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H_∞ Stabilization of Markovian Jumping Singularly Perturbed Delayed Systems

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Abstract

This paper deals with the class of Markovian singularly perturbed linear continuous-time systems with time varying and mode-dependent time-delay. The stochastic stability and H_∞ performance and the H_∞ state feedback stabilization are tackled. Sufficient conditions in the LMI setting are established for both the problems. A numerical example is provided to show the effectiveness of the developed results.

Résumé

Cet article traite de la classe des systèmes linéaire à sauts markoviens, perturbation singulière et retard dépendant du temps et du mode du système. La stabilité et la commande H_∞ sont considérées. Des conditions dépendantes du retard en forme d'inégalités linéaires matricielles sont établies pour la stabilité et la stabilisation H_∞ . Un exemple numérique est présenté pour montrer l'efficacité des résultats développés.

1 Introduction

Deterministic and stochastic systems with time-delay have received considerable attention in the last two decades and for more details on this subject we refer the reader to Boukas and Liu [5] and the references therein. Many problems have been tackled and interesting results have been reported in the literature. Among these results we quote those related to H_∞ state feedback stabilization. Boukas and Liu [5] and Cao and Lam [3] have considered the H_∞ state feedback stabilization for Markovian jumping systems with time-delay. In these two references we can find LMI-based delay-independent and delay-dependent conditions.

On the other hand, most physical systems inherently contain multiple time-scale phenomena and the singularly perturbation approach has proven to be a powerful analytic tool for such systems. In a singularly perturbed system, a small positive parameter multiplies the time derivatives of some of the states in the state model. The presence of the small parameter makes the system stiff and unwieldy. During recent years a large amount of attention has been paid to the problem of analysis and synthesis of singularly perturbed systems (see [11, 13, 23]). H_∞ control problem for standard singularly perturbed systems has been extensively studied for the past decade. A popular approach is based on the exact decomposition of the full-order Riccati equation [12]. The zero-sum differential game approach was proposed in [20]. Shi and Dragan [25] investigated the H_∞ control problem for singularly perturbed systems with parametric uncertainty. For non-standard singularly perturbed systems, Tan [27] proposed a descriptor system approach to design the dynamic output feedback controller. Another paper that dealt with the non-standard case is [19], which proposed an iterative algorithm with quadratic convergence property to solve the Riccati equation related H_∞ control problem. Recently, several authors used the linear matrix inequality (LMI) approach for the analysis and synthesis of singularly perturbed systems (see [13, 14, 15]). It is well known that the LMI approach seems to have a promising perspective due to its efficiency in computations.

The control of Markovian jumping singularly perturbed systems has been a research subject and attracted a lot of interest during the past decade. [24] gave a recursive algorithm for the regulator design, and [2] proposed a parallel algorithm for the optimal controller design, which can yield arbitrary orders of accuracy. In the case of H_∞ control, [21] studied the design of robust reduced-order controllers in the context of piecewise-deterministic systems exhibiting distinguishable slow and fast modes via standard reduced-order techniques. In [11], the authors discussed H_∞ control using the bounded real property, but the results are in the form of a set of coupled algebraic Riccati equations that is difficult to solve. Moreover, their results apply only to standard cases. Recently, an LMI approach is developed to study the control problem [1]. However, we can see that in [1], the matrix in Eq. (15) is not symmetric and the results are very difficult to use. Furthermore, neither of the above literatures investigated the time-delayed cases. Ref. [6] has proposed an approach, but in that literature, the input matrix is required to be square and invertible, which is a very strong constraint.

The goal in this paper is to study the combination of the class of Markovian jumping systems with time-delay and the class of singularly perturbed systems that we call the class of Markovian jumping singularly systems delayed systems (MJSPDS). The time-delay in the system is assumed to dependent on the system mode. This idea has been used for the first time by Boukas and Liu [5] for the continuous-time systems and [4] for the discrete-time systems. Similar works can also be found in [9], [8], [30] and [18].

To the best of our knowledge the case we are treating here has never been studied. And the proposed approach is different from that of [6] since in this paper, the input matrices are not required to be square. We will focus of the stochastic stability and the H_∞ performance. The H_∞ state feedback stabilization is also considered.

