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Abstract

In this paper, we study a decentralized supply chain in which the manufacturer sells a short lifecycle product using wholesale price (only) contracts to a price-setting retailer, who in turn sells it to the market. Our two-period framework captures the salient features of this product: Price-sensitive stochastic retail demand; non-stationary demand and cost parameters; correlated demands which enable updating of demand characteristics; and limited, but possibly more than one, pricing, replenishment and wholesale pricing (contracting) opportunities. First, we develop a benchmark model where the integrated chain can price and order at the beginning of each period. We then model four decentralized decision-making paradigms with increasing degrees of decision flexibilities: Neither pricing nor ordering recourse; only pricing recourse; pricing and ordering recourses; and finally, pricing, ordering and contracting recourses. A novel transformation technique allows us to analytically characterize all the five models, and reduce the complex profit maximization problems in each case to a mere one-dimensional search. Subsequently, based on a numerical study, we systematically compare the values and behaviors of the optimal decisions for the five models. In addition, we offer managerial insights as to how optimal decisions behave temporally and how they are affected by system characteristics like price elasticity, demand correlation and demand uncertainty. A more detailed investigation comparing the optimal profits for the five models allows us to identify the values of pricing, ordering and contracting flexibilities from the viewpoint of the two channel partners. Our analysis generates managerial suggestions as to which decision-making paradigm might be most suitable for the chain depending on the business environment and on the lifecycle phase of the product.

Résumé

Dans cet article, nous étudions une chaîne logistique décentralisée dans laquelle le manufacturier vend un produit à court cycle de vie, sur une base contractuelle de prix de gros seulement, à un distributeur qui lui, revend au marché à un prix ajustable. Notre formulation à deux périodes représente adéquatement les caractéristiques dominantes du produit : une demande au distributeur stochastique et sensible au prix; une demande non stationnaire et des paramètres de coût; des demandes corrélées qui permettent de mettre à jour les caractéristiques de la demande; et des possibilités limitées d'ajuster de manière différente le prix client, la fréquence de placement des commandes de même que le prix de gros du manufacturier. Au départ, nous développons un modèle de référence dans lequel la chaîne intégrée peut placer la commande au début de chaque période. Nous formulons ensuite quatre autres modèles comportant un niveau de flexibilité croissant dans la prise de décision : aucun ajustement de prix et aucun renouvellement de commande possible; ajustement de prix possible; ajustement de prix et renouvellement de commande possibles; ajustement de prix, renouvellement de commande et renégociation du prix de gros possibles. Une technique de transformation nouvelle nous permet de caractériser analytiquement les cinq modèles, et de réduire les problèmes multidimensionnels de maximisation du profit à une recherche unidimensionnelle. Par la suite, sur la base d'un cas numérique particulier, nous comparons systématiquement les valeurs de profit et les natures des décisions dans les cinq modèles. De plus, nous développons des intuitions de gestion concernant les prises de décision optimales et la manière dont elles sont affectées par les caractéristiques du système incluant élasticité du prix, corrélations de la demande ainsi qu'incertitude de la demande. Plus loin, une investigation détaillée comparant les profits des cinq modèles nous permet d'identifier les valeurs de prix, et les divers schémas de flexibilité tels que perçus par chacun des deux partenaires. Notre analyse débouche sur des suggestions de gestion quant au paradigme décisionnel le plus adéquat pour la chaîne pour un environnement d'affaires donné et un cycle de vie donné du produit.

1 Introduction and Motivation

In this era of ever-changing technology and consumer tastes, short lifecycle products have become a fact of life in the business world. These products are characterized by high demand uncertainties and limited number of decision-making opportunities. In analyzing operational issues related to such products, the literature commonly adopts the newsvendor framework. That is, there is a single period of sales, along with a single replenishment and pricing opportunity for retailers. In reality, however, the selling period (product lifecycle) might be long enough to allow multiple production/buying opportunities for firms. Furthermore, demand characteristics and operating costs can change during the course of the selling season as the product progresses in its lifecycle. As a result, manufacturers/retailers often adjust prices during the selling period. For example, Taylor (2001) indicates that technology-related industries face sharp declines in product pricing due to the rapid rate of product introductions, which forces channel members to renegotiate the terms of their trade agreements. In such industries, less price-sensitive customers purchase early in the selling period. As the product matures rapidly, the costs and prices drop, triggering demand also from more price-sensitive customers. Moreover, the demand signal obtained from early period sales can help to set the adjusted prices and additional replenishment quantities (Fisher and Raman 1996). As Fisher et al. (2001) point out, in the fashion garment industry, the extent of correlation between early sales and remaining demand can be quite significant (0.95 in their application). These realities naturally call for a multi-period, temporal analysis with some correlation between early and late demands.

Recently, we have also witnessed a dramatic rise in outsourcing and resulting fragmentation of supply chains. Decisions of chain members in such decentralized settings are often uncoordinated; there is a trading mechanism (contract) in place and each firm tends to act independently with their own profit-maximization motive in mind. The most prevalent contract used in practice is the *wholesale price-only* type (Cachon 2003), which involves the retailer paying a per-unit price for the quantity it orders from the manufacturer.

Motivated by the above salient characteristics, we focus on a decentralized, two-echelon supply chain operating under a price-only contract in a two-period setting with the ability for demand information updating. Specifically, we deal with a single product whose demand is uncertain and price-sensitive. Our primary objective in studying this setting is to determine the impact of dynamic decision-making flexibilities (in terms of retail pricing, replenishment, and wholesale pricing/contracting) on the optimal decisions and profits of each of the channel partners, as well as on that of the entire chain.

Considerable research has been undertaken on issues related to our study. Our objective here is not to provide a comprehensive review, but rather to position our work with respect to this vast, ever-growing literature, and briefly review the most related works. Figure 1 visualizes the positioning of our study with representative examples. As seen in Figure 1, three main research thrusts orthogonally extend the traditional newsvendor framework. One line of work recognizes the strategic importance of making integrated operations-marketing decisions and incorporates *pricing* as an endogenous variable. Another direction (and perhaps the most popular one) is the inclusion of upstream members to analyze the

A: Classical Newsvendor
Porteus (2002)

B: Price-Setting Newsvendor
Petruzzi & Dada (1999)

C: Selling to a Newsvendor
Lariviere & Porteus (2001)

D: Selling to a Price-Setting Newsvendor
Cachon (2003), Petruzzi (2004)

E: Multi-Period, Single-Location Inventory Models
Zipkin (2000)

F: Multi-Period Joint Inventory Pricing Models
Federgruen & Heching (1999), Petruzzi & Dada (2001)

G: Multi-Period Selling to a Newsvendor
Barnes-Schuster et al (2002), Taylor (2001)

H: Our Models

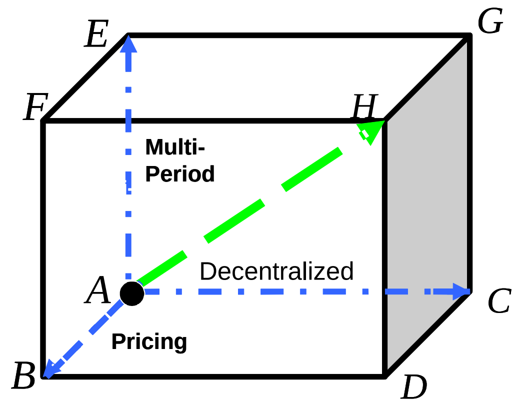

Figure 1: Positioning of our study with respect to the existing literature

supply chain in a *decentralized* setting. A third avenue is the allowance of multiple decision-making opportunities (*multi-period*).

Three main research categories combine issues around two dimensions. The models within each category differ significantly with respect to their assumptions and objectives. Hence, here we only provide a brief overview of each stream, without going into specific details, and highlight their relationship to our study:

• **Pricing + Decentralized:** Cachon (2003, Sections 3 and 5) and Bernstein and Federgruen (2005) present a detailed analysis of both competitive (vertical and horizontal, respectively) and centralized supply chains in price-sensitive *newsvendor* settings. They also study various contracts, including wholesale pricing, for decentralized chains under similar settings and concentrate on coordination issues.[1] Petruzzi (2004) and Wang et al. (2004), on the other hand, analyze wholesale price and consignment contracts, respectively, in decentralized settings, but focus on determining the optimal decisions from the perspective of the channel member offering the contract. As such, the multiple decision-making and demand-updating aspects of our study are not considered in this research stream. It should be noted that papers dealing with decision postponements (e.g., Van Mieghem and Dada 1999; Erhun et al. 2004) may also be included in this category, although they allow an extra pricing and ordering opportunity (once before demand is realized and once after). Such papers aim to determine the value of postponing the procurement and/or pricing decision by one of the channel partners or the integrated channel. As we will show later on, the dynamic pricing opportunity by *both* channel partners, which is ignored by this stream, plays a significant role in our paper. Moreover, in our setting, all decisions are

[1] That is, designing contracts that can allow a decentralized chain attain the profit performance of a centralized one.

made in a random environment, although the latter ones with additional information about the actual demand situation.

• **Multi-Period + Pricing:** A second body of research tackles retail pricing and/or ordering decisions in a multi-period setting (e.g., Cachon and Kok 2004; Chen et al. 2005; Federgruen and Heching 1999; Ferguson and Koenigsberg 2003; Monahan et al. 2004; Petruzzi and Dada 2001; Bernstein and Federgruen 2003). All of these studies are based on a *centralized* environment, and hence some of the defining elements of our paper – decentralized analysis, wholesale pricing and related issues – are not addressed.

• **Decentralized + Multi-Period:** The stream of research that conducts decentralized analysis in a multi-period (typically two-period) setting assumes a *price-insensitive* demand environment (e.g., Kouvelis and Gutierrez 1997; Donohue 2000; Taylor 2001; Barnes-Schuster et al. 2002). These papers are normally based on a particular contract setting (e.g., price-only, buyback, option), and the primary goal is to select the appropriate contract parameters so as to coordinate the chain. Hence, retail pricing and related issues (e.g., price elasticity) are not studied. We remark that there are similar works that assume all prices, retail and wholesale, to be exogenous (e.g., Fisher et al. 2001; Gurnani and Tang 1999; Milner and Kouvelis 2002).

To the best of our knowledge, no explicit attempt to address all three directions simultaneously has yet been undertaken. The models studied in this paper accomplish precisely this. In this sense, we synthesize and generalize most of the existing literature, albeit in a two-period setting.[2]

The remainder of this paper is organized as follows. § 2 provides a description of our framework and models. Sections 3 and 4 contain a comprehensive analysis of the models. In § 5 we conduct a numerical study to compare the optimal decisions and profits for different models, and generate managerial insights on the values of decision flexibilities. A summary of the main results and our concluding remarks are presented in § 6. An extended appendix contains a glossary of notation (Appendix A), detailed proofs of all results presented in the paper (Appendix B), and some additional formulas related to the numerical study (Appendix C).

2 Description of the Framework and Models

The investigative framework of this paper is a manufacturer-retailer supply chain selling a single, short lifecycle product for two periods. The demand in each period, which occurs at the retail site, is price-sensitive as well as stochastic (in a multiplicative form). Furthermore, the demand in the second period is correlated with early demand in the first period, which is fully observable. We assume a decentralized Stackelberg setting (led by

[2] Anand et al. (2003) also has addressed all three issues, but in a *deterministic* setting. They show that even if all the classical reasons for holding inventory are eliminated, the retailer might still decide to carry inventories purely for "strategic" reason of reducing the monopoly pricing power of the manufacturer in the second period.

the manufacturer) with a wholesale price contract in place. The retailer makes (at most) two decisions in each period: Retail price p_i and order-up-to level y_i, $i = 1, 2$. The manufacturer produces the retail orders instantaneously at cost c_i and sets the wholesale price w_i for $i = 1, 2$. Note that the manufacturer does not keep any stock for retail orders. The initial stock at the retailer is zero. Any demand that is not satisfied in either period is *lost*, and any leftover inventory from the first period (denoted as $x \geq 0$) is *carried over* to the second period. For simplicity, we assume that the holding costs for carried over inventory and the salvage costs/values of the leftovers at the end of second period are zero (like in Monahan et al. 2004). In line with the inherent characteristics of most short lifecycle products, we focus on the scenario in which the second period faces more elastic demand ($k_2 \geq k_1$) and benefits from lower production costs ($c_1 \geq c_2$). This scenario also aids in expositional clarity. All parameters are common knowledge and both channel partners are risk-neutral. We comment more on our key assumptions in our concluding discussion.

The above general framework helps analysis of a wide variety of decision-making contexts. Specifically, we are able to study several scenarios that differ in the degree of flexibility in making pricing and/or ordering decisions. In decreasing order of flexibilities, they are:

Dynamic Supply Chain Model (DSCM): In this fully dynamic model, the retailer makes pricing and ordering decisions and the manufacturer wholesale-pricing decision at the beginning of each period (i.e., *dynamic contract*).

Dynamic Retail Pricing and Ordering Model (DRPOM): This is the DSCM with a *static* contract, i.e., the manufacturer decides on a single wholesale price for two periods at the beginning of period one.

Dynamic Retail Pricing Model (DRPM): This is the DRPOM except that the retailer is constrained to place only one order, at the beginning of period one.

Static Model (SM): Both channel partners make decisions only at the beginning of period one. The same retail price is applied at the beginning of period two.

We also study the integrated scenario, namely the **Centralized Model (CM)**, which serves as a benchmark. Note that the difference in profits between DRPM and SM provides the value of *retail pricing flexibility*, while DRPOM and DRPM profit difference renders the (incremental) value of *retail ordering flexibility*, and DSCM and DRPOM profit difference gives the (incremental) value of *wholesale pricing flexibility* (or equivalently, the value of *dynamic contracting*).

From a technical standpoint, our primary contribution is the analytical characterization of the above models. A particular transformation technique enables us to tackle the dynamic analysis of three decisions simultaneously. Specifically, we are able to express the rather complicated profit expression for each model as a single variable optimization problem. Consequently, the optimal decision variables and profits – for each channel partner, as well as the integrated system – can be determined by a simple one-dimensional search procedure under some relatively mild conditions. Subsequently, we address the effects of system features like price elasticity, demand correlation and demand uncertainty in a numerical study. Comparing the optimal decisions, we identify the systematic differences in the optimal pricing and ordering strategies under each operating regime. Specifically, analysis of stationary settings reveals a number of interesting behavior. Non-dynamic contracts (i.e., all models except DSCM) always result in the same constant optimal wholesale price; dynamic contracts (DSCM), however, show a high-low pattern - the optimal wholesale price is higher than the constant value in the first period and lower in the second, with the average value being less than non-dynamic cases. The optimal retail prices also usually demonstrate a high-low pattern for dynamic contracts, but a low-high structure for their non-dynamic counterparts. Moreover, the optimal retail price and the optimal order quantity exhibit conflicting behaviors. The former, in general, increases with decentralization and decreases with "flexibility", while the latter acts inversely. We also show how the optimal decisions behave in non-stationary settings and the effects of the three system characteristics on the optimal values.

We then focus on the comparison of optimal profits for the different decision-making paradigms. Generally speaking, both parties' profits improve with greater decision flexibility. However, we also noticed exceptions, e.g., when the retailer might be worse-off because of the manufacturer's ability to change wholesale prices. Although profit improvements from decision flexibilities are significant, a fully "flexible" decentralized chain might still be substantially less profitable than a centralized system. The comparisons also enable us to identify the values of different decision flexibilities. The relative improvements from retail pricing and ordering flexibilities are normally equal for both channel partners, but wholesale pricing flexibility provides more value to the manufacturer. In majority of the cases, from the retailer's viewpoint, the most important concern is the ability to change its retail price in the second period; whereas for the manufacturer, it is to set its wholesale price dynamically. So, under certain situations, selecting a mutually acceptable operating paradigm might itself create conflict. However, there are situations when dynamic contracting is valuable for the retailer, and dynamic retail pricing for the manufacturer. Retail ordering flexibility is of value to both parties only for very uncertain demand environments. We can generate managerial insights as to which decision-making paradigm might be suitable for the supply chain, depending on the business environment or on the stage of the product in its lifecycle.

3 Model Preliminaries

We assume a multiplicative demand form[3] $D_i = d_i(p_i)\epsilon_i, i = 1, 2$, where ϵ_i is a positive random variable and $d_i(p_i) = p_i^{-k_i}$ ($k_i > 1$ is price elasticity). We allow the demand distributions for the two periods to be non-identical with any form of correlation between them. But the actual demand in the first period must be fully observable regardless of the stocking quantity. We make this assumption primarily for analytical tractability, and note that this is reasonable in many cases (e.g., on-line firms), where firms are able to track demand even when there is no inventory (and hence no sales). The pdf and cdf of

[3] A comprehensive list of notations is provided in Appendix A.

