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The petrol station replenishment problem: A case study from West Africa

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Abstract : This paper describes a planning problem faced by a fast-growing petroleum company in West Africa. The problem is a complex variant of the petrol station replenishment problem. It integrates the truck loading problem and the inventory routing problem. We tackle this problem in two steps. In the first step, we use *Branch-and-Price* to solve the weekly case where each petrol station can be visited more than once each week within the horizon. In the second step, we use Benders decomposition to solve up to four months horizon. Numerical results on a real network with up to four depots, four types of petrol products, four main groups of petrol stations, and a heterogeneous fleet of highly compartmented tank trucks prove the effectiveness and high potential of the proposed approach. They also highlight optimization's impact on Africa and the petrol station replenishment practice on the continent.

Keywords: Petrol station replenishment, Inventory routing, truck loading, Benders decomposition, column generation, branch-and-price, OR in developing countries

1 Introduction

Despite the increasing adoption of electric vehicles (Collett et al., 2021; Er Raqabi and Li, 2023), Africa still relies heavily on petrol products for transport, even with the emergence of other energies (e.g., electric, hydrogen). For petrol distribution companies, this comes with a need to invest heavily in the infrastructure, the petrol tank truck fleet, and the business analytics tools that can ensure better performance for all stakeholders in the petroleum distribution sectors (e.g., distribution companies, petrol stations, governments, and customers).

The petrol station replenishment process has three parts: the loading, the routing, and the unloading. The trip starts from a depot, where a multi-compartment tank truck is loaded with petrol products. Following the loading, the tank truck visits some petrol stations. These petrol stations usually have underground tanks where the petrol products are unloaded. Once the trip is completed, the tank truck returns to the depot. This generic problem, with many variants in the literature, is referred to as the petrol station replenishment problem (PSRP). Mathematically, it integrates the tank truck loading problem (TTLP) and the inventory routing problem (IRP). Traditionally, the TTLP and IRP have been tackled sequentially, with tank truck loading being an input for the routing problem. The sequential procedure leads to suboptimal and sometimes infeasible solutions. Furthermore, when considering inventory management at the depots and petrol stations, attempts to tackle the PSRP variant obtained have been mainly based on heuristics rather than exact methods.

1.1 Contributions

Using operations research (OR) techniques, Bani et al. (2023) tackled exactly a complex and realistic variant of the PSRP, referred to as the multi-depot multi-period petrol station replenishment problem (MDMPPSRP). Furthermore, Bani et al. (2024) solve exactly another complex and realistic variant of the PSRP, referred to as the multi-depot multi-period petrol station replenishment problem with inventory management (MDMPPSRPIM). Our study has two main contributions:

- Leveraging insights from Bani et al. (2023) and Bani et al. (2024), we highlight a gradual approach to tackle a real-life MDMPPSRPIM from West Africa. In the first step of the approach, we tackle the weekly PSRP using Column Generation (CG). In the second step of the approach, we tackle the MDMPPSRPIM by combining Benders decomposition (BD) and CG.
- We use this gradual approach to conduct extensive experiments, which highlight the following: (1) Among two popular policies, we highlight that the direct billing policy is more costly than the standard billing policy, (2) The oil price fluctuations can inform buying and stocking decisions. In particular, if the oil price fluctuates significantly, stocking becomes profitable for the Petroleum Company (PC), (3) At petrol stations, there are optimal tank capacities. In our experiments, we found that increasing the tank capacities by 15% ensures a significant decrease in total cost, and (4) Among all available types of tank trucks (more than 40), less than 14 types are used frequently.

The remainder of this paper is organized as follows. We discuss the relevant studies in Section 2. We describe the problem and the methods in Sections 3 and 4, respectively. The results and the discussion are highlighted in Section 5, followed by conclusions in Section 6.

2 Literature review

Considered as the most studied variant of the multi-compartment vehicle routing problem (Derigs et al., 2011), the PSRP has received substantial attention in the literature over the last decades. Several variants have been studied after the problem was formulated for the first time by Brown and Graves (1981). For a comprehensive overview of the history of the PSRP, the papers published by

Cornillier et al. (2012), Benantar et al. (2016), and Lima et al. (2016) provide additional background on the PSRP. Many researchers created models to address real-world problems to help organizations reduce their logistic costs (Ng et al., 2008; Triki, 2013; Benantar et al., 2016; Al-Hinai and Triki, 2020).

2.1 Tank Truck Loading Problem

In the context of the PSRP, tank trucks are used for the petrol products distribution. However, due to compatibility constraints, it is only possible to load one petrol product into each compartment Kaabi and Jabeur (2015). Coelho and Laporte (2015) have introduced several mixed integer linear programming (MILP) models to address this issue, categorizing them as fractional and non-fractional delivery. Fractional delivery allows for the distribution of compartment contents among multiple petrol stations, while non-fractional delivery necessitates the complete delivery of compartment contents to a single petrol station. Furthermore, it is worth noting that previous studies often assumed that the tank trucks don't have flow meters, resulting in the requirement to empty the compartment at each petrol station (Macedo et al., 2011; Cornillier et al., 2008a,b; Benantar et al., 2016; Popović et al., 2012; Vidović et al., 2014). This absence of flow meters does not allow accurately measuring the quantity of product delivered to each petrol station. In such a case, a constraint is needed to ensure at most one petrol product from one petrol station can be loaded in a tank truck compartment. This type of constraint is hard to solve because we must solve a series of branching nodes.

In their study, Wang et al. (2020) propose a novel approach for the PSRP, which involves using a different TTLP with similar compartments. These compartments are equipped with flow meters, enabling the assignment of multiple orders of the same petrol product to each compartment. This feature simplifies the problem by transforming the TTLP into assigning petrol quantities to the compartments rather than petrol station orders' quantities. In this case, the TTLP can be formulated as a pure linear program, and given its small size, it solves very quickly.

Recently, in Bani et al. (2023), even if the tank trucks are equipped with flow meters, for management simplification and other administrative reasons, a compartment is assigned in its entirety at most to a single petrol station. They solve the MDMPPSRP using CG. The latter involves solving an elementary shortest path problem with resource constraints in the CG subproblems, where hundreds of millions of extensions are performed. Each extension includes the resolution of a TTLP that ensures the feasibility of loading the petrol station's orders into the tank truck at each extension. To solve the TTLP, Bani et al. (2023) use a hashing technique that they employ in each iteration of their solution for the TTLP. This technique allows the memorization of solved TTLPs to leverage them again in subsequent iterations. By reducing the number of MILP problems that need to be solved in each iteration, the hashing technique significantly decreases the execution time of the CG subproblem labeling algorithm. Additionally, the authors demonstrate that examining the total petrol stations' orders allows the optimization of various resources, such as the tank truck capacity and the compartment number.

2.2 Inventory Routing Problem

In recent studies, researchers have examined the introduction of IRP in PSRP to reduce both routing and inventory costs. Popović et al. (2011, 2012); Vidović et al. (2014) have all explored this approach.

The underlying assumption in the problems above is the vendor-managed inventory (VMI) concept. Under this concept, the PC has access to information regarding inventory levels and is responsible for making decisions regarding replenishment. Decisions to take simultaneously include the delivery time and quantities for each petrol station, and the most efficient routes for petrol station service. This arrangement benefits both the PC and vendors, as it helps minimize distribution costs and eliminates the need for vendors to allocate resources towards inventory management (Coelho et al., 2014).

There are various inventory policies to determine the quantities to deliver. The two inventory policies are the *Maximum-Level* policy and the *Order-Up-to-Level* policy. Under the *Maximum-Level* policy, the order quantity is flexible but lower than the maximum capacity. On the other hand, the *Order-Up-to-Level* policy utilizes a fixed order quantity, which ensures that the inventory is maintained beyond specific level (Coelho et al., 2014). The most commonly used policy for the PSRP is the *Maximum-Level* policy. This policy has been applied by Cornillier et al. (2008a) and Popović et al. (2011), who set limits on the delivery quantities based on the petrol station tank capacity. Li et al. (2014), on the other hand, implemented an *Order-Up-to-Level* policy where the delivery quantity is set to the remaining tank capacity at the time of delivery.

