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Abstract

This work is devoted to the study of linear continuous-time systems with Markovian jumping parameters and constrained control. The constraints used in this paper are of nonsymmetrical inequality type. The approach of positively invariant sets is used to obtain a sufficient condition of exponential stability for this class of systems with constraints.

Résumé

Cet article porte sur l'étude de la commande des systèmes à sauts Markoviens avec contraintes sur la commande. Ces contraintes sont assymétriques de type inegalité. Le concept d'invariance positif est utilisé pour établir des conditions suffisantes pour garantir la stabilité exponentielle de la classe des systèmes étudiés. Un exemple numérique est donné pour montrer la validité des résultats proposés.

1 Introduction

Linear systems with Markovian jumping parameters offers the advantage to model a large varieties of physical phenomena. This class of systems has been used successfully to model manufacturing systems, power systems, economic systems, etc. We refer the reader, for example to [5]-[15] for discrete-time and continuous-time systems. It is well known that all these physical systems admit inputs limitation which are modeled by constraints of inequality type. The regulator problem for linear systems with constrained control was widely studied during these two decades. The tool of positive invariance was successfully applied to almost all the deterministic systems with constrained control, see for example [1], [3] and the references therein. However, to the best of our knowledge, the problem of stochastic stability of linear systems with both Markovian jumping parameters and constrained control has only been investigated for discrete-time systems [4]. In the previous work [4], the notion of stochastic positive invariance is introduced and applied on the common set between all the modes. However, this procedure in not followed in this paper since the deterministic positive invariance is still applied for this common set. This solution is based on the simple fact that, if the common set is positively invariant with respect to each mode, then all the trajectories of the system with Markovian jumping parameters, emanating from this set, can not leave it at any random jump. This technique enables us to deal with non symmetrical constraints.

The aim of this paper is to study the regulator problem for continuous-time systems with Markovian jumping parameters and constrained control by using the positive invariance approach. In this work, non symmetrical constraints are considered. A sufficient condition for exponential stability of the system is obtained.

This paper is organized as follows: The studied problem is presented in Section 2. Section 3 deals with definitions and preliminary results as the necessary and sufficient condition of positive invariance and the sufficient condition of exponential stability for the free system. In Section 4, these results are used to design a law control ensuring to the control to remain admissible. The exponential stability is also guaranteed. An algorithm together with an example illustrating this design is also presented.

2 Problem Formulation

We consider the continuous-time linear system with Markovian jumping parameters defined by:

$$(\Sigma) : \begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) \\ x(0) = x_0, r(0) = r_0 \end{cases} \tag{1}$$

Where $x \in R^n$ is the state vector, $u \in R^m$ is the control vector. The system state is a function of the discrete parameter $r(t)$ which is a continuous-time homogeneous Markov process with finite discrete state space $S = \{1, 2, \ldots, s\}$.
The previous equation can be regarded as the result of the following s equations:

$$\dot{x}(t) = A_\alpha x(t) + B_\alpha u(t) \quad , \quad 1 \leq \alpha \leq s \tag{2}$$

switching from one to the others according to the evolution of the Markov process. The transition probability of $r(t)$ is defined by:

$$P\left[r(t+h)=\beta|r(t)=\alpha\right] = \begin{cases} \lambda_{\alpha\beta}h+o(h) & \text{if, } \alpha\neq\beta \\ 1+\lambda_{\alpha\alpha}h+o(h) & \text{if, } \alpha=\beta \end{cases} \tag{3}$$
$$\sum_{\beta}\lambda_{\alpha\beta} = 0;\lambda_{\alpha\beta}\geq 0 \text{ if, } \alpha\neq\beta$$

The $\lambda_{\alpha\beta}$'s are the transition probability rates from the αth mode to the βth mode. The set of constraints for the control $u(t)$ is given for each mode α by:

$$\Omega(\alpha)=\left\{u\in R^{m}/-q_2(\alpha)\leq u\leq q_1(\alpha);q_1(\alpha),q_2(\alpha)\in R^{m+}\right\} \tag{4}$$

In this paper, we consider the problem of finding a controller which respects the constraints and exponentially stabilizes the system Σ by using a control law of the form:

$$u(t)=F(r(t))x(t) \tag{5}$$

The closed-loop system is given by,

$$\dot{x}(t)=(A(r(t))+B(r(t))F(r(t))x(t)=A_c(r(t))x(t) \tag{6}$$

3 Preliminary Results

In this section, a new exponential stability condition of the system (6) is given in the unconstrained case. We first begin by recalling the definition of exponential stability as known in the literature.