ϵ_1 and $(\epsilon_2|\epsilon_1)$ are denoted as $f_1(u)$, $F_1(u)$ and $f_2(u|\epsilon_1)$, $F_2(u|\epsilon_1)$ respectively. For ease of exposition, we assume that ϵ_1 and $(\epsilon_2|\epsilon_1)$ have supports over $[0,\infty)$, although our analysis readily extends to finite supports. The key assumption we make is that ϵ_1 and $(\epsilon_2|\epsilon_1)$ have increasing generalized failure rates (IGFR),[4] i.e.,

$$r_1(u) = \frac{uf_1(u)}{1-F_1(u)} \quad \text{and} \quad r_2(u|\epsilon_1) = \frac{uf_2(u|\epsilon_1)}{1-F_2(u|\epsilon_1)}$$

are increasing over $[0,\infty)$. Furthermore, we assume the following to hold:

$$lim_{u\to+\infty} u[1-F_1(u)] = 0 \quad \text{and} \quad lim_{u\to+\infty} u[1-F_2(u|\epsilon_1)] = 0 \ \text{ for any given } \epsilon_1.$$

Note that the above are rather standard assumptions in the related literature. The multiplicative demand form with an iso-elastic deterministic price function has been used extensively in the literature and is supported by empirical studies (Cachon and Kok, 2004, Petruzzi and Dada, 1999, and references therein). The IGFR property, on the other hand, is satisfied by most of the theoretical distributions used in the operations management (OM) literature including Uniform, Gamma with shape parameter ≥ 1, Beta with both parameters ≥ 1, Normal, Exponential and Left-truncated (at 0) Normal (refer to Lariviere and Porteus, 2001, for more details).

Defining $\mu_1 = E[\epsilon_1]$ and $\mu_2(\epsilon_1) = E[\epsilon_2|\epsilon_1]$, we introduce several functions that are used throughout the paper:

$$\Theta_1(z) = \int_z^{+\infty}(u-z)f_1(u)du, \qquad \Lambda_1(z) = \int_0^z (z-u)f_1(u)du, \qquad V_1(z) = \frac{\mu_1 - \Theta_1(z)}{\int_0^z uf_1(u)du},$$
$$\Theta_2(z|\epsilon_1) = \int_z^{+\infty}(u-z)f_2(u|\epsilon_1)du, \qquad \Lambda_2(z|\epsilon_1) = \int_0^z (z-u)f_2(u|\epsilon_1)du, \qquad V_2(z|\epsilon_1) = \frac{\mu_2(\epsilon_1) - \Theta_2(z|\epsilon_1)}{\int_0^z uf_2(u|\epsilon_1)du},$$

for any $z \geq 0$. It is easy to verify that for any $z \geq 0$,

$$\mu_1 - \Theta_1(z) = \int_0^z uf_1(u)du + z[1-F_1(z)], \qquad zF_1(z) - \Lambda_1(z) = \int_0^z uf_1(u)du,$$
$$\mu_2(\epsilon_1) - \Theta_2(z|\epsilon_1) = \int_0^z uf_2(u|\epsilon_1)du + z[1-F_2(z|\epsilon_1)], \qquad zF_2(z|\epsilon_1) - \Lambda_2(z|\epsilon_1) = \int_0^z uf_2(u|\epsilon_1)du.$$

The functions $V_i(\cdot)$, $i = 1, 2$, are crucial to our analysis. The next lemma characterizes the basic properties of these functions. For brevity, we omit the proof of this lemma, and refer the interested reader to Song et al. (2005):

Lemma 1 *$V_1(z)$ is decreasing, $lim_{z\to0^+}V_1(z) = +\infty$, and $lim_{z\to+\infty}V_1(z) = 1$. Also, for any specific realization of ϵ_1, $V_2(z|\epsilon_1)$ is decreasing, $lim_{z\to0^+}V_2(z|\epsilon_1) = +\infty$, and $lim_{z\to+\infty}V_2(z|\epsilon_1) = 1$. Let $U(z) = \mu_1 - \Theta_1(z) - k_1\int_0^z uf_1(u)du$; $U(z)$ is unimodal on $(0,+\infty)$ and there exists a unique positive Z_0 such that $U(z) = 0$.* □

[4] Throughout the paper we use increasing and decreasing in the weak sense, unless otherwise stated.

For any realization of ϵ_1 in period one, we define $H_2(\epsilon_1)$ as the unique, positive solution of $V_2(z|\epsilon_1) = k_2$. Note that the uniqueness follows from Lemma 1 since $k_2 > 1$. Based on this, we also define

$$B(\epsilon_1) = H_2(\epsilon_1)^{\frac{1-k_2}{k_2}}[\mu_2(\epsilon_1) - \Theta_2(H_2(\epsilon_1)|\epsilon_1)]. \tag{1}$$

4 Analysis of the Models

In this section, we provide a comprehensive characterization of the optimal decisions and profits for the five models. Note that each model requires a separate, non-trivial analysis, although seemingly they are special cases of each other. In fact, even the “simplest” one (static) has not been analyzed before in the literature. Our common result is that the optimizations of these models – with up to six decision variables – can each be reduced to a one-variable optimization problem. To establish this result, in addition to the notation introduced earlier, we occasionally make use of the stocking-factor transformation z_i, where $z_i = \frac{y_i}{d_i(p_i)}$ for $i = 1, 2$. We also point out that we use the superscripts M, R to denote the profits of the manufacturer and the retailer respectively, while the profit expressions for the two-period problem have no subscripts and those for the centralized chain have no superscripts. We start our analysis with CM and then continue with the decentralized models.

4.1 Analysis of the Centralized Model (CM)

The CM serves as the benchmark in our subsequent analysis. In this setting, there is a single decision maker, who sets the retail prices (p_1 and p_2) and order-up-to quantities (y_1 and y_2) to maximize overall supply chain profits. To derive these quantities, we proceed with the analysis in two steps. First, we study the second-period profit for a given initial stock level $x(\geq 0)$ and a realization of ϵ_1, and determine the resulting optimal supply chain profit $\pi_2(x, \epsilon_1)$. We then investigate the properties of the two-period profit function $\pi(p_1, z_1)$.

Consider a given $x(\geq 0)$ and a realization of ϵ_1. For any order-up-to level $y_2 \geq x$ and a price p_2, the expected supply chain profit in period two can be expressed as

$$\begin{aligned}\pi_2(p_2, y_2, x, \epsilon_1) &= p_2 E[Min\,\{y_2, d_2(p_2)(\epsilon_2|\epsilon_1)\}] - c_2(y_2 - x) \\ &= p_2 d_2(p_2)[\mu_2(\epsilon_1) - \Theta_2(\frac{y_2}{d_2(p_2)}|\epsilon_1)] - c_2(y_2 - x), \end{aligned} \tag{2}$$

where the first term in (2) represents the revenue, and the second term the purchase/production cost. As $\frac{\partial \pi_2^R(p_2,y_2,x,\epsilon_1)}{\partial p_2} = d_2(p_2)(\int_0^{\frac{y_2}{d_2(p_2)}} u f_2(u|\epsilon_1)du)\{V_2(\frac{y_2}{d_2(p_2)}|\epsilon_1) - k_2\}$, from Lemma 1, it follows that π_2 is unimodal in p_2. Hence, for any given order-up-to inventory level $y_2(\geq x)$, the unique optimal retail price $p_2(y_2)$ satisfies $\frac{\partial \pi_2^R(p_2,y_2,x,\epsilon_1)}{\partial p_2} = 0$, which, after some manipulation, can be expressed as

$$p_2(y_2) = H_2(\epsilon_1)^{\frac{1}{k_2}} y_2^{-\frac{1}{k_2}}. \tag{3}$$

Substituting $p_2(y_2)$ in (2), we obtain $\pi_2(y_2, x, \epsilon_1) = B(\epsilon_1) y_2^{\frac{k_2-1}{k_2}} - c_2(y_2 - x)$, where $B(\epsilon_1)$ is as defined in (1). Note that $\pi_2(y_2, x, \epsilon_1)$ is concave in y_2. Hence the optimal order-up-to inventory level is given as:

$$y_2(x, \epsilon_1) = \begin{cases} S(\epsilon_1) & \text{if } x \in [0, S(\epsilon_1)) \\ x & \text{if } x \in [S(\epsilon_1), +\infty) \end{cases}, \tag{4}$$

where $S(\epsilon_1) = (\frac{k_2-1}{k_2})^{k_2} (\frac{B(\epsilon_1)}{c_2})^{k_2}$. Substitution of $y_2(x, \epsilon_1)$ in $\pi_2(y_2, x, \epsilon_1)$ yields the optimal profit for period two:

$$\pi_2(x, \epsilon_1) = \begin{cases} \frac{c_2}{k_2-1} S(\epsilon_1) + c_2 x & \text{if } x \in [0, S(\epsilon_1)) \\ B(\epsilon_1) x^{\frac{k_2-1}{k_2}} & \text{if } x \in [S(\epsilon_1), +\infty). \end{cases} \tag{5}$$

Observe that $\pi_2(x, \epsilon_1)$ is increasing and piece-wise concave in the initial inventory level x.

We can now represent the total two-period expected supply chain profit under CM as:

$$\pi(p_1, z_1) = p_1 d_1(p_1)[\mu_1 - \Theta_1(z_1)] + \int_0^{+\infty} \pi_2(x, u) f_1(u) du - c_1 d_1(p_1) z_1, \tag{6}$$

where $x = d_1(p_1)(z_1 - u)$ if $z_1 > u$, and $x = 0$ otherwise. We have the following result about $\pi(p_1, z_1)$.[5]

Theorem 1 *For any given z_1, $\pi(p_1, z_1)$ is unimodal in p_1, and has a unique optimizer $p_1(z_1)$ which satisfies*

$$\frac{k_1 - 1}{k_1} p_1[\mu_1 - \Theta_1(z_1)] + \int_0^{z_1} \frac{\partial \pi_2(x, u)}{\partial x}[z_1 - u] f_1(u) du - c_1 z_1 = 0. \tag{7}$$

Consequently, the optimization of CM can be reduced to a one-variable optimization problem in terms of z_1, i.e., $\pi(p_1(z_1), z_1)$. Furthermore, $p_1(z_1)$ is increasing in z_1. □

Under certain mild conditions, we can even prove the unimodality of $\pi(p_1(z_1), z_1)$.

Theorem 2 *Let S_1 be the overall optimal order quantity of the profit function $I_1(p_1, y_1) = p_1 d_1(p_1)[\mu_1 - \Theta_1(\frac{y_1}{d_1(p_1)})] + c_2 d_1(p_1) \Lambda_1(\frac{y_1}{d_1(p_1)}) - c_1 y_1$. If $S_1 \leq S(0)$, then $\pi(p_1(z_1), z_1)$, and hence CM, has a unique maximizer.* □

Note that I_1 is the expected price-setting newsvendor profit function with zero initial inventory and a salvage value of c_2. Clearly, based on the definition of $S(0)$, $S_1 \leq S(0)$ is a comparison between two optimal order-up-to levels. It is noteworthy that this is a sufficient condition which only depends on system parameters, and can easily be checked numerically. Theorem 2 establishes that optimizing CM boils down to finding the unique optimal stocking factor z_1^* of $\pi(p_1(z_1), z_1)$. The remaining optimal decisions can then be easily backtracked. The following corollary summarizes the optimal decisions and the resulting profit.

[5] Proofs for all Lemmas and Theorems are provided in Appendix B.

Corollary 1 *In the first period, it is optimal to charge $p_1^* = p_1(z_1^*)$ to end customers and order $y_1^* = z_1^* d_1(p_1^*)$. In period two, it is optimal to charge $p_2(y_2(x, \epsilon_1))$ (as in (3)) and set the optimal order-up-to level as $y_2(x, \epsilon_1)$, and actually place an order $(y_2(x, \epsilon_1) - x)$ if $x \in [0, S(\epsilon_1))$ and 0 if $x \in [S(\epsilon_1), +\infty)$, where $x = y_1^* - d_1(p_1^*)\epsilon_1$ if $\epsilon_1 \le \frac{y_1^*}{d_1(p_1^*)}$, and 0 otherwise. The optimal total supply chain profit is given as $\pi(p_1^*, z_1^*)$.* □

4.2 Analysis of the Decentralized System

The basis of analysis for the decentralized system is a Stackelberg game with the manufacturer as the leader and the retailer as the follower. However, interaction between the manufacturer and the retailer differs in each of the models. For this reason, each decentralized model needs to be analyzed separately. In this section, we establish the main properties, and using these properties characterize the optimal decisions for each model. From a temporal standpoint, the analysis follows the same two steps as CM. However, for decentralized analysis, in each step we start by optimizing the retailer's profit for a given set of manufacturer's decisions. This establishes the link between the manufacturer's decision and the retailer's optimal response. To determine the manufacturer's optimal decision, we use a slightly different approach than incorporating the retailer's optimal response functions into the manufacturer's profit function. Instead, we invert the relationship to express the manufacturer's decision as a function of retail decisions, and then optimize over the retailer's decisions. Once the optimal values are characterized, using the same relationships in the reverse order we can infer the manufacturer's optimal decisions, and finally compute the resulting expected profits. We present our analysis in decreasing order of flexibility (DSCM, DRPOM, DRPM, and SM).

4.2.1 Dynamic Supply Chain Model (DSCM)

DSCM is the focal model of our paper and the one that represents the highest degree of decision making flexibility. Specifically, it involves dynamic contracting on the part of the manufacturer, as well as a dynamic pricing and ordering opportunity for the retailer. Consider the second period. For any wholesale price w_2 set by the manufacturer and any given initial stock x and a realization of first period demand ϵ_1, the retailer determines the optimal order-up-to level $y_2 (\ge x)$ and the retail price p_2 to maximize her expected profit in period two:

$$\pi_2^R(p_2, y_2, x, \epsilon_1) = p_2 d_2(p_2)[\mu_2(\epsilon_1) - \Theta_2(\frac{y_2}{d_2(p_2)}|\epsilon_1)] - w_2(y_2 - x). \tag{8}$$

Note that (8) is the same as the second-period CM profit (2) with c_2 replaced by w_2, and hence possesses an optimal (p_2, y_2) pair. It also stands to reason that the manufacturer will set her wholesale price (as long as it is $\ge c_2$) to induce a positive ordering quantity from the retailer, which means that for any such wholesale price the retailer's first order conditions $\frac{\partial \pi_2^R(p_2,y_2,x,\epsilon_1)}{\partial p_2} = 0$ and $\frac{\partial \pi_2^R(p_2,y_2,x,\epsilon_1)}{\partial y_2} = 0$ should be satisfied. This implies that:

$$V_2(\frac{y_2}{d_2(p_2)}|\epsilon) = k_2 \text{ and } w_2 = p_2[1 - F_2(\frac{y_2}{d_2(p_2)}|\epsilon_1)]. \tag{9}$$

Recall that the solution to the first equation in (9) is defined as $H_2(\epsilon_1)$. Also, for a given order-up-to inventory level $y_2(\geq x)$, the expression for the optimal retail price $p_2(y_2)$ is identical to that in CM, given by (3). Substituting $p_2(y_2)$ into the second equation in (9), the manufacturer's optimal wholesale price can be expressed as

$$w_2(y_2) = A(\epsilon_1)y_2^{-\frac{1}{k_2}}, \tag{10}$$

where $A(\epsilon_1) = H_2(\epsilon_1)^{\frac{1}{k_2}}[1 - F_2(H_2(\epsilon_1)|\epsilon_1)]$. Consequently, we can express the manufacturer's second-period expected profit function $(w_2 - c_2)(y_2 - x)$ in terms of y_2 only:

$$\pi_2^M(y_2, x, \epsilon_1) = [A(\epsilon_1)y_2^{-\frac{1}{k_2}} - c_2](y_2 - x). \tag{11}$$

Analyzing the above profit function we can conclude that:

Lemma 2 *The retailer's optimal order-up-to level $y_2(x, \epsilon_1)$ in the second period is the unique positive solution of*

$$\frac{k_2 - 1}{k_2}A(\epsilon_1)y_2^{-\frac{1}{k_2}} + \frac{1}{k_2}A(\epsilon_1)xy_2^{-\frac{k_2+1}{k_2}} = c_2. \tag{12}$$

Moreover, $y_2(x, \epsilon_1)$ is increasing and concave in $x(\geq 0)$. Hence, $w_2(x, \epsilon_1) = w_2(y_2(x, \epsilon_1))$ and $p_2(x, \epsilon_1) = p_2(y_2(x, \epsilon_1))$ are decreasing and convex in x on $[0, +\infty)$. $\square$

Lemma 2 characterizes the optimal decisions of the channel partners in the second period under the assumption that the manufacturer can always extract positive profit. Note from Lemma 2 that this will be true for a certain range of initial stock levels. In particular, defining $S(\epsilon_1) = (\frac{A(\epsilon_1)}{c_2})^{k_2}$, from the characterization of $w_2(y_2)$ in (10) we see that for any $x > S(\epsilon_1)$, the manufacturer can only offer $w_2 = c_2$ and obtain zero profit. Hence, it is optimal for the manufacturer to offer the following contract to the retailer:

$$w_2(x, \epsilon_1) = \begin{cases} A(\epsilon_1)y_2(x, \epsilon_1)^{-\frac{1}{k_2}} & \text{if } x \in [0, S(\epsilon_1)] \\ c_2 & \text{if } x \in (S(\epsilon_1), +\infty), \end{cases} \tag{13}$$

The optimal retail price is $p_2(x, \epsilon_1) = H_2(\epsilon_1)^{\frac{1}{k_2}}y_2(x, \epsilon_1)^{-\frac{1}{k_2}}$. The optimal order-up-to level for the retailer is $y_2(x, \epsilon_1)$ (refer to Lemma 2) for any initial stock level $x \in [0, S(\epsilon_1)]$ and x otherwise. The actual order quantity then follows. The ensuing expected profits for the second period and their behavior are derived in the next result.