2.3 Petrol station replenishment problem

Many variants of the problem have emerged, differing in whether planning takes place over one or multiple periods, whether tank trucks are homogeneous, and whether there are time windows to model delivery times.

PSRP Variants: i) The standard PSRP consists of one depot, a single period, and no time windows (Brown and Graves, 1981). ii) The MPPSRP, a multi-period PSRP, has been studied by Malépart et al. (2003); Triki (2013); Archetti et al. (2015). iii) The PSRPTW is a PSRP with time windows, as discussed by Cornillier et al. (2009). iv) The MDPSRPTW is a multi-depot PSRPTW, which has been addressed by Cornillier et al. (2012) using heuristics. v) The MDMPPSRP, a multi-depot MPPSRP, has been tackled by Carotenuto et al. (2018) and Bani et al. (2023).

Heuristic/matheuristic approaches. Most surveyed papers have proposed heuristic or metaheuristic approaches to address the PSRP variants. This preference stems from the challenges associated with implementing exact methods, which are often more complex, and their limited suitability for solving large-scale instances. As pointed out by Laporte (1992), heuristic and metaheuristic approaches are considered more practical in such cases. The frequently utilized metaheuristics include cluster first (which considers geographic limitations and expert considerations) followed by route determination, adapted VNS introduced by Hansen and Mladenović (2001), load first (which involves assigning orders to tank trucks) followed by route determination and tailored heuristics. It is important to note that for the problems addressed, neither lower bounds nor optimal solutions are currently known, making it challenging to assess the quality of the obtained solutions.

Exact models/approaches. Only a few papers have attempted to develop exact models or solution methods for solving small instances of the PSRP. For instance, Cornillier et al. (2008a) decomposed the standard PSRP into two subproblems, namely the TTLP and the routing problem (RP), using a CG scheme. The TTLP assigns orders to compartments to maximize profit, while the RP focuses on selecting routes that minimize overall transportation costs. On the other hand, Avella et al. (2004) proposed a *Branch-and-Price* algorithm for the standard PSRP, allowing for multiple petrol stations per route. They use a heuristic approach to generate an initial set of columns for the algorithm and test real-world data with specific characteristics. Additionally, Benantar et al. (2016) solved small instances of the MPPSRP using CPLEX. In another study, Cornillier et al. (2008a) presented an exact model for the MDPSRPTW, which aimed to select a subset of feasible trips that satisfy demand while maximizing overall daily net revenue. However, due to the large number of possible trips, the authors proposed an alternative heuristic approach. Bani et al. (2023) solve the integrated TTLP and RP simultaneously using an exact *Branch-and-Price* algorithm with multiple petrol stations per route.

2.4 State of practice

America. One of the pioneering studies in the field of petroleum distribution is the study of Brown and Graves (1981) in the United States. Their research focused on a real-life case for the PSRP and aimed to automate the dispatching of tank trucks. This study laid the foundation for future advancements in optimizing the distribution process. In a subsequent study, Brown et al. (1987) explored a planning optimization approach for a PC in the United States. The objective of this research was to enhance personnel productivity and reduce costs. By implementing their proposed optimization model, the PC improved significantly efficiency and cost savings. Kazemi and Szmerekovsky (2015) examined petrol distribution in the United States provinces as an IRP. Cornillier et al. (2008a) further expanded on petroleum distribution network optimization. Their study proposed a heuristic approach to solve the problem at hand. The authors successfully demonstrated the benefits of optimization to the PC, leading to the integration of the proposed algorithm into the planning systems of the distribution company. This integration increased awareness about optimization advantages and facilitated the practical implementation of the algorithm in real-world scenarios.

Europe. A notable contribution to the field was made by Van der Bruggen et al. (1995) through their consultancy study conducted in the Netherlands. Their research focused on optimizing the distribution network of a large PC. Specifically, they aimed to improve the efficiency of transporting petrol products from the depot to petrol stations. The findings of this study provided valuable insights for the management of the PC, aiding them in restructuring their distribution network.

Asia. Several researchers have developed various models to analyze real-life scenarios. In one study, Ng et al. (2008) focused on enhancing the petrol distribution within a network in Hong Kong. The authors implement the VMI concept. The proposed approach proved beneficial for the PC as it resulted in an increase in delivery volume and a reduction in driver costs. In another investigation, Triki (2013) utilizes real-life cases to evaluate the effectiveness of solution methods proposed by the authors for solving the PSRP. The most efficient method yielded a remarkable 17.7% savings compared to planning conducted by a human operator. Additionally, they explore the PSRP in the context of a PC in Oman. They present a MILP model for bidding in transportation procurement auctions. Li et al. (2014) focused on a large PC in China and modeled the distribution of petrol products across provinces as an IRP. They developed a heuristic approach that demonstrated near-optimal solutions. They employed a MILP model to determine the optimal supply chain design, incorporating multi-modal transportation methods and facility locations.

Africa. The case study conducted by Benantar et al. (2016) evaluated the distribution process for an Algerian PC. The authors proposed a model for the PSRP that incorporated compartmented tank trucks and time windows. Their method outperformed the company's existing solution in the number of tank trucks required and the total travel distance.

Globally. Petrol product prices have been fluctuating significantly over recent years. For instance, during the COVID pandemic, the world witnessed historic petrol product price fluctuation (Le et al., 2021). To face fluctuations, governments implement and reform subsidy systems to ensure the smooth supply of petrol products. For instance, major reforms to the subsidy system in Morocco began on September 16th, 2013, with the decision to reactivate the price indexation mechanism for petrol products, including *Gasoline*, *Diesel*, and *Fuel oil* (Verme and Araar, 2017). The highest petrol barrel price was $147.27 in July 2008. As of March 1st, 2024, the petrol barrel price is $79.81 per barrel. The petrol barrel prices significantly affect the petrol products purchase and transport costs. The purchase cost is directly linked to the raw material price, and the transport cost is also directly linked since *Gasoline* is the main energy used for transport. Both *Gasoline* and *Diesel* are stored in large cylindrical underground tanks. Each tank has a capacity of 21,000 to 22,000 liters. The number of tanks depends on the number of dispensing machines on the ground. A typical petrol station has a tank capacity of 110,000 to 150,000 liters (Evans, 2009). For billing policies, we distinguish direct billing and standard billing. The direct billing policy is billing the petrol station based on the kilometer (km) distance from the depot. The standard billing scenario is billing the petrol station based on the distance in km from the previous stop (e.g., another petrol station served just before).

3 Problem

The problem under investigation centers on the PC's decision-making process regarding the types of petrol products to be delivered and the petrol product volumes transported by contracted distributors over a specified planning horizon. These private transport companies operate a diverse fleet of multi-compartment tank trucks, each with limited capacity, to replenish two distinct groups of petrol products for three different classes of petrol stations. The objective is to schedule efficiently the weekly replenishment for these petrol stations. During each planning horizon, the PC faces constraints on the available quantity of petrol products (stored in multiple depots).

Planning horizon. The planning horizon is up to four months. We decompose the planning horizon into weeks, each having seven days.

Petrol products. There are a total of four petrol products: Two unmarked (*Super Gasoline* and *Gasoline*) and two marked (*Marine Zoom* and *Diesel*). These petrol products are chemically marked to detect fraud because the state subsidizes them. Additional customs escort fees apply to marked petrol products delivered to marine stations, and the amounts may vary between destinations. Here, we are considering the case where the total supply of each petrol product type is greater than or equal to the total demand for that type.

Depots. The petrol products depots are in the capital and have a limited capacity. Some of them specialize in specific types of petrol products they store. We want to determine the distribution of petrol product volumes that each transport company must distribute to the petrol stations. The goal is to minimize the distribution cost.