Definition 1 *[14] The system (6) is said:*

1. *stochastically stable if there exists a scalar η such that: $E\left[\int_0^{\infty}\|x(t)\|^2dt\right]\leq\eta(r_0,x_0)$.*
2. *exponentially stable if there exists two scalars $\eta>0,\rho>0$ such that: $E\left[\|x(t)\|\right]\leq\eta e^{-\rho t}$.*
3. *Mean square stable if $\lim_{t\to\infty}E[\|x(t)\|^2]=0$.*

Theorem 1 *[10] The unconstrained system (6) is stochastically stable if there exist symmetric and positive definite matrices $P(\alpha)$, $\alpha\in S$ $(P(\alpha)\succ 0)$, such that the following holds:*

$$A_c^T(\alpha)P(\alpha)+P(\alpha)A_c(\alpha)+\sum_{\beta\in S}\lambda_{\alpha\beta}P(\beta)\prec 0 \textit{ for all } \alpha\in S \tag{7}$$

Note that this result was obtained by using the quadratic Lyapunov function and is a necessary and sufficient condition of mean square stability [9]. A second result can also be obtained by using a non quadratic Lyapunov function.

Theorem 2 *The unconstrained system (6) is exponentially stable if for all* $\alpha \in S$ *there exist positive vectors* $w(\alpha) \in R^{n+}; \alpha = 1, \ldots, s$ *such that:*

$$\hat{A}_c(\alpha)w(\alpha) + \left[\sum_{\beta=1}^{s} \lambda_{\alpha\beta} \max_{1 \le i \le n} \left(\frac{w_i(\alpha)}{w_i(\beta)}\right)\right] w(\alpha) < 0; \forall \alpha \in S \tag{8}$$

with,

$$\hat{A}_c(\alpha)(i,j) = \begin{cases} |A_c(i,j)(\alpha)| & \text{if, } i \neq j \\ A_c(i,i)(\alpha) & \text{if, } i = j \end{cases} \tag{9}$$

Proof: Consider the following Lyapunov function candidate:

$$V(x(t), r(t)) = max_i \frac{|x_i(t)|}{w_i(r(t))} \tag{10}$$

The infinitesimal operator is computed by:

$$\begin{aligned} \mathcal{A}V(x(t),\alpha) &= \lim_{h\to 0} \frac{1}{h} \left\{E\left[V(x(t+h), r(t+h))|r(t)=\alpha\right] - V(x(t),\alpha)\right\} \\ &= \lim_{h\to 0} \frac{1}{h} \left\{E\left[max_i \frac{|x_i(t+h)|}{w_i(r(t+h))}|r(t)=\alpha\right] - V(x(t),\alpha)\right\} \end{aligned}$$

We use the fact that,

$$\begin{aligned} x_i(t+h) &= x_i(t) + h\left[A_c(r(t))x(t)\right]_i + o_i(h) \\ &= \left[L(r(t),h)x(t)\right]_i + o_i(h) \end{aligned}$$

where $L(r(t),h) = I_n + hA_c(r(t))$, I_n is the $n \times n$ identity matrix and $o_i(h)$ a scalar which satisfies $\lim_{h\to 0} \frac{o_i(h)}{h} = 0$. It follows that one can write,

$$\mathcal{A}V(x(t),\alpha) \le \lim_{h\to 0} \frac{1}{h} \left\{E\left[max_i \frac{|\left[L(\alpha,h)x(t)\right]_i|}{w_i(r(t+h))}|r(t)=\alpha\right] - V(x(t),\alpha)\right\}$$

Taking (3) into account, one obtains,

$$\begin{aligned} \mathcal{A}V(x(t),\alpha) \le \lim_{h\to 0} \frac{1}{h} \Bigg\{ & \sum_{\beta} h\lambda_{\alpha\beta} max_i \frac{|\left[L(\alpha,h)x(t)\right]_i|}{w_i(\beta)} \\ & + max_i \frac{|\left[L(\alpha,h)x(t)\right]_i|}{w_i(\alpha)} - V(x(t),\alpha) \Bigg\} \end{aligned}$$