Theorem 3 *For any given realization of ϵ_1 in period one and initial stock level $x(\geq 0)$ in period two, the retailer's and the manufacturer's expected profit functions can be expressed respectively as:*

$$\pi_2^M(x, \epsilon_1) = \begin{cases} c_2\frac{(y_2 - x)^2}{(k_2 - 1)y_2 + x}|_{\{y_2 = y_2(x, \epsilon)\}} & \text{if } x \in [0, S(\epsilon_1)] \\ 0 & \text{if } x \in (S(\epsilon_1), +\infty), \end{cases} \tag{14}$$

$$\text{and} \quad \pi_2^R(x,\epsilon_1) = \begin{cases} \frac{k_2c_2}{k_2-1}\frac{y_2[y_2+(k_2-1)x]}{(k_2-1)y_2+x}\big|_{\{y_2=y_2(x,\epsilon_1)\}} & \text{if } x \in [0, S(\epsilon_1)] \\ \frac{k_2}{k_2-1}A(\epsilon_1)x^{\frac{k_2-1}{k_2}} & \text{if } x \in (S(\epsilon_1), +\infty). \end{cases} \tag{15}$$

Furthermore, the following properties hold true:

(1) $\pi_2^M(x,\epsilon_1)$ *is decreasing and convex with respect to* $x \in [0,+\infty)$; $\pi_2^R(x,\epsilon_1)$ *is increasing and concave with respect to* $x \in [0,+\infty)$.

(2) Both $\frac{\partial \pi_2^M(x,\epsilon_1)}{\partial x}$ *and* $\frac{\partial \pi_2^R(x,\epsilon_1)}{\partial x}$ *are continuous with respect to* x *on* $[0,+\infty)$.

(3) $\frac{\partial \pi_2^R(x,\epsilon_1)}{\partial x} + k_2 x \frac{\partial^2 \pi_2^R(x,\epsilon_1)}{\partial x^2} > 0$ *for any* $x \in [0,+\infty)$. □

The Two-Period Problem: The above results pave the way for the two-period analysis. For any given wholesale price w_1 offered by the manufacturer at the beginning of period one, the retailer optimizes her total expected profit

$$\pi^R(p_1,z_1) = p_1 d_1(p_1)[\mu_1 - \Theta_1(z_1)] - w_1 z_1 d_1(p_1) + \int_0^{+\infty} \pi_2^R(x,u) f_1(u) du, \tag{16}$$

where $x = d_1(p_1)(z_1 - u)$ if $z_1 > u$ and 0 otherwise (recall that $z_1 = \frac{y_1}{d_1(p_1)}$ is the stocking factor). The first order conditions for the retailer's optimal response decisions can then be written as

$$\begin{aligned} &\frac{\partial \pi^R(p_1,z_1)}{\partial p_1} \\ &= \frac{d_1(p_1)}{p_1}\{(1-k_1)[\mu_1 - \Theta_1(z_1)]p_1 + k_1 w_1 z_1 - k_1 \int_0^{z_1} \frac{\partial \pi_2^R(x,u)}{\partial x}(z_1 - u) f_1(u) du\} = 0, \end{aligned}$$

$$\text{and} \quad \frac{\partial \pi^R(p_1,z_1)}{\partial z_1} = d_1(p_1)\{p_1[1 - F_1(z_1)] + \int_0^{z_1} \frac{\partial \pi_2^R(x,u)}{\partial x} f_1(u) du - w_1\} = 0.$$

From the first condition we derive a unique w_1 satisfying the retailer's FOC:

$$w_1(p_1,z_1) = \frac{1}{z_1}\{\frac{k_1 - 1}{k_1} p_1[\mu_1 - \Theta_1(z_1)] + \int_0^{z_1} \frac{\partial \pi_2^R(x,u)}{\partial x}(z_1 - u) f_1(u) du\}. \tag{17}$$

Furthermore, combining the two conditions, the retailer's optimal (p_1, z_1) should satisfy

$$L(p_1,z_1) = p_1\{k_1 \int_0^{z_1} u f_1(u) du - [\mu_1 - \Theta_1(z_1)]\} - k_1 \int_0^{z_1} \frac{\partial \pi_2^R(x,u)}{\partial x} u f_1(u) du = 0. \tag{18}$$

Utilizing (17), the manufacturer's total expected profit over the two-period planning horizon which is given by,

$$\pi^M(w_1,p_1,z_1) = (w_1 - c_1) d_1(p_1) z_1 + \int_0^{+\infty} \pi_2^M(x,u) f_1(u) du,$$

can be expressed in terms of (p_1, z_1) only as:

$$\begin{aligned}\pi^M(p_1, z_1) &= \frac{k_1 - 1}{k_1} p_1 d_1(p_1)[\mu_1 - \Theta_1(z_1)] \\ &+ d_1(p_1) \int_0^{z_1} \frac{\partial \pi_2^R(x, u)}{\partial x}(z_1 - u) f_1(u) du - c_1 d_1(p_1) z_1 \\ &+ \int_0^{+\infty} \pi_2^M(x, u) f_1(u) du. \end{aligned} \tag{19}$$

Clearly, the manufacturer needs to maximize $\pi^M(p_1, z_1)$ taking into account the fact that (p_1, z_1) should also satisfy (18). It is obvious that for any $z_1 \in [0, Z_0]$, there is no p_1 such that (18) is satisfied, where Z_0 is the unique positive solution of $U(z_1) = 0$ (refer to Lemma 1 for definition of $U(z_1)$). Carrying out this analysis, we claim that:

Theorem 4 *For any given $z_1 \in (Z_0, +\infty)$, there exists a unique $p_1(z_1)$ satisfying constraint (18). Substituting $p_1(z_1)$ in (19), the optimization of DSCM can be reduced to a one-variable optimization problem in terms of z_1.* □

Theorem 4 establishes that optimizing DSCM requires a search over z_1. Once the optimal stocking factor z_1^* is determined, the remaining optimal decisions and the resulting profits can be backtracked as summarized in the following corollary.

Corollary 2 *The optimal contract for the manufacturer is $w_1^* = w_1(p_1(z_1^*), z_1^*)$ (as in (17)) and $w_2(x, \epsilon_1)$ (as in (13)), where the leftover inventory at the end of period one $x = d_1(p_1^*)(z_1^* - \epsilon_1)$ if $\epsilon_1 \leq z_1^*$ and $x = 0$ if $\epsilon_1 > z_1^*$. The optimal strategy for the retailer in response to the contract is to charge $p_1^* = p_1(z_1^*)$ to customers and order $y_1^* = z_1^*(d_1(p_1^*))$ from the manufacturer in period one. In the second period, it is optimal for the retailer to charge $p_2(x, \epsilon_1)$ and set the order-up-to level as $y_2(x, \epsilon_1)$, and actually place an order $(y_2(x, \epsilon_1) - x)$ if $x \in [0, S_2(\epsilon_1))$ and 0 if $x \in [S_2(\epsilon_1), +\infty)$. The retailer's optimal total profit over the two periods is given as $\pi^R(p_1^*, z_1^*)$, while that for the manufacturer is $\pi^M(p_1^*, z_1^*)$.* □

4.2.2 Dynamic Retail Pricing and Ordering Model (DRPOM)

In this model, the wholesale price is set by the manufacturer only once at the beginning of the first period, and the manufacturer commits to charging the same wholesale price w_1 in the second period (i.e., static contract). The retailer, however, has the flexibility to price and order dynamically.

Consider a fixed wholesale price w_1. The retailer's second period expected profit, for given ϵ_1 and x, is similar to those analyzed for CM and DSCM:

$$\pi_2^R(p_2, y_2, x, \epsilon_1) = p_2 d_2(p_2)[\mu_2(\epsilon_1) - \Theta_2(\frac{y_2}{d_2(p_2)}|\epsilon_1)] - w_1(y_2 - x).$$

As in those models, the retailer's optimal price for a fixed y_2 is $p_2(y_2)$, given by (3). Substituting $p_2(y_2)$ into $\pi_2^R(p_2, y_2, x, \epsilon_1)$ and carrying out essentially the same analysis as for CM/DSCM, it is easy to establish that the retailer's optimal order-up-to inventory level is

$$y_2(x,\epsilon_1) = \begin{cases} S(\epsilon_1) & \text{if } x \in [0, S(\epsilon_1)) \\ x & \text{if } x \in [S(\epsilon_1), +\infty), \end{cases} \tag{20}$$

where $S(\epsilon_1) = (\frac{k_2-1}{k_2})^{k_2}(\frac{B(\epsilon_1)}{w_1})^{k_2}$. The resulting optimal second-period profit is

$$\pi_2^R(x,\epsilon_1) = \begin{cases} \frac{w_1}{k_2-1}S(\epsilon_1) + w_1 x & \text{if } x \in [0, S(\epsilon_1)) \\ B(\epsilon_1)x^{\frac{k_2-1}{k_2}} & \text{if } x \in [S(\epsilon_1), +\infty). \end{cases} \tag{21}$$

Observe further, that for any fixed w_1, the retailer's two-period total profit $\pi^R(p_1, z_1)$ is identical to that in CM, with c_1 and c_2 replaced with w_1. From Theorem 2 it then follows that under certain condition (see below) there exists a unique optimal stocking factor $z_1(w_1)$ and optimal price $p_1(z_1(w_1))$ for the retailer. Consequently, the manufacturer's total expected profit can be expressed only in terms of w_1 as follows:

$$\pi^M(w_1) = (w_1 - c_1)y_1(w_1) + (w_1 - c_2)\int_0^{+\infty}[y_2(x,u) - x]f_1(u)du,$$

where $y_1(w_1) = z_1(w_1)d_1(p_1(z_1(w_1)))$ is the retailer's optimal order-up-to level for period one and x is the initial stock level at the beginning of the second period. Hence, we can claim that:

Theorem 5 *For any given $w_1(> c_1)$, let $S_1(w_1)$ be the optimal order quantity of the expected profit function $I_1(p_1, y_1) = (p_1 - w_1)d_1(p_1)[\mu_1 - \Theta_1(\frac{y_1}{d_1(p_1)})]$. If $S_1(w_1) \le S(0)$, then the optimization of DRPOM can be reduced to a one-variable optimization problem in terms of w_1.* □

Note that, like in Theorem 2, $S_1(w_1) \le S(0)$ is a sufficient condition that can be checked numerically. Once the optimal wholesale price w_1^* (for both periods) is determined via search, all remaining optimal decisions can be backtracked. The following corollary summarizes the optimal decisions and the resulting profit for DRPOM.

Corollary 3 *The optimal strategy for the retailer in response to the static contract w_1^* is to charge $p_1^* = p_1(z_1(w_1^*))$ to customers and order $y_1^* = z_1(w_1^*)(d_1(p_1^*))$ from the manufacturer in period one. In the second period, it is optimal for the retailer to charge $p_2(y_2(x,\epsilon_1))$ and set the order-up-to level as $y_2(x,\epsilon_1)$, and actually place an order $(y_2(x,\epsilon_1) - x)$ if $x \in [0, S_2(\epsilon_1))$ and 0 if $x \in [S_2(\epsilon_1), +\infty)$. The retailer's optimal total profit is given as $\pi^R(p_1^*, z_1^*)$. The manufacturer's optimal total profit is given as $\pi^M(w_1^*)$.* □

4.2.3 Dynamic Retail Pricing Model (DRPM)

This model differs from DRPOM in that the retailer is constrained to order only once, at the beginning of the first period. However, the retailer still has the flexibility to dynamically change the retail price.

Consider a wholesale price w_1. The retailer's second-period expected profit for given ϵ_1 and x is given as

$$\pi_2^R(p_2, x, \epsilon_1) = p_2 d_2(p_2)[\mu_2(\epsilon_1) - \Theta_2(\frac{x}{d_2(p_2)}|\epsilon_1)].$$

Note that this function is identical to the DSCM profit (8) with y_2 replaced by x. It is then straightforward to verify that, for any fixed x, there exists a $p_2(x)$ which maximizes $\pi_2^R(p_2, x, \epsilon_1)$, and substituting this into $\pi_2^R(p_2, x, \epsilon_1)$ we have $\pi_2^R(x, \epsilon_1) = B(\epsilon_1) x^{\frac{k_2-1}{k_2}}$. Consequently, the retailer's total profit $\pi^R(p_1, z_1)$ can be expressed as

$$\pi^R(p_1, z_1) = p_1 d_1(p_1)[\mu_1 - \Theta_1(z_1)] + d_1(p_1)^{\frac{k_2-1}{k_2}} \int_0^{z_1} B(u)(z_1-u)^{\frac{k_2-1}{k_2}} f_1(u) du - w_1 d_1(p_1) z_1.$$

For any given w_1, the retailer's optimal decisions should clearly satisfy the first order conditions $\frac{\partial \pi^R(p_1,z_1)}{\partial z_1} = 0$ and $\frac{\partial \pi^R(p_1,z_1)}{\partial p_1} = 0$, which can respectively be written as

$$w_1(p_1, z_1) = p_1[1 - F_1(z_1)] + \frac{k_2-1}{k_2} d_1(p_1)^{-\frac{1}{k_2}} \int_0^{z_1} B(u)(z_1-u)^{-\frac{1}{k_2}} f_1(u) du, \text{ and} \tag{22}$$

$$p_1\{[\mu_1 - \Theta_1(z_1)] - k_1 \int_0^{z_1} u f_1(u) du\} + \frac{k_1}{k_2}(k_2-1) d_1(p_1)^{-\frac{1}{k_2}} \int_0^{z_1} B(u)(z_1-u)^{-\frac{1}{k_2}} u f_1(u) du = 0. \tag{23}$$

Substituting w_1 given by (22) into the manufacturer's total expected profit $(w_1–c_1)d_1(p_1)z_1$, we obtain:

$$\begin{aligned}\pi^M(p_1, z_1) = {} & p_1 d_1(p_1) z_1 [1 - F_1(z_1)] \\ & + \frac{k_2-1}{k_2} d_1(p_1)^{\frac{k_2-1}{k_2}} z_1 \int_0^{z_1} B(u)(z_1-u)^{-\frac{1}{k_2}} f_1(u) du - c_1 d_1(p_1) z_1. \end{aligned}\tag{24}$$

Hence, the manufacturer's problem is reduced to maximizing $\pi^M(p_1, z_1)$ subject to the fact that the optimal (p_1, z_1) should satisfy (23). Carrying out the analysis, we claim that:

Theorem 6 *For any given z_1, there exists a unique $p_1(z_1)$ satisfying constraint (23). Substituting this $p_1(z_1)$ in $\pi^M(p_1, z_1)$, the optimization of DRPM can be reduced to a one-variable optimization problem in terms of z_1.* □

As before, once the optimal stocking factor z_1^* is determined by search, the remaining optimal decisions can be backtracked. The following corollary summarizes the optimal decisions and the resulting profit for DRPM. [6]

[6] Specifically for $k_1 = k_2 = k$, we can show that $w_1^* = \frac{k}{k-1} c_1$ and $\frac{\Pi^M(p_1^*, z_1^*)}{\Pi^R(p_1^*, z_1^*)} = \frac{k-1}{k}$ (refer to the proof of Theorem 6 in Appendix B).

