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Abstract

This paper deals with the class of continuous-time singular linear systems with random abrupt changes. The state feedback stabilization and its robustness for this class of systems with norm bounded uncertainties are tackled. Sufficient conditions for designing either a stabilizing or a robust stabilizing controller are developed in the LMI setting. The developed sufficient conditions are used to synthesis the state feedback controller that guarantees either the nominal system or the uncertain system is piecewise regular, impulse free and stochastically stable or robust stochastically stable.

Key Words: Singular systems, Jump linear systems, Linear matrix inequality, Stochastic stabilization, Robust stochastic stabilization, State feedback controller.

Résumé

Cet article traite de la commande des systèmes singuliers avec des changements brusques et aléatoires. Le problème de stabilisation avec retour d'état et sa robustesse pour des incertitudes de type borné en norme est aussi considéré. Des conditions suffisantes en forme d'inégalités linéaires matricielles sont développées pour le design du correcteur par retour d'état pour le système nominal et le système incertain pour assurer que le système en boucle fermée est régulier sans impulsion par morceau et stochastiquement stable.

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1 Introduction

The stochastic class of systems driven by continuous-time Markov chains, called also Markovian jump systems, has been used to model many practical systems, where random failures and repairs and sudden environment changes may occur. For more detail on what it has been done on the subject, we refer the reader to Refs. 1–2 and the references therein. This class of systems has also attracted a lot of researchers from both mathematical and control community. Many results on stochastic stability and stochastic stabilization have been reported to the literature. For more details on these results we refer the reader to Refs. 3–7 and the references therein, where different approaches have been used. The $\mathscr{H}_\infty$ control problem was investigated in Refs. 8–9, where sufficient conditions for the solvability of this problem was proposed. For the Markovian jump systems with time delays, the results on stability analysis and $\mathscr{H}_\infty$ control were also reported in Refs. 10–12 for different types of time delays. For more detail on Markovian jumping systems with time delay, we refer the reader to Ref. 1 and the references therein.

In parallel, there have been also considerable research efforts on the study of singular systems. This is due to the extensive applications of this class of systems in many practical systems, such as circuits boundary control systems, chemical processes, and other areas see for more details Refs. 13–16. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems (see Refs. 13, 15). A great number of fundamental notions and results in control and systems theory based on state-space systems have been successfully extended to singular systems; see, e.g., Refs. 17–26, and the references therein.

To the best of our knowledge, the class of singular systems with random abrupt changes has not yet been fully investigated and this will be the goal of this paper. Most of the result developed for this class of systems are not easily tractable for more details we refer the reader to Refs. 27–29, where different approaches have been used to get the established results that are totally different from the one of this paper. In this paper, we will mainly concentrate of the stochastic stabilizability and the robust stochastic stabilizability of such class of systems. Firstly using results on stochastic stability of Boukas Ref. 30 a sufficient condition, in the linear matrix inequality (LMI) setting is developed to design a state feedback controller that makes the closed-loop dynamics of a given system of this class of systems, piecewise regular, impulse-free and stochastic stability. Based on this, a sufficient condition is developed to design a robust state feedback controller that guarantees that the closed-loop dynamics of the uncertain system of the class under consideration to be piecewise regular, impulse-free and stochastic stability is also proposed. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed methods.

The rest of this paper is organized as follows. In Section 2, the stabilization problem is stated and some results are recalled. In Section 3, the main results are given and these include results on stochastic stabilizability and robust stochastic stabilizability.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$\top$" denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{I}$ is the identity matrices with compatible dimensions. $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respective to some probability measure $\mathcal{P}$.

2 Problem Statement

Consider a stochastic switching system with N modes, i.e., $\mathscr{S} = \{1, 2, \ldots, N\}$. The mode switching is assumed to be governed by a continuous-time Markov process $\{r_t, t \geq 0\}$ taking values in the state space $\mathscr{S}$ and having the following infinitesimal generator

$$\Lambda = (\lambda_{ij}), i, j \in \mathscr{S},$$

where $\lambda_{ij} \geq 0, \forall j \neq i, \lambda_{ii} = -\sum_{j\neq i} \lambda_{ij}$.

