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Home health care delivery with consistency consideration in a stochastic environment

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Abstract : We study the integration of multi-period assignment, routing, and scheduling of care workers for home health care services. In such a context, it is important to ensure service consistency, where a designated care worker must visit each patient at a specific time and in a consistent manner based on an established route and schedule. The challenge in maintaining service consistency and quality lies in the fact that a care agency must determine consistent and efficient schedules of visits to patient locations for multiple care workers despite uncertainty in travel and service times. To this end, we extend the home health care routing and scheduling problem (HHCRSP) presented in the literature and introduce a stochastic optimization model to incorporate service level constraints under stochastic travel and service times. We propose the solution framework based on two representations: a discrete scenario set and an extreme value theory-based (EVT-based) approximation. To tackle instances of practical size, we employ branch-and-check (B&Ch), a variant of the logic-based Benders decomposition (LBBD) method, where the subproblem is efficiently solved using constraint programming (CP). The results show that the stochastic approaches, especially with the EVT-based approximation model, can efficiently handle practical benchmark instances while producing schedules with significantly higher service levels than the deterministic approach. We also demonstrate the effectiveness of scenario-based and EVT-based models under different types of uncertainty.

Keywords : Service consistency, home health care routing and scheduling, stochastic travel and service times, logic-based Benders decomposition, constraint programming

1 Introduction

Due to aging populations and governments' plans to decrease hospitalization costs (Lanzarone and Matta 2014, Restrepo, Rousseau, and Vallée 2020), home health care (HHC) has received significant attention during the past decade and has become a highly active area of research (Wang et al. 2022b, Xie et al. 2023). For example, in Canada, Deloitte (2021) estimates an increase in HHC needs by 615,479 cases by 2031, which corresponds to a 53% increase in demand since 2019. These include a broad range of health-related services, from basic ones, such as housekeeping, to professional ones, such as medical care at people's homes. In HHC, the quality of service plays an important role, and service consistency is necessary to ensure proper follow-ups and monitoring of patients (Freeman and Hughes 2010). Woodward et al. (2004) studied the importance of service consistency in HHC and its aspects by interviewing home care clients and their care workers. They state that consistency in care is supported by personnel consistency and consistent timing. Notwithstanding the preeminent role of consistency in HHC, very few works in HHC scheduling consider it in their studies (Yang, Ni, and Yang 2021).

Personnel consistency means assigning the same care worker or a limited number of different care workers to each client during their presence in the HHC system. By doing so, the clients and their assigned care workers develop trust over time, increasing the quality of their communication. In addition, consistent timing means that service delivery to clients occurs at regular times at each visit. Receiving the service regularly is essential to some clients, especially those who need time-sensitive services, such as insulin injections, or, generally, ones with time-specific routines.

Many operation research (OR) problems are defined around HHC planning issues in each strategic, tactical, and operational decision. One of the central and challenging problems at the operational level is the home health care routing and scheduling problem (HHCRSP) (Cissé et al. 2017). In the HHCRSP, a health institution must decide the routes and times of providing health services at clients' homes for their care workers. This problem is a practical extension of the vehicle routing problem (VRP) (Di Mascolo, Espinouse, and Hajri 2017). In the planning phase of HHCRSP, due to its complexity, decision-makers often assume deterministic values for travel and service times (TST) even when they face uncertainty in TST in the execution phase due to traffic congestion, weather conditions and unpredictable treatment times (Shi, Boudouh, and Grunder 2019). Ignoring the uncertainty of TST could result in inefficient (or even infeasible) schedules and service delays. In HHCRSP, with visit time consistency, a service delay could result in their dissatisfaction or even be risky for patients' health, whose timely treatments are necessary. For example, patients who require medication provision at specific times could be severely hurt by late service (Fikar and Hirsch 2017).

This is illustrated in Figure 1. It shows the routes and schedules for a care worker generated by deterministic and stochastic models. The deterministic model generates the same route and schedules even when there is a higher uncertainty level in travel times between nodes. However, the stochastic model adapts the routes and schedules to uncertainty levels, providing more robust solutions. The average probability of delay for solution (1) when the uncertainty is the same as case (2) is 23.4%, whereas when the uncertainty is the same as case (3), the average probability of delay is 33%. In contrast, the stochastic solution can keep its performance in both cases (2) and (3) below 1%. To our best knowledge, despite the importance of incorporating TST uncertainty in HHCRSP with service consistency, such research topics have not received significant attention.