Petrol stations. The network of petrol stations, comprising 12 regular petrol stations, 37 bakeries, and 11 marine stations, is distributed throughout Senegal (see Figures 1). The PC owns some petrol stations, while others are private. The regular petrol stations are equipped with two *Super Gasoline* tanks and 1 *Gasoline* tank. The Bakeries generally have a tank capacity of 1000, 2000, or 5000 liters for *Diesel*. Maritime stations typically have a single tank with a capacity ranging from 14000 to 50000 liters, except some have two tanks. The average demand for each product in a petrol station is constant and deterministic, providing known daily consumption quantities before optimization. A discount of 40% on custom escort fees is applicable when two maritime stations are visited successively. Multiple tank trucks can visit the petrol station during the same week for supplies of different petrol products. Some petrol stations are close to the petrol products depots, while others are very far away, some in cities with regulated access, others in rural areas with less easy access, making the routing problem more challenging and richer.

Tank trucks. There are four private transport companies, each maintaining fleets composed of heterogeneous multi-compartment tank trucks located at the transport company depots. The number of compartments varies from one tank truck to another, ranging from 4 to 15. The total capacity of a tank truck is the sum of the capacities of its compartments. There are 44 tank trucks, but not all may be available simultaneously, as other PCs allocate them based on petrol station demands for tasks in different locations. All tank trucks are assumed to travel at the same average speed, resulting in equal travel times for the same distance. A tank truck is active as soon as at least one compartment is in use, even though it may travel with empty compartments, which should be minimized. Only tank trucks available within the planning horizon are considered, and once a delivery is completed, the private transport company incurs fixed rental costs for the tank truck. Some tank trucks are exclusively dedicated to transporting marked petrol products. These features should be considered in the search strategy for feasible solutions. Transportation costs per kilometer are fixed due to government price equalization for petrol products. Some tank trucks may receive preferential treatment if painted in PC colors (mobile advertising), agree to pay partially for petrol products, or are close to high daily consumption petrol stations.

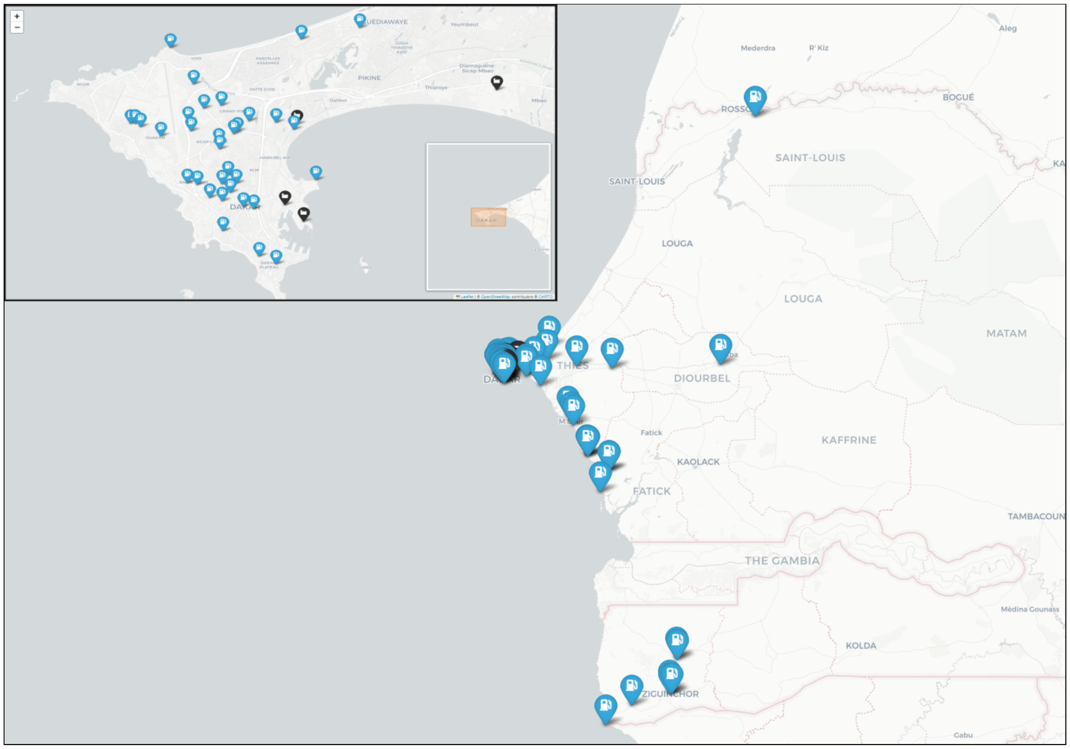

Figure 1: PC Petrol Stations (Blue) and Depots (Black)

Routes. The starting point for each route is the petrol products depot, with the endpoint being the transport company's depot. Petrol station routes are categorized into two families: routes for marked petrol products ordered by maritime stations and bakeries and routes for unmarked petrol products ordered by general petrol stations. Some routes for unmarked petrol products may consist of only one service point and are termed singleton routes. For each singleton petrol station, the tank truck supplying it must return directly to the depot immediately after the supply. All empty return trips to the transport company's depot are not invoiced to the PC. Only the travel time is considered to ensure the feasibility of the routes. Once a tank truck is parked in the loading area, the loading time for petrol products at the depot is assumed to be 1 hour. The unloading time at a petrol station depends on the diameter of the hoses used. Still, an average time can be established: 1 hour and 30 minutes for bakeries and 30 minutes for general petrol stations.

Planning decisions. Allocation of distribution requests to the transport companies involves ensuring that all tank trucks used to supply each petrol station are accurately identified. Then, petrol volumes are assigned to specific tank trucks and their compartments, resulting in a comprehensive truck-loading plan. Details of the supplied petrol stations, including the tanks that have been replenished and the corresponding quantities received, are recorded. Additionally, the routes taken by the tank trucks, specifying the order of visits to the petrol stations, are documented for tracking and logistical analysis.

Petrol station constraints. We distinguish the following petrol station constraints: i) Meeting the demand of each petrol station for every type of petrol product (i.e., preventing any petrol station from running out of stock for each period). ii) Adhering to the tank capacity of each petrol station. iii) Ensuring the petrol station is not visited more than once per period to deliver the same petrol product type. iv) Imposing a constraint that determines the inventory for each petrol station at the end of each period for each type of petrol product. v) As the visited petrol stations are supplied with at least one petrol product, it is possible to introduce an additional constraint to ensure that the petrol station visited during the same route does not exceed the number of compartments in a tank truck.

Tank truck constraints. We distinguish the following tank truck constraints: i) Adhere to the capacities of tank truck compartments. ii) Each compartment is restricted to containing only one petrol product from one petrol station for each tank truck route. iii) A tank truck cannot simultaneously transport marked and unmarked petrol products on the same route. iv) Some tank trucks are exclusively designated for transporting the marked petrol products. v) A tank truck transporting marked petrol products can visit at most two maritime stations during the same route. vi) Ensure that a compartment is assigned to deliver a petrol product to a petrol station only if it needs to be visited. vii) Guarantee that if a petrol station is visited during the period, it will be served at least one petrol product.

Depots constraints. We distinguish the following depot constraints: i) Ensure the availability of petrol products in each depot for each period. ii) Impose a constraint that determines the inventory for each depot and petrol product at the end of each period for each petrol product type.

The presented problem is a rich PSRP variant, which results in a complex model with millions of constraints and variables, among which several are integers. The mathematical formulation without inventory management is available in Bani et al. (2023) while the mathematical formulation with inventory management is available in Bani et al. (2024). Both problems have a large size. Solving them with generic commercial solvers is untractable, even for small instances. Thus, we design a two-step method highlighted next.

4 Method

In this section, we present the framework. Then, we describe each step of the two-step method.

4.1 Framework

We highlight the two-step method in Figure 2. In the first step, we tackle the MDMPPSRP. In the second step, we tackle the MDMPPSRPIM. The insight behind the two-step method is solving first the weekly MDMPPSRP using CG. Then, to solve the MDMPPSRPIM, we use BD to decompose the MDMPPSRPIM into several weekly MDMPPSRP that we solve efficiently using *Step 1*.

