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Abstract

WDM networks offer a technology that can transferred several optical signals into a single optical fiber. This allows for more efficient use of the huge capacity of optical fibers but it also poses new network design and management problems such as routing and wavelength planning. This paper discusses the RWA problem, i.e., the routing and wavelength assignment problem in WDM networks. Given a set of requests and the number of available wavelengths the fibers support, we are to find, through the network, a route for each connection and assign wavelengths to them. In this paper, we consider the objective of minimizing the number of connections that have to be denied. We propose a multi-phase heuristic algorithm called RWABOU. It begins with an initialization step in which we compute for each connection, r-shortest paths from each source to their destination. It is followed by an algorithm with two interactive phases. The first phase integrates a Tabu Search heuristic for the wavelength assignment followed by and interacting with a partial rerouting heuristic phase for the blocked connections. Computational experience shows that the RWABOU heuristic outperforms some recently published work on traffic instances run on the NSF and the EON networks. Moreover, comparing the lower bound of the RWABOU heuristic with the best known upper bound allows to conclude to the optimality of the RWA solution for several benchmark problems and to the very high quality of several others.

Key Words: WDM network, single-hop, graph coloring, blocking probability, wavelength assignment, routing.

Résumé

Les réseaux WDM offrent une technologie qui permet de transférer plusieurs signaux optiques dans une seule fibre optique. Cela permet une utilisation plus efficace de la capacité considérable des fibres optiques mais cela pose aussi de nouveaux problèmes de conception et de gestion des réseaux, ainsi que de planification du routage et des longueurs d'onde requises. Ce rapport discute du problème RWA, c'est-à-dire du problème de routage et d'affectation de longueurs d'onde dans les réseaux optiques WDM. Étant donné un ensemble de requêtes de connexion et le nombre disponible de longueurs d'onde dont chaque fibre dispose, nous devons trouver, à travers le réseau, une route pour chaque connexion et lui affecter une longueur d'onde. Dans ce rapport, nous considérons l'objectif de minimiser le nombre de connexions qui sont refusées. Nous proposons une heuristique avec plusieurs phases appelée RWABOU. Elle commence avec une étape d'initialisation dans laquelle nous calculons pour chaque connexion, les r plus courts chemins de chaque source à chaque destination. Cette étape est suivie par un algorithme avec deux phases interactives. La première phase intègre une heuristique de type recherche tabou pour l'affectation des longueurs d'onde suivie d'une phase heuristique de reroutage partiel pour les connexions bloquées. Les expériences de calcul montrent que la procédure RWABOU se comporte mieux que certaines procédures publiées récemment sur des instances de trafic avec les réseaux NSF et EON. De plus, en comparant la borne inférieure fournie par la procédure RWABOU avec la meilleure borne supérieure connue, nous pouvons conclure à l'optimalité de la solution pour plusieurs instances, et à l'excellente qualité de plusieurs autres solutions.

1 Introduction

All-optical wavelength-division multiplexed (WDM) networks are considered as highly promising for the next generation of wide-area backbone networks that will need to support hundreds and thousands of users, each operating at gigabit-per-second speed. In this paper, we consider a WDM single hop optical network architecture with wavelength routers interconnected by point-to-point optical links, where connections can be set-up by appropriately choosing a wavelength continuous route subject to some grade (GoS) or quality of service (QoS) constraints or objectives. We concentrate here on the static problem of routing and wavelength assignment (RWA) with the objective of minimizing the blocking probability, sometimes called max-RWA problem.

Numerous papers have already appeared on the max-RWA problem, see, e.g., Dutta and Rouskas [22], Zang, Jue and Mukherjeee [11], Leonardi, Mellia and Marsan [5] for three surveys. Most of the earliest papers consider two-phase solution approaches where the routing and the wavelength assignment are decoupled. The first phase corresponds to a routing phase, either a fixed route strategy based on the computation of shortest routing paths or an alternate route strategy with, e.g., an ordered list of the r-shortest paths or a list of link-disjoint paths. The second phase deals with the wavelength assignment problem, very often reformulated as a graph coloring problem solved using different heuristic schemes, see, e.g., Hyytiä and Virtamo [15] for a comparison of some of them.

Some authors have also attempted to reformulate the max-RWA problem using integer programming approaches, with the advantage that the routing and wavelength assignment are then coupled. For instance, Krishnaswamy and Sivarajan [18] proposed two 0-1 linear programming formulations of the max-RWA problem which however contains too many variables in order to be solved efficiently in practice. For this reason, they explore the use of the linear relaxations of these formulations both for deriving upper bounds and for deducing heuristic solutions using a rounding-off procedure. An analytical and a computational comparison of the two formulations of Krishnaswamy and Sivarajan [18] and their upper bounds with other formulations proposed by, e.g., Coudert and Rivano [4], can be found in Jaumard *et al.* [16].

Another approach has been proposed by Chen and Banerjee [3] with a *layered-graph* model where routing and wavelength assignment steps are also tightly coupled. However, using this model for solving the max-RWA problem is still difficult as it reduces to an edge-disjoint path problem, reformulated as an integer multicommodity flow problem. This last reformulation can be used both for obtaining upper bounds and designing efficient heuristics, and computational experience reported by the authors show that it leads to better results than some heuristics with a two-phase scheme.

We propose a new algorithm, called RWABOU, corresponding to an iterative two-phase heuristic, made of a routing phase interacting with a wavelength assignment phase. Both phases are called recursively until no further improvement can be obtained on the blocking rate. Each phase uses the solution output by the previous phase to improve on the blocking rate. The wavelength assignment phase, which corresponds to a W-stability graph problem that differs slightly from the classical graph coloring formulation used by most of the previous authors, is solved by a Tabu Search meta-heuristic, a well known and efficient technique for solving highly combinatorial problems.

The paper is organized as follows. In Section 2 we detail the statement of the problem. The RWABOU heuristic is described in Section 3 with two variants. Computational and comparative results are presented in the last section.

2 Network Model and Problem Formulation

2.1 Notation

We represent the optical fiber network by a multigraph $\mathcal{N} = (N, L)$ where each vertex of N corresponds to a node (e.g., multiplexers or routers) in the physical network and each edge or link $\ell \in L$ to a fiber between two nodes. Physical links may support several fibers, and if it is the case, we assume that the definition of the routing paths identifies for each link the fiber that is used. This entails that two identical paths in terms of physical links may lead to two different routing paths in the multigraph.

Let D be the traffic matrix that specifies the traffic flow D_{sd} between each pair $\{N_s, N_d\}$ of source-destination nodes. Each matrix element D_{sd} is defined as a set of T_{sd} aggregated flows denoted $\phi^t_{sd}, t = 1, 2, ..., T_{sd}$, where it is assumed that each flow ϕ^t_{sd} can be carried out on a single wavelength. We will consider both symmetrical and asymmetrical traffic matrices. For symmetrical traffic, we will assume that $\mathcal{N}$ is an undirected multigraph and that each wavelength is full-duplex, although such theoretical full-duplex wavelengths might be split into two directional wavelengths in practice. For asymmetrical traffic, $\mathcal{N}$ is a directed multigraph with usually two fibers per link, one for each direction.

We also assume that flows are not bifurcated, i.e., all packets of a given traffic flow are routed along the same path. Moreover, each lightpath uses the same wavelength along the whole path (wavelength continuity constraint), i.e., there is no wavelength conversion.

We assume that the physical topology of the network is given and in particular that the link capacities are given. We denote by C_ℓ the capacity of link ℓ expressed by the number of fibers per link and the number of wavelengths per fiber. Let W be the number of wavelengths per fiber that is available in the network. Let $\Phi = \{\phi^t_{sd} : N_s, N_d \in N, t = 1, 2, \ldots, T_{sd}\}$ be the set of all connections, i.e., the set of all aggregated flows. Each aggregated flow ϕ^t_{sd} corresponds to a set of sessions (or individual flows) between the same source-destination pair denoted by $\{N_s, N_d\}$ for symmetrical traffic (or from a source to a destination denoted by (N_s, N_d) for asymmetrical traffic).