This paper considers an HHCRSP with personnel and visit time consistency under travel and service time uncertainties. The problem is similar to the one presented by Heching, Hooker, and Kimura (2019), but we extend the scope of the problem to incorporate service level constraints under stochastic travel and service times. The main goal of this research is to investigate the interactions between uncertainty in TST and consistency in the context of Home Health Care. Specifically, the contributions are as follows: (1) We propose a chance-constrained stochastic programming approach to model TST uncertainty for an HHCRSP with hard personnel and visit time consistency. In other

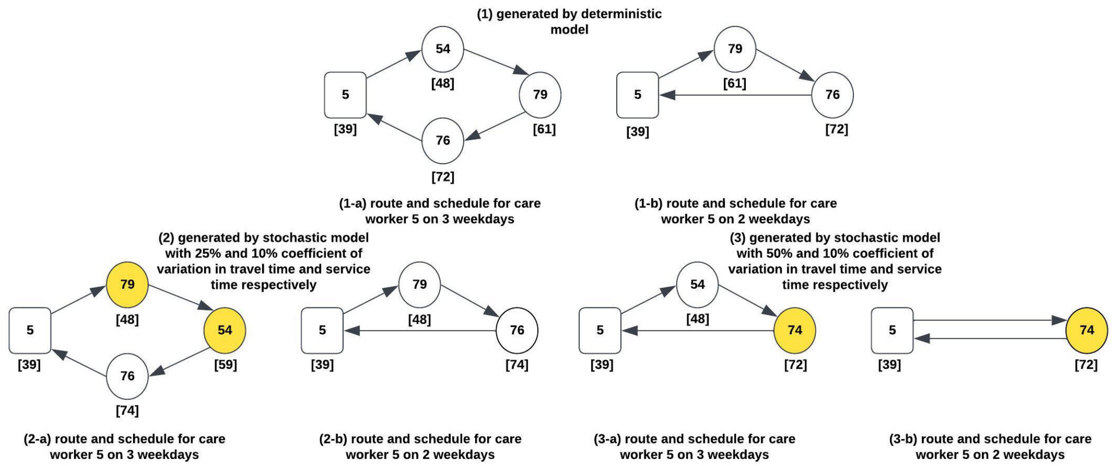

Figure 1: Robustness of routes and schedules generated by stochastic model The square shape indicates the care worker. The circle represents a patient. Additionally, the numbers in the circles indicate node numbers, whereas the numbers in the brackets indicate the scheduled visit times.

words, routes to serve patients must respect the consistency requirements in personnel and visit time with the probability of service delay not exceeding the guaranteed service level. (2) We present two mathematical formulations to deal with the complex uncertainty of arrival time arising from dependent sequences of each route (due to the fact that the service cannot begin before the starting time of the designated time window): one based on a discrete scenario set and another based on a distributional approximation via extreme value theory (EVT). (3) To efficiently solve these formulations, we propose a tailored Branch-and-Check (B&Ch) framework, a variant of the logic-based Benders decomposition (LBBD), in which the subproblem is formulated as a constraint programming (CP) model that allows us to handle the nonlinear EVT-based model. (4) We numerically validate the performances of the two modeling frameworks and the benefits of considering uncertainty at the planning level by comparing the service quality and optimal value between the deterministic model based on the work of Heching, Hooker, and Kimura (2019) and stochastic ones. Our experiments demonstrate that the solutions from the stochastic models can guarantee high service quality, while the stochastic framework does not require a significantly greater computational effort than the deterministic one. The EVT-based model can generally produce high-quality solutions more efficiently than the scenario-based one, which suffers from scalability issues when the number of scenarios is very large.

The remainder of this paper is organized as follows. Section 2 discusses the recent literature on this topic. Section 3 presents mathematical formulations and solution approaches to solve the stochastic HHCRSP. Section 4 presents the experimental results. Finally, a summary and conclusion of our work are provided in Section 5.