Let $l_{ij}(\alpha,h), i = 1,\ldots,n, j = 1,\ldots,n$ be the components of matrix $L(\alpha,h)$, that is,

$$l_{ij}(\alpha,h) = \begin{cases} 1 + hA_c(i,j)(\alpha) & \text{if, } i = j \\ hA_c(i,j)(\alpha) & \text{if, } i \neq j \end{cases} \tag{11}$$

It follows that,

$$\begin{aligned}\mathcal{A}V(x(t),\alpha) &\leq \sum_{\beta\in S}\lambda_{\alpha\beta}max_i\frac{|x_i(t)|}{w_i(\beta)}\\ &\quad +lim_{h\to 0}\frac{1}{h}\left\{max_i\frac{\sum_j |l_{ij}(\alpha,h)||x_j(t)|}{w_i(\alpha)} - V(x(t),\alpha)\right\}\\ &\leq \sum_{\beta\in S}\lambda_{\alpha\beta}max_i\frac{w_i(\alpha)}{w_i(\beta)}V(x(t),\alpha) + lim_{h\to 0}\frac{1}{h}\upsilon(h)V(x(t),\alpha)\end{aligned}$$

With $\upsilon(h) = max_i\frac{\sum_j |l_{ij}(\alpha,h)|w_j(\alpha)}{w_i(\alpha)} - 1$

Using (11),

$$\begin{aligned}\sum_j\frac{|l_{ij}(\alpha,h)|w_j(\alpha)}{w_i(\alpha)} &= \sum_{j\neq i}h\frac{|A_c(i,j)(\alpha)|}{w_i(\alpha)}w_j(\alpha) + |1+hA_c(i,i)(\alpha)|\\ &= \sum_{j\neq i}h\frac{|A_c(i,j)(\alpha)|}{w_i(\alpha)}w_j(\alpha) + 1 + hA_c(i,i)(\alpha),\ \text{for a small } h\end{aligned}$$

According to (9), this latter enables one to have,

$$\mathcal{A}V(x(t),\alpha) \leq \left\{\sum_{\beta\in S}\lambda_{\alpha\beta}max_i\left(\frac{w_i(\alpha)}{w_i(\beta)}\right) + max_i\frac{\left(\hat{A}_c(\alpha)w(\alpha)\right)_i}{w_i(\alpha)}\right\}V(x(t),\alpha) \tag{12}$$

Pose,

$$\rho(\alpha) = -\left\{\sum_{\beta\in S}\lambda_{\alpha\beta}max_i\left(\frac{w_i(\alpha)}{w_i(\beta)}\right) + max_i\frac{\left(\hat{A}_c(\alpha)w(\alpha)\right)_i}{w_i(\alpha)}\right\} \tag{13}$$

Note that if condition (8) holds, $\rho(\alpha) > 0$. This leads to,

$$\mathcal{A}V(x(t),\alpha) < -\rho(\alpha)V(x(t),\alpha) \tag{14}$$

That is,

$$E\left[\mathcal{A}V(x(t),\alpha)\right] < -\rho E\left[V(x(t),\alpha)\right]$$

with

$$\rho = \min_{\alpha\in S}\rho(\alpha)$$

Applying the Dynkin's formula,

$$V(x(t),r(t)) = V(x_0,r_0) + \int_0^t \mathcal{A}V(x(t),r(s))ds$$

one obtains,

$$\begin{aligned} E\left[V(x(t), r(t)\right] &\leq E\left[V(x_0, r_0\right] + \int_0^t E\left[\mathcal{A}V(x(s), r(s))\right] ds \\ &< E\left[V(x_0, r_0\right] - \rho \int_0^t E\left[V(x(s), r(s))\right] ds \end{aligned}$$

By virtue of Gronwall lemma, we have,

$$E\left[V(x(t), r(t)\right] < e^{-\rho t} E\left[V(x_0, r_0)\right] \tag{15}$$

Finally, (15) implies that $E\left[\|x(t)\|\right] < \eta e^{-\rho t}$ with:
$\|x(t)\| = \max_i |x_i(t)|$ and $\eta = \max_{\alpha \in S} \min_i w_i(\alpha) E\left[V(x_0, r_0)\right]$ $\nabla\nabla\nabla$