2.2 Problem Formulation

In this paper, we propose to study the following planning problem. Given the physical topology of an optical network $\mathcal{N} = (N, L)$ together with link capacities, a set D of aggregated traffic flows, we want to establish as many lightpaths as possible for the set Φ of aggregated traffic flows in order to minimize the blocking rate. The lightpath definition must satisfy some constraints: *(i)* two different lightpaths using the same fiber and link must have distinct wavelengths and *(ii)* the number of lightpaths using the same link and fiber should not exceed the capacity of the fiber, or in other words the maximum number of wavelengths per fiber.

The grade of service (GoS) will be evaluated through the blocking probabilities, that will be estimated in turn through blocking rates, and the objective will be to maximize the GoS, i.e., the number of traffic flows that can be accepted in the network. A traffic flow will be blocked if no wavelength is available on any path satisfying the capacity or lightpath constraints described in the previous paragraph. The blocking rate might depend on the services. Assuming that we have, e.g., both voice and video traffic requests, we can calculate a blocking rate for each service, $P^{\text{voice}}_{\text{blocking}}$ and $P^{\text{video}}_{\text{blocking}}$.

The objective function will therefore be written

$$\min \sum_{s \in \text{Services}} P^s_{\text{blocking}},$$

where the blocking probability will be defined as follows:

$$P^s_{\text{blocking}} = \frac{\text{number of denied flows of class } s}{\text{number of requests for flows of class } s}.$$

Assuming priorities to exist, the objective function will be rewritten as follows:

$$\min \sum_{s \in \text{Services}} p_s \times P^s_{\text{blocking}},$$

where p_s represents a weight for service s that takes its relative priority into account.

3 The RWABOU Algorithm

3.1 Outline of the RWABOU algorithm

We propose a two phase algorithm, called RWABOU, with a routing phase alternating and interacting with a wavelength assignment phase until no improvement can be obtained with respect to the objective function.

The algorithm starts with an initialization step that consists in computing a set $\mathcal{P}_{sd}$ of potential routes in the network $\mathcal{N} = (N, L)$ for each pair of source and destination nodes using a r-shortest path algorithm (see, e.g., Eppstein [6], Yen [23], Martins and Pascoal [20]). If the traffic is symmetrical, paths are undirected ($\mathcal{P}_{sd} = \mathcal{P}_{ds}$), otherwise they are directed ($\mathcal{P}_{sd} \neq \mathcal{P}_{ds}$).

The core of the algorithm consists in two alternating phases: a routing phase in which we select a routing path for each connection in the first routing phase and modify some paths in the subsequent ones, followed by a wavelength assignment phase. Several rerouting strategies will be explored. The wavelength assignment phase consists in assigning a wavelength to the maximum number of connections subject to the selected routing paths: this corresponds to the W-stability graph problem, i.e., to the problem of minimizing the number of uncolored vertices given an undirected graph and only W colors for coloring its nodes.

We will explore two solution schemes to solve the W-stability graph problem associated with the wavelength assignment phase. Both solution schemes will be color exchange procedures based on a Tabu Search scheme in which we change the color of one vertex at each iteration (or equivalently the wavelength assigned to a given connection). In the first scheme, the basic idea is to start from an infeasible W-coloring, minimize the number of edge conflicts (i.e., the number of adjacent nodes identically colored), and then remove iteratively vertices until the W-coloring becomes feasible. In the second scheme, we introduce a dummy color $W + 1$ for the vertices that cannot be colored with the first W colors. No coloring constraint applies for the vertices colored with $W + 1$: no edge conflict is counted for adjacent nodes identically colored $W + 1$. The objective is then to minimize the number of vertices colored with $W + 1$.

A Tabu Search method can be described as follows. It is an iterative procedure in which we go from one solution to another through elementary moves towards a given objective or evaluation function (sometimes constraints are relaxed and introduced with a fixed or dynamic penalty factor in the objective, leading to a so-called evaluation function). For each solution s, we define a neighbor $\mathcal{N}(s)$. For the coloring problem, $\mathcal{N}(s)$ is often defined as the set of solutions derived from s by changing the color of one vertex. At each iteration the best move is usually defined as the one going from s to the best solution s' in $\mathcal{N}(s)$. The best move is defined in absolute value: if there is no solution in $\mathcal{N}(s)$ that can improve the current value of the objective or the evaluation function, we go on with the solution leading to the smallest deterioration of the objective/evaluation function. In order to avoid going back to a previous solution, we use a Tabu list in which we insert the deteriorating moves (in some version all moves) for a given number of iterations corresponding to the length of the Tabu list. The Tabu Search has been independently introduced by Glover [8] [9] and Hansen and Jaumard [12] and has since then been widely used successfully for solving difficult combinatorial problems. Moreover, many refinements have been proposed that significantly improve on the basic version of Tabu Search in many cases, e.g., aspiration criterion or diversification strategy, see, e.g., Glover and Laguna [10] for more details and references.

The W-coloring problem has been widely studied, and several efficient greedy heuristics and metaheuristics have been proposed to solve it, see, e.g., Brelaz [2] and Leighton [19] for the most popular greedy schemes, and Hertz and de Werra [13], Hertz, Taillard and de Werra [14], Galinier and Hao [7] and Avanthay, Hertz and Zuffery [1] for some of the most current efficient metaheuristics. Although close to the W-coloring problem, the W-stability graph problem has not been studied much, and no efficient heuristic has yet been proposed to solve it.

Note that if there exists several connections between a given pair of nodes, they can use similar or distinct routing paths as long as the traffic flow is not bifurcated.

3.2 The Wavelength Graph and its Subgraphs

Before describing the Tabu Search algorithms for solving the wavelength assignment problem, let us first introduce the wavelength graph. It is a graph $G = (V, E)$ defined as follows. Each node $v_{sd,p,t} \in V$ is associated with a potential routing path p for the flow ϕ^t_{sd}. An edge exists between two nodes $v_{sd,p,t}$ and $v_{s'd',p',t'}$ if the two routing paths p and p' share at least one edge (symmetric traffic) or arc (asymmetric traffic).

In Figure 1, we describe a network where we consider a traffic matrix with four connection requests: three requests (flows ϕ^1_{12}, ϕ^2_{12} and ϕ^3_{12}) between N_1 and N_2 with three potential routing paths, one request between N_2 and N_3 (flow ϕ^1_{23}) with again three potential routing paths. In Figure 2, we outline the structure of the wavelength graph. We next detailed all the edges of the wavelength graph in Figure 3 assuming that three potential routing paths exist for each pair of origin-destination nodes.

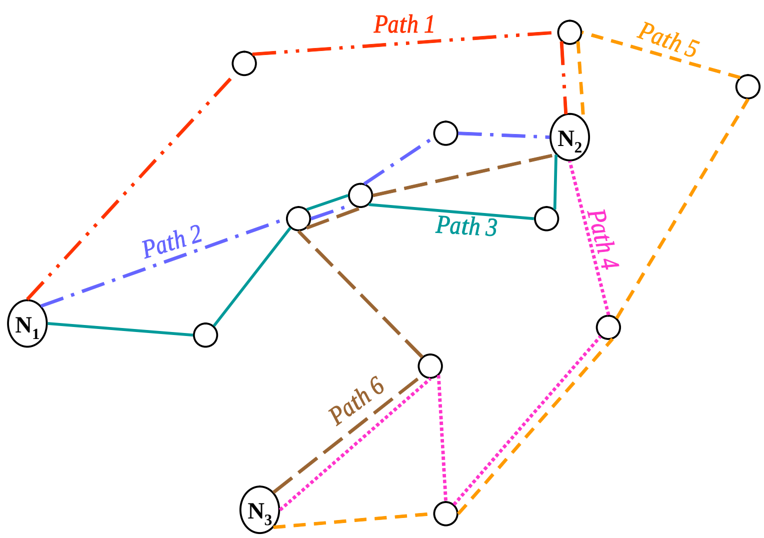


Figure 1: A small network with a set of potential routing paths

The wavelength assignment problem can be defined as follows on the wavelength graph $G = (V, E)$: find a W-coloring which aims at coloring as much as possible at least one vertex per set V^t_{sd}, where V^t_{sd} is the set of vertices associated with a given aggregated flow ϕ^t_{sd} and each vertex $v_{sd,p,t}$ of V^t_{sd} is associated with a potential routing path p for this flow.