2 Literature review

The HHCRSP models differ from each other based on the features considered in them. Although there is no standard version of the HHCRSP, it is closely related to the VRP (Grenouilleau, Lahrichi, and Rousseau 2020, Di Mascolo, Espinouse, and Hajri 2017). The service consistency feature in the HHCRSP makes the model similar to a consistent vehicle routing problem (ConVRP) (Kovacs et al. 2014). Nevertheless, in the HHCRSP (Heching, Hooker, and Kimura 2019), one must consider specific cases where skill-requirement constraints, health care personnel, and visit time consistency should be guaranteed for each patient in the rolling schedule, which is updated periodically. In this section, we survey papers dealing with consistent VRP and HHCRSP with consistency considerations. Table 1 summarizes relevant papers reviewed in this section. For the papers dealing with uncertain parameters, we indicate the stochastic parameters in the *Stochastic Information* column where "-" indicates that the paper considers a deterministic solution approach with no uncertain parameters.

Table 1: Summary of related literature on ConVRP and HHCRSP

Author(s)	Consistency		Solution Type		Stochastic Information
	Time	Personnel	Exact	Heuristic	
Groër, Golden, and Wasil (2009)	Hard	Hard	-	RTR[1]	-
Sungur et al. (2010)	-	Soft	-	TS[2]	Customer and Service Time
Tarantilis, Stavropoulou, and Repoussis (2012)	Hard	Hard	-	TS	-
Kovacs, Parragh, and Hartl (2014)	Hard	Hard	-	ALNS[3]	-
Kovacs et al. (2015)	Soft	Hard	-	LNS[4]	-
Spliet and Gabor (2015)	Hard	-	BPC[5]	-	Demand
Kovacs, Parragh, and Hartl (2015)	Soft	Soft	ϵ[6]	MDLNS[7]	-
Jabali et al. (2015)	Soft	-	-	TS/LP[8]	Travel Times
Subramanyam and Gounaris (2018)	Hard	-	Decomp[9]	-	-
Subramanyam, Wang, and Gounaris (2018)	Hard	-	Decomp	-	Demand and Travel Times
Goeke, Roberti, and Schneider (2019)	Hard	Hard	CG[10]	LNS	-
Heching, Hooker, and Kimura (2019)	Hard	Hard	LBBD/B&Ch	-	-
Yang, Ni, and Yang (2021)	Soft	Soft	-	Artificial bee colony	Travel and Service Times
Wang et al. (2022a)	Hard	Hard	CG[10]	-	Demand
Cappanera and Scutellà (2022)	Hard	Hard	-	Pattern-based	Demand
This paper	Hard	Hard	B&Ch	-	Travel and Service Times

[1] Record-To-Record, [2] Tabu Search, [3]Adaptive Large Neighborhood Search, [4] Large Neighborhood Search, [5] Branch-Price-and-Cut,[6] ϵ-constraint,[7] Multi-Directional LNS,[8] Linear Programming, [9] Decomposition,[10] Column Generation

In routing problems in which customers require services periodically over time, consistency in visit time can play a significant role in improving service quality. Groër, Golden, and Wasil (2009) introduced a new variant of the periodic VRP in which service consistency is incorporated into the problem and called it ConVRP. In this multiday VRP, in addition to the constraints on vehicle capacity and route length, there are additional consistency requirements, i.e., each customer must be served by the same driver (personnel consistency) at approximately the same time on each day (time consistency) when the service takes place. Several studies, e.g., Groër, Golden, and Wasil (2009), Tarantilis, Stavropoulou, and Repoussis (2012), and Kovacs, Parragh, and Hartl (2014), proposed heuristics to determine a template-based solution with consistency considerations in which a set of template routes (also called priority routes) including only customers requiring service on multiple days (frequent customers) is generated. Then, for each day, the daily routes are constructed by removing the customers who do not ask for service on that day and by inserting customers who require service on only that day (non-frequent customers). A generalized consistent vehicle routing problem, GenConVRP, was introduced by Kovacs et al. (2015) in which the maximum difference in arrival times is penalized in the objective function instead of it being considered a hard constraint. To solve their problem, they proposed a large neighborhood search (LNS) heuristic, which outperforms the template-based heuristics of Groër, Golden, and Wasil (2009), Tarantilis, Stavropoulou, and Repoussis (2012), and Kovacs, Parragh, and Hartl (2014) in terms of both travel cost and time consistency. A multi-objective ConVRP that combines consistency and cost objectives was considered by Kovacs, Parragh, and Hartl (2015), and they proposed a multi-directional large neighborhood search heuristic to solve it. Goeke, Roberti, and Schneider (2019) proposed an efficient exact solution approach based on a column-and-cut generation (CCG) procedure to solve the ConVRP with driver and time consistencies. They also developed an LNS heuristic to tackle large instances. Recently, Wang et al. (2022a) considered the ConVRP with route consistency and proposed an exact solution approach based on set partitioning-based models and CCG techniques to solve it.

