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Abstract

Price-matching-guarantees (PMGs) are offers by firms whereby they assure customers that they will match any lower price offered by the competition for an identical product. Extant economics/marketing literature assumes that a simple proof of the competitor's lower price is sufficient to grant the price match. Under this form of PMG and deterministic customer demand, it has been shown that PMGs may actually lead to tacit collusion such that all retailers set their price and order levels like a monopolist (in a competitive setting). However, firms nowadays reserve the right to *verify the availability* of the product at the competitor location and *decline to match* the lower price if the product is not available there. This option brings stocking decisions of the firms, and hence demand uncertainty, into play. Our study focuses on these elements. We model two identical newsvendor retailers that offer PMGs in a stochastic, price-sensitive demand environment. The retailers compete by simultaneously selecting prices and stocking quantities. We present a comprehensive characterization of the equilibrium prices and stocking levels for the case when the firms verify availability before matching the price, and the case when they do not. We also investigate how retail prices and order quantities are influenced by PMGs in an uncertain demand environment, as well as how uncertainty in demand affects structural properties related to "deterministic" PMGs.

Our results demonstrate that the tacit collusion outcome of deterministic models extends to a stochastic demand environment if a simple proof is deemed sufficient for a price match. However, we also show that demand uncertainty enables the retailer to price discriminate by verifying the availability of the product at competing stores before granting any price-match request. In such a case, one firm increases its price and quantity *even beyond monopoly levels*. By comparing the equilibria, we identify product/market characteristics under which verifying the availability is desirable to the retailers. We find that the opportunity cost of not verifying the availability increases with the uncertainty in the market. From a managerial perspective, our results indicate that verifying availability can be a significant profit-enhancing mechanism and show how the benefits vary with the product/market characteristics. We find that verifying the availability is most profitable for highly anticipated, relatively more costly, innovative products which face relatively more uncertain demand. Geographical proximity between competing retailers is another factor that makes price discrimination and verification of availability a valuable retail pricing strategy.

Key Words: price-matching guarantees, pricing, inventory, product availability, stochastic demand, newsvendor.

Résumé

Les garanties de plus bas prix (GBP) sont une promesse qu'une compagnie fait à ses clients de leur garantir le prix le plus bas que la concurrence pourrait leur offrir pour tout produit identique. La littérature de marketing suppose qu'une preuve quelconque établissant qu'un concurrent vend le produit à un prix plus bas, est suffisante pour engendrer une baisse équivalente du côté de la compagnie en question. Il a été démontré qu'appliquées sous cette forme, et en présence d'une demande déterministe pour un produit donné, les GBP débouchent sur de la collusion entre distributeurs qui agissent alors collectivement comme un monopole (dans un contexte concurrentiel). Cependant, dans le contexte actuel, les compagnies se réservent le droit de vérifier la disponibilité du produit chez le concurrent, et refusent de baisser leur prix si le produit n'y est pas disponible. Cette variante force les compagnies à tenir compte des questions d'inventaires et de ce fait, le niveau d'incertitude dans la demande devient un facteur influent. Notre étude est centrée sur ces éléments. Nous considérons le cas de deux vendeurs de journaux qui offrent des GBP dans un contexte stochastique de demande élastique. Les vendeurs jouent sur deux facteurs, les prix et les niveaux d'inventaire, pour tenter d'améliorer leur position. Nous présentons une caractérisation détaillée des prix et inventaires à l'équilibre dans les cas distincts où les compagnies choisissent soit de vérifier, soit de ne pas vérifier la disponibilité d'inventaires chez leurs concurrents. Nous étudions en particulier l'impact de l'incertitude dans la demande et dans l'environnement, sur les propriétés structurelles des équilibres atteints par rapport au cas purement déterministe.

Nos résultats indiquent que l'effet de monopole établi dans le cas déterministe persiste dans le contexte stochastique si le critère de baisse de prix consiste en une simple preuve de prix de vente chez le concurrent. Dans le cas contraire, les équilibres sont modifiés et une des deux compagnies aura tendance à augmenter ses prix et quantités même au-delà des niveaux associés au monopole. Par comparaison des divers équilibres, nous identifions les caractéristiques marché/produit pour lesquelles la vérification de disponibilité par les compagnies est la stratégie la plus indiquée. Nous trouvons que la désirabilité de vérifier augmente avec l'incertitude dans le marché. D'un point de vue de gestion, nos résultats indiquent que la vérification de disponibilité peut devenir un moyen de faire augmenter les profits et démontrent comment les bénéfices varient avec les caractéristiques marché/produit. Ainsi, la vérification sera d'autant plus indiquée que le produit est relativement plus coûteux, hautement innovateur et attendu sur le marché. La proximité géographique entre concurrents est un autre facteur militant en faveur de la stratégie de vérification de disponibilité et de différentiation des prix.

1 Introduction and Overview of Literature

A price-matching guarantee (PMG) is a popular retail pricing strategy[1] under which the retail store promises not to be undersold by matching any lower price offered by a competitor for the same merchandise. Under this strategy, customers have the opportunity to receive the lowest price in the marketplace at their preferred store. Consider a customer who prefers to shop with a particular retailer offering PMG but realizes that a competing store is offering a lower price. The customer can then get the lower price from her/his preferred retailer.[2] PMGs are common in most retail sectors such as home and office appliances, auto supplies, and consumer electronics. Most of the retail giants such as Sears, BestBuy, Circuit City, Staples, and OfficeMax offer PMGs. In fact, a survey of the top 20 national consumer electronics retailers in U.S. and Canada (excluding warehouse clubs) reveals that 75% of them offer PMGs.[3] Note that a PMG shows the determination of the retailer to be competitive in price by assuring the "best deal". However, it does not necessarily guarantee that the retailer's price itself is the lowest one in the market. PMGs only provide an option of receiving the lowest price if customers are willing and are *able* to claim it.

Earlier studies in economics and marketing literature assume that all customers are granted the price-match if they prove the existence of a lower price at a competing retailer. This proof can be in the form of weekly flyers, magazine advertisement, and website information. We refer to this case as simple price-matching, PM in short. In reality, however, retailers ask for more than a proof of the lower price. For example, Sears, a multinational retail store, indicates clearly that it will match the price only if its staff can verify that the same product is *in stock and available* for immediate sale/delivery by the competitor at the advertised price (see Figure 9 in Appendix). In other words, retailers match the price and agree to a lower profit margin only if the customer requesting the match has a credible alternative location to make the purchase. If the product is not available at the competing retailer, the price-match request will be declined. We refer to this phenomenon as price-matching based on availability (PMA in short).

The existence of PMA type policies shows the important role of product availability in PMGs. The value that retailers attach to availability is borne by the fact that 100% of the top 20 national consumer electronics retailers that offer PMGs consider the verification of availability at the competing retailer as a prerequisite for matching the price. Obviously, inventory availability depends on the quantity stocked and the demand realized. In this regard, the deterministic price-demand relationship in marketing/economics literature undervalues the role of product availability. The importance of availability/stocking decision becomes apparent only in an uncertain demand environment. Under such circumstances,

[1] A search for "price matching" in Google results in 682,000 results.

[2] Retailers sometimes refund more than the difference, i.e., they "beat" the price of the competitor. For ease of exposition, we only consider the case where retailers "match" the competitor's price.

[3] Based on Dealerscope's Top 101 Consumer Electronics Retailers list for March 2007, available at http://www.dealerscope.com

the prices charged by the retailers, and hence their profits, depend on the mismatch between supply and demand, as well as the policy in place (PM or PMA).

The primary objective of this paper is to identify the effects of demand uncertainty and the verification of product availability on the decisions and profits of retailers. Specifically, we focus on a setting that involves two price-matching newsvendor retailers, both selling an identical product and deciding on their price and inventory levels. Our study is related to two distinct streams of literature: *i*) papers in economics and marketing domain which concentrate on analyzing PM policy in a competitive environment, but ignore issues related to product availability and demand uncertainty (hence, PMA policy), and *ii*) models in operations management (OM) literature which deal with price and/or inventory decisions of competing price-setting newsvendors, but without any price-matching considerations. By analyzing PMGs for the first time in an operations framework, we bridge the gap between the two streams. In what follows, we provide an overview of the related literature. For a clear understanding of how this paper fits within the literature, we provide Table 1 where we refer to this study as Price-matching Newsvendor in the right-lower corner.

Deterministic Models:

We start with a brief overview of the deterministic models (the top row in Table 1). Monopoly (DM) and Bertrand competition (DB) scenarios with deterministic demand models are well established in the economics literature (Tirole, 1988); therefore, within deterministic demand models, we focus on the papers related to price-matching (DPM).

There is a substantial body of research on price-matching in economics/marketing literature. Most related to our work (due to similar demand assumptions) is the stream of research that studies the Bertrand price-competition between two price-matching firms selling an identical product with constant (marginal) costs. Customers know the price of each firm and make their decisions accordingly. It is well-known that price-only-competition (i.e., no PMGs) leads to lower retail prices in this setting. In this case, any firm can steal customers from the competitor and create more sales by reducing the price (compared to monopoly ones). If the increase in sales is large enough, profits will increase. As an

Table 1: Positioning of this study with respect to the related literature.

	Monopoly	**Price Competition**	**Competitive Price Matching**
Deterministic Demand (Marketing/Economics Literature)	Monopoly **(DM)**	Bertrand Competition **(DB)**	Price-matching **(DPM)**
Stochastic Demand (Operations Literature)	Price-setting Newsvendor **(N)**	Competitive Price- setting Newsvendor **(C)**	**Price-matching Newsvendor (PM), (PMA)**

alternative scenario, suppose that one of the firms offer PMG. In that case, the competitor firm can not steal customers by reducing its price. Customers, all of whom are informed about the PMG offer, will ask for price-matching at their preferred retailer instead of switching to the competitor. That is, a price-matching firm will not be undersold by the competitor. Consequently, there is no incentive for the competitor to reduce its price. If both firms offer PMGs, then none will have an incentive to deviate from monopoly prices (Hay et al. 1980, Salop 1986). This tacit collusion outcome continues to hold in various other settings such as oligopolistic markets (Doyle 1988, Logan and Lutter 1989, Corts 1995), sequential play between firms (Belton, 1987), customer heterogeneity in information level (Baye and Kovenock, 1994). In summary, comparing different market scenarios with deterministic demand functions (top row in Table 1), we can conclude that competition reduces retail prices compared to the monopoly case; however, with the introduction of PMGs, prices increase and monopoly prices prevail even under competitive market settings. We complement this stream of literature by studying the effect of PMGs on retail decisions and the robustness of the tacit collusion outcome in an uncertain demand environment.[4]

Stochastic Models:

In the OM literature, the newsvendor model is the building block for models incorporating demand uncertainty. Most directly related to our paper are its two extensions: price-setting newsvendor models in monopolistic and competitive settings. The monopolist problem, in which a single retailer decides on the price and order quantity simultaneously to satisfy uncertain price-sensitive demand, has been studied extensively in recent years (see Petruzzi and Dada 1999 for an excellent review). The competitive model replaces the single retailer with a set of price-competing ones. In this setting, it is shown that retailers can increase profitability by coordinating their pricing and production decisions (Parlar and Weng, 2006), and price-competition results in under-pricing and over-stocking compared to centralized decision making with multiple retailers (Chen and Yao, 2004). Bernstein and Federgruen (2005) design contractual agreements to coordinate the equilibrium behavior in a (finite-horizon) decentralized framework. Moreover, Bernstein and Federgruen (2003) identify the conditions for the existence of equilibrium under which each retailer adopts a single stationary price and a base-stock policy with a single stationary stock level in the infinite-horizon extension. They also show that certain monotonic properties of the expected profits with respect to uncertainty, valid in centralized systems, may breakdown in a competitive, infinite-horizon setting.

Comparison of different market scenarios in an uncertain demand environment (Table 1 bottom row) has not been a focal point in OM literature. To the best of our knowledge,

[4]Other streams of research, less related to our work, questions the robustness of the tacit collusion with respect to customer heterogeneity and firm asymmetry. These studies show that PMGs can: *i*) be price discriminating devices when customers differ in their information level and on their evaluations of the product (Png and Hirshleifer 1987, Corts 1996), *ii*) serve as a signal of low prices and low service levels (Jain and Srivastava 2000, Moorthy and Winter 2006, Moorthy and Zhang 2006).

Zhao and Atkins (2007) is the only study that compares price decisions in monopoly and competitive settings and shows that the introduction of price-competition leads to lower equilibrium prices (for a specific form of the demand function).[5] We compare the price and order quantities under monopoly and competitive scenarios for a more general class of demand functions, and more importantly, bring the price-matching scenario into the comparison. On the other hand, a column-wise analysis of Table 1 illustrates the effects of demand uncertainty. Various papers in OM literature study monopolistic and competitive settings (Mills 1959, Karlin and Carr 1962, Petruzzi and Dada 1999, Bernstein and Federgruen 2003, Chen and Yao 2004, Li and Atkins 2005). We extend these results by demonstrating the effects of demand uncertainty in the presence of PMGs.

Overall, in this paper, our integrated operations-marketing approach reflects on the interaction between demand uncertainty and PMGs, addressing the following research questions:

- What role do price-matching guarantees play in a stochastic demand environment? In particular, does the tacit collusion outcome (monopoly price and order quantity decisions at equilibrium) related to PMGs in a deterministic environment continue to hold in the presence of demand uncertainty?
- In a PMG context, what is the effect of verifying the availability on price and inventory decisions and profits of the retailers? Is it always beneficial for retailers to verify the availability before matching the price? If not, under which conditions it makes sense to do so?
- How does demand uncertainty affect equilibrium decisions and profits when retailers offer price-matching guarantees? In other words, does the effect of demand uncertainty under monopoly and competitive scenarios continue to hold if firms offer PMGs?

To address these issues, we analyze a horizontal price-competition model in a single-period setting with two retailers offering PMGs (PM or PMA type), and selling an identical product. Retailers are symmetric in the sense that they have identical cost and demand parameters. Price and inventory decisions are set simultaneously before the demand is realized. Demand at each retailer is uncertain, as well as sensitive to the price decisions of the two retailers. All customers are knowledgeable about the prices as well as PMG offers and make their decisions accordingly.

