising only one DC and one retailer, without considering the other important roles of the DC such as "risk pooling", "economies of scale", etc. However, it can be seen in later sections that our analysis and findings do not conflict with or weaken the other functions of the DC. In addition, as our analysis provides a new insight into the role of the DC and reveals the full potential of the VMI system, the findings are significant in terms of their theoretical contributions and potential application in different kinds of VMI systems.

3 Problem Formulation

We consider a two-echelon VMI system consisting of one DC and one retailer. By following a single-period ordering strategy, one kind of product supplied by a manufacturer is delivered from the DC to the retailer. There is no backlogging at the retailer, which means that the retailer will forgo the profit when the product is not available. Denote D_t as the demand faced by the retailer in period $t, t = 1, 2, \dots$, which following Lee et al. (2000) is assumed to be autocorrelated and modeled as follows

$$D_t = d + \rho D_{t-1} + \varepsilon_t, \quad (1)$$

where $d > 0$ is a prior estimate of the average demand at period 1; $-1 \leq \rho \leq 1$ is a constant coefficient expressing the degree of correlation between the demand in the present period and

the demand and the retailers action in the previous period; and ε_t is the error term, which is *i.i.d.* according to the normal distribution with mean 0 and variance σ^2 . We assume that ε_t is sufficiently smaller than d so that the probability of the demand being negative is negligible.

Denote h_v and h_r as the cost of holding one unit of the product per unit time at the DC and the retailer, respectively. Since the unit holding cost at the retailer normally exceeds that at the DC, we assume that $h_r > h_v$. We further assume that no fixed ordering cost is incurred when placing an order in time period $t, t = 1, 2, \dots$, because the ordering cost has no effect on the optimal solution for a single-period inventory problem.

In addition to the supply from the manufacturer, it is assumed that the DC can also obtain some units from an “alternative” source of supply if it does not have enough stock to meet the quantities required by the retailer, which will incur an additional cost being the penalty for the shortfall.

A similar problem has been considered by other researchers (see, e.g., Lee et al., 2000; Cheng and Wu, 2005), where the supplier and the manufacturer in these studies are referred to as the manufacturer and the DC, respectively, in this paper. However, we address the problem in a different way. First, the VMI mode is taken into account, i.e., the DC makes both ordering decisions (from the manufacturer) and replenishment decisions (for the retailer); second, the DC can make a choice of whether or not to use the “alternative” source of supply. In other words, the following alternative strategies are considered in this paper.

Case 1. The replenishment (delivery) decisions are characterized by the fact that the DC makes no change to the quantities of the product ordered and received from the manufacturer. We call the ordering decisions under Case 1 the “one-stage decision” (OSD) strategy. Denote L as the lead time of the ordering and delivery decisions. See Figure 1 for reference, where M represents the manufacturer, DC represents the distribution center, and R represents the retailer.

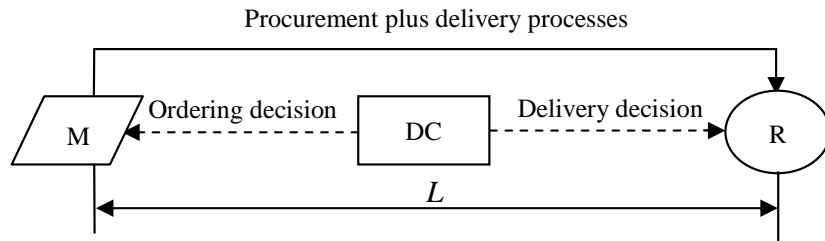


Figure 1: Illustration of the OSD strategy

Under the OSD strategy, the shortage and inventory costs in the considered VMI system occur at the retailer only. Denote c_r as the unit shortage cost at the retailer, which is taken as the profit of the SC system from selling one unit of the product.

Case 2. When the quantities of the product supplied by the manufacturer arrive, depending on the DC’s delivery decisions, some units may be stocked at the DC, or some quantities of the product may be supplied from an “alternative” source that has an unlimited capacity to supply the product. We call the ordering decisions under Case 2 the “two-stage decision” (TSD) strategy. Denote l_v and l_r as the lead time of the ordering decisions and the delivery decisions, respectively. See Figure 2 for reference. It is assumed that $l_v + l_r$ is constant, and it is reasonable to assume that $l_v + l_r > L$.

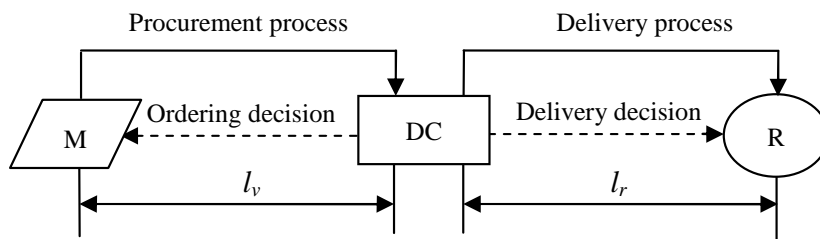


Figure 2: Illustration of the TSD strategy

As mentioned above, it is assumed that the DC will incur a higher cost to acquire the product from the “alternative” source than from the manufacturer, and let c_v be the difference between the two costs. So c_v is the unit shortage cost at the DC. As the delivery decisions for the retailer are made from the point of view of the SC system, it is obvious that the unit shortage cost at the retailer under the TSD strategy is c_r , too. In addition, it is reasonable to assume that $c_r - c_v > 0$, i.e., the supply chain will benefit from selling the products obtained from the “alternative” source of supply.

