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Abstract

This paper deals with the problem of guaranteed cost control for uncertain neutral stochastic systems. The parameter uncertainties are assumed to be time-varying but norm-bounded. Dynamic output feedback controllers are designed such that, for all admissible uncertainties, the resulting closed-loop system is mean-square asymptotically stable and an upper bound on the closed-loop value of the cost function is guaranteed. By employing a linear matrix inequality (LMI) approach, a sufficient condition for the solvability of the underlying problem is obtained. A numerical example is provided to demonstrate the potential of the proposed techniques.

Key Words: Guaranteed cost control, linear matrix inequality, neutral stochastic systems, output feedback, uncertain systems.

Résumé

Cet article traite de la commande des systèmes stochastiques avec incertitudes de type borné en norme. Un correcteur de type retour de sortie dynamique est synthétisé de manière à ce que la boucle-fermée soit stable et garantie que le coût choisi est borné pour toutes les incertitudes admissibles. Les conditions suffisantes établies pour la synthèse de ce contrôleur sont en forme de LMIs. Un exemple numérique est proposé pour montrer la validité des résultats proposés.

Mots clés : Commande avec coût garanti, inégalités matricielles linéaires, systèmes à retard, correcteur par retour de sortie dynamique, systèmes incertains.

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1 Introduction

Guaranteed cost control for uncertain systems has been a research topic of recurring interest in recent years. The purpose is the design of a controller such that the resulting closed-loop system is asymptotically stable while an upper bound on the closed-loop value of an integral quadratic cost function is guaranteed. Various approaches have been developed and a great number of results on this topic have been reported in the literature. For example, by a Riccati equation approach, guaranteed cost controllers were designed for uncertain continuous-time systems in Ref. 13. The corresponding results for uncertain discrete-time systems can be found in Refs. 6 and 16, respectively. In the case when both parameter uncertainties and time delays appear in a control system, the guaranteed cost control problem was considered in Ref. 3, where a linear matrix inequality (LMI) approach was developed and a sufficient condition for the solvability of this problem was presented. It is noted that in Ref. 3 state feedback controllers were designed. When not all the states are available directly for feedback, dynamic output feedback controllers were constructed in Ref. 4 to solve the guaranteed cost control problem.

On the other hand, it has been shown that many dynamic systems not only depend on the present and the past states but also involve the derivatives with delays. Such systems can be modelled by functional differential equations of the neutral type. For example, in the study of the loseless transmission problem, a partial differential equation can be transformed to a delay equation of the neutral type (Ref. 2). Stability analysis and stabilization for such systems have been investigated; see, e.g., Refs. 9,12,14,18,15, and the references therein. When the environment disturbances are taken into account in the study of neutral systems, neutral stochastic systems are introduced in the literature (Ref. 8). Results on asymptotic stability and exponential stability in the mean-square sense of neutral stochastic systems have been proposed by adopting different approaches (Refs. 7,8,10,20). It should be pointed out that these results cannot provide an adequate level of performance of a neutral stochastic system. One approach to this problem is the guaranteed cost control approach. It is noted that in the deterministic case, some guaranteed cost control results were obtained in Ref. 19 via the LMI approach. However, for uncertain neutral stochastic systems, to the authors' best knowledge, no results on the guaranteed cost control problem are available in the literature.

In this paper, we consider the guaranteed cost control problem for uncertain neutral stochastic systems. The parameter uncertainties are assumed to be time-varying but norm-bounded. The time delay is assumed to appear in both the state and measurement equations. A linear quadratic cost function is defined as a performance measure for the closed-loop system. Attention is focused on the design of a dynamic output feedback controller which ensures not only the mean-square asymptotic stability of the closed-loop system but also an upper bound on the closed-loop value of the cost function. A sufficient condition for the solvability of this problem is obtained. It has been shown that a desired dynamic output feedback controller can be constructed by solving an LMI, which can be handled easily by using the recently developed algorithms (Ref. 1). Finally, we provide a numerical example to demonstrate the effectiveness and applicability of the proposed approach.

2 Problem Formulation

Consider a class of neutral stochastic systems with state delay and parameter uncertainties described by

$$\begin{aligned}
(\Sigma):\quad & d\left[x(t) - Dx(t-\tau)\right] \\
& = \left[(A+\Delta A(t))\,x(t) + (A_d+\Delta A_d(t))\,x(t-\tau) + (B_1+\Delta B_1(t))\,u(t)\right]dt \\
& \quad + \left[(E+\Delta E(t))\,x(t) + (E_d+\Delta E_d(t))\,x(t-\tau)\right]d\omega(t), \qquad (1)\\
& dy(t) \\
& = \left[(C+\Delta C(t))\,x(t) + (C_d+\Delta C_d(t))\,x(t-\tau) + (B_2+\Delta B_2(t))\,u(t)\right]dt \\
& \quad + \left[(H+\Delta H(t))\,x(t) + (H_d+\Delta H_d(t))\,x(t-\tau)\right]d\omega(t), \qquad (2)\\
& x(t) \\
& = \varphi(t), \quad \forall t \in [-\tau, 0], \qquad (3)
\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $z(t) \in \mathbb{R}^q$ is the controlled output; $\omega(t)$ is a zero-mean real scalar Wiener process on a probability space (Ω, $\mathcal{F}$, $\mathcal{P}$) relative to an increasing family $(\mathcal{F}_t)_{t>0}$ of σ-algebras $\mathcal{F}_t \subset \mathcal{F}$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. We assume

$$\mathcal{E}\left\{d\omega(t)\right\} = 0, \quad \mathcal{E}\left\{d\omega(t)^2\right\} = dt, \qquad (4)$$

where $\mathcal{E}\left\{\cdot\right\}$ is the expectation operator.