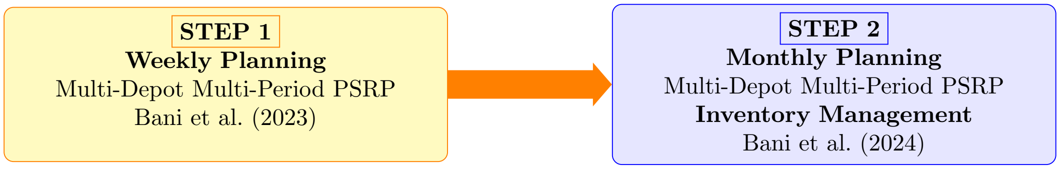

Figure 2: Two-Step Approach

4.2 First step

The first step addresses the MDMPPSRP. In this step, we solve the complex PSRP variant with an exact *Branch-and-Price* approach and some derived heuristics. An important feature is that we can model this problem as a set partitioning type problem with low to moderate density (the number of ones per column, i.e., petrol stations to visit, is not large). The set partitioning type problems have some interesting polyhedral properties to consider for favoring integrality. Furthermore, some case-specific complex handling rules apply. A natural way is to address them in the CG subproblem as an elementary shortest path problem with resource constraints, which constitutes the major bottleneck. To succeed in tackling the latter, we need to design some sophisticated techniques: i) for branching to profit from the polyhedral properties and ii) for solving the CG subproblem. The direct use of on-the-shelf algorithms does not work, unfortunately. More details about the problem and the mathematical formulation can be found in Bani et al. (2023).

4.3 Second step

Driven by a *decoupling* intuition, we develop an exact two-phase solution approach that combines BD and CG for the MDMPPSRPIM. This intuition comes from the fact that the inventory management constraints couple the planning horizon weeks because they link all buying, storage, and transport decisions in a given week with subsequent weeks. Furthermore, keeping these constraints in the model makes solving very difficult since the model is enormous. The BD (Rahmaniani et al., 2017; Er Raqabi et al., 2023b) allows us to achieve the *decoupling* intuition. Once the inventory variables (and the corresponding linking constraints) are fixed using the solution of the BD master problem, the MDMPPSRPIM becomes decoupled. The BD allows decomposing the MDMPPSRPIM into $|\mathcal{W}|$ MDMPPSRPs, which are solved using the first step insights. In the first phase, we solve the relaxed (integrality) BD subproblems using CG until the inventory levels stabilize. In the second phase, we solve the BD subproblems using CG embedded in a branch-and-bound framework. We enhance the two-phase solution approach with acceleration strategies, including warm-start, parallelism, a hashing technique, and a primal diving heuristic. More details about the problem and the mathematical formulation can be found in Bani et al. (2024).

5 Results

In this section, we run computational experiments on realistic instances. We first describe the experimental design. After that, we highlight the computational results.

5.1 Experimental design

For our tests, we consider 48 realistic instances introduced by Bani et al. (2024). These instances belong to two seasons (2016 and 2017). The features of these instances presented in Table 1 are: the number of client-products ($|\mathcal{N}|$), where a client-product represents a petrol product $p \in \mathcal{P}$ requested by a petrol station, the number of petrol stations (#C), the number of petrol products ($\mathcal{P}$), the number of weeks/days ($|\mathcal{W}|/|\mathcal{D}|$), the number of depots ($|\mathcal{E}|$), and the instance name. In total, we have four types of petrol products. The instance name includes the number of weeks, the number of client-products, the number of depots, and the season. For example, instance **P4-C47-D1-S1** has four weeks, 47 client-products, one depot, and belongs to season 1 (2016). We classify instances into two classes based on $|\mathcal{N}|$.

Table 1: Instances Properties

$|\mathcal{N}|$	#C	$\mathcal{P}$	$|\mathcal{W}|/|\mathcal{D}|$	$|\mathcal{E}|$	Inst.	$|\mathcal{N}|$	#C	$\mathcal{P}$	$|\mathcal{W}|/|\mathcal{D}|$	$|\mathcal{E}|$	Inst.
47	47	$[2,3]$	4/28	1	**P4-C47-D1-S1**	77	65	$[0,1,2,3]$	4/28	1	**P4-C77-D1-S1**
					P4-C47-D1-S2						**P4-C77-D1-S2**
				4	**P4-C47-D4-S1**					4	**P4-C77-D4-S1**
					P4-C47-D4-S2						**P4-C77-D4-S2**
			8/56	1	**P8-C47-D1-S1**				8/56	1	**P8-C77-D1-S1**
					P8-C47-D1-S2						**P8-C77-D1-S2**
				4	**P8-C47-D4-S1**					4	**P8-C77-D4-S1**
					P8-C47-D4-S2						**P8-C77-D4-S2**
			12/84	1	**P12-C47-D1-S1**				12/84	1	**P12-C77-D1-S1**
					P12-C47-D1-S2						**P12-C77-D1-S2**
				4	**P12-C47-D4-S1**					4	**P12-C77-D4-S1**
					P12-C47-D4-S2						**P12-C77-D4-S2**

The tank trucks' features, including the capacity (Ca.) in m^3, the compartment configuration (Config.), the number of compartments (Co.), if the tank truck is suitable for marked and unmarked petrol products (F.), and if the tank truck is jumbo or not (J.) are highlighted in Table 2.

Table 2: Tank Trucks Properties

Id	Ca.	Config.	Co.	F.	J.
k_1	13	$[2, 1, 4, 6]$	4	☑	☐
k_2	13	$[1, 2, 4, 6]$	4	☑	☐
k_3	13	$[3, 1, 1, 1, 1, 2, 2, 2]$	8	☑	☐
k_4	13	$[2, 1, 1, 1, 1, 5, 2]$	7	☑	☐
k_5	13	$[3, 2, 2, 3.5, 3]$	5	☑	☐
k_6	14	$[2, 1, 1, 4, 4, 2]$	6	☐	☐
k_7	14	$[2, 2, 3, 1, 2, 2, 2]$	7	☑	☐
k_8	14	$[4, 2, 3, 2, 3]$	5	☑	☐
k_9	14	$[2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2]$	11	☐	☐
k_{10}	14	$[2, 1, 1, 1, 1, 2, 3, 3]$	8	☑	☐
k_{11}	14	$[4, 2, 1, 1, 4, 2]$	6	☑	☐
k_{12}	18	$[3, 2, 2, 3, 2, 6]$	6	☐	☐
k_{13}	18	$[5, 2, 2, 2, 3, 4]$	6	☑	☐
k_{14}	18	$[4, 2, 5, 4, 3]$	5	☐	☐
k_{15}	18	$[2, 2, 2, 2, 1, 1, 1, 1, 3, 3]$	10	☐	☐
k_{16}	18	$[2, 1.5, 1, 1, 1.5, 4, 4, 3]$	8	☐	☐
k_{17}	18	$[2, 1.5, 1, 1, 1.5, 4, 4, 3]$	8	☐	☐
k_{18}	19	$[5, 1, 1, 5, 5, 2]$	6	☐	☐
k_{19}	20	$[6, 5, 5, 2, 2]$	5	☐	☑
k_{20}	20	$[3, 3, 3, 3, 2, 2, 2, 2]$	8	☐	☑
k_{21}	20	$[2, 4, 3, 4, 5, 1, 1]$	7	☐	☑
k_{22}	20	$[2, 4, 3, 4, 5, 1, 1]$	7	☐	☑
k_{23}	21	$[2, 5, 5, 5, 4]$	5	☑	☑
k_{24}	21	$[2, 3, 1, 1, 2, 3, 5, 4]$	8	☐	☑
k_{25}	21	$[6, 4, 6, 2, 3, 5, 4, 2, 6]$	9	☐	☑
k_{26}	22	$[2, 2, 1, 3, 2, 1, 2, 3, 2, 2, 2]$	11	☑	☑
k_{27}	30	$[7, 7, 2, 7, 7]$	5	☐	☑
k_{28}	33	$[5, 3, 4, 1.5, 1.5, 1, 1, 2, 3, 2, 4, 3, 2]$	13	☑	☑
k_{29}	33	$[5, 4, 3, 2, 1, 1, 2, 2, 3, 4, 6]$	11	☐	☑
k_{30}	33	$[5, 4, 3, 2, 1, 1, 2, 2, 3, 4, 6]$	11	☐	☑
k_{31}	33	$[4, 5, 4, 3, 2, 3, 2, 5, 5]$	9	☐	☑
k_{32}	35	$[6, 4, 5, 2, 2, 2, 3, 8, 3]$	9	☑	☑
k_{33}	35	$[4, 6, 2, 4, 3, 4, 3, 6, 3]$	9	☐	☑
k_{34}	35	$[6, 4, 3, 1, 1, 2, 5, 6, 7]$	9	☐	☑
k_{35}	36	$[4, 2, 6, 2, 4, 2, 4, 3, 6, 3]$	10	☐	☑
k_{36}	37	$[5, 2, 5, 2, 2, 1, 1, 2, 2, 2, 2, 5, 2, 2, 2]$	**15**	☑	☑
k_{37}	37	$[6, 3, 6, 4, 6, 6, 6]$	7	☑	☑
k_{38}	37	$[10, 3, 4, 2, 5, 6, 7]$	7	☑	☑
k_{39}	38	$[6, 4, 1, 1, 4, 2, 4, 6, 4, 3, 5]$	11	☑	☑
k_{40}	38	$[8, 4, 3, 2, 2, 1, 1, 2, 5, 4, 6]$	11	☑	☑
k_{41}	38	$[6, 4, 6, 2, 3, 5, 4, 2, 6]$	9	☑	☑
k_{42}	40	$[7, 5, 4, 1, 1, 4, 2, 5, 2, 3, 6]$	11	☐	☑
k_{43}	40	$[7, 5, 4, 1, 1, 4, 2, 5, 2, 3, 6]$	11	☐	☑
k_{44}	40	$[6, 4, 1, 1, 4, 2, 4, 6, 4, 3, 5]$	11	☐	☑