[5] Another extension of the fixed-price newsvendor, less related to our work, investigates the quantity decisions of retailers in order to capture the *stock-out based substitution* of demand among competing retailers. This stream assumes that a constant fraction of unsatisfied demand of one retailer visits another retailer in search of the product. Retailers decide on their inventory level simultaneously to capture excess demand and set higher levels compared to the monopoly case (Parlar 1988, Lippman and McCardle 1997, Netessine and Rudi 2003, Mishra and Raghunathan 2004). See Zhao and Atkins (2007) and Krishnan and Winter (2007) for models studying both price-competition and stock-out based substitution to identify their combined effects.

We present a comprehensive equilibrium analysis of the game and confirm that price-matching guarantees inflate retail prices in an uncertain demand environment as well. Specifically, PMGs lead to tacit collusion among retailers if simple proofs are considered to be sufficient for granting price-match requests of customers (i.e., under PM policy). As a result, both retailers offer monopoly price and quantity to the market. However, we show that demand uncertainty opens the door for price discrimination and the retailers can exploit this opportunity by verifying the availability of the product at the competing store before accepting a price matching request. Under certain conditions (which we specify in the analysis), one retailer offers the monopoly price and order quantity, whereas the other retailer differentiates itself by setting a price and ordering level *higher than monopoly levels*. This result signifies that *demand uncertainty can act as a means for price discrimination* even in a perfectly symmetric environment.

Our analysis highlight the fact that it is crucial for store managers to understand the implications of demand uncertainty, demand correlation and availability verification, and adjust their price and inventory decisions as well as their price-matching policies accordingly. To this end, verification of availability is particularly profitable for innovative, highly anticipated products, as well as for products at the introductory stage of their life cycle. Our results also signify that, as the uncertainty in demand increases, the value of verifying the availability also increases. In addition, high demand correlation between competing retailers, which is more likely to occur when competing price matching stores are closely located, strengthens the incentive for verifying the availability.

The remainder of this paper is as follows. In § 2, we model our horizontal retail price-quantity competition. We derive the equilibrium prices in § 3, and analyze retailers' preferences between PM and PMA policies in § 4. The effects of PMGs are presented in § 5. The effects of demand uncertainty and correlation are presented in § 6. § 7 presents the concluding remarks. The proofs of all propositions are in the appendix.

2 Model Framework

Consider a market with two competing retail stores, $S1$ and $S2$, selling an identical short life cycle product. The demand at each retailer is uncertain, and sensitive to the price offered by the two retailers. Retailers simultaneously decide on their individual strategies, which consists of a list price, p_i, and an order quantity, q_i, prior to observing the (uncertain) demand. They are also undifferentiated in terms of unit costs (c) and all other aspects (e.g., quality of service offered). We assume that customers do not know the stocking level at the retailers before their visits, but they are all perfectly informed about the list prices announced by the retailers. Let $\mathbf{p} = (p_1, p_2)$ and $\mathbf{q} = (q_1, q_2)$. The demand for each retailer is given by:

$$\begin{aligned} D_1(\mathbf{p}) &= d_1(p_1|p_2) + \epsilon_1 \doteq d(p_1) + s(p_1, p_2) + \epsilon_1 \\ D_2(\mathbf{p}) &= d_2(p_2|p_1) + \epsilon_2 \doteq d(p_2) - s(p_1, p_2) + \epsilon_2 \end{aligned}$$

In the above framework, the uncertainty is modeled in an additive fashion by independent and identically distributed (*i.i.d.*) positive continuous random variables ϵ_1 and ϵ_2. The deterministic component of the demand has two parts. The first part, $d(\cdot)$, is a decreasing function representing the relation between price and demand in the absence of competition. The second part, $s(\cdot)$, represents the switchers due to price competition (i.e., price difference). Since retailers are identical and customers make their decisions based solely on the prices, switching behavior is symmetric in price decisions, i.e., $d_1(x|y) = d_2(y|x)$. Consequently, there will be no switching if retailers offer equal prices, i.e., $d_1(x|x) = d_2(x|x) = d(x)$. Excess demand is lost, which means that customers leave the market if the product is not available at the first-visited retailer. The expected profit function of retailer i, in this Bertrand price-competition setting, is given by,

$$\pi_i^C(\mathbf{p}, \mathbf{q}) \quad = \quad p_i E\big[\min\{q_i, D_i(\mathbf{p})\}\big] - cq_i \ , \quad i = 1, 2. \tag{1}$$

Note that the order quantity of one retailer has no effect on the profit of the other; given the prices, retailers can optimize their order quantity independent of the competitor's quantity decision. Let $q_i(\mathbf{p})$ be the optimal order quantity for Si. The expected profit for each player is then given by,

$$\pi_i^C(\mathbf{p}) \quad = \quad p_i E\big[\min\{q_i(\mathbf{p}), D_i(\mathbf{p})\}\big] - cq_i(\mathbf{p}) \ , \quad i = 1, 2. \tag{2}$$

Price Matching Guarantee:

In the presence of PMGs, customers, who are knowledgeable about the prices and guarantees, will make their purchase decisions according to the minimum price in the market. So, customers will make store choices based on their retailer preferences only; the price difference between the retailers will not play any role. This implies that the demand for each retailer will be a function of the "effective price" in the market, p^e, which is the minimum of the two list prices, i.e., $p^e \doteq \min\{p_1, p_2\}$. In this case, the demand at the two retailers can be expressed as:

$$\begin{aligned} D_1(\mathbf{p}) &\doteq d_1(p^e|p^e) + \epsilon_1 = d(p^e) + \epsilon_1, \\ D_2(\mathbf{p}) &\doteq d_2(p^e|p^e) + \epsilon_2 = d(p^e) + \epsilon_2. \end{aligned}$$

Note that not all of this demand will be served at the effective price p^e. Depending on the policy offered (PM or PMA), some customers may be charged the list price. In order to simplify the notation, we drop the index in demand, suppress the price decision of the competitor (it is also the effective price) and use the random variable $D(p^e) = d(p^e) + \epsilon$ to denote the demand of each retailer. The density, distribution, and survival function of demand $D(p)$ are denoted by $g_p(\cdot)$, $G_p(\cdot)$, and $\overline{G_p}(\cdot)$, respectively. Furthermore, we denote the reciprocal of the failure rate of the demand as $h_p(\cdot) = \overline{G_p}(\cdot)/g_p(\cdot)$.

Each customer first visits her/his preferred retailer expecting to buy at price p^e. For the time being focus on customers visiting $S1$. If the product is sold-out at $S1$, then they

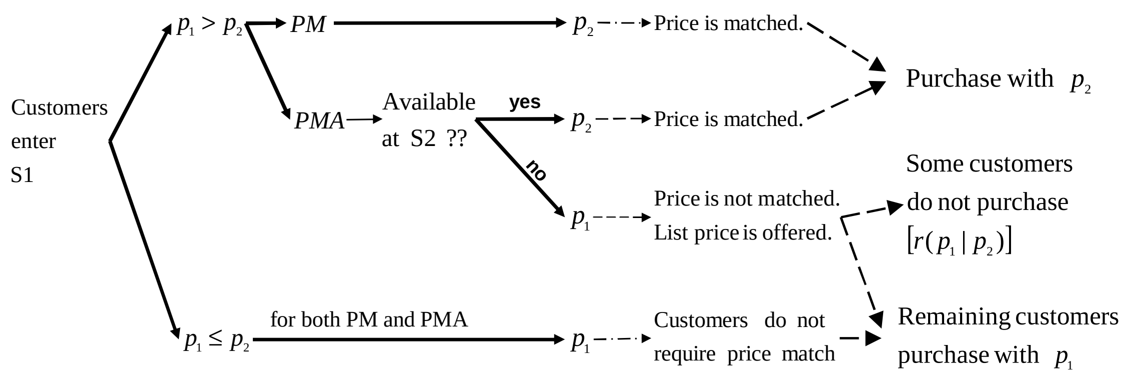


Figure 1: Prices charged by $S1$ based on the price-matching strategy.

leave the market. If the product is available and $p_1 \leq p_2$ (i.e., $p^e = p_1$), they purchase with list price p_1. On the other hand, if the product is available but $p_1 > p_2$ (i.e., $p^e = p_2$), then they request a price-match; we assume that there is no hassle cost associated with this request to the customer.[6] A schematic representation of the prices is provided in Figure 1. If $S1$ is offering a simple price-matching guarantee (PM), then the price will be matched for all customers. The profit for $S1$ under PM is then,

$$\pi_1^{PM}(\mathbf{p}) \quad = \quad p^e E\big[\min\{q_1(p^e), D(p^e)\}\big] - cq_1(p^e). \tag{3}$$

On the other hand, if the retailer is offering a price-matching guarantee based on availability (PMA), then it will check the availability at $S2$. Store managers do this either through the website of the competitor or via phone pretending to be a customer. We assume that the cost of verifying the availability is negligible. If the product is in-stock at $S2$, then the price will be matched, and the customer will make a purchase paying p_2. If, however, $S2$ has a stock-out, then $S1$ will refuse to match the price and offer its list price p_1. Since customers visited $S1$ hoping to buy at $p^e = p_2$ and are now charged $p_1 > p_2$, some of them may decide not to buy. We represent these customers by the function $r(p_1|p^e)$. Naturally, more customers will decide not to purchase as the list price increases or as the price difference between the effective price and list price increases. Therefore, we assume that $r(x|y)$ is increasing-convex in x, decreasing in y, and $r(x|y) = 0$ for $x \leq y$, i.e., all customers visiting the retailer make a purchase if they are charged the effective price.[7]

Note that the prices charged depend on when the competitor stocks out. In order to model this time dimension in a static (single-period) setting, we assume that the customers visit the retailers at an equal rate. This is a reasonable assumption since retail characteristics are identical, the demand at each retailer is a function of only the effective price and

[6] We discuss the implications of this and other key modeling assumptions in § 7.

[7] Results continue to hold as long as $x\frac{\partial r(x|y)}{\partial x}$ is increasing.

there is no switching of customers due to stock-outs. To clarify the implications of this assumption, suppose that $S1$ sets a list price of p_1 and orders q_1 units, where $q_1 > q_2$ and $p_1 > p_2$ (so, $p^e = p_2$). In that case, $S1$ has an opportunity to charge its list price to some customers if $S2$ has a stock-out. Let D_1 and D_2 be the demand realizations at $S1$ and $S2$, respectively.

1. If $D_2 < q_2$, then $S1$ has to match the price for all customers since $S2$ does not stock-out.
2. If $D_2 > q_2$, then $S1$ can charge p_1 to those customers who visit $S1$ after the q_2^{th} customer of $S2$ (if there are any). There can be two subcases of this scenario.
   i. If $D_1 < q_2 < D_2$, then all the customers of $S1$ will visit $S1$ before $S2$ stocks out. Therefore $S1$ will have to charge p^e to all of them.
   ii. If $D_1 > q_2$ and $D_2 > q_2$, then some customers of $S1$ will visit the retailer after $S2$ sells all its stock. Based on our assumption of equal rates, there will be $D_1 - q_2$ such customers. That is, for $D_1 - q_2$ of its customers, $S1$ will decline to match the price p_2, due to the unavailability at $S2$, and offer its list price p_1. Note that $r(p_1|p_2)$ of them will walk away without a purchase, and the remaining customers (if any) will purchase the item with price p_1.

The expected profit for $S1$ under PMA policy can then be written as follows ($[x]^+ = \max\{0, x\}$):

$$\begin{aligned}\pi_1^{PMA}(\mathbf{p},\mathbf{q}) = p^e E\Big[\min\big\{q_1, q_2, D(p^e)\big\}\Big] + p^e Pr\big\{D(p^e) \le q_2\big\} E\Big[\min\big\{q_1, D(p^e)\big\} - q_2\Big]^+ \\ + p_1 Pr\{D(p^e) > q_2\} E\left[\min\left\{[q_1 - q_2]^+, \Big[D(p^e) - q_2 - r(p_1|p^e)\Big]^+\right\}\right] - cq_1.\end{aligned} \tag{4}$$

The first two terms in (4) represents the revenue from customers who visit $S1$ early and get the effective price. The third term is for customers who arrive at $S1$ after $S2$ has sold all its products. Observe that, unlike the existing price competition models in OM literature which do not model PMGs, the quantity of $S2$ has a direct effect on the expected profit of $S1$. In other words, when price-matching is based on availability, quantity decisions become a strategic component of competition among the retailers, *even in the absence of stock-out based substitution*. As a result, we are effectively analyzing a game where retailers compete on both price and quantity decisions.

Before proceeding with the analysis, note that a unique profit maximizing strategy $\{p^N, q^N\}$ for the monopolist price-setting newsvendor problem, $\max_{p,q} \big\{pE\big[\min\{q, D(p)\}\big] - cq\big\}$, is guaranteed under the following assumptions.

ASSUMPTION A1:

1. $D(p) = d(p) + \epsilon$ where ϵ is a positive random variable with an increasing failure rate (IFR).
2. $d(p)$ is a decreasing function defined on $[c, p^u]$, where p^u is the "null price", and $d(c) > 0$.
3. The price elasticity of the riskless demand $(d(p) + \mu)$ defined by $\eta(p) = \frac{-pd'(p)}{d(p)+\mu}$ is increasing.
4. Marginal riskless profit is negative at the null price, i.e., $\mu + (p^u - c)d'(p^u) < 0$.

IFR assumption is satisfied by many log-concave density functions frequently used in the OM literature, e.g., uniform, normal, truncated normal, exponential, gamma (with shape parameter ≥ 1), and beta (with both parameters ≥ 1). Property 2 rules out the possibility of infinite price. Property 3 indicates that price elasticity is increasing in retailers own price decisions. This assumption is satisfied by typical demand functions used in the literature such as linear, power, iso-elastic and concave forms. Property 4 guarantees an interior solution. In the sequel we assume that **A1** holds.