Under both strategies, we assume that the order-up-to-level policy is adopted by the DC whenever it makes ordering decisions or delivery decisions, since such a policy minimizes the total discounted holding and shortage costs over the infinite horizon (Heyman and Sobel, 1984). It should be pointed out that the other roles of the DC such as “risk pooling” and “economies of scale” can be realized under either the OSD strategy or the TSD strategy, and it is possible for the DC to adopt other strategies with regard to the ordering and delivery decisions. However, to focus on assessing the value of the DC’s modification ability, we confine our study to the above two strategies without considering the other important functions of the DC.

4 The OSD and the TSD Strategies

We analyze the OSD and TSD strategies, and derive the order-up-to-level (OUL), the quantities of the product delivered to the retailer, and the cost and profit due to the decisions made in time period t under each of these two strategies in Section 4.1 and Section 4.2, respectively. In Section 4.3 we derive the means and variances of the demands corresponding to the DC’s ordering and delivery decisions, and we show that the variance of the demand under the TSD strategy is larger than that under the OSD strategy.

4.1 The OSD strategy

(1) The order-up-to-level

Under the OSD strategy, denote the optimal OUL in time period t as S_t^1 . According to Heyman and Sobel (1984), we have

$$S_t^1 = e_t^1 + k_1 \sqrt{\vartheta_t^1}, \quad (2)$$

where e_t^1 and ϑ_t^1 are the mean and variance of $\sum_{i=1}^{L+1} D_{t+i}$, respectively; $k_1 = F_s^{-1}\left(\frac{c_r}{c_r + h_r}\right)$, where $F_s^{-1}(\cdot)$ is the inverse cumulative standard normal distribution function.

(2) The quantity delivered to the retailer

Denote y_t^1 as the quantities of the product ordered by the DC in time period t , which will be supplied by the manufacturer and delivered from the DC to the retailer without any modification. According to the OUL policy, we have

$$y_t^1 = D_t + S_t^1 - S_{t-1}^1. \quad (3)$$

(3) The cost due to the ordering decision made in time period t

Since it is common that the mean and standard deviation of the demand are always on the positive side of the ordinate, we assume without loss of generality that $\int_0^a f(x)dx = \int_{-\infty}^a f(x)dx$ in this paper, where $f(x)$ is the probability density function of the demand. Under the OSD strategy, the minimum expected cost of the SC system due to the decision made in time period t can be deduced as follows (see, e.g., Lee et al., 2000):

$$\begin{aligned} G(S_t^1) &= h_r \int_0^{S_t^1} (S_t^1 - x)f_1(x)dx + c_r \int_{S_t^1}^{\infty} (x - S_t^1)f_1(x)dx \\ &= (c_r + h_r)[k_1 F_s(k_1) + f_s(k_1)]\sqrt{\vartheta_t^1} - c_r k_1 \sqrt{\vartheta_t^1} \\ &= (c_r + h_r)f_s(k_1)\sqrt{\vartheta_t^1}, \end{aligned} \quad (4)$$

where $f_1(x) = \frac{1}{\sqrt{2\pi\vartheta_t^1}} \exp\left[-\frac{(x-e_t^1)^2}{2\sqrt{\vartheta_t^1}}\right]$; $F_s(\cdot)$ and $f_s(\cdot)$ are the cumulative standard normal distribution function and the standard normal probability density function, respectively.

(4) The profit due to the ordering decision made in time period t

Under the OSD strategy, the decisions made in time period t are to meet the demand in time period $t + L + 1$, so the maximum expected profit of the SC system due to the ordering and delivery decisions made in time period t is:

$$W(S_t^1) = \int_0^{\infty} c_r x \phi_1(x) dx - (c_r + h_r) f_s(k_1) \sqrt{\vartheta_t^1},$$

where $\phi_1(x)$ is the probability density function of D_{t+L+1} .

It can be deduced from Eq.(1) that

$$D_{t+L+1} = d \frac{1 - \rho^{L+1}}{1 - \rho} + \rho^{L+1} D_t + \sum_{i=0}^L \rho^{L-i} \varepsilon_{t+1+i}.$$

Denote e_{t+L+1} and ϑ_{t+L+1} as the mean and variance of D_{t+L+1} , respectively. So

$$e_{t+L+1} = d \frac{1 - \rho^{L+1}}{1 - \rho} + \rho^{L+1} D_t,$$

and

$$\vartheta_{t+L+1} = \sigma^2 \left(\frac{1 - \rho^{L+1}}{1 - \rho} \right)^2.$$

It can be further deduced that

$$W(S_t^1) = c_r e_{t+L+1} - (c_r + h_r) f_s(k_1) \sqrt{\vartheta_t^1}. \quad (5)$$

4.2 The TSD strategy

Under the TSD strategy, both ordering and delivery decisions are made based on the OUL policy (while the latter one can be called the delivery-up-to-level (DUL) accordingly), which means the quantities of the product delivered to the retailer in time period t may be different from the quantities of the product received by the DC. Compared with the OSD strategy, the delivery decision made under the TSD strategy is based on the latest information about demand in the market. In addition, the product can be stored at the DC at a lower cost than at the retailer, and the DC is able to supply the product from the alternative resource at a lower shortage cost than at the retailer. So, intuitively, the TSD strategy may outperform the OSD strategy due to the DC's ability to modify delivery decisions. However, a formal analysis to quantify the difference between the two strategies and the factors that influence the DC's modification ability is needed.