The coding language is C++, and tests are conducted using version 22.1.1 of the IBM ILOG CPLEX solver. All experiments were carried out on a 3.20GHz Intel(R) Core(TM) i7-700 processor, with 64GiB System memory, running on Oracle Linux Server release 7.7. We use real-time to measure runtime.

5.2 Computational results

In this section, we compare between direct billing and standard billing. Then, we check the impact of price fluctuations. After that, we evaluate the effect of capacities before discussing minimal configurations.

5.2.1 Comparison between direct billing and standard billing

We compare two popular billing scenarios: direct billing and standard billing. We recall that the direct billing scenario is billing the petrol station based on the number of kilometers (km) from the depot, while the standard billing scenario is billing the petrol station based on the number of kilometers (km) from the previous petrol stop (e.g., another petrol station served just before).

Table 3 compares the number of visits (#Vis), the number of routes (#Ro), the number of tank trucks (#Tr), and the tank trucks free space percentage (F) for the two scenarios. Furthermore, we report the transport cost (local currency) for the direct billing and the percentage of savings in transportation costs (R) under the standard billing compared to the direct billing. Direct billing requires more visits than standard billing, uses more routes and tank trucks on average, and maintains a higher free space percentage. This is because each tank truck serves a single petrol station more frequently under this scenario. Compared to direct billing, standard billing ensures around 3.7% and 1.5% reduction in transportation costs for the first class ($|\mathcal{N}| = 47$) and second class ($|\mathcal{N}| = 77$), respectively. Under the standard billing scenario, the PC charges and serves each petrol station often separately.

Table 3: Direct billing vs Standard billing

			Direct Billing					Standard Billing				
$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{W}\rvert$	Inst.	#Vis	#Ro	#Tr	F(%)	Transport Cost	#Vis	#Ro	#Tr	F(%)	R(%)
47	4	**P4-C47-D1-S1**	95	55	44	8.48	21339956	94	55	42	8.06	1.36
47	4	**P4-C47-D4-S1**	88	54	41	7.49	18009887	87	51	40	6.84	2.52
47	4	**P4-C47-D1-S2**	95	55	44	8.48	21339956	94	55	42	8.06	1.36
47	4	**P4-C47-D4-S2**	88	54	41	7.23	18009887	87	51	40	6.84	4.38
47	8	**P8-C47-D1-S1**	197	140	96	9.27	43422166	195	139	93	8.21	7.76
47	8	**P8-C47-D4-S1**	197	135	92	7.35	43091682	193	132	91	7.02	3.42
47	8	**P8-C47-D1-S2**	207	139	96	10.01	43422166	205	138	93	8.95	7.76
47	8	**P8-C47-D4-S2**	194	135	91	7.89	42366889	195	132	87	7.07	4.25
47	12	**P12-C47-D1-S1**	297	212	143	7.72	64696920	293	206	142	6.53	1.48
47	12	**P12-C47-D4-S1**	302	208	148	6.09	65239896	299	204	147	5.78	1.45
47	12	**P12-C47-D1-S2**	299	211	149	6.43	62336922	295	205	148	6.28	5.85
47	12	**P12-C47-D4-S2**	306	203	138	6.31	65239896	303	201	143	6.02	2.14
		Avg	**197**	**133**	**94**	**7.73**	**42376352**	**195**	**131**	**92**	**7.14**	**3.65**
77	4	**P4-C77-D1-S1**	158	102	57	10.48	38356235	161	103	64	9.29	3.18
77	4	**P4-C77-D4-S1**	156	105	56	14.30	35173229	146	98	57	11.64	1.89
77	4	**P4-C77-D1-S2**	158	102	57	10.48	37796242	164	105	65	9.48	1.67
77	4	**P4-C77-D4-S2**	157	106	63	14.14	34523208	144	97	55	11.56	0.36
77	8	**P8-C77-D1-S1**	325	244	135	9.51	86125302	327	241	125	8.97	0.74
77	8	**P8-C77-D4-S1**	335	230	133	9.43	76229624	321	228	126	8.04	0.92
77	8	**P8-C77-D1-S2**	324	244	131	9.80	85905135	327	242	131	9.30	3.68
77	8	**P8-C77-D4-S2**	332	227	135	9.36	78249658	323	231	130	8.53	2.79
77	12	**P12-C77-D1-S1**	490	367	203	7.81	121436837	495	372	202	7.90	0.50
77	12	**P12-C77-D4-S1**	511	354	206	8.12	112664792	495	350	198	6.91	1.12
77	12	**P12-C77-D1-S2**	490	368	201	7.84	120206621	495	372	202	7.90	0.61
77	12	**P12-C77-D4-S2**	517	356	200	7.70	114175404	494	350	195	6.75	0.91
		Avg	**329**	**234**	**131**	**9.91**	**78403524**	**324**	**232**	**129**	**8.86**	**1.53**

5.2.2 Impact of price fluctuations

We evaluate the impact of price fluctuations. Figures 3 and 4 highlight how the quantities bought and stocked change when the prices/costs are constant or variable, respectively.