Depending on the set of potential routing paths that have been identified at the outset and on the number of requested connections, the wavelength graph might be quite large. Therefore, at each iteration k, we consider the subgraph $G^k = (V^k, E^k)$ deduced from G in which $V^k \subseteq V$ contains only one vertex $v_{sd,p,t}$ for each traffic flow ϕ^t_{sd}: we retain only one routing path for each traffic flow. On these subgraphs, the wavelength assignment problem

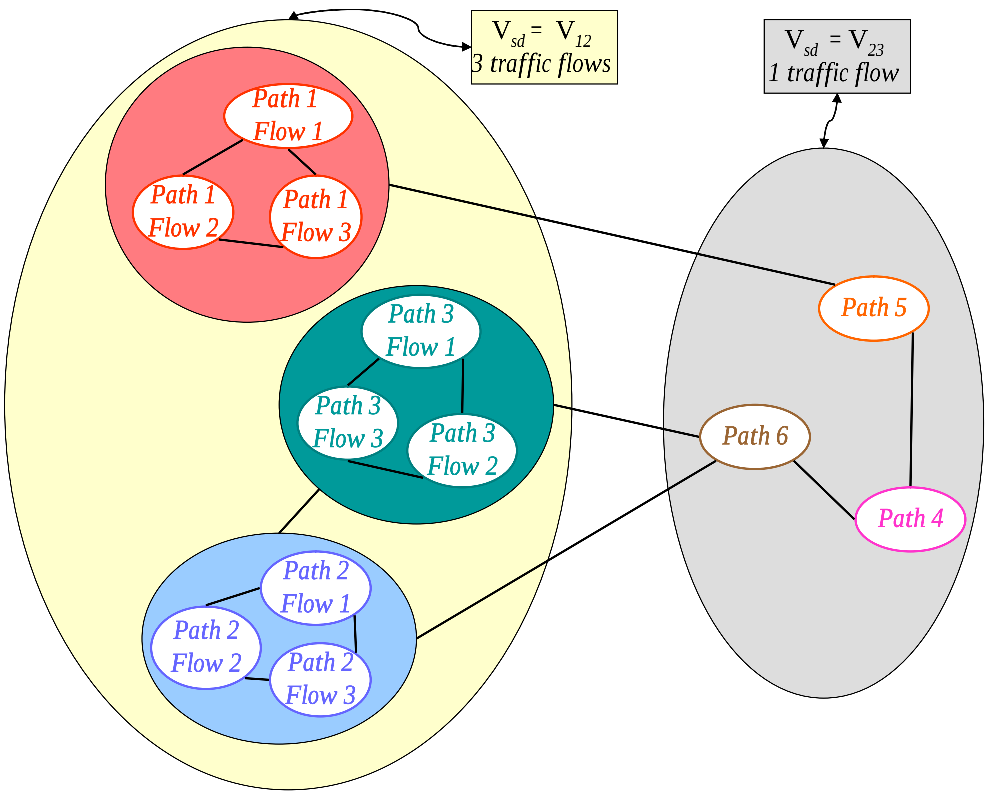


Figure 2: Structure of the wavelength graph

is easier to solve and corresponds exactly to the W-stability graph problem. Observe however that solving the special W-coloring problem on G as stated in the previous paragraph could lead to a one-phase solution scheme for the RWA problem.

Examples of wavelength subgraphs are given in Figure 4. In the first one G^1, it is easy to find a feasible wavelength assignment with two wavelengths while in the second one G^2, we need three wavelengths if, e.g., we select the same routing path for the flows between N_1 and N_2, or otherwise a traffic flow is blocked if only two wavelengths are available.

3.3 Wavelength Assignment and W-coloring

We describe below a first Tabu Search algorithm, called RWABOU1, for solving the wavelength assignment problem.

As explained in Section 3.1, the first solution scheme for the wavelength assignment phase consists in starting with a possibly infeasible W-coloring. In this case, the first routing phase is combined with a wavelength assignment procedure that proceeds as follows.

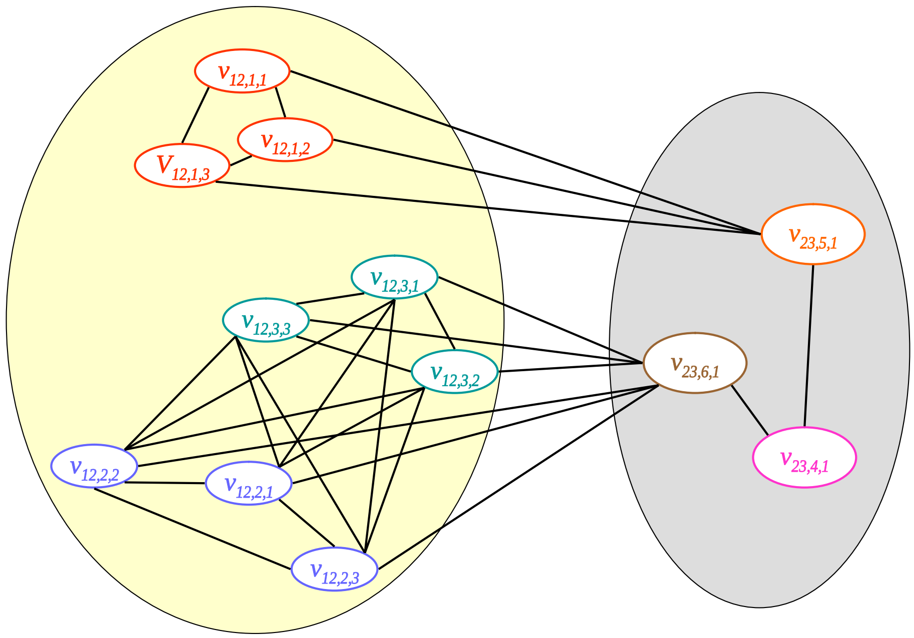


Figure 3: Wavelength graph G

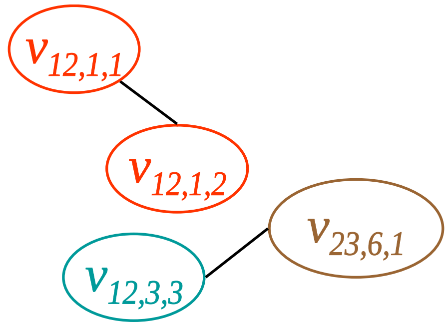
We consider each traffic flow ϕ_{sd}^t in turn. For each of them, we select a routing path in the set $\mathcal{P}_{sd}$, starting with the shortest ones, and considering only those that satisfy the link and fiber capacity constraints. We build an initial wavelength subgraph G^0 along with the selection of routing paths. If at some point, no feasible routing path can be found for a given connection ϕ_{sd}^t (i.e., no vertex $v_{sd,p,t}$ for $p \in \mathcal{P}_{sd}$ can be added with a proper color), we select the path leading to the smallest number of edge conflicts in the current wavelength subgraph.

For the subsequent wavelength assignment phases, we assume the routing phase to be completed and we proceed in two steps. We first look for a feasible W-coloring if possible. If we don't succeed in finding one, we go on with an iterative process to eliminate the minimum number of vertices in G in order to remain with a wavelength subgraph that has a feasible W-coloring. The wavelength assignment procedure can be summarized as follows.

Input. A wavelength subgraph $G^k = (V^k, E^k)$ associated with the path selection of the previous routing phase.

Step 1. Define an initial wavelength assignment using a greedy procedure similar to the Leighton's RLF algorithm [19].

Step 2. Using a Tabu Search similar to the TABUCOL heuristic of Hertz and de Werra [13], find the W-coloring with the smallest number of edge conflicts.



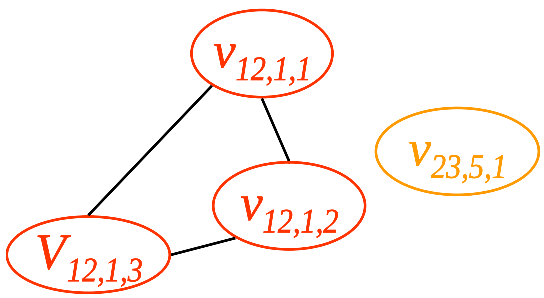


Figure 4: Two Wavelength Subgraphs: G^1 and G^2

Step 3. If no edge conflict remains, exit the wavelength assignment phase.