Remark 1 One can note the similarity between the results of Theorem 1 and Theorem 2. In case of $n = 1$, condition (8) becomes $a_c(\alpha) + \sum_\beta \lambda_{\alpha\beta} \frac{w(\alpha)}{w(\beta)} < 0$, where the term $\sum_\beta \lambda_{\alpha\beta} \frac{w(\alpha)}{w(\beta)}$ is not necessarily positive and condition (7), $a_c(\alpha) + \frac{1}{2} \sum_\beta \lambda_{\alpha\beta} \frac{p(\beta)}{p(\alpha)} < 0$. That is, condition (8) is more restrictive than (7), however, it needs the computation of only $s\ n$ parameters instead of $s\ n^2$ parameters for (8). Besides, if $s = 1$, condition (8) becomes $\hat{A}_c w < 0$ which is a sufficient condition of asymptotic stability [2].

In the next part, we recall a result obtained by using the concept of positive invariance for the constrained regulator problem. For the details see [2].
Consider the following linear continuous-time stationary system:

$$\dot{z}(t) = H z(t) \tag{16}$$

Where the state $z \in \mathcal{R}^m$ is subject to the constraints of the form,

$$\mathcal{D} = \left\{ z \in \mathcal{R}^m | -\theta_2 \leq z \leq \theta_1 \ ; \ \theta_1, \theta_2 \in \mathcal{R}_+^m \right\} \tag{17}$$

Definition 2 *A domain $\mathcal{D} \subset \mathcal{R}^m$ is said to be positively invariant w.r.t the system* (16) *if, for any initial state $z_0 \in \mathcal{D}$, the trajectory $z(z_0, t) \in \mathcal{D}$, $\forall t \geq 0$.*

Theorem 3 *[2] The domain $\mathcal{D}$ is positively invariant w.r.t system* (16) *if and only if the following condition holds:*

$$\tilde{H}\theta \leq 0 \tag{18}$$

where matrix $\tilde{H}$ and vector θ are given by,

$$\tilde{H} = \begin{bmatrix} H_1 & H_2 \\ H_2 & H_1 \end{bmatrix}, \ \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{19}$$

$$H_1 = \begin{cases} H_1(i,i) = H(i,i) \ , if\ i = j \\ H_1(i,j) = sup\left[H(i,j), 0\right] \ , if\ i \neq j \end{cases} ,$$

$$H_2 = \begin{cases} H_2(i,i) = 0 \, , if i = j \\ H_2(i,j) = sup\left[-H(i,j), 0\right] \ if i \neq j \end{cases}$$

4 Main Results

In this section, the obtained results in the previous section allow us to deal with the problem of continuous-time systems with Markovian jumping parameters and constrained control as presented in the first section.

Recall that the control law is given by (5) and each domain $\Omega(\alpha)$ generates by this feedback law a polyhedral domain in the state space $\mathcal{K}(\alpha)$ defined by:

$$\mathcal{K}(\alpha) \quad = \quad \left\{x(t) \in R^n / - q_2(\alpha) \leq F(\alpha)x \leq q_1(\alpha); q_1(\alpha), q_2(\alpha) \in R^{m+}\right\} \tag{20}$$

Let $\mathcal{K}_c$ be the common set of all the modes,

$$\mathcal{K}_c = \bigcap_{\alpha \in S} \mathcal{K}(\alpha) \tag{21}$$

The closed-loop system is given by (6). Let us introduce the following change of variable,

$$z(t) = F(\alpha)x(t) \;\; , \quad \alpha \in S \tag{22}$$

In this case, for each $\alpha \in S$ the domain $\mathcal{K}(\alpha)$ is transformed to the following domain,

$$\mathcal{D}(\alpha) = \left\{z \in \mathcal{R}^m | - q_2(\alpha) \leq z \leq q_1(\alpha) \, , \, q_1(\alpha), q_2(\alpha) \in \mathcal{R}^m_+\right\}$$

Let,

$$\begin{aligned} \mathcal{D}_c \quad &= \quad \bigcap_{\alpha \in S} \mathcal{D}(\alpha) \\ &= \quad \{z \in \mathcal{R}^m | - \theta_2 \leq z \leq \theta_1\} \end{aligned}$$

where,

$$\theta_j = \min_{\alpha \in S} q_j(\alpha) \; ; j = 1, 2 \; ; \; \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{23}$$