The mode transition probabilities are described as follows:

$$P[r_{t+\Delta} = j | r_t = i] = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & j \neq i \\ 1 + \lambda_{ii}\Delta + o(\Delta), & j = i \end{cases} \tag{1}$$

where $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$.

Let the class of singular systems random abrupt changes be defined in a fundamental probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and assume that its behavior is described by the following dynamics:

$$E\dot{x}(t) = A(r_t, t)x(t) + B(r_t, t)u(t), x(s) = x_0 \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the system state a time t, $u(t) \in \mathbb{R}^m$ is the control input of the system a time t, $A(r_t, t)$ and $B(r_t, t)$ are assumed to have uncertainties, i,e.: $A(r_t, t) = A(r_t) + D_A(r_t)F_A(r_t)E_A(r_t)$ and $B(r_t, t) = B(r_t) + D_B(r_t)F_B(r_t)E_B(r_t)$with $D_A(i)$, $E_A(i)$, $D_B(i)$ and $E_B(i)$ are known real matrices with appropriate dimensions for each $i \in \mathscr{S}$ and $F_A(r_t)$ and $F_B(r_t)$ satisfy $F_A^\top(i)F_A(i) \leq \mathbb{I}$ and $F_B^\top(i)F_B(i) \leq \mathbb{I}$ for each $i \in \mathscr{S}$, the matrix E may be singular, and we assume $0 \leqslant \operatorname{rank}(E) = n_E < n$.

Remark 2.1 *When the uncertainties are equal to zero the system will be referred to as nominal system. The uncertainties that satisfies the previous conditions are referred to as admissible. The uncertainties we are considering in this paper are known in the literature as norm bounded uncertainties. When the matrix E is nonsingular the system (2) is referred to as normal system.*

Definition 2.1 *(see Ref. 13)*

i. Nominal system (2) is said to be regular if the characteristic polynomial, $\det(sE - A(i))$ *is not identically zero for each mode* $i \in \mathscr{S}$.

ii. Nominal system (2) is said to be impulse free, i.e. the $\deg(\det(sE - A(i))) = rank(E)$ *for each mode* $i \in \mathscr{S}$.

For more details on other properties and the existence of the solution of system (2), we refer the reader to Ref. Ref. 19, and the references therein. In general, the regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

For the system (2), we have the following definitions:

Definition 2.2 *Nominal system (2) is said to be stochastically stable (SS) if there exists a constant* $T(r_0, \phi(\cdot))$ *such that*

$$\mathbb{E}\left[\int_0^\infty \|x(t)\|^2 dt \middle| r_0, x(0) = x_0\right] \leq T(x_0, r_0); \tag{3}$$

Definition 2.3 *Uncertain system (2) is said to be robust stochastically stable (RSS) if there exists a constant* $T(x_0, r_0)$ *such that (3) holds for all admissible uncertainties.*

The goal of this paper is to design a controller of the following form:

$$u(t) = K(r_t)x(t) \tag{4}$$

where $K(i)$ is a design parameter that has to be determined for every $i \in \mathscr{S}$, that renders the closed loop dynamics of the nominal system or the uncertain system piecewise regular, impulse-free and stochastically stable.

Definition 2.4 *Nominal system (2) is said to be stabilizable in the stochastic sense if there exists a control law of the form (4) such that the closed-loop dynamics of the nominal system is piecewise regular, impulse free and stochastically stable.*

Definition 2.5 *Uncertain system (2) is said to be robust stabilizable in the stochastic sense if there exists a control law of the form (4) such that the closed-loop dynamics of the uncertain system is piecewise regular, impulse free and stochastically stable for all admissible uncertainties.*

Remark 2.2 *Notice that we have used the word piecewise in all our definitions, this is due to the fact that the class of systems we are considering has inherent discontinuities at each jump of the Markov process* $\{r_t, t \geq 0\}$.

