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Abstract

Dynamic programming is applied to the economic dispatch problem with spinning reserve constraint. A first algorithm, based on a direct application of Bellman's equation with two state variables turns out to be slow for large systems. A second algorithm exploiting state dominance is much more efficient. Computational results are presented. Moreover, we show that well-known heuristic approaches do not provide optimal solutions even for small systems.

Key Words: Economic dispatch, spinning reserve, dynamic programming.

Résumé

Pour le problème de chargement optimal avec contrainte de réserve tournante, on utilise la programmation dynamique. Un premier algorithme, basé sur une application directe du principe de Bellman, avec deux variables d'état, s'avère trop lent pour de grands systèmes. Un second algorithme, exploitant la dominance entre états est développé et s'avère beaucoup plus éfficace, comme le montrent des expériences de calcul. De plus, on montre que certaines approches heuristiques développées dans la littérature ne sont pas optimales même pour de petits systèmes.

Mots clés : chargement optimal, réserve tournante, programmation dynamique.

1 Introduction

Production and distribution of electricity are economic activities of paramount importance. The cost of a large power plant can indeed be in billions of dollars, and the annual production cost of electricity in 100 millions of dollars [13]. The need for optimization of design and management of power plants is therefore considerable. The goal of this paper is to determine an optimal policy for the production of several turbo-alternator groups with a one period spinning reserve constraint. This constraint arises when the startup time of the groups is rather moderate. A breakdown of one of the groups or any other random event in the network often triggers a sudden change in the demand for production. As a consequence, a powerful spinning reserve has to be envisaged and distributed to a sufficient number of groups. In this paper, we assume that the thermo-electrical group have several vapor inlets. This entails a piecewise concave production cost function for each of them (see Figure 1).

Several heuristic methods for solving the optimal loading problem were proposed in the literature, prominent among which are the Lagrange multiplier method [7, 13] that assumes the convexity of the input-output function, the mixed-integer programming approach, the dual method of optimization, the classic optimization [5, 10], the simulated annealing [14] and the genetic search [2, 12, 15].

Another way of tackling the optimal loading problem is to use dynamic programming. This approach can be defined as a way to solve sequential decision problems that relies on the Bellman optimality principle. When applied to the optimal loading problem, the Bellman principle can be expressed as follows: 'any optimal production policy for N groups can only have optimal sub-policies for any subset of these groups'. In this paper, we propose a dynamic programming formulation of the optimal loading problem with spinning reserve.

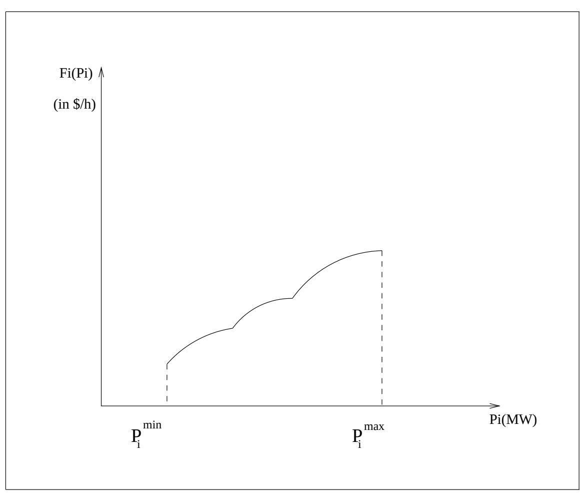

Figure 1: Cost function of a turbo-alternator group with several vapor inlet valves

It generalizes the standard dynamic programming formulation of the optimal loading problem with minimum and maximum power constraints which relies on the following recursive equation:

$$F_i^*(D_i) = Min_{P_i}(\, F_{i-1}^*(D_i - P_i) + \, F_i(P_i)) \tag{1}$$

where

- $F_i^*(D_i)$: denotes the minimum cost for satisfying a demand D_i (in MW while using the groups $1, 2, \ldots, i$,
- P_i: denotes the power of group i.

Moreover, we show that exploiting state dominance drastically improves this approach. The paper is organized as follows: the economic dispatch problem with spinning reserve is stated mathematically in the next section. Two standard heuristics for its solution are presented and discussed in Section 3. Our dynamic programming method is given in Section 4, in two versions, not exploiting and exploiting state dominance. Comparative computational experience is reported in Section 5.