According to (22), we obtain,

$$\dot{z}(t) = F(\alpha) \left[A(\alpha) + B(\alpha)F(\alpha)\right] x(t) \tag{24}$$

Then, if there exists matrix $H(\alpha) \in \mathcal{R}^{m \times m}$, for each $\alpha \in S$ such that:

$$F(\alpha) \left[A(\alpha) + B(\alpha)F(\alpha)\right] = H(\alpha)F(\alpha) \tag{25}$$

the dynamical system (6), for each mode $r(t) = \alpha$, ($\alpha \in S$), is transformed by the use of (22) to the following dynamical system:

$$\dot{z}(t) = H(\alpha)z(t) \tag{26}$$

Note that the variable $z(t)$ is identical to the control $u(t)$ while the set $\mathcal{D}(\alpha)$ is the same as the set of constraints $\Omega(\alpha)$. At this step, if domain $\mathcal{D}_c$ is positively invariant w.r.t system (26) for each mode $\alpha \in S$, then the control (5) will be always admissible, i.e., $u(t) \in \Omega, \forall t \geq 0$ and therefore, the linear behaviour of the system in the closed-loop (6) remains valid.

We now state the main stability result of this paper.

Theorem 4 *If, for each $\alpha \in S$, there exist matrices $H(\alpha) \in \mathbb{R}^{m\times m}$ and positive definite matrices $P(\alpha) \in \mathbb{R}^{n\times n}$, such that:*

(i)

$$F(\alpha)\left[A(\alpha) + B(\alpha)F(\alpha)\right] = H(\alpha)F(\alpha), \tag{27}$$

(ii)

$$\tilde{H}(\alpha)\theta \leq 0, \tag{28}$$

(iii)

$$A_c^T(\alpha)P(\alpha) + P(\alpha)A_c(\alpha) + \sum_{\beta\in S}\lambda_{\alpha\beta}P(\beta) \prec 0 \tag{29}$$

where θ is given by (23) and $\tilde{H}$ by (19).
Then, system (1) *with* (5) *is stochastically stable while the control is admissible* $\forall x_0 \in \mathcal{K}_c$.

Proof: Equation (27) allows to transform system (6) for each mode $r(t) = \alpha$, $\alpha \in S$ to system (26). According to Theorem 2, condition (28) guarantees the positive invariance of domain $\mathcal{D}_c$ w.r.t system (26). That is, the positive invariance of domain $\mathcal{K}_c$ w.r.t system (6) is also guaranteed for each mode $r(t) = \alpha$, $\alpha \in S$. Further, by virtue of Theorem 1, condition (29) ensures the stochastic stability of the system in the closed-loop (6) which is always valid since the control by state feedback is always admissible. $\nabla\nabla\nabla$

Corollary 1 *If , for each $\alpha \in S$, there exist matrices $H(\alpha) \in \mathbb{R}^{m\times m}$ and vectors $w(\alpha) \in \mathbb{R}^{n+}$, such that conditions (27), (28) and (8) hold, then, system (1) with (5) is exponentially stable while the control is admissible $\forall x_0 \in \mathcal{K}_c$.*

Proof: Obvious.

Comment 1

- It is worth noting that two possibilities can be followed to use the results of Theorem 4 and Corollary 1. The first consists in computing feedback controls dependent of the jumping process by using the LMI (29), computing matrices $H(\alpha)$ according to (27) and testing (28) for each mode. The second consists in giving s matrices $H(\alpha)$ according to (27), computing the feedback controls by resolving equations (28) which lead to gain feedback initially independent of the jumping process. The obtained matrices in closed-loop are then used to compute s definite positive matrices $P(\alpha)$ for (29) (i.e., $s\ n^2$ parameters) or only s positive vectors $w(\alpha)$ for (8) (i.e., $s\ n$ parameters). The resolution of the LMI (29) is available in MATLAB.
- Note that no assumption on the stabilizability of each mode is needed even in presence of saturation. This means that if the stochastic stability is ensured for the system with Markovian jumping parameters, one or more mode can always be unstable with admissible controls ensured by (28). For the unstable mode, the resolution of equation (27) leads to a spectrum in closed-loop composed with m stable eigenvalues of matrix H and $n - m$ non necessary all stable eigenvalues of matrix A (see [1]).

The steps of constructing such controllers is summarized in the following algorithm.