3 Model Analysis

3.1 Analysis of Simple Price-Matching (PM) Model

Recall that if the retailer adopts a simple price-matching guarantee policy, i.e., PM, then it agrees to match the price for all customers. The profit function for retailer i in this case is given by

$$\pi_i^{PM}(\mathbf{p}) \quad = \quad p^e E\big[\min\{q_i(p^e), D(p^e)\}\big] - cq_i(p^e), \; i = 1, 2. \tag{5}$$

Let $q^F(p) = \overline{G}_p^{-1}(c/p)$ denote the optimal order quantity of the fixed-price newsvendor problem for a given p. Deriving the best response function of each retailer, we arrive at the following.

Proposition 1 *There exists a continuum of equilibrium solutions (p, p) for $p \in [c, p^N]$ to the PM game. The equilibrium point (p^N, p^N) is Pareto dominant among them. At this equilibrium, each retailer charges optimal monopoly price p^N, orders the optimal monopoly order quantity $q^N = q^F(p^N)$, and earns the optimal monopoly profit.*

Note that Proposition 1 extends the tacit collusion outcome of deterministic PMG literature to an uncertain demand environment; if retailers do not verify the availability before matching the price, then monopoly decisions become the Pareto dominant equilibrium. There are no incidences of price matching since retailers announce equal prices, and all customers are charged the monopoly price which is the effective price.

3.2 Analysis of Price-Matching based on Availability (PMA) Model

For the PMA policy, we first investigate the profit function and best response strategy of the two retailers (since the two are symmetric, we focus on $S1$), and then we characterize the equilibrium.

Expected profit function:

As discussed previously, the expected profit of $S1$ under PMA policy, given by (4), has a different behavior depending on the magnitude of price and order quantity decisions of the two retailers. Therefore, we analyze the game by dividing the state space into four different regions. These four regions are defined as follows: $R1 \equiv \{(\mathbf{p},\mathbf{q})|p_1 \leq p_2, q_1 \leq q_2\}$, $R2 \equiv \{(\mathbf{p},\mathbf{q})|p_1 \geq p_2, q_1 \leq q_2\}$, $R3 \equiv \{(\mathbf{p},\mathbf{q})|p_1 \geq p_2, q_1 \geq q_2\}$, $R4 \equiv \{(\mathbf{p},\mathbf{q})|p_1 \leq p_2, q_1 \geq q_2\}$ (see Figure 2).

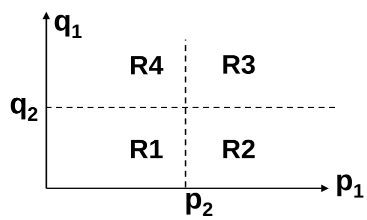


Figure 2: Four regions in the strategy space for different profit functions under PMA policy.

Profit function in each of these regions is as follows:

$$\pi_1(\mathbf{p},\mathbf{q}) = \begin{cases} p_1 E\Big[\min\big\{q_1, D(p_1)\big\}\Big] - cq_1 & (\mathbf{p},\mathbf{q}) \in R1, R4 \\ p_2 E\Big[\min\big\{q_1, D(p_2)\big\}\Big] - cq_1 & (\mathbf{p},\mathbf{q}) \in R2 \\ p_2 E\Big[\min\big\{q_2, D(p_2)\big\} + p_2 G_{p_2}(q_2) E\Big[\min\big\{q_1, D(p_2)\big\} - q_2\Big]^+ & \\ +p_1 \overline{G}_{p_2}(q_2) E\Bigg[\min\Big\{q_1 - q_2, \Big[D(p_2) - q_2 - r(p_1|p_2)\Big]^+\Big\}\Bigg] - cq_1 & (\mathbf{p},\mathbf{q}) \in R3 \end{cases}$$

A careful observation of the above profit function shows that:

1. In regions $R1$ and $R4$, $S1$ is the lower priced retailer. Hence, the quantity decision of $S2$ does not influence $S1$, which is identical to a monopolistic price-setting newsvendor.
2. In regions $R2$ and $R3$, the effective price is set by $S2$; so $S1$ is a price-taker.
3. In region $R2$ we further have $q_1 \leq q_2$. Therefore, $S2$ will not stock-out before $S1$, and so the quantity decision q_2 does not influence the profit for $S1$. The profit function in $R2$ is actually identical to that of the standard newsvendor model with exogenous price p_2.

Response strategy:

In this part we derive the response strategy of $S1$, for a given price decision p of $S2$. We start by examining the response strategy in $R1$ and $R4$, and then continue with $R2$ and $R3$. Proposition 2 follows from observation 1 on the expected profit function.

Proposition 2 *Suppose that the strategy of $S2$ is given by $\{p, q\}$. The best response strategy of $S1$ in $R1$ and $R4$ is then $\{p, q^F(p)\}$ for $p \leq p^N$ and $\{p^N, q^N\}$ for $p > p^N$.*

Since the retailers are symmetric, the best response strategy of $S2$ in $R2$ and $R3$ (i.e., for $p_1 \geq p_2$) is identical to the best response strategy of $S1$ in $R1$ and $R4$ (i.e., for $p_1 \leq p_2$). This has an immediate implication on the strategy space of $S2$.

Corollary 1 *In $R2$ and $R3$ any strategy $\{p, q\}$ of $S2$ is dominated by the strategy $\{p, q^F(p)\}$.*

Corollary 1 suggests that in $R2$ and $R3$, $S2$ is better off by optimizing its order quantity given its own price decision. While studying the best response of $S1$ in $R2$ and $R3$, we will utilize this information and suppose that the strategy of $S2$ is $\{p, q^F(p)\}$.

Lemma 1 *Given the strategy $\{p, q^F(p)\}$ of $S2$, the best response of $S1$ in $R2$ is $\{p, q^F(p)\}$.*

In $R2$, all customers of $S1$ visit the retailer before $S2$ stocks out. Consequently, $S1$ is a price taker and has to match the price for all of them. Therefore, $S1$ orders as a fixed-price newsvendor with price p. Next we consider the response strategy of $S1$ in $R3$. This analysis is divided into two parts. First, in Proposition 3, we determine the optimal order quantity given the prices of $S1$ and $S2$.

Proposition 3 *Consider $S1$ in $R3$. Given the strategy of competitor, $\{p, q^F(p)\}$, and its own price decision $p_1 \geq p$, the optimal order quantity for $S1$ is given by*

$$\begin{cases} q_1(p_1|p) & \text{if } p_1\overline{G}_p\Big(q^F(p) + r(p_1|p)\Big) - c > 0 \\ q^F(p) & \text{o/w} \end{cases} \tag{6}$$

where $q_1(p_1|p)$ is unimodal in p_1 and is the unique q_1 satisfying the first order condition

$$pG_p\Big(q^F(p)\Big)\overline{G}_p\Big(q_1\Big) + p_1\overline{G}_p\Big(q^F(p)\Big)\overline{G}_p\Big(q_1 + r(p_1|p)\Big) = c. \tag{7}$$

The optimal order quantity $q_1(p_1|p)$ in (7) equates the marginal revenue of stocking another unit to the marginal cost of acquiring it. When selling this unit, $S1$ will have to match the price of the competitor if the following two events occur simultaneously: there is no stock-out at $S2$, and $S1$ has enough demand to sell the last unit. On the other hand, $S1$ will charge its own list price if there is a stock-out at $S2$ and if there are remaining customers when $S1$ refuses to match the price and offers its list price. The expectation gives us the marginal revenue of ordering another unit.

By substituting the optimal order quantity as a function of price, we can rewrite the expected profit function of $S1$ in terms of p_1 and the strategy of $S2$. The next proposition combines the findings for regions $R2$ and $R3$. Note that we define $r'(p_1|p) \doteq \frac{\partial r(p_1|p)}{\partial p_1}$.

Proposition 4 *Given the strategy $\{p, q^F(p)\}$ of $S2$, the best response of $S1$ in $R2$ and $R3$ is*

$$= \begin{cases} \hat{p_1}(p), q_1(\hat{p_1}(p)|p) & \text{if } h_p(q^F(p)) \geq p.r'(p|p) \\ p, q^F(p) & o/w \end{cases}$$

where $\hat{p_1}(p) > p$ is the unique p_1 satisfying $\int\limits_{q^F(p)+r(p_1|p)}^{q_1(p_1|p)+r(p_1|p)} \left[\overline{G}_p(x) - p_1 r'(p_1|p) g_p(x)\right] dx = 0.$

Proposition 4 tells us that in $R3$, the response of $S1$ will have a higher price and a higher inventory level compared to $S2$ under a certain condition. Otherwise $S1$ will charge a price equal to that of $S2$. Note that the required condition holds if and only if

$$\Phi(p) \doteq z'(p) - r'(p|p) \geq 0, \tag{8}$$

where $z(p) = q^F(p) - d(p)$ is the stocking factor.[8]

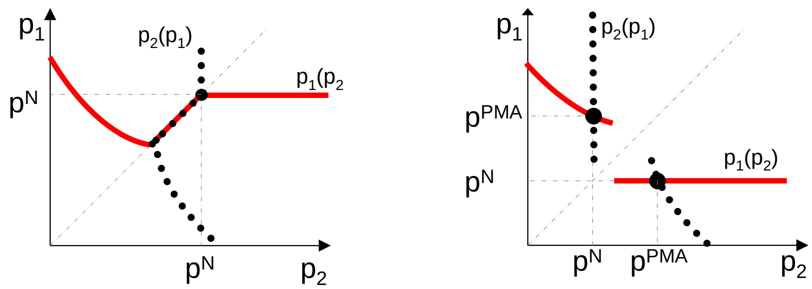
The intuition for (8) is as follows. Suppose that $S1$ slightly increases its price from p to $p + \xi$. In this case, $S1$ will also need to increase its stocking factor (or safety stock) approximately by $\xi.z'(p)$ units. This is also the number of customers that the retailer will decline the price match request and offer its list price. However, approximately $\xi.r'(p|p)$ of these customers will decide not to buy. If condition (8) is satisfied, then it means that there are still some customers who purchase at the list price. Otherwise, there are no customers to purchase with the list price, and the retailer has no incentive to offer a higher price and order quantity. It is relatively straightforward to show that $\Phi(p)$ is a decreasing function.[9] Consequently, everything else being constant, the retailer is more likely to differentiate its response strategy when the competing retailer has *relatively low price levels*. Figure 3 illustrates the *price* decisions of the response strategy under PMA policy.

Equilibrium:

Above we derived the response strategy for $S1$ on the basis of four regions. Since the retailers are symmetric, the response of each player is identical. Therefore, for any equilibrium point in $R2+R3$, there exists an equilibrium point in $R1+R4$ in which players interchange their price and quantity decisions. The following proposition formalizes the equilibria for the PMA game.

[8] Since $q^F(p)$ satisfies $\overline{G_p}(q^F(p)) = c/p$ we have $\frac{h_p(q^F(p))}{p} = \frac{\overline{G_p}(q^F(p))}{p.g_p(q^F(p))} = \frac{\partial q^F(p)}{\partial p} - d'(p) = z'(p)$.

[9] $z(p)$ satisfies $F(z(p)) = \frac{p-c}{p}$, where $F(\cdot)$ is the distribution of the random variable ϵ.



(a) PMA when $\Phi(p^N) \leq 0$ (b) PMA when $\Phi(p^N) > 0$

Figure 3: Price decisions of response strategy under PMA policy.

Theorem 1 *Suppose that the retailers verify the availability before granting any price-match request (i.e., adopt PMA policy). If $\Phi(p^N) > 0$, then there are two pure-strategy equilibria: (a) one in which retailer $S2$ sets list price p^N and order quantity q^N, and $S1$ sets its list price p^{PMA} and order quantity q^{PMA} where $\{p^{PMA}, q^{PMA}\} \equiv \{\hat{p_1}(p^N), q_1(\hat{p_1}(p^N)|p^N)\}$; (b) another where the strategies are interchanged between the retailers. Further, the retailer with strategy $\{p^{PMA}, q^{PMA}\}$ has higher profit than the competitors profit, which is equal to the monopoly profit of a price-setting newsvendor.*

On the other hand if $\Phi(p^N) \leq 0$, then there exists a continuum of pure-strategy equilibria where retailers set equal prices p and order quantity $q^F(p)$ for some $p \in [c, p^N]$. Pure-strategy equilibrium in which both retailers set their strategy as $\{p^N, q^N\}$ is the Pareto dominant equilibrium among them. Both retailers earn optimal monopoly profits at this Pareto dominant equilibrium.

If $\Phi(p^N) \leq 0$ (Figure 3a), then the equilibrium structure is identical to the PM policy; that is, PM and PMA policies are then effectively equivalent. On the other hand, if $\Phi(p^N) > 0$ (Figure 3b), then verifying the availability creates the opportunity for price discrimination. In the presence of demand uncertainty, there is always a chance that the competitor sells out, making a particular retailer the sole provider of the product. This gives the incentive to post a price (p^{PMA}) even higher than the monopoly price (p^N), and extract a premium when there is scarcity in the market. In this sense, p^{PMA} can be interpreted as the "scarcity monopoly price". In order to take advantage of this opportunity, the price discriminating retailer also orders more ($q^{PMA} > q^N$). In the following section, we investigate the conditions under which this situation is likely to occur.