(1) The delivery and ordering decisions

As the delivery and ordering decisions are all based on the OUL policy and can be made separately, both processes are similar to that under the OSD strategy, with differences only in the related parameters. Let the superscripts $2r$ and $2v$ denote the parameters for the delivery and order decisions, respectively. Following the derivation in Section 4.1, we have

$$S_t^{2m} = e_t^{2m} + k_{2m} \sqrt{\vartheta_t^{2m}}, m = r, v, \quad (6)$$

where e_t^{2r} and ϑ_t^{2r} are the mean and variance of $\sum_{i=1}^{l_r+1} D_{t+i}$, respectively; e_t^{2v} and ϑ_t^{2v} are the mean and variance of $\sum_{i=1}^{l_v+1} y_{t+i}^{2r}$, respectively; $k_{2r} = k_1 = F_s^{-1}\left(\frac{c_r}{c_r+h_r}\right)$; $k_{2v} = F_s^{-1}\left(\frac{c_v}{c_v+h_v}\right)$; and

$$y_t^{2r} = D_t + S_t^{2r} - S_{t-1}^{2r}, \quad (7)$$

$$y_t^{2v} = y_t^{2r} + S_t^{2v} - S_{t-1}^{2v}, \quad (8)$$

where y_t^{2r} and y_t^{2v} are the quantities delivered and ordered in time period t , respectively. The expected costs due to the delivery and ordering decisions are respectively

$$\begin{aligned} G(S_t^{2m}) &= h_m \int_0^{S_t^{2m}} (S_t^{2m} - x) f_{2m}(x) dx + c_m \int_{S_t^{2m}}^{\infty} (x - S_t^{2m}) f_{2m}(x) dx \\ &= (c_m + h_m) f_s(k_{2m}) \sqrt{\vartheta_t^{2m}}, m = r, v, \end{aligned} \quad (9)$$

where $f_{2m}(x) = \frac{1}{\sqrt{2\pi\vartheta_t^{2m}}} \exp\left[-\frac{(x-e_t^{2m})^2}{2\sqrt{\vartheta_t^{2m}}}\right]$.

(2) The profit from the decisions made in time period t

From the point of view of the SC system, profit is realized only when the product is sold by the retailer. So the expected profit of the SC system due to the ordering and delivery decisions made in time period t is equal to the excess of the corresponding revenue over cost, which is expressed as follows:

$$W(S_t^{2r} + S_t^{2v}) = \int_0^{\infty} c_r x \phi_{21}(x) dx - G(S_t^{2r}) - G(S_t^{2v}),$$

where $\phi_{21}(x)$ is the probability density function of D_{t+l_r+1} .

Similar to the analysis in Section 4.1, it can be deduced that

$$W(S_t^{2r} + S_t^{2v}) = c_r e_{t+l_r+1} - (c_r + h_r) f_s(k_{2r}) \sqrt{\vartheta_t^{2r}} - (c_v + h_v) f_s(k_{2v}) \sqrt{\vartheta_t^{2v}}, \quad (10)$$

where $e_{t+l_r+1} = d \frac{1-\rho^{l_r+1}}{1-\rho} + \rho^{l_r+1} D_t$ is the mean of D_{t+l_r+1} .

It can be seen from above results that, under the OSD strategy, the expected profit of the SC system due to the ordering and delivery decisions made in time period t is related to c_r , h_r, k_1 and ϑ_t^1 , while such profit under the TSD strategy is related to $c_m, h_m, k_{2m}, \vartheta_t^{2m}$, where $m = r, v$. Since c_m, h_m, k_{2m} , where $m = r, v$, have direct relations with the cost parameters given, we analyze in the following the means and variances of the quantities of the product ordered and delivered under the two strategies, which have an impact on the modification ability of the DC.

4.3 Means and variances of the quantities of the product ordered and delivered under the OSD and TSD strategies

(1) Mean and variance under the OSD strategy

The OUL decision made under the OSD strategy is based on $\sum_{j=1}^{L+1} D_{t+j}$. It can be deduced from Eq.(1) that

$$\sum_{j=1}^{L+1} D_{t+j} = d \sum_{j=1}^{L+1} \frac{1-\rho^j}{1-\rho} + \sum_{j=1}^{L+1} \rho^j D_t + \sum_{j=1}^{L+1} \sum_{i=0}^{j-1} \rho^{j-1-i} \varepsilon_{t+1+i}.$$

When the ordering decision is made in time period t , D_t and ε_t are known. So

$$e_t^1 = d \sum_{j=1}^{L+1} \frac{1-\rho^j}{1-\rho} + \sum_{j=1}^{L+1} \rho^j D_t = \frac{d}{1-\rho} \left[L+1 - \frac{\rho(1-\rho^{L+1})}{1-\rho} \right] + \frac{\rho(1-\rho^{L+1})}{1-\rho} D_t,$$

and

$$\vartheta_t^1 = \frac{\sigma^2}{(1-\rho)^2} \sum_{i=1}^{L+1} (1-\rho^i)^2.$$

(2) Means and variances under the TSD strategy

The DUL decision and the OUL decision made under the TSD strategy are based on $\sum_{i=1}^{l_r+1} D_{t+i}$ and $\sum_{j=1}^{l_v+1} y_{t+j}^{2r}$, respectively. It can be deduced from Eq.(1) to Eq.(3) that

$$\sum_{j=1}^{l_r+1} D_{t+j} = d \sum_{j=1}^{l_r+1} \frac{1-\rho^j}{1-\rho} + \sum_{j=1}^{l_r+1} \rho^j D_t + \sum_{j=1}^{l_r+1} \sum_{i=0}^{j-1} \rho^{j-1-i} \varepsilon_{t+1+i},$$

and

$$\sum_{j=1}^{l_v+1} y_{t+j}^{2r} = \frac{d}{1-\rho} \left[l_v+1 - \frac{\rho^{l_r+2}(1-\rho^{l_v+1})}{1-\rho} \right] + \frac{\rho^{l_r+2}(1-\rho^{l_v+1})}{1-\rho} D_t + \sum_{i=1}^{l_v+1} \frac{1-\rho^{l_v+l_r+3-i}}{1-\rho} \varepsilon_{t+i}.$$