Corollary 4 *The optimal contract for the manufacturer is $w_1^* = w_1(p_1(z_1^*), z_1^*)$ with ensuing profit over two periods $\pi^M(p_1(z_1^*), z_1^*)$. The optimal strategy for the retailer in response to the contract is to charge $p_1^* = p_1(z_1^*)$ to customers and order $y_1^* = z_1^* d(p_1^*)$ from the manufacturer in period one. In the second period, it is optimal for the retailer to charge $p_2^* = H_2(\epsilon_1)^{\frac{1}{k_2}} x^{-\frac{1}{k_2}}$, where $x = d_1(p_1^*)(z_1^* - \epsilon_1)$ if $\epsilon_1 \leq z_1^*$ and $x = 0$ if $\epsilon_1 > z_1^*$. The retailer's optimal total profit is given as $\pi^R(p_1^*, z_1^*)$.* □

4.2.4 Static Model (SM)

In this model, all decision variables are set at the beginning of the first period. Specifically, the retailer charges the same price in both periods, and is allowed to order only once at the beginning of the first period. The retailer's second-period expected profit, for a given ϵ_1, x and retail price p_1, is given as

$$\pi_2^R(p_1, x, \epsilon_1) = p_1 d_2(p_1)[\mu_2(\epsilon_1) - \Theta_2(\frac{x}{d_2(p_1)}|\epsilon_1)].$$

The total two-period expected profit of the retailer then becomes

$$\pi^R(p_1, z_1) = p_1 d_1(p_1)[\mu_1 - \Theta_1(z_1)] + p_1 d_2(p_1) \int_0^{z_1} [\mu_2(u) - \Theta_2(\frac{d_1(p_1)}{d_2(p_1)}(z_1 - u)|u)] f_1(u) du - w_1 d_1(p_1) z_1.$$

As before, for any given w_1, the retailer's optimal decisions should satisfy the first order conditions $\frac{\partial \pi^R(p_1,z_1)}{\partial z_1} = 0$ and $\frac{\partial \pi^R(p_1,z_1)}{\partial p_1} = 0$. The first condition can be expressed as

$$w_1(p_1, z_1) = p_1\{[1 - F_1(z_1)] + \int_0^{z_1} [1 - F_2(\frac{d_1(p_1)}{d_2(p_1)})(z_1 - u)|u)] f_1(u) du\}. \tag{25}$$

On the other hand, combining the two conditions, we obtain the following constraint

$$L(q(p_1), z_1) = 0, \tag{26}$$

where $q(p_1) = \frac{d_1(p_1)}{d_2(p_1)}$ and $L(q, z_1) = k_1 z_1 q\{[1 - F_1(z_1)] + \int_0^{z_1} [1 - F_2(q(z_1 - u)|u)] f_1(u) du\} - (k_1 - 1) q[\mu_1 - \Theta_1(z_1)] - (k_2 - 1) \int_0^{z_1} [\mu_2(u) - \Theta_2(q(z_1 - u)|u)] f_1(u) du - (k_1 - k_2) q \int_0^{z_1} [1 - F_2(q(z_1 - u)|u)] f_1(u) du$.

Substituting w_1 given by (25) into the manufacturer's total expected profit $(w_1 - c_1) d_1(p_1) z_1$, we derive

$$\pi^M(p_1, z_1) = p_1 d_1(p_1) z_1 \{[1 - F_1(z_1)] + \int_0^{z_1} [1 - F_2(q(p_1))(z_1 - u)|u)] f_1(u) du\} - c_1 d_1(p_1) z_1.$$

Note that the manufacturer's optimal (p_1, z_1) should satisfy constraint (26). As per the previous cases, we find:

Theorem 7 *For any given z_1, there exists a unique $p_1(z_1)$ satisfying constraint (26). Consequently, substitution of $p_1(z_1)$ in $\pi^M(p_1, z_1)$ reduces the optimization of SM into a one-variable optimization problem in terms of z_1.* □

The following corollary summarizes the optimal decisions and the resulting profit for SM based on z_1^*. [7]

Corollary 5 *The optimal contract for the manufacturer is $w_1^* = w_1(p_1^* = p_1(z_1^*), z_1^*)$ with ensuing profit over two periods $= \pi^M(p_1^*, z_1^*)$. The optimum response strategy for the retailer is to charge p_1^* to customers in both periods and order $y_1^* = z_1^* d(p_1^*)$ from the manufacturer in period one. The retailer's optimal total profit is $\pi^R(p_1^*, z_1^*)$.* □

5 Numerical Study and Managerial Insights

The previous section focused on the analytical characterization of the profit functions and optimal decisions. However, any further comparison is intractable due to the complex first order conditions involved. Consequently, we focus on generating managerial insights through extensive numerical tests. In this section, we first examine the optimal decision values under each model and their sensitivities to key market conditions. These comparative statics allow us to identify the structural differences between the optimal decisions, which shed light on how managers should adapt their decisions to changes in decision-making paradigms and business environments. We then proceed to compare the optimal profits, and isolate and quantify the values of retail pricing, retail ordering and wholesale pricing (or equivalently, dynamic contracting) flexibilities from the perspective of the channel partners. Hence, we are able to indicate to managers the conditions under which each form of flexibility is of most value. We also explore the *efficiency* of the decentralized chain by comparing the profit of CM with the decentralized models.

For the purpose of this paper we define the market conditions in terms of three characteristics: Price-elasticity, correlation of demand between the two periods and natural demand uncertainty. To provide some perspective about our insights, we first discuss how these characteristics vary depending on products/markets.

Price elasticity: The value of this particular characteristic depends largely on the product type. For example, "functional" products like basic apparel and groceries are likely to have higher price elasticities than "innovative" products like fashion apparel, or high-end telecom products (Fisher 1997; Ray et al. 2005). However, depending on the target market niche, innovative products (e.g., DVD players) can also be aimed at price-elastic customers. Furthermore, price elasticities of most products dynamically increase over time, especially as a product progresses towards its maturity stage. We believe that the extent of increase will be greater for fashion products (impulsive purchases) like cellphones, than for industrial products (planned purchases) like telecom networks.

[7] As in DRPM, when $k_1 = k_2 = k$, we have $w_1^* = \frac{k}{k-1}c_1$ and $\frac{\Pi^M(p_1^*, z_1^*)}{\Pi^R(p_1^*, z_1^*)} = \frac{k-1}{k}$.

Demand correlation: For most products, the demand profile over time displays some correlation, although the extent again depends on product/market characteristics. Fisher et al. (2001) indicate that for fashion apparels the correlation between early and late demands is extremely high. Other fashion items like toys, cellphones, books are also likely to exhibit high demand correlation. On the other hand, for more mature products (e.g., basic apparel) or for products whose purchases are carefully planned (e.g., most industrial products), the demand correlation might be comparatively smaller.

Natural Demand Uncertainty: In our stochastic price-sensitive setting, uncertainty in final customer demand is influenced both by retail prices and by the inherent variability in end-customer demand. We refer to the second element as the natural demand uncertainty, which depends on the particular product type and the amount/quality of demand information available to the firm. For example, functional products like groceries tend to have rather predictable demand, while innovative high-end telecom products face highly variable demand. Similarly, online firms with faster access to better demand information can be more accurate in their demand forecasting compared to their offline competitors (Ray et al. 2005). Moreover, the natural randomness of demand also changes over time and, once again, the degree of change depends on market and product characteristics. Although the demand uncertainty in the second period is expected to fall below than that in the first, we also study reverse scenarios in order to provide more general insights.

Numerical Experiment Setting: We conduct our numerical study utilizing a truncated (at zero) normal distribution for the random part of the demand (which satisfies IGFR requirement), and incorporating correlation between the two periods. We let ϵ_1 denote the truncated version of $\overline{\epsilon}_1$ and $(\epsilon_2|\epsilon_1)$ be the truncated version of $(\overline{\epsilon}_2|\overline{\epsilon}_1)$ over $[0, +\infty)$, where $\overline{\epsilon}_1$ and $\overline{\epsilon}_2$ are correlated and their joint distribution is bivariate normal. Let $\overline{\mu}_i$ and $\overline{\sigma}_i$ be the mean and standard deviation of $\overline{\epsilon}_i,\ i = 1, 2$, respectively. We can then express the distributions $F_1(u)$ and $F_2(u \mid \epsilon_1)$ of ϵ_1 and $(\epsilon_2|\epsilon_1)$ respectively, in terms of $\overline{\mu}_i$, $\overline{\sigma}_i,\ i = 1, 2$, and the correlation coefficient ρ (for details refer to Appendix C).

The basic data set for our numerical study is as follows: $\overline{\mu}_1 = 100$, $\overline{\sigma}_1 = 40$, $\overline{\mu}_2 = 100$, $\overline{\sigma}_2 = 40$, $k_1 = 1.5$, $k_2 = 1.5$, $\rho = 0.5$ and $c_1 = c_2 = c = 1$. We capture the effects of absolute levels of price elasticities by assuming $k_1 = k_2 = k$, and then changing k from 1.5 to 2.5. On the other hand, in order to represent the relative difference in price elasticity levels for the two periods (specifically, $k_2 \geq k_1$) we fix $k_1 = 1.5$ and vary k_2 from 1.5 to 2.5.[8] The effects of demand correlation between the two periods are studied by changing the value of ρ from 0 (independent demands) to 0.9 (highly correlated demands). Lastly, we examine the effects of natural demand uncertainty by varying $\overline{\sigma}_i,\ i = 1, 2$. By assuming $\overline{\sigma}_1 = \overline{\sigma}_2 = \overline{\sigma}$ and varying $\overline{\sigma}$ from 10 to 100, we investigate the effects of differences in demand uncertainty levels between products. The temporal aspect of demand uncertainty is analyzed by keeping $\overline{\sigma}_1$ fixed at 40 and only changing $\overline{\sigma}_2$ from 10 to 100. Note that in all our experiments the single-dimensional profit function turned out to be unimodal, thus enabling us to evaluate the optimal decision variables and profit values.

[8]The selected price elasticity values have empirical justification; refer to Cachon and Kok (2004).

5.1 Optimal Decision Variables

In this section, we present a comparative study of the behavior of optimal wholesale prices, retail prices, and retail orders under each modelling framework.

5.1.1 Wholesale Prices

The effects of the three market characteristics on the optimal wholesale prices for the different models are shown in Figures 2–3.[9] We have pointed out in the previous section that, when $k_1 = k_2 = k$, the optimal wholesale prices for DRPM and SM models are identical ($w_1^* = \frac{k}{k-1}c$). In fact, Figure 2 suggests that this is valid even for the DRPOM model. Furthermore, the optimal value is the same as that of a single-period newsvendor problem with the same demand function as our first period (Petruzzi 2004). This means that whenever the manufacturer employs a *non-dynamic* contract (all models except DSCM) and the price elasticity of customers does not change over time, it is optimal to charge a price that maximizes *myopic* profits. Under a dynamic contract (DSCM), however, the manufacturer should charge a higher price in the first period and lower (in the expected sense) in the second one. This *mark-down* pricing strategy discourages the retailer from procuring significantly in the first period and carrying the excess inventory to the second period to curtail the monopoly pricing power of the manufacturer (refer also to Anand et al. 2003). Moreover, as evident from Figure 2, the average wholesale price for DSCM (average of first period and expected second-period wholesale prices) is lower than the optimal wholesale price for non-dynamic contracts. As we will show later on, this induces the retailer to order significantly more in DSCM. In summary, the optimal wholesale prices for $k_1 = k_2 = k$ are ordered as follows:

$$DSCM_{i=1} > DRPOM = DRPM = SM > DSCM_{i=2}.$$

When $k_1 \neq k_2$, the high-low pricing is still optimal for DSCM, but the optimal wholesale prices for the other models need not be equal. In particular, the manufacturer charges a lower price under SM compared to DRPM or DRPOM so as to counterbalance the lack of retail pricing or ordering recourses. Clearly, for all models, the optimal wholesale prices are shaped primarily by price elasticities and are decreasing in k_i of either period.

5.1.2 Retail Prices

As far as optimal retail prices are concerned, we first focus on the symmetric scenarios, i.e., $k_1 = k_2 = k$ and $\overline{\sigma}_1 = \overline{\sigma}_2 = \overline{\sigma}$, and medium values of ρ (Figures 4–5). In those cases the behavior of the optimal pricing policy for the retailer is governed by the degree of decision flexibility and the (vertical) competition in the system. Expectedly, CM with full decision flexibility and no competition results in lowest retail prices in both periods. The

[9] For brevity, in all figures we denote the five models by the following: DSCM - D, DRPOM - SD, DRPM - P, SM - S, CM - C. Also, σ_i refers to the natural demand uncertainty $\bar{\sigma}_i$, $i = 1, 2$. Furthermore, we present the *expected* values of the second-period optimal wholesale and retail prices, since their actual values will depend on the realization of the first period demand.

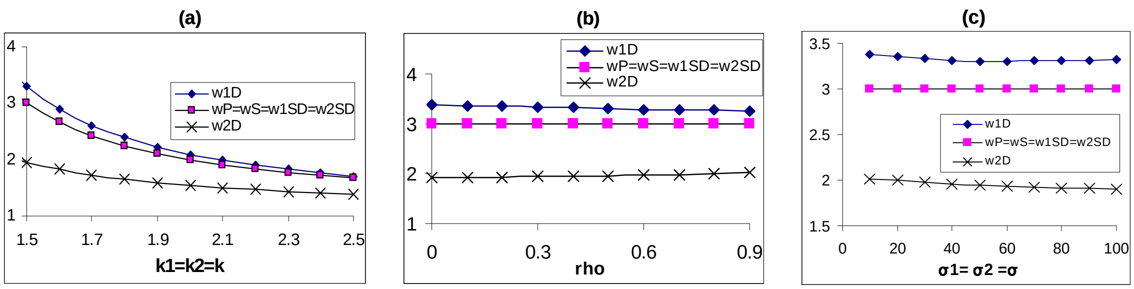

Figure 2: Optimal wholesale prices

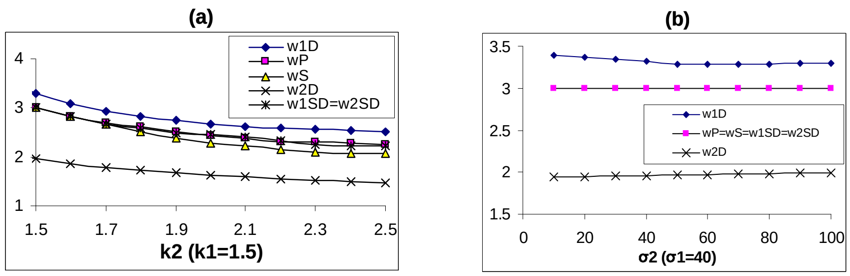

Figure 3: Optimal wholesale prices under asymmetric conditions

decentralized models have inherent double-marginalization, so must price higher than CM. In order to gain an understanding of the retail prices in a broad sense, we have computed and compared *average* retail prices, i.e., average of first-period and expected second-period retail prices, for decentralized models. Although not shown, it can be inferred that average retail prices are inversely related to the extent of flexibility ($SM > DRPM > DRPOM > DSCM$). This is intuitive since the fewer levers the firm has to match supply and demand, the more conservative it becomes in its pricing policy.

A closer examination of retail prices in Figures 4–5 shows that under a dynamic contract (DSCM), it is optimal for the retailer to employ *mark-down* pricing, which essentially mimics the manufacturer's wholesale pricing strategy. In contrast, under a non-dynamic contract with dynamic retail pricing (DRPOM, DRPM), *mark-up*[10] pricing is optimal for the retailer, even though the entire inventory is procured at the same wholesale price. Since CM shares the same characteristics, mark-up pricing is also optimal for that scenario. The reason is that the lower price in the first period (and resulting expected higher demand) lessens the chance of the retailer having substantial leftover inventory at the end of the planning horizon. However, by the second period, the retailer will be able to reduce its stock and collect information about demand characteristics. The risk of leftovers is then comparatively lower and the retailer can increase the price. Since SM does not possess such

[10]In the sense that the expected optimal retail price is higher in the second period. This pricing strategy is akin to the concept of *penetrative pricing* in the marketing literature (e.g., Tellis 1988), and should not be confused with cost-plus pricing.