The aim of this paper is to develop LMI conditions that can be used to design a state feedback controller that guarantees that the closed loop dynamics of the nominal system or the uncertain system is piecewise regular, impulse free and stochastically stable robust stochastically stable for all admissible norm bounded uncertainties. Our methodology in this paper will be mainly based on the Lyapunov theory and some algebraic results. The conditions we will develop here will be in terms of the solutions to linear matrix inequalities that can be easily obtained using LMI control toolbox.

Before closing this section, let us recall some lemmas that we will be using in the rest of the paper.

Lemma 2.1 *(see Ref. 1) Let* H *and* E *be given matrices with appropriate dimensions and* F *satisfying* $F^\top F \leq \mathbb{I}$. *Then, we have For any* $\varepsilon > 0$,

$$HFE + E^\top F^\top H^\top \leq \varepsilon HH^\top + \frac{1}{\varepsilon} E^\top E.$$

Lemma 2.2 *(see Ref. 1) The linear matrix inequality*

$$\begin{bmatrix} H & S^\top \\ S & R \end{bmatrix} > 0$$

is equivalent to

$$R > 0, H - S^\top R^{-1} S > 0$$

where $H = H^\top$, $R = R^\top$ *and* S *is a matrix with appropriate dimension.*

Lemma 2.3 *(see Ref. 31) The nominal singular Markovian jump system (2) is piecewise regular, impulse-free and stochastically stable if there exists a set of nonsingular matrices* $P = (P(1), \cdots, P(N))$ *such that the following set of coupled LMIs holds for each* $i \in \mathscr{S}$:

$$E^\top P(i) = P^\top(i) E \geq 0 \tag{5}$$

$$P^\top(i) A(i) + A^\top(i) P(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top P(j) < 0. \tag{6}$$

3 Main Results

In this section, we start by developing results that can be used to design a state feedback controller that assures that the nominal system (2) is piecewise regular, impulse free and stochastically stable. Then, these results will be extended to the case of uncertain systems. LMI conditions are established to design a state feedback controller that guarantees that either a nominal system or an uncertain system of the class we are considering is piecewise regular, impulse free and stochastically stable.

Let us now consider the nominal system and see how we can design a state feedback controller of the form (4) that guarantees that the closed loop dynamics will be piecewise regular, impulse free and stochastically stable. For this purpose, plugging controller (4) in the nominal dynamics (2) gives:

$$\dot{x}(t) = A_{cl}(r_t)x(t), x(s) = x_0 \tag{7}$$

where $A_{cl}(r_t) = A(r_t) + B(r_t)K(r_t)$.

Based on Lemma 2.3, we know that the dynamics (7) will be piecewise regular, impulse-free and stochastically stable if there exits a set of nonsingular $P = (P(1), \cdots, P(N))$ such that the following hold for each $i \in \mathscr{S}$:

$$\begin{aligned} &E^\top P(i) = P^\top(i)E \geq 0 \\ &P^\top(i)A(i) + A^\top(i)P(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top P(j) < 0. \end{aligned}$$

Notice that we have:

$$\sum_{j=1}^{N} \lambda_{ij} E^\top P(j) = \lambda_{ii} E^\top P(i) + \sum_{j=1, j\neq i}^{N} \lambda_{ij} E^\top P(j).$$

If we assume that the following holds for a given $\varepsilon_P > 0$:

$$\varepsilon_P \left[P(i) + P^\top(i) \right] \geq E^\top P(i)$$

then the conditions of Lemma 2.3 become:

$$\begin{aligned} &\varepsilon_P \left[P(i) + P^\top(i) \right] \geq E^\top P(i) = P^\top(i)E \geq 0 \\ &P^\top(i)A_{cl}(i) + A_{cl}^\top(i)P(i) + \lambda_{ii}E^\top P(i) + \sum_{j=1, j\neq i}^{N} \varepsilon_P \lambda_{ij} \left[P(j) + P^\top(j) \right] < 0. \end{aligned}$$

