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# Delay-Dependent Static Output Feedback Stabilization for Singular Linear Systems

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**Abstract**

This paper deals with the class of continuous-time singular linear systems with time delay in the state vector. Delay-independent and delay dependent sufficient conditions on static output feedback stabilization are developed. A design algorithm for a memoryless static output feedback controllers which guarantee that the closed-loop dynamics will be regular, impulse free and stable is proposed in terms of the solutions to linear matrix inequalities (LMIs). Two numerical examples are given to show the effectiveness of the developed results.

**Key Words:** Singular systems, Continuous-time linear systems, Linear matrix inequality, Stability, Stabilizability, Static output feedback controller.

**Résumé**

Cet article traite de la classe des systèmes singuliers à sauts markoviens à retard. Des conditions suffisantes (indépendantes et dépendantes du retard) de stabilisation par retour de sortie sont établies. Un algorithme de design d'un contrôleur par retour de sortie, qui garantie que la boucle fermée du système est regulière, sans impulsion et stochastiquement stable, est developpé sous forme d'inégalités matricielles linéaires. Des exemples numériques sont donnés pour montrer la validité des résultats.

## 1 Introduction

The class of singular continuous-time linear systems is an important class of systems that has attracted a lot researchers from mathematics and control communities. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems [4, 9]. It was shown in many studies that the class of singular systems is more appropriate to describe the behavior of some practical systems in different fields ranging from chemical processes to robotics (see [4] and some references therein). Many problems for this class of systems either in the continuous-time and discrete-time have been tackled and interesting results have been reported in the literature. Among these contributions we quote those of [13, 19, 17, 5, 14, 15, 16, 12, 7, 8, 10, 11, 3], and the references therein.

Some practical systems that can be modelled by the class of singular systems that we are considering here may have time-delay in their dynamics which may be the cause of instability and performance degradation of such systems (see [2]). Therefore, more attention should be paid to these class of systems. To the best of our knowledge, the class of continuous-time singular systems with time delays has not yet been fully investigated. Particularly delay-dependent sufficient conditions for stabilization are few even not existing in the literature.

This paper deals with the problem of stabilization for singular continuous-time linear systems with time delays. Using the measurement of the output system a static output feedback controller is design to render the closed-loop regular, impulse-free and stable. Firstly, we recall a delay-dependent sufficient condition, which guarantees that the system is regular, impulse free and stable that was developed in Boukas [1]. Based on this, a delay-dependent sufficient condition for the existence of a static output feedback controller guaranteeing that the closed-loop dynamics is regular, impulse free and stable is proposed. Delay-independent results are also developed. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed results. All the developed results are in the LMI framework which makes them more interesting since the solutions are easily obtained using existing powerful tools like the LMI toolbox of Matlab or any equivalent tool.

The rest of this paper is organized as follows. In Section 2, the problem is stated and the goal of the paper is stated. In Section 3, the main results are given and they include results on output stabilizability. A memoryless static output feedback controller is used in this paper and a design algorithm in terms of the solutions to linear matrix inequalities is proposed to synthesize the controller gains we are using. Delay independent and delay dependent sufficient conditions are developed to design the appropriate controller that makes the closed-loop dynamics regular, impulse-free and stable. Section 4 presents two numerical examples to show the effectiveness of the proposed results.

**Notation.** Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the $n$ dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$\mathsf{T}$" denotes matrix transposition and the notation $X \geq Y$(respectively, $X > Y$) where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{I}$ is the identity matrices with compatible dimensions. $\mathscr{L}_2$ is the space of integral vector over $[0, \infty)$. $\| \cdot \|$ will refer to the Euclidean vector norm whereas $\| \cdot \|$ denotes the $\mathscr{L}_2$-norm over $[0, \infty)$ defined as $\|f\|^2 = \int_0^\infty f^T(t)f(t)\ dt$.

## 2 Problem statement

Let $x(t) \in \mathbb{R}^n$ be the physical state of the studied system, which is assumed to satisfy the following dynamics:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_1 x(t-h) + Bu(t), \\ y(t) = Cx(t), \\ x(s) = \phi(s), -h \leq s \leq 0 \end{cases} \tag{1}$$

where $u(t) \in \mathbb{R}^m$ is the control input system, $y(t) \in \mathbb{R}^p$ is the measured output system, $A$, $A_1$, $B$, and $C$ are known real matrices with appropriate dimensions, the matrix $E$ may be singular, and we assume that $rank(E) = n_E \leq n$, $h > 0$ represents the system delay, and $\phi(t)$ is a smooth vector-valued initial function in $[-h, 0]$ representing the initial condition of the system such that $x(s) = \phi(s) \in \mathscr{L}_2[-h, 0] \overset{\Delta}{=} \{f(\cdot) | \int_0^\infty f^\top(t)f(t)dt < \infty\}$.