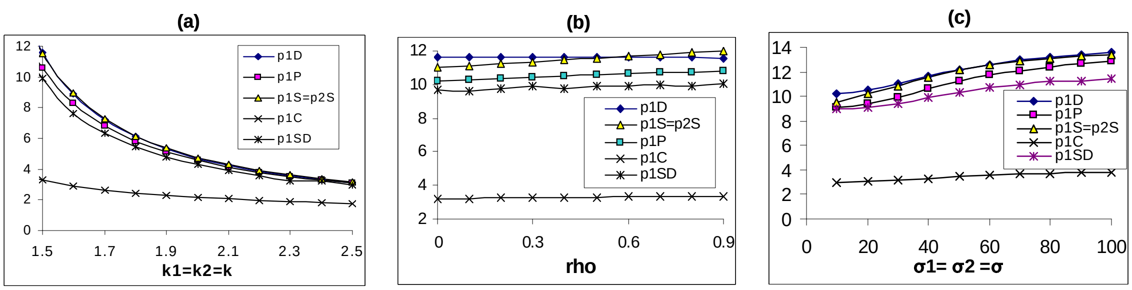

Figure 4: Optimal retail prices for the first period

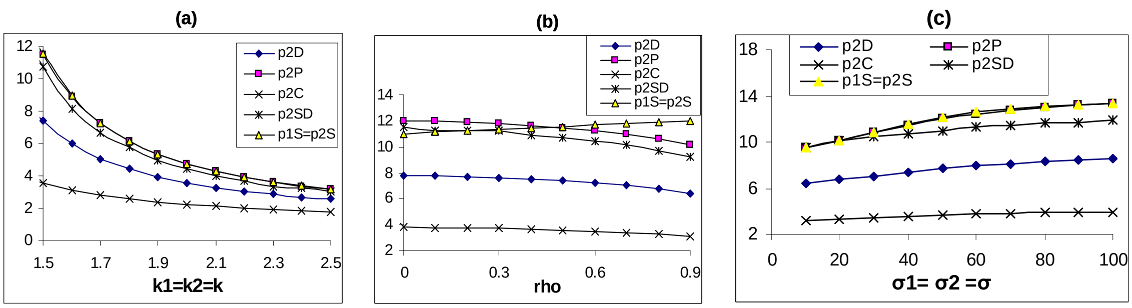

Figure 5: Optimal retail prices for the second period

flexibility, it charges a high price in both periods. Although mark-down pricing of fashion products has garnered most attention, empirical literature has shown that mark-up pricing as a means of penetrating the market is also employed regularly by retailers (Elmaghraby and Keskinocak 2003; Tellis 1988). To summarize, under symmetric conditions, the first- and second-period optimal retail prices are ordered as:

$$DSCM_i \approx SM_i > DRPM_i > DRPOM_i > CM_i, i = 1;$$

$$DRPM_i \approx SM_i > DRPOM_i > DSCM_i > CM_i, i = 2.$$

The above ordering of retail prices across the models holds (approximately) true even for an asymmetric system ($k_1 \neq k_2$ or $\overline{\sigma}_1 \neq \overline{\sigma}_2$ or low/high values of ρ), as seen from Figures 4b, 5b, 6 and 7. However, our previous conclusions regarding the optimal retail prices of the two periods for a particular model might no longer be valid. In particular, since optimal prices decrease with elasticity, when $k_2 \gg k_1$ (e.g., fashion products), the second-period optimal retail price might be lower than the first-period one for DRPOM, DRPM and CM. Hence, mark-down pricing can still be optimal under a non-dynamic contract when there is retail pricing flexibility. With respect to overall demand uncertainty, recall that it is driven by both the natural demand variability $\overline{\sigma}_i$ and the retail prices p_i; so optimal prices increase with $\overline{\sigma}_i$ to restrain the increase of overall demand uncertainty (Figures 4c, 5c, 7). This enables the retailer to reduce inventory costs, as we will demonstrate shortly. When

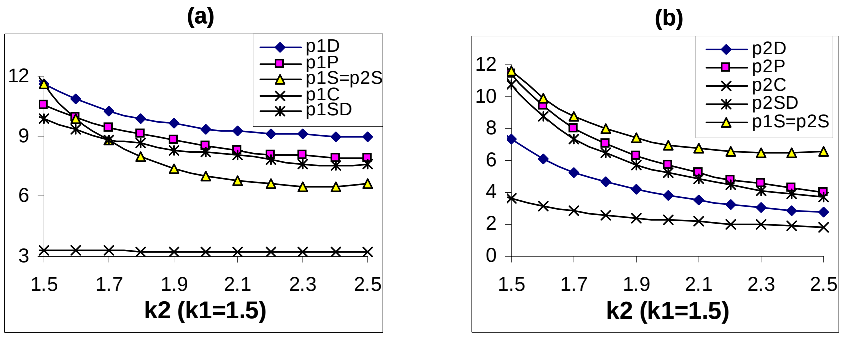

Figure 6: Optimal retail prices under asymmetric price elasticities

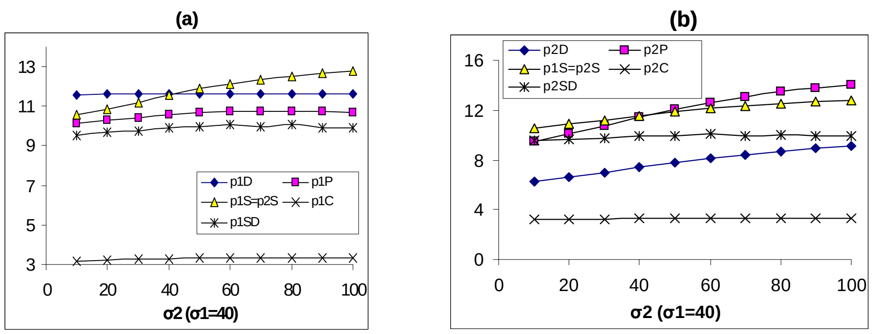

Figure 7: Optimal retail prices under asymmetric natural demand uncertainties

there is no retail pricing flexibility (i.e., SM), the retailer obviously has to accomodate the characteristics of both periods when setting the uniform price. This balancing act explains why the retailer charges a lower price in the first period and a higher one in the second period compared to other decentralized models when $k_2 \gg k_1$, as evident from Figure 6 (the behavior for $\overline{\sigma}_2 \gg \overline{\sigma}_1$ or $\overline{\sigma}_2 \ll \overline{\sigma}_1$ can be similarly explained). In order to understand the effects of the demand correlation ρ, note that higher values of ρ represent an increase in the total variability of the system (over two periods), but an almost deterministic demand in the second period after the realization of ϵ_1. It is known that (Petruzzi and Dada 1999), under a multiplicative demand function, deterministic demand results in a lower optimal retail price than stochastic one for the same expected demand value. In line with this fact, for all models with dynamic retail pricing, the second-period retail prices decrease with ρ (Figure 5b). This enables the retailer to slightly increase prices in the first period (Figure 4b). For SM, there is no pricing recourse; hence the retailer increases prices with ρ so as to curb variability (Figures 4b and 5b).

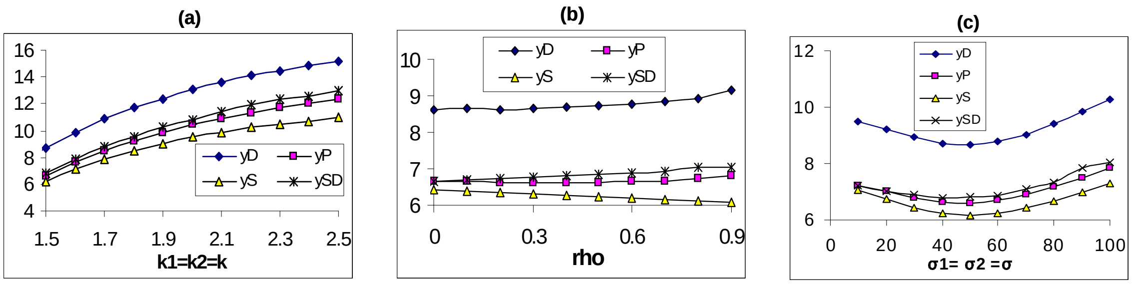

Figure 8: Total (expected) retail orders

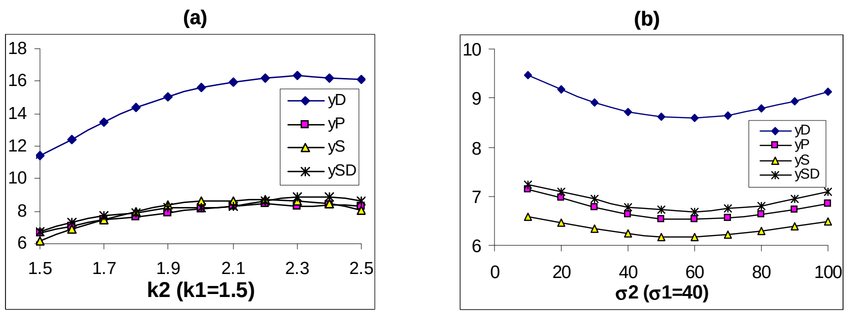

Figure 9: Total (expected) retail orders under asymmetric conditions

5.1.3 Retail Orders

The total amount of inventory ordered by the retailer reflects the demand effect of retail prices. As illustrated in Figure 8, in symmetric scenarios and all correlation levels, the higher the average retail prices, the lower the total expected retail orders over two periods, i.e., they are ordered as

$$CM > DSCM > DRPOM > DRPM > SM.$$ [11]

The increasing behavior of the orders with respect to k is intuitive. Note, however, the non-monotone effects of $\overline{\sigma}$ in Figure 8c. As natural demand uncertainty increases, so does the optimal retail price, which counteracts variability. The joint effect results in the non-monotone nature of the total retail orders (Figures 8c, 9b). In the case of demand correlation ρ (Figures 4b, 5b), however, our numerical studies indicate that the retail price effect dominates. Hence, for SM, the total order size decreases with ρ; whereas, in all other models, the total orders increase with ρ. As evident from Figure 9, most of the above relationships also hold true for asymmetric cases.

[11] Note that we do not show the order quantity of CM in Figures 8–9. The behavior in that case is exactly the same as in others, except that the order size is much larger. For example, in the base case it is around 35.

A number of other observations deserve attention. First, the decentralized environment considerably reduces the inventory availability compared to a centralized system. This is, of course, driven largely by double marginalization. Within decentralized models, the total amount ordered in SM, DRPM, and DRPOM is quite similar, due to the similar wholesale prices charged for these models. However, inventory availability for DSCM is substantially higher, primarily because of the lower average wholesale prices. Thus, the manufacturer cannot induce the retailer to purchase more by simply allowing her to adjust prices or place multiple orders; it is essential that the manufacturer also offers sufficient price incentives (i.e., reductions). Finally, although not shown, we observed that in DSCM, the expected order size is, in general, smaller in the second period than in the first, in spite of the fact that the wholesale price is actually higher in the first period. This signifies the retailer's motivation to carry inventory from the first period to restrict the manufacturer's pricing power.

5.2 Values of Decision Flexibilities

In this section, we compare the profits under different models and provide a detailed account of the values of pricing and ordering flexibilities to each channel member, as well as the supply chain as a whole. Note that, later on in §6 we synthesize the results of this section (as well as §5.1) to generate further managerial insights.

5.2.1 Values of Decision Flexibilities to the Manufacturer

We first briefly discuss the behavior of the manufacturer's total profit with respect to the three market characteristics for the decentralized models. Like before, we focus on symmetric scenarios (Figure 10) and on asymmetric cases (Figure 11a).[12] Figures 11b and 11c depict the retailer's total profits for asymmetric conditions. Observe from these figures that the manufacturer's profit improves with the degree of decision-making flexibility:

$$DSCM > DRPOM > DRPM > SM.$$

Evident from Figure 10a, customers' price sensitivity has a non-monotone effect on the manufacturer's profits. This is somewhat counter-intuitive because, as customers become more price concerned, we would expect both retail and manufacturer profits to decrease. However, as we know from the preceding section, as k increases, the wholesale price decreases, but the retailer's order quantity increases. When k is low, e.g., products in the growth phase of the lifecycle, the effect of retail orders dominates, whereas for higher k, e.g., mature products, the wholesale price effect matters more. The impact of natural demand uncertainty (Figure 10c) follows the behavior of the retail orders shown in Figure 8c, as the wholesale price is relatively independent of $\bar{\sigma}$. Interestingly, this means that the manufacturer prefers either a highly deterministic (e.g., commodity products) or highly uncertain demand environment (e.g., very fashionable products). On the other hand, as ρ increases, the total demand variability increases for SM, which reduces the manufacturer's profit (Figure 10b). For all other models, higher values of ρ allow more informed

[12] The behavior for $k_2 > k_1$ is structurally similar to Figure 10a and, hence, is not presented.

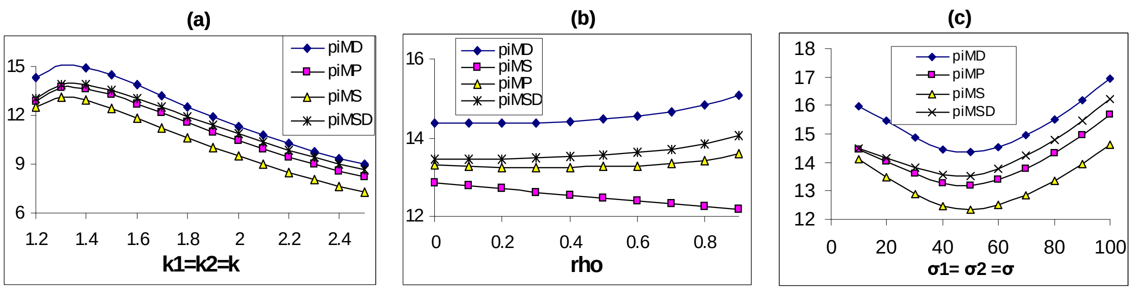

Figure 10: Manufacturer's profit

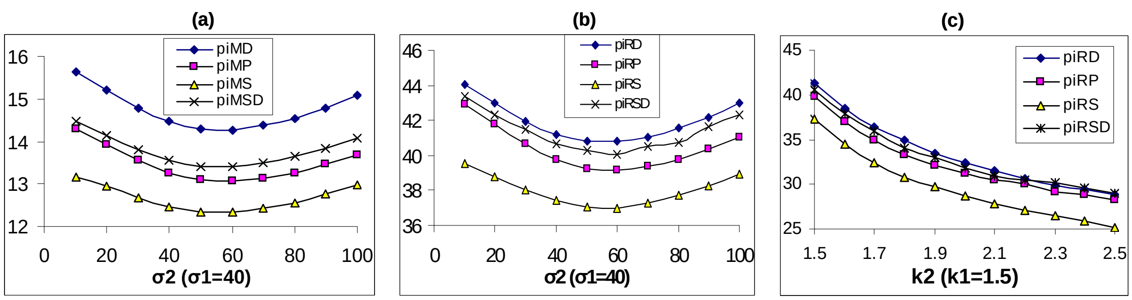

Figure 11: Manufacturer and retailer profits under asymmetric conditions

decision-making in the second period, which benefits the manufacturer. Clearly, the extent of benefits increases with ρ and with the degree of flexibility.

Although the manufacturer's profit increases with degree of decision-making flexibility, the extent of the improvement is governed by operating/market conditions. For this reason, it is important to quantify the distinct values of wholesale pricing, retail pricing and ordering flexibilities and how they are influenced by the above conditions. We can then identify the most desirable conditions for each decision-making paradigm, from the manufacturer's perspective. Since the profit for SM is the lowest, we quantify the values of the flexibilities by comparing the profits under DRPM, DRPOM and DSCM against SM profit. In Figures 12 and 13, the lines RPM, ROM and WPM show the percent profit improvement for the manufacturer for DRPM, DRPOM and DSCM scenarios respectively, compared to SM. Clearly, RPM denotes the value of *retail pricing flexibility*, (ROM - RPM) denotes the (incremental) value of *retail ordering flexibility*, and (WPM - ROM) denotes the (incremental) value of *wholesale pricing flexibility* (or of *dynamic contracting*), from the manufacturer's perspective.[13]

[13] The corresponding percent improvements for the retailer are illustrated by the lines RPR, ROR, and WPR, respectively, and the decision flexibilities can be deduced in a similar fashion.