Pre- and post-multiply the matrix inequalities respectively by $P^{-\top}(i)$ and $P^{-1}(i)$ give:

$$\begin{aligned}
&\varepsilon_P\left[P^{-1}(i)+P^{-\top}(i)\right]\geq P^{-\top}(i)E^\top=EP^{-1}(i)\geq 0\\
&A_{cl}(i)P^{-1}(i)+P^{-\top}(i)A_{cl}^\top(i)+\lambda_{ii}P^{-\top}(i)E^\top\\
&\qquad+\sum_{j=1,j\neq i}^N\varepsilon_P\lambda_{ij}P^{-\top}(i)\left[P(j)+P^\top(j)\right]P^{-1}(i)<0.
\end{aligned}$$

Let $X(i)=P^{-1}(i)$, $Y(i)=K(i)X(i)$, and using the expression of $A_{cl}(i)$ and noticing that $X^{-1}(j)+X^{-\top}(j)\leq\mathbb{I}+\left(X^\top(j)X(j)\right)^{-1}$ holds for any $j\in\mathscr{S}$, we get:

$$\begin{aligned}
&\varepsilon_P\left[X(i)+X^\top(i)\right]\geq X^\top(i)E^\top=EX(i)\geq 0\\
&A(i)X(i)+X^\top(i)A^\top(i)+B(i)Y(i)+Y^\top(i)B^\top(i)+\lambda_{ii}X^\top(i)E^\top\\
&\qquad+\sum_{j=1,j\neq i}^N\varepsilon_P\lambda_{ij}X^\top(i)\left[\mathbb{I}+\left(X^\top(j)X(j)\right)^{-1}\right]X(i)<0.
\end{aligned}$$

Noticing that $X^\top(i)X(i)\geq X^\top(i)+X(i)-\mathbb{I}$ and defining:

$$\begin{aligned}
\mathcal{X}_i(X)&=\mathrm{diag}\Big[X^\top(1)+X(1)-\mathbb{I},\cdots,X^\top(i-1)+X(i-1)-\mathbb{I},X^\top(i+1)+X(i+1)-\mathbb{I},\\
&\qquad\cdots,X^\top(N)+X(N)-\mathbb{I}\Big]\\
\mathcal{S}_i(X)&=\left[\sqrt{\varepsilon_P\lambda_{i1}}X^\top(i),\cdots,\sqrt{\varepsilon_P\lambda_{ii-1}}X^\top(i),\sqrt{\varepsilon_P\lambda_{ii+1}}X^\top(i),\cdots,\sqrt{\varepsilon_P\lambda_{iN}}X^\top(i)\right]
\end{aligned}$$

we have:

$$\sum_{j=1,j\neq i}^N\varepsilon_P\lambda_{ij}X^\top(i)X(i)=\mathcal{S}_i(X)\mathcal{S}_i^\top(X)$$

$$\sum_{j=1,j\neq i}^N\varepsilon_P\lambda_{ij}X^\top(i)\left(X^\top(i)X(i)\right)^{-1}X(i)=\mathcal{S}_i(X)\mathcal{X}_i^{-1}(X)\mathcal{S}_i^\top(X).$$

Using this and the previous development, the design for a state feedback controller that will guarantees that the closed-loop dynamics of the nominal system will be piecewise regular, impulse-free and stochastically stable is given by the results of the following theorem.