The following definitions will be used in the rest of this paper. For more details on the singular systems properties, we refer the reader to [4] and the references therein.

**Definition 2.1** *[4]*

*i. System (1) is said to be regular if the characteristic polynomial,* $\det(sE - A)$ *is not identically zero.*

*ii. System (1) is said to be impulse free, i.e. the* $\deg(\det(sE - A)) = rank(E)$.

For more details on other properties and the existence of the solution of system (1), we refer the reader to [14], and the references therein. In general, the regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

This paper studies the stabilization of the class of systems (1). Our main objective in this paper is to design a static output feedback controller guaranteeing that the closed-loop is regular, impulse free and stable. In the rest of this paper, we will assume that we don't have complete access to the system state which renders the state feedback stabilization not possible. To overcome this, an alternative to design a static output feedback controller is proposed. Our methodology in this paper will be mainly based on the Lyapunov theory and some algebraic results. The conditions we will develop here will be in terms of the solutions to linear matrix inequalities that can be easily obtained using LMI control toolbox. These conditions are delay-dependent, which makes them less conservative compared to delay independent conditions. The stabilizing controller that we would like to design has the following form:

$$u(t) = Ky(t) \tag{2}$$

where $K$ is a design parameter that has to be determined.

Before closing this section let us give the following results that will be used in the proofs of our results.

**Lemma 2.2** *(Boukas [1]) The free singular linear system (1) is regular, impulse-free and stable if there exist a nonsingular matrix $P$ and symmetric and positive-definite matrix $Q > 0$, such that the following set of LMIs holds:*

$$E^\top P = P^\top E \geq 0 \tag{3}$$

$$\begin{bmatrix} P^\top A + A^\top P + Q & P^\top A_1 \\ A_1^\top P & -Q \end{bmatrix} < 0. \tag{4}$$

**Remark 2.3** *To prove the results of this lemma, we choose the following Lyapunov functional:*

$$V(x) = x^\top E^\top P x(t) + \int_{t-h}^{t} x^\top(s) Q x(s) ds$$

*and following the same steps as in Boukas [1].*

**Lemma 2.4** *(Boukas [1]) The free singular linear system (1) is regular, impulse-free and stable if there exist a nonsingular matrix $P$ and symmetric and positive-definite matrices $Q > 0$, $R > 0$, and $T > 0$, such that the following set of LMIs holds:*

$$E^\top P = P^\top E \geq 0 \tag{5}$$

$$\begin{bmatrix} R & P^\top \\ P & T \end{bmatrix} \geq 0 \tag{6}$$

$$\begin{bmatrix} J & P^\top A_1 - P^\top E & hA^\top T \\ A_1^\top P - E^\top P & -Q & hA_1^\top T \\ hTA & hTA_1 & -hT \end{bmatrix} < 0. \tag{7}$$

*where $J = P^\top A + A^\top P + Q + P^\top E + E^\top P + hR$.*

## 3 Main results

In this section, we will design a static output feedback controller of the form (2) that guarantees the closed-loop dynamics of system (1) is regular, impulse free and stable goal. Delay independent and delay dependent sufficient conditions are developed in the LMI setting.

Let us firstly develop delay independent sufficient conditions for static output feedback stabilization. Let us now concentrate on the design of a static output feedback controller of the form (2) which guarantees that the closed-loop will be regular, impulse free and stable. For this purpose, plugging the controller (2) in the dynamics (1) gives:

$$\begin{cases} \dot{x}(t) = A_{cl}x(t) + A_1 x(t-h) \\ z(t) = Cx(t), \\ x(s) = \phi(s), -h \le s \le 0 \end{cases} \tag{8}$$

where $A_{cl} = A + BK$.