2 Formulation

The economic dispatch problem with spinning reserve constraint may be expressed mathematically as follows:

$$Min \ \sum_{i=1}^{N} F_i(P_i) \tag{2}$$

subject to the constraints

$$\sum_{i=1}^{N} P_i = P_R \tag{3}$$

$$\sum_{i=1}^{N} S_i \geq S_R \tag{4}$$

$$P_i^{min} \leq P_i \leq P_i^{max} \tag{5}$$

$$S_i = Min\,(P_i^{max} - P_i, S_i^{max}) \tag{6}$$

where symbols are defined as follows:

- $F_i(P_i)$: cost function of the i^{th} turbo-alternator unit (in $\$/h$);
- N: number of units;
- P_i: power of unit i^{th} (in MW);
- P_i^{min}, P_i^{max}: minimum and maximum power of the i^{th} unit (in MW);
- P_R: power demand (in MW);
- S_i: contribution of the i^{th} unit to the spinning reserve (in MW);

- S_R: minimal value of the spinning reserve (en MW);
- S_i^{max}: maximum contribution of the i^{th} unit to the spinning reserve.

The objective function expresses that the total cost is additive over the units, constraint (2) that the demand must be exactly satisfied, constraint (3) that the spinning reserve must be at least equal to a given minimum value, constraints (4) that any unit must produce a power within a given range (we assume here that all units will produce; the more general case where unit commitment is made together with economic dispatch, or in other words where those units which will produce are chosen together with the power they are assigned to produce, is a straightforward extension e.g. see Hansen and Mladenović [6] for its treatment without spinning reserve constraint). Constraints (5) are a consequence of the following pairs of constraints:

$$S_i \leq P_i^{max} - P_i \tag{7}$$

and

$$0 \leq S_i \leq S_i^{max} \tag{8}$$

which express that the contribution of the i^{th} unit to the spinning reserve cannot exceed the maximum increase in power of that unit, nor a given maximum value, imposed in order to spread out the spinning reserve over the units.

3 Heuristic solution methods

A first heuristic for problem (1)–(5) is due to Lee and Breipohl [9]. It consists in decoupling the problem of allocating the spinning reserve and the problem of economic dispatch. So, it solves two dynamic programming problems with a single state variable.

The first problem is the following:

$$Min \ \sum_{i=1}^{N} F_i(P_i) \tag{9}$$

subject to

$$\begin{aligned} \sum_{i=1}^{N} P_i &= \sum_{i=1}^{N} P_i^{max} - S_R & (10) \\ P_i^{max} - S_i^{max} &\leq P_i \leq P_i^{max} \qquad i = 1, 2, \ldots, N. & (11) \end{aligned}$$

This problem expresses that one proceeds to produce a power equal to the maximum which can be obtained minus the lower bound on the spinning reserve. In other words, one proceeds to distribute the spinning reserve among the groups assuming the production to be the largest possible. If the power demanded P_R is equal to this power, the allocation of spinning reserve to the units is optimal; however if P_R is smaller this may not be the case anymore.

The contribution to the spinning reserve of the units are obtained, by solving problem (9)–(11), noted $S_i^* = P_i^{max} - P_i^*$ for $i = 1, 2, \ldots, N$, and kept fixed in the second problem:

$$Min \sum_{i=1}^{N} F_i(P_i) \tag{12}$$

subject to

$$\sum_{i=1}^{N} P_i = P_R \tag{13}$$

$$P_i^{min} \leq P_i \leq P_i^{max} - S_i^* \qquad i = 1, 2, \ldots, N \tag{14}$$

The recurrence relation used in both cases is the following:

$$F_1^*(D_1) = F_1(D_1)$$

$$F_i^*(D_i) = Min_{P_i}(\ F_{i-1}^*(D_i - P_i) +\ F_i(P_i)),$$

for $i = 2, 3, \ldots, N$.

A second heuristic for problem (2)–(6) was proposed by Wood [17]. Again, it is a two stage process. First, the problem is solved while ignoring the constraint on spinning reserve. Then it is checked weither this constraint is satisfied or not by the solution found. If it is, the process stop otherwise, feasibility of the constrained problem is checked and local modifications brought to the solution to transform it at least cost into a feasible one.