4 When to Verify the Availability?

We have shown that in equilibrium retailers may have symmetric or asymmetric strategies depending on the problem parameters if they adopt PMA policy. If $\Phi(p^N)$ is positive under PMA policy, then retailers offer differentiated prices and the equilibrium strategy of players are mirror images of each other. This makes it impossible to favor one of them based on the commonly used selection criteria such as Pareto Dominance or Risk Dominance (Harsanyi and Selten, 1988). Nevertheless, we are not concerned about equilibrium choice in this study. Given the symmetry, the analysis and managerial insights of each equilibrium point will lead to the same conclusions. For this reason, we proceed with the implicit assumption that the players have some mechanism by which they arrive at one of these focal equilibria (Schelling, 1980). Our main objective in this section is to identify the market characteristics that increase the incentive to verify the availability and thus price discriminate (i.e., increases the value of $\Phi(p^N)$) by comparing this equilibrium with the Pareto dominant solution of the PM game. Note that the decision to verify the availability and to price discriminate are inseparable from a retailer's perspective; therefore, we use the them interchangeably from this point on. We define the key market characteristics as the market size, price sensitivity, unit cost, price sensitivity of customers when the price-matching offer is refused, and uncertainty in demand. In order to demonstrate these effects clearly, we suppose that $D(p) = \alpha - \beta p + \epsilon$ and $r(x|y) = \gamma[x-y]^+$. Under this linear demand form, we can analytically identify the effects of market size (α), price sensitivity (β) and price sensitivity when price-match request is declined (γ).

Proposition 5 *A decrease in α, and γ or an increase in β results in an increase in $\Phi(p^N)$, i.e., strengthen the incentive for retailers to price discriminate by verifying the availability before granting a price-match request.*

Suppose that the strategy of $S2$ is $\{p^N, q^N\}$. A smaller market size or a higher price sensitivity leads to a decrease in the optimal monopoly price. As a result, $S2$ will lower its safety stock; $S1$ will then have more customers to offer the higher list price, and therefore, more incentive to verify the availability. Moreover, as γ decreases, customers become less sensitive to the price increase in case their price-match request is declined, hence, $S1$ has more incentive for price discrimination.

The effect of unit cost and demand uncertainty, however, is not straightforward. A change in unit cost or demand uncertainty has two counter-acting effects: a direct effect on $\Phi(\cdot)$, and an indirect effect through p^N. It is analytically difficult to ascertain which effect dominates, so we rely on numerical experiments. In these numerical experiments, we assume that ϵ is normally distributed with a mean μ, and standard deviation σ. We have done extensive tests and observed consistent results with respect to the effects of unit cost and demand uncertainty. We present an illustrative example in Figure 4, where we provide the indifference curve between PM and PMA policies.[10] To the right of these points the retailer has higher profits when the availability of the product at the competitor is verified before matching the price; to the left of these points the two price matching policies, are identical in terms of equilibrium strategy.

When the unit cost increases, the direct effect is a decrease in the safety stock of $S2$ which leads to an increase in the expected number of customers who are offered the list

[10]In the illustrative example $\alpha = 210, \beta = 6, \gamma = 0.5, \mu = 120$, $c \in [4, 14]$ and $\sigma \in [0, 50]$.

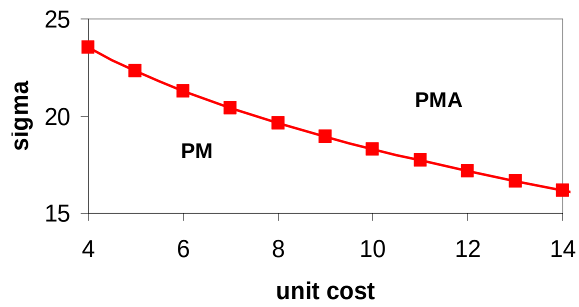


Figure 4: The indifference curve for the retailer as a function of unit cost and variance of demand.

price at $S1$. Moreover, the increase in unit cost also increases the monopoly price. The indirect effect is then an increase in the safety stock of $S2$. The overall effect is an increase in the expected number of customers who purchase with the list price i.e., *more incentive for price discrimination*. When the demand variability increases, we also observe the same conflicting effects; however, in reverse directions. The dominant effect of price leads to *higher incentive for* $S1$ *to verify the availability*.

Table 2 summarizes our findings on the market characteristics that *strengthen the incentive to verify the availability and price discriminate* for retailers offering PMGs.

The market characteristics that we are considering depend mostly on the product type. Fisher (1997) classifies products into two categories, functional and innovative, based on their demand patterns. Innovative products are likely to have shorter life cycles, unpredictable demand, and high stock out rates than functional products. Furthermore, consumer research literature shows that the consumers are more informed about the prices and product characteristics if the perceived risk (economical and psychological) of satisfaction and the price of the product is high (Blodgett et al., 1995) which are common characteristics for innovative products. Table 2 indicates to managers that it makes more sense to adopt a PMA policy for innovative products such as high-end consumer electronics products (PC's, cellphones, games). High unit cost and high relative scale of uncertainty associated with high-end innovative products such as flat-screen, high definition TV's present

Table 2: Impact of market characteristics.

Market characteristics	$\alpha \searrow$	$\beta \nearrow$	$\gamma \searrow$	$c \nearrow$	$\sigma \nearrow$
Optimal price for monopolistic price-setting newsvendor, p^N	$\searrow$	$\searrow$	$-$	$\nearrow$	$\searrow$
Optimal stocking factor for monopolistic price-setting newsvendor, $z(p^N)$	$\searrow$	$\searrow$	$-$	$\searrow$	$\nearrow$
Incentive for verifying the availability, $\Phi(p^N)$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$

stronger cases for adopting PMA. It is also natural to assume that customers who are eager to obtain a product will have less price sensitivity in case of a stock-out at one of the retailers. This is particularly true for highly-anticipated innovative products such as XBox or Playstation. Our findings also suggest that managers should adopt PMA policy for these highly anticipated products. On the other hand, for functional products with longer life cycles, and relatively more stable and predictable demand, such as basic apparel and durable consumers goods (e.g. groceries), a simple proof should be sufficient to grant price-matching requests.

5 Comparison of Different Competitive Scenarios

In this section we compare the equilibrium strategies of the retailers under different competition scenarios (monopoly, Bertrand competition, PMG). As noted earlier (see § 1), comparisons among these scenarios under deterministic demand setting are well-studied in the literature: Bertrand price-competition intensifies the competition by reducing market prices below the monopoly level, while introduction of PMGs reduces competitive intensity and restores monopoly decisions. We investigate whether similar relationship remains valid under stochastic demand. This comparison also enables us to quantify the impact of demand uncertainty, which we will separately discuss in §6. In order to do the comparison, we need to first derive an intermediate result regarding the equilibrium under Bertrand price-competition (i.e., no PMG) between price-setting newsvendors. The existence and uniqueness of the equilibrium is already established for models with multiplicative demand uncertainty (Bernstein and Federgruen 2003, Chen and Yao 2004). Bernstein and Federgruen (2003) remark that the existence of the equilibrium can be proved analogously in an additive uncertainty model, but do not mention the uniqueness. In what follows, we formally establish the uniqueness of the equilibrium under additive demand. To this end we introduce a set of assumptions, namely **A2**, extending the monopoly assumptions (**A1** in § 2) to a duopoly setting. Note that **A2** is the additive demand equivalent of the assumptions in Chen and Yao (2004).

ASSUMPTION A2:

1. $D_i(\mathbf{p}) = d_i(\mathbf{p}) + \epsilon_i$ where ϵ_i is a positive random variable with an increasing failure rate (IFR).
2. $d_i(\mathbf{p})$ is a continuous, bounded and differentiable in the strategy space $p_i \in [c, p_i^u]$, where $p_i^u = \min\{p_i | d_i(p_i, p_{-i}) = 0\}$ is the "null price" and $\frac{\partial d_i(\mathbf{p})}{\partial p_i} < 0$, $\frac{\partial d_i(\mathbf{p})}{\partial p_j} \geq 0$ $i = 1, 2$, $j \neq i$.
3. The price elasticity of the riskless demand defined by $\eta_i(\mathbf{p}) = \frac{-p_i}{d_i(\mathbf{p})+\mu_i} \frac{\partial d_i(\mathbf{p})}{\partial p_i}$ is increasing.
4. Marginal riskless profit is negative at the null price: $\mu_i + (p_i^u - c)\frac{\partial d_i(\mathbf{p})}{\partial p_i}\Big|_{p_i=p_i^u} < 0$.

5. If the competitor increases its price, the demand will increase, $\frac{\partial d_i(\mathbf{p})}{\partial p_j} > 0$, $i = 1, 2$, $j \neq i$.
6. An increase in competitors price decreases the price elasticity, $\frac{\partial \eta_i(\mathbf{p})}{\partial p_j} < 0$, $i = 1, 2$, $j \neq i$.
7. Domination conditions, guaranteeing that local price effect of a price change dominates the cross price effect on local price change, $|\frac{\partial d_i(\mathbf{p})}{\partial p_i}| > |\frac{\partial d_i(\mathbf{p})}{\partial p_j}|$ and $|\frac{\partial \eta_i(\mathbf{p})}{\partial p_i}| > |\frac{\partial \eta_i(\mathbf{p})}{\partial p_j}|$ for $i = 1, 2$ $j \neq i$.

Proposition 6 *Suppose that* **A2** *holds and that none of the retailers offer PMG. Then, the expected profit of each retailer is given by (2), and there exists a unique symmetric equilibrium solution to this game.*[11] *Both retailers play with strategy* $\{p^C, q^C\}$ *where* $q^C = q^F(p^C)$ *and* p^C *satisfies the first order conditions,* $\frac{\pi_i^C(\mathbf{p})}{\partial p_i} = 0$ *for* $i = 1, 2$.

When symmetric retailers compete on price in an uncertain demand environment, the equilibrium solution is also symmetric. Both retailers charge the same price, and make their order quantity decisions accordingly. We are now ready to compare the decisions under different market scenarios, which is formalized in the next proposition and is illustrated in Figure 5.

Proposition 7 *Suppose that* **A1** *and* **A2** *holds. Then the following are true:*

Price Comparisons
- $p^{DM} = p^{DPM} > p^{DB}$
- $p^{PMA} \geq p^{PM} = p^N > p^C$
- $p^{DM} > p^N$, $p^{DB} > p^C$, $p^{DPM} > p^{PM}$
- *There are cases where* $p^{PMA} > p^{DM} = p^{DPM}$.

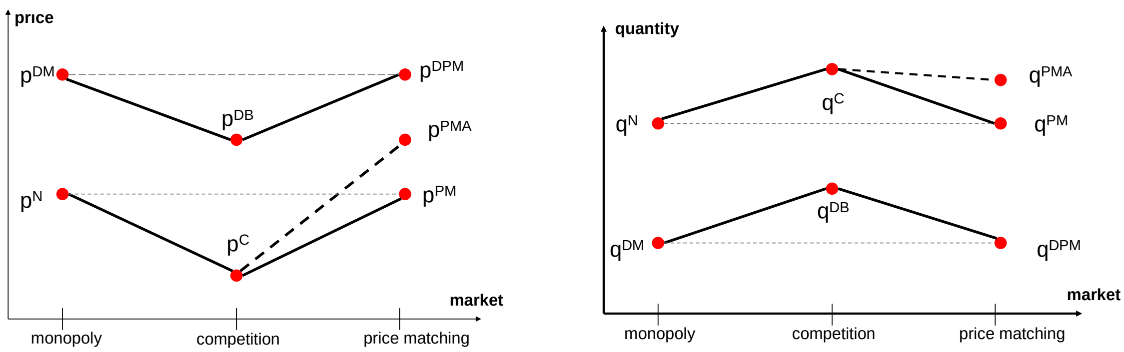
Quantity Comparisons
- $q^{DB} > q^{DM} = q^{DPM}$
- $q^C > q^N = q^{PM}$
- $q^{PMA} \geq q^{PM}$

We have already pointed out the effects of price-competition and the tacit collusion outcome under deterministic demand (i.e., $p^{DM} = p^{DPM} \geq p^{DB}$); the quantity comparison follows from the price comparison (i.e., $q^{DM} = q^{DPM} \leq q^{DB}$). In case of uncertain demand, we have three observations/results based on the effects of competition and the existence of PMGs.

Observation 1: Price-competition increases the intensity of competition by reducing prices and increasing order quantities compared to the monopoly scenario (i.e., $p^N \geq p^C$ and $q^N \leq q^C$).

Observation 2: Price-matching guarantees reduce the intensity of price-competition in a stochastic demand environment. Specifically, *tacit collusion outcome of the deterministic*

[11]Note that the uniqueness result can be extended to the *oligopoly* case with N retailers which are *asymmetric in demand* and *unit costs*. In Appendix we prove the uniqueness of the equilibrium for this general case.



(a) Price comparisons (b) Quantity comparisons

Figure 5: Comparisons of decision variables under different competition and demand models.

PMGs continues to hold ($p^{PM} = p^N, q^{PM} = q^N$) under PM policy in an uncertain demand environment.

Observation 3: By offering PMA policy, retailers' use the availability of the product (at the competitor) as a medium for price discrimination. *This results in an increase in the price and and quantity decision of one retailer to even higher than monopoly levels.*

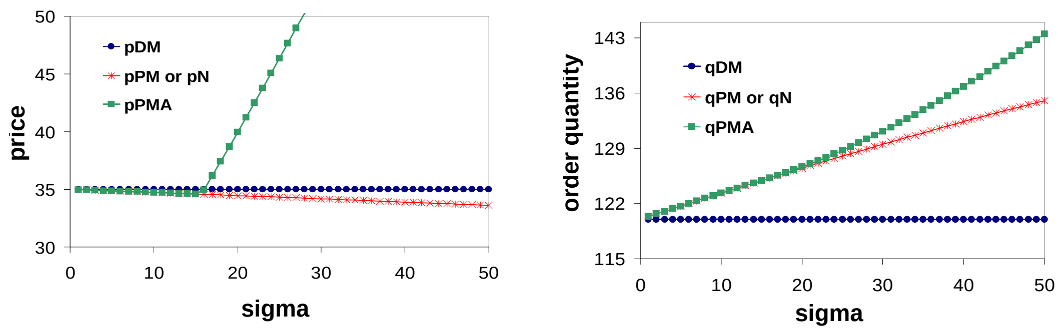
6 The Effect of Demand Uncertainty

Another important managerial issue is to understand how demand uncertainty shapes equilibrium outcomes. We know that in monopoly and competitive settings, the price levels decrease with the introduction of additive demand uncertainty ($p^{DM} \geq p^N, p^{DB} \geq p^C$). In the following, we identify the effects of demand uncertainty when retailers offer PMGs.