Most of the studies in the ConVRP consider the deterministic case when all the parameters are assumed to be perfectly known. In other words, information related to customer demands, customer presence, service times, and travel times is often considered fully available prior to optimization. There are, however, a few papers that explore the approaches under stochastic information. For example, Sungur et al. (2010) studied a variant of VRP with soft time windows (VRPTW), called the courier delivery problem (CDP), in which customers' presence and their service times are uncertain. They modeled uncertainties in service times and the presence of customers through robust optimization and scenario-based stochastic programming approaches. Nevertheless, they did not consider service consistency explicitly, but such a consistency is encouraged by maximizing the similarity of routes across multiple scenarios. A tabu search heuristic was employed to solve this problem.

A similar VRP variant to ConVRP is the time window assignment vehicle routing problem (TWAVRP) (Spliet and Gabor 2015). In this problem, the authors assumed that demands are not known when the decision-maker assigns time windows (with fixed-size width) to their customers. Once the demand is realized, they decide on a vehicle routing schedule to satisfy the demand of each customer at the assigned time window. In the TWAVRP, however, personnel consistency is not considered, and only time consistency is imposed. Later on, Subramanyam, Wang, and Gounaris (2018) extended the uncertainty vector to incorporate demand and travel time uncertainties through a set of discrete scenarios. They employed a two-stage stochastic programming framework in which the first-stage decisions comprise time window assignments, and the second-stage decisions comprise routes to serve customers based on the assigned time windows. They adapted the decomposition algorithm of Subramanyam and Gounaris (2018) to solve their TWAVRP in an exact manner. Jabali et al. (2015) studied a self-imposed TWAVRP in which the provider decides the time window for customers. In this case, time windows are considered endogenous to the routing problem. They called this problem a vehicle routing problem with self-imposed time windows. Their model assumes that travel times are uncertain when time windows are assigned. They applied a buffer allocation model in order to protect scheduled time windows against travel time uncertainty. To solve this problem, they applied a tabu search heuristic to determine routing decisions. Afterward, they used a linear programming model to optimize the schedules for customers based on the routing decisions previously determined in the first step.

Consistency is one of the main features considered in HHCRSP models. Despite its practical importance, it has not been extensively studied in the literature, and it only started gaining attention from 2018 (Di Mascolo, Martinez, and Espinouse 2021). Personnel consistency, which is often referred to as continuity of care (Borsani et al. 2006), has been studied in the literature more than time consistency (Cappanera and Scutellà 2022). Di Mascolo, Martinez, and Espinouse (2021) emphasize the lack of approaches concerning consistency in time. In other words, there are only a few papers that study HHCRSP models with both personnel and time consistency simultaneously.

Heching, Hooker, and Kimura (2019) studied a home health care delivery problem and they proposed an exact optimization method, called LBBD, to maximize the number of new patients accepted while respecting service consistency for existing patients. The problem they addressed concerned updating a home hospice care company's weekly schedule in response to patient population changes to predict their staffing needs. Nevertheless, Heching, Hooker, and Kimura (2019) assumed deterministic TST. However, the routes and schedules determined by the deterministic model could lead to significant delays in services and treatments in the presence of travel and service times uncertainty. In our work, we aim to extend this context to consider stochastic TST and demonstrate the value of the stochastic optimization approach, which can improve the service level for patients through reduced risks of service delays.