We next recall the precise rules of this heuristic.

(a) Solve problem (2) (3) (5). Let P_1^*, $P_2^*, \ldots, P_N^*$ denote the solution so obtained. Check if the constraint on spinning reserve is satisfied or not, using

$$\sum_{i=1}^{N} Min(P_i^{max} - P_i^*, S_i^{max}) \geq S_R \tag{15}$$

If it is, stop, the optimal solution being found.

(b) Otherwise, let δ denote the amount by which this constraint is violated, i.e., the difference between the right and left-hand sides of (14). Then partition the units in two classes:

(b_1) those for which the production $P_i = P_i^*$ can be marginally augmented without changing their contribution to the spinning reserve (which is presently maximum):

$$I_1 = \{i/P_i^* < P_i^{max} - S_i^{max}\}; \tag{16}$$

(b_2) those for which a marginal decrease of the production $P_i = P_i^*$ would increase their contribution to the spinning reserve:

$$I_2 = \{i/P_i^* > P_i^{max} - S_i^{max}\}. \tag{17}$$

The production P_i^* of units for which $P_i^* = P_i^{max} - S_i^{max}$ will not be changed.

Compute the maximum total increase in power of units with index in I_1 which will not change their contribution to the spinning reserve:

$$UP = \sum_{i \in I_1} (P_i^{max} - S_i^{max} - P_i^*) \tag{18}$$

and the maximum total decrease in power of units with index in I_2 which will increase their contribution to the spinning reserve (by the same amount):

$$DOWN = \sum_{i \in I_2} (P_i^* - P_i^{max} + S_i^{max}) \tag{19}$$

If $UP < \delta$ or $DOWN < \delta$, stop, the problem being infeasible.

(c) Otherwise, augment by δ the production of units with index in I_1 at least cost by solving the problem:

$$Min \sum_{i \in I_1} F_i(P_i) \tag{20}$$

subject to

$$\sum_{i \in I_1} P_i = \sum_{i \in I_1} P_i^* + \delta \tag{21}$$

$$P_i^{min} \leq P_i \leq P_i^{max} \qquad i \in I_1 \tag{22}$$

then decrease by δ the production of units with index in I_2 at least cost (or largest benefit) by solving the problem:

$$Min \sum_{i \in I_1} F_i(P_i) \tag{23}$$

subject to

$$\sum_{i \in I_2} P_i = \sum_{i \in I_2} P_i^* - \delta \tag{24}$$

$$P_i^{min} \leq P_i \leq P_i^{max} \qquad i \in I_2 \tag{25}$$

As mentioned in the introduction neither of these heuristics does always provide an optimal solution, even for small instances. To show this is indeed the case, we consider an example

Table 1: Example data: breakpoints of unit cost functions and corresponding costs

unit i	breakpoint j	1	2	3	4
	$P_j(MW)$	50	100	150	200
1	$F_i(P_j)(\$/h)$	400	700	900	1150
2	$F_i(P_j)(\$/h)$	450	600	1100	1300
3	$F_i(P_j)(\$/h)$	200	400	700	1100

with three units where the cost functions are piecewise concave. In [1] we show that in such a case at most one unit will have an optimal production elsewhere than at a breakpoint of its cost function. Moreover, to simplify matters, we choose breakpoints regularly and consider values of the demand for which all units will produce at breakpoints. Data for the three units are given in Table 1.