In system (Σ), the scalar $\tau > 0$ is the time delay of the system, which is unknown, $\varphi(t)$ is the initial condition, A, A_d, B_1, B_2, C, C_d, E, E_d, H and H_d are known real constant matrices, and $\Delta A(t)$, $\Delta A_d(t)$, $\Delta B_1(t)$, $\Delta B_2(t)$, $\Delta C(t)$, $\Delta C_d(t)$, $\Delta E(t)$, $\Delta E_d(t)$, $\Delta H(t)$ and $\Delta H_d(t)$ represent the parameter uncertainties of the system, which are assumed to be of the form

$$\begin{aligned}
&\begin{bmatrix} \Delta A(t) & \Delta A_d(t) & \Delta B_1(t) & \Delta E(t) & \Delta E_d(t) \\ \Delta C(t) & \Delta C_d(t) & \Delta B_2(t) & \Delta H(t) & \Delta H_d(t) \end{bmatrix} \\
&\qquad = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(t) \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & N_5 \end{bmatrix}, \qquad (5)
\end{aligned}$$

where M_i, N_j, $i = 1, 2$, $j = 1, \ldots, 5$, are known real constant matrices and $F(\cdot) : \mathbb{R} \to \mathbb{R}^{k \times l}$ is an unknown time-varying matrix function satisfying

$$F(t)^T F(t) \leq I, \quad \forall t \tag{6}$$

The parameter uncertainties $\Delta A(t)$, $\Delta A_d(t)$, $\Delta B_1(t)$, $\Delta B_2(t)$, $\Delta C(t)$, $\Delta C_d(t)$, $\Delta E(t)$, $\Delta E_d(t)$, $\Delta H(t)$ and $\Delta H_d(t)$ are said to be admissible if both (5) and (6) hold.

Associated with system (Σ) is the cost function defined as

$$J = \mathcal{E}\left\{ \int_0^{\infty} \left[x(t)^T Q_1 x(t) + u(t)^T Q_2 u(t) \right] dt \right\}, \tag{7}$$

where Q_1 and Q_2 are positive matrices.

Throughout the paper, we make the following assumption on the matrix D in (1).

Assumption 2.1 *The matrix D in (1) satisfies*

$$\rho\,(D) < 1, \tag{8}$$

where $\rho\,(D)$ denotes the spectral radius of D.

Now, we consider a dynamic output feedback controller given as

$$(\Sigma_K): \qquad d\xi(t) = A_K \xi(t) dt + B_K dy(t), \tag{9}$$

$$u(t) = C_K \xi(t), \tag{10}$$

where $\xi(t) \in \mathbb{R}^n$ is the controller state, A_K, B_K and C_K are matrices to be determined.

Then, applying the controller in (9) and (10) to the uncertain stochastic neutral system (Σ), we obtain the closed-loop system as

$$\begin{aligned} d\left[\eta(t) - \bar{D} L \eta(t-\tau)\right] = {} & \left[A_{cK}(t)\eta(t) + A_{cKd}(t) L \eta(t-\tau)\right] dt \\ & + \left[E_{cK}(t) L \eta(t) + E_{cKd}(t) L \eta(t-\tau)\right] d\omega(t), \end{aligned} \tag{11}$$

where

$$\eta(t) = \begin{bmatrix} x(t)^T & \xi(t)^T \end{bmatrix}^T,$$

and

$$A_{cK}(t) = A_{cK} + \Delta A_{cK}(t), \quad A_{cdK}(t) = A_{cdK} + \Delta A_{cdK}(t), \tag{12}$$

$$E_{cK}(t) = E_{cK} + \Delta E_{cK}(t), \quad E_{cKd}(t) = E_{cKd} + \Delta E_{cKd}(t), \tag{13}$$

$$A_{cK} = \begin{bmatrix} A & B_1 C_K \\ B_K C & A_K + B_K B_2 C_K \end{bmatrix}, \quad A_{cKd} = \begin{bmatrix} A_d \\ B_K C_d \end{bmatrix}, \tag{14}$$

$$E_{cK} = \begin{bmatrix} E \\ B_K H \end{bmatrix}, \quad E_{cKd} = \begin{bmatrix} E_d \\ B_K H_d \end{bmatrix}, \tag{15}$$

$$\Delta A_{cK}(t) = \begin{bmatrix} \Delta A(t) & \Delta B_1(t) C_K \\ B_K \Delta C(t) & B_K \Delta B_2(t) C_K \end{bmatrix}, \quad \Delta A_{cKd}(t) = \begin{bmatrix} \Delta A_d(t) \\ B_K \Delta C_d(t) \end{bmatrix}, \tag{16}$$

$$\Delta E_{cK}(t) = \begin{bmatrix} \Delta E(t) \\ B_K \Delta H(t) \end{bmatrix}, \quad \Delta E_{cKd}(t) = \begin{bmatrix} \Delta E_d(t) \\ B_K \Delta H_d(t) \end{bmatrix}, \tag{17}$$

$$\bar{D} = \begin{bmatrix} D \\ 0 \end{bmatrix} \quad L = \begin{bmatrix} I & 0 \end{bmatrix}. \tag{18}$$

The guaranteed cost control problem to be addressed in this paper can be formulated as follows: design a dynamic output feedback controller in (9) and (10) such that, for all admissible uncertainties, the closed-loop system in (11) is mean-square asymptotically stable and the value of the cost function in (7) satisfies

$$J \leq J^* \tag{19}$$

for some given scalar $J^* > 0$.

3 Main Results

In this section, an LMI approach will be developed to solve the guaranteed cost control problem formulated in the previous section. We first introduce the following results which will be used in the proof of our main results.

Lemma 3.1 *(Ref. 17) Let $\mathcal{A}$, $\mathcal{S}$ and $\mathcal{W}$ be real matrices of appropriate dimensions with $\mathcal{W} > 0$. Then, we have*

$$2x^T \mathcal{A}\mathcal{S}y \leq x^T \mathcal{A}\mathcal{W}\mathcal{A}^T x + y^T \mathcal{S}^T \mathcal{W}^{-1} \mathcal{S}y.$$

Now, we present our first result in this paper.

