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Supplementary material

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Supplementary material

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Abstract : Bus scheduling problem is a core optimization problem for public transit agencies. Given a set of timetabled trips to cover during a day and a homogeneous bus fleet assigned to multiple depots, the multiple-depot vehicle scheduling problem (MDVSP) consists in finding least-cost feasible schedules that cover each trip exactly once. To solve large-scale MDVSPs, we develop a column generation (CG) heuristic that is applied to a block-based set-partitioning model, where a block starts and ends at a depot without intermediate returns. To reduce degeneracy and improve performance, we combine CG with an improved dynamic constraint aggregation procedure (IDCA). We further devise a hybrid dual-disaggregation (HDD) step to accelerate convergence. Computational results on real-world instances with up to 6,296 trips show significant speed ups resulting from i) using a block-based model rather than a schedule-based model, ii) integrating CG with IDCA, and iii) applying HDD. Together, the these techniques yield an average speed up factor of 12.4 compared to CG alone applied to a schedule-based model, with only marginal degradation in the solution cost.

Keywords : Public transportation; bus scheduling; integer programming; column generation; dynamic constraint aggregation

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1 Scheduled-based column generation heuristic

Here, we present the components of the schedule-based CG heuristic that differ from those used for the block-based CG algorithm. First, we present a schedule-based formulation for the MDVSP. Then, we describe the CG subproblem and the decisions made in the diving heuristic.

1.1 Schedule-based formulation

Let S_d be the set of feasible schedules for the buses in depot $d \in D$. Denote by c_s the total cost of bus schedule $s \in S_d$, including the fixed and variable costs, and by $\alpha_{s,t}$ a binary parameter equal to 1 if schedule s covers trip $t \in T$, and 0 otherwise. Finally, let y^d_s be a binary variable equal to 1 if a bus from depot d is assigned to schedule $s \in S_d$, and 0 otherwise. Using this notation, the MDVSP can be formulated as follows:

$$\min \quad \sum_{d\in D}\sum_{s\in S_d} c_s y^d_s \tag{1a}$$

$$\text{s.t.} \quad \sum_{d\in D}\sum_{s\in S_d} \alpha_{s,t} y^d_s = 1, \qquad \forall t \in T, \tag{1b}$$

$$y^d_s \in \{0,1\}, \quad \forall d \in D, s \in S_d. \tag{1c}$$

Objective function (2) minimizes the total cost of the selected bus schedules. Constraints (3) ensure that each trip $t \in T$ is covered exactly once. Binary requirements (4) define the domain of the variables.

1.2 Pricing subproblems

Similar to block-based model, we consider a SP for each depot $d \in D$ to generate schedule variables y^d_s, $s \in S_d$. The SP is defined on an acyclic network $\mathcal{G}^d = (\mathcal{N}^d, \mathcal{A}^d)$ that allows intermediate returns to the depot (see Figure 1). Node set $\mathcal{N}^d$ comprises a source node $\bar{o}^d$, a sink node $\underline{o}^d$, a node for each trip $t \in T$, and a node for each time point M^d_i, $i \in I$. Note that we also apply time-point aggregation, as described in Section 3.3, but leave it out of the current presentation for clarity.

Arc set $\mathcal{A}^d$ contains six arc types: a start arc $(\bar{o}^d, M^d_1)$; an end arc $(M^d_{2n}, \underline{o}^d)$; pull-out arcs $(M^d_{i^{\text{out}}(t)}, t)$, $t \in T$ and its corresponding out-time-point index $i^{\text{out}}(t) \in I$; pull-in arcs $(t, M^d_{i^{\text{in}}(t)})$, $t \in T$ and its corresponding in-time-point index $i^{\text{in}}(t) \in I$; connection arcs (t_1, t_2), $t_1, t_2 \in T$; and waiting arcs (M^d_i, M^d_{i+1}), $i \in I \setminus \{2n\}$. The latter type allows to wait at the depot before the first block, after the last block, or between two blocks. The adjusted cost of an arc $a \in \mathcal{A}^d$ is given by:

$$\bar{c}_a = \begin{cases} c_a - \pi_{t^{\text{tail}}_a} & \text{if } a \text{ is a connection or a pull-in arc} \\ c_a & \text{otherwise.} \end{cases} \tag{2}$$