Theorem 3.1 *Let ε_P be a given positive scalar ($\varepsilon_P>0$). There exists a state feedback controller of the form (4) such that the closed-loop dynamics of the nominal system (2) is* *piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices* $X = (X(1), \cdots, X(N))$*, and a set of matrices* $Y = (Y(1), \cdots, Y(N))$*, such that the following set of coupled LMIs holds for each* $i \in \mathscr{S}$*:*

$$\varepsilon_P \left[X(i) + X^\top(i) \right] \geq X^\top(i) E^\top = E X(i) \geq 0 \tag{8}$$

$$\begin{bmatrix} \widehat{J}(i) & \mathcal{S}_i(X) & \mathcal{S}_i(X) \\ \mathcal{S}_i^\top(X) & -\mathbb{I} & 0 \\ \mathcal{S}_i^\top(X) & 0 & -\mathcal{X}_i(X) \end{bmatrix} < 0, \tag{9}$$

where

$$\begin{aligned} \widehat{J}(i) &= A(i)X(i) + X^\top(i)A^\top(i) + B(i)Y(i) + B^\top(i)Y^\top(i) + \lambda_{ii} X^\top(i) E^\top \\ \mathcal{X}_i(X) &= \operatorname{diag}\left[X^\top(1) + X(1) - \mathbb{I}, \cdots, X^\top(i-1) + X(i-1) - \mathbb{I}, X^\top(i+1) + X(i+1) - \mathbb{I}, \right. \\ &\qquad \left. \cdots, X^\top(N) + X(N) - \mathbb{I} \right] \\ \mathcal{S}_i(X) &= \left[\sqrt{\varepsilon_P \lambda_{i1}} X^\top(i), \cdots, \sqrt{\varepsilon_P \lambda_{ii-1}} X^\top(i), \sqrt{\varepsilon_P \lambda_{ii+1}} X^\top(i), \cdots, \sqrt{\varepsilon_P \lambda_{iN}} X^\top(i) \right] \end{aligned}$$

The stabilizing controller gain is given by $K(i) = Y(i)X^{-1}(i)$*,* $i \in \mathscr{S}$*.*

Let us now concentrate on the design of robust state feedback controller of the form (4) that can guarantee that the closed-loop dynamics of our uncertain system will be piecewise regular, impulse free and stochastically stable. For this purpose, using the results of Theorem 3.1, for a given positive scalar ($\varepsilon_P > 0$), there exists a state feedback controller of the form (4) such that the closed-loop system (2) is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$, and a set of matrices $Y = (Y(1), \cdots, Y(N))$, such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$ for all admissible uncertainties:

$$\varepsilon_P \left[X(i) + X^\top(i) \right] \geq X^\top(i) E^\top = E X(i) \geq 0$$

$$\begin{bmatrix} \widehat{J}(i) & \mathcal{S}_i(X) & \mathcal{S}_i(X) \\ \mathcal{S}_i^\top(X) & -\mathbb{I} & 0 \\ \mathcal{S}_i^\top(X) & 0 & -\mathcal{X}_i(X) \end{bmatrix} < 0,$$

Using Lemma 2.1, for any $\varepsilon_A(i) > 0$ and $\varepsilon_B(i) > 0$, for each $i \in \mathscr{S}$ we have:

$$\begin{aligned} & X^\top(i)E_A^\top(i)F_A^\top(i)D_A^\top(i) + D_A(i)F_A(i)E_A(i)X(i) \\ & \quad \leq \varepsilon_A^{-1}(i)X^\top(i)E_A^\top(i)E_A(i)X(i) + \varepsilon_A(i)D_A(i)D_A^\top(i) \\ & Y^\top(i)E_B^\top(i)F_B^\top(i)D_B^\top(i) + D_B(i)F_B(i)E_B(i)Y(i) \\ & \quad \leq \varepsilon_B^{-1}(i)Y^\top(i)E_B^\top(i)E_B(i)Y(i) + \varepsilon_B(i)D_B(i)D_B^\top(i) \end{aligned}$$

Using now these inequalities and Lemma 2.2, we get the following result for the design of a robust state feedback controller that guarantees that the closed-loop dynamics of the uncertain system is piecewise regular, impulse-free and stochastically stable.