Observation 4: If the retailers adopt PM policy, then prices decrease with the introduction of demand uncertainty, $p^{DPM} \geq p^{PM}$. However, verifying the availability increases the price, $p^{PMA} \geq p^{PM}$. In fact, based on problem parameters, the high-priced retailer may set a price higher then the deterministic monopoly price ($p^{PMA} \geq p^{DPM}$).

Observation 5: Under monopoly and competitive scenarios, demand uncertainty increases order quantities (i.e., $q^N > q^{DM}$, $q^C > q^{DB}$). Similarly, under both types of PMG policies (i.e., PM and PMA), demand uncertainty increases order quantities (i.e., $q^{PMA} > q^{PM} > q^{DPM}$). In that sense, the directional effect of uncertainty on order quantities is the same for all three market scenarios.[12]

[12] The comparison of order quantities under deterministic and uncertain demand models rely on our numerical experiments. We use the same numerical framework introduced in § 4. All other quantity/price comparisons are based on analytical proofs.



(a) Effects on price. (b) Effects on quantity.

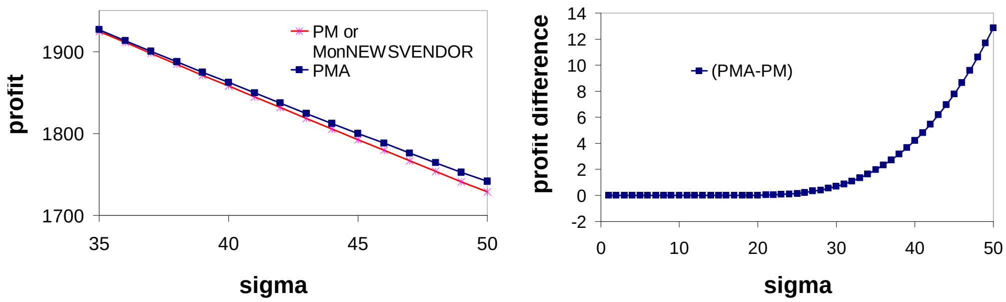
Figure 6: Effects of variability on equilibrium decisions.

The above analysis investigates the effect of demand variability compared to the extreme case with zero variance. We also explore the directional effects of uncertainty via the numerical framework introduced in § 4. We illustrate the results in Figures 6 and 7.

Figure 6a presents the equilibrium price decisions with respect to demand uncertainty. As expected, the optimal monopoly price under deterministic demand (i.e., p^{DM}) is invariant, where as the optimal price of the monopoly newsvendor (i.e., p^N) is decreasing in demand uncertainty. For low levels of demand uncertainty, prices for PM and PMA policies are identical to the monopoly price. After a certain threshold value of σ, however, the uncertainty in demand motivates the retailers to verify the availability. The low-priced retailer continues to charge the monopoly price; the high-priced retailer, however, increases its price as uncertainty increases, thereby taking advantage of stock-outs at the competing retailer. As seen in Figure 6b, an increase in demand uncertainty leads to an increase in the order quantity under monopoly and PM scenarios. Under PMA policy, the high-priced retailer increases its order quantity as the variability increases in order to capture the higher number of customers visiting after competitor's stock-out.

Figure 7 illustrates the effects of demand uncertainty on equilibrium profits. For low levels of demand uncertainty, PM and PMA policies are identical to the monopoly newsvendor; thus, profits decrease with uncertainty. As seen in Figure 7a, under PMA policy, the price discriminating retailer gains higher profits than the competitor (as shown in Theorem 1). The overall effect of uncertainty on the profit is still negative, meaning that her profits are decreasing with uncertainty. Interestingly, the *profit gain* by verifying the availability increases in a convex fashion (see Figure 7b).

Observation 6: *The value of verifying the availability increases with demand uncertainty.*



(a) Effects on profits. (b) Effect on profit difference.

Figure 7: Effect of variability on equilibrium profits.

6.1 The Effect of Demand Correlation

In practice, most retailers require the competitor stores to be located in close proximity (i.e., same geographical region) for matching prices (see Figure 9 in Appendix). In such cases, it is plausible that there is some correlation between the retailers' demands. This section investigates how demand correlation affect equilibrium strategies. In order to do so, we relax the independence assumption of the random shocks, and assume that they follow a bivariate normal distribution function. To identify and highlight the effects of correlation, we suppose that the *marginal* distribution functions of random variables are identical.

The curve in Figure 8 illustrates the points where the retailer is indifferent between verifying the availability and accepting simple proofs before price-match. Note that, the figure generalizes our previous result; for a given value of coefficient of correlation, an increase in demand variance provides an incentive to offer PMA policy as discussed in the previous section.

Observation 7: *An increase in correlation increases retailer's incentive to offer PMA policy.*

The intuition is as follows. Suppose that $S2$ adopts strategy $\{p^N, q^N\}$, whereas $S1$ adopts strategy $\{p^{PMA}, q^{PMA}\}$. Let the demand realized at $S1$ and $S2$ be D_1 and D_2, respectively. Consider the following two events: i) there is excess demand at $S2$ ($D_2 > q_2$) and excess inventory at $S1$ ($D_1 < q_1$), and ii) there is excess inventory at $S2$ ($D_2 < q_2$) and excess demand at $S1$ ($D_1 > q_1$). In both these cases, $S1$ will have to match the price of $S2$ for all its customers. In other words, $S1$ will not be able price discriminate customers even though there is demand mismatch between $S2$ and itself.[13] When the additive demand shocks are independent, these events occur with positive probability. As

[13] Of course, $S1$ will have to match the price for all of its customers even when $D_1 < q_1$ and $D_2 < q_2$, but we do not consider this as a mismatch between demands.

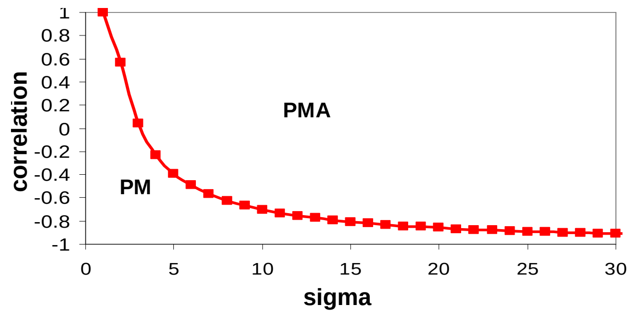


Figure 8: Indifference curve as a function of demand variance and correlation.

the demand correlation increases (decreases), the probability of these events occurring decrease (increase), which increases (decreases) the incentive for price discrimination.

In order to highlight the effects of demand correlation on equilibrium strategies, we consider the two extreme cases of independent and perfect positively correlated demand distributions. The next proposition analytically compares the equilibrium prices and order quantities under these cases.

Proposition 8 *When the demand shocks have perfect positive correlation, there is less price difference and more quantity difference between retailers compared to the independent case.*

As noted previously, when we move from the independent case to the perfect positive correlation case, the probability of missing an opportunity to price discriminate decreases. This results in an increase in the marginal profit of acquiring another unit, thus an increase in the equilibrium quantity of the discriminating retailer. The high-priced retailer also decreases its price to increase the sales with the list price.

7 Concluding Remarks

Retailers offering PMGs promise to match any lower price at the competitor for the same product. In this paper, we consider two types of guarantees: *i*) PM policy, under which a simple proof such as a weekly flyer or website information is sufficient to grant the price match, and *ii*) PMA policy, under which retailers demand that the product is available at the competitor store before satisfying any price-match request. We analyzed PMGs in an uncertain demand environment by studying the price-competition between two price-setting newsvendors selling an identical product and offering PMGs. Retailers simultaneously make their joint pricing and inventory decisions before the demand is realized. In order to highlight the effects of demand uncertainty and verification of availability, we assumed that retailers are symmetric and that all customers are knowledgeable about the prices in the market and about the PMG offers.

The major contributions of this paper are two-fold. Theoretically, we are able to characterize and compare the equilibria for the duopoly game when the retailers adopt various PMG policies as well as the case without any PMGs (Bertrand competition). From these comparisons we are able to generate practical insights as to when it is beneficial to verify the availability before matching prices. Furthermore, we shed light on the effects of certain market characteristics, specifically demand uncertainty, on retailers price-matching policies, price decisions, order quantities and profits.

Our analysis shows that if retailers accept simple proofs to match the price, then there exists a Pareto dominant solution where both retailers act like monopolies. On the other hand, if the availability is verified before matching the price, then there can be symmetric or asymmetric equilibria. The symmetric solution is identical to the Pareto dominant one under PM policy. In the asymmetric equilibria case, we have two pure strategy equilibria that are mirror images of each other in terms of price and quantity decisions. At each of these equilibria, one of the retailers act like a monopolist, whereas the other one opts for higher-than-monopoly price and order quantity.

We also report on the competitive effects of PMGs. When customers are informed about the prices and the guarantees, offering PMGs without verifying the availability leads to tacit collusion and increase prices from competitive levels to monopoly levels. This result verifies that the tacit collusion outcome of economics literature with deterministic demand models extends to the case of uncertain demand. More importantly, we identify another aspect of PMGs that is not present under deterministic demand. We show that verifying the availability before matching the price can increase the prices even beyond monopoly levels. In particular, when one of the retailers faces a stock-out, the competitor retailer can take advantage of the scarcity of the product in the marketplace by refusing to match the price and offering the higher list price. This means that *uncertainty in demand acts as a tool for price-discrimination even in a perfectly symmetric setting.* Price matching based on availability is a particular mechanism for retailers to exploit price discrimination opportunity induced by the uncertainty in demand. Note that in order to do so, price discriminating retailer also needs to order more than its competitor.

The effect of demand uncertainty depends on the PMG policy that the retailers employ. Under PM policy, an increase in demand uncertainty results in a decrease in prices and increase in quantities (as in the case of a monopoly price-setting newsvendor). If retailers verify the availability (PMA policy), then equilibrium prices, quantity and profit of the price discriminating retailer increases with uncertainty. Interestingly, the value of verifying the availability increase as the level of demand uncertainty in the market increases. Furthermore, we show that an increase in the correlation between the demands of the retailers also strengthens the incentive to verify the availability.

From a practical perspective, our results indicate to managers that verifying availability can indeed be a profit-enhancing mechanism. We find that this is most beneficial when the demand uncertainty is high, demand correlation is high, and the unit cost of the product is high. That is, retail managers should verify the availability (i.e., adopt PMA policy) for innovative products (especially high-end ones) or which are in the early stages of the life cycle. Highly anticipated products, where customers are eager to obtain the product and less sensitive to price changes in case of scarcity, are also cases where it makes more sense to verify the availability. Geographical proximity between the competing retailers can be a sign of high demand correlation which in turn makes price discrimination and verification of availability more valuable. On the other hand, a simple proof may be sufficient for functional and mature products with relatively stable demand (this avoids any cost of verifying the availability). Consequently, it is not at all surprising to see that most electronics retailers, dealing with innovative products, adopt PMA policy.

Some of our assumptions deserve further discussion. Our analysis identify the conditions under which retailers earn higher profits by verifying the availability. The benefits we report have to be carefully traded-off against the cost of verifying the availability, if it exists, in order to reach a better decision. We also assume that there are no hassle costs associated with price-matching requests. In other words, invoking a price-match request is costless to customers. This is a reasonable assumption, considering the vast information availability via website of the retailers. Furthermore, increasing service competition among retailers may force them to take actions in reducing the hassle cost related to PMGs. For example, Tweeter, a national sound systems retailer, offers an automatic price protection under which Tweeter itself monitors competitive prices and reimburses its customers who have purchased the product at a higher price than the competitors (Coughlan and Shaffer, 2003). As a concluding point, we remark that if hassle costs are to be incorporated, special attention has to be paid to the particular PMG policy in place, because customers are likely to incur different hassle costs depending on whether firms offer PM or PMA policy.

A A Price-Matching Guarantee Example

Price-Matching Guarantee by Sears available at http://www.sears.ca

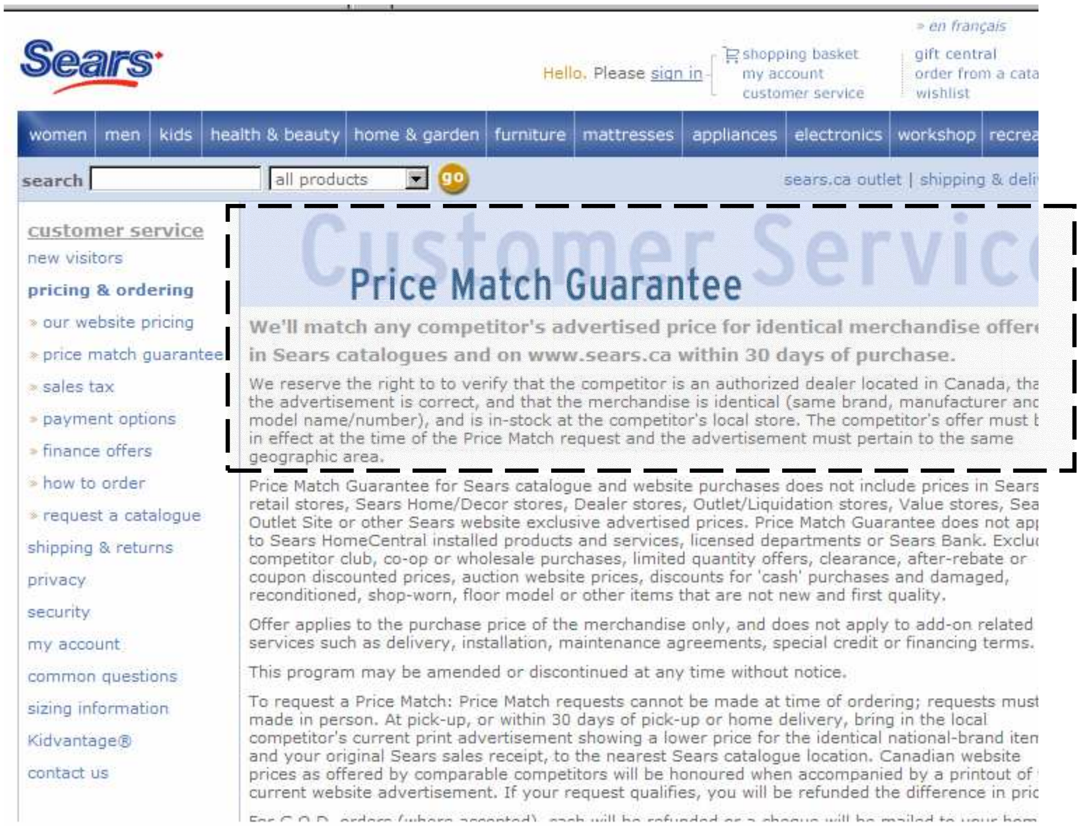


Figure 9: Price-Matching Guarantee offered by Sears, www.sears.ca.