When the delivery decision is made in time period t , D_t and ε_t are known. So

$$e_t^{2r} = \frac{d}{1-\rho} \left[l_r + 1 - \frac{\rho(1-\rho^{l_r+1})}{1-\rho} \right] + \frac{\rho(1-\rho^{l_r+1})}{1-\rho} D_t,$$

$$\vartheta_t^{2r} = \frac{\sigma^2}{(1-\rho)^2} \sum_{i=1}^{l_r+1} (1-\rho^i)^2.$$

And,

$$e_t^{2v} = \frac{d}{1-\rho} \left[l_v + 1 - \frac{\rho^{l_r+2}(1-\rho^{l_v+1})}{1-\rho} \right] + \frac{\rho^{l_r+2}(1-\rho^{l_v+1})}{1-\rho} D_t,$$

$$\vartheta_t^{2v} = \frac{\sigma^2}{(1-\rho)^2} \sum_{i=1}^{l_v+1} (1-\rho^{l_r+1+i})^2.$$

Property 1 $\vartheta_t^{2r} + \vartheta_t^{2v} > \vartheta_t^1$.

Proof. Note that

$$\vartheta_t^{2r} + \vartheta_t^{2v} = \frac{\sigma^2}{(1-\rho)^2} \sum_{i=1}^{l_r+1} (1-\rho^i)^2 + \frac{\sigma^2}{(1-\rho)^2} \sum_{i=1}^{l_v+1} (1-\rho^{l_r+1+i})^2 = \frac{\sigma^2}{(1-\rho)^2} \sum_{i=1}^{l_v+l_r+2} (1-\rho^i)^2.$$

Since $l_v + l_r > L$, we have

$$\vartheta_t^{2r} + \vartheta_t^{2v} > \frac{\sigma^2}{(1-\rho)^2} \sum_{i=1}^{L+1} (1-\rho^i)^2 = \vartheta_t^1.$$

5 Analysis of the DC's Modification Ability

5.1 Definition of the DC's modification ability

In this paper the DC's modification ability originates from a comparison of the OSD strategy with the TSD strategy, and such an ability is shown by the influence of the DC's delivery decisions on the retailer. If the profit under the TSD strategy is higher than that under the OSD strategy, the modification ability is considered to be *positive*; otherwise, it is considered to be *negative*. This leads to the following definition.

Definition 1. If $W(S_t^{2r} + S_t^{2v}) > W(S_t^1)$, then the DC's ability to modify the retailer's replenishment decisions is positive; else, if $W(S_t^{2r} + S_t^{2v}) < W(S_t^1)$, then the DC's ability to modify the retailer's replenishment decisions is negative.

The identification of the DC's modification ability is significant in practice. If the modification ability is positive, the VMI system should adopt the TSD strategy. Otherwise, the DC should adopt the OSD strategy and act as a cross-dock only, without making any modification to the quantities of the product ordered. Under both strategies, other benefits of the DC such as "risk pool" and "economies of scale" can be realized at the same time. In addition to the choice of strategy with regard to ordering and delivery decisions, the identification of the DC's modification ability is relevant to the consideration of the DC's location, and should therefore be a decision factor in determining the DC's location.

5.2 Influence of cost parameters on the DC's modification ability

Since the DC's modification ability is related to the inventory or supply ability of the DC, it is related to both c_v and h_v . In addition, since $W(S_t^1)$ has no relation with either c_v or h_v , in order to show the impact of c_v , h_v on the DC's modification ability, we need only to analyze the change in $W(S_t^{2r} + S_t^{2v})$ as a result of changes in c_v and h_v .

Property 2 *The DC's modification ability decreases with an increase in c_v .*

The proof is given in Appendix 1.

Property 3 *The DC's modification ability decreases with an increase in h_v .*

The proof is given in Appendix 2.

Properties 2 and 3 indicate that lower unit inventory cost and unit shortage cost in the DC are the sources of the DC's modification ability, which is enabled by the TSD strategy. However, as the advantage of lower costs in the DC deteriorates with increasing variances of the ordering quantities under the TSD strategy (see Property 1), it is easy to see that the modification ability of the DC decreases with increasing c_v or h_v , and the ability will eventually become negative for sufficiently large values of c_v or h_v .

5.3 Analysis of the DC's location decision

In addition to the cost parameters discussed in Section 5.2., l_v and l_r are two factors that influence the DC's modification ability, too. As mentioned in Section 3, it is assumed in this paper that $l_v + l_r$ is constant, so l_v and l_r are related to the site at which the DC is located. It is significant to discuss the DC's location decision with consideration of its modification ability.

(1) Cost parameters are constant

In this subsection we discuss the case where the cost parameters have no relations with the site at which the DC is located.

Property 4 *The DC's modification ability is different on different points at which it is located, and is minimized at a certain point.*

The proof is given in Appendix 3.

Property 4 shows that when the cost parameters h_v and c_v have no relations with l_v , the location that maximizes the DC's modification ability should be either near the manufacturer or near the retailer.