Algorithm 1

Step 1: Compute matrices $H_1(\alpha)$ and $H_2(\alpha)$ satisfying (28) by resolving the following linear programming for each $\alpha \in S$:

$$(LP1)\begin{cases} \max \epsilon / \\ \begin{bmatrix} H_1 & H_2 \\ H_2 & H_1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \leq -\epsilon \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \\ \epsilon > 0 \\ H_1(i,i) < 0, H_2(i,i) = 0; H_1(i,j) > 0, H_2(i,j) > 0, i \neq j \end{cases} \tag{30}$$

where φ_1 and φ_2 are design positive vectors. If matrices $H(\alpha) = H_1(\alpha) - H_2(\alpha)$ are obtained according to the required assumptions of the resolution of equation (27) (i.e., $\sigma(H)(\alpha) \bigcap \sigma(A(\alpha)) = \emptyset$) continue, else change φ_1 and φ_2.

Step 2: Compute the gain matrices $F(\alpha)$ solutions of equations (27) by using the method given in [1] and matrices $A_c(\alpha)$ of the system in closed-loop. Go to Step 3 or Step 4.

Step 3: Compute matrices $P(\alpha)$ solutions of the LMI's (29). If the LMI's are not feasible go to Step 1 to modify the vectors $\varphi(\alpha)$.

Step 4: Compute vectors $w(\alpha)$ solution of the following linear programming for each $\alpha \in S$:

$$(LP2)\left\{\begin{array}{l} \max \eta / \\ \hat{A}_c(\alpha)w(\alpha) + \left(\sum_{\beta=1}^{s} \lambda_{\alpha\beta}\Gamma(\alpha,\beta)\right) w(\alpha) < -\eta\,\psi(\alpha) \\ w(\alpha) \leq \Gamma(\alpha,\beta)\,w(\beta); \forall \beta \in S \\ \eta > 0 \end{array}\right. \tag{31}$$

where vectors $\psi(\alpha)$ and matrix Γ are design positive parameters with $\Gamma(i,i) = 1; \Gamma(i,j)$ $\Gamma(j,i) > 1; i = 1, \ldots s; j = 1, \ldots s$. If the problem is feasible stop, if not go to Step 1 to obtain a new matrix H.

Example 1 Consider the following linear continuous-time system with Markovian jumping parameters with 2 modes:

- mode 1: $A(1) = \begin{bmatrix} 2 & 1.5 \\ 0 & -2.5 \end{bmatrix}, B(1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}; q_1(1) = 5, q_2(1) = 10$
- mode 2: $A(2) = \begin{bmatrix} 3 & 0 \\ 0.1 & -2.1 \end{bmatrix}, B(2) = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}; q_1(2) = 10, q_2(2) = 20$

The Markovian process is described by its matrix transition given by:

$$\Pi = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$$

For this example, we have $n - m = 1$. The resolution of the linear programming (LP1) leads to the following scalars $H(\alpha)$, $\alpha = 1, 2$ with $\varphi(1) = \begin{bmatrix} 200 \\ 2.4 \end{bmatrix}$ and $\varphi(2) = \begin{bmatrix} 155 \\ 22 \end{bmatrix}$:

$$H(1) = -0.495\ ,\ H(2) = -1.97$$

$\alpha = 1, 2$.
The resolution of the algebraic equations $XA + XBX = HX$ gives the following gain matrices $F(\alpha)$ and $A_c(\alpha)$:

$$F(1) = \begin{bmatrix} -3.7425 & -1.2475 \end{bmatrix},\ F(2) = \begin{bmatrix} -9.94 & 0 \end{bmatrix}$$

$$A_c(1) = \begin{bmatrix} -1.7425 & 0.2525 \\ 3.7425 & -1.2525 \end{bmatrix},\ A_c(2) = \begin{bmatrix} -1.97 & 0 \\ -9.84 & -2.1 \end{bmatrix}$$

The resolution of the LMI's (29) gives the following matrices:

$$P(1) = \begin{bmatrix} 1.6502 & 0.3849 \\ 0.3849 & 0.6423 \end{bmatrix}\ ,\ P(2) = \begin{bmatrix} 2.2398 & -0.455 \\ -0.455 & 0.4630 \end{bmatrix}$$