Using the results of Lemma 2.2, the closed-loop dynamics of the singular linear system (8) is regular, impulse-free and stable if there exist a nonsingular matrix $P$ and symmetric and positive-definite matrix $Q > 0$, such that the following set of LMIs holds:

$$\begin{array}{l} E^\top P = P^\top E \ge 0 \\ \begin{bmatrix} P^\top A_{cl} + A_{cl}^\top P + Q & P^\top A_1 \\ A_1^\top P & -Q \end{bmatrix} < 0. \end{array}$$

Now, pre- and post-multiplying the last LMI respectively by $\text{diag}\left(P^{-\top}, P^{-\top}\right)$ and its transpose, we get:

$$\begin{bmatrix} A_{cl}P^{-1} + P^{-\top}A_{cl}^\top + P^{-\top}QP^{-1} & A_1 P^{-1} \\ P^{-\top}A_1^\top & -P^{-\top}QP^{-1} \end{bmatrix} < 0,$$

Using the expression of $A_{cl}$ and letting $X = P^{-1}$, we get:

$$\begin{array}{l} EX = X^\top E^\top \ge 0 \\ \begin{bmatrix} AX + X^\top A^\top + BKCX + X^\top C^\top K^\top B^\top + X^\top QX & A_1 X \\ X^\top A_1^\top & -X^\top QX \end{bmatrix} < 0, \end{array}$$

Let $K = NM^{-1}$ with $M$ a nonsingular matrix and $N$ a matrix that have to be determined. Assuming $MC = CX$ holds, and letting $Z = X^\top QX$, then we get the following results.

**Theorem 3.1** *There exists a static output feedback controller of the form (2) such that the closed-loop system (1) is regular, impulse-free and stable if there exist nonsingular matrices $X$, $M$ and a matrix $N$, and symmetric and positive-definite matrix $Z > 0$ such that the following set of LMIs holds:*

$$X^\top E^\top = EX \ge 0 \tag{9}$$

$$MC = CX \tag{10}$$

$$\begin{bmatrix} AX + X^\top A^\top + BNC + C^\top N^\top B^\top + Z & A_1 X \\ X^\top A_1^\top & -Z \end{bmatrix} < 0, \tag{11}$$

*The stabilizing controller gain is given by $K = NM^{-1}$.*

The results of this theorem are delay independent and consequently when the LMIs are feasible this means that the controller will stabilize the system for any delay. These results are restrictive and therefore sufficient conditions that are function of the delay of the system are of interest and are less conservative. In the rest of this section, we will develop results that are delay dependent.

Based on the results of Lemma 2.4, the closed-loop dynamics (8) will be regular, impulse free and stable if there exist a nonsingular matrix $P$ and symmetric and positive-definite matrices $Q > 0$, $R > 0$, and $T > 0$ such the following set of LMIs holds:

$$E^\top P = P^\top E \geq 0$$
$$\begin{bmatrix} R & P^\top \\ P & T \end{bmatrix} > 0$$
$$\begin{bmatrix} J_0 & P^\top A_1 - P^\top E & hA_{cl}^\top T \\ A_1^\top P - E^\top P & -Q & hA_1^\top T \\ hTA_{cl} & hTA_1 & -hT \end{bmatrix} < 0.$$

where $J = P^\top A_{cl} + A_{cl}^\top P + Q + P^\top E + E^\top P + hR$.

Now, pre- and post-multiplying the last LMI respectively by diag $\left(P^{-\top}, P^{-\top}, T^{-1}\right)$ and its transpose, we get:

$$\begin{bmatrix} \tilde{J} & A_1 P^{-1} - EP^{-1} & hP^{-\top} A_{cl}^\top \\ P^{-\top} A_1^\top - P^{-\top} E^\top & -P^{-\top} Q P^{-1} & hP^{-\top} A_1^\top \\ hA_{cl} P^{-1} & hA_1 P^{-1} & -hT^{-1} \end{bmatrix} < 0, \tag{12}$$

where

$$\tilde{J} = A_{cl} P^{-1} + P^{-\top} A_{cl}^\top + P^{-\top} Q P^{-1} + EP^{-1} + P^{-\top} E^\top + hP^{-\top} R P^{-1}.$$

Let $K = NM^{-1}$. Using, now the expression of $A_{cl}$, letting $X = P^{-1}$, $Z = P^{-\top} Q P^{-1}$, $W = P^{-\top} R P^{-1}$, $U = T^{-1}$, and noting that $\begin{bmatrix} R & P^\top \\ P & T \end{bmatrix} \geq 0$ can be rewritten after pre- and post-multiplying the left hand side respectively by diag $\left(P^{-\top}, T^{-1}\right)$ and its transpose as: $\begin{bmatrix} W & U \\ U & U \end{bmatrix} \geq 0$, and in similar way the relations $E^\top P = P^\top E \geq 0$ can be rewritten as $X^\top E^\top = EX \geq 0$, and assuming that $MC = CX$ holds, we get the following results for the stabilization for our class of systems.