(2) Cost parameters are related to the site

In reality, it is usual that h_v and c_v have relations with l_v , and the closer the DC is to the retailer, the higher h_v and c_v are. Without loss of generality, we assume that $h_v = \frac{\beta l_v}{l_v + l_r} h_r$ and $c_v = \frac{\beta l_v}{l_v + l_r} c_r$, where β is a constant parameter. It is evident that the optimal point can be determined by solving the following optimization problem:

$$\text{maximize } W(S_t^{2r} + S_t^{2v}), \quad (11)$$

where

$$\begin{aligned}
 W(S_t^{2r} + S_t^{2v}) &= c_r e_{t+l_r+1} - \frac{\beta l_v}{l_v + l_r} (c_r + h_r) \sqrt{\vartheta_t^{2v}} f_s \left[F_s^{-1} \left(\frac{\beta l_v}{l_v + l_r} \frac{c_r}{c_r + h_r} \right) \right] \\
 &\quad - (c_r + h_r) \sqrt{\vartheta_t^{2r}} f_s \left[F_s^{-1} \left(\frac{c_r}{c_r + h_r} \right) \right].
 \end{aligned}$$

We can apply iteration methods to find the optimal l_v .

6 Computational Examples

To make better sense of the practical implications of the theoretical results presented in the previous sections, we provide some computational examples in this section. The computational experiments were conducted from two perspectives. First, we analyzed the influence of the cost parameters h_v and c_v on the DC's modification ability. Second, under each of the two cases where the cost parameters are related and not related to the DC's site, we studied the DC's optimal location decision.

Without loss of generality, we assume that $L = l_v + l_r - 1$, and the basic parameters used in all the examples are given in Table 1.

Table 1: The basic parameters used in the examples

Parameters	d	ρ	σ	D_t	ε_t	$l_v + l_r$	l_v	c_r	h_r	c_v	h_v
Values	100	0.3	10	135	8	15	10	200	10	50	5

6.1 Influence of cost parameters on the DC's modification ability

To analyze the influence of cost parameters c_v and h_v on the DC's modification ability, we constructed several test problems based on the parameters given in Table 1. The computational results of these problems are presented in Table 2 and Table 3, respectively, where $\Delta W = W(S_t^{2r} + E_t^{2v}) - W(S_t^1)$. Figures 3 and 4 show changes in the two profits with increases in c_v and h_v , respectively.

It can be seen from Table 2 and Figure 3 that, under the TSD strategy, an increase in c_v , while increasing S_t^{2v} accordingly, will steadily decrease the expected profit of the SC system. The computational results indicate that the DC's modification ability decreases with an increase in c_v . When $c_v = 60$, ΔW is negative, which means that the TSD strategy is inferior to the OSD strategy, so the DC should act as a cross-dock only, without making modification decisions to the quantities of the product received from the manufacturer.

From Table 3 and Figure 4, it can be seen that with an increase in h_v , the OUL of the DC decreases, and so does the expected profit of the SC system. When $h_v = 6$, the DC's modification ability is negative, so the TSD strategy is inferior to the OSD strategy in this case.

The computational results in Table 2 and Table 3 are in line with Properties 2 and 3, which are reasonable in reality. c_v and h_v are opportunity costs in the DC's delivery decisions. If any of them increases, the advantage of the TSD strategy with regard to modifying the delivery

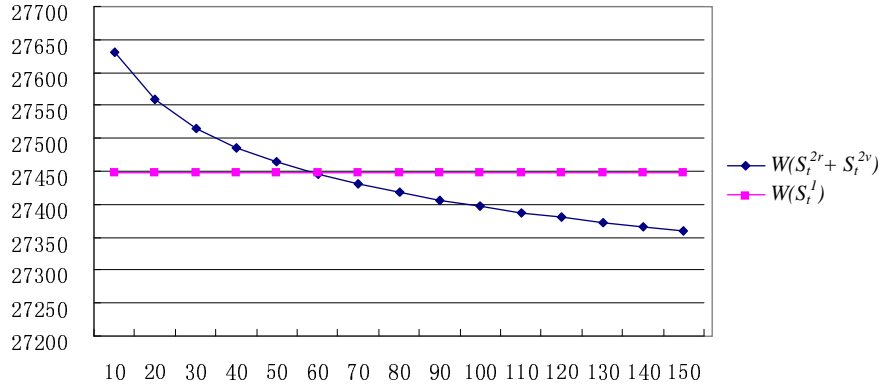


Figure 3: Profits of the VMI system under two strategies with respect to changes in c_v

Table 2: Computational results with respect to changes in c_v

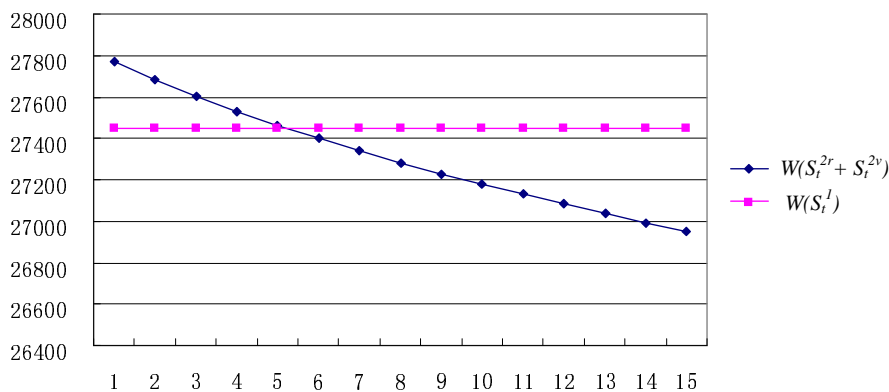
c_v	S_t^l	$W(S_t^l)$	S_t^{2r}	S_t^{2v}	$W(S_t^{2r} + S_t^{2v})$	ΔW
10	2372	27448	908	1449	27631	182
20	2372	27448	908	1468	27557	109
30	2372	27448	908	1479	27515	66
40	2372	27448	908	1486	27485	37
50	2372	27448	908	1492	27463	14
60	2372	27448	908	1496	27445	-4
70	2372	27448	908	1500	27429	-19
80	2372	27448	908	1503	27417	-32
90	2372	27448	908	1505	27405	-43
100	2372	27448	908	1508	27395	-53
110	2372	27448	908	1510	27387	-62
120	2372	27448	908	1512	27379	-70
130	2372	27448	908	1513	27371	-77
140	2372	27448	908	1515	27365	-84
150	2372	27448	908	1516	27358	-90