The rest of this paper is organized as follows. In Section 2, the problem is stated and some definitions are given. Section 3 contains the results on stochastic stability and H_∞ performance for our class of systems. The results are in the LMI setting. In Section 4, the design of memoryless state feedback controller is performed and LMIs conditions are developed for this purpose.

Notation. Throughout this paper, \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript “ T ” denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrices with compatible dimensions. \mathcal{L}_2 is the space of integral vector over $[0, \infty)$. $\|\cdot\|$ will refer to the Euclidean vector norm whereas $\|\cdot\|$ denotes the \mathcal{L}_2 -norm over $[0, \infty)$ defined as $\|f\|^2 = \int_0^\infty f^T(t)f(t) dt$. For a symmetric block matrix, we use “ $*$ ” as an ellipsis for the terms that are introduced by symmetry. Finally, $diag\{\}$ stands for a block-diagonal matrix.

2 Problem formulation

The Markovian jumping singularly perturbed delayed system (MJSPDS) is a class of singularly perturbed systems with Markovian jumping parameters and time-delays. Here we will consider the MJSPDS with mode-dependent singular perturbation parameters and time-delays.

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, \mathcal{F} is the algebra of events and \mathcal{P} is the probability measure defined on \mathcal{F} . We consider a class of Markovian jump singularly perturbed systems with mode-dependent delays:

$$\begin{cases} E(r(t))\dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t - \tau(r(t))) \\ \quad + B(r(t))u(t) + D(r(t))w(t) \\ z(t) = G(r(t))x(t) + H(r(t))u(t) + L(r(t))w(t) \\ x(t) = \varphi(t), t \in [-\mu, 0] \end{cases} \quad (1)$$

where $E(r(t)) = \text{diag}\{I_{n_1}, \varepsilon(r(t))I_{n_2}\}$, $x(t) \in \mathbb{R}^n$ is the state variable ($n = n_1 + n_2$); $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^q$ is the disturbance input, which is a square integrable vector function over $[0, +\infty)$; $z(t) \in \mathbb{R}^p$ is the output. The parameter $r(t)$ is a continuous-time Markovian process taking values in a finite set $\mathbf{S} = \{1, 2, \dots, s\}$, with transition matrix $\Pi := \{\pi_{ij}\}$ and the transition probability is given by

$$\Pr\{r(t + \Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j, \end{cases}$$

where $\Delta > 0$, $\pi_{ij} \geq 0$ for $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j \neq i}^s \pi_{ij}$ for each mode i . It is assumed that the jump process is accessible for every $t > 0$. For each possible value of $r(t) = i \in \mathbf{S}$, we have $\varepsilon(r(t)) = \varepsilon_i$ and $A(r(t)) = A_i$, $A_d(r(t)) = A_{di}$, $B(r(t)) = B_i$, $D(r(t)) = D_i$, $G(r(t)) = G_i$, $H(r(t)) = H_i$, $L(r(t)) = L_i$ which are known constant matrices of appropriate dimensions. Also, we have $\tau(r(t)) = \tau_i(t)$, which satisfy

$$0 < \tau_i(t) \leq \mu_i < +\infty, \quad \dot{\tau}_i(t) \leq h_i < 1, \quad \forall i \in \mathbf{S}$$

where μ_i and h_i are known constants, and $\mu = \max\{\mu_i, i \in \mathbf{S}\}$.

For singularly perturbed systems, there exists a small scalar $\varepsilon_0 > 0$ satisfying $\varepsilon_i < \varepsilon_0 \ll 1$.

Remark 2.1 *System (1) is called a standard MJSPDS when the matrix A_{22i} , for all $i \in \mathbf{S}$ is invertible, otherwise it is called nonstandard MJSPDS.*

For $r(t) = i$, $i \in \mathbf{S}$, we can rewrite (1) as

$$\begin{cases} E_{\varepsilon_i} \dot{x}(t) = A_i x(t) + A_{di} x(t - \tau_i(t)) + B_i u(t) + D_i w(t) \\ z(t) = G_i x(t) + H_i u(t) + L_i w(t) \\ x(t) = \varphi(t), t \in [-\mu, 0] \end{cases}$$

where $E_{\varepsilon_i} = \text{diag}\{I_{n_1}, \varepsilon_i I_{n_2}\}$.