Let $S_i^{max} = 50MW$ for $i = 1, 2, 3$, $P_R = 400MW$ and $S_R = 100MW$. Lee and Breipohl's heuristic is then used to solve problem (1)–(5) with $P_R = \sum_{i=1}^{3} P_i^{max} - S_R = 500MW$. The optimal solution is $P_1^* = 150MW$, $P_2^* = 200MW$ and $P_3^* = 150MW$ with a total cost equal to 2900\$$/h$. We next compute contributions S_i^* to the spinning reserve corresponding to this solution, i.e., $S_1^* = 50MW$, $S_2^* = 0$, $S_3^* = 50MW$, and use them to obtain new upper bounds on the powers P_i^{max}, i.e., $P_1^{max} = 150MW$, $P_2^{max} = 200MW$, $P_3^{max} = 150MW$. We then solve the problem for an initial demand equal to $400MW$. We find that the optimal solution is: $P_1^* = 150MW$, $P_2^* = 100$ and $P_3^* = 150MW$ with a total cost equal to 2200\$$/h$. Since, as is easily checked, the optimal solution for the loading problem with a demand $P_R = 400MW$ and a contribution to the spinning reserve of $100MW$ is $P_1^* = 200MW$, $P_2^* = 100MW$ and $P_3^* = 100MW$ with a total cost equal to 2150\$$/h$, this illustrates the non-optimality of the Lee and Breipohl approach [9].

For the heuristic developed by Wood [17], we consider the same example of three groups, with a power demand of $450MW$ and a spinning reserve demand of $100MW$. The approach of Wood [18] solves the problem without taking into account the spinning reserve. This yields the following optimal solution: $P_1^* = 50MW$, $P_2^* = 200MW$ and $P_3^* = 200MW$ with a total cost equal to 2400\$$/h$. However, this solution violates the spinning reserve constraint. We overcome this drawback by forming two classes of groups. The first class contains groups whose production cannot be increased without decreasing their contribution to the spinning reserve whereas the second one contains groups whose production cannot be decreased without increasing the contribution to the spinning reserve. It then comes that $I_1 = 1, I_2 = \{2, 3\}$, $UP = 100MW$ and $DOWN = 100MW$. The power of the first class will be increased by $50MW$ and the production of the second one decreased by $50MW$ at the lowest cost. The new solution is $P_1^* = 100MW$, $P_2^* = 150MW$ and $P_3^* = 200MW$ with a total cost equal to 2500\$$/h$ whereas the optimal solution is $P_1^* = 150MW$, $P_2^* = 150MW$ and $P_3^* = 150MW$ with a total cost equal to 2450\$$/h$. This illustrates the non optimality of the Wood approach [17].

4 A Dynamic Programming Algorithm

The economic dispatch problem with spinning reserve constraint can be solved by dynamic programming, extending the standard algorithm for the case where there is no such constraint (see e.g. Wood and Wollenberg [17] for an exposition of the latter). Nevertheless, it is worth observing that this problem involves two state variables, D_i and R_i. These state variables characterize groups 1 to i's production policy:

$$\sum_{k=1}^{i} P_k = D_i \tag{26}$$

and

$$\sum_{k=1}^{i} S_k = R_i \tag{27}$$

The Bellman equation is then expressed as follows:

$$F_i^*(D_i, R_i) = Min_{P_i,S_i}[F_{i-1}^*(D_i - P_i, R_i - S_i) + F_i(P_i)] \tag{28}$$

where:

$$P_i^{min} \leq P_i \leq Min(P_i^{max}, D_i - D_{i-1}^{min}) \tag{29}$$

with P_{i-1}^{min} equal to the sum of minimum production for units $1, 2, \ldots, i-1$ and

$$0 \leq S_i \leq Min(S_i^{max}, P_i^{max} - P_i) \tag{30}$$

The algorithm works as follows:

1. Discretize the output and the contribution to the spinning reserve of unit i with a dicretization step of $\Delta\ MW$.
2. Discretize the demand $D_i \in [D_i^{min}, D_i^{max}]$ which may be satisfied by the i first units, with

   $$D_i^{min} = \sum_{k=1}^{i} P_k^{min}$$

   and

   $$D_i^{max} = \sum_{k=1}^{i} P_k^{max}$$

   with the same step as above.

3. For $i = 1$, set $F_i^*(D_i, S_i) = F_1(P_1)$ for
$P_1 \in [D_i^{min}, D_i^{max}]$ and $0 \leq S_1 \leq Min(S_1^{max}, P_1^{max} - D_1)$ and set $F_1^*(D_1, S_1) = M$ (an arbitrarily large value) otherwise. Then compute $F_i^*(D_i, R_i)$ for $D_i \in [D_i^{min}, D_i^{max}]$ and $0 \leq P_i \leq P_i^{max}$ where

$$R_i^{max} = Min(S_R, \sum_{k=1}^{i} S_i^{max}),$$

using equation (27), where the values of the P_i and S_i to the taken into account are those of (28) and (29).