Theorem 3.1 *If there exists a matrix $P > 0$ such that the following matrix inequality holds for some matrix $Z > 0$ and scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$,*

$$\begin{bmatrix} \Upsilon_K & PA_{cKd} & A_{cK}^T & L^T E_{cK}^T & P\tilde{M}_{1K} & 0 & \tilde{N}_{1K}^T & \tilde{N}_4^T & \tilde{C}_K^T \\ A_{cKd}^T P & \bar{D}^T P \bar{D} - Z & A_{cKd}^T & E_{cKd}^T & 0 & 0 & N_2^T & N_5^T & 0 \\ A_{cK} & A_{cKd} & -P^{-1} & 0 & \tilde{M}_{1K} & 0 & 0 & 0 & 0 \\ E_{cK}L & E_{cKd} & 0 & -P^{-1} & 0 & \tilde{M}_{1K} & 0 & 0 & 0 \\ \tilde{M}_{1K}^T P & 0 & \tilde{M}_{1K}^T & 0 & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_{1K}^T & 0 & -\epsilon_2 I & 0 & 0 & 0 \\ \tilde{N}_{1K} & N_2 & 0 & 0 & 0 & 0 & -\epsilon_1^{-1} I & 0 & 0 \\ \tilde{N}_4 & N_5 & 0 & 0 & 0 & 0 & 0 & -\epsilon_2^{-1} I & 0 \\ \tilde{C}_K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{Q}^{-1} \end{bmatrix} < 0, \tag{20}$$

where

$$\begin{aligned} \Upsilon_K &= PA_{cK} + A_{cK}^T P + L^T Z L, \\ \tilde{C}_K &= \begin{bmatrix} I & 0 \\ 0 & C_K \end{bmatrix}, \quad \tilde{Q} = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}, \\ \tilde{M}_{1K} &= \begin{bmatrix} M_1 \\ B_K M_2 \end{bmatrix}, \quad \tilde{N}_{1K} = \begin{bmatrix} N_1 & N_3 C_K \end{bmatrix}, \quad \tilde{N}_4 = \begin{bmatrix} N_4 & 0 \end{bmatrix}, \end{aligned}$$

then, the closed-loop system in (11) is mean-square asymptotically stable for all admissible uncertainties. In this case, the value of the cost function in (7) satisfies

$$J \le \mathcal{E}\Big\{ \left[\eta(0) - \bar{D}L\eta(-\tau)\right]^T P \left[\eta(0) - \bar{D}L\eta(-\tau)\right] + \int_{-\tau}^{0} \eta(s)^T L^T Z L \eta(s) ds \Big\}. \tag{21}$$

Proof. From (20), it is easy to see that there exists a scalar $\lambda > 0$ such that

$$\begin{bmatrix} \Upsilon_K + \lambda I & PA_{cKd} & A_{cK}^T & L^T E_{cK}^T & P\tilde{M}_{1K} & 0 & \tilde{N}_{1K}^T & \tilde{N}_4^T & \tilde{C}_K^T \\ A_{cKd}^T P & \bar{D}^T P \bar{D} - Z & A_{cKd}^T & E_{cKd}^T & 0 & 0 & N_2^T & N_5^T & 0 \\ A_{cK} & A_{cKd} & -P^{-1} & 0 & \tilde{M}_{1K} & 0 & 0 & 0 & 0 \\ E_{cK}L & E_{cKd} & 0 & -P^{-1} & 0 & \tilde{M}_{1K} & 0 & 0 & 0 \\ \tilde{M}_{1K}^T P & 0 & \tilde{M}_{1K}^T & 0 & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_{1K}^T & 0 & -\epsilon_2 I & 0 & 0 & 0 \\ \tilde{N}_{1K} & N_2 & 0 & 0 & 0 & 0 & -\epsilon_1^{-1} I & 0 & 0 \\ \tilde{N}_4 & N_5 & 0 & 0 & 0 & 0 & 0 & -\epsilon_2^{-1} I & 0 \\ \tilde{C}_K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{Q}^{-1} \end{bmatrix} < 0,$$

which, by the Schur complement equivalence, gives

$$\begin{bmatrix} \Upsilon_K + \lambda I + \tilde{C}_K^T \tilde{Q} \tilde{C}_K & PA_{cKd} & A_{cK}^T & L^T E_{cK}^T \\ A_{cKd}^T P & \bar{D}^T P \bar{D} - Z & A_{cKd}^T & E_{cKd}^T \\ A_{cK} & A_{cKd} & -P^{-1} & 0 \\ E_{cK} L & E_{cKd} & 0 & -P^{-1} \end{bmatrix}$$
$$+ \epsilon_1 \begin{bmatrix} \tilde{N}_{1K}^T \\ N_2^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{N}_{1K}^T \\ N_2^T \\ 0 \\ 0 \end{bmatrix}^T + \epsilon_1^{-1} \begin{bmatrix} P\tilde{M}_{1K} \\ 0 \\ \tilde{M}_{1K} \\ 0 \end{bmatrix} \begin{bmatrix} P\tilde{M}_{1K} \\ 0 \\ \tilde{M}_{1K} \\ 0 \end{bmatrix}^T$$
$$+ \epsilon_2 \begin{bmatrix} \tilde{N}_4^T \\ N_5^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{N}_4^T \\ N_5^T \\ 0 \\ 0 \end{bmatrix}^T + \epsilon_2^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{M}_{1K} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{M}_{1K} \end{bmatrix}^T < 0. \tag{22}$$