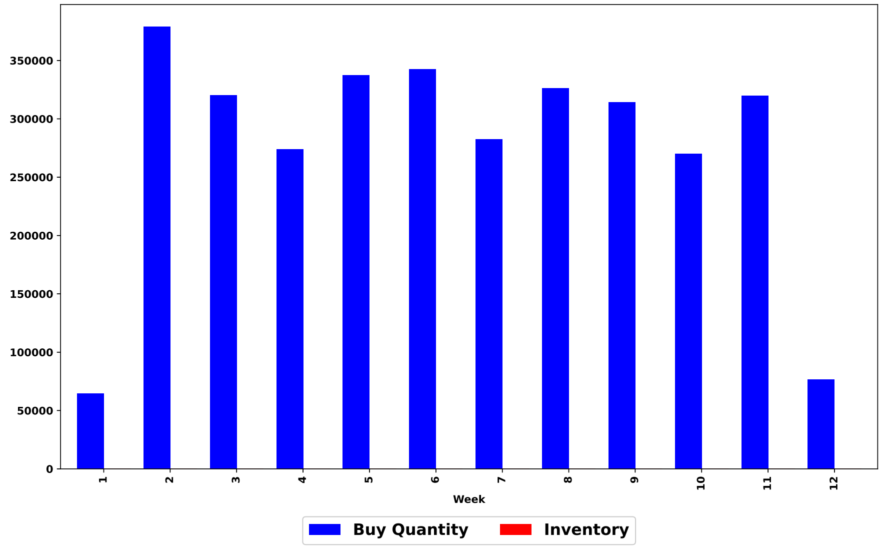

Figure 3: Constant Prices/Costs

When the prices/costs are constant, the PC purchases and delivers without keeping inventory at the depots. It is due to eliminating inventory costs. These costs add to the buying costs. We highlight this case in Figure 3. When the prices/costs are variable, buying and stocking occur. The PC will buy when the prices are low and stock when the prices are high. We highlight the second case in Figure 4. The PC buys two times, with the best prices, in two weeks during the horizon demand and stocks throughout the four-month horizon.

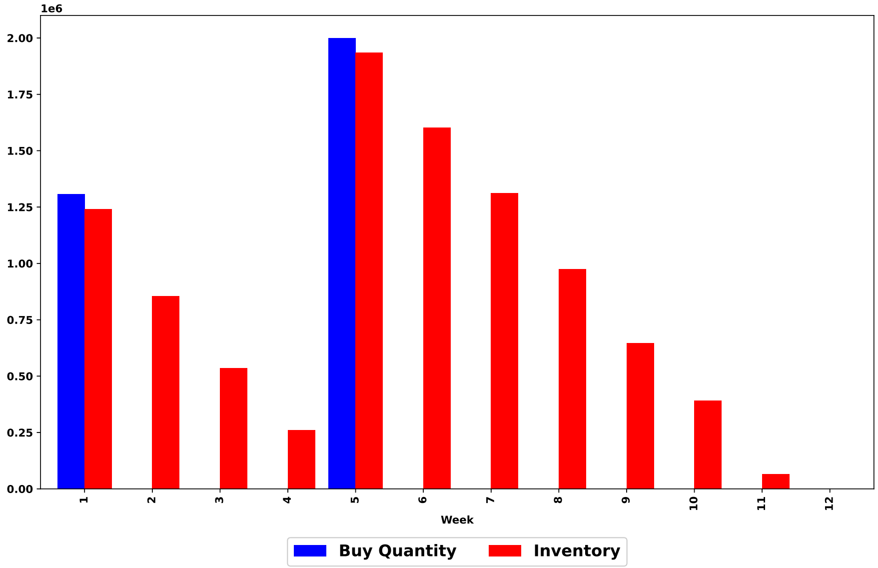

Figure 4: Variable Prices/Costs

5.2.3 Impact of capacities

We check the impact of adjusting petrol stations tank capacities. Table 4 shows the impact of increasing by 15% the tank capacity for frequently visited petrol stations under the standard billing scenario. We refer to this policy as the adjusted standard billing and compare it to the direct billing scenario. We report the number of visits, the number of routes, the number of tank trucks, the tank trucks free space percentage, and the saving percentage in transportation costs compared to direct billing.

Under the adjusted standard billing, the number of visits decreases, the number of routes and tank trucks decreases, and the free space percentage increases for the first class. For the second class, the number of visits decreases, the number of routes and tank trucks increases slightly, and the free space percentage decreases. The main difference is the number of visits, which decreases significantly for both classes. This is due to the increased tank capacity. When the tank capacity at the petrol stations increases, the transport company delivers higher quantities since the tank capacity is higher, thus reducing the number of visits to the petrol stations. Furthermore, compared to direct billing, the adjusted standard billing ensures a reduction of around 8.7% and 6.7% in total cost for the 47 and 77 classes, respectively.

We push the analysis further and conduct additional tests where we vary the petrol station tank capacity increase percentage. Figure 5 shows the total cost variation when the petrol stations' capacities are increased by 5%, 15%, 20%, and 25%. We observe that the total cost decreases strictly and significantly from 0% (basic) to 5% and from 5% to 15%. After that, while the decrease remains strict, it is no longer significant. It might suggest that increasing tank capacities by 15% is optimal.

Table 4: Increase Petrol Stations' Tanks for Frequently Visited Petrol Stations

			Direct Billing					Adjusted Standard Billing				
$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{W}\rvert$	Inst.	#Vis	#Ro	#Tr	F(%)	Total Cost	#Vis	#Ro	#Tr	F(%)	R(%)
47	4	**P4-C47-D1-S1**	95	55	44	8.48	528596571	90	56	44	8.78	10.08
47	4	**P4-C47-D4-S1**	88	54	41	7.49	492292241	86	54	41	9.27	4.32
47	4	**P4-C47-D1-S2**	95	55	44	8.48	536612958	90	56	44	8.78	7.77
47	4	**P4-C47-D4-S2**	88	54	41	7.23	515343240	86	54	41	9.27	6.21
47	8	**P8-C47-D1-S1**	197	140	96	9.27	1339204257	189	132	93	8.23	7.64
47	8	**P8-C47-D4-S1**	197	135	92	7.35	1260730999	189	127	89	8.37	9.79
47	8	**P8-C47-D1-S2**	207	139	96	10.01	1297574207	188	133	90	8.12	7.64
47	8	**P8-C47-D4-S2**	194	135	91	7.89	1280828575	188	128	91	8.17	8.81
47	12	**P12-C47-D1-S1**	297	212	143	7.72	2088190180	286	208	150	6.18	9.16
47	12	**P12-C47-D4-S1**	302	208	148	6.09	2037352514	294	201	135	6.37	11.19
47	12	**P12-C47-D1-S2**	299	211	149	6.43	2056481848	285	208	145	5.88	8.82
47	12	**P12-C47-D4-S2**	306	203	138	6.31	2038004458	293	198	140	6.35	13.66
		Avg	**197**	**133**	**94**	**7.73**	**1289267671**	**189**	**130**	**92**	**7.81**	**8.76**
77	4	**P4-C77-D1-S1**	158	102	57	10.48	881749096	155	106	61	10.16	6.65
77	4	**P4-C77-D4-S1**	156	105	56	14.30	855580632	140	96	54	10.98	11.97
77	4	**P4-C77-D1-S2**	158	102	57	10.48	924302869	155	106	62	10.00	5.09
77	4	**P4-C77-D4-S2**	157	106	63	14.14	832848269	140	96	54	10.98	10.02
77	8	**P8-C77-D1-S1**	325	244	135	9.51	2162855951	320	241	133	9.00	10.85
77	8	**P8-C77-D4-S1**	335	230	133	9.43	2068869342	315	238	127	10.15	2.92
77	8	**P8-C77-D1-S2**	324	244	131	9.80	2147779341	320	243	135	9.15	10.78
77	8	**P8-C77-D4-S2**	332	227	135	9.36	2066854673	315	237	125	9.85	0.44
77	12	**P12-C77-D1-S1**	490	367	203	7.81	3417761012	482	371	212	7.55	6.92
77	12	**P12-C77-D4-S1**	511	354	206	8.12	3323517723	481	360	205	7.64	2.96
77	12	**P12-C77-D1-S2**	490	368	201	7.84	3435103033	482	368	209	7.43	7.99
77	12	**P12-C77-D4-S2**	517	356	200	7.70	3315566988	481	358	203	7.54	4.76
		Avg	**329**	**234**	**131**	**9.91**	**2119399077**	**316**	**235**	**132**	**9.20**	**6.78**