B Proofs for Lemmas, Propositions and Theorems

For expositional clarity, denote the expected profit function of the monopolistic price-setting newsvendor as $\pi^N(p,q) = pE\big[\min\{q, D(p)\}\big] - cq$. The profit function is concave in q. Therefore, for a given price, the optimal order quantity satisfies $G_p(q) = (p-c)/p$ and is denoted by $q^F(p)$. The expected profit function evaluated at the optimal order quantity for a given price is denoted by $\pi^N(p) = pE\big[\min\{q^F(p), D(p)\}\big] - cq^F(p)$. It can be shown that the expected profit function $\pi^N(p)$ is unimodal in p under **A1**. We denote the optimal price decision by p^N and the optimal quantity decision by q^N where $q^N = q^F(p^N)$.

Proposition 1 Let us first derive the response strategy of $S1$. The expected profit function $\pi_1^{PM}(\mathbf{p}, \mathbf{q})$, is concave with respect to q_1 and the order quantity of the competitor has no effect on the profit of $S1$. Therefore the optimal order quantity is given by $q^F(p^e)$. Plugging in the optimal order quantity, the game can be reduced to a price game, $\pi_1^{PM}(p_1, p_2) = p^e E\big[\min\{q^F(p^e), D(p^e)\}\big] - cq^F(p^e)$. Note that $\pi_1^{PM}(p_1, p_2) = \pi^N(p^e)$. Therefore, for $p \leq p_2$ and $p_2 \in [c, p^N]$, we have $\pi_1^{PM}(p_2, p_2) = \pi^N(p_2) \geq \pi^N(p) = \pi_1^{PM}(p, p_2)$ since $\pi^N(p)$ is increasing for $p \leq p_2 \leq p^N$. On the other hand if $p_2 \geq p^N$ then $\pi_1^{PM}(p^N, p_2) = \pi^N(p^N) \geq \pi^N(p) = \pi_1^{PM}(p, p_2)$ for any p, due to the optimality of (p^N). Thus the response strategy if $S1$ is, $p_1(p_2) = \min\{p_2, p^N\}$. It can be shown that the response strategy of $S2$ is identical, i.e., $p_2(p_1) = \min\{p_1, p^N\}$. Therefore any point (p, p) where $p \in [c, p^N]$ is an equilibrium point. In order to show the pareto dominance consider $\pi_1^{PM}(p^N, p^N) = \pi^N(p^N) \geq \pi^N(p) = \pi_1^{PM}(p, p)$ for any $p \leq p^N$. □

Proposition 2 Note that in regions $R1$ and $R4$ (i.e., when $p_1 \leq p$) the expected profit of $S1$ is independent of any decision of $S2$. In other words $S1$ solves the price-setting newsvendor problem, $\max_{p_1, q_1} \pi^N(p_1, q_1)$. The expected profit function is unimodal in p_1, maximized at p^N, hence increasing for $p_1 \leq p^N$. Thus, if the strategy of $S2$ satisfies $p \leq p^N$, then the best response (to p in $R1$ and $R4$) is to set $p_1 = p$. On the other hand when $p^N \leq p$ then the best response for $S1$ (to p in $R1$ and $R4$) is to set $p_1 = p^N$. □

Corollary 1 Note that in $R2$ and $R3$ (i.e., when $p_2 \leq p_1$), the expected profit function of $S2$ is independent of the strategy of $S1$ and identical to $\pi^N(p_2, q_2)$. Therefore, $\pi^N(p_2, q^F(p_2)) > \pi^N(p_2, q_2)$, that is, the optimal order quantity $q^F(p_2)$ dominates all other possible order quantity decisions. Furthermore, strategy $\{p^N, q^N\}$ dominates any other strategy with a higher price. □

Lemma 1 In $R2$, the profit of $S1$ is equal to the fixed-price newsvendor problem with price p, i.e., $\pi_1^{R2}(p_1, p, q_1, q^F(p)) = \pi^N(p, q)$. Therefore, the optimal quantity decision at this price is $q^F(p)$. □

Proposition 3 We first derive the response quantity in $R3$, ($p_1 > p$ and $q_1 > q$). The *profit* for $S1$ is,

$$
\begin{aligned}
&= p^e \min\Big\{D(p^e), q\Big\} + p^e\Big[\min\{q_1, D(p^e)\} - q\Big]^+ 1_{[D \leq q]} \\
&\qquad\qquad + p_1 \min\Big\{q_1 - q, [D(p^e) - q - r(p_1|p^e)]^+\Big\} 1_{[D > q]} - cq_1 \\
&= p \min\Big\{D(p), q\Big\} + p\Big[\min\{q_1, D(p)\} - q\Big]^+ 1_{[D(p) \leq q]} \\
&\qquad\qquad + p_1 \min\Big\{q_1 - q, [D(p) - q - r(p_1|p)]^+\Big\} 1_{[D(p) > q]} - cq_1 \\
&= \Pi_1^{R3}(p, p, q, q) + p\Big[\min\{q_1, D(p)\} - q\Big]^+ 1_{[D(p) \leq q]} \\
&\qquad\qquad + p_1 \min\Big\{q_1 - q, [D(p) - q - r(p_1|p)]^+\Big\} 1_{[D(p) > q]} - c(q_1 - q)
\end{aligned}
$$

The expected profit in this range is;

$$\pi_1^{R3}(p_1,p,q_1,q) = pG_p(q)\int_q^{q_1}[x-q]dG_p(x) + \overline{G}_p(q)\int_{q+r(p_1|p)}^{q_1+r(p_1|p)} p_1[x-q-r(p_1|p)]dG_p(x)$$
$$+\pi_1^{R3}(p,p,q,q) + [q_1-q]\Big[pG_p(q)\overline{G}_p(q_1) + p_1\overline{G}_p(q)\overline{G}_p\Big(q_1+r(p_1|p)\Big) - c\Big]$$

$\pi_1^{R3}(p_1,p,q_1,q)$ is concave in quantity decision, q_1 for given p_1, p and q. First order condition with respect to the order quantity q_1 provides us ;

$$\frac{\partial \pi_1^{R3}(p_1,p,q_1,q)}{\partial q_1} = pG_p\big(q\big)\overline{G}_p\big(q_1\big) + p_1\overline{G}_p\big(q\big)\overline{G}_p\big(q_1+r(p_1|p)\big) - c = 0 \tag{9}$$

Let us analyze the conditions such that the order quantity $q_1(p_1|p)$ satisfying the first order condition (9) also satisfies $q_1(p_1|p) > q^F(p)$ (that is, also in $R3$). The expected profit function is concave in q_1 for given p_1, p, q. If, for a given $p_1 > p$, the first derivative w.r.t. q_1 is positive at q then, the optimal quantity is higher than q. According to this, we need $\left.\frac{\partial \pi_1^{R3}(p_1,p,q_1,q^F(p))}{\partial q_1}\right|_{q_1=q^F(p)} > 0$ which can be rewritten as (recall that $p\overline{G}_p\big(q^F(p)\big) = c$); $p_1\overline{G}_p\big(q^F(p)+r(p_1|p)\big) > c$. In other words the optimal order quantity is in $R3$ is given by

$$= \begin{cases} q_1(p_1|p) & \text{if } p_1\overline{G}_p\big(q^F(p)+r(p_1|p)\big) > c \\ q^F(p) & o/w \end{cases}$$

Next we will show that the order quantity satisfying (9) is unimodal in p_1. Differentiating both sides of (9) (or by implicit differentiation) we obtain

$$q_1'(p_1|p) = \frac{\overline{G}_p(q^F(p))g_p\big(q_1(p_1|p)+r(p_1|p)\big)\Big[h_p\big(q_1(p_1|p)+r(p_1|p)\big) - r'(p_1|p)p_1\Big]}{pG_p\big(q^F(p)\big)g_p\big(q_1(p_1|p)\big) + p_1\overline{G}_p\big(q^F(p)\big)g_p\big(Q_1(p_1|p)+r(p_1|p)\big)}.$$

Thus $q_1'(p_1|p) = 0$ if and only if $h_p\big(q_1(p_1)+r(p_1|p)\big) = r'(p_1|p)p_1$. The sign of the second derivative at this extreme point is negative implying the unimodality,

$$\left. q_1''(p_1|p)\right|_{q_1'=0}$$
$$= \frac{\overline{G}_p(q^F(p))g_p\big(q_1(p_1|p)+r(p_1|p)\big)\Big[r'(p_1|p)h_p'\big(q_1(p_1|p)+r(p_1|p)\big) - \big[r'(p_1|p)+p_1r''(p_1|p)\big]\Big]}{pG_p\big(q^F(p)\big)g_p\big(q_1(p_1|p)\big) + p_1\overline{G}_p\big(q^F(p)\big)g_p\big(q_1(p_1|p)+r(p_1|p)\big)} < 0$$

The last inequality is due to the fact that, $r(p_1|p)$ is increasing in p_1, $h_p(x)$ is decreasing in x and since $r(p_1|p)$ is convex-increasing in p_1. □

Proposition 4 If the best order quantity of $S1$ (in $R3$) is $q^F(p)$, then we can see from the expected profit function that the retailer has the same profit for any $p_1 > p$, therefore, the best response price is p. We concentrate on the remaining case, the case where $q_1(p_1|p, q^F(p)) > q^F(p)$. That is, when $p_1\overline{G}_p(q^F(p) + r(p_1|p)) > c$. Lemma 2 (presented below) summarizes the conditions for that to happen. We present Lemma 2 and its proof after this proposition. According to Lemma 2, we have $q_1(p_1|p) > q^F(p)$ if and only if $h_p\big(q^F(p)\big) \geq p.r'(p|p)$ and $p_1 \in [p, p^m(p)]$ where $p^m(p)$ is the largest root of $p_1\overline{G}_p\big(q^F(p) + r(p_1|p)\big) = c$. We investigate the profit function under these two conditions. First we show that the profit function is unimodal in price. The first and second order derivatives are (by plugging in the value of $q_1'(p_1|p)$ and utilizing integration by parts),

$$\frac{\partial \pi^{R3}(p_1, p, q_1(p_1|p), q^F(p))}{\partial p_1} = \overline{G}_p(q^F(p)) \int\limits_{q^F(p)+r(p_1|p)}^{q_1(p_1|p)+r(p_1|p)} \overline{G}_p(x) - p_1 r'(p_1|p) g_p(x) dx$$

$$\begin{aligned}\frac{\partial^2 \pi^{R3}(p_1, p, q_1(p_1|p), q^F(p))}{\partial {p_1}^2} = &\Big\{ r'(p_1|p) g_p\big(q^F(p) + r(p_1|p)\big)\Big[p_1 r'(p_1|p) - h_p\big(q^F(p) \\ &+ r(p_1|p)\big)\Big] + \Big[q_1'(p_1|p) + r'(p_1|p)\Big] g_p\big(q_1(p_1|p) + r(p_1|p)\big) \\ &\Big[h_p\big(q_1(p_1|p) + r(p_1|p)\big) - p_1 r'(p_1|p)\Big] \\ &- r'(p_1|p)\Big[G_p\big(q_1(p_1|p) + r(p_1|p)\big) \\ &- G_p\big(q^F(p_2) + r(p_1|p)\big)\Big]\Big\}\overline{G}_p\Big(q^F(p)\Big)\end{aligned}$$

We can show that $q_1'(p_1|p) + r'(p_1|p) > 0$, therefore $\frac{\partial^2 \pi^{R3}(p_1,p,q_1(p_1|p),q^F(p))}{\partial {p_1}^2} < 0$ for p_1 satisfying

$h_p\Big(q_1(p_1|p) + r(p_1|p)\Big) \leq p_1.r'(p_1|p) \leq h_p\Big(q^F(p) + r(p_1|p)\Big)$. Note that first order condition can be rewritten as $p_1.r'(p_1|p) = \frac{\int_{q^F(p)+r(p_1|p)}^{q_1(p_1|p)+r(p_1|p)} \overline{G}_p(x)dx}{G_p\Big(q_1(p_1|p)+r(p_1|p)\Big) - G_p\Big(q^F(p)+r(p_1|p)\Big)}$ and due to the IFR property of $g(\cdot)$ we have[14]

[14] If the density $f(\cdot)$ has an increasing failure rate then for $A \leq x \leq B$ we have $\frac{f(A)}{\overline{F}(A)} \leq \frac{f(x)}{\overline{F}(x)} \leq \frac{f(B)}{\overline{F}(B)}$. We can easily show that $\frac{\overline{F}(A)}{f(A)} \geq \frac{\overline{F}(x)}{f(x)} \geq \frac{\overline{F}(B)}{f(B)} \Rightarrow \frac{\overline{F}(A)f(x)}{f(A)} \geq \overline{F}(x) \geq \frac{\overline{F}(B)f(x)}{f(B)} \Rightarrow \frac{\overline{F}(A)\big[F(B)-F(A)\big]}{f(A)} \geq \int_A^B \overline{F}(x)dx \geq \frac{\overline{F}(B)\big[F(B)-F(A)\big]}{f(B)} \Rightarrow \frac{\overline{F}(A)}{f(A)} \geq \frac{\int_A^B \overline{F}(x)dx}{F(B)-F(A)} \geq \frac{\overline{F}(B)}{f(B)}$