In this paper, we will assume that the state variables, $x(t)$, and the mode system, $r(t)$, are accessible for feedback.

Definition 2.1 ([3, 5]) *The unforced system (1) (setting $u(t) = 0$ and $w(t) = 0$) is said to be stochastically stable if, for all continuous function $\varphi(t)$ defined on $[-\mu, 0]$ and initial mode $r_0 \in \mathbf{S}$, there exists one positive constant ρ , such that*

$$\lim_{T_f \rightarrow \infty} \mathbf{E} \left\{ \int_0^{T_f} x^T(t, \varphi(t), r_0) x(t, \varphi(t), r_0) dt | \varphi(t), r_0 \right\} \leq \rho$$

Definition 2.2 ([3, 5]) *For a given real number $\gamma > 0$, the system (1) is said to be stochastically stable with γ -disturbance attenuation if, for all continuous function $\varphi(t)$ defined on*

$[-\mu, 0]$ and initial mode $r_0 \in \mathbf{S}$, it is stochastically stable and in the case of $\varphi(t) = 0$, the signal $z(t)$ satisfies

$$\mathbf{E} \left\{ \int_0^{T_f} z^T(t)z(t)dt \right\} < \gamma^2 \int_0^{T_f} w^T(t)w(t)dt$$

for all $T_f \in (0, +\infty)$.

Remark 2.2 In Definition 2.2, we assume that the initial conditions are equal to zero. If the initial conditions are not equal to zero, the reader is invited to consult Boukas and Liu [5] how we treat such case.

In the next sections we will treat the stochastic stability and the H_∞ performance and the H_∞ state feedback stabilization. The sufficient conditions in the LMI setting will be established.

3 Stochastic stability and H_∞ performance

In this section, a set of sufficient conditions on the stochastic stability and H_∞ performance for MJSPDS (1) are provided.

Theorem 3.1 Given the open-loop system (1) (setting $u(t) \equiv 0$), if for $i = 1, 2, \dots, s$, there exist matrices $P_{11i} > 0$, $P_{22i} > 0$, P_{21i} , $Q_i > 0$ and $Q > 0$ with appropriate dimensions satisfying

$$H_i \equiv \begin{bmatrix} \Xi_i & * & * & * \\ A_{di}^T P_i & -(1-h_i)Q_i & * & * \\ D_i^T P_i & 0 & -\gamma^2 I & * \\ G_i & 0 & L_i & -I \end{bmatrix} < 0$$

$$Q_i < Q \tag{2}$$

where

$$\Xi_i = A_i^T P_i + P_i^T A_i + Q_i + \eta\mu Q + \sum_{j=1}^s \pi_{ij} E P_j$$

$P_i = \begin{bmatrix} P_{11i} & 0 \\ P_{21i} & P_{22i} \end{bmatrix}$, $E = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix}$ and $\eta = \max\{|\pi_{ii}|, i \in \mathbf{S}\}$, then there exists an $\varepsilon^* > 0$ such that the open-loop system (1) is stochastically stable with γ -disturbance attenuation for $\varepsilon_i \in (0, \varepsilon^*]$.

Proof: Since we have $P_{11i} > 0$ and $P_{22i} > 0$, we have that there exists an $\hat{\varepsilon}_0^* > 0$ such that $P_{11i} - \varepsilon_i P_{21i}^T P_{22i}^{-1} P_{21i} > 0$ for $\varepsilon_i \in (0, \hat{\varepsilon}_0^*]$, which is equivalent to

$$E_{\varepsilon_i} P_{\varepsilon_i} = P_{\varepsilon_i}^T E_{\varepsilon_i} = \begin{bmatrix} P_{11i} & \varepsilon_i P_{21i}^T \\ \varepsilon_i P_{21i} & \varepsilon_i P_{22i} \end{bmatrix} > 0$$

where

$$P_{\varepsilon i} = \begin{bmatrix} P_{11i} & \varepsilon_i P_{21i}^T \\ P_{21i} & P_{22i} \end{bmatrix}$$

Note that $\{(x(t), r(t)), t \geq 0\}$ is not a Markov process. to cast this model into the framework of Markov process, we define a new process $\{(x_t, r(t)), t \geq 0\}$ by

$$x_t(s) = x(t+s) \quad t - \tau_{r(t)}(t) \leq s \leq t.$$

Then, similar to [10] and [29], we can verify that $\{(x_t, r(t)), t \geq 0\}$ is a Markov process with initial state $(\varphi(t), r_0)$.