Noting the expressions in (16) and (17) and using Lemma 3.1, we have

$$\begin{bmatrix} P\Delta A_{cK}(t) + \Delta A_{cK}(t)^T P & P\Delta A_{cKd}(t) & \Delta A_{cK}(t)^T & 0 \\ \Delta A_{cKd}(t)^T P & 0 & \Delta A_{cKd}(t)^T & 0 \\ \Delta A_{cK}(t) & \Delta A_{cKd}(t) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} P\tilde{M}_{1K} \\ 0 \\ \tilde{M}_{1K} \\ 0 \end{bmatrix} F(t) \begin{bmatrix} \tilde{N}_{1K} & N_2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \tilde{N}_{1K}^T \\ N_2^T \\ 0 \\ 0 \end{bmatrix} F(t)^T \begin{bmatrix} \tilde{M}_{1K}^T P & 0 & \tilde{M}_{1K}^T & 0 \end{bmatrix}$$
$$\leq \epsilon_1 \begin{bmatrix} \tilde{N}_{1K}^T \\ N_2^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{N}_{1K}^T \\ N_2^T \\ 0 \\ 0 \end{bmatrix}^T + \epsilon_1^{-1} \begin{bmatrix} P\tilde{M}_{1K} \\ 0 \\ \tilde{M}_{1K} \\ 0 \end{bmatrix} \begin{bmatrix} P\tilde{M}_{1K} \\ 0 \\ \tilde{M}_{1K} \\ 0 \end{bmatrix}^T, \tag{23}$$

and

$$\begin{bmatrix} 0 & 0 & 0 & L^T \Delta E_{cK}(t)^T \\ 0 & 0 & 0 & \Delta E_{cKd}(t)^T \\ 0 & 0 & 0 & 0 \\ \Delta E_{cK}(t) L & \Delta E_{cKd}(t) & 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{M}_{1K} \end{bmatrix} F(t) \begin{bmatrix} \tilde{N}_4 & N_5 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \tilde{N}_4^T \\ N_5^T \\ 0 \\ 0 \end{bmatrix} F(t)^T \begin{bmatrix} 0 & 0 & 0 & \tilde{M}_{1K}^T \end{bmatrix}$$

$$\leq \epsilon_2 \begin{bmatrix} \tilde{N}_4^T \\ N_5^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{N}_4^T \\ N_5^T \\ 0 \\ 0 \end{bmatrix}^T + \epsilon_2^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{M}_{1K} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{M}_{1K} \end{bmatrix}^T . \tag{24}$$

It follows from (22)–(24) that

$$\begin{bmatrix} \begin{array}{c} PA_{cK}(t) + A_{cK}(t)^T \\ P + L^T Z L + \\ \tilde{C}_K^T \tilde{Q} \tilde{C}_K + \lambda I \end{array} & PA_{cKd}(t) & A_{cK}(t)^T & L^T E_{cK}(t)^T \\ A_{cKd}(t)^T P & \bar{D}^T P \bar{D} - Z & A_{cKd}(t)^T & E_{cKd}(t)^T \\ A_{cK}(t) & A_{cKd}(t) & -P^{-1} & 0 \\ E_{cK}(t) L & E_{cKd}(t) & 0 & -P^{-1} \end{bmatrix} < 0. \tag{25}$$

Applying the Schur complement equivalence to (25) results in

$$\Theta(t) + \begin{bmatrix} \tilde{C}_K^T \tilde{Q} \tilde{C}_K + \lambda I & 0 \\ 0 & 0 \end{bmatrix} < 0 \tag{26}$$

$$\begin{aligned} \Theta(t) = & \begin{bmatrix} PA_{cK}(t) + A_{cK}(t)^T P + L^T Z L & PA_{cKd}(t) \\ A_{cKd}(t)^T P & \bar{D}^T P \bar{D} - Z \end{bmatrix} \\ & + \begin{bmatrix} A_{cK}(t)^T \\ A_{cKd}(t)^T \end{bmatrix} P \begin{bmatrix} A_{cK}(t)^T \\ A_{cKd}(t)^T \end{bmatrix}^T \\ & + \begin{bmatrix} L^T E_{cK}(t)^T \\ E_{cKd}(t)^T \end{bmatrix} P \begin{bmatrix} L^T E_{cK}(t)^T \\ E_{cKd}(t)^T \end{bmatrix}^T . \end{aligned} \tag{27}$$

Now, choose the Lyapunov function candidate for the closed-loop system in (11) as:

$$\begin{aligned} V(\eta_t, t) = & \left[\eta(t) - \bar{D}L\eta(t-\tau)\right]^T P \left[\eta(t) - \bar{D}L\eta(t-\tau)\right] \\ & + \int_{t-\tau}^{t} \eta(s)^T L^T Z L \eta(s) ds, \end{aligned} \tag{28}$$

where

$$\eta_t = \eta(t+\theta), \quad -\tau \leq \theta \leq 0.$$

Then, by Itô's formula, the stochastic differential $dV(x_t, t)$ can be obtained as (see Refs. 8 and 11):

$$dV(\eta_t,t) = \mathcal{L}V(\eta_t,t)dt + 2\left[\eta(t) - \bar{D}L\eta(t-\tau)\right]^T \\ P\left[E_{cK}(t)L\eta(t) + E_{cKd}(t)L\eta(t-\tau)\right]d\omega(t),$$

where

$$\begin{aligned}
&\mathcal{L}V(\eta_t,t) \\
&\quad = 2\left[\eta(t) - \bar{D}L\eta(t-\tau)\right]^T P\left[A_{cK}(t)\eta(t) + A_{cKd}(t)L\eta(t-\tau)\right] \\
&\qquad + \left[E_{cK}(t)L\eta(t) + E_{cKd}(t)L\eta(t-\tau)\right]^T P\left[E_{cK}(t)L\eta(t) + E_{cKd}(t)L\eta(t-\tau)\right] \\
&\qquad + \eta(t)^T L^T Z L\eta(t) - \eta(t-\tau)^T L^T Z L\eta(t-\tau) \\
&\quad = 2\eta(t)^T P\left[A_{cK}(t)\eta(t) + A_{cKd}(t)\delta(t-\tau)\right] \\
&\qquad - 2\delta\left(t-\tau\right)^T \bar{D}^T P\left[A_{cK}(t)\eta(t) + A_{cKd}(t)\delta(t-\tau)\right] \\
&\qquad + \left[E_{cK}(t)L\eta(t) + E_{cKd}(t)\delta(t-\tau)\right]^T P\left[E_{cK}(t)L\eta(t) + E_{cKd}(t)\delta(t-\tau)\right] \\
&\qquad + \eta(t)^T L^T Z L\eta(t) - \delta(t-\tau)^T Z\delta(t-\tau),
\end{aligned} \tag{29}$$

where

$$\delta\left(t-\tau\right) = L\eta(t-\tau).$$

Using Lemma 3.1 again, we have

$$\begin{aligned}
&-2\delta\left(t-\tau\right)^T \bar{D}^T P\left[A_{cK}(t)\eta(t) + A_{cKd}(t)\delta(t-\tau)\right] \\
&\qquad \le \delta\left(t-\tau\right)^T \bar{D}^T P\bar{D}\delta\left(t-\tau\right) + \left[A_{cK}(t)\eta(t) + A_{cKd}(t)\delta(t-\tau)\right]^T \\
&\qquad\qquad P\left[A_{cK}(t)\eta(t) + A_{cKd}(t)\delta(t-\tau)\right].
\end{aligned} \tag{30}$$