1.3 Diving heuristic

We use the same diving heuristic as for the block-based solution approach (see Section 4.3), except that schedule variables y^d_s are fixed instead of block variables x^d_b, and the connection arc-flow fixing is replaced by a more general inter-trip fixing technique that allows to fix indirect connections, i.e., connections including an intermediate return to the depot. More precisely, for every pair of distinct trips t_1 and t_2 in T, and bus schedule $s \in S_d$, $d \in D$, let $\sigma^s_{t_1,t_2}$ be a binary parameter equal to 1 if schedule s covers t_1 and t_2, and t_2 is the next trip covered after t_1 in s. Then, denote by $f_{t_1,t_2} = \sum_{d\in D}\sum_{s\in S_d} \sigma^s_{t_1,t_2} y^d_s$ the total flow on the schedules that cover t_2 immediately after t_1, possibly visiting the depot between these two trips. When it is not possible to fix schedule variables with a sufficiently large fractional value, we fix to 1 the inter-trip flow variables f_{t_1,t_2} that take a fractional value greater than or equal to a given threshold (set to 0.7 for our test). These decisions are directly imposed in the SPs by modifying the labeling algorithm as described in Irnich and Desaulniers (2005).

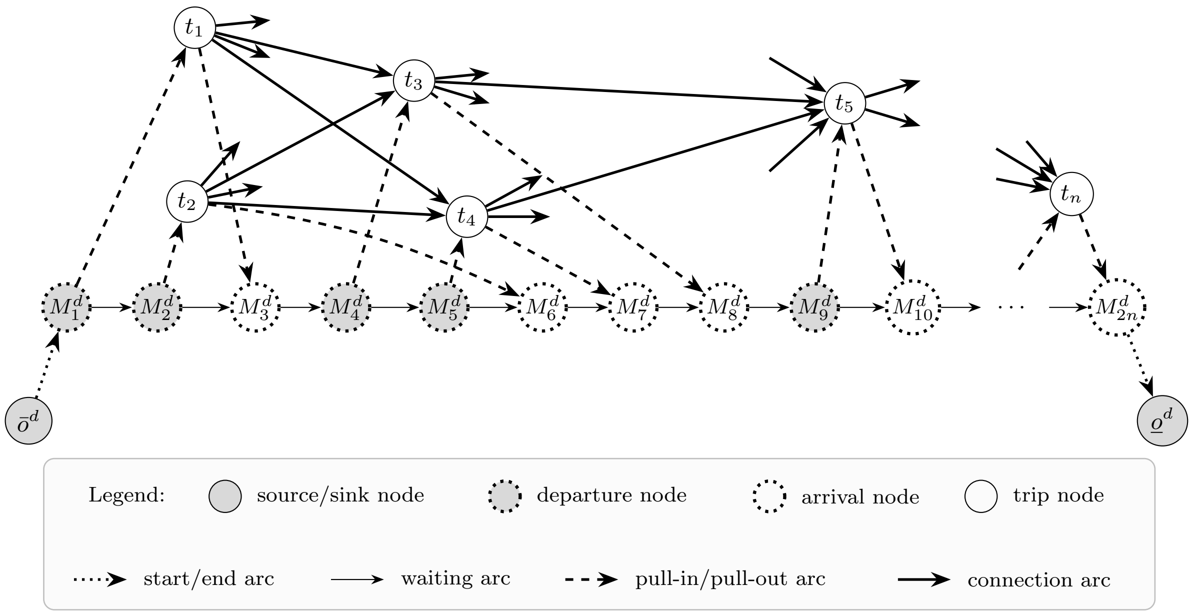

Figure 1: Subproblem Network for Schedule-Based CG Heuristic

2 Details results for the instances with additional forbidden or penalized connections

In this section, we report the detailed results obtained by BM/CG and BM/CG-IDCA-HDD on the SDVSP and MDVSP instances with additional forbidden/penalized connections. As mentioned in Section 5.1 of the paper, 15% or 30% of the connections arcs involving a deadhead is removed in these instances, yielding SP networks of reduced size. The results are provided in eight tables, namely, for all combinations of SDVSP or MDVSP, forbidden or penalized, and 15% or 30%. The content of these tables is as described in Section 5.2 of the paper.