Theorem 3.2 *Let ε_P be a given positive scalar ($\varepsilon_P > 0$). There exists a state feedback controller of the form (4) such that the closed-loop system (2) is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$, a set of matrices $Y = (Y(1), \cdots, Y(N))$, and sets of positive scalars $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$ and $\varepsilon_B = (\varepsilon_B(1), \cdots, \varepsilon_B(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$ and for all admissible uncertainties:*

$$\varepsilon_P \left[X(i) + X^\top(i)\right] \geq X^\top(i)E^\top = EX(i) \geq 0 \tag{10}$$

$$\begin{bmatrix} \widetilde{J}(i) & X^\top(i)E_A^\top(i) & Y^\top(i)E_B^\top(i) & \mathcal{S}_i(X) & \mathcal{S}_i(X) \\ E_A(i)X(i) & -\varepsilon_A(i)\mathbb{I} & 0 & 0 & 0 \\ E_B(i)Y(i) & 0 & -\varepsilon_B(i)\mathbb{I} & 0 & 0 \\ \mathcal{S}_i^\top(X) & 0 & 0 & -\mathbb{I} & 0 \\ \mathcal{S}_i^\top(X) & 0 & 0 & 0 & -\mathcal{X}_i(X) \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned} \widetilde{J}(i) &= A(i)X(i) + X^\top(i)A^\top(i) + B(i)Y(i) + B^\top(i)Y^\top(i) + \varepsilon_A(i)D_A(i)D_A^\top(i) \\ &\quad + \varepsilon_B(i)D_B(i)D_B^\top(i) + \varepsilon_P\lambda_{ii}X^\top(i)E^\top \\ \mathcal{Z}_i(Z) &= \text{diag}\Big[X^\top(1) + X(1) - \mathbb{I}, \cdots, X^\top(i-1) + X(i-1) - \mathbb{I}, X^\top(i+1) + X(i+1) - \mathbb{I}, \\ &\quad \cdots, X^\top(N) + X(N) - \mathbb{I}\Big] \\ \mathcal{S}_i(X) &= \Big[\sqrt{\varepsilon_P\lambda_{i1}}X^\top(i), \cdots, \sqrt{\varepsilon_P\lambda_{ii-1}}X^\top(i), \sqrt{\varepsilon_P\lambda_{ii+1}}X^\top(i), \cdots, \sqrt{\varepsilon_P\lambda_{iN}}X^\top(i)\Big] \end{aligned}$$

The stabilizing controller gain is given by $K(i) = Y(i)X^{-1}(i)$, $i \in \mathscr{S}$.

The results we developed in this paper extend those developed for the normal linear Markovian switching systems on stochastic stabilization and robust stochastic stabilization.

Remark 3.1 *The results we developed depend of the parameter ε_P that we have chosen to be mode-independent. Our results can be extended to the case of ε_P mode-independent.*

4 Numerical Example

To show the validness of our results, let us consider a numerical example with two-mode singular system and state space in $\mathbb{R}^3$. The data of this system are as follows:

- mode # 1:

$$A(1) = \begin{bmatrix} -1.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 1.0 \\ 0.0 & -1.0 & -1.0 \end{bmatrix}, B(1) = \begin{bmatrix} 0.3 & 0.0 \\ 0.0 & 0.1 \\ 0.2 & 1.0 \end{bmatrix},$$

$$D_A(1) = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}, E_A(1) = \begin{bmatrix} 0.3 & 0.2 & 0.1 \end{bmatrix},$$

$$D_B(1) = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.2 \end{bmatrix}, E_B(1) = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},$$