By (29) and (30), it can be deduced that

$$\mathcal{L}V(\eta_t,t) \le \left[\begin{array}{cc} \eta(t)^T & \delta(t-\tau)^T \end{array}\right] \Theta(t) \left[\begin{array}{c} \eta(t) \\ \delta(t-\tau) \end{array}\right], \tag{31}$$

where $\Theta(t)$ is given in (27). Then, by (26) and (31), we obtain

$$\mathcal{L}V(\eta_t,t) < -\lambda\left|\eta(t)\right|^2. \tag{32}$$

Considering this inequality and Assumption 2.1 and the stability results in Refs. 8,11, we have that the closed-loop system in (11) is mean-square asymptotically stable for all admissible uncertainties.

Next, we shall show that the inequality in (21) is satisfied. To this end, we note that by (26) and (31), it can also be deduced that

$$\mathcal{L}V(\eta_t,t) < \left[\begin{array}{cc} \eta(t)^T & \delta(t-\tau)^T \end{array}\right] \left[\begin{array}{cc} -\tilde{C}_K^T\tilde{Q}\tilde{C}_K & 0 \\ 0 & 0 \end{array}\right] \left[\begin{array}{c} \eta(t) \\ \delta(t-\tau) \end{array}\right].$$

Integrating both sides from 0 to ∞ and then taking expectation we have

$$-\mathcal{E}\left\{\left[\eta(0)-\bar{D}L\eta(-\tau)\right]^T P\left[\eta(0)-\bar{D}L\eta(-\tau)\right]+\int_{-\tau}^{0}\eta(s)^T L^T Z L\eta(s)ds\right\}$$
$$\leq -\mathcal{E}\left\{\int_0^\infty \left[\eta(t)^T \tilde{C}_K^T \tilde{Q}\tilde{C}_K \eta(t)\right] dt\right\}.$$

Considering this and the relationship

$$J=\mathcal{E}\left\{\int_0^\infty \left[\eta(t)^T \tilde{C}_K^T \tilde{Q}\tilde{C}_K \eta(t)\right] dt\right\},$$

it is easy to show that (21) follows. This completes the proof. □

Now, we are in a position to present the solvability condition for the guaranteed cost control problem.

Theorem 3.2 *Consider the uncertain neutral stochastic system* (Σ) *with the cost function in (7). Then there exists a dynamic output feedback controller in (9) and (10) such that the closed-loop system in (11) is mean-square asymptotically stable and (19) is satisfied for some scalar* $J^*>0$ *if there exist matrices* $X>0$, $Y>0, \Lambda$, Φ *and* Ψ *such that the following LMI holds for some matrix* $Z>0$ *and scalars* $\epsilon_1>0$, $\epsilon_2>0$,

$$\begin{bmatrix} \Gamma_1 & \Omega_1^T & 0 & \Omega_2^T & \Omega_3^T \\ \Omega_1 & \Gamma_2 & \Omega_4^T & \Omega_5^T & 0 \\ 0 & \Omega_4 & -X & \Omega_6^T & \Omega_7^T \\ \Omega_2 & \Omega_5 & \Omega_6 & \Gamma_3 & 0 \\ \Omega_3 & 0 & \Omega_7 & 0 & \Gamma_4 \end{bmatrix} < 0, \tag{33}$$

where

$$\begin{aligned}
\Gamma_1 &= AY+YA^T+B_1\Phi+\Phi^T B_1^T,\\
\Gamma_2 &= \begin{bmatrix} XA+A^TX+\Psi C+C^T\Psi^T & XA_d+\Psi C_d & A^T & A^TX+C^T\Psi^T & E^T \\ A_d^TX+C_d^T\Psi^T & D^TXD-Z & A_d^T & A_d^TX+C_d^T\Psi^T & E_d^T \\ A & A_d & -Y & -I & 0 \\ XA+\Psi C & XA_d+\Psi C_d & -I & -X & 0 \\ E & E_d & 0 & 0 & -Y \end{bmatrix},\\
\Gamma_3 &= \mathrm{diag}\left(-\epsilon_1 I, -\epsilon_2 I, -\epsilon_1^{-1}I, -\epsilon_2^{-1}I, -Q_1^{-1}, -Q_2^{-1}, -Z^{-1}\right),\\
\Gamma_4 &= \mathrm{diag}\left(-I,-I\right),
\end{aligned}$$

$$\begin{aligned}
\Omega_1 &= \begin{bmatrix} \Lambda + A^T \\ A_d^T \\ AY + B_1\Phi \\ \Lambda \\ EY \end{bmatrix}, \ \Omega_2 = \begin{bmatrix} M_1^T \\ 0 \\ N_1 Y + N_3\Phi \\ N_4 Y \\ Y \\ \Phi \\ Y \end{bmatrix}, \ \Omega_3 = \begin{bmatrix} Y \\ 0 \end{bmatrix}, \\
\Omega_4 &= \begin{bmatrix} XE + \Psi H & XE_d + \Psi H_d & 0 & 0 & -I \end{bmatrix}, \\
\Omega_5 &= \begin{bmatrix} M_1^T X + M_2^T \Psi^T & 0 & M_1^T & M_1^T X + M_2^T \Psi^T & 0 \\ 0 & 0 & 0 & 0 & M_1^T \\ N_1 & N_2 & 0 & 0 & 0 \\ N_4 & N_5 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Omega_6 &= \begin{bmatrix} 0 \\ M_1^T X + M_2^T \Psi^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
\Omega_7 &= \begin{bmatrix} 0 \\ E^T X + H^T \Psi^T \end{bmatrix}.
\end{aligned}$$