Let the mode at time t be i ; that is, $r(t) = i \in \mathbf{S}$. Take the stochastic Lyapunov functional to be

$$\begin{aligned} V(x_t, r(t) = i) &= x^T(t) E_{\varepsilon i} P_{\varepsilon i} x(t) + \int_{t-\tau_i(t)}^t x^T(\sigma) Q_i x(\sigma) d\sigma \\ &+ \eta \int_{-\mu t+\theta}^0 \int_{-\mu t+\theta}^t x^T(\sigma) Q x(\sigma) d\sigma d\theta \end{aligned}$$

Let \mathcal{A} be the weak infinitesimal operator of the stochastic process $\{x_t, r(t), t \geq 0\}$, then we have

$$\begin{aligned} \mathcal{A}V(x_t, r(t) = i) &= x^T(t) [A_i^T P_{\varepsilon i} + P_{\varepsilon i}^T A_i + \sum_{j=1}^s \pi_{ij} E_{\varepsilon i} P_{\varepsilon j}] x(t) \\ &+ 2x^T(t) P_{\varepsilon i}^T A_{di} x(t - \tau_i(t)) \\ &+ w^T(t) D_i^T P_{\varepsilon i} x(t) + x^T(t) P_{\varepsilon i}^T D_i w(t) \\ &+ x^T(t) Q_i x(t) \\ &- (1 - \dot{\tau}_i(t)) x^T(t - \tau_i(t)) Q_i x(t - \tau_i(t)) \\ &+ \sum_{j=1}^s \pi_{ij} \int_{t-\tau_j(t)}^t x^T(\sigma) Q_j x(\sigma) d\sigma \\ &+ \eta \mu x^T(t) Q x(t) - \eta \int_{t-\mu}^t x^T(\sigma) Q x(\sigma) d\sigma \end{aligned}$$

It is obvious that

$$\begin{aligned} &2x^T(t) P_{\varepsilon i}^T A_{di} x(t - \tau_i(t)) \\ &\leq (1 - h_i) x^T(t - \tau_i(t)) Q_i x(t - \tau_i(t)) \\ &\quad + (1 - h_i)^{-1} x^T(t) P_{\varepsilon i}^T A_{di} Q_i^{-1} A_{di}^T P_{\varepsilon i} x(t) \end{aligned}$$

Noting $\tau_j(t) \leq \mu$, $Q_i < Q$, $\pi_{ij} \geq 0$ for $i \neq j$, and $-\eta \leq \pi_{ii} \leq 0$, we have

$$\begin{aligned}
& \sum_{j=1}^s \pi_{ij} \int_{t-\tau_j(t)}^t x^T(\sigma) Q_j x(\sigma) d\sigma \\
& \leq \sum_{j=1}^s \pi_{ij} \int_{t-\mu}^t x^T(\sigma) Q_j x(\sigma) d\sigma \\
& \leq \sum_{j=1, j \neq i}^s \pi_{ij} \int_{t-\mu}^t x^T(\sigma) Q_j x(\sigma) d\sigma \\
& \leq \sum_{j=1, j \neq i}^s \pi_{ij} \int_{t-\mu}^t x^T(\sigma) Q x(\sigma) d\sigma \\
& = -\pi_{ii} \int_{t-\mu}^t x^T(\sigma) Q x(\sigma) d\sigma \\
& \leq \eta \int_{t-\mu}^t x^T(\sigma) Q x(\sigma) d\sigma
\end{aligned}$$

Then,

$$\begin{aligned}
\mathcal{A}V(x_t, r(t) = i) & \leq x^T(t) \Lambda_{\varepsilon i} x(t) + w^T(t) D_i^T P_{\varepsilon i} x(t) \\
& \quad + x^T(t) P_{\varepsilon i}^T D_i w(t)
\end{aligned}$$

where

$$\begin{aligned}
\Lambda_{\varepsilon i} & = A_i^T P_{\varepsilon i} + P_{\varepsilon i}^T A_i + \sum_{j=1}^s \pi_{ij} E_{\varepsilon i} P_{\varepsilon j} + Q_i + \eta \mu Q \\
& \quad + (1 - h_i)^{-1} P_{\varepsilon i}^T A_{di} Q_i^{-1} A_{di}^T P_{\varepsilon i}
\end{aligned}$$