To the best of our knowledge, no literature on HHCRSP tackles the challenge of stochasticity in TST. Cappanera and Scutellà (2022) addressed both time consistency and personnel consistency for home care services optimization when demand uncertainty is present. They solved this problem through a pattern-based heuristic framework. This framework determines the scheduling decisions by using the concept of patterns, which are possible templates used for scheduling multiple visits over a planning horizon. Yang, Ni, and Yang (2021) considered a multi-objective consistent home health care routing and scheduling problem with uncertain travel and service times in which consistencies were enforced through soft constraints. Three objectives were considered in this work: minimizing routing cost, increasing service consistency, and improving workload balance. The authors solved this problem heuristically via a multi-objective artificial bee colony algorithm. In addition to the fact that our focus is on the exact algorithmic framework, one distinct aspect of the problem considered in our work versus the problem in Yang, Ni, and Yang (2021) is that we incorporate complex uncertainty of arrival times that arise from hard time window restrictions in our stochastic framework.

3 Consistent HHCRSP formulation and solution framework

The problem considered in this work is an extension of the deterministic problem presented by Heching, Hooker, and Kimura (2019), which was derived from a real-world HHCRSP. In a given week comprising five weekdays, a set of patients P must be scheduled throughout the week. Since a rolling schedule is executed every week, there exists a set of *existing* patients who are already scheduled for visits. In addition, the planner can accept and add *new* patients to the schedule. To respect the service consistency for existing patients, their assigned care workers and scheduled service days must be kept unchanged. In contrast, the exact visit time to each of the existing patients can be modified as long as the modified visit remains consistent. For newly admitted patients, the planner must assign a responsible care worker, visit days, and visit times to them. The number of required visit days for all patients is in the set $\{1, 2, 3, 4, 5\}$. In case a patient asks for multiple visits (between 2 to 5 days) in a week, they must be served at the same time each day (time consistency) and by the same care worker (personnel consistency). For patients with less frequent visits, such as two or three times a week, there could be some restrictions on the visit days assigned to them. For example, for a patient with two weekly visits, close visits like Mondays and Tuesdays must be avoided through a separation constraint. We incorporate such constraints into our mathematical formulation, and we elaborate on them in Section 4. The time constraint is hard, i.e., the service cannot start earlier than the visit time, and the care worker must wait in case of early arrival (Bertels and Fahle 2006). Respecting given time windows for care workers and patients and the maximum weekly working time for care workers are two other important constraints in the model that influence the number of accepted patients in the objective function. The ultimate objective of this problem is to maximize the number of newly admitted patients while respecting consistency and service level constraints.

Table 2: Description of the notation used in the consistent HHCRSP model

Set	Description
P	Set of all patients
P_{new}	Set of new patients
P_{existing}	Set of existing patients
A	Set of care workers
D	Set of days
Q_p	Set of required qualifications to serve patient p
Q_a	Set of qualifications of care worker a
D_p	Set of pre-assigned days to patient p
K_p	Set of vectors of a feasible combination of visit days to patient p
Parameter	**Description**
w_p	Number of visits required for patient p
$[r_p, d_p]$	Time window of patient p
$\bar{s}_p$	Nominal duration per visit for patient p
a_p	Pre-assigned care worker to patient p
$[r_a, d_a]$	Time window of care worker a
l_a	Location of care worker a
U_a	Maximum work time over the week of care worker a
$\bar{t}_{i,j}$	Nominal travel time between location i and j
L	Maximum allowable delay
α	Minimum acceptable service level
Pr	Probability function

In the following section, a mathematical formulation for this problem is presented. The descriptions of the sets and parameters are provided in Table 2. The formulation is described by a master problem (MP) that comprises patient acceptance, care worker assignment, and visit days assignment decisions and a set of subproblems (SPs) in which each SP comprises visit time and routing decisions for each care worker.