In this case, a desired dynamic output feedback controller in (9) and (10) can be obtained with parameters as

$$\begin{aligned}
A_K &= W_1^{-1} \left(\Lambda - XAY - \Psi CY - XB_1\Phi - \Psi B_2\Phi\right) W_2^{-T}, &(34)\\
B_K &= W_1^{-1}\Psi, \quad C_K = \Phi W_2^{-T}, &(35)
\end{aligned}$$

where W_1 and W_2 are any nonsingular matrices satisfying

$$W_1 W_2^T = I - XY. \tag{36}$$

Furthermore, the corresponding value of the cost function in (7) satisfies

$$J \leq \mathcal{E}\Bigg\{ \left[\eta(0) - \bar{D}L\eta(-\tau)\right]^T \begin{bmatrix} X & W_1 \\ W_1^T & \Xi \end{bmatrix} \left[\eta(0) - \bar{D}L\eta(-\tau)\right] + \int_{-\tau}^{0} \eta(s)^T L^T Z L \eta(s) ds \Bigg\}, \tag{37}$$

where

$$\Xi = W_1^T \left(X - Y^{-1}\right)^{-1} W_1. \tag{38}$$

Proof. It follows from (33) that

$$\begin{bmatrix} -Y & -I \\ -I & -X \end{bmatrix} < 0,$$

which implies

$$X - Y^{-1} > 0.$$

Therefore, $I - XY$ is nonsingular. Then, under the condition of the theorem, there always exist nonsingular matrices W_1 and W_2 such that (36) is satisfied. Now, set

$$\Pi_1 = \begin{bmatrix} Y & I \\ W_2^T & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} I & X \\ 0 & W_1^T \end{bmatrix}. \tag{39}$$

Then, it is easy to see that both Π_1 and Π_2 are nonsingular. Define

$$P = \Pi_2 \Pi_1^{-1}. \tag{40}$$

Following a similar argument as in Ref. 5, we can deduce that

$$P = \begin{bmatrix} X & W_1 \\ W_1^T & \Xi \end{bmatrix} > 0.$$

On the other hand, by applying Schur complement equivalence to (33), we have

$$\begin{bmatrix} \Gamma_1 + YY & \Omega_1^T & 0 & \Omega_2^T \\ \Omega_1 & \Gamma_2 & \Omega_4^T & \Omega_5^T \\ 0 & \Omega_4 & (XE + \Psi H)(XE + \Psi H)^T - X & \Omega_6^T \\ \Omega_2 & \Omega_5 & \Omega_6 & \Gamma_3 \end{bmatrix} < 0. \tag{41}$$

By Lemma 3.1, it can be shown that

$$\begin{bmatrix} 0 \\ 0 \\ XE + \Psi H \\ 0 \end{bmatrix} \begin{bmatrix} Y & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} Y \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & (XE + \Psi H)^T & 0 \end{bmatrix}$$

$$\leq \begin{bmatrix} 0 \\ 0 \\ XE + \Psi H \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ XE + \Psi H \\ 0 \end{bmatrix}^T + \begin{bmatrix} Y \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} Y \\ 0 \\ 0 \\ 0 \end{bmatrix}^T.$$

This, together with (41), gives

$$\begin{bmatrix} \Gamma_1 & \Omega_1^T & Y\left(XE+\Psi H\right)^T & \Omega_2^T \\ \Omega_1 & \Gamma_2 & \Omega_4^T & \Omega_5^T \\ \left(XE+\Psi H\right)Y & \Omega_4 & -X & \Omega_6^T \\ \Omega_2 & \Omega_5 & \Omega_6 & \Gamma_3 \end{bmatrix} < 0. \tag{42}$$

Considering (34), (35) and (40), the matrix inequality in (42) can be re-written as

$$\left[\begin{array}{cccccccccc} \Pi_1^T\left(PA_{cK}+A_{cK}^TP\right)\Pi_1 & \Pi_1^TPA_{cKd} & \Pi_1^TA_{cK}^T\Pi_2 & \Pi_1^TL^TE_{cK}^T\Pi_2 & \Pi_1^TP\tilde{M}_{1K} & 0 & \Pi_1^T\tilde{N}_{1K}^T & \Pi_1^T\tilde{N}_4^T & \Pi_1^T\tilde{C}_K^T & \Pi_1^TL^T \\ A_{cKd}^TP\Pi_1 & \bar{D}^TP\bar{D}-Z & A_{cKd}^T\Pi_2 & E_{cKd}^T\Pi_2 & 0 & 0 & N_2^T & N_5^T & 0 & 0 \\ \Pi_2^TA_{cK}\Pi_1 & \Pi_2^TA_{cKd} & -\Pi_2^TP^{-1}\Pi_2 & 0 & \Pi_2^T\tilde{M}_{1K} & 0 & 0 & 0 & 0 & 0 \\ \Pi_2^TE_{cK}L\Pi_1 & \Pi_2^TE_{cKd} & 0 & -\Pi_2^TP^{-1}\Pi_2 & 0 & \Pi_2^T\tilde{M}_{1K} & 0 & 0 & 0 & 0 \\ \tilde{M}_{1K}^TP\Pi_1 & 0 & \tilde{M}_{1K}^T\Pi_2 & 0 & -\epsilon_1I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_{1K}^T\Pi_2 & 0 & -\epsilon_2I & 0 & 0 & 0 & 0 \\ \tilde{N}_{1K}\Pi_1 & N_2 & 0 & 0 & 0 & 0 & -\epsilon_1^{-1}I & 0 & 0 & 0 \\ \tilde{N}_4\Pi_1 & N_5 & 0 & 0 & 0 & 0 & 0 & -\epsilon_2^{-1}I & 0 & 0 \\ \tilde{C}_K\Pi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{Q}^{-1} & 0 \\ L\Pi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Z^{-1} \end{array}\right] < 0, \tag{43}$$