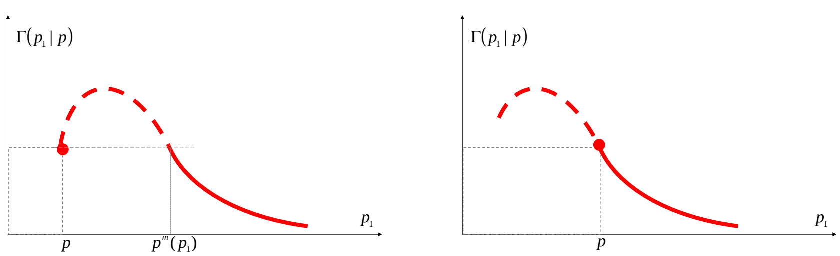
$h_p\Big(q_1(p_1|p)+r(p_1|p)\Big) \leq \frac{\int_{q^F(p)+r(p_1|p)}^{q_1(p_1|p)+r(p_1|p)} \overline{G}_p(x)dx}{G_p\Big(q_1(p_1|p)+r(p_1|p)\Big)-G_p\Big(q^F(p)+r(p_1|p)\Big)} \leq h_p\Big(q^F(p)+r(p_1|p)\Big)$, implying the unimodality. □

Lemma 2 *$q_1(p_1|p) > q^F(p)$ if and only if $h_p\Big(q^F(p)\Big) \geq r'(p|p)p$ and $p_1 \in [p, p^m(p)]$ where $p^m(p)$ is the largest root of $p_1\overline{G}_p\Big(q^F(p)+r(p_1|p)\Big) = c$.*

Lemma 2 Proof Recall that $q_1(p_1|p) > q^F(p)$ for some $p_1 \geq p$ if and only if $p_1\overline{G}_p\Big(q^F(p)+r(p_1|p)\Big) \geq c$. Let $\Gamma(p_1|p) \doteq p_1\overline{G}_p\Big(\Phi(p_1)\Big) = p_1\overline{G}_p\Big(q^F(p)+r(p_1|p)\Big)$. We first identify the conditions under which $\Gamma(p_1|p) \geq c$. We show that $\Gamma(p_1|p)$ is a unimodal function provided that $G_p(\cdot)$ has increasing failure rate.

$\Gamma(p_1|p) = p_1\overline{G}_p(\Phi(p_1))$. $\Gamma'(p_1|p) = \overline{G}_p\big(\Phi(p_1)\big) - p_1\Phi'(p_1)g_p\big(\Phi(p_1))\big)$. $\Gamma'(p_1|p) = 0 \Leftrightarrow h_p\big(\Phi(p_1)\big) = \Phi'(p_1)p_1$.

$\Gamma''(p_1|p) = -2\Phi'(p_1)g_p\big(\Phi(p_1)\big) - p_1\Phi''(p_1)g_p\big(\Phi(p_1)\big) - p_1\big[\Phi'(p_1)\big]^2 g_p'\big(\Phi(p_1)\big)$. By the first order condition we obtain, $\Gamma''(p_1|p)\Big|_{\Gamma'(p_1)=0} = -\big[\Phi'(p_1)+p_1\Phi''(p_1)\big]g_p\big(\Phi(p_1)\big) - \Phi'(p_1)\big[1-h_p'\big(\Phi(p_1)\big)\big] < 0$, since $h_p(x)$ is decreasing in x and we have $\Phi'(p_1)+p_1\Phi''(p_1) = r'(p_1|p) + p_1 r''(p_1|p) > 0$ by assumption. That is, $\Gamma(\cdot)$ is unimodal. Let $p^\Gamma(p)$ denote the maxima satisfying the first order condition given by $h_p\big(\Phi(p_1)\big) = \Phi'(p_1)$. We can show that $\Gamma(0|p) = 0$, $\lim_{p_1\to\infty}\Gamma(p_1|p) = 0$ and, $\Gamma(p|p) = p\overline{G}_p(q^F(p)) = c$. Therefore we are guaranteed that $\Gamma(p_1|p) = c$ has two roots. In other words, if $\Gamma'(p|p) > 0$, that is, if $h_p\big(q^F(p)\big) \geq r'(p|p)p$ holds, then there exists a unique $p^m(p, q^F(p)) > p$ such that $\Gamma(p_1|p, q^F(p)) \geq c$ for $p_1 \in [p, p^m(p, q^F(p)]$ (Figure 10). On the other hand, if this condition does not hold, then $\Gamma(p_1|p, q^F(p)) < c$ for $p_1 > p$. □



(a) $h_p(q^F(p)) > r'(p_1|p)p$ (b) $h_p(q^F(p)) < r'(p_1|p)p$

Figure 10: Possible cases

Theorem 1 Let $\Omega(p) \doteq h_p(q^F(p)) - p.r'(p|p)$. Recall that $h_p(q^F(p))$ is decreasing in p due to the IFR property and the fact that $q^F(p)$ is increasing. In addition, $r(p|p)$ is an increasing-convex function thus $p.r'(p|p)$ is an increasing function. Thus there is a unique price decision (if it exists), say $p^\Omega = \{p|\Omega(p) = 0\}$. Moreover, $p < p^\Omega \Leftrightarrow \Omega(p) > 0$. It follows that if $\Omega(p^N) > 0 \Rightarrow \Omega(p) > \Omega(p^N) > 0$ for $p < p^N$. By Proposition 4, if $\Omega(p^N) > 0$ for any $p \in [c, p^N)$ the response of $S1$ will satisfy $\hat{p_1}(p) > p$. On the other hand for any $p \in [p^N, \infty)$ the response of $S1$ is to set its price equal to the optimal monopoly price, p^N. The only equilibrium in $R2 + R3$ is at point $(p^{PMA}, p^N, q^{PMA}, q^N)$. By symmetry, $(p^N, p^{PMA}, q^N, q^{PMA})$ is also an equilibrium point.

On the other hand if $\Omega(p^N) < 0$, then for $p \in [p^\Omega, p^N]$ we have $\Omega(p) < 0$ and based on Proposition 4 we will have $\hat{p_1}(p) = p$ for $p \in [p^\Omega, p^N]$, and there are a continuum of equilibrium points (p, p) for $p \in [p^\Omega, p^N]$. When both retailers charge equal prices the expected profit of each retailer is identical to that of the monopolistic price-setting newsvendor. Therefore, both retailers will be better of when the set their price equal to p^N, i.e., (p^N, p^N) is the pareto dominant one among the continuum of equilibrium solutions (p, p) for $p \in [p^\Omega, p^N]$. □

Proposition 5 Recall that $\Phi(p) = z'(p) - r'(p)$, and that $F(\cdot)$ and $\overline{F(\cdot)}$ are the distribution and complementary distribution function for the random variable respectively. Under linear demand assumptions we have $\Phi(p) = z'(p) - \gamma = \frac{\overline{F}(z(p))}{p.f(z(p))} - \gamma$. Therefore, $\Phi'(p) = z''(p) = \frac{\partial}{\partial w}[\frac{\overline{F}(w)}{f(w)}]\Big|_{w=z(p)} z'(p)\frac{1}{p} - \frac{\overline{F}(z(p))}{f(z(p))}\frac{-1}{p^2} < 0$ due to the IFR property of the random variable. That is, $\Phi(p)$ is a monotone decreasing function of p. On the other hand, $\frac{\partial \Phi(p,\alpha)}{\partial \alpha} = 0$, $\frac{\partial \Phi(p,\beta)}{\partial \beta} = 0$, $\frac{\partial \Phi(p,\gamma)}{\partial \gamma} < 0$. The price of the monopolistic price-setting newsvendor p^N satisfies ${\pi^N}'(p) = \alpha + \beta.c - 2\beta p + \Theta\big(z(p)\big) = 0$ where $\Theta(x) = \int\limits_0^x \overline{F}(y)dy$. Further, the profit function is unimodal stating that ${\pi^N}''(p^N) < 0$. Let us consider the effect of market characteristics on the monopoly price

- $$\frac{\partial p^N}{\partial \alpha} = -\frac{\frac{\partial^2 \pi^N(p)}{\partial p \partial \alpha}}{{\pi^N}''(p)}\Big|_{p=p^N} = \frac{-1}{{\pi^N}''(p^N)} > 0.$$
$$\text{Thus } \frac{\partial \Phi(p^N, \alpha)}{\partial \alpha} = \frac{\partial \Phi(p)}{\partial \alpha} + \Phi'(p^N)\frac{\partial p^N(\alpha)}{\partial \alpha} = \Phi'(p^N)\frac{\partial p^N(\alpha)}{\partial \alpha} < 0.$$

- $$\frac{\partial p^N}{\partial \beta} = -\frac{\frac{\partial^2 \pi^N(p)}{\partial p \partial \beta}}{{\pi^N}''(p)}\Big|_{p=p^N} = \frac{2p^N - c}{{\pi^N}''(p^N)} < 0.$$
$$\text{Thus } \frac{\partial \Phi(p^N, \beta)}{\partial \beta} = \frac{\partial \Phi(p)}{\partial \beta} + \Phi'(p^N)\frac{\partial p^N(\beta)}{\partial \beta} = \Phi'(p^N)\frac{\partial p^N(\beta)}{\partial \beta} > 0.$$

- $\dfrac{\partial p^N}{\partial \gamma} = -\dfrac{\frac{\partial^2 \pi^N(p)}{\partial p \partial \gamma}}{\pi^{N''}(p)}\Big|_{p=p^N} = 0.$

$$\text{Thus } \frac{\partial \Phi(p^N,\gamma)}{\partial \gamma} = \frac{\partial \Phi(p)}{\partial \gamma} + \Phi'(p^N)\frac{\partial p^N(\gamma)}{\partial \beta} = \frac{\partial \Phi(p)}{\partial \gamma} < 0.$$

That is an increase in α and γ results in a decrease in the value of $\Phi(p^N)$ whereas an increase in β result in an increase in $\Phi(p^N)$. □

Proposition 6 Note that we provide the proof for the general case with N asymmetric players. In this case the expected profit for retailer i is,

$$\pi_i(\mathbf{p},\mathbf{q}) = p_i E\Big[\min\{q_i, D_i(\mathbf{p})\}\Big] - c_i q_i$$

where $D_i(\mathbf{p}) = d_i(\mathbf{p}) + \epsilon_i$. The density and the distribution of ϵ_i is denoted by $f_i(\cdot)$ and $F_i(\cdot)$ respectively. Since the expected profit of one retailer is not effected by the quantity decision of the other retailers, we can reduce the original game into a game where retailers compete on price alone.

$$\pi_i(\mathbf{p}) = p_i E\Big[\min\{q_i(\mathbf{p}), D_i(\mathbf{p})\}\Big] - c_i q_i(\mathbf{p}).$$

Let $z_i(\mathbf{p}) = q_i(\mathbf{p}) - d_i(\mathbf{p})$ be the stocking factor. The optimal stocking factor satisfies the first order condition, $\overline{F_i}(z_i(\mathbf{p})) = c_i/p_i$. Note that by implicit differentiation we obtain $\frac{\partial z_i(\mathbf{p})}{\partial p_i} = \frac{\overline{F}_i(z_i(\mathbf{p}))}{p_i f_i(z_i(\mathbf{p}))}$. We plug in the optimal stocking factor into the profit function.

$$\begin{aligned}\pi_i(\mathbf{p}) = (p_i - c_i)d_i(\mathbf{p}) + p_i E\big[\min\{z_i(\mathbf{p}), \epsilon_i\}\big] - c_i z_i(\mathbf{p}) \\ = (p_i - c_i)[d_i(\mathbf{p}) + z_i(\mathbf{p})] - p_i \Lambda_i\big(z_i(\mathbf{p})\big)\end{aligned}$$

where $\Lambda_i\big(z\big) = \int\limits_0^z F_i(x)dx$ is the expected number of leftovers at the end of the period.

We will make two observations. Denote $p_i^* = \{p_i | \frac{\partial \pi_i(\mathbf{p})}{\partial p_i} = 0\}$.

Observation 1 $\frac{\partial q_i(\mathbf{p}^*)}{\partial p_i} \leq 0$. Due to the IFR property, for $x < z$ we have $\frac{f(z)}{\overline{F}(z)} \geq \frac{f(x)}{\overline{F}(x)} \Rightarrow \overline{F}(x) \geq \frac{f(x)\overline{F}(z)}{f(z)} \Rightarrow \int\limits_0^z \overline{F}(x)dx \geq \int\limits_0^z \frac{f(x)\overline{F}(z)}{f(z)}dx$. Denoting $\Theta_i(z) = z - \Lambda_i(z)$ we have $\Theta_i\big(z_i(\mathbf{p})\big) = \int\limits_0^{z_i(\mathbf{p})} \overline{F_i}(x)dx \geq \int\limits_0^{z_i(\mathbf{p})} \frac{f_i(x)\overline{F_i}(z_i(\mathbf{p}))}{f_i(z_i(\mathbf{p}))}dx = \frac{F_i(z_i(\mathbf{p}))\overline{F_i}(z_i(\mathbf{p}))}{f_i(z_i(\mathbf{p}))} = p_i F_i\big(z_i(\mathbf{p})\big)\frac{\partial q_i(\mathbf{p})}{\partial p_i} = (p_i - c_i)\frac{\partial q_i(\mathbf{p}^*)}{\partial p_i}$. It follows from here that,

$$\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} = d(\mathbf{p}) + (p_i - c_i)\frac{\partial d(\mathbf{p})}{\partial p_i} + z_i(\mathbf{p}) - \Lambda_i(z_i(\mathbf{p})) > d_i(p_i) + (p_i - c_i)\frac{\partial q_i(\mathbf{p})}{\partial p_i} \Rightarrow \frac{\partial q_i(\mathbf{p}^*)}{\partial p_i} \leq 0$$

Observation 2 $F_i\big(z_i(\mathbf{p}^*)\big)\eta_i\big((\mathbf{p}^*)\big) \leq 1$. Denote $\Psi_i(z) = \int\limits_z^\infty \overline{F_i}(x)dx$ and note that $z_i(\mathbf{p}) - \Lambda(z_i(\mathbf{p})) = \mu_i - \Psi_i(z(\mathbf{p})) < \mu_i$. Thus, $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} < d_i(\mathbf{p}) + (p_i - c_i)\frac{\partial d_i(\mathbf{p})}{\partial p_i} + \mu_i = \big[d_i(\mathbf{p}) + \mu_i\big]\big[1 - F_i\big(z_i(\mathbf{p})\big)\eta_i(\mathbf{p})\big] \Rightarrow F_i\big(z_i(\mathbf{p}^*)\big)\eta_i(\mathbf{p}^*) \leq 1$.