Define

$$J(T_f) \equiv \mathbf{E} \left\{ \int_0^{T_f} [z^T(t) z(t) - \gamma^2 w^T(t) w(t)] dt \right\}$$

From Dynkin' formula, we have

$$\begin{aligned}
& \mathbf{E} \{ V(x_{T_f}, r(T_f)) \} - V(x_0, r_0) \\
& = \mathbf{E} \left\{ \int_0^{T_f} \mathcal{A}V(x_t, r(t)) dt \right\}
\end{aligned}$$

Assume the initial condition $\varphi(t) = 0$, then, $V(\mathbf{x}_0, r_0) = 0$ and therefore,

$$\begin{aligned} & J(T_f) \\ &= \mathbf{E} \left\{ \int_0^{T_f} [z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \mathcal{A}V(x_t, r(t))] dt \right\} \\ &\quad - \mathbf{E} \{ V(x_{T_f}, r(T_f)) \} \\ &\leq \mathbf{E} \left\{ \int_0^{T_f} [z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \mathcal{A}V(x_t, r(t))] dt \right\} \\ &= \mathbf{E} \left\{ \int_0^{T_f} x_w^T(t) M_{\varepsilon_i} x_w(t) dt \right\} \end{aligned}$$

where $x_w(t) = [x^T(t), w^T(t)]^T$, and

$$M_{\varepsilon_i} = \begin{bmatrix} M_{\varepsilon_{11i}} & * \\ D_i^T P_{\varepsilon_i} + G_i^T L_i & -\gamma^2 I + L_i^T L_i \end{bmatrix}$$

with

$$\begin{aligned} M_{\varepsilon_{11i}} &= A_i^T P_{\varepsilon_i} + P_{\varepsilon_i}^T A_i + \sum_{j=1}^s \pi_{ij} E_{\varepsilon_i} P_{\varepsilon_j} + Q_i + \eta \mu Q \\ &\quad + (1 - h_i)^{-1} P_{\varepsilon_i}^T A_{di} Q_i^{-1} A_{di}^T P_{\varepsilon_i} + G_i^T G_i \end{aligned}$$

On the other hand, define

$$H_{\varepsilon_i} \equiv \begin{bmatrix} H_{\varepsilon_{11i}} & * & * & * \\ A_{di}^T P_{\varepsilon_i} & -(1 - h_i) Q_i & * & * \\ D_i^T P_{\varepsilon_i} & 0 & -\gamma^2 I & * \\ G_i & 0 & L_i & -I \end{bmatrix}$$

where

$$H_{\varepsilon_{11i}} = A_i^T P_{\varepsilon_i} + P_{\varepsilon_i}^T A_i + Q_i + \eta \mu Q + \sum_{j=1}^s \pi_{ij} E_{\varepsilon_i} P_{\varepsilon_j}$$

It is obvious that $H_{\varepsilon_i} = H_i + \Delta H_{\varepsilon_i}$, where ΔH_{ε_i} is the product of the small parameter ε_i and some matrix with appropriate dimension[17].

Since $H_i < 0$, by continuity, there exists an $\hat{\varepsilon}^* > 0$ such that $H_{\varepsilon_i} < 0$ for $i = 1, 2, \dots, s$ and $\varepsilon_i \in (0, \hat{\varepsilon}^*]$. Using Schur complement, it is equivalent to $M_{\varepsilon_i} < 0$.

Therefore, we have $J(T_f) < 0$ for all $T_f > 0$, that is

$$\mathbf{E} \left\{ \int_0^{T_f} z^T(t)z(t) dt \right\} < \gamma^2 \int_0^{T_f} w^T(t)w(t) dt$$

for $\varepsilon_i \in (0, \varepsilon^*]$, where $\varepsilon^* = \min\{\hat{\varepsilon}_0^*, \hat{\varepsilon}^*\}$.