decisions deteriorates accordingly. Eventually, the TSD strategy will become inferior to the OSD strategy, considering the higher variations of the quantities of the product resulting from the decisions under the TSD strategy (see Property 1).

6.2 The DC's location decision

When the cost parameters c_v and h_v have no relations with the DC's location, the optimal l_v and the corresponding profit of the SC system for the problems constructed by changing the value of h_r in Table 1 are shown in Table 4.

The computational results given in Table 4 were obtained by taking l_v as a variable in Eq. (10) and selecting the optimal value $l_v = l_v^*$ that maximizes the profit. It can be seen that, under all the conditions, the optimal location of the DC is closed to the retailer, which follows the conclusion of Property 4. In addition, with an increase in h_r , ΔW increases steadily, showing the importance of the DC's modification ability. The increment of h_r approximates the decrement of h_v , so the results also follow the conclusion of Property 3.

Figure 4: Profits of the VMI system under two strategies with respect to changes in h_v Table 3: Computational results with respect to changes in h_v

h_v	S_t^1	$W(S_t^1)$	S_t^{2r}	S_t^{2v}	$W(S_t^{2r} + S_t^{2v})$	ΔW
1	2372	27448	908	1526	27774	325
2	2372	27448	908	1512	27683	235
3	2372	27448	908	1504	27603	155
4	2372	27448	908	1497	27530	82
5	2372	27448	908	1492	27463	14
6	2372	27448	908	1487	27399	-49
7	2372	27448	908	1484	27340	-109
8	2372	27448	908	1480	27283	-165
9	2372	27448	908	1477	27230	-219
10	2372	27448	908	1474	27179	-270
11	2372	27448	908	1472	27130	-319
12	2372	27448	908	1470	27083	-366
13	2372	27448	908	1467	27038	-411
14	2372	27448	908	1465	26994	-454
15	2372	27448	908	1463	26952	-496

When it was assumed that $h_v = \frac{\beta l_v}{l_v + l_r} h_r$ and $c_v = \frac{\beta l_v}{l_v + l_r} c_r$, the optimal l_v and the corresponding expected profit of the SC system for the problems constructed by changing the value of β and the parameters in Table 1 are shown in Table 4.

The computational results given in Table 5 were obtained by taking l_v as a variable in Eq.(11) and selecting the optimal $l_v = l_v^*$ that maximizes the profit. It can be seen that, with an increase in β , the optimal location of the DC is increasingly closed to the manufacturer. Since l_r decreases with an increase in l_v , such results in fact show the trade-off between the costs incurred by the ordering decision and by the delivery decision. In addition, it can be seen from the results that, with an increase in β , the DC's maximum modification ability ΔW decreases accordingly. As c_v and h_v increase along with an increase in β , the results also follow the conclusions of Property 2 and Property 3.

7 Conclusions

We studied a two-level vendor managed inventory system comprising a distribution center and a retailer. Both the DC's and the retailers replenishment decisions follow the order-up-to-level policy and aim at maximizing the profit of the overall system. We proposed and compared

Table 4: Optimal l_v and the corresponding decision variables and profit.

h_r	S_t^{2r}	S_t^{2v}	$W(S_t^{2r} + S_t^{2v})$	l_v	$W(S_t^1)$	ΔW
10	310	2703	27592	14	27448	144
30	301	2703	27134	14	25942	1192
50	297	2703	26785	14	24798	1987
70	293	2703	26499	14	23857	2642
90	290	2703	26255	14	23053	3202
150	286	2703	25680	14	21164	4516

Table 5: The optimal l_v and the corresponding profit.

β	S_t^{2r}	S_t^{2v}	$W(S_t^{2r} + S_t^{2v})$	l_v	$W(S_t^1)$	ΔW
0.9	2083	327	27427	2	27448	-21
0.8	1937	476	27435	3	27448	-13
0.7	1790	625	27449	4	27448	1
0.6	611	1800	27507	12	27448	59
0.5	461	1946	27599	13	27448	111
0.4	461	1946	27696	13	27448	248

two ordering and delivery strategies in order to examine the DC's ability to modify delivery decisions. We classified the DC's modification ability as positive or negative according to whether it increases or decreases the expected profit of the supply chain system, and we identified and quantified the cost factors that influence it. Our findings offer a new viewpoint on the DC's location decision. We presented computational examples to demonstrate the practical implications of the theoretical results.

The findings of this paper are significant in practice. We showed that the role of the DC should be broadened by considering its ability to modify delivery decisions. If the modification ability is positive, the VMI system should adopt the TSD strategy. Otherwise, the DC should adopt the OSD strategy and act as a cross-dock only, without making any modification to the quantities of the product ordered. Under both strategies, other benefits of the DC such as "risk pooling" and "economies of scale" can be realized at the same time. While many past studies on the DC's location were mainly concerned with cost factors, we established in this paper a relationship between the DC's location and its modification ability, and showed the trade-off between the DC's modification ability and related costs.