Let us next evaluate the number of computation required by this algorithm. This depends on N, the number of units, and the number of possible states for each value of $i = 1, 2, \ldots, N$. For simplicity, we assume that the lower and upper bounds on the power produced and the contribution to the spinning reserve are the same for all units, i.e., $P_i^{min} = P_{min}, P_i^{max} = P_{max}$ and $S_i^{max} = S_{max}$ for all $i = 1, 2, \ldots, N$. The number of states for D_i is then

$$\frac{i(P_{max} - P_{min})}{\Delta} + 1$$

and the number of states for the spinning reserve

$$\frac{i(S_{max})}{\Delta} + 1.$$

The maximum number of computations for a value of D_i is at most $(P_{max} - P_{min})$. The number of computations for a value of R_i is at most S_{max}. Hence, assuming that $\Delta = 1MW$ the total number of computations is

$$\sum_{i=1}^{N} i^2 (P_{max} - P_{min})^2 S_{max}^2.$$

Setting $P_{max} - P_{min} = E$ and $S_{max} = S$, the order of magnitude of the number of computations is in

$$O(N^3 E^2 S^2).$$

Based on this expression, the dynamic programming algorithm for economic dispatch with spinning reserve constraint turns out to be both time and space consuming.

Indeed, the use of a second state variable drastically increases the number of computations and the set of optimal solution for the sub-policies that use the i first groups. This clearly shows that this method can only be applied for small size problems. This difficulty can be alleviated however as the use of some dominance rules allows us to reduce the number of computations at each step and substantially improve the efficiency of the dynamic programming algorithm. The dominance rule can be defined as follows:

State j dominates state i if and only if:

$$F_k(D_i, R_i) \geq F_k(D_j, R_j),$$

$$D_i \leq D_j$$

and

$$R_i \leq R_j.$$

We then have

Proposition 1 *Any dominated solutions for sub-policies that uses i groups can only generate dominated solutions for sub-policies that use $i+1$ groups.*

Proof Consider two states j and k implied by the Bellman's equation for i groups such that

$$F_i^*(D_k, R_k) \geq F_i^*(D_j, R_j),$$

$$R_k \leq R_j.$$

In state $i+1$ we get:

$$F_{i+1}^*(D_k + P_{i+1}, R_k + S_{i+1}) \geq F_{i+1}^*(D_j + P_{i+1}, R_j + S_{i+1})$$

and

$$R_k + S_{i+1} \leq R_j + S_{i+1}.$$

Thus any solution computed for $i+1$ groups from the dominated solution for i groups is itself dominated. ■

By taking into account this rule, we reduce the number of computations by eliminating the dominated solution at each step. Moreover, because the optimal solution satisfies the power and spinning reserve demands in state N at the lowest cost and therefore cannot be dominated, it remains unchanged.

The dynamic programming algorithm with state dominance proceeds as follows:

1. Discretize the output of group i using a discretization step Δ in MW.
2. Discretize the demand $D_i \in [D_i^{min}, D_i^{max}]$ that can be satisfied for the i first group using the same discretization Δ
3. If $i = 1$ then $F_1^*(D_1, S_1) = F_1(P_1)$ for $D_1 \in [D_1^{min}, D_1^{max}]$, otherwise set $F_1^*(D_1, S_1) = M$, where M is a large number and $S_1 = Min(S_1^{max}, P_1^{max} - D_1)$.

Else, compute $F_i^*(D_i, R_i)$ as in (28) for $D_i \in [D_i^{min}, D_i^{max}]$ and $0 \leq R_i \leq R_i^{max}$ where

$$R_i^{max} = Min(S_R, \sum_{k=1}^{i} S_i^{max})$$

and the values of P_i et S_i in the expression are

$$P_i^{min} \leq P_i \leq Min(P_i^{max}, D_i - D_{i-1}^{min})$$

and

$$0 \leq S_i \leq Min(R_i, S_i^{max}, P_i^{max} - P_i).$$

If

$$F_i(D_i^j, R_i^j) \geq F_i(D_i^k, R_i^k),$$
$$D_i^j \leq D_i^k$$

and

$$R_i^j \leq R_i^k,$$

then eliminate the state (D_i^j, R_i^j) of the non dominated solutions for the i groups and iterate this step until $i = N$.