Table 1: Detailed Results for the Combination SDVSP/Forbidden/15%

			Time (min)			Total	Gap	
Instance	Itr.	Nodes	(A)RMP	SPs	Total	Cost	(%)	Buses
BM/CG								
Depot50	187	8	26.0	1.0	27.0	2,124,425	0.001	210
Depot54	142	11	10.3	0.4	10.7	2,124,759	0.000	210
Depot55	99	3	5.1	0.2	5.4	1,324,027	0.001	131
Depot56	162	9	22.8	0.9	23.8	2,094,343	0.001	207
Depot57	67	3	1.1	0.1	1.2	1,124,365	0.000	111
Depot58	166	10	8.2	0.3	8.5	2,223,272	0.000	220
Depot59	147	7	7.0	0.4	7.5	1,174,314	0.003	116
Depot60	200	19	3.0	0.1	3.2	1,021,768	0.001	101
BM/CG-IDCA-HDD								
Depot50	507	35	10.2	1.5	12.3	2,124,580	0.008	210
Depot54	355	30	6.1	0.7	7.2	2,124,837	0.004	210
Depot55	210	14	0.8	0.4	1.1	1,324,147	0.010	131
Depot56	418	33	12.8	1.5	15.1	2,094,528	0.009	207
Depot57	221	15	0.5	0.1	0.6	1,124,392	0.003	111
Depot58	231	27	2.6	0.5	3.6	2,223,420	0.007	220
Depot59	267	15	1.3	0.4	2.0	1,174,372	0.008	116
Depot60	383	28	1.1	0.2	1.4	1,021,868	0.010	101

Table 2: Detailed Results for the Combination SDVSP/Forbidden/30%

			Time (min)			Total	Gap	
Instance	Itr.	Nodes	(A)RMP	SPs	Total	Cost	(%)	Buses
BM/CG								
Depot50	174	10	23.0	0.7	23.7	2,145,035	0.001	212
Depot54	110	7	9.0	0.2	9.3	2,145,257	0.000	212
Depot55	96	3	4.9	0.1	5.0	1,334,410	0.000	132
Depot56	165	10	22.1	0.6	22.7	2,094,629	0.001	207
Depot57	91	6	1.3	0.1	1.4	1,124,682	0.000	111
Depot58	211	15	9.3	0.3	9.9	2,223,574	0.014	220
Depot59	123	6	6.5	0.3	6.9	1,174,841	0.003	116
Depot60	176	16	2.8	0.1	2.9	1,021,949	0.000	101
BM/CG-IDCA-HDD								
Depot50	535	41	11.1	1.2	12.9	2,145,064	0.003	212
Depot54	375	32	5.6	0.6	6.6	2,145,328	0.003	212
Depot55	207	15	0.5	0.2	0.8	1,334,586	0.013	132
Depot56	454	41	15.1	1.3	17.3	2,094,673	0.003	207
Depot57	208	12	0.5	0.1	0.7	1,124,735	0.005	111
Depot58	285	22	1.9	0.4	2.6	2,223,735	0.021	220
Depot59	296	20	1.3	0.4	2.0	1,174,937	0.011	116
Depot60	440	38	1.1	0.1	1.5	1,021,999	0.005	101

Table 3: Detailed Results for the Combination SDVSP/Penalized/15%

			Time (min)			Total	Gap	
Instance	Itr.	Nodes	(A)RMP	SPs	Total	Cost	(%)	Buses
BM/CG								
Depot50	233	17	25.0	1.1	26.1	2,124,329	0.002	210
Depot54	123	8	10.7	0.4	11.1	2,104,675	0.000	208
Depot55	114	6	5.4	0.2	5.6	1,324,019	0.001	131
Depot56	160	9	21.1	1.0	21.7	2,094,283	0.001	207
Depot57	81	4	1.4	0.1	1.5	1,124,350	0.000	111
Depot58	145	10	10.5	0.5	11.0	2,213,287	0.001	219
Depot59	120	4	7.7	0.6	8.3	1,174,301	0.005	116
Depot60	179	15	3.0	0.1	3.2	1,021,779	0.002	101
BM/CG-IDCA-HDD								
Depot50	472	30	9.1	1.8	11.6	2,124,475	0.009	210
Depot54	359	31	6.0	0.9	7.5	2,104,704	0.002	208
Depot55	221	15	0.6	0.2	1.1	1,324,175	0.013	131
Depot56	447	34	17.7	2.1	20.5	2,094,353	0.004	207
Depot57	213	15	0.5	0.1	0.8	1,124,376	0.003	111
Depot58	322	26	2.1	0.6	3.1	2,213,418	0.007	219
Depot59	309	18	1.6	0.6	2.6	1,174,324	0.007	116
Depot60	420	34	1.1	0.2	1.7	1,021,836	0.007	101