To sum up, this study provides a new insight through careful modeling and analysis of the role of the DC and reveals the full potential of the VMI system. Our findings and their practical implications, demonstrated with the aid of computational examples, are helpful for enhancing the practice of VMI at both strategic and operational levels.

Our research can be extended to consider problems with more retailers and/or supply chain systems with more levels. Integrated studies of the modification ability of the DC with other abilities discussed in prior research on VMI and distribution systems are worth undertaking. In addition to the OUL policy, other ordering and delivery policies should be explored to study the DC's modification ability under other circumstances, especially in the case where the DC cannot acquire the product from an "alternative" source.

Appendix 1. Proof of Property 2

Proof. Taking c_v as a variable and differentiating $W(S_t^{2r} + S_t^{2v})$ with respect to c_v yields

$$\frac{d}{dc_v} W(S_t^{2r} + S_t^{2v}) = -f_s(k_{2v})\sqrt{\vartheta_t^{2v}} - (c_v + h_v)\sqrt{\vartheta_t^{2v}} \left[-k_{2v}f_s(k_{2v})\frac{dk_{2v}}{dc_v} \right]. \quad (12)$$

Denote $y = k_{2v} = F_s^{-1}\left(\frac{c_v}{c_v + h_v}\right)$, we have

$$\frac{d}{dc_v} \left(\frac{c_v}{c_v + h_v} \right) = \frac{d}{dy} F_s(y) \frac{dy}{dc_v} = f_s(y) \frac{dy}{dc_v}.$$

So

$$\frac{dk_{2v}}{dc_v} = \frac{dy}{dc_v} = \frac{1}{f_s(y)} \frac{d}{dc_v} \left(\frac{c_v}{c_v + h_v} \right) = \frac{1}{f_s(k_{2v})} \cdot \frac{h_v}{(c_v + h_v)^2}. \quad (13)$$

Substituting $\frac{dk_{2v}}{dc_v}$ into Eq.(12) with Eq.(13), it can be deduced that

$$\frac{d}{dc_v} W(S_t^{2r} + S_t^{2v}) = -\sqrt{\vartheta_t^{2v}} \left[f_s(k_{2v}) - k_{2v}(1 - F_s(k_{2v})) \right]. \quad (14)$$

If $\sqrt{\vartheta_t^{2v}} [f_s(k_{2v}) - k_{2v}(1 - F_s(k_{2v}))] \geq 0$, then it indicates that $W(S_t^{2r} + S_t^{2v})$ decreases with an increase in c_v , which means that the DC's modification ability decreases with an increase in c_v . In the following we discuss the implications of $\frac{d}{dc_v} W(S_t^{2r} + S_t^{2v})$.

Denote $z = \sqrt{\vartheta_t^{2v}} [f_s(k_{2v}) - k_{2v}(1 - F_s(k_{2v}))]$, and take k_{2v} as a variable. Then

$$\frac{dz}{dk_{2v}} = \sqrt{\vartheta_t^{2v}} [-k_{2v}f_s(k_{2v}) - 1 + F_s(k_{2v}) + k_{2v}f_s(k_{2v})] = \sqrt{\vartheta_t^{2v}} [F_s(k_{2v}) - 1],$$

and

$$\frac{d^2z}{dk_{2v}^2} = \sqrt{\vartheta_t^{2v}} f_s(k_{2v}) \geq 0.$$

So the function $z(k_{2v})$ is convex, and there is only one minimum solution that satisfies $\sqrt{\vartheta_t^{2v}} [F_s(k_{2v}) - 1] = 0$, and the minimum value of z is $z_{min} = 0$. So $z \geq 0$, i.e., $\frac{d}{dc_v} W(S_t^{2r} + S_t^{2v}) \leq 0$, which means that the DC's modification ability decreases with an increase in c_v .

Appendix 2. Proof of Property 3

Proof. Taking h_v as a variable and differentiating $W(S_t^{2r} + S_t^{2v})$ with respect to h_v yields

$$\frac{d}{dh_v} W(S_t^{2r} + S_t^{2v}) = -f_s(k_{2v})\sqrt{\vartheta_t^{2v}} + (c_v + h_v)\sqrt{\vartheta_t^{2v}} \left[k_{2v}f_s(k_{2v})\frac{dk_{2v}}{dh_v} \right]. \quad (15)$$

Denote $y = k_{2v} = F_s^{-1}\left(\frac{c_v}{c_v + h_v}\right)$. So $\frac{d}{dh_v} \left(\frac{c_v}{c_v + h_v} \right) = \frac{d}{dy} F_s(y) \frac{dy}{dh_v} = f_s(y) \frac{dy}{dh_v}$.

It follows that

$$\frac{dk_{2v}}{dh_v} = \frac{dy}{dh_v} = \frac{1}{f_s(y)} \frac{d}{dh_v} \left(\frac{c_v}{c_v + h_v} \right) = \frac{-1}{f_s(k_{2v})} \cdot \frac{c_v}{(c_v + h_v)^2}. \quad (16)$$

Substituting $\frac{dk_{2v}}{dh_v}$ into Eq.(15) with Eq.(16), we have

$$\frac{d}{dh_v} W(S_t^{2r} + S_t^{2v}) = -f_s(k_{2v}) \sqrt{\vartheta_t^{2v}} - \sqrt{\vartheta_t^{2v}} \left[k_{2v} \frac{c_v}{c_v + h_v} \right] < 0. \quad (17)$$

Eq.(17) indicates that $W(S_t^{2r} + S_t^{2v})$ decreases with an increase in h_v , i.e., the DC's modification ability decreases with an increase in h_v .