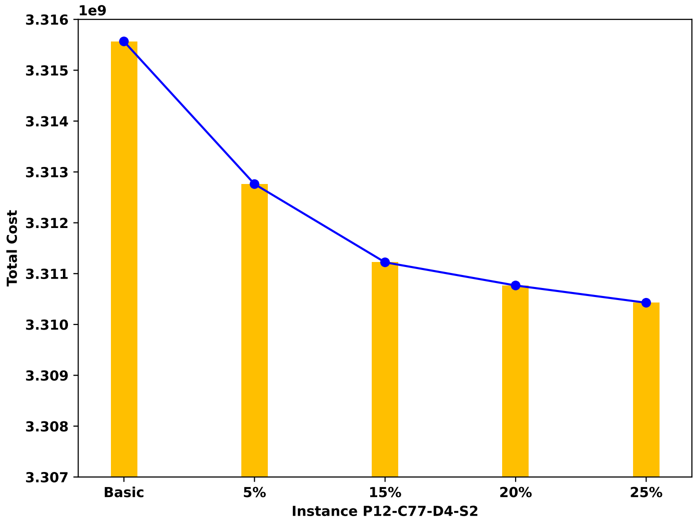

Figure 5: Total Cost when Changing Petrol Stations' Tank Capacities

5.2.4 Minimal tank truck configurations

We also check the tank trucks used for delivery among all available ones. Figure 6 shows the tank trucks used in the optimal solution (lowest cost).

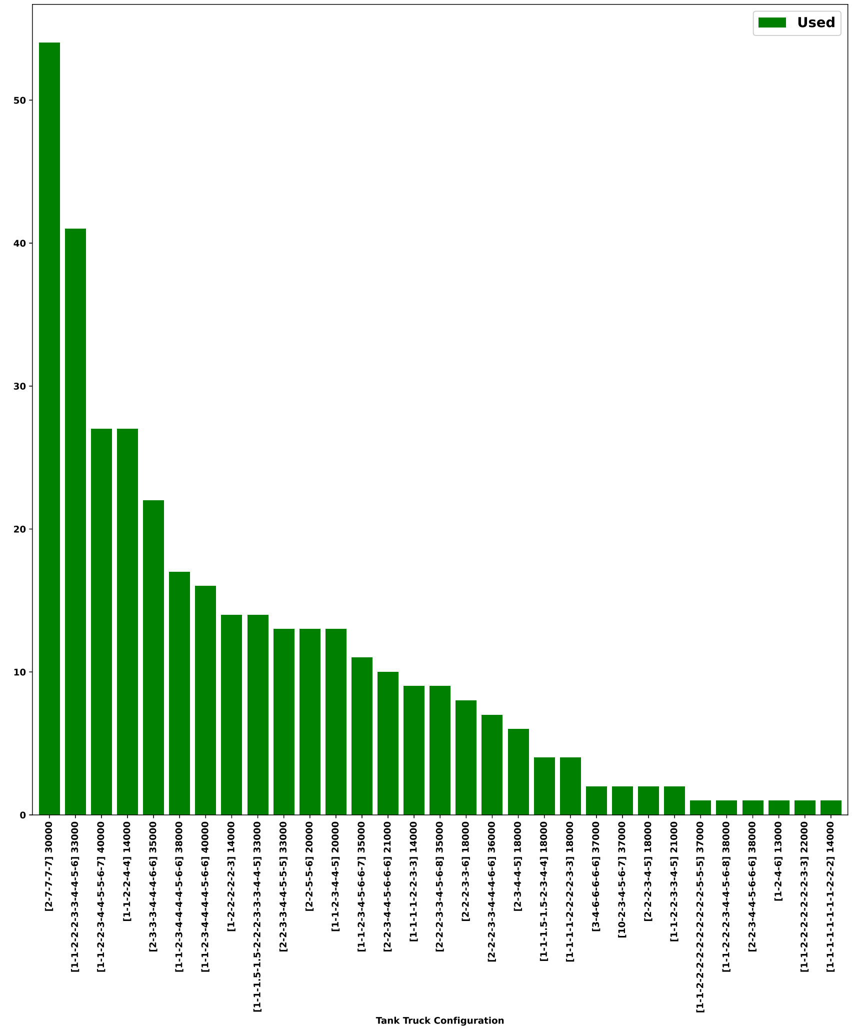

Figure 6: Minimal Number of Tank Truck Configurations for the Optimal Cost

Among all the available tank truck configurations, we observe that 14 tank truck configurations are used more than ten times, and five are used between 5 and 10 times. All remaining tank trucks (configurations) are used less than three times. When the number of tank trucks (configurations) is large, the optimization problem is very complex. If it is possible to reduce the tank truck fleet, the optimization becomes quick, and the management in the real world as well.

Given that not all tank truck configurations are used, we measure the impact of reducing the fleet size on the total cost. Table 5 shows the impact of reducing the fleet size and the impact of the total cost (Δ Total Cost).

Table 5: Impact of Reducing Fleet Size on Total Cost

			Standard Billing					
$\|\mathcal{N}\|$	$\|\mathcal{W}\|$	**Inst.**	**#Vis**	**#Ro**	**#Tr**	**F(%)**	**Fleet Size**	**ΔTotal Cost(%)**
77	12	P12-C77-D4-S2	494	350	195	6.75	44	Basic
77	12	P12-C77-D4-S2	501	362	189	8.82	37	+0.05
77	12	P12-C77-D4-S2	490	362	172	9.56	26	+0.09
77	12	P12-C77-D4-S2	486	367	178	9.48	23	+0.10
77	12	P12-C77-D4-S2	487	365	176	9.88	20	+0.13

We observe that when we decrease the fleet size, the total cost slightly increases. To enhance management capabilities with the PC, the latter might accept a slight cost increase to have a smaller fleet to manage. In our case, the PC might reduce its fleet size by up to 24 while accepting an increase of 0.13% in total cost.

5.3 Discussion

The results above highlight that optimization can help reduce costs significantly without any investment in the infrastructure. With the quick optimization capability, optimization becomes an efficient decision-making tool to check, control, simulate, and re-optimize various what-if scenarios, especially when the manual optimization is intractable, as in our case. This makes the supply chain more resilient to unexpected events and risks.

The optimization tool is interesting and practical. Furthermore, it is useful in various ways. Given price forecasts, petrol distribution companies can use the optimization tool to inform various strategic, tactical, and operational decisions. For strategic decisions, the tool can orient future design and investments by petroleum distribution companies. Examples include the petrol stations tank capacities and the preferred tank truck fleet. The optimal routes obtained from the optimization can inform potential depot locations and potential tank truck sizes with the number of compartments in each. For tactical and operational decisions, the tool can be used to inform buying and stocking decisions based on price fluctuations in the market. States can also use the optimization tool to inform sponsorship incentives, evaluate petroleum distribution policies, and check various strategies. For instance, direct billing increases costs significantly and leads distribution companies to achieve higher profits. States can then enforce a standard billing policy that will balance profits achieved by the petroleum distribution companies with the costs incurred by petrol stations.

This work with other OR success stories in Africa (e.g., Becker et al., 2022; Gibson et al., 2023; Er Raqabi et al., 2023a) highlights the benefits of using OR in Africa, especially that in several cases, organizations can achieve important gains without investing a single dollar.

6 Conclusion

This paper presents a complex variant of the petrol station replenishment problem. Using a two-step approach, we can tackle it to near-optimality in a few minutes. This paper adds to the practice of operations research in Africa, specifically in the context of transport. The quick optimization allows for testing several what-if scenarios, re-optimizing quickly, boosting decision-making capabilities, and comparing various policies. In particular, we compare two popular policies: direct and standard billing. We observe that the direct billing is more costly and less efficient than the standard billing. We also highlight the impact of various cost and capacity fluctuations on the optimal solution. The insights presented in this paper could incentivize African states to sponsor some policies over others. These insights can also be used to criticize and evaluate existing approaches in practice and invest more in infrastructure and analytics tools to boost operations management.

References

Al-Hinai, N. and Triki, C. (2020). A two-level evolutionary algorithm for solving the petrol station replenishment problem with periodicity constraints and service choice. Annals of Operations Research, 286(1–2):325–350.