Step 4. Otherwise, modify the current wavelength graph G^k and eliminate recursively the vertex with the largest number of edge conflicts in the best W-stability graph solution found in Step 2 until obtaining a feasible W-coloring problem (we assume that we compute at each iteration of the Tabu Search algorithm a feasible W-coloring using an iterative vertex removal procedure as described earlier).

During the experiments, we observe that the best W-stability graph solution was not necessarily corresponding to the solution with the smallest number of edge conflicts and that it was often more interesting to consider the best W-stability graph solution in Step 4 rather than the solution with the smallest number of conflicts.

In order to improve the efficiency of the TABUCOL heuristic, a couple of modifications were brought with respect to the management of the Tabu list and the stopping criterion. We use a dynamic length Tabu list instead of the fixed size Tabu list of length 7 of the authors and a stopping criterion with a maximum number of iterations without improvement of the incumbent value. Moreover, a move was inserted in the Tabu list only when it had led to a deterioration of the current value of the objective.

3.4 Wavelength Assignment and the W-stability Graph Problem

Observe that the two step process of the wavelength phase described in Section 3.3, that is first determine if there exist a W-coloring, and then remove the smallest number of nodes in order to find a feasible wavelength assignment does not necessarily lead to the wavelength assignment with the minimum blocking rate. Let us consider the example described below in Figure 5. It corresponds to an optical network with a traffic matrix made of 7 source-destination pairs leading to the wavelength graph depicted in Figure 5. Two wavelength assignments are described with two wavelength conflicts for each of them (corresponding to the dashed links). While in the first case we need to deny only one flow (e.g., deny ϕ_{14} between N_1 and N_4) in order to get a feasible wavelength assignment, it is necessary to deny two flows (deny, e.g., ϕ_{12} between N_1 and N_2 and ϕ_{35} between N_3 and N_5) in the second case. In order to overcome this drawback, we made the following modification in the wavelength assignment phase of the RWABOU1 algorithm of the previous section.

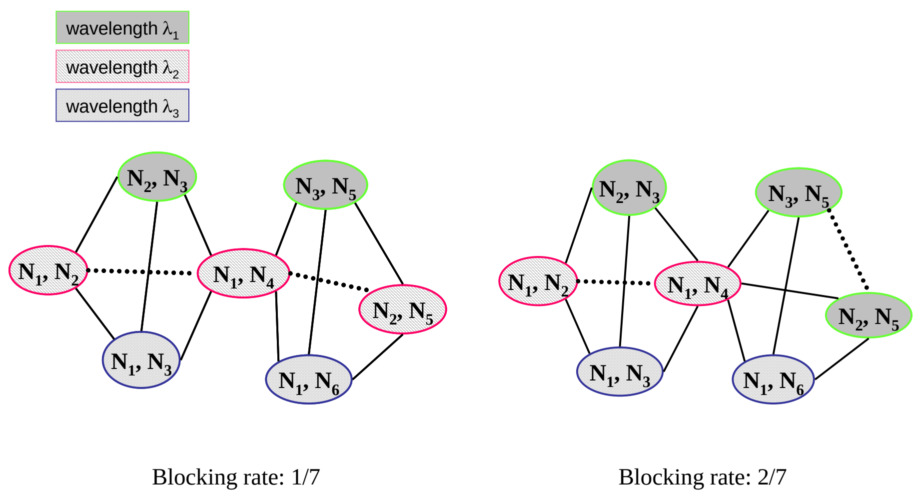


Figure 5: Illustration of some drawback of the two step wavelength assignment procedure

Instead of minimizing the number of wavelength conflicts as in the RWABOU1 Tabu Search procedure, we consider directly the objective of minimizing the blocking rate. It corresponds to a constrained Tabu Search procedure, called RWABOU2, for which several adjustments are needed.

Let us first introduce some notation. Denote by $E(\lambda^\ell)$ the set of edges in G^k with identically colored vertices. The wavelength assignment is feasible if

$$\sum_{\ell=1}^{W} |E(\lambda_\ell)| = 0. \tag{1}$$

Let us introduce a dummy wavelength λ_{W+1} and consider the following optimization problem

$$\min\{|V^k(\lambda_{W+1})| : \sum_{k=1}^{W} |E(\Lambda_k)| = 0\},$$

where $V^k(\lambda_{W+1})$ defines the set of nodes that have been assigned λ_{W+1}. This corresponds to the problem of finding the wavelength assignment with the smallest blocking rate.

In order to solve this last optimization problem, we consider again a Tabu Search procedure in which we start with a wavelength assignment with W wavelengths which might be infeasible. Indeed, we have two choices, starting with a feasible or an infeasible wavelength assignment. In the attempts we made, it turned out that it was very difficult to design an efficient Tabu Search procedure starting with a feasible wavelength assignment. We only describe below the most successful attempt. In order to take the feasibility condition (1) into account, we consider the following evaluation function

$$f(\Lambda) = |V^k(\lambda_{W+1})| + w^{\mathsf{penal}} \sum_{\ell=1}^{W} |E(\lambda_\ell)|,$$

where Λ refers to the current wavelength assignment and w^{penal} denotes a penalty factor that is adjusted throughout the Tabu Search procedure in order to end up with a feasible wavelength assignment with respect to the first W wavelengths. The penalty is increased (e.g., $w^{\mathsf{penal}} \leftarrow w^{\mathsf{penal}} + 5$ every iteration) when the wavelength assignment becomes too infeasible, and is decreased (e.g., $w^{\mathsf{penal}} \leftarrow w^{\mathsf{penal}}/2$ every 10 iterations) when it is feasible: if the wavelength assignment always remains feasible, we observe that the value of $f(\Lambda)$ very quickly stabilized and that a large fraction of the solution set is unexplored. Therefore, going infeasible while controlling the infeasibility level leads to a good strategy for exploring the solution set and finding the best possible blocking rate.

It remains to specify how we proceed with the color exchange at each step of the Tabu Search procedure. We consider two ways. In the first one, called RWABOU2a, we select at random either a vertex involved in an edge conflict or a vertex colored with $W+1$. We next evaluate with $f(\Lambda)$ the best alternate color in $\{1, 2, ..., W, W+1\}$ if the vertex is involved in an edge conflict or in $\{1, 2, ..., W\}$ otherwise. In the second variant, called RWABOU2b, we select the vertex involved in the largest number of edge conflicts and evaluate with $f(\Lambda)$ the best alternate color.

3.5 Routing Phase

Several rerouting strategies have been considered. Although we consider one strategy with several reroutings, we observe that the most efficient strategies consist in rerouting a small fraction of blocked connections. After each wavelength assignment phase, we order the blocked connections with respect to their decreasing number of edge conflicts in a list $\mathcal{L}$. We explore the following four strategies:

S_Next: select the first blocked connection of $\mathcal{L}$ and select the next routing path in its list of potential paths,

S_Best: select the first blocked connection of $\mathcal{L}$ and select the first routing path, if there exists one, that leads to a reduction in the number of edge conflicts. If none exists, select the alternate one leading to the smallest number of edge conflicts,

S_Disjoint: select the first blocked connection of $\mathcal{L}$ and select the routing path that is the most edge disjoint (i.e., fiber disjoint) with the current routing path,

S_Mul: select the first 5 connections of $\mathcal{L}$such that, and for each of them, select the best routing path, i.e., the route leading to the best reduction in the number of edge conflicts.

3.6 Summary of the RWABOU algorithms

We summarize below the RWABOU algorithm with the flow chart of Figure 6. In the first variant, the wavelength assignment phase is solved using a W-coloring process as described in Section 3.3, leading to the so-called RWABOU1 algorithm. In the second variant, called RWABOU2 algorithm, we consider a Tabu Search algorithm aiming at solving the W-stability graph formulation in a one-step process as described in Section 3.4.