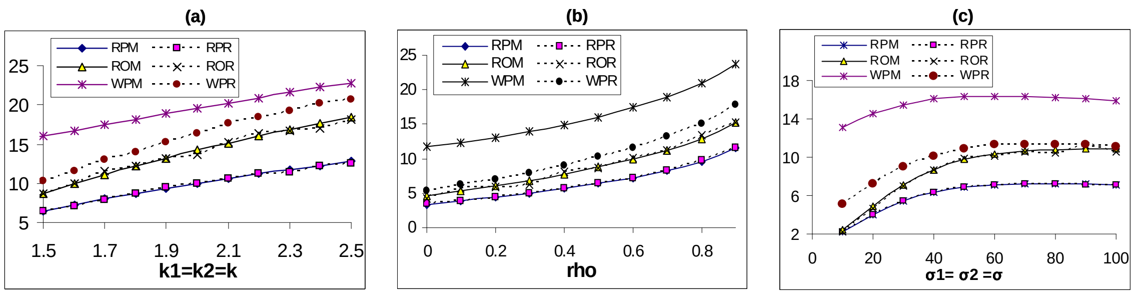

Figure 12: Values of decision flexibilities

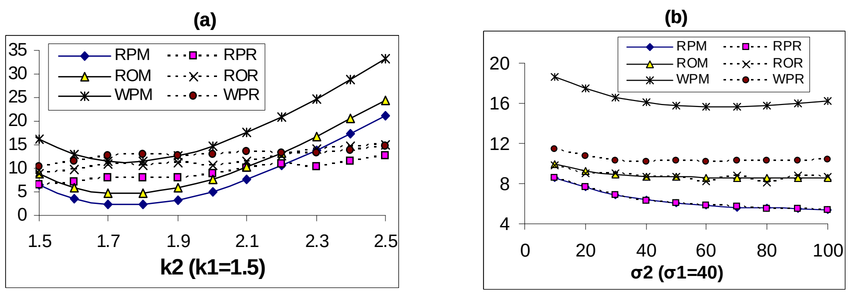

Figure 13: Values of decision flexibilities under asymmetric conditions

• ***Effects of Price Elasticity:*** From Figure 12a we observe that as $k(=k_1 \approx k_2)$ increases, the values of retail pricing and ordering flexibilities increase, while that for the wholesale pricing flexibility decreases. When customers are generally less price-concerned (e.g., for high-tech products), the manufacturer benefits most from dynamic contracting, whereas in the opposite scenario (e.g., basic apparel), the retailer's dynamic pricing captures most of the profit gains. On the other hand, Figure 13a shows that as k_2 (hence, price elasticity differential) increases, the value of retail pricing flexibility initially decreases and then increases, while the incremental values of the other two flexibilities remain relatively stable. Consequently, if the late demand proves significantly more price-sensitive (e.g., for cellphones), even the manufacturer gains most from the ability of the retailer to dynamically set prices.

• ***Effects of Demand Correlation:*** As the extent of correlation ρ increases, so does the value of information updating, resulting in an increase in the profit gain from all three recourse opportunities (Figure 12b). For low values of ρ (e.g., planned industrial purchases), the manufacturer benefits most from dynamically changing the contract. However, as ρ increases, retail pricing flexibility also becomes important for improving the manufacturer's profit.

• ***Effects of Natural Demand Uncertainty:*** From Figure 12c we note that as $\overline{\sigma}(=\overline{\sigma}_1 = \overline{\sigma}_2)$ increases, the values of retail pricing and ordering flexibilities both increase, while that for wholesale pricing falls. Evidently, for products with relatively known demand, the profit gain to the manufacturer accrues mainly from the ability to dynamically change the wholesale price. On the other hand, for highly variable demand, all three recourse decisions (especially retail pricing) might be important. For asymmetric conditions, we can conclude from Figure 13b, that when the second period demand is quite certain, dynamic wholesale and retail pricing flexibilities contribute most to the manufacturer's profit. But if second-period demand proves to be significantly more uncertain, then the retailer's flexibility to place a second order also becomes crucial.

5.2.2 Values of Decision Flexibilities to the Retailer

The structural behavior of the retail profits for symmetric scenarios is similar to those for the manufacturer and hence are not presented. As Figure 11b shows, the same is true when second-period demand uncertainty is different than that of the first-period. The main difference, as illustrated in Figure 11c, is that the retailer's profit strictly decreases (rather than being non-monotone) as the second-period demand becomes more price sensitive. This is intuitive since the retailer directly faces the price elasticity of the customers. In general, it is true that the retailer earns more profit when there is more decision-making flexibility, but exceptions are actually possible. A close look at Figure 11c shows that when $k_2 \gg k_1$, e.g., fashion apparel or cellphones, DRPOM profit is actually higher than DSCM. This proves that although dynamic contracting is always beneficial to the manufacturer, it might actually hurt the retailer. In these cases, a single static wholesale price provides more favorable terms to the retailer as opposed to the dynamic mark-down wholesale price scheme.

From a managerial perspective, it is perhaps more important to assess the values of different recourse decisions to the retailer under various business conditions. Figures 12 and 13 serve this purpose. Observe that, except for the case when early and late purchasers exhibit different price elasticities, the magnitude of gains from retail pricing and ordering flexibilities are almost *identical* for the manufacturer and for the retailer. Wholesale pricing flexibility, however, provides more profit gains to the manufacturer than to the retailer.

• ***Effects of Price Elasticity:*** Figure 12a shows that as $k(= k_1 \approx k_2)$ increases, the values of all decision flexibilities also increase. For innovative products, when customers are generally less price-concerned, dynamic retail pricing captures most of the profit gains for the retailer; the added benefit of second ordering and dynamic contracting is almost negligible. Only when customer price elasticity is high, do the added values of the other two flexibilities become observable. Note that retail pricing remains the most important element of flexibility even when the second-period customers turn out to be significantly more price sensitive, but (as indicated earlier) the manufacturer's dynamic contracting flexibility might deteriorate the retailer's profits.

• ***Effects of Demand Correlation:*** The impact of demand correlation on the retailer's profits is identical to that on the manufacturer's profits. Clearly, for all values of ρ, the major benefit for the retailer stems from retail pricing flexibility (Figure 12b). The retailer's ordering or the manufacturer's pricing recourse does not add much incremental profit, except the latter for almost independent demands.

• ***Effects of Natural Demand Uncertainty:*** The behavior of the values of decision flexibilities to the retailer as a function of natural demand uncertainty is similar to that to the manufacturer. However, since dynamic contracting confers considerably less profit gain to the retailer, the comparative values of the three recourse decisions are altered accordingly. Consequently, the retailer's ability to dynamically change prices is critical in most situations. In particular, when the second-period demand is highly certain (Figure 13b), this decision flexibility by far adds most value among the three elements. As evident from Figure 12c, for products with generally known demand, dynamic contracting is important for the retailer (even more than retail pricing flexibility), whereas for products with highly variable demand, the profit gains from this flexibility prove to be almost negligible. In highly variable demand environments, the opportunity to place a second order remains a valuable option for the retailer.

5.2.3 Values of Decision Flexibilities to the Chain

The magnitude and behavior of decision flexibilities for the supply chain as a whole are largely governed by the relative share of the profit which each channel member garners. We note that in our multiplicative demand setting, the retailer usually earns a larger share of profit. This benefit is comparatively low when k is high or $k_2 \gg k_1$.[14] The results of this section become clear when this fact is taken into perspective. For this reason, we omit detailed illustrations and directly provide the key insights.

As expected, the supply chain *always* benefits from higher decision making flexibility. In most cases, dynamic retail pricing is the most significant of all three elements. In market conditions when dynamic retail pricing is also significantly profitable for the manufacturer (high k, $k_2 \gg k_1$, high ρ, high $\overline{\sigma}$, or $\overline{\sigma}_2 \ll \overline{\sigma}_1$), this flexibility indeed accounts for almost the entire profit gain for the chain. However, when dynamic contracting is substantially profitable for the manufacturer ($k_1 \approx k_1$ and low, low ρ, low $\overline{\sigma}$, or $\overline{\sigma}_2 \gg \overline{\sigma}_1$), it might be so for the entire chain. The benefits of the second ordering opportunity are maximized when the second-period demand uncertainty is the most distinct market characteristic (high ρ, high $\overline{\sigma}$ or $\overline{\sigma}_2 \gg \overline{\sigma}_1$).

When analyzing chain profits, it might also be constructive to investigate the extent of profit loss due to decentralized decision making. Figure 14 serves that purpose. Clearly, a decentralized chain suffers from substantial profit loss even when all decisions are made dynamically. Only when customers are generally less price-sensitive and the product has relatively certain demand, e.g., pharmaceutical products, are the losses comparatively lower. Note that these are the same conditions (as well as independent demand scenarios) when the overall profit improvement from dynamic decision-making, compared to the static one (SM), is relatively small.

[14] Recall from Sections 4.2.3 and 4.2.4 that when $k_1 = k_2 = k$, $\Pi^M/\Pi^R = (k-1)/k$.

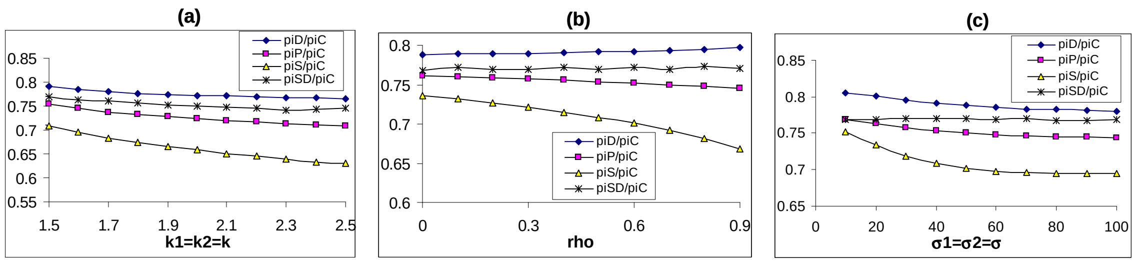

Figure 14: Comparison of Supply Chain Profits

6 Concluding Discussion and Future Research Opportunities

In this paper, we developed and analyzed four different models to investigate the effects of dynamic decision making in a decentralized supply chain facing non-stationary, price-sensitive stochastic demand. We also studied a centralized model as a benchmark for evaluating the inefficiencies created by decentralized decision making. This paper's contribution is two-fold. From a technical standpoint, we are able to characterize the main analytical properties of the optimal decisions (up to six in total) and profits for each of the models. From a managerial perspective, our comparison of the optimal decisions and the resulting profits under each model yields key insights regarding the effects of different decision-making paradigms. Since the four decentralized models differ in the number of recourse decisions allowed for the channel members, we are able to distinguish the value of each recourse decision for the channel members and the supply chain as a whole.

Our main technical result is that, under mild parameter restrictions and distributional assumptions, the optimization of each model can be reduced to a search over a single decision variable. We obtain our main result utilizing a series of transformations of the decision variables as well as changes to the optimization spaces. We strongly believe that such techniques can form the building blocks for the analysis of multi-period versions of our models. For example, the multi-period version of CM corresponds to the joint inventory-pricing problem with iso-elastic, price-sensitive demand and multiplicative form of uncertainty. To the best of our knowledge, characterizations of the optimal form of the policy, as well as of the optimal inventory and pricing decisions have not been carried out for this rather fundamental demand modelling framework. [15]

Our results highlight some structural differences in the optimal decisions under each operating regime. When the manufacturer employs a non-dynamic wholesale contract, it sets the optimal wholesale price myopically, with little consideration to the retailer's ordering and pricing flexibilities. Hence, the optimal wholesale prices are very similar (and in certain cases provably identical). In a dynamic contract setting, however, we find

[15] Such results exist for other demand models such as the linear demand function with additive uncertainty (Chen et al. 2005).

that it is optimal for the manufacturer to adopt a mark-down strategy, which results in higher prices in the first period and lower prices in the second period compared to non-dynamic scenarios. Moreover, average wholesale prices are also lower for dynamic contracts compared to non-dynamic ones. With a mark-down pricing strategy, the manufacturer aims at enhancing its pricing power in the second period by curbing the amount of inventory that the retailer carries over from the first period. Naturally, the retailer's response to a dynamic wholesale contract is to also pursue a mark-down retail pricing strategy. In contrast, under a non-dynamic wholesale price contract, although all units are procured at the same unit price, the retailer's tendency is to follow a mark-up (penetrative) pricing strategy, except when the late purchasers are significantly more price-sensitive. The initial low price enhances demand and reduces the probability of excess stock in either period. When the retailer has the option to adjust decisions, she does so with renewed demand information, which makes higher prices justifiable. Generally speaking, the average retail price (over two periods) decreases with accrued decision-making flexibility to the chain. The demand effects of retail prices have an immediate consequence on total retail orders, which increase when the supply chain has more flexibility. The most distinct increase, however, is due to dynamic wholesale pricing. This implies that giving pricing or ordering recourse to the retailer does not necessarily induce higher orders for the manufacturer, unless the wholesale price is likewise revised.

As for the impact of recourse decisions on profit, while there are exceptions, it is generally true that profits for both channel partners improve with the degree of decision-making flexibility in the chain. Furthermore, the scale of profit improvement due to dynamic retail pricing or ordering is quite similar for the two parties, whereas wholesale price adjustments provide comparatively more gains to the manufacturer. As a matter of fact, dynamic wholesale pricing can even be detrimental to the retailer (i.e., the exception), particularly when late purchasers are significantly more price-sensitive. The supply chain, on the other hand, always benefits from higher decision-making flexibility. This benefit is most significant for products which exhibit high levels of price elasticity (especially $k_2 \gg k_1$), variability (especially $\sigma_2 \ll \sigma_1$) and correlation. But the effects of double-marginalization are strong, regardless. In our numerical experiments, the optimal profit of the best decentralized model does not capture more than 85% of the optimal centralized profits.

We also shed light on the relative importance of the three decision flexibilities to the manufacturer and to the retailer. In particular, on the basis of price elasticity of demand, demand correlation between periods, and (natural) demand uncertainty, we identify scenarios under which the contribution of each decision flexibility to the overall profit improvement is maximal. Without repeating these results but synthesizing them, we can infer that these conditions are not necessarily the same for the manufacturer as for the retailer. Hence, in certain cases, deciding on a particular contracting scheme itself might be a source of conflict within the supply chain.

For the retailer, pricing flexibility is the most critical decision flexibility in majority of the cases. In fact, under certain conditions, the ability to dynamically price captures the potential profit gains almost in its entirety for the retailer. These conditions are: i) When customer price sensitivity in both periods is similar and low, or ii) the late demand is much more price-concerned or more predictable or independent of early demand. Unfortunately, many of these conditions do not constitute a favorable environment for the manufacturer to be satisfied with only retail pricing flexibility. A favorable scenario arises when late purchasers are significantly more price-sensitive than early purchasers, e.g., fashion products. Under some other previously described conditions, especially low $k_1 \approx k_2$ and low ρ, the manufacturer would insist on renegotiating the contract, since this flexibility would enable her to garner the highest profit gains. Such dynamic contracting is also important for the manufacturer in relatively certain demand environments, or when demand uncertainty increases over the planning horizon. On the other hand, in addition to the condition $k_2 \gg k_1$, retail pricing flexibility can be valuable to the manufacturer: i) If the demand is quite uncertain, or ii) price elasticities are generally high, or iii) demands are highly correlated. Finally, in general, ordering flexibility is relatively less valuable (compared to the other two) to both channel partners. The only conditions under which this flexibility is relevant are aligned for the two parties and are indeed rather intuitive: When uncertainty in both periods or in the second period is high.

While decision flexibilities add to the profits of the channel partners, their administration may incur significant costs (Elmaghraby and Keskinocak 2003). Hence, the ensuing benefits must be able to counterbalance these costs in order to give credence to the flexibilities. Against this backdrop, we can suggest the decision-making paradigms which might be most favorable for the channel partners (and consequently the chain) under different business environments, as summarized below.

• **Absolute Price Elasticity ($k_1 \approx k_2$):** *High* (e.g., basic apparel or mature products) – Static contract with retail pricing and ordering flexibility (DRPOM); *Low* (e.g., high-tech industrial products) - Dynamic wholesale price contract with retail pricing and ordering flexibility (DSCM).

• **Relative Price Elasticity ($k_2 - k_1$):** *High* (e.g., fashion apparel, cellphones) - Only retail pricing flexibility (DRPM).

• **Demand Correlation (ρ):** *High* (e.g., fashion apparel) - DRPOM; *Low* (e.g., planned industrial purchases) - DSCM.

• **Absolute Demand Uncertainty ($\bar{\sigma}_1 \approx \bar{\sigma}_2$):** *High* (e.g., growth phase industrial products) - DRPOM; *Low* (e.g., mature products) - DSCM.

• **Relative Demand Uncertainty:** *High* – for $\bar{\sigma}_2 \ll \bar{\sigma}_1$ (e.g., fashion products), DRPM; for $\bar{\sigma}_2 \gg \bar{\sigma}_1$, DSCM.

• **Static decision-making paradigm (SM):** Only for very low (and similar) price elasticities for the two periods, low demand correlation and relatively certain demand (e.g., pharmaceutical products).