Archetti, C., Jabali, O., and Speranza, M. G. (2015). Multi-period vehicle routing problem with due dates. Computers & Operations Research, 61:122–134.

Avella, P., Boccia, M., and Sforza, A. (2004). Solving a fuel delivery problem by heuristic and exact approaches. European Journal of Operational Research, 152(1):170–179.

Bani, A., El Hallaoui, I., Corréa, A. I., and Tahir, A. (2023). Solving a real-world multi-depot multi-period petrol replenishment problem with complex loading constraints. European Journal of Operational Research, 311(1):154–172.

Bani, A., Er Raqabi, E. M., El Hallaoui, I., and Corréa, A. I. (2024). Combining Benders decomposition and column generation for the petrol station replenishment problem with inventory management. Les Cahiers du GERAD, G–2024–18:1–31.

Becker, A., Benabderrazik, K., Bertsimas, D., Chtinna, N., El Majdoub, N., Mahboubi, E. M., Kitane, D. L., Kokkotos, S., Mourtzinou, G., and Rakhis, I. (2022). Toward global food security: Transforming ocp through analytics. INFORMS Journal on Applied Analytics, 52(1):90–107.

Benantar, A., Ouafi, R., and Boukachour, J. (2016). A petrol station replenishment problem: new variant and formulation. Logistics Research, 9(1):6.

Brown, G. G., Ellis, C. J., Graves, G. W., and Ronen, D. (1987). Real-time, wide area dispatch of mobil tank trucks. Interfaces, 17(1):107–120.

Brown, G. G. and Graves, G. W. (1981). Real-time dispatch of petroleum tank trucks. Management Science, 27(1):19–32.

Carotenuto, P., Giordani, S., Massari, S., and Vagaggini, F. (2018). A multi-depot periodic vehicle routing model for petrol station replenishment. In Advanced Concepts, Methodologies and Technologies for Transportation and Logistics, pages 421–437. Springer.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2014). Thirty years of inventory routing. Transportation Science, 48(1):1–19.

Coelho, L. C. and Laporte, G. (2015). Classification, models and exact algorithms for multi-compartment delivery problems. European Journal of Operational Research, 242(3):854–864.

Collett, K. A., Hirmer, S. A., Dalkmann, H., Crozier, C., Mulugetta, Y., and McCulloch, M. D. (2021). Can electric vehicles be good for sub-saharan africa? Energy Strategy Reviews, 38:100722.

Cornillier, F., Boctor, F., and Renaud, J. (2012). Heuristics for the multi-depot petrol station replenishment problem with time windows. European Journal of Operational Research, 220(2):361–369.

Cornillier, F., Boctor, F. F., Laporte, G., and Renaud, J. (2008a). An exact algorithm for the petrol station replenishment problem. Journal of the Operational Research Society, 59(5):607–615.

Cornillier, F., Boctor, F. F., Laporte, G., and Renaud, J. (2008b). A heuristic for the multi-period petrol station replenishment problem. European Journal of Operational Research, 191(2):295–305.

Cornillier, F., Laporte, G., Boctor, F. F., and Renaud, J. (2009). The petrol station replenishment problem with time windows. Computers & Operations Research, 36(3):919–935.

Derigs, U., Gottlieb, J., Kalkoff, J., Piesche, M., Rothlauf, F., and Vogel, U. (2011). Vehicle routing with compartments: applications, modelling and heuristics. OR spectrum, 33(4):885–914.

Er Raqabi, E. M., Beljadid, A., Bennouna, M. A., Bennouna, R., Boussaadi, L., El Hachemi, N., El Hallaoui, I., Fender, M., Jamali, A., Si Hammou, N., and Soumis, F. (2023a). Ocp optimizes its supply chain for africa. Les Cahiers du GERAD, G–2023–29.

Er Raqabi, E. M., El Hallaoui, I., and Soumis, F. (2023b). The Primal Benders Decomposition. Les Cahiers du GERAD, G–2023–27.

Er Raqabi, E. M. and Li, W. (2023). An electric vehicle transitioning framework for public fleet planning. Transportation Research Part D: Transport and Environment, 118:103732.

Evans, D. (2009). A review of underground fuel storage events and putting risk into perspective with other areas of the energy supply chain. Geological Society, London, Special Publications, 313(1):173–216.

Gibson, E., Deo, S., Jónasson, J. O., Kachule, M., and Palamountain, K. (2023). Redesigning sample transportation in malawi through improved data sharing and daily route optimization. Manufacturing & Service Operations Management, 25(4):1209–1226.

Hansen, P. and Mladenović, N. (2001). Variable neighborhood search: Principles and applications. European Journal of Operational Research, 130(3):449–467.

Kaabi, H. and Jabeur, K. (2015). Hybrid algorithm for solving the multi-compartment vehicle routing problem with time windows and profit. In 2015 12th International Conference on Informatics in Control, Automation and Robotics (ICINCO), volume 1, pages 324–329. IEEE.

Kazemi, Y. and Szmerekovsky, J. (2015). Modeling downstream petroleum supply chain: The importance of multi-mode transportation to strategic planning. Transportation Research Part E: Logistics and Transportation Review, 83:111–125.

Laporte, G. (1992). The vehicle routing problem: An overview of exact and approximate algorithms. European Journal of Operational Research, 59(3):345–358.

Le, T.-H., Le, A. T., and Le, H.-C. (2021). The historic oil price fluctuation during the covid-19 pandemic: What are the causes? Research in International Business and Finance, 58:101489.

Li, K., Chen, B., Sivakumar, A. I., and Wu, Y. (2014). An inventory–routing problem with the objective of travel time minimization. European Journal of Operational Research, 236(3):936–945.

Lima, C., Relvas, S., and Barbosa-Póvoa, A. P. F. (2016). Downstream oil supply chain management: A critical review and future directions. Computers & Chemical Engineering, 92:78–92.

Macedo, R., Alves, C., de Carvalho, J. V., Clautiaux, F., and Hanafi, S. (2011). Solving the vehicle routing problem with time windows and multiple routes exactly using a pseudo-polynomial model. European Journal of Operational Research, 214(3):536–545.

Malépart, V., Boctor, F., Renaud, J., and Labillois, S. (2003). Nouvelles approches pour l'approvisionnement des stations d'essence. Revue Française de Gestion Industrielle, 22(2):15–31.

Ng, W. L., Leung, S., Lam, J., and Pan, S. (2008). Petrol delivery tanker assignment and routing: a case study in hong kong. Journal of the Operational Research Society, 59(9):1191–1200.

Popović, D., Bjelić, N., and Radivojević, G. (2011). Simulation approach to analyse deterministic irp solution of the stochastic fuel delivery problem. Procedia-Social and Behavioral Sciences, 20:273–282.

Popović, D., Vidović, M., and Radivojević, G. (2012). Variable neighborhood search heuristic for the inventory routing problem in fuel delivery. Expert Systems with Applications, 39(18):13390–13398.

Rahmaniani, R., Crainic, T. G., Gendreau, M., and Rei, W. (2017). The benders decomposition algorithm: A literature review. European Journal of Operational Research, 259(3):801–817.

Triki, C. (2013). Solution methods for the periodic petrol station replenishment problem. The Journal of Engineering Research [TJER], 10(2):69–77.

Van der Bruggen, L., Gruson, R., and Salomon, M. (1995). Reconsidering the distribution structure of gasoline products for a large oil company. European Journal of Operational Research, 81(3):460–473.

Verme, P. and Araar, A. (2017). The quest for subsidy reforms in the middle east and north africa region. Washington DC: World Bank.

Vidović, M., Popović, D., and Ratković, B. (2014). Mixed integer and heuristics model for the inventory routing problem in fuel delivery. International Journal of Production Economics, 147:593–604.

Wang, L., Kinable, J., and Van Woensel, T. (2020). The fuel replenishment problem: A split-delivery multi-compartment vehicle routing problem with multiple trips. Computers & Operations Research, 118:104904.