Appendix 3. Proof of Property 4

Proof. It can be seen that ϑ_t^{2r} and ϑ_t^{2v} vary with points at which the DC is located. However, $A_t = \vartheta_t^{2r} + \vartheta_t^{2v}$ is a constant (see the proof of Property 1). Denote

$$\begin{aligned} y &= (c_v + h_v) \sqrt{\vartheta_t^{2v}} f_s(k_{2v}) + (c_r + h_r) \sqrt{\vartheta_t^{2r}} f_s(k_{2r}) \\ &= (c_v + h_v) \sqrt{\vartheta_t^{2v}} f_s(k_{2v}) + (c_r + h_r) \sqrt{A_t - \vartheta_t^{2v}} f_s(k_{2r}). \end{aligned}$$

In the following we take ϑ_t^{2v} as a variable, and evaluate the change of y with respect to ϑ_t^{2v} . Differentiating y with respect to ϑ_t^{2v} yields

$$\frac{dy}{d\vartheta_t^{2v}} = \frac{(c_v + h_v) f_s(k_{2v})}{2\sqrt{\vartheta_t^{2v}}} - \frac{(c_r + h_r) f_s(k_{2r})}{2\sqrt{A_t - \vartheta_t^{2v}}}$$

Taking ϑ_t^{2v} as a variable and differentiating $\frac{dy}{d\vartheta_t^{2v}}$ with respect to ϑ_t^{2v} yields

$$\frac{d^2y}{d(\vartheta_t^{2v})^2} = -\frac{(c_v + h_v) f_s(k_{2v})}{4(\vartheta_t^{2v})^{\frac{3}{2}}} - \frac{(c_r + h_r) f_s(k_{2r})}{4(A_t - \vartheta_t^{2v})^{\frac{3}{2}}} < 0. \quad (18)$$

Eq.(18) indicates that y is a concave function, and there is only one maximum solution at the point ϑ_t^{2v*} that satisfies

$$\frac{dy}{d\vartheta_t^{2v*}} = \frac{(c_v + h_v) f_s(k_{2v})}{2\sqrt{\vartheta_t^{2v*}}} - \frac{(c_r + h_r) f_s(k_{2r})}{2\sqrt{A_t - \vartheta_t^{2v*}}} = 0.$$

That is,

$$\vartheta_t^{2v*} = \frac{A_t [(c_v + h_v) f_s(k_{2v})]^2}{[(c_v + h_v) f_s(k_{2v})]^2 + [(c_r + h_r) f_s(k_{2r})]^2}.$$

So, the DC's modification ability is minimized at the point ϑ_t^{2v*} .

References

- [1] Angulo A., H. Nachtmann and M.A. Waller. 2004. Supply chain information sharing in a vendor managed inventory partnership. *Journal of Business Logistics*, 25 101–120.
- [2] Aviv Y. and A. Federguen. 1998. The operational benefits of information sharing and vendor managed inventory (VMI) programs. Technical Report, Columbia University, New York.
- [3] Bertazzi L, G. Paletta and M.G. Speranza. 2005. Minimizing the total cost in an integrated vendor-Managed inventory system. *Journal of Heuristics*, 11 393–419.
- [4] Bordley R., M. Beltramo and D. Blumenfeld. 1999. Consolidating distribution centers can reduce lost sales. *International Journal of Production Economics*, 58 57–61.
- [5] Chen M.C., C.L. Huang, K.Y. Chen and H.P. Wu. 2005. Aggregation of orders in distribution centers using data mining. *Expert Systems with Applications*, 28 453–460.
- [6] Cheng T.C.E. and Y.N. Wu. 2005. The impact of information sharing in a two-level supply chain with multiple retailers. *Journal of the Operational Research Society*, 56 1159–1165.
- [7] Disney S.M., A.T. Potter and B.M. Gardner. 2003. The impact of vendor managed inventory on transport operations. *Transportation Research Part E*, 39 363–380.
- [8] Eskigun E., R. Uzsoy, P.V. Preckel, G. Beaujon, S. Krishnan and J.D. Tew. 2005. Outbound supply chain network design with mode selection, lead times and capacitated vehicle distribution centers. *European Journal of Operational Research*, 165 182–206.
- [9] Gurbuz M.C., K. Moinzadeh and Y.P. Zhou. 2007. Coordinated replenishment strategies in inventory/distribution systems. *Management Science*, 53 293–307.
- [10] Heyman D. and M. Sobel. 1984. *Stochastic Models in Operations Research, Vol. II*. McGraw-Hill, New York.
- [11] Jaruphongsa W., S. Cetinkaya and C.Y. Lee. 2004. Warehouse space capacity and delivery time window considerations in dynamic lot-sizing for a simple supply chain. *International Journal of Production Economics*, 92 169–180.
- [12] Lee H.L., K.C. So and C.S. Tang. 2000. The value of information sharing in a two-level supply chain. *Management Science*, 46 626–643.
- [13] Neubert G., Y. Ouzrout and A. Bouras. 2004. Collaboration and integration through information technologies in supply chains. *International Journal of Technology Management*, 28 259–273.
- [14] Shen Z.J.M., C. Coullard and M.S. Daskin. 2003. A joint location-inventory model. *Transportation Science*, 37 40–55.
- [15] Snyder L.V. 2006. Facility location under uncertainty: a review. *IIE Transactions*, 38 537–554.