Pre- and post multiplying (43) by diag $\left(\Pi_1^{-T}, I, \Pi_2^{-T}, \Pi_2^{-T}, I, I, I, I, I, I\right)$ and its transpose, and then using the Schur complement formula, we obtain

$$\begin{bmatrix} \begin{matrix} PA_{cK} \\ +A_{cK}^T P \\ +L^T Z L \end{matrix} & PA_{cKd} & A_{cK}^T & L^T E_{cK}^T & P\tilde{M}_{1K} & 0 & \tilde{N}_{1K}^T & \tilde{N}_4^T & \tilde{C}_K^T \\ A_{cKd}^T P & \bar{D}^T P \bar{D} - Z & A_{cKd}^T & E_{cKd}^T & 0 & 0 & N_2^T & N_5^T & 0 \\ A_{cK} & A_{cKd} & -P^{-1} & 0 & \tilde{M}_{1K} & 0 & 0 & 0 & 0 \\ E_{cK} L & E_{cKd} & 0 & -P^{-1} & 0 & \tilde{M}_{1K} & 0 & 0 & 0 \\ \tilde{M}_{1K}^T P & 0 & \tilde{M}_{1K}^T & 0 & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_{1K}^T & 0 & -\epsilon_2 I & 0 & 0 & 0 \\ \tilde{N}_{1K} & N_2 & 0 & 0 & 0 & 0 & -\epsilon_1^{-1} I & 0 & 0 \\ \tilde{N}_4 & N_5 & 0 & 0 & 0 & 0 & 0 & -\epsilon_2^{-1} I & 0 \\ \tilde{C}_K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{Q}^{-1} \end{bmatrix} < 0,$$

By this inequality and Theorem 3.1, the desired result follows immediately. □

Remark 3.1 *Note that under Assumption 2.1, Theorem 3.2 provides a sufficient condition for the solvability of the guaranteed cost control problem for uncertain neutral stochastic system. A desired guaranteed cost controller can be constructed by (34) and (35) when (33) is feasible. In this case, a corresponding upper bound on the closed-loop cost function can be obtained by (37). It is noted that (33) is an LMI for some fixed matrix $Z > 0$ and scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$, which can be easily handled by resorting to recently developed algorithms solving LMIs (Ref. 1).*

Now, we consider the nominal neutral stochastic system of (Σ); that is,

$$\begin{aligned} (\Sigma_1): \quad & d\left[x(t) - Dx(t-\tau)\right] \\ & \quad = \left[Ax(t) + A_d x(t-\tau) + B_1 u(t)\right] dt + \left[Ex(t) + E_d x(t-\tau)\right] d\omega(t), \\ & dy(t) \\ & \quad = \left[Cx(t) + C_d x(t-\tau) + B_2 u(t)\right] dt + \left[Hx(t) + H_d x(t-\tau)\right] d\omega(t), \\ & x(t) \\ & \quad = \varphi(t), \quad \forall t \in [-\tau, 0], \end{aligned}$$

Then, by Theorem 3.2, it is easy to have the following corollary.

Corollary 3.1 *Consider the neutral stochastic system (Σ_1) with the cost function in (7). Then there exists a dynamic output feedback controller in (9) and (10) such that the closed-loop system*

$$\begin{aligned} d\left[\eta(t) - \bar{D}L\eta(t-\tau)\right] = & \left[A_{cK}\eta(t) + A_{cKd}L\eta(t-\tau)\right] dt \\ & + \left[E_{cK}L\eta(t) + E_{cKd}L\eta(t-\tau)\right] d\omega(t), \end{aligned}$$

is mean-square asymptotically stable and (19) is satisfied for some scalar $J^* > 0$ *if there exist matrices* $X > 0$*,* $Y > 0, \Lambda$*,* Φ *and* Ψ *such that the following LMI holds for some matrix* $Z > 0$*,*

$$\begin{bmatrix} \Gamma_1 & \Omega_1^T & 0 & \tilde{\Omega}_2^T & \Omega_3^T \\ \Omega_1 & \Gamma_2 & \Omega_4^T & \tilde{\Omega}_5^T & 0 \\ 0 & \Omega_4 & -X & 0 & \Omega_7^T \\ \tilde{\Omega}_2 & \tilde{\Omega}_5 & 0 & \tilde{\Gamma}_3 & 0 \\ \Omega_3 & 0 & \Omega_7 & 0 & \Gamma_4 \end{bmatrix} < 0,$$

where

$$\begin{aligned} \tilde{\Gamma}_3 &= \operatorname{diag}\left(-Q_1^{-1}, -Q_2^{-1}, -Z^{-1}\right), \\ \tilde{\Omega}_2 &= \begin{bmatrix} Y \\ \Phi \\ Y \end{bmatrix}, \ \tilde{\Omega}_5 = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

and Γ_1*,* Γ_2*,* Γ_4*,* Ω_1*,* Ω_3*,* Ω_4 *and* Ω_7 *are given in Theorem 3.2. In this case, a desired dynamic output feedback controller in (9) and (10) can be obtained with parameters as*

$$\begin{aligned} A_K &= W_1^{-1}\left(\Lambda - XAY - \Psi CY - XB_1\Phi - \Psi B_2\Phi\right)W_2^{-T}, \\ B_K &= W_1^{-1}\Psi, \quad C_K = \Phi W_2^{-T}, \end{aligned}$$

where W_1 *and* W_2 *are any nonsingular matrices satisfying*

$$W_1 W_2^T = I - XY.$$

Furthermore, the corresponding value of the cost function in (7) satisfies

$$J \leq \mathcal{E}\Bigg\{ \left[\eta(0) - \bar{D}L\eta(-\tau)\right]^T \begin{bmatrix} X & W_1 \\ W_1^T & \Xi \end{bmatrix} \left[\eta(0) - \bar{D}L\eta(-\tau)\right] + \int_{-\tau}^{0} \eta(s)^T L^T Z L \eta(s) ds \Bigg\},$$

where

$$\Xi = W_1^T\left(X - Y^{-1}\right)^{-1} W_1.$$

4 Numerical Example

In this section, we provide an example to demonstrate the effectiveness of the proposed method.