Although the number of computations in the dynamic programming algorithm combined with state dominance is in worst case of the same order of magnitude as in the previous algorithm, the use of the dominance rule reduces substantially the computation time in practice.

The efficiency of the algorithm can be further improved, in some cases, by adding a simple test. The optimal solution is first computed omitting the spinning reserve constraint; if this latter is satisfied, the algoritm stops. Otherwise, the dynamic programming algorithm with state dominance is used to sove the problem.

5 Results

In order to evaluate their performance, the classical dynamic programming algorithm for the economic dispatch problem with spinning reserve as well as the version using state dominance were programmed in C and tested on a SUN ULTRA 2 computer (300 MHz).

Both algorithms were tested on several examples constructed from the data of Bakirtzis et al. [2]. Tables 2 and 3 give the minimum and maximum power, their cost and the maximum contribution to spinning reserve for 4 units and 9 units examples.

In Tables 4 and 5 optimal solutions for there two problems are given, when the demand in power is of 800 MW and in spinning reserve 100 MW for 4 units, and 2500 MW and 250 MW respectively for 9 units. These optimal solutions have a cost of 15556.5$\$/h$ and of 57292$\$/h$. Table 6 compares the computing times for the two algorithms. It appears that state dominance reduces the computational burden quite drastically, i.e., by a factor of about 3000 for 4 units and of about 80 for 9 units.

Further examples with 8 and 16 units were constructed by doubling once or twice the Bakirtzis et al. [2] data for 4 units. The demand and spinning reserve were also doubled accordingly. Table 7 presents the cost of the optimal solution and the corresponding computing times. To conclude, it appears that large economic dispatch problem with spinning reserve constraint may be solved exactly by dynamic programming with two state variables. To render this method feasible in practice (in particular if the problem is to be solved repeatedly at short intervals in time) one must exploit dominance between states: computation times are then cut by a factor which can exceed 1000.

Table 2: Example data: breakpoints of 4 units cost functions and corresponding costs

unit i	1	2	3	4
$P^{max}(MW)$	600	220	560	360
$P^{min}(MW)$	190	70	50	170
$S^{max}(MW)$	100	50	100	60
$F(P^{min})$($/h)	4000	1600	3900	2000
$F(P^{max})$($/h)	10927.5	5005	3910537.5	4300

Table 3: Example data: breakpoints of 9 units cost functions and corresponding costs

unit i	1	2	3	4	5	6	7	8	9
$P^{max}(MW)$	560	185	220	570	360	640	640	600	600
$P^{min}(MW)$	50	50	70	210	170	130	130	190	190
$S^{max}(MW)$	100	100	100	100	100	100	100	100	100
$F(P^{min})$($/h)	2000	1400	1600	4300	3200	7500	7500	4000	3900
$F(P^{max})$($/h)	10690	5045	5005	11035	7186	26985	26205	10927.5	10537.5

Table 4: Optimal solution for 4 units example

unit i	1	2	3	4	total
$P_i(MW)$	360	70	200	170	2500
$S_i(MW)$	100	0	0	0	100
F_i($/h)	6339.50	1600	4417	3200	15556.53

Table 5: Optimal solution for 9 units example

unit i	1	2	3	4	5	6	7	8	9	total
$P_i(MW)$	460	50	145	215	170	130	130	600	600	2500
$S_i(MW)$	50	50	50	50	50	0	0	0	0	250
F_i($/h)	8765	1400	3055	4407	3200	7500	7500	10927.50	10537.50	57291.96

Table 6: Comparing the efficacity of dominance programming algorithm without and with state dominance

Number of units	**Dynamic programming** (sec CPU)	**Dynamic programming with state dominance** (sec CPU)
4	2176.28	0.7
9	87120	1475.41

Table 7: Computational time for dynamic programming with state dominance

Number of units	P_R (MW)	S_R (MW)	time CPU	Optimal value ($/h)
4	800	100	0.25	15978.89
8	1600	200	17.05	31952.68
16	3200	400	3991.21	63882.708

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