Table 4: Detailed Results for the Combination SDVSP/Penalized/30%

Instance	Itr.	Nodes	Time (min) (A)RMP	Time (min) SPs	Time (min) Total	Total Cost	Gap (%)	Buses
BM/CG								
Depot50	208	12	26.6	1.2	27.9	2,124,727	0.000	210
Depot54	124	8	9.6	0.4	10.0	2,105,196	0.000	208
Depot55	126	7	5.2	0.2	5.4	1,324,367	0.001	131
Depot56	162	10	20.8	0.9	21.7	2,094,501	0.000	207
Depot57	97	7	1.2	0.1	1.3	1,124,667	0.000	111
Depot58	143	8	8.7	0.3	9.1	2,213,599	0.001	219
Depot59	126	6	6.7	0.5	7.3	1,174,757	0.003	116
Depot60	173	14	3.0	0.1	3.1	1,021,901	0.000	101
BM/CG-IDCA-HDD								
Depot50	517	36	9.4	1.7	12.1	2,124,899	0.008	210
Depot54	362	29	6.6	0.9	7.4	2,105,249	0.003	208
Depot55	211	13	0.6	0.2	1.1	1,324,503	0.011	131
Depot56	431	39	16.9	1.9	19.6	2,094,573	0.004	207
Depot57	197	11	0.5	0.1	0.8	1,124,701	0.003	111
Depot58	289	22	1.8	0.5	2.8	2,213,726	0.007	219
Depot59	319	20	1.6	0.5	2.5	1,174,805	0.007	116
Depot60	412	32	1.2	0.2	1.8	1,021,958	0.006	101

Table 5: Detailed Results for the Combination MDVSP/Forbidden/15%

Instance	Itr.	Nodes	Time (min) (A)RMP	Time (min) SPs	Time (min) Total	Total Cost	Gap (%)	Buses
BM/CG								
Depot50_58	355	16	115.0	3.3	118.5	4,254,724	0.004	421
Depot50_60	642	70	108.3	2.9	111.9	3,092,254	0.002	306
Depot55_57	151	8	24.2	0.7	24.9	2,405,600	0.002	238
Depot55_58	205	10	62.9	1.9	64.9	3,434,020	0.001	340
Depot55_59	218	7	69.2	2.9	72.3	2,465,924	0.004	244
Depot57_59	184	9	35.4	1.7	37.3	2,245,866	0.004	222
Depot55_57_58	218	10	128.2	3.2	132.0	4,526,538	0.007	448
Depot55_57_59	248	13	144.3	4.6	149.4	3,548,582	0.009	351
Depot57_58_59	197	9	145.6	5.0	150.9	4,387,862	0.008	434
BM/CG-IDCA-HDD								
Depot50_58	483	18	28.2	5.5	35.2	4,255,735	0.027	421
Depot50_60	593	27	45.5	4.2	50.8	3,092,456	0.008	306
Depot55_57	369	17	11.8	1.7	12.5	2,405,714	0.006	238
Depot55_58	341	14	11.8	3.0	16.1	3,434,608	0.018	340
Depot55_59	426	24	14.7	3.1	19.0	2,466,125	0.012	244
Depot57_59	438	23	13.3	2.8	16.9	2,245,901	0.005	222
Depot55_57_58	419	18	45.7	6.8	54.4	4,527,194	0.021	448
Depot55_57_59	470	28	38.9	6.9	47.8	3,548,906	0.018	351
Depot57_58_59	444	19	48.1	7.8	57.9	4,388,487	0.022	434

Table 6: Detailed Results for the Combination MDVSP/Forbidden/30%

			Time (min)			Total	Gap	
Instance	Itr.	Nodes	(A)RMP	SPs	Total	Cost	(%)	Buses
BM/CG								
Depot50_58	432	26	122.4	2.7	125.3	4,255,084	0.002	421
Depot50_60	612	66	120.8	2.3	123.4	3,092,781	0.004	306
Depot55_57	170	11	29.2	0.7	30.0	2,405,871	0.001	238
Depot55_58	190	10	57.5	1.4	59.0	3,434,470	0.003	340
Depot55_59	219	10	57.5	1.6	59.2	2,476,394	0.004	245
Depot57_59	152	6	33.6	1.3	35.0	2,246,201	0.004	222
Depot55_57_58	197	7	193.6	2.9	197.4	4,527,028	0.017	448
Depot55_57_59	224	10	176.0	3.8	180.8	3,549,071	0.023	351
Depot57_58_59	274	16	204.5	3.9	209.0	4,388,346	0.019	434
BM/CG-IDCA-HDD								
Depot50_58	582	28	36.3	4.9	43.7	4,256,033	0.024	421
Depot50_60	601	24	57.0	3.8	62.7	3,092,878	0.008	306
Depot55_57	356	14	11.6	1.4	13.9	2,405,991	0.006	238
Depot55_58	355	15	13.3	2.5	17.2	3,434,736	0.011	340
Depot55_59	417	20	15.1	2.3	18.7	2,476,544	0.010	245
Depot57_59	363	11	13.5	1.9	16.5	2,246,254	0.007	222
Depot55_57_58	416	18	40.9	5.6	48.8	4,527,722	0.033	448
Depot55_57_59	472	22	53.7	6.1	61.2	3,549,228	0.027	351
Depot57_58_59	483	24	48.3	6.8	56.9	4,388,956	0.033	434