Consider an uncertain neutral stochastic system in (Σ) with parameters as follows:

$$
\begin{aligned}
A &= \begin{bmatrix} -0.7 & 0.1 & 0.3 \\ -0.1 & -0.8 & 0.2 \\ 0 & 0.1 & 0.1 \end{bmatrix}, A_d = \begin{bmatrix} -0.2 & -0.1 & 0.2 \\ 0.1 & 0.1 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1.8 & -1.3 & 0 \\ 1.7 & 0.2 & 1 \\ 2 & 1 & 1.5 \end{bmatrix}, \\
E &= \begin{bmatrix} -0.1 & 0 & 0.1 \\ 0.1 & -0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, E_d = \begin{bmatrix} 0.1 & 0.2 & 0 \\ -0.1 & 0.1 & -0.2 \\ 0.1 & 0 & 0.1 \end{bmatrix}, C = \begin{bmatrix} -1.2 & 0.5 & 0.4 \\ 0.5 & -1.2 & 2.5 \end{bmatrix}, \\
C_d &= \begin{bmatrix} 0.1 & -0.1 & 0.3 \\ 0 & 0.1 & 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & 0 & -0.5 \\ -0.5 & 1 & 0.6 \end{bmatrix}, H = \begin{bmatrix} -0.3 & 0 & 0.1 \\ 0 & -0.1 & 0.2 \end{bmatrix}, \\
H_d &= \begin{bmatrix} 0.1 & -0.1 & 0 \\ 0.3 & 0.1 & 0.3 \end{bmatrix}, D = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0 & -0.1 & 0.1 \\ 0 & 0 & -0.2 \end{bmatrix}, \\
M_1 &= \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix}, M_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \\
N_1 &= \begin{bmatrix} 0.1 & 0 & 0.1 \end{bmatrix}, N_2 = \begin{bmatrix} 0 & 0.1 & -0.1 \end{bmatrix}, N_3 = \begin{bmatrix} 0 & 0.2 & 0.1 \end{bmatrix}, \\
N_4 &= \begin{bmatrix} -0.1 & 0.2 & 0 \end{bmatrix}, N_5 = \begin{bmatrix} 0.3 & -0.2 & 0.1 \end{bmatrix}
\end{aligned}
$$

Associated with this system (Σ), the cost function is given in (7) with

$$
Q_1 = \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 \\ 0 & 0.1 & 0.1 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.3 & 0.1 & -0.1 \\ 0.1 & 0.2 & 0.1 \\ -0.1 & 0.1 & 0.2 \end{bmatrix}.
$$

In order to design a guaranteed cost controller, we choose

$$
Z = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}, \quad \epsilon_1 = 1.8, \quad \epsilon_2 = 2.5.
$$

Then, by using the Matlab LMI Control Toolbox to solve the LMI in (33), we obtain the solution as follows:

$$
\begin{aligned}
X &= \begin{bmatrix} 10.8337 & -0.9139 & 1.9247 \\ -0.9139 & 10.4069 & -0.5412 \\ 1.9247 & -0.5412 & 18.1286 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.1946 & 0.0054 & 0.0001 \\ 0.0054 & 0.2182 & 0.0011 \\ 0.0001 & 0.0011 & 0.1624 \end{bmatrix}, \\
\Lambda &= \begin{bmatrix} -0.7675 & 0.1389 & -0.0101 \\ -0.2645 & -0.8251 & -0.1562 \\ 0.1131 & 0.3643 & -0.5178 \end{bmatrix}, \quad \Phi = \begin{bmatrix} -0.0243 & 0.0027 & -0.0616 \\ -0.0007 & 0.0160 & -0.0525 \\ 0.0293 & -0.0267 & 0.0415 \end{bmatrix},
\end{aligned}
$$

$$\Psi = \begin{bmatrix} -1.3029 & -1.2653 \\ 0.7482 & -0.0227 \\ -2.8424 & -5.5320 \end{bmatrix}.$$

Now, we set a nonsingular matrix W_1 as

$$W_1 = \begin{bmatrix} 2 & 1.6 & -3 \\ 1.2 & -3.2 & 1.5 \\ 2.5 & 2 & 1.8 \end{bmatrix},$$

From (36), we can find W_2 as

$$W_2 = \begin{bmatrix} -0.2440 & -0.0447 & 0.1812 \\ -0.2032 & 0.3121 & -0.0154 \\ -0.3476 & -0.2856 & -0.2799 \end{bmatrix}.$$

Therefore, by Theorem 3.2, a desired guaranteed cost controller can be chosen as

$$d\xi(t) = \begin{bmatrix} -1.2390 & -0.7406 & -0.2785 \\ -0.0010 & -1.2490 & -0.2864 \\ -0.1645 & -0.5015 & -1.0719 \end{bmatrix} \xi(t)dt + \begin{bmatrix} -0.5465 & -1.1070 \\ -0.5413 & -0.7417 \\ -0.2187 & -0.7118 \end{bmatrix} dy(t), \quad (44)$$

$$u(t) = \begin{bmatrix} 0.1002 & 0.0750 & 0.0193 \\ 0.0360 & 0.0777 & 0.0637 \\ -0.0600 & -0.1223 & 0.0509 \end{bmatrix} \xi(t). \quad (45)$$

In this case, if we assume $\tau = 0.5$, and $\eta(t) = \begin{bmatrix} 1 & 1 & -1 & 0 & 1 & 0.5 \end{bmatrix}^T$ for $t \in [-0.5, 0]$, then, by Theorem 3.2, the closed-loop cost function in (7) satisfies

$$J \leq 39.4961.$$

5 Conclusions

The problem of guaranteed cost control for uncertain neutral stochastic systems with time-varying and norm-bounded parameter uncertainties has been studied in this paper. Dynamic output feedback controllers have been designed, which guarantee not only the mean-square asymptotic stability of the closed-loop system but also an upper bound on the closed-loop value of the cost function for all admissible uncertainties. An LMI approach has been developed and a numerical example has been provided to demonstrate the effectiveness and applicability of the proposed approach.

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