- mode # 2:

$$A(2) = \begin{bmatrix} 1.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & -1.0 \end{bmatrix}, B(2) = \begin{bmatrix} 0.1 & 0.0 \\ 0.0 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},$$

$$D_A(2) = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}, E_A(2) = \begin{bmatrix} 0.3 & 0.2 & 0.1 \end{bmatrix},$$

$$D_B(2) = \begin{bmatrix} 0.3 \\ 0.1 \\ 0.2 \end{bmatrix}, E_B(2) = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}.$$

The transition matrix rates, Λ, and the singular matrix, E, are given by:

$$\Lambda = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Solving the LMIs (8)-(9) with $\varepsilon_P = 2$, we get:

$$X(1) = \begin{bmatrix} 0.2324 & -0.0049 & 0.0 \\ -0.0049 & 0.1701 & 0.0 \\ 0.0026 & 0.0405 & 0.1441 \end{bmatrix}, X(2) = \begin{bmatrix} 0.2723 & -0.0000 & 0.0 \\ -0.0000 & 0.2580 & 0.0 \\ 0.0000 & 0.0140 & 0.1624 \end{bmatrix},$$

$$Y(1) = \begin{bmatrix} 2.2393 & -0.0080 & 1.4247 \\ -1.0102 & 0.2438 & 0.0000 \end{bmatrix}, Y(2) = \begin{bmatrix} -1.9137 & -0.1396 & -0.0000 \\ 0.1448 & -1.8438 & 0.3313 \end{bmatrix}.$$

which give the following gains for the state feedback controller:

$$K(1) = \begin{bmatrix} 9.4808 & -2.1261 & 9.8838 \\ -4.3202 & 1.3089 & 0.0000 \end{bmatrix}, K(2) = \begin{bmatrix} -7.0282 & -0.5411 & -0.0000 \\ 0.5318 & -7.2568 & 2.0398 \end{bmatrix}.$$

For the robust stochastic stabilization, solving the LMIs (10)-(11) with $\varepsilon_A(1) = \varepsilon_A(2) = 0.1$ and $\varepsilon_P = 2$, we get:

$$X(1) = \begin{bmatrix} 0.0747 & -0.0013 & 0.0 \\ -0.0013 & 0.0771 & 0.0 \\ 0.0016 & 0.0166 & 0.0536 \end{bmatrix}, X(2) = \begin{bmatrix} 0.1098 & -0.0034 & 0.0 \\ -0.0034 & 0.1082 & 0.0 \\ -0.0030 & 0.0288 & 0.0570 \end{bmatrix},$$

$$Y(1) = \begin{bmatrix} 0.3712 & -0.0797 & 0.0018 \\ -0.1568 & 0.1128 & 0.0995 \end{bmatrix}, Y(2) = \begin{bmatrix} -0.7562 & -0.4581 & -0.7535 \\ 0.3775 & 0.1162 & 0.3982 \end{bmatrix}.$$

which give the following gains for the state feedback controller:

$$K(1) = \begin{bmatrix} 4.9499 & -0.9604 & 0.0336 \\ -2.1192 & 1.0291 & 1.8561 \end{bmatrix}, K(2) = \begin{bmatrix} -7.2765 & -0.9451 & -13.2264 \\ 3.6071 & -0.6718 & 6.9893 \end{bmatrix}.$$

As it can be seen either for the nominal system or the uncertain system, the developed LMIs sufficient conditions give feasible solution that determines the state feedback controller that guarantees that the corresponding closed-loop dynamics is piecewise regular, impulse-free and stochastically stable.

5 Conclusions

This paper dealt with the class of continuous-time singular linear systems with random abrupt changes. Results on stochastic stabilization and its robustness are developed. The state feedback controller is designed to assure that the closed-loop dynamics of the nominal system or the uncertain systems is piecewise regular, impulse-free and stochastically stable. The LMI framework is used to establish the different results on stochastic stabilization and its robustness. The results we developed here can easily be solved using any LMI toolbox like the one of Matlab or the one of Scilab.

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