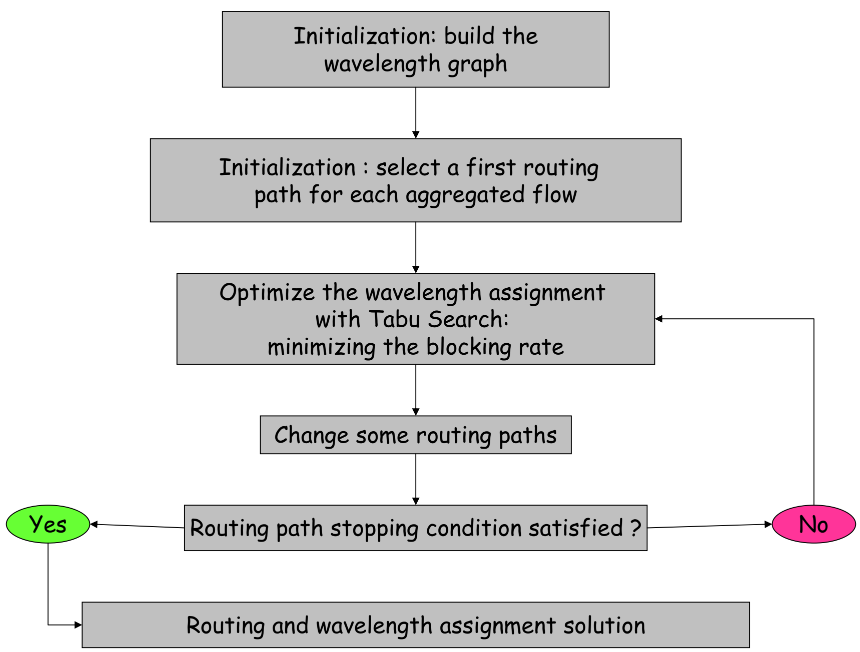


Figure 6: Flowchart of the RWABOU algorithms

4 Computational and Comparative Results

We describe the computational experience we have performed with the RWABOU algorithms proposed in Section 3 and discuss their performances in comparison with some previous heuristics proposed in the literature. We also study the deterioration of the grade of service with incremental traffic.

4.1 Optical Networks

The RWABOU algorithms have been tested on two data sets of the literature. We consider two optical networks widely used in the literature, the NSF and the EON networks. The NSF network is recalled in Figure 7, it is a network with 14 nodes and 21 optical links, with an average of 3 links per node. The EON network is described in, e.g., Mahony *et al.* [21] and also in Figure 8 below. It is a network with 20 nodes and 39 optical links, with an average of 4 links per node, see Figure 8.

4.2 Traffic Data

We used the asymmetric traffic matrices of Krishnaswamy [17] that we reproduce below in Figures 9 and 10. For the experimental results with symmetrical traffic, we modify those matrices and use $\max\{D_{sd}, D_{ds}\}$ for the number of connections between a pair of source-destination nodes $\{s, d\}$. The resulting symmetric matrices lead to 191 connections for the NSF instance and to 270 for the EON one.

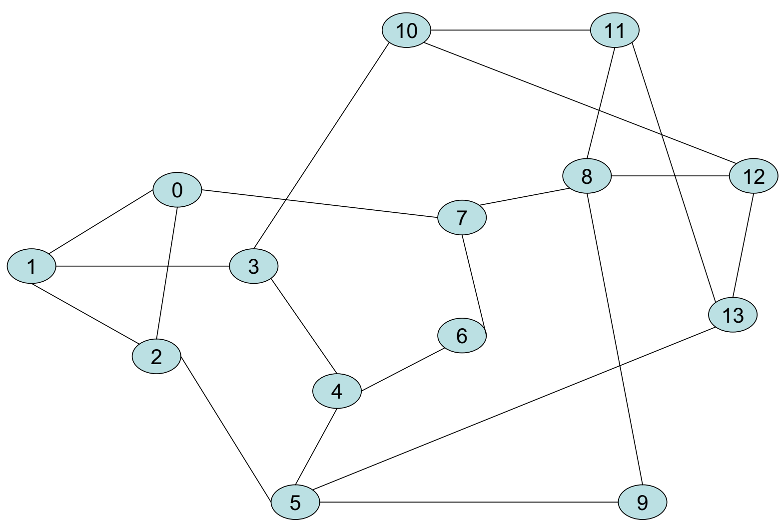


Figure 7: NSF (National Science Foundation Network) Network

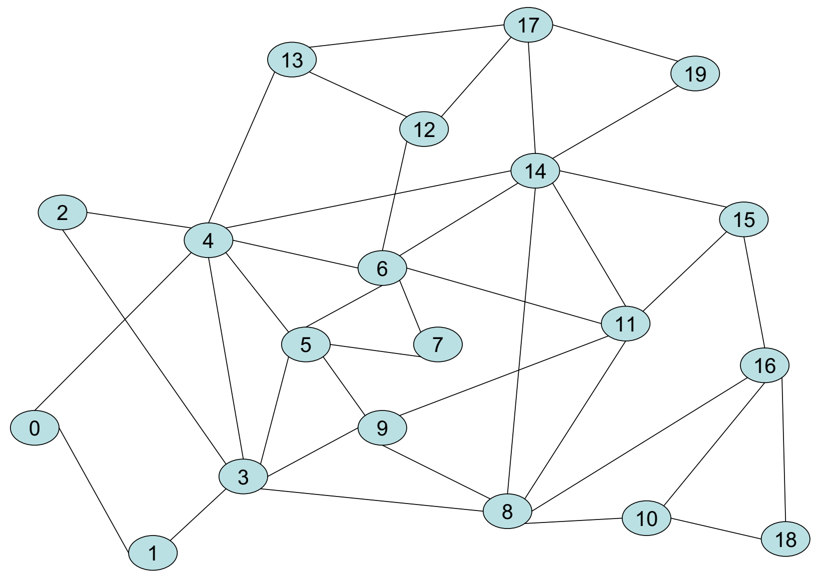


Figure 8: EON (European Optical Network) Network

4.3 Computational Results

We present below some computational and comparative results for the RWABOU algorithms.

4.3.1 Behavior of the Tabu Search Algorithms We illustrate on Figures 11 and 12 some of the typical behaviors of the RWABOU1 and RWABOU2 Tabu Search procedures respectively. We use the NSF network with asymmetrical traffic and 10 wavelengths. For the RWABOU1 algorithm, we drew two curves, one for the number of edge conflicts, one for the number of denied connections. This last number is evaluated as follows: at each iteration, we use a greedy procedure that sequentially removes vertices until all edge conflicts have disappeared, considering for each removal the vertex involved in the largest number of edge conflicts. We can observe on Figure 11 how the two numbers are varying along with the iterations: while the number of edge conflicts is decreasing, the number of denied connections is increasing. When the stopping criterion applies for the RWABOU1 algorithm, we consider the incumbent solution in terms of number of denied connections, and apply again the vertex removal greedy procedure. The curve of the number of edge conflicts is slightly increasing as the incumbent solution does not correspond to the best solution in terms of edge conflicts.

For the RWABOU2 algorithm, we observe that once we reach a feasible coloring, it is rather difficult to reduce further the number of vertices colored $W+1$, even if we allow for some controlled infeasibility. The variable penalty factor plays a crucial role in reaching or returning to a feasible solution, and it must take sometimes quite high value before

-	1	2	3	4	5	6	7	8	9	10	11	12	13	14	total
1	0	1	3	1	1	1	3	0	2	0	1	2	0	3	18
2	0	0	0	2	2	2	1	1	1	2	1	0	1	3	16
3	3	2	0	3	0	1	2	3	1	3	1	2	2	0	23
4	3	1	0	0	1	1	2	3	2	2	1	2	1	3	22
5	1	3	0	2	0	1	0	2	0	3	0	1	1	3	17
6	1	2	1	3	2	0	1	3	3	1	0	1	1	2	21
7	2	2	3	1	3	3	0	0	3	1	2	0	3	3	26
8	3	1	2	3	1	0	1	0	0	3	2	0	3	0	19
9	3	0	1	3	3	3	1	0	0	2	1	1	1	0	19
10	0	0	0	1	2	0	2	0	1	0	1	0	0	3	10
11	1	0	0	2	0	3	0	1	0	3	0	3	1	3	17
12	2	3	1	1	3	2	3	2	2	2	2	0	1	3	27
13	2	0	1	2	0	1	2	0	3	0	2	1	0	3	17
14	1	1	0	2	1	0	1	3	0	1	2	1	3	0	16