For the analytical results in Section 4 we assumed $k_2 \geq k_1$ and $c_1 \geq c_2$. Note that these are sufficient conditions; numerical experiments have shown that the unimodality property of the profit functions hold true even if they are not satisfied. We do not experiment with $c_1 \neq c_2$ in Section 5, as otherwise the profit comparison between the models would not be justified (recall that for SM and DRPM all units are procured in the first period, while for others they are procured in both periods). Note also that we ignore the holding costs for carryover inventories and the salvage values of the final leftovers. These assumptions are indeed common in the related literature for technical tractability (e.g., Monahan et al. 2004; Wang et al. 2004). We believe that our key qualitative results will not be affected if these costs are taken into consideration, although the precise values of the optimal decisions and profits will surely change. Notice that the two cost elements will have a counteracting effect on the retailer's overall order quantity - while the first one will force the retailer to procure less, the second one will encourage her to purchase more. However, depending on their values, these costs might cause the relative amount of inventories procured in the two periods to change considerably (there will be an incentive to procure comparatively more in the second period). As a result, we can surmise that the ordering flexibility would most probably become more relevant in the presence of these costs.

Dynamic decision-making is an issue of considerable recent interest for managers, especially those in online firms, and will remain so for the foreseeable future. In fact, a number of software applications have been developed specifically for this purpose (Elmaghraby and Keskinocak 2003). Under these circumstances, our paper which addresses such dynamic decisions in three dimensions - retail pricing, retail ordering and contracting - simultaneously, should be of considerable significance. However, there are still a number of possible future extensions of our framework. Specifically we identify four avenues which may benefit from future research: i) Examination of how results will change if the lost sales are unobservable, ii) investigation of a competitive setting either at retail or manufacturer level, iii) extension of our model to analyze T-period ($T > 2$) or infinite horizon problems, and iv) incorporation of inventory decisions at the manufacturer level.

Appendix A: Glossary of Notation[16]

k_i	=	price elasticity of the deterministic part of the demand in period i
d_i, ϵ_i	=	deterministic price sensitive and random non-price dependent parts of the demand in period i
$\mu_1, \mu_2(\epsilon_1)$	=	mean values of ϵ_1 and $(\epsilon_2 \vert \epsilon_1)$
$f_1(u), F_1(u)$	=	density and distribution functions of ϵ_1, respectively
$f_2(u \vert \epsilon_1), F_2(u \vert \epsilon_1)$	=	density and distribution functions of $(\epsilon_2 \vert \epsilon_1)$, respectively
p_i, y_i	=	retail price per unit and order-up-to level in period i
c_i, w_i	=	manufacturer's per unit purchasing cost and wholesale price in period i
$x(\geq 0)$	=	$= y_1 - d_1(p_1)\epsilon_1$ if $y_1 > d_1(p_1)\epsilon_1$, and 0 otherwise; initial inventory level at the beginning of period two
z_1	=	stocking factor in period one, $z_1 = \frac{y_1}{d_1(p_1)}$

[16] $i = 1, 2$ in all notation represents the two periods

$\pi_2(x, \epsilon_1)$	=	centralized system's expected profit in period two for a given x and demand realization of ϵ_1 in period one
$\pi(p_1, z_1)$	=	centralized system's expected profit over the two-period planning horizon
$\pi_2^M(x, \epsilon_1)$	=	manufacturer's expected profit in period two for a given x and ϵ_1
$\pi_2^R(x, \epsilon_1)$	=	retailer's expected profit in period two for a given x and ϵ_1
$\pi^M(w_1, p_1, z_1)$	=	manufacturer's expected profit over the planning horizon for a given (w_1, p_1, z_1)
$\pi^R(p_1, z_1)$	=	retailer's expected profit over the planning horizon for a given (p_1, z_1)

Appendix B: Proofs of Lemmas and Theorems

Proof of Theorem 1:

For any given $z_1(> 0)$, taking partial derivatives of (6) with respect to p_1, we get

$$\frac{\partial \pi(p_1, z_1)}{\partial p_1} = -k_1 \frac{d_1(p_1)}{p_1} \{ \frac{k_1 - 1}{k_1} p_1 [\mu_1 - \Theta_1(z_1)] + \int_0^{z_1} \frac{\partial \pi_2(x, u)}{\partial x} [z_1 - u] f_1(u) du - c_1 z_1 \},$$

$$\begin{aligned} \frac{\partial^2 \pi(p_1, z_1)}{\partial p_1^2} \Big|_{\{\frac{\partial \pi(p_1, z_1)}{\partial p_1} = 0\}} &= -k_1 \frac{d_1(p_1)}{p_1} \{ \frac{k_1 - 1}{k_1} [\mu_1 - \Theta_1(z_1)] \\ &\quad - k_1 \frac{d_1(p_1)}{p_1} \int_0^{z_1} \frac{\partial^2 \pi_2(x, u)}{\partial x^2} [z_1 - u]^2 f_1(u) du \} \\ &\leq -(k_1 - 1) d_1(p_1) [\mu_1 - \Theta_1(z_1)] < 0. \end{aligned}$$

Hence, for any given $z_1(> 0)$, $\pi(p_1, z_1)$ is unimodal. The unique optimizer $p_1(z_1)$ is then the solution of the first order condition $\frac{\partial \pi(p_1, z_1)}{\partial p_1} = 0$, which simplifies to (7). Substituting this $p_1(z_1)$ in (6), we can express the CM profit function only in terms of z_1. Observe from (7) that

$$c_1 z_1 - \frac{k_1 - 1}{k_1} p_1 [\mu_1 - \Theta_1(z_1)] = \int_0^{z_1} \frac{\partial \pi_2(x, u)}{\partial x} [z_1 - u] f_1(u) du \leq c_2 [z_1 F_1(z_1) - \int_0^{z_1} u f_1(u) du].$$

Since $c_1 \geq c_2$, $\frac{k_1 - 1}{k_1} p_1 [\mu_1 - \Theta_1(z_1)] \geq c_1 z_1 - c_2 [z_1 F_1(z_1) - \int_0^{z_1} u f_1(u) du] \geq c_2 [\mu_1 - \Theta_1(z_1)]$, implying $p_1(z_1) \geq \frac{k_1}{k_1 - 1} c_2$.

To show that $p_1(z_1)$ is increasing, we define $L(p_1, z_1) = \frac{k_1 - 1}{k_1} p_1 [\mu_1 - \Theta_1(z_1)] + \int_0^{z_1} \frac{\partial \pi_2(x, u)}{\partial x}$ $[z_1 - u] f_1(u) du - c_1 z_1$. Note that $p_1(z_1)$ satisfies $L(p_1, z_1) = 0$. Clearly $\frac{d p_1(z_1)}{d z_1} =$

$-\frac{\partial L(p_1,z_1)}{\partial z_1}(\frac{\partial L(p_1,z_1)}{\partial p_1})^{-1}$ from the implicit function theorem. As $\frac{\partial L(p_1,z_1)}{\partial p_1}|_{\{L(p_1,z_1)=0\}} = \frac{k_1-1}{k_1}[\mu_1 - \Theta_1(z_1)] - k_1\frac{d_1(p_1)}{p_1}\int_0^{z_1}\frac{\partial^2\pi_2(x,u)}{\partial x^2}[z_1-u]^2 f_1(u)du > 0$, we only need to show $\frac{\partial L(p_1,z_1)}{\partial z_1}|_{\{L(p_1,z_1)=0\}} \leq 0$. It is easy to verify that

$$z_1\frac{\partial L(p_1,z_1)}{\partial z_1} = \frac{k_1-1}{k_1}p_1 z_1[1-F_1(z_1)] + z_1\int_0^{z_1}\frac{\partial \pi_2(x,u)}{\partial x}f_1(u)du + d_1(p_1)z_1\int_0^{z_1}\frac{\partial^2\pi_2(x,u)}{\partial x^2}[z_1-u]f_1(u)du - c_1 z_1.$$

For $L(p_1,z_1)=0$, we also know that

$$\frac{k_1-1}{k_1}p_1 z_1[1-F_1(z_1)] - c_1 z_1 = -\frac{k_1-1}{k_1}p_1\int_0^{z_1}uf_1(u)du - \int_0^{z_1}\frac{\partial \pi_2(x,u)}{\partial x}[z_1-u]f_1(u)du.$$

Substituting the above into the expression for $z_1\frac{\partial L(p_1,z_1)}{\partial z_1}$, we get

$$z_1\frac{\partial L(p_1,z_1)}{\partial z_1}|_{\{L(p_1,z_1)=0\}} = -\int_0^{z_1}[\frac{k_1-1}{k_1}p_1 - \frac{\partial \pi_2(x,u)}{\partial x}]uf_1(u)du + d_1(p_1)z_1\int_0^{z_1}\frac{\partial^2\pi_2(x,u)}{\partial x^2}[z_1-u]f_1(u)du.$$

It now follows from (5) and $p_1(z_1) \geq \frac{k_1}{k_1-1}c_2$, that $\frac{\partial L(p_1,z_1)}{\partial z_1}|_{\{L(p_1,z_1)=0\}} \leq 0$. □

Proof of Theorem 2:

Let $I_1(p_1,y_1) = p_1 d_1(p_1)[\mu_1 - \Theta_1(\frac{y_1}{d_1(p_1)})] + c_2 d_1(p_1)\Lambda_1(\frac{y_1}{d_1(p_1)}) - c_1 y_1$ and $I_2(p_1,y_1) = \int_0^{+\infty}[\pi_2(x,u) - c_2 x]f_1(u)du$ where $x = y_1 - d_1(p_1)u$ if $y_1 > d_1(p_1)u$ and $x=0$ otherwise. It is obvious that the total expected supply chain profit under CM can be rewritten as $\pi(p_1,\frac{y_1}{d_1(p_1)}) = I_1(p_1,y_1) + I_2(p_1,y_1)$ (note that $\frac{y_1}{d_1(p_1)} = z_1$).

First, we show that $S(u)$ is increasing in terms of u, the realization of ϵ_1. In order to show this, it is sufficient to show that $B(u)$ is increasing in terms of u (by the definition of $S(u)$). Based on the expression of $\pi_2(y_2,x,u)$, we get $B(u) = \pi_2(1,1,u) = Max_{p_2>0}\pi_2(p_2,y_2 = 1, x = 1, u) = Max_{p_2>0}\{p_2 E[Min\{y_2 = 1, d_2(p_2)(\epsilon_2|u)\}]\}$ and it is obvious that $B(u)$ is increasing if $(\epsilon_2|u)$ is stochastically increasing in terms of u.

Secondly, as $S(u) \geq S(0)$ by the above result and $S(0) \geq S_1$ based on our assumption, so $I_2(p_1(y_1),y_1)$ is constant if $y_1 \leq S_1$, where $p_1(y_1)$ is the maximizer of $\pi(p_1,\frac{y_1}{d_1(p_1)})$, for a given y_1. Hence, $\pi(p_1(y_1),\frac{y_1}{d_1(p_1(y_1))})$ is increasing on $[0,S_1]$.

Thirdly, we show that $\pi(p_1(y_1),\frac{y_1}{d_1(p_1(y_1))})$ is decreasing on $[S_1,+\infty)$. By the result in Song et al. (2005, Property 2), for any given $S_1 \leq y^1 \leq y^2$ and p^2,[17] there exists a p^1 such

[17] Note that p^i and y^i are used to denote particular points (i is not an exponent). We use this notation to distinguish it from functions.

that $I_1(p^1, y^1) > I_1(p^2, y^2)$ and $max(0, y_1 - d_1(p^1)u) \leq max(0, y^2 - d_1(p^2)u)$ for any $u > 0$. Hence, $\pi(p_1(y_1), \frac{y_1}{d_1(p_1(y_1))})$ is decreasing on $[S_1, +\infty)$.

Clearly, $\pi(p_1(y_1), \frac{y_1}{d_1(p_1(y_1))})$ is then unimodal and S_1 is the unique maximizer.

We now show that there is a one-to-one relationship between y_1 and z_1. Note that the centralized system needs to maximize (6), subject to the fact that the two first order conditions (FOCs) in terms of p_1 and z_1 are satisfied. Combining these two conditions we get the following equation:

$$p_1\{k_1 \int_0^{z_1} uf_1(u)du - [\mu_1 - \Theta_1(z_1)]\} - k_1 \int_0^{z_1} \frac{\partial \pi^R(x,u)}{\partial x} uf_1(u)du = 0. \qquad (27)$$

So, CM needs to maximize (6) keeping in mind that the optimal p_1 and z_1 must satisfy (27). We can then show that for $z_1 \in [0, Z_0]$, there is no p_1 such that (27) is satisfied, where Z_0 is the unique positive solution of $U(z_1) = 0$ (DSCM model also uses a similar argument). Hence, we only focus on $z_1 \in (Z_0, +\infty)$, and show that $y_1(z_1) = z_1 d_1(p_1(z_1))$ is increasing in that range, where $p_1(z_1)$ is as defined in Theorem 1. As $y_1'(z_1) = d_1(p_1)[1 - k_1 z_1 \frac{p_1'(z_1)}{p_1(z_1)}]$, it is equivalent to showing that $k_1 z_1 p_1'(z_1) < p_1(z_1)$. Based on the expression of $p_1'(z_1)$ in the proof of Theorem 1, the above inequality can be further reduced to

$$\frac{k_1 - 1}{k_1} p_1(z_1)[\mu_1 - \Theta(z_1)] - (k_1 - 1)p_1(z_1) \int_0^{z_1} uf_1(u)du$$
$$+ k_1 d_1 \int_0^{z_1} \frac{\partial^2 \pi_2(x,u)}{\partial x^2}(z_1 - u)uf_1(u)du + k_1 \int_0^{z_1} \frac{\partial \pi_2(x,u)}{\partial x} uf_1(u)du > 0.$$

This is obvious since $[\mu_1 - \Theta(z_1)] - k_1 \int_0^{z_1} uf_1(u)du > 0$ on $(Z_0, +\infty)$ and $\int_0^{z_1} [\frac{\partial \pi_2(x,u)}{\partial x} + k_1 x \frac{\partial^2 \pi_2(x,u)}{\partial x^2}] uf_1(u)du \geq 0$.

Since $\pi(p_1(y_1), \frac{y_1}{d_1(p_1(y_1))})$ is unimodal in y_1 (recall that $z_1 = \frac{y_1}{d_1(p_1(y_1))}$) and there is a one-to-one relation between y_1 and z_1, $\pi(p_1(z_1), z_1)$ is also unimodal. Furthermore, note that Theorem 5 then also holds true since the retailer's problem in DRPOM is equivalent to CM with $c_1 = c_2 = w_1$. □

Proof of Lemma 2:

Taking partial derivatives of (11), we have

$$\frac{\partial \pi_2^M(y_2, x, \epsilon_1)}{\partial y_2} = \frac{k_2 - 1}{k_2} A(\epsilon_1) y_2^{-\frac{1}{k_2}} + \frac{1}{k_2} A(\epsilon_1) x y_2^{-\frac{k_2+1}{k_2}} - c_2,$$

$$\frac{\partial^2 \pi_2^M(y_2, x, \epsilon_1)}{\partial y_2^2} = -\frac{k_2 - 1}{k_2^2} A(\epsilon_1) y_2^{-\frac{k_2+1}{k_2}} - \frac{k_2 + 1}{k_2^2} A(\epsilon_1) x y_2^{-\frac{2k_2+1}{k_2}} < 0.$$

Hence, $\pi_2^M(y_2, x, \epsilon_1)$ is concave in y_2. The optimal $y_2(x, \epsilon_1)$ is given by the solution of the first order condition which yields (12). Using implicit differentiation, we can establish

$$\frac{\partial y_2(x,\epsilon)}{\partial x} = \frac{k_2 y_2}{(k_2-1)y_2 + (k_2+1)x}|_{\{y_2=y_2(x,\epsilon_1)\}}$$

$$\text{and} \quad \frac{\partial^2 y_2(x,\epsilon_1)}{\partial x^2} = -\frac{k_2(k_2+1)[xy_2 + (k_2-1)y_2^2]}{[(k_2-1)y_2 + (k_2+1)x]^3}|_{\{y_2=y_2(x,\epsilon_1)\}}.$$

Hence $y_2(x, \epsilon)$ is increasing concave in $x(\geq 0)$ (note that $k_2 > 1$). Since both $p_2(y_2)$ and $w_2(y_2)$ are decreasing convex in y_2, the other results follow. □

Proof of Theorem 3:

We need to consider two cases: $x \in [0, S(\epsilon_1)]$ and $x \in (S(\epsilon_1), +\infty)$.