On the other hand, when we consider the stochastic stability, we can set $w(t) = 0$. In this case, we have

$$\mathcal{A}V(x_t, r(t) = i) \leq x^T(t) \Lambda_{\varepsilon i} x(t)$$

Obviously $M_{\varepsilon i} < 0$ implies $\Lambda_{\varepsilon i} < 0$. Let $\lambda_0 = \min\{\lambda_{\min}(-\Lambda_{\varepsilon i}), i \in \mathbf{S}\}$. Then, by using Dynkin' formula, we can get

$$\begin{aligned} & \mathbf{E}\{V(x_{T_f}, r(T_f))\} - V(x_0, r_0) \\ &= \mathbf{E}\left\{\int_0^{T_f} \mathcal{A}V(x_\sigma, r(\sigma)) d\sigma\right\} \\ &\leq -\lambda_0 \mathbf{E}\left\{\int_0^{T_f} \|x(\sigma)\|^2 d\sigma\right\} \end{aligned}$$

Taking the limit as $T_f \rightarrow +\infty$, we have

$$\lim_{T_f \rightarrow +\infty} \mathbf{E}\left\{\int_0^{T_f} \|x(\sigma)\|^2 d\sigma\right\} \leq \frac{1}{\lambda_0} V(x_0, r_0)$$

which implies that the open-loop system (1) is stochastically stable for $\varepsilon \in (0, \varepsilon^*]$. This completes the proof. \square

Remark 3.1 Notice that we don't have to assume that the initial conditions are zero. In this case we will have for the disturbance attenuation the following expression (see Boukas and Liu [5]):

$$\|z\|_2^2 \leq \gamma^2 \|w\|_2^2 + V(x_0, r_0)$$

Remark 3.2 In the Lyapunov functional (3), V_1 and V_2 are mode-dependent functionals, whereas V_3 is a mode-independent functional.

Remark 3.3 The conditions in (2) are LMIs and can be solved efficiently by the famous interior-point algorithm.

Remark 3.4 In this paper, no assumption is made regarding whether A_{22i} is invertible or not, so the results proposed in this paper apply not only to standard, but also to nonstandard MJSPDS.

4 Controller design

In this section, we consider the design of a memoryless H_∞ state-feedback controller

$$u(t) = K(r(t))x(t) \tag{3}$$

such that the closed-loop system

$$\begin{cases} E_\varepsilon(r(t))\dot{x}(t) = (A(r(t)) + B(r(t))K(r(t)))x(t) \\ \quad + A_d(r(t))x(t - \tau(r(t))) + D(r(t))w(t) \\ z(t) = (G(r(t)) + H(r(t))K(r(t)))x(t) + L(r(t))w(t) \\ x(t) = \varphi(t), t \in [-\mu, 0] \end{cases} \quad (4)$$

is stochastically stable with γ -disturbance attenuation for sufficiently small ε_i s, where $K(r(t))$ is a matrix function of the random jumping process $\{r(t)\}$ and $K(r(t)) = K_i \in \mathbb{R}^{m \times n}$ when $r(t) = i$.

Theorem 4.1 *If for $i = 1, 2, \dots, s$, there exist matrices $X_{11i} > 0$, $X_{22i} > 0$, X_{21i} , $S_i > 0$, $S > 0$ and $W_i > 0$ with appropriate dimensions satisfying*

$$\begin{bmatrix} \Phi_i & * & * & * & * \\ X_i^T A_{di}^T & -(1 - h_i)S_i & * & * & * \\ D_i^T & 0 & -\gamma^2 I & * & * \\ G_i X_i + H_i Y_i & 0 & L_i & -I & * \\ N_i & 0 & 0 & 0 & -M_i \end{bmatrix} < 0 \quad (5)$$

$S_i < S$
 $EX_i < W_i$

where

$$\Phi_i = A_i X_i + X_i^T A_i^T + B_i Y_i + Y_i^T B_i^T + S_i + \eta \mu S + \pi_{ii} X_i^T E,$$

$$X_i = \begin{bmatrix} X_{11i} & 0 \\ X_{21i} & X_{22i} \end{bmatrix}, E = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\eta = \max\{\pi_{ii}, i \in \mathbf{S}\},$$

$$N_i = [\sqrt{\pi_{i1}} X_i^T, \dots, \sqrt{\pi_{i,i-1}} X_i^T, \sqrt{\pi_{i,i+1}} X_i^T, \dots, \sqrt{\pi_{i,i+1}} X_i^T, \dots, \sqrt{\pi_{is}} X_i^T]^T$$

and

$$M_i = \text{diag} [X_1 + X_1^T - W_1, \dots, X_{i-1} + X_{i-1}^T - W_{i-1}, \dots, X_s + X_s^T - W_s]$$

then there exists an $\varepsilon^* > 0$ such that the closed-loop system (4) under the controller (3) is stochastically stable with γ -disturbance attenuation for $\varepsilon_i \in (0, \varepsilon^*]$, where the controller gains are given as $K_i = Y_i X_i^{-1}$.