3.1 Master problem (MP)

In the master problem, three sets of decision variables are defined: binary variable δ_p to determine whether patient p is accepted, binary variable $x_{a,p}$ to determine whether patient p is assigned to the care worker a, and binary variable $y_{a,p,d}$ to determine whether patient p is visited by care worker a on the day d. Additionally, $\boldsymbol{y}$ is a vector of variables $y_{a,p,d}$ that is described by a feasible set K_p. The master problem can be stated as follows:

$$MP : \max \sum_{p \in P} \delta_p \tag{1}$$

$$\text{s.t.} \quad \sum_{a \in A | Q_p \subseteq Q_a} x_{a,p} = \delta_p \qquad \forall p \in P, \tag{2}$$

$$y_{a,p,d} \leq x_{a,p} \qquad \forall a \in A, \forall p \in P, \forall d \in D, \tag{3}$$

$$\sum_{a \in A} \sum_{d \in D} y_{a,p,d} = w_p \delta_p \qquad \forall p \in P, \tag{4}$$

$$\delta_p = 1 \qquad \forall p \in P_{\text{existing}}, \tag{5}$$

$$x_{a,p} = 1 \qquad \forall p \in P_{\text{existing}}, a = a_p, \tag{6}$$

$$y_{a,p,d} = 1 \qquad \forall p \in P_{\text{existing}}, a = a_p, \forall d \in D_p, \tag{7}$$

$$\boldsymbol{y} \in K_p \qquad \forall p \in P, \tag{8}$$

$$\sum_{d} \sum_{p \in P_{SP_{a,d}}} (1 - y_{a,p,d}) \geq 1 \qquad \forall a \in A, \tag{9}$$

$$\delta_p, x_{a,p}, y_{a,p,d} \in \{0, 1\} \qquad \forall a \in A, \forall p \in P, \forall d \in D. \tag{10}$$

The objective (1) maximizes the number of accepted patients. Constraints (2) ensure that, if a new patient is accepted, one dedicated care worker will be assigned to that patient. Constraints (3) enforce that patients are only visited by care workers assigned to them. Constraints (4) ensure that the number of visits required by each accepted patient is satisfied. Constraints (5), (6), and (7) impose the inclusion, care worker assignment, and visit days assignment, respectively, for existing patients based on existing consistency requirements. Constraints (8) enforce that visits to patients must respect the feasible visit days described by set K_p. Note that this constraint is consistent with presented by Heching, Hooker, and Kimura (2019), and we explain in Section 4 how it can be described using a set of linear inequalities. Constraints (9) are no-good cuts that eliminate infeasible routing and scheduling decisions. These constraints will be generated by solving the SPs iteratively in the B&Ch procedure. Here, $P_{SP_{a,d}}$ includes the set of assigned new patients to care worker a on day d that makes their schedule infeasible.

3.2 Subproblem (SP)

The subproblem generates a feasible schedule of visits that is compatible with time windows, maximum work time, and service level constraints for each care worker based on the assignment decisions made in the MP. The SP is modeled as a constraint satisfaction problem (CSP) with (non-linear) service level constraints. We use constraint programming techniques to solve the SP, as CP algorithms can tackle such a complex feasibility problem efficiently.

Given a solution vector $\boldsymbol{y}$ determined by the MP, we define a set of patients assigned to a care worker a on the day d as $\bar{P}_{a,d}$. The cardinality of $\bar{P}_{a,d}$ (defined as $n_{a,d}$) equals the number of patients that care worker a will visit on day d. Integer variable $\pi_{d,v}$ denotes patient v^{th} who is visited on day d.

For each patient p the variables a_p, v_p, and ss_p represent the arrival time, assigned visit time, and actual starting time of the service, respectively. If the SP determines that there is no feasible solution, we add no-good cuts (9) to the master problem to eliminate the current solution from the feasible space of the MP. Note that the bar signs above parameters $s_{\pi_{d,v}}$ and $t_{\pi_{d,v},\pi_{d,v+1}}$ indicate the expected values of these parameters. The SP for care worker a can be defined as follows:

$$SP_a : \max \ 0 \tag{11}$$

$$\text{s.t. } \bar{P}_{a,d} = \{p | \, \bar{y}_{a,p,d} = 1\} \qquad n_{a,d} = |\bar{P}_{a,d}| \qquad \forall d \in D, \tag{12}$$