Figure 9: Traffic matrix for the NSF network, total of 268 connections

-	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	total
1	0	1	2	1	1	0	2	0	1	0	1	2	0	2	0	0	1	1	1	0	16
2	1	0	0	2	0	0	1	2	2	1	2	0	1	1	2	0	2	0	1	1	19
3	0	2	0	0	0	1	1	1	2	1	1	1	1	0	2	1	2	0	1	0	17
4	0	1	0	0	2	0	0	0	2	1	2	0	2	2	1	2	2	1	0	1	19
5	0	2	2	1	0	2	1	2	2	0	2	1	1	0	2	2	2	1	2	2	27
6	1	0	1	0	2	0	1	0	2	0	2	0	0	2	2	2	1	0	1	0	17
7	0	0	0	0	0	0	0	1	2	0	1	0	1	1	0	0	2	1	0	0	9
8	1	0	2	0	1	0	2	0	2	1	2	2	2	1	1	2	2	2	2	1	26
9	2	1	0	2	1	0	1	1	0	0	1	1	0	2	0	2	0	2	1	0	17
10	0	1	0	0	0	2	0	0	1	0	0	2	0	2	2	2	1	0	2	0	15
11	1	2	2	1	2	0	2	1	2	1	0	2	1	2	2	0	2	0	1	0	24
12	1	1	0	1	1	2	1	0	1	0	0	0	0	0	2	1	0	2	0	0	13
13	2	2	2	2	0	0	1	1	1	0	1	2	0	0	0	1	1	0	2	1	19
14	0	0	2	2	0	2	0	0	2	1	2	1	1	0	2	1	1	0	0	1	19
15	1	0	2	0	1	0	0	1	0	2	2	2	0	2	0	2	2	1	2	1	21
16	1	0	1	0	1	1	2	0	0	2	2	0	1	1	2	0	1	2	1	2	20
17	0	0	1	2	2	1	1	2	0	0	1	2	0	2	2	1	0	1	1	1	20
18	0	1	2	0	2	2	2	0	1	2	2	0	2	1	0	1	0	0	2	0	20
19	1	0	1	0	2	2	1	0	2	1	2	1	0	2	0	1	1	1	0	2	20
20	1	2	2	0	1	0	0	0	0	1	0	0	0	2	2	0	1	2	2	0	16

Figure 10: Traffic matrix for the EON network, total of 374 connections

we reach a first feasible coloring solution. But the behavior is as expected, when $f(\Lambda)$ is increasing due to the penalty factor, the feasibility is improving and the number of vertices colored $W+1$ is increasing. Depending on the instance that was considered, the solution provided by the RWABOU1 algorithm was sometimes better, sometimes worse than the one obtained with RWABOU2.

4.3.2 Adaptative Routing Before comparing the RWABOU algorithms with the results obtained by Krishnaswamy and Sivarajan [18], we study the parameters of the adaptive routing strategy, and in particular the impact of changing the routing paths. The list of potential routing paths has been built using Eppstein algorithm [6] with some modifications for eliminating cycles and was limited to the first 15-shortest paths for all instances. Results have been obtained with the rerouting strategy S_best and are summarized in Table 1 below. Performance of the RWABOU algorithms is measured with the number of edge conflicts with respect to the W-coloring (only in the RWABOU1 algorithm) and the grade of service GoS, i.e., the number of accepted connections, after several rerouting phases.

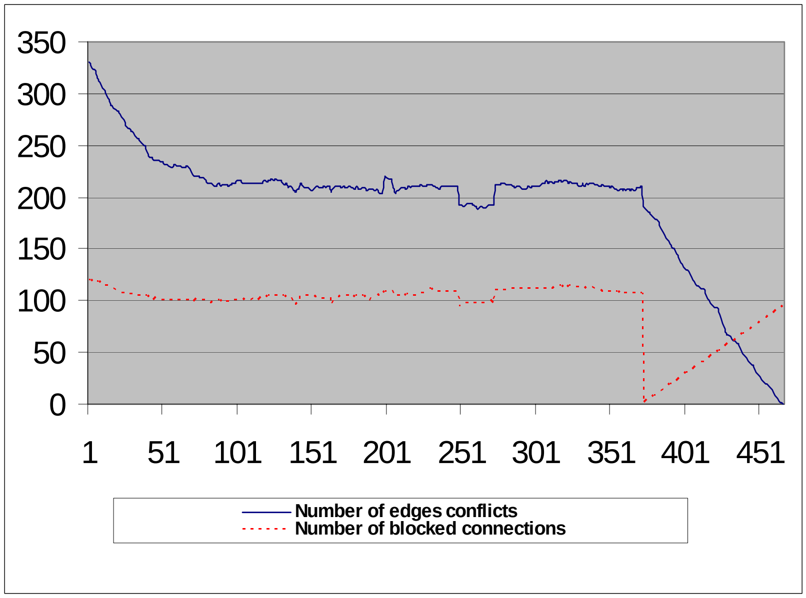


Figure 11: Typical behavior of RWABOU1

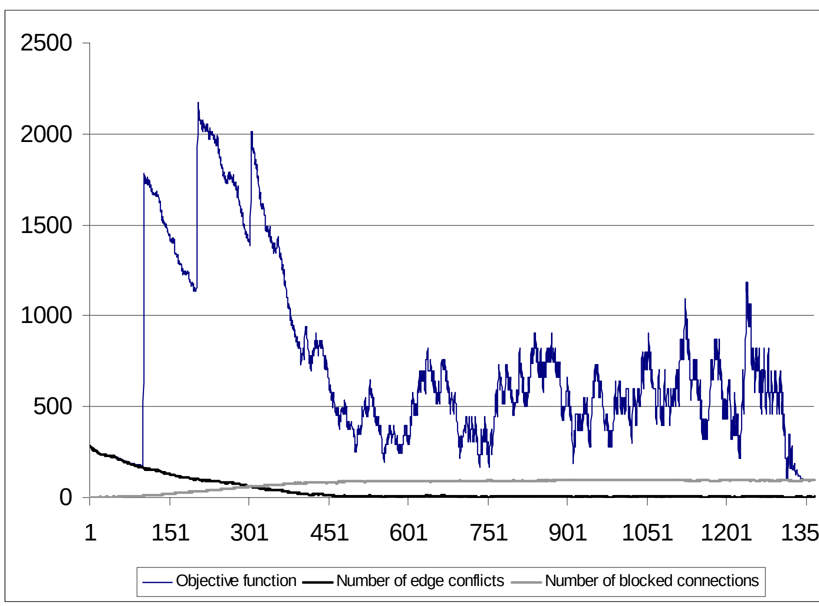


Figure 12: Typical behavior of RWABOU2

Table 1: Impact of the rerouting phases

Network instance	Performance indicators	RWABOU1 # Rerouting Phases 1	100	200	300	400	RWABOU2a # Rerouting Phases 1	100	200	300	400	RWABOU2b # Rerouting Phases 1	100	200	300	400
NSFNET/10	# edge conflicts	284	197	197	197	197										
	GoS	183	187	187	187	187	183	185	186	186	186	183	185	186	187	187
NSFNET/12	# edge conflicts	125	95	95	95	95										
	GoS	205	206	206	206	206	205	208	209	209	209	205	206	207	209	209
NSFNET/14	# edge conflicts	68	51	51	39	37										
	GoS	223	225	226	227	227	223	228	229	229	229	223	224	227	227	229
NSFNET/16	# edge conflicts	45	17	16	16	16										
	GoS	243	245	246	246	246	243	246	247	248	248	243	247	248	248	248
EONNET/10	# edge conflicts	340	257	257	257	257										
	GoS	274	278	278	278	278	274	275	276	276	276	274	276	276	276	276
EONNET/12	# edge conflicts	149	122	122	122	122										
	GoS	304	306	306	306	306	304	306	306	306	306	304	305	306	306	306
EONNET/14	# edge conflicts	84	64	64	64	64										
	GoS	327	329	329	329	329	327	328	329	329	329	327	327	328	329	329
EONNET/16	# edge conflicts	55	29	28	28	28										
	GoS	347	347	348	348	348	347	347	347	347	347	347	347	347	347	347

We observe that both versions of the RWABOU2 algorithms are comparable to the RWABOU1 one. Very few improvements were obtained with the RWABOU algorithms after 100 or 200 rerouting phases, showing that it is difficult to identify the connections to be rerouted in order to improve the GoS.