Proof: From the structure of X_j we can see $EX_j = X_j^T E < W_j$, and therefore $EX_j < X_j^{-T} W_j X_j$. Further, using the relation:

$$X_j W_j^{-1} X_j^T \geq X_j + X_j^T - W_j$$

which can be easily deduced from the fact that $(X_j W_j^{-1/2} - W_j^{-1/2})(X_j W_j^{-1/2} - W_j^{-1/2})^T \geq 0$, we can get

$$(X_j + X_j^T - W_j)^{-1} \geq X_j^{-T} W_j X_j^{-1} > E X_j^{-1}$$

By using Schur complement to (5) yields

$$\begin{bmatrix} (\Phi_i + N_i^T M_i^{-1} N_i) & * & * & * \\ X_i^T A_{di}^T & -(1 - h_i) S_i & * & * \\ D_i^T & 0 & -\gamma^2 I & * \\ G_i X_i + H_i Y_i & 0 & L_i & -I \end{bmatrix} < 0 \quad (6)$$

with

$$\begin{aligned} N_i^T M_i^{-1} N_i &= \sum_{j=1, j \neq i}^s \pi_{ij} X_i^T (X_j + X_j^T - W_j)^{-1} X_i \\ &> \sum_{j=1, j \neq i}^s \pi_{ij} X_i^T (E X_j^{-1}) X_i \end{aligned}$$

therefore, we have

$$\begin{bmatrix} \hat{\Phi}_i & * & * & * \\ X_i^T A_{di}^T & -(1 - h_i) S_i & * & * \\ D_i^T & 0 & -\gamma^2 I & * \\ G_i X_i + H_i Y_i & 0 & L_i & -I \end{bmatrix} < 0 \quad (7)$$

where

$$\begin{aligned} \hat{\Phi}_i &= A_i X_i + X_i^T A_i^T + B_i Y_i + Y_i^T B_i^T + S_i + \eta \mu S \\ &\quad + \sum_{j=1}^s \pi_{ij} X_i^T (E X_j^{-1}) X_i \end{aligned}$$

Pre- and Post-multiplying the above inequality by $\text{diag}\{X_i^{-T}, X_i^{-T}, I, I\}$, $\text{diag}\{X_i^{-1}, X_i^{-1}, I, I\}$ respectively and letting $P_i = X_i^{-1}$, $Q_i = P_i^T S_i P_i$, $Q = P_i^T S P_i$ and $K_i = Y_i P_i$, we get

$$\begin{bmatrix} \Theta_i & * & * & * \\ A_{di}^T P_i & -(1 - h_i) Q_i & * & * \\ D_i^T P_i & 0 & -\gamma^2 I & * \\ G_i + H_i K_i & 0 & L_i & -I \end{bmatrix} < 0 \quad (8)$$

where

$$\Theta_i = (A_i + B_i K_i)^T P_i + P_i^T (A_i + B_i K_i) + Q_i + \eta \mu Q + \sum_{j=1}^s \pi_{ij} E P_j$$

Also, we can note that P_i have the following structure

$$P_i = \begin{bmatrix} P_{11i} & 0 \\ P_{21i} & P_{22i} \end{bmatrix}$$

where $P_{11i} > 0$ and $P_{22i} > 0$. Furthermore, the relation $S_i < S$ is equivalent to $Q_i < Q$.

Therefore, from Theorem 1, we can see that there exists an $\varepsilon^* > 0$ such that the closed-loop system (4) under the controller (3) is stochastically stable with γ -disturbance attenuation for any $\varepsilon_i \in (0, \varepsilon^*]$. This completes the proof.