$$\pi_{d,1} = l_a, \quad \pi_{d,n_{a,d}+2} = l_a \qquad \forall d \in D, \tag{13}$$

$$\text{all-different}\{\pi_{d,v} | v = 1, ..., n_{a,d} + 2\} \qquad \forall d \in D, \tag{14}$$

$$r_p \leq v_p \leq d_p - \bar{s}_p \qquad \forall p \in \bar{P}_{a,d}, \tag{15}$$

$$v_{\pi_{d,v}} + \bar{s}_{\pi_{d,v}} + \bar{t}_{\pi_{d,v},\pi_{d,v+1}} \leq v_{\pi_{d,v+1}} \qquad \forall d \in D, v = 1, ..., n_{a,d} + 1, \tag{16}$$

$$\sum_{d \in D} (v_{\pi_{d,n_{a,d}-1}} + \bar{s}_{\pi_{d,n_{a,d}-1}} - v_{\pi_{d,2}}) \leq U_a, \tag{17}$$

$$ss_p \geq v_p \qquad \forall p \in \bar{P}_{a,d}, \tag{18}$$

$$Pr\{a_p \leq v_p + L\} \geq \alpha \qquad \forall p \in \bar{P}_{a,d}, \tag{19}$$

$$a_p, v_p \in \mathbf{R}, \ \pi_{d,v} \in \bar{P}_{a,d} \cup l_a \qquad \forall p \in \bar{P}_{a,d}, \forall d \in D, v = 1, ..., n_{a,d} + 2. \tag{20}$$

The objective (11) is zero because we only search for a feasible solution. Constraints (13) state that the care worker always starts from their corresponding node of origin and returns to this node at the end of the day. Constraints (14) are *global constraints* in CSP which indicate that the value of each $\pi_{d,v}$ variable must be distinct from other values in the variable set (Heching, Hooker, and Kimura 2019). We also adopt constraints (15), (16) and (17) which are used by (Heching, Hooker, and Kimura 2019) to impose that the expected travel and service time constraints must be satisfied. This is to ensure that the model will satisfy the same TST constraints as the deterministic model. More specifically, constraints (15) impose that the assigned visit time must respect the time window of the patient. Expected travel and service times constraints are considered in constraints (16). Constraint (17) limits the maximum expected work time of the care worker. Constraints (18) ensure that the service does not happen sooner than the assigned visit time. Service level constraints are defined in constraints (19) and will be described in the subsequent sections. Finally, constraints (20) define the variables' domains.

Note that, constraints (19) take the form of probabilistic chance constraints, which limit the risk of delays at an acceptable level based on the decision maker (Gendreau, Jabali, and Rei 2014). More specifically, constraints (19) guarantee that, for each patient, care worker a will arrive on time with a minimum probability α. Therefore, $(1 - \alpha)$ is the maximum level of risk of arriving late that the decision-maker can accept. The parameter L represents an acceptable buffer of delay. In other words, an actual arrival after $v_p + L$ is considered a delay. Note that this term is used to provide additional flexibility, and L can indeed be set to 0 when the arrival must be strictly within v_p. To respect time consistency, care workers can not start their service sooner than the assigned visit time, as patients may not be available before their scheduled visit time. We propose to model the chance constraint using a scenario-based model and an EVT-based model, which are described in the next subsections.

3.2.1 Scenario-based model

In the scenario-based model, we assume the uncertainty can be represented using a discrete set of scenarios, N, which can be generated from travel and service time distributions through a Monte Carlo sampling approach. This approach allows us to deal with dependencies in the uncertain parameters due to the fact that the realized values of the uncertain parameters can be calculated independently for each scenario. Since optimization is agnostic to the scenario generation approach in this model, one can incorporate complex scenario generation methods that also consider dependencies and correlations among the uncertain parameters (i.e., travel and service times in our case). In the SP, we substitute constraints (19) with the sample average approximation (SAA) from the samples generated using this method.