Further, the exponential stability condition (8) of Corollary 1 is also satisfied with

$$w(1) = \begin{bmatrix} 0.585 & 3 \end{bmatrix}^T, w(2) = \begin{bmatrix} 0.571 & 2.67 \end{bmatrix}^T$$

That is,

$$max_i \left(\frac{[\hat{A}_c(1)w(1)]_i}{w_i(1)} \right) + \sum_{\beta \in S} \lambda_{1\beta} max_i \left(\frac{w_i(1)}{w_i(\beta)} \right) = -0.297$$

$$max_i \left(\frac{[\hat{A}_c(2)w(2)]_i}{w_i(2)} \right) + \sum_{\beta \in S} \lambda_{2\beta} max_i \left(\frac{w_i(2)}{w_i(\beta)} \right) = -0.171$$

The results of simulation are given as follows: Figure 1 presents the common set of positive invariance and stochastic stability $\mathcal{K}_c$ with a trajectory of the system, Figure 2 concerns the corresponding jumping Markovian process while Figure 3 deals with the admissible control.

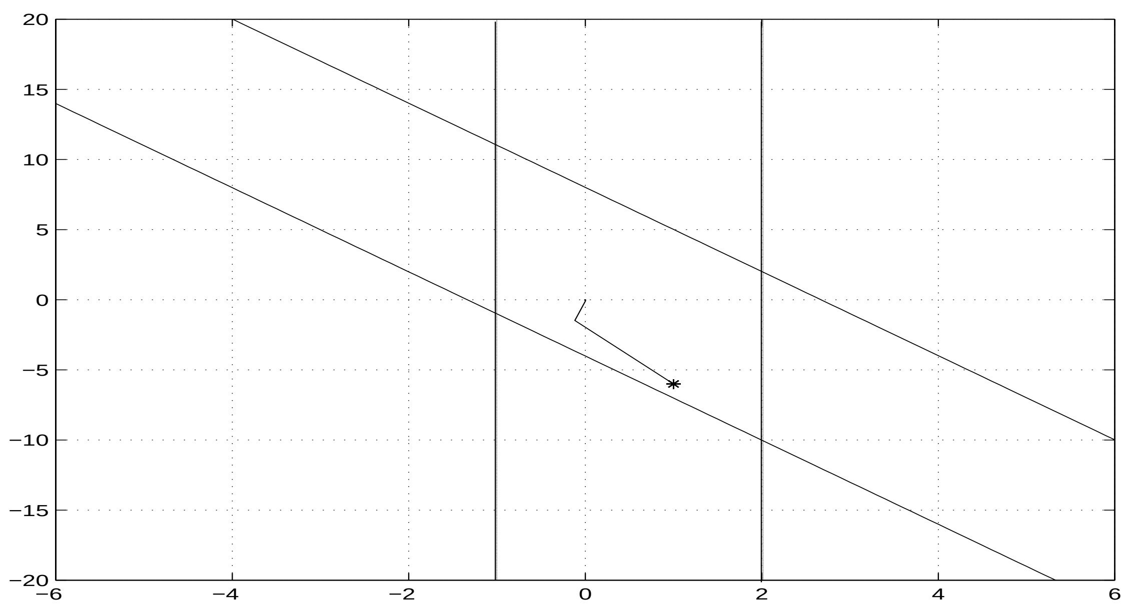

Figure 1: This figure presents the set $\mathcal{K}_c$ of positive invariance and exponential stability of the system with Markovian jumping parameters and constrained control.