Remark 4.1 *The conditions in (5) are also LMIs and can be solved efficiently by interior-point algorithms.*

Remark 4.2 *The synthesis results also apply both standard and nonstandard MJSPDS since no assumption is made regarding whether A_{22i} is invertible or not,*

Remark 4.3 *Compared with the results of [6], the approach proposed in this paper do not require the input matrix B_i to be square.*

5 Numerical examples

To show the validness of our results let us consider a four states system with three modes. The dynamics of this described by (1) with the following data:

$$A_1 = \begin{bmatrix} -0.366 & 0.271 & 0.188 & -0.4555 \\ 0.482 & -1.01 & 0.24 & -0.4 \\ 0.100 & 0.0 & 0 & 0 \\ -0.407 & 0.0 & 1.0 & 0 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.1 & 0.02 & 0 & 0 \\ 0.1 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.566 & 0.271 & 0.188 & -0.4555 \\ 0.482 & -1.01 & 0.24 & -0.4 \\ 0.100 & 0.3 & 0 & 0 \\ -0.807 & 0.0 & 1.0 & 0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.1 & 0 & 0 & 0 \\ 0.1 & -0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & -0.2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.166 & 0.271 & 0.188 & -0.4555 \\ 0.482 & -1.01 & 0.24 & -0.4 \\ 0.100 & 0.5 & 0 & 0 \\ -0.207 & 0.0 & 1.0 & 0 \end{bmatrix},$$

$$A_{d3} = \begin{bmatrix} -0.1 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0 & 0 \\ 0.1 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & -0.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, B_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, D_2 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, D_3 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

$$G_1 = G_2 = G_3 = [1 \quad 0.5 \quad 1 \quad 0.5],$$

$$H_1 = H_2 = H_3 = 0.5, L_1 = L_2 = L_3 = 0$$

$$\Pi = \begin{bmatrix} -0.1341 & 0.1007 & 0.0334 \\ 0.1007 & -0.1007 & 0 \\ 0.0354 & 0 & -0.0354 \end{bmatrix}$$

$$n_1 = n_2 = 2$$

$$\tau_1(t) = 3 + 0.2\sin(t),$$

$$\tau_2(t) = 4 + 0.3\sin(t),$$

$$\tau_3(t) = 5 + 0.4\sin(t),$$

From above we can get

$$\mu_1 = 3.2, \mu_2 = 4.3, \mu_3 = 5.4,$$

$$h_1 = 0.2, h_2 = 0.3, h_3 = 0.4$$

Selecting $\gamma = 2.6$ and solving the LMIs (5), we get:

$$P_1 = \begin{bmatrix} 23.1563 & 2.9052 & 0 & 0 \\ 2.9052 & 2.9400 & 0 & 0 \\ -33.0678 & -5.9137 & 29.5819 & 27.7189 \\ -30.4738 & -9.7837 & 27.7189 & 38.3195 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 28.1510 & 2.9711 & 0 & 0 \\ 2.9711 & 1.5568 & 0 & 0 \\ -33.8429 & -4.7175 & 27.2948 & 35.9038 \\ -50.6250 & -8.47130 & 35.9038 & 75.3236 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 25.2293 & 2.0553 & 0 & 0 \\ 2.0553 & 1.1363 & 0 & 0 \\ -29.1126 & -2.2427 & 28.9431 & 30.6781 \\ -38.6246 & -2.4091 & 30.6781 & 41.2128 \end{bmatrix},$$

$$K_1 = [147.9097 \quad 38.4059 \quad -231.1862 \quad -265.1340],$$

$$K_2 = [199.4889 \quad 27.4152 \quad -254.7818 \quad -445.8874],$$

$$K_3 = [143.3570 \quad -4.2496 \quad -240.4672 \quad -288.5424],$$

which construct a set of feasible solutions. Under this controller, the resulting closed-loop system can remain stochastic stability and H_∞ performance. The upper-bound ε^* can be evaluated by using numerical simulations.

6 Conclusion

This paper deals with the class Markovian jumping singularly perturbed delayed systems. The delays are on the state vector and are mode dependent. The stability and the stabilizability problems are tackled and LMIs conditions are established. Our conditions do not require the classical standard assumptions on singularly perturbed systems. A memoryless state feedback controller is used in our study. This controller uses the slow and the fast part of the state vector of the system.

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