**Theorem 3.2** *There exists a static output feedback controller of the form (2) such that the closed-loop system (1) is regular, impulse-free and stable if there exist a nonsingular matrices $X$, and $M$, a matrix $N$, and symmetric and positive-definite matrices $Z > 0$,*

*$W > 0$, and $U > 0$ such that the following set of LMIs holds:*

$$X^\top E^\top = EX \geq 0 \tag{13}$$

$$MC = CX \tag{14}$$

$$\begin{bmatrix} W & U \\ U & U \end{bmatrix} \geq 0 \tag{15}$$

$$\begin{bmatrix} \widehat{J} & A_1X - EX & h\,(AX + BNC)^\top \\ X^\top A_1^\top - X^\top E^\top & -Z & hX^\top A_1^\top \\ h\,(AX + BNC) & hA_1X & -hU \end{bmatrix} < 0, \tag{16}$$

*where*

$$\widehat{J} = AX + X^\top A^\top + BNC + C^\top N^\top B^\top + EX + X^\top E^\top + Z + hW$$

*The stabilizing controller gain is given by $K = NM^{-1}$.*

**Remark 3.3** *The LMIs of Theorem 3.2 are delay dependent which make them more realistic and less conservative. The conditions we developed in this theorem can be extended in a straightforward way to the case of time varying delays in the system when the appropriate assumptions on the delay are satisfied.*

## 4 Numerical example

To show the effectiveness of our results, let us consider two numerical examples of a singular system with state space in $\mathbb{R}^3$.

**Example 4.1** *The data of the system we are considering in this example are as follow:*

$$A = \begin{bmatrix} 0.1 & 0.0 & 1.0 \\ 0.0 & -2.0 & 1.0 \\ 0.0 & -1.0 & -1.0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.1 & 0.0 & 0.1 \\ 0.0 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.3 \end{bmatrix},\ B = \begin{bmatrix} 0.0 & 0.2 \\ 1.0 & 0.0 \\ -0.1 & 1.0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 1.0 \\ 0.5 & 1.0 & 0.0 \end{bmatrix}.$$

*The singular matrix $E$ is given by the following expression:*

$$E = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

*Solving the LMIs (9)-(11), we get:*

$$Z = \begin{bmatrix} 1.0065 & -0.0003 & -0.0009 \\ -0.0003 & 1.0194 & 0.0033 \\ -0.0009 & 0.0033 & 1.0057 \end{bmatrix},\ X = \begin{bmatrix} 0.9463 & -0.0090 & 0.0000 \\ -0.0090 & 0.9328 & 0.0000 \\ -0.9235 & -0.0471 & 0.0509 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.0509 & -0.0561 \\ -0.0000 & 0.9283 \end{bmatrix},\ N = \begin{bmatrix} 0.4184 & 0.8841 \\ -0.9555 & 0.4974 \end{bmatrix},$$

*which gives the following stabilizing gain:*

$$K = \begin{bmatrix} 8.2175 & 1.4493 \\ -18.7682 & -0.5993 \end{bmatrix}.$$

*Solving the LMIs (13)-(16) with* $h = 0.45$, *we get:*

$$Z = \begin{bmatrix} 0.4296 & 0.1342 & 0.2657 \\ 0.1342 & 0.5118 & -0.1382 \\ 0.2657 & -0.1382 & 0.4188 \end{bmatrix},$$

$$X = \begin{bmatrix} 0.2597 & -0.0730 & 0.0000 \\ -0.0730 & 0.1503 & 0.0000 \\ -1.0781 & 0.0201 & -0.7919 \end{bmatrix}, U = \begin{bmatrix} 0.8270 & 0.1027 & 0.4506 \\ 0.1027 & 1.0354 & -0.3471 \\ 0.4506 & -0.3471 & 1.1122 \end{bmatrix},$$

$$W = \begin{bmatrix} 1.0493 & 0.2167 & 0.6167 \\ 0.2167 & 1.4228 & -0.4278 \\ 0.6167 & -0.4278 & 1.3264 \end{bmatrix}, M = \begin{bmatrix} -0.7919 & -0.0529 \\ 0.0000 & 0.1138 \end{bmatrix},$$

$$N = \begin{bmatrix} 1.4460 & -1.1949 \\ -1.8474 & 0.4712 \end{bmatrix},$$

*which gives the following stabilizing gain:*

$$K = \begin{bmatrix} -1.8260 & -11.3532 \\ 2.3330 & 5.2274 \end{bmatrix}.$$

*For each* $h \in [0, 0.45]$ *the set of LMIs given in our results are feasible. Beyond the value* 0.45 *the LMIs are not feasible and we can say anything regarding the stabilizability of the considered system since our conditions are only sufficient.*