Table 7: Detailed Results for the Combination MDVSP/Penalized/15%

			Time (min)			Total	Gap	
Instance	Itr.	Nodes	(A)RMP	SPs	Total	Cost	(%)	Buses
BM/CG								
Depot50_58	322	14	119.1	3.7	123.1	4,254,587	0.001	421
Depot50_60	707	76	108.5	3.4	112.3	3,092,336	0.006	306
Depot55_57	131	6	25.5	0.8	26.4	2,395,571	0.001	237
Depot55_58	218	9	44.7	1.7	46.1	3,434,081	0.003	340
Depot55_59	254	11	56.8	2.6	59.5	2,465,899	0.003	244
Depot57_59	124	3	32.9	1.9	34.8	2,245,829	0.003	222
Depot55_57_58	172	6	116.1	3.1	119.6	4,526,529	0.006	448
Depot55_57_59	204	11	101.4	3.8	105.6	3,548,554	0.008	351
Depot57_58_59	200	7	114.2	4.6	119.2	4,387,863	0.008	434
BM/CG-IDCA-HDD								
Depot50_58	510	21	37.6	7.3	46.7	4,255,461	0.021	421
Depot50_60	577	27	42.4	5.0	48.8	3,092,550	0.013	306
Depot55_57	358	14	12.1	2.0	14.7	2,395,738	0.008	237
Depot55_58	399	23	13.4	3.7	18.9	3,434,438	0.014	340
Depot55_59	455	25	15.9	4.0	20.8	2,466,023	0.008	244
Depot57_59	438	23	13.6	2.8	17.2	2,245,901	0.006	222
Depot55_57_58	395	18	31.5	7.6	41.5	4,527,563	0.029	448
Depot55_57_59	461	24	40.5	8.2	50.4	3,548,880	0.017	351
Depot57_58_59	445	26	45.9	9.2	51.2	4,388,454	0.022	434

Table 8: Detailed Results for the Combination MDVSP/Penalized/30%

Instance	Itr.	Nodes	Time (min) (A)RMP	SPs	Total	Total Cost	Gap (%)	Buses
BM/CG								
Depot50_58	328	16	123.5	3.8	127.4	4,255,006	0.002	421
Depot50_60	611	59	108.9	3.2	112.2	3,092,572	0.004	306
Depot55_57	162	11	25.0	0.9	25.9	2,395,896	0.002	237
Depot55_58	191	8	41.1	1.4	42.6	3,434,363	0.001	340
Depot55_59	230	10	64.0	3.0	67.1	2,466,356	0.004	244
Depot57_59	162	7	32.2	1.7	33.9	2,246,181	0.005	222
Depot55_57_58	260	18	189.6	4.3	194.7	4,526,950	0.016	448
Depot55_57_59	245	13	191.3	5.9	197.8	3,549,054	0.022	351
Depot57_58_59	243	13	188.2	6.5	195.5	4,388,286	0.018	434
BM/CG-IDCA-HDD								
Depot50_58	559	26	42.5	7.3	51.4	4,255,629	0.017	421
Depot50_60	598	22	50.6	5.3	57.1	3,092,732	0.009	306
Depot55_57	371	18	12.4	1.9	14.9	2,395,992	0.006	237
Depot55_58	400	22	14.5	3.6	19.2	3,434,653	0.010	340
Depot55_59	382	14	17.5	3.7	22.1	2,466,562	0.012	244
Depot57_59	376	12	14.8	2.9	18.3	2,246,337	0.011	222
Depot55_57_58	426	20	45.6	7.7	55.7	4,527,567	0.029	448
Depot55_57_59	480	25	52.1	8.8	62.7	3,549,242	0.028	351
Depot57_58_59	491	28	53.2	9.5	65.1	4,388,982	0.034	434

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