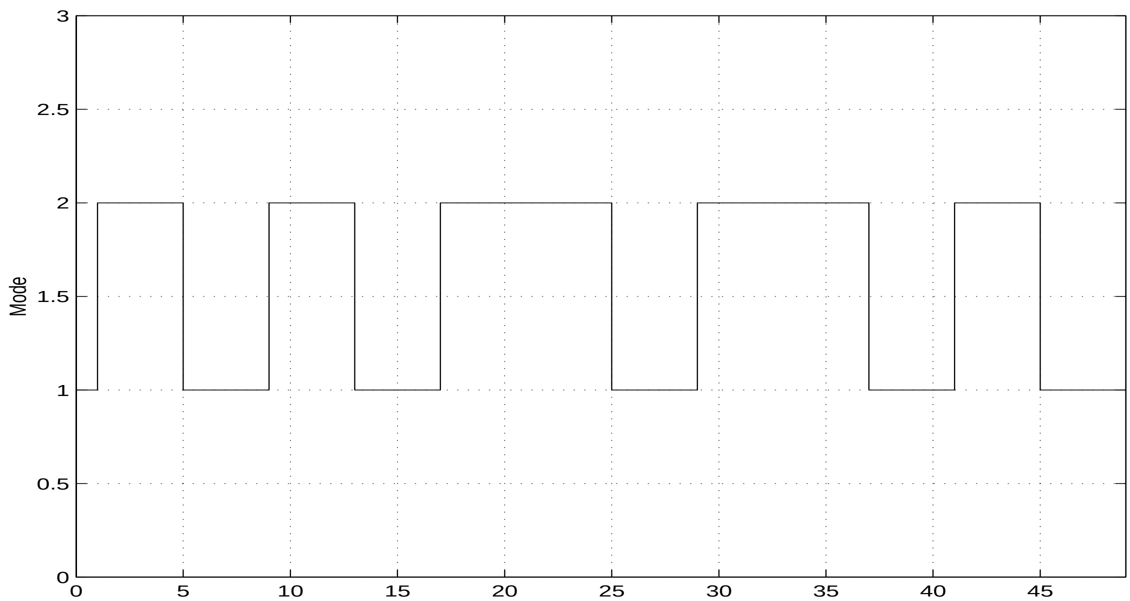

Figure 2: This figure presents the evolution of the jumping Markovian process of the system.

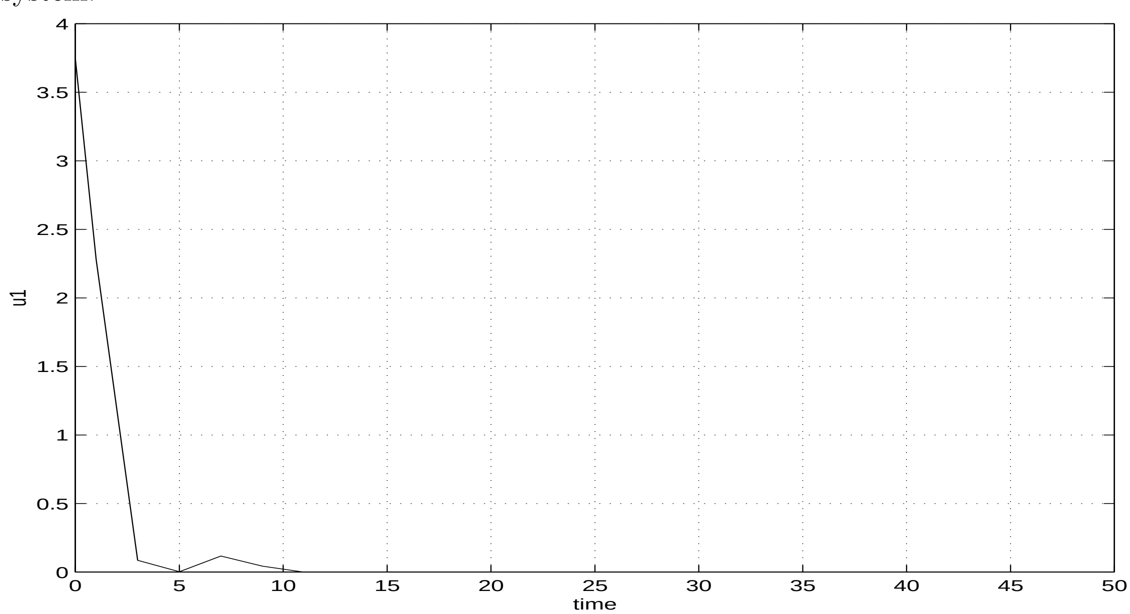

Figure 3: This figure presents the evolution of the control of the system with Markovian jumping parameters and constrained control.

5 Conclusion

In this paper, a sufficient condition of exponential stability is obtained by using the non quadratic Lyapunov function as is usually the case in the problems with constraints of inequality type. The result obtained in [2] is used to guarantee the positive invariance of the common domain $\mathcal{K}_c$ w.r.t all the mode of the system. However, the quadratic condition of stochastic stability can also be used leading to a useful algorithm based on the resolution of the LMI's (29) available in Matlab. The complete formulation of the main result of this paper under LMI's is now under consideration. Finally, an illustrative example is also presented.

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