4.3.3 Comparison with Krishnaswamy and Sivarajan [18]

We now compare the RWABOU heuristics with the two algorithms of Krishnaswamy and Sivarajan [18]. These two algorithms correspond to two different rounding off heuristics, both associated with a reformulation of the RWA problem as a 0-1 linear program. In the last column of the two tables, we indicate the upper bound on the optimal GoS obtained by [18] with the optimal solution of the linear relaxation of their first mathematical programming reformulation.

The comparative results are summarized in Tables 2 and 3 below. In the first set of results for RWABOU1, the routing paths have been built considering only a shortest path strategy. In the second set with adaptive routing strategies, we compare the four rerouting strategies described in Section 3.5. We observe that while S_Disjoint seems to perform better on the NSF instances, it is the S_Best that performs best on the EON instances for the RWABOU1 algorithm with one path rerouted at each iteration: we therefore use the S_Mul with a selection of the best alternate path for the five rerouted connections. There is no clear conclusion for the comparative performance of the RWABOU1 and RWABOU2 algorithms, results vary from one data set to the next. Comparing with the results of Krishnaswamy and Sivarajan [18], we observe that we obtain much better GoS for both all NSF and EON instances. Both RWABOU1 and RWABOU2 improve easily on the results of [18] for all rerouting strategies except for the shortest path strategy.

Moreover, we can conclude to the optimality of the solution found by the RWABOU algorithms for several instances: the last three instances for both the NSF and the EON networks. For the remaining instances, the gap has been significantly reduced as it is less than or around 1% for the NSF and EON instances between 14 and 18 wavelengths while it was on the average around 10% with the solutions obtained by Krishnaswamy and Sivarajan [18] for the same number of wavelengths.

Table 2: Comparative results on the NSFNET asymmetric network

	Algorithm RWABOU1										Algorithm RWABOU2		Krishnaswamy and Sivarajan [18]			Gap
	Shortest Path Routing		Adaptive Routing										Algo A	Algo B	Upper bound	
	GoS		Next SP GoS		Disjoint SP GoS		Best SP GoS		Mul SP GoS		GoS		GoS	GoS		
W	#	%	#	%	#	%	#	%	#	%	#	%	#	#		
10	179	66.8%	187	69.8%	187	69.8%	187	69.8%	187	69.8%	186	69.4%	177	150	198	5.5%
12	202	75.4%	206	76.9%	206	76.9%	206	76.9%	212	79.1%	209	78.0%	195	187	218	2.7%
14	219	81.7%	227	84.7%	235	87.7%	227	84.7%	234	87.3%	229	85.4%	215	214	238	1.3%
16	235	87.7%	247	92.2%	253	94.4%	246	91.8%	248	92.5%	248	92.5%	233	226	258	1.9%
18	248	92.5%	264	98.5%	265	98.9%	261	97.4%	265	98.9%	265	98.9%	242	250	267	0.7%
20	256	95.5%	268*	100%	268*	100%	268*	100%	268*	100%	268*	100%	256	263	268	0%
22	264	98.5%	268*	100%	268*	100%	268*	100%	268*	100%	268*	100%	264	267	268	0%
24	268*	100%	268*	100%	268*	100%	268*	100%	268*	100%	268*	100%	268	268	268	0%

Table 3: Comparative results on the EONNET asymmetric network

	Algorithm RWABOU1										Algorithm RWABOU2		Krishnaswamy and Sivarajan [18]			Gap
	Shortest Path Routing		Adaptive Routing										Algo A	Algo B	Upper bound	
	GoS		Next SP GoS		Disjoint SP GoS		Best SP GoS		Mul SP GoS		GoS		GoS	GoS		
W	#	%	#	%	#	%	#	%	#	%	#	%	#	#		
10	248	66.3%	277	74.1%	276	73.8%	278	74.3%	278	74.3%	276	73.8%	262	250	285	2.4%
12	272	72.7%	305	81.6%	305	81.6%	306	81.8%	306	81.8%	306	81.8%	284	278	317	3.5%
14	288	77.0%	328	87.7%	328	87.7%	329	87.9%	329	88.0%	329	87.9%	310	308	337	2.4%
16	303	81.0%	347	92.8%	347	92.8%	348	93.0%	349	93.3%	347	92.8%	319	318	350	0.3%
18	314	83.9%	361	96.5%	361	96.5%	361	96.5%	361	96.5%	359	96.0%	339	334	362	0.3%
20	324	86.6%	369	98.7%	370*	98.9%	370*	98.9%	370*	98.9%	370*	98.9%	341	340	370	0%
22	334	89.3%	374*	100%	374*	100%	374*	100%	374*	100%	374*	100%	355	352	374	0%
24	340	90.9%	374*	100%	374*	100%	374*	100%	374*	100%	374*	100%	364	364	374	0%

4.4 Symmetrical vs Asymmetrical Traffic

In this section, we perform the same comparisons that in the previous section for symmetric traffic with the aim of evaluating the variation of bandwidth utilization whether we assume the traffic to be symmetrical or asymmetrical. We therefore first present in Tables 4 and 5 the performance of the various RWABOU algorithms, with an upper bound evaluated again with the optimal solution of the continuous relaxation of a 0-1 linear programming formulation of the RWA problem proposed by Jaumard *al* [16] for symmetrical traffic. Among the different adaptive routing strategies, the S_Best and S_Mul perform uniformly better than the other ones for the RWABOU1 algorithm. Although on the NSF instances, RWABOU1 clearly outperforms the RWABOU2 algorithm, this is not the case on the EON instances where their performances are rather equal taking into account the fact that the number of reroutings in the S_mul strategy of RWABOU1 has been increased to 1000, while it remained limited to 300 in RWABOU2.

Using upper bounds proposed by Jaumard *et al.* [16], we can evaluate the quality of the heuristic solution provided by the RWABOU algorithms. For the largest values of W (i.e., 22 and 24) for instances on both networks, we obtain the optimal solutions. For the intermediate number of W, the gap is quite small (less than 1%), while it goes up to 5% for smaller values, i.e. for $W = 10$.

Table 4: Comparative results on the NSFNET symmetric network

	Algorithm RWABOU1										Algorithm RWABOU2		Upper bound [16]	Gap
	Shortest Path Routing		Adaptive Routing											
			Next SP		Disjoint SP		Best SP		Mul SP					
Number of wavelengths	GoS		GoS		GoS		GoS		GoS		GoS		GoS	
	#	%	#	%	#	%	#	%	#	%	#	%	#	
10	106	55.5%	107	56.0%	109	57.1%	109	57.1%	109	57.1%	107	56.0%	115	5.2%
12	118	61.8%	121	63.4%	121	63.6%	121	63.4%	122	63.8%	119	62.3%	129	5.4%
14	128	67.0%	132	69.1%	132	69.1%	132	69.1%	135	70.7%	131	68.6%	143	5.6%
16	139	72.8%	144	75.4%	142	74.3%	145	75.9%	146	76.4%	144	75.4%	153	4.6%
18	149	78.0%	156	81.7%	153	80.1%	156	81.7%	158	82.7%	153	80.1%	161	1.9%
20	158	82.7%	164	85.9%	163	85.3%	166	86.9%	166	86.9%	163	85.3%	169	1.8%
22	167	87.4%	177	92.7%	174	91.1%	177	92.7%	177	92.7%	174	91.1%	177	0%
24	173	90.5%	183	95.8%	184	96.3%	184	96.3%	185	96.8%	182	95.3%	185	0%

Table 5: Comparative results on the EONNET symmetric network

	Algorithm RWABOU1										Algorithm RWABOU2		Upper bound [16]	Gap
	Shortest Path Routing		Adaptive Routing											
			Next SP		Disjoint SP		Best SP		Mul SP					
Number of wavelengths	GoS		GoS		GoS		GoS		GoS		GoS		GoS	
	#	%	#	%	#	%	#	%	#	%	#	%	#	
10	155	57.4%	166	61.5%	163	60.4%	168	62.2%	168	62.2%	168	62.2%	176	4.5%
12	170	63.0%	183	67.7%	182	67.4%	185	68.5%	185	68.5%	183	67.8%	194	4.6%
14	183	67.8%	198	73.3%	199	73.7%	199	73.7%	204	75.5%	199	73.7%	212	3.8%
16	198	73.3%	218	80.7%	215	79.6%	219	81.1%	220	81.5%	219	81.1%	225	2.2%
18	209	77.4%	232	85.9%	231	85.5%	233	86.3%	235	87.0%	233	86.3%	237	0.8%
20	218	80.7%	245	90.7%	242	89.6%	245	90.7%	247	91.5%	245	90.7%	249	0.8%
22	227	84.1%	253	93.7%	253	93.7%	254*	94.1%	254*	94.1%	254*	94.1%	254	0%
24	235	87.0%	260	96.3%	261	96.7%	262*	97.0%	262*	97.0%	260*	96.3%	262	0%