**Example 4.2** *As a second example, let us consider a system with state in* $\mathbb{R}^3$ *with the following data:*

$$A = \begin{bmatrix} 2.0 & -1.0 & 1.0 \\ 0.0 & -6.0 & 1.2 \\ 0.0 & -0.2 & -6.5 \end{bmatrix}, A_1 = \begin{bmatrix} -0.1 & -0.4 & 1.3 \\ 0.0 & -1.6 & -0.5 \\ 0.3 & 0.0 & -1.5 \end{bmatrix}, B = \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 0.2 \\ 0.30.1 & \end{bmatrix},$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 1.0 \\ 0.5 & 1.0 & 0.0 \end{bmatrix},$$

*The singular matrix* $E$ *is given by the following expression:*

$$E = \begin{bmatrix} 2.0 & 1.0 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}.$$

*Solving the LMIs (9)-(11), we get:*

$$Z = \begin{bmatrix} 1.5405 & -0.1267 & -0.2407 \\ -0.1267 & 1.9640 & 0.1286 \\ -0.2407 & 0.1286 & 1.4750 \end{bmatrix}, \; X = \begin{bmatrix} 0.7693 & 0.0027 & 0.0000 \\ 0.0027 & 0.7733 & 0.0000 \\ -0.2921 & 0.2306 & 0.3605 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.3605 & 0.2333 \\ -0.0000 & 0.7747 \end{bmatrix}, \; N = \begin{bmatrix} 23.0352 & 12.4806 \\ -62.1232 & -20.5990 \end{bmatrix},$$

*which gives the following stabilizing gain:*

$$K = \begin{bmatrix} 63.8900 & -3.1292 \\ -172.3039 & 25.2976 \end{bmatrix}.$$

*For the delay dependent stabilization, solving the LMIs (13)-(16) with* $h = 0.2$*, we get:*

$$Z = \begin{bmatrix} 2.1994 & 1.3212 & 1.8878 \\ 1.3212 & 3.1503 & 0.6624 \\ 1.8878 & 0.6624 & 4.3231 \end{bmatrix},$$

$$X = \begin{bmatrix} 0.7129 & 0.1838 & 0.0000 \\ 0.1838 & 0.9886 & 0.0000 \\ 0.5956 & 0.6080 & 0.9127 \end{bmatrix}, \; U = \begin{bmatrix} 6.4804 & 1.3306 & 5.3779 \\ 1.3306 & 4.7856 & -0.9318 \\ 5.3779 & -0.9318 & 6.5324 \end{bmatrix},$$

$$W = \begin{bmatrix} 8.4331 & 2.2395 & 6.8558 \\ 2.2395 & 8.0817 & -1.6320 \\ 6.8558 & -1.6320 & 8.4124 \end{bmatrix}, \; M = \begin{bmatrix} 0.9127 & 0.7918 \\ 0.0000 & 1.0805 \end{bmatrix},$$

$$N = \begin{bmatrix} 39.9564 & 55.5709 \\ -110.5821 & -142.0872 \end{bmatrix},$$

*which gives the following gain:*

$$K = \begin{bmatrix} 43.7804 & 19.3492 \\ -121.1654 & -42.7132 \end{bmatrix}.$$

*In a similar way we can determine the maximum delay for which the system will be regular, impulse-free and stable. Our computations show that for each* $h \in [0, 0.28]$ *the set of LMIs given in our results are feasible. Beyond the value* 0.28 *the LMIs are not feasible and we can say anything as before regarding the stabilizability of the considered system.*

## 5 Conclusion

This paper dealt with the class of continuous-time singular linear systems with time-delay in the state vector. Results on output stabilization are developed. The LMI framework is used to establish the different results on output stabilizability. The conditions we developed are delay independent and delay dependent. The results we developed can easily be solved using any LMI toolbox like the one of Matlab or the one of Scilab.

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