Case 1. $x \in [0, S(\epsilon_1)]$**:** As both the optimal retail price p_2 and the wholesale price w_2 can be expressed in terms of y_2 for any given realization of ϵ_1, the retailer's total expected profit in period two can be expressed in terms of y_2 as:

$$\pi_2^R(y_2, x, \epsilon_1) = H_2(\epsilon_1)^{\frac{1-k_2}{k_2}}[\mu_2(\epsilon_1) - \Theta_2(H_2(\epsilon_1)|\epsilon_1)]y_2^{\frac{k_2-1}{k_2}} - A(\epsilon_1)y_2^{\frac{k_2-1}{k_2}} + A(\epsilon_1)xy_2^{-\frac{1}{k_2}}.$$

Since $\mu_2(\epsilon_1) - \Theta_2(H_2(\epsilon_1)|\epsilon_1) = \int_0^z uf_2(u|\epsilon_1)du + z[1 - F_2(z|\epsilon_1)]$, $\pi_2^R(y_2, x, \epsilon_1)$ can be simplified as

$$\pi_2^R(y_2, x, \epsilon_1) = H_2(\epsilon_1)^{\frac{1-k_2}{k_2}}(\int_0^{H_2(\epsilon_1)} uf_2(u|\epsilon_1)du)y_2^{\frac{k_2-1}{k_2}} + A(\epsilon_1)xy_2^{-\frac{1}{k_2}}.$$

From the definition of $H_2(\epsilon_1)$, this can be further simplified as

$$\pi_2^R(y_2, x, \epsilon_1) = A(\epsilon_1)y_2^{-\frac{1}{k_2}}\{\frac{y_2}{k_2-1} + x\}. \tag{28}$$

Clearly, $\pi_2^R(x, \epsilon_1) = \pi_2^R(y_2(x, \epsilon_1), x, \epsilon_1)$ and $\pi_2^M(x, \epsilon_1) = \pi_2^M(y_2(x, \epsilon_1), x, \epsilon_1)$, where $\pi_2^M(y_2(x, \epsilon_1), x, \epsilon_1)$ is given in (11). Substituting $y_2(x, \epsilon_1)$ characterized in (12) and noting that $A(\epsilon_1)y_2^{-\frac{1}{k_2}}|_{\{y_2=y_2(x,\epsilon_1)\}} = \frac{k_2c_2y_2}{(k_2-1)y_2+x}|_{\{y_2=y_2(x,\epsilon_1)\}}$, we get

$$\pi_2^M(x, \epsilon_1) = c_2\frac{(y_2-x)^2}{(k_2-1)y_2+x}|_{\{y_2=y_2(x,\epsilon)\}}$$

$$\text{and} \quad \pi_2^R(x, \epsilon_1) = \frac{k_2c_2}{k_2-1}\frac{y_2[y_2+(k_2-1)x]}{(k_2-1)y_2+x}|_{\{y_2=y_2(x,\epsilon_1)\}}.$$

Case 2. $x \in (S(\epsilon_1), +\infty)$**:** In this case, the optimal order-up-to level $y_2(x, \epsilon_1) = x$ and the optimal wholesale price $w_2(x, \epsilon_1) = c_2$, which results

$$\pi_2^M(x, \epsilon_1) = 0 \quad \text{and} \quad \pi_2^R(x, \epsilon_1) = \frac{k_2}{k_2 - 1} A(\epsilon_1) x^{\frac{k_2-1}{k_2}}.$$

Combining the results for the two cases, we get the profit expressions in Theorem 3.

In order to establish the properties of $\pi_2^R(x, \epsilon_1)$ and $\pi_2^M(x, \epsilon_1)$, we only need to focus on the interval $[0, S(\epsilon_1)]$ (for $[S(\epsilon_1), +\infty]$ it is obvious). Taking partial derivatives of $\pi_2^M(x, \epsilon_1)$ with respect to x, we get

$$\frac{\partial \pi_2^M(x, \epsilon_1)}{\partial x} = -\frac{c_2(y_2 - x)}{(k_2 - 1)y_2 + x}|_{\{y_2 = y_2(x,\epsilon_1)\}} \leq 0.$$

Hence, $\pi_2^M(x, \epsilon_1)$ is decreasing with respect to x on $[0, S(\epsilon_1)]$. In order to prove the convexity of $\pi_2^M(x, \epsilon_1)$ in terms of x on $[0, S(\epsilon_1)]$, it suffices to show that

$$\{(y_2' - 1)[(k_2 - 1)y_2 + x]\}|_{\{y_2 = y_2(x,\epsilon)\}} \leq \{(y_2 - x)[(k_2 - 1)y_2' + 1]\}|_{\{y_2 = y_2(x,\epsilon)\}}.$$

By the expression of y_2' in Lemma 2, the above is equivalent to proving that $\{(k_2 - 1)y_2 + x\}|_{\{y_2 = y_2(x,\epsilon)\}} \geq 0$, which is obviously true.

On the other hand, taking partial derivatives of $\pi_2^R(x, \epsilon_1)$ with respect to x, we get

$$\frac{\partial \pi_2^R(x, \epsilon_1)}{\partial x} = \frac{k_2^2 c_2 (x + y_2) y_2}{[(k_2 - 1)y_2 + x][(k_2 - 1)y_2 + (k_2 + 1)x]}|_{\{y_2 = y_2(x,\epsilon_1)\}} \geq 0.$$

Hence, $\pi_2^R(x, \epsilon_1)$ is increasing with respect to x on $[0, S(\epsilon_1)]$. Furthermore,

$$\frac{\partial^2 \pi_2^R(x, \epsilon_1)}{\partial x^2} = -\frac{k_2^2 y_2 c_2}{[(k_2 - 1)y_2 + x][(k_2 - 1)y_2 + (k_2 + 1)x]^3}\{3(k_2 - 1)y_2^2 + (k_2 + 1)x^2 + 2(k_2 + 1)xy_2\} < 0.$$

Finally, $\frac{\partial \pi_2^R(x,\epsilon_1)}{\partial x} + k_2 x \frac{\partial^2 \pi_2^R(x,\epsilon_1)}{\partial x^2} = (k_2 - 1)xy_2^2 + (k_2 + 1)x^3 + (k_2 - 1)x^2 y_2 + (k_2 - 1)^2 y_2^3 > 0$ for any $x \in [0, +\infty)$, which proves part (3). □

Proof of Theorem 4:

It is easy to check that

$$\frac{\partial L(p_1, z_1)}{\partial p_1}|_{\{L(p_1,z_1)=0\}} = \frac{k_1}{p_1} \int_0^{z_1} [\frac{\partial \pi_2^R(x, u)}{\partial x} + k_1 x \frac{\partial^2 \pi_2^R(x, u)}{\partial x^2}] u f_1(u) du.$$

By the result of part (3) in Theorem 3 we get $\frac{\partial L(p_1,z_1)}{\partial p_1}|_{\{L(p_1,z_1)=0\}} > 0$ for any $z_1 (> 0)$ if $k_1 \leq k_2$. For any $z \in (Z_0, +\infty)$, as $lim_{p_1 \to 0^+} L(p_1, z_1) < 0$ and $lim_{p_1 \to +\infty} L(p_1, z_1) > 0$, there exists a unique $p_1(z_1)$ such that $L(p_1, z_1) = 0$. Substituting this $p_1(z_1)$ in (19), we can express the profit function only in terms of z_1. □

Proof of Theorem 6:

If $k_1 \neq k_2$, for any given feasible $z_1(>0)$ we get the closed form expression of $p_1(z_1)$ by (23):

$$p_1(z_1) = \frac{k_2}{k_1(k_2-1)}\{-\frac{[\mu_1 - \Theta_1(z_1)] - k_1\int_0^{z_1} uf_1(u)du}{\int_0^{z_1} B(u)(z_1-u)^{-\frac{1}{k_2}}uf_1(u)du}\}^{\frac{k_2}{k_1-k_2}}.$$

Substituting this $p_1(z_1)$ in (24), we can express the profit function only in terms of z_1.

If $k_1 = k_2 = k$, let $d_1(p_1) = d_2(p_1) = d(p_1)$ and $z_1 = \frac{y_1}{d(p_1)}$. The retailer's expected two-period profit can be simplified as $\pi^R(p_1, z_1) = p_1 d(p_1) I(z_1) - w_1 d(p_1) z_1$ where $I(z_1) = \mu_1 - \Theta_1(z_1) + \int_0^{z_1} B(u)(z_1-u)^{\frac{k-1}{k}} f_1(u)du$. The manufacturer's total expected profit can be simplified as: $\pi^M(p_1, z_1) = \frac{k-1}{k} p_1 d(p_1) I(z_1) - c_1 d(p_1) z_1$, under the constraint of $kz_1 I'(z_1) = (k-1)I(z_1)$. For any given $z_1(\geq 0)$, from $\frac{\partial \pi^M(p_1,z_1)}{\partial p_1} = k\frac{d(p_1)}{p_1}\{-(\frac{k-1}{k})^2 I(z_1)p_1 + c_1 z_1\} = 0$, the optimal p_1 is given by:

$$p_1(z_1) = (\frac{k}{k-1})^2 \frac{c_1 z_1}{I(z_1)}. \tag{29}$$

Thus, the manufacturer's total expected profit over the two-period planning horizon can be expressed solely in terms of z_1 as $\pi^M(p_1(z_1), z_1) = \frac{(k-1)^{2k-1}}{c_1^{k-1}k^{2k}} \frac{I(z_1)^k}{z_1^{k-1}}$. The optimal z_1^* follows from $\frac{\partial \pi^M(p_1(z_1),z_1)}{\partial z_1} = 0$ via any one-variable search technique.

Note that, based on $\frac{\partial \pi^R(p_1,z_1)}{\partial p_1} = 0$, the wholesale price $w_1 = \frac{k-1}{k} p_1 \frac{I(z_1)}{z_1}$. Combining this and (29), we get $w_1^* = \frac{k}{k-1}c_1$. Substituting $w_1 = \frac{k-1}{k} p_1 \frac{I(z_1)}{z_1}$ into $\pi^R(p_1, z_1)$, we get $\pi^R(p_1, z_1) = \frac{1}{k} p_1 I d(p_1)(z_1)$. On the other hand, by (29) we get $c_1 z_1 = (\frac{k-1}{k})^2 I(z_1) p_1$. Substituting this into $\pi^M(p_1, z_1)$, we get $\pi^M(p_1, z_1) = \frac{k-1}{k^2} p_1 I d(p_1)(z_1)$. Hence, $\frac{\Pi^M}{\Pi^R} = \frac{k-1}{k}$ follows. □

Proof of Theorem 7:

For any feasible $z_1(>0)$, we get

$$\begin{aligned}
\frac{\partial L(q,z_1)}{\partial q} &= k_1 z_1[1 - F_1(z_1)] - (k_1-1)[\mu_1 - \Theta_1(z_1)] \\
&\quad + q\int_0^{z_1} [-k_1 z_1 + (k_1 - k_2)] f_1(u) f_2(q(z_1-u))[z_1 - u]du \\
&\quad + \int_0^{z_1} [k_1 z_1 - (k_2-1)(z_1-u) - (k_1-k_2)](1-F_2) f_1 du,
\end{aligned}$$

$$\text{and} \quad \frac{\partial^2 L(q,z_1)}{\partial q^2} = -\int_0^{z_1} [(k_2-1)u + (k_2+1)z_1] f_1(u) f_2(q(z_1-u))[z_1-u]du < 0.$$

As $L(0, z_1) = F_1(z_1) > 0$ if $k_1 < k_2$ and $L(+\infty, z_1) = F_1(z_1) > 0$ if $k_1 > k_2$, for any feasible $z_1(> 0)$ there exists a unique $p_1(z_1)$ satisfying $L(q, z_1) = 0$. If $k_1 = k_2$, the proof is almost identical to the one of Theorem 6. □

Appendix C: Numerical Experiment Setting Details

Note that $(\overline{\epsilon}_2|\overline{\epsilon}_1)$ is normally distributed with mean $\mu(\overline{\epsilon}_1 = x_1) = \overline{\mu}_2 + \rho\frac{\overline{\sigma}_2}{\overline{\sigma}_1}[x_1 - \overline{\mu}_1]$ and standard deviation $\sigma(\overline{\epsilon}_1 = x_1) = \overline{\sigma}_2\sqrt{1-\rho^2}$, where ρ is the correlation coefficient between the demands for the two periods. As the density function $\overline{f}_1(x_1)$ of $\overline{\epsilon}_1$ can be expressed as $\int_{-\infty}^{+\infty} \overline{f}(x_1, x_2)dx_2$, i.e., $\overline{f}_1(x_1) = \frac{1}{\sqrt{2\pi}\overline{\sigma}_1}e^{-\frac{1}{2}\{\frac{x_1-\overline{\mu}_1}{\overline{\sigma}_1}\}^2}$, and the density function $\overline{f}_2(x_2|x_1)$ can be expressed as $\frac{\overline{f}(x_1,x_2)}{\overline{f}_1(x_1)}$, i.e., $\overline{f}_2(x_2|x_1) = \frac{1}{\sqrt{2\pi}\sigma(x_1)}e^{-\frac{1}{2}\{\frac{x_2-(\mu(x_1))}{\sigma(x_1)}}^2$, we get $f_1(x_1) = \frac{1}{N_1}\overline{f}_1(x_1)$ and $f_2(x_2|x_1) = \frac{1}{N_2}\overline{f}_2(x_2|x_1)$ for any $x_1 \in [0, +\infty)$ and any $x_2 \in [0, +\infty)$ where N_1 and N_2 are normalizing factors. They can be expressed as: $N_1 = 1 - \Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})$ $N_2 = 1 - \Phi(-\frac{\mu(x_1)}{\sigma(x_1)})$. Next we can calculate the mean values μ_1 and $\mu_2(x_1)$ of ϵ_1 and $(\epsilon_2|\epsilon_1 = x_1)$, respectively, as:

$$\mu_1 = \overline{\mu}_1 + \frac{\overline{\sigma}_1\phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})}{1-\Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})} \quad \text{and} \quad \mu_2(x_1) = \mu(x_1) + \frac{\sigma(x_1)\phi(-\frac{\mu(x_1)}{\sigma(x_1)})}{1-\Phi(-\frac{\mu(x_1)}{\sigma(x_1)})}.$$

The standard deviation σ_1 of ϵ_1 can be expressed as:

$$\sigma_1 = \overline{\sigma}_2^2\{1 - \frac{\phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})}{1-\Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})}[\frac{\phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})}{1-\Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})} + \frac{\overline{\mu}_1}{\overline{\sigma}_1}]\}.$$

Note that

$$F_1(z) = \frac{1}{[1-\Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})]}\{\Phi(\frac{z-\overline{\mu}_1}{\overline{\sigma}_1}) - \Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})\},$$

$$\text{and} \int_0^z uf_1(u)du = \frac{1}{[1-\Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})]}\{\overline{\sigma}_1[\phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1}) - \phi(\frac{z-\overline{\mu}_1}{\overline{\sigma}_1})] + \overline{\mu}_1[\Phi(\frac{z-\overline{\mu}_1}{\overline{\sigma}_1}) - \Phi(-\frac{\overline{\mu}_1}{\overline{\sigma}_1})]\}.$$

Thus, based on Lemma 1, we can express $\Theta_1(z)$ and $\Lambda_1(z)$ in terms of $\phi(x)$ and $\Phi(x)$. Similarly, $\Theta_2(z|x_1) = \mu_2(x_1) - \int_0^z uf_2(u|x_1)du - z[1 - F_2(z|x_1)]$ and $\Lambda_2(z|x_1) = zF_2(z|x_1) - \int_0^z uf_2(u|x_1)du$ for any $z \in [0, +\infty)$ and any $x_1 \in [0, +\infty)$. Now

$$F_2(z|x_1) = \frac{1}{[1-\Phi(-\frac{\mu(x_1)}{\sigma(x_1)})]}\{\Phi(\frac{z-\mu(x_1)}{\sigma(x_1)}) - \Phi(-\frac{\mu(x_1)}{\sigma(x_1)},$$

$$\text{and} \quad \int_0^z uf_2(u|x_1)du = \frac{1}{[1-\Phi(-\frac{\mu(x_1)}{\sigma(x_1)})]}\{\sigma[\phi(-\frac{\mu(x_1)}{\sigma(x_1)}) - \phi(\frac{z-(\mu(x_1))}{\sigma(x_1)})]$$

$$+ (\mu(x_1))[\Phi(\frac{z - (\mu(x_1))}{\sigma(x_1)}) - \Phi(-\frac{\mu(x_1)}{\sigma(x_1)})]\}.$$

Hence, we also can express $\Theta_2(z|x_1)$ and $\Lambda_2(z|x_1)$ in terms of $\phi(x)$ and $\Phi(x)$. Consequently, both $V_1(z)$ and $V_2(z|x_1)$ can be expressed only in terms of $\phi(x)$ and $\Phi(x)$, which facilitate numerical computations.

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