Comparison of bandwidth utilization estimation is done in Table 6 depending on assuming the traffic to be symmetrical or asymmetrical. For each network instance, we have reported the best GoS that has been obtained and we have estimated the variation in the bandwidth utilization. In order to facilitate the comparison, we have converted the number of symmetrical connections, i.e. bidirectional connections, in a number of directional connections, i.e., multiplied by 2. In the third column of each network instance, we measure the traffic increase resulting from assuming the traffic to be symmetrical, and the variation in the bandwidth utilization on the fourth column. The bandwidth utilization rate is measured as follows:

$$\tau_B = \frac{\sum\limits_{e \in E} n_e^\lambda}{m \times W},$$

where n_e^λ is the number of wavelengths used on edge e. We observe that as the number of wavelengths and therefore the GoS are increasing, τ_B is increasing with its increase corresponding to the additional bandwidth requested by approximating the asymmetrical traffic with a symmetrical traffic.

Table 6: Impact of Symmetrical vs Asymmetrical Traffic on the Bandwidth Utilization Rate

	NSF Network				EON Network			
# wavelengths	GoS		traffic	τ_B	GoS		traffic	τ_B
	sym	asym	increase	variation	sym	asym	increase	variation
10	218	187		5.9%	336	278		8.7%
12	244	212		4.9%	370	306		12.4%
14	270	235		4.7%	408	329		15.2%
16	292	253	42.5%	5.6%	440	349	44.4%	17.2%
18	316	265		10.3%	470	361		19.2%
20	332	268		17.3%	494	370		23.1%
22	354	268		25.6%	508	374		24.5%
24	370	268		28.0%	524	374		29.6%

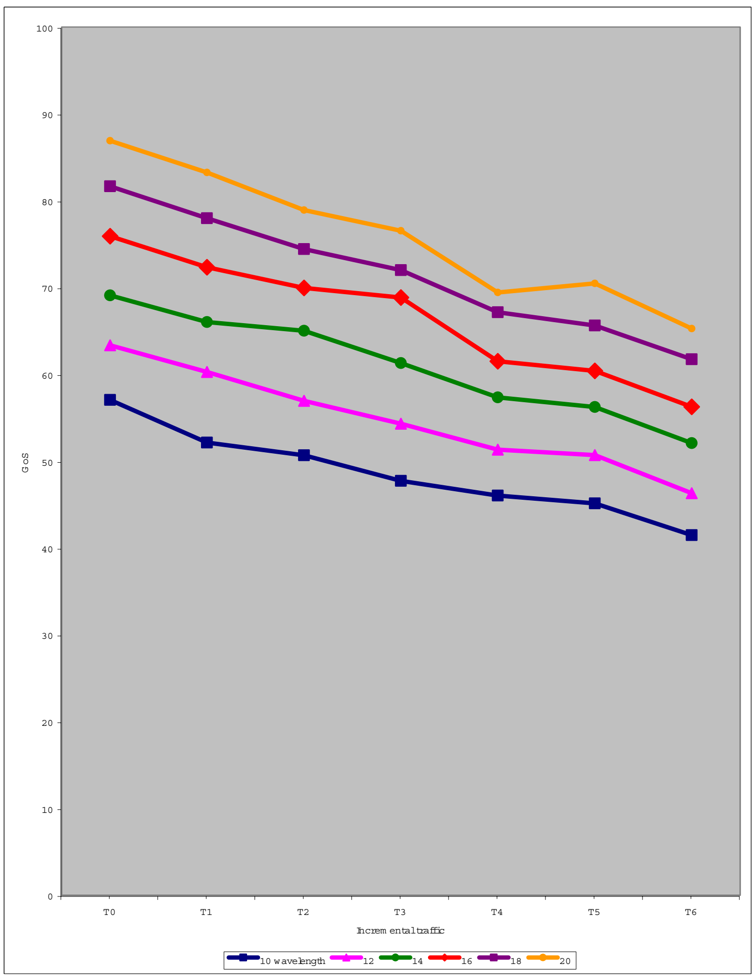


Figure 13: Variations of the GoS

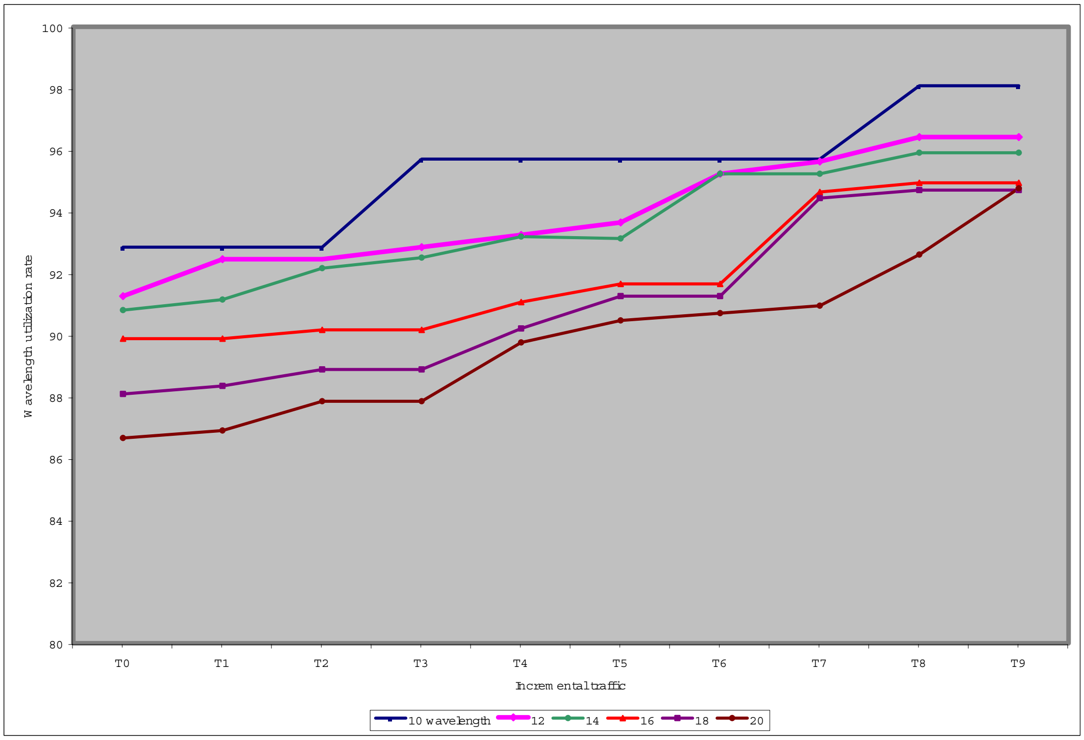


Figure 14: Bandwidth utilization rate

4.5 Performance of the rwabou algorithm on incremental traffic

We next consider the evolution of the GoS and the average number of required wavelengths (denoted $\overline{\lambda}$) when the traffic is increasing. Denote by $T^0 = T$ the initial traffic symmetrical matrix for the NSF network, we build incremental traffic matrices as follows. Given a traffic matrix T_i, we define T^{i+1} with a 5% increase for the number of connections. For each pair $\{N_s, N_d\}$ of source and destination nodes, we randomly generate a random number α between 0 and 3, and increase the number of connections between this node pair by α. This process is repeated until we reach the 5% increase in the number of connections.

The results are summarized in the graphs depicted in Figures 13 and 14. We observe that as the traffic increases, the GoS is decreasing at an increasing rate, while the bandwidth utilization rate is stabilizing after some time, and more quickly when the number is wavelengths is small.

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