Existence of the equilibrium follows from the supermodularity of the game by (Topkis, 1979). In order to show that we have a supermodular game we need, $\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j} \geq 0$.

$$\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j} = (p_i - c_i)\frac{\partial^2 d_i(\mathbf{p})}{\partial p_i \partial p_j} + \frac{\partial d_i(\mathbf{p})}{\partial p_j} = F_i(z_i(p_i))p_i\Big[\frac{\partial^2 d_i(\mathbf{p})}{\partial p_i \partial p_j} + \frac{1}{F_i(z_i(p_i))p_i}\frac{\partial d_i(\mathbf{p})}{\partial p_j}\Big]$$
$$\geq F_i(z_i(p_i))p_i\Big[\frac{\partial^2 d_i(\mathbf{p})}{\partial p_i \partial p_j} + \frac{\eta_i(\mathbf{p})}{p_i}\frac{\partial d_i(\mathbf{p})}{\partial p_j}\Big] = -F_i(z_i(p_i))\big[d_i(\mathbf{p}) + \mu_i\big]\Big[\frac{\eta_i(\mathbf{p})}{\partial p_j}\Big] \geq 0$$

For the uniqueness, following the index theory approach (Vives, 1999), we require,

$$-\left[\frac{\partial^2 \pi_i(\mathbf{p})}{\partial {p_i}^2} + \sum_{j \neq i}\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j}\right]\Bigg|_{\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}=0} > 0 \tag{10}$$

Note that we have

$$\begin{aligned}
\frac{\partial^2 \pi_i(\mathbf{p})}{\partial {p_i}^2} + \sum_{j \neq i}\frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i \partial p_j} = 2\frac{\partial d_i(\mathbf{p})}{\partial p_i} &+ (p_i - c_i)\frac{\partial^2 d_i(\mathbf{p})}{\partial {p_i}^2} + z_i'(p_i)\overline{F_i}(z_i(p_i)) \\
&+ \sum_{j \neq i}\frac{\partial d_i(\mathbf{p})}{\partial p_j} + (p_i - c_i)\sum_{j \neq i}\frac{\partial d_i(\mathbf{p})}{\partial p_i \partial p_j} \\
= -F_i(z_i(p_i))[d_i(\mathbf{p}) + \mu_i]&\left[\frac{\partial \eta_i(\mathbf{p})}{\partial p_i} + \sum_{j \neq i}\frac{\partial \eta_i(\mathbf{p})}{\partial p_j}\right] \\
&+ \Big[\frac{\partial d_i(\mathbf{p})}{\partial p_i} + \sum_{j \neq i}\frac{\partial d_i(\mathbf{p})}{\partial p_j}\Big]\Big[1 - F_i(z_i(p_i))\eta_i(\mathbf{p})\Big] \\
&+ \overline{F_i}(z_i(p_i))\Big[\frac{\partial q_i(p_i|p_{-i})}{\partial p_i}\Big]
\end{aligned}$$

At equilibrium, the first part of the right hand side is negative by Assumption 2 in § 5. Furthermore we know that $\Big[1 - F_i(z_i(\mathbf{p}^*))\eta_i(\mathbf{p}^*)\Big] > 0$ at equilibrium. The second part is negative by assumption in § 5 and Observation 2. The last part is negative at equilibrium by Observation 1. Thus inequality in (10) is satisfied proving the uniqueness. □

Proposition 7

Stochastic: Monopoly v.s. Competition Note that due to substitution effect the competitive model will be more sensitive to price changes, that is we have $\frac{\partial d_i(p_i|p_j)}{\partial p_i} \leq <$

$d'(p_i)$. On the other hand we have a symmetric solution in the competition case, therefore, $d(p^C) = d_1(p^C|p^C) = d_2(p^C|p^C)$. Recall that monopolistic newsvendor decides the price based on the first order condition $\pi'(p) = (p-c)d'(p)+d(p)+\Theta\big(z(p)\big) = 0$ where $\overline{F}(z(p)) = c/p$ and $\Theta\big(z\big) = \int_0^z \overline{F}(x)dx$. Furthermore the solution of the competitive scenario satisfies $\frac{\partial \pi_1^C(p_i|p_j)}{\partial p_i} = \frac{\partial d_i(p_i|p_j)}{\partial p_i} + d_i(p_i|p_j) + \Theta\big(z_i(p_i)\big) = 0$ In order to compare the prices we evaluate the first order derivative of the monopolist at the equilibrium price of the competitive case. That is we will evaluate

$$
\begin{aligned}
\pi'(p^C) &= (p^C-c)d'(p^C) + d(p^C) + \Theta\big(z(p^C)\big) \\
&= (p^C-c)\frac{\partial d_1(p_1|p^C)}{\partial p_1}\Big|_{p_1=p^C} + d_1(p^C|p^C) + \Theta\big(z(p^C)\big) \\
&\qquad + (p^C-c)\left[d'(p^C) - \frac{\partial d_1(p_1|p^C)}{\partial p_1}\Big|_{p_1=p^C}\right] + [d(p^C) - d_1(p^C|p^C)] \\
&= \frac{\partial d(p_1|p^C)}{\partial p_1}\Big|_{p_1|p^C} + (p^C-c)\left[d'(p^C) - \frac{\partial d_1(p_1|p^C)}{\partial p_1}\Big|_{p_1=p^C}\right] \\
&= (p^C-c)\left[d'(p^C) - \frac{\partial d_1(p_1|p^C)}{\partial p_1}\Big|_{p_1=p^C}\right] > 0
\end{aligned}
$$

Thus $\pi'(p^C) > 0$, recall that $\pi'(\cdot)$ changes sign only once and the sign change is from positive to negative, therefore we can conclude that $p^N > p^C$.

Stochastic vs. Deterministic: Monopoly/ Competition We know from Petruzzi and Dada (1999) that the optimal price decision of the riskless retailer is higher than the optimal price of the newsvendor retailer in a monopolistic setting. It can be easily shown that (based on the first order conditions) the same holds for the riskless and newsvendor competition models. Thus $p^{DM} < p^N$ and $p^{DB} < p^C$.

Price-Matching, Newsvendor vs. Riskless We know from the basic literature on price-matching that in a deterministic game with only informed customers, both retailers will set their price equal to the deterministic monopolistic price and earn the profits of the monopolist. In the stochastic framework, we have two equilibrium solutions. These two solutions are mirror image of each other. In the equilibrium one retailer sets its price and quantity equal to that of a pricing newsvendor and the other retailer sets a higher price and a higher order quantity level. If $r(p_1|p_2)$ is constant and sufficiently small then there exists a constant number of customers who is willing to purchase the product what ever the list price of the product is. In this case the list price for $S1$ approaches to infinity. The list price offered by $S1$ is higher than the deterministic monopoly price. □

Proposition 8:

Let

$$\begin{aligned}\Upsilon^I(q_1) = \frac{\partial \pi_1^I(p_1, p, q_1, q^F(p))}{\partial q_1} &= pG_p\Big(q^F(p)\Big)\overline{G}_p\Big(q_1\Big) \\ &\quad + p_1\overline{G}_p\Big(q^F(p)\Big)\overline{G}_p\Big(q_1 + r(p_1|p)\Big) - c, \\ \Upsilon^{PC}(q_1) = \frac{\partial \pi_1^{PC}(p_1, p, q_1, q^F(p))}{\partial q_1} &= p_1\overline{G}_p\Big(q_1 + r(p_1|p)\Big) - c,\end{aligned}$$

where the superscripts I and PC denote the independent and perfect correlation cases, respectively. It can be shown that $\Upsilon^{PC}(q_1) > \Upsilon^I(q_1)$. That is, the marginal profit of acquiring an additional unit increases as a result of the perfect correlation. Since $\frac{\partial \pi_1^I(p_1,p,q_1(p_1|p),q^F(p))}{\partial p_1} = \frac{\partial \pi_1^{PC}(p_1,p,q_1(p_1|p),q^F(p))}{\partial q_1}$ we have, $\frac{\partial \pi_1^I(p_1,p,q_1^I(p_1|p),q^F(p))}{\partial p_1} = \frac{\partial \pi_1^{PC}(p_1,p,q_1^{PC}(p_1|p),q^F(p))}{\partial q_1}$ which proves that the optimal price under PC case is less than the independent case. This further strengthens the increase in the optimal order quantity. □

References

Baye, M., D. Kovenock. 1994. How to Sell a Pickup Truck:Beat-or-Pay. *Advertisements as Facilitating Devices, International Journal of Industrial Organization* **12** 21–33.

Belton, T.M. 1987. A model of duopoly and meeting or beating competition. *International Journal of Industrial Organization* **5** 399–417.

Bernstein, F., A. Federgruen. 2003. Pricing and Replenishment Strategies in a Distribution System with Competing Retailers. *Operations Research* **51**(3) 409–426.

Bernstein, F., A. Federgruen. 2005. Decentralized Supply Chains with Competing Retailers Under Demand Uncertainty. *Management Science* **51**(1) 18–29.

Blodgett, J.G., D. Hill, G. Stone. 1995. A Model of the Determinants of Retail Search. *Advances In Consuer Research* **22** 518–518.

Chen, F.Y., H.L. Yao. 2004. A newsvendor pricing game. *Systems, Man and Cybernetics, Part A, IEEE Transactions on* **34**(4) 450–456.

Corts, K.S. 1995. On the robustness of the argument that price-matching is anti-competitive. *Economics Letters* **47**(3) 417–421.

Corts, K.S. 1996. On the competitive effects of price-matching policies. *International Journal of Industrial Organization* **15**(3) 283–299.

Coughlan, A.T., G.E. Shaffer. 2003. Price-Matching Guarantees, Retail Competition, and Product-Line Assortment. Tech. rep., Simon Business School Working Paper No. FR 03-29.

Doyle, C. 1988. Different Selling Strategies in Bertrand Oligopoly. *Economics Letters* **28**(4) 387–390.

Fisher, M.L. 1997. What is the right supply chain for your product. *Harvard Business Review* **75**(2) 105–116.

Harsanyi, J.C., R. Selten. 1988. *A general theory of equilibrium selection in games*. MIT Press.

Hay, G.A., Center for the Study of American Political Economy, Cornell University. 1980. *Oligopoly, Shared Monopoly and Antitrust Law*. Center for the Study of American Political Economy, Cornell University.

Jain, S., J. Srivastava. 2000. An Experimental and Theoretical Analysis of Price-Matching Refund Policies. *Journal of Marketing Research* **37**(3) 351–362.

Karlin, S., C.R. Carr. 1962. Prices and optimal inventory policy. *Studies in Applied Probability and Management Science* 159–172.

Krishnan, H., R. Winter. 2007. Vertical Control of Price and Inventory, Forthcoming. *American Economic Review* .

Li, Q., D. Atkins. 2005. On the effect of demand randomness on a price/quantity setting firm. *IIE Transactions* **37**(12) 1143–1153.

Lippman, S.A., K.F. McCardle. 1997. The Competitive Newsboy. *Operations Research* **45**(1) 54–65.

Logan, J., R. Lutter. 1989. Guaranteed Lowest Prices: Do They Facilitate Collusion. *Economics Letters* **31**(2) 189–92.

Mills, E.S. 1959. Uncertainty and Price Theory. *The Quarterly Journal of Economics* **73**(1) 116–130.

Mishra, B.K., S. Raghunathan. 2004. Retailer- vs. Vendor-Managed Inventory and Brand Competition. *Management Science* **50**(4) 445–457.

Moorthy, S., R. Winter. 2006. Price-Matching Guarantees. *RAND Journal of Economics* **37**(2) 449–465.

Moorthy, S., X. Zhang. 2006. Price Matching by Vertically Differentiated Retailers: Theory and Evidence. *Journal of Marketing Research* **43** 156–167.

Netessine, S., N. Rudi. 2003. Centralized and Competitive Inventory Models with Demand Substitution. *Operations Research* **51**(2) 329.

Parlar, M. 1988. Game theoretic analysis of the substituable product inventory problem with random demands. *Naval research logistics* **35**(3) 397–409.

Parlar, M., Z.K. Weng. 2006. Coordinating pricing and production decisions in the presence of price competition. *European journal of operational research* **170**(1) 228–240.

Petruzzi, N.C., M. Dada. 1999. Pricing and the Newsvendor Problem: A Review with Extensions. *Operations Research* **47**(2) 183–194.

Png, I., D. Hirshleifer. 1987. Price Discrimination Through Offers to Match Price. *The Journal of Business* **60**(3) 365–383.

Salop, S. 1986. Practices that (Credibly) Facilitate Oligopoly Coordination. J. Stiglitz and F. Mathewson, eds., New Developments in the Analysis of Market Structure.

Schelling, T.C. 1980. *The Strategy of Conflict*. Cambridge, MA.

Tirole, J. 1988. *The Theory of Industrial Organization*. MIT Press.

Topkis, D.M. 1979. Equilibrium Points in Nonzero-Sum$ n$-Person Submodular Games. *SIAM Journal on Control and Optimization* **17** 773.

Vives, X. 1999. *Oligopoly Pricing: Old Ideas and New Tools*. Mit Pr.

Zhao, X., D. Atkins. 2007. Newsvendors under Price and Inventory Competition, Forthcoming. *Manufacturing and Service Operations Management* .