To do so, we define a new variable for arrival time and the start-service time for patient p in sample n, that is, a_p^n and ss_p^n when $n \in N$. Given TST realization for each scenario, we can check whether or not the inequality $a_p^n \leq v_p + L$ is satisfied. To keep track of the number of on-time arrivals for patient p, the binary variable γ_p^n is set to one when we arrive on time; otherwise, it is set to zero. For patient p, we find the sample average of γ_p^n, which is equal to or greater than α, that is, $\frac{1}{|N|} \sum_{n \in N} \gamma_p^n \geq \alpha \quad \forall p \in \bar{P}_{a,d}$. In integer programming, to find the value of binary variables ($\boldsymbol{\gamma}$), we need to use big-M in the following way:

$$a_p^n \leq v_p + L + M(1 - \gamma_p^n)$$

However, thanks to CP logical constraints, we can readily find the value of binary variables ($\boldsymbol{\gamma}$) by applying inference algorithms (Hooker 2002) without needing to use big-M in the following way:

$$a_p^n \leq v_p + L \Rightarrow \gamma_p^n = 1,$$
$$a_p^n > v_p + L \Rightarrow \gamma_p^n = 0.$$

Despite the generalizability and flexibility of a scenario-based model, the model can suffer from scalability issues when the number of scenarios becomes large. To tackle large-scale instances more efficiently, we propose another modeling framework that does not directly rely on a discrete set of scenarios but rather on the (parametric) approximation of the underlying probabilistic distributions of TST in the next subsection.

3.2.2 EVT-based model

In VRP under stochastic travel and service times with hard time windows, service is not allowed to start prior to the beginning of the time window of each patient. As a consequence, the arriving care worker must wait until the beginning of the time window in case of early arrival. Unlike the case of soft time windows (e.g., see Hoogeboom et al. 2021), since the probability distributions of the arrival times at customers are truncated with the presence of waiting times, it is not directly possible to derive formulae for the convolution of the arrival time distributions (Gendreau, Jabali, and Rei 2014). Indeed, the distribution of a_p depends not only on travel times of the arcs traversed and service times of patients visited before arrival at the patient's location p but also on waiting times at prior patients. As a result, to find the mean and variance of a_p, applying the convolution of distributions such as normal or gamma to sum the means and variances of the service times of the nodes and travel times of the arcs traversed before p is not directly applicable. By leveraging extreme value theory, Ehmke, Campbell, and Urban (2015) proposed a method to approximate each customer's start-service time and arrival time distributions when waiting times must be considered under the hard time window constraints. Even though the derivation is based on the assumption that travel times are stochastic and follow normal distributions independently, the method is also applicable to instances with non-normal travel time distributions. This EVT-based method is then integrated as part of our stochastic optimization framework.

In our problem, each patient is assigned a consistent visit time v_p when patients are scheduled for the care worker. If the care worker arrives at patient p before v_p, they must wait until v_p to begin service. Otherwise, if they arrive later than v_p, they will begin the service immediately. Recall that a_p and ss_p represent the arrival time and the starting time of service for patient p, respectively. In the EVT-based model, the travel times between patients are assumed to be statistically independent. This is in line with the empirical results presented by Park and Rilett (1999) and Ehmke, Campbell, and Urban (2015), which demonstrated low correlations between arc travel times.

To ensure that the probability of on-time arrival time at each patient p by $v_p + L$ is greater than or equal to an acceptable level α, we need to derive a probabilistic distribution of a_p for each patient.

The distribution of a_p depends on the likelihood that the care worker had to wait at prior patients. As a consequence, the arrival time at patient p equals the sum of the random travel times of the arcs traversed before the arrival at p, the random waiting times, and the random service times of all the prior patients. An illustration of different arrival time distribution possibilities when the visit time window limits the start of service can be found in Figure 2. The first case at the top shows the case when the arrival time distribution is very likely within the time window of the patient, and, hence, there is a low probability that a care worker must wait at a patient's location. The second possibility is the case in which a care worker arrives too early and very likely needs to wait at a patient's location. Finally, the last subfigure shows the case when part of the arrival time distribution is prior to the starting time window and the remaining part is within the time window. Indeed, the arrival time variance at a patient p can be derived after determining the service time of patient $p-1$ and travel time between $p-1$ and p. Correspondingly, the arrival time distribution of the next patient a_{p+1} can be calculated from the sum of the random parameters ss_p, s_p, and $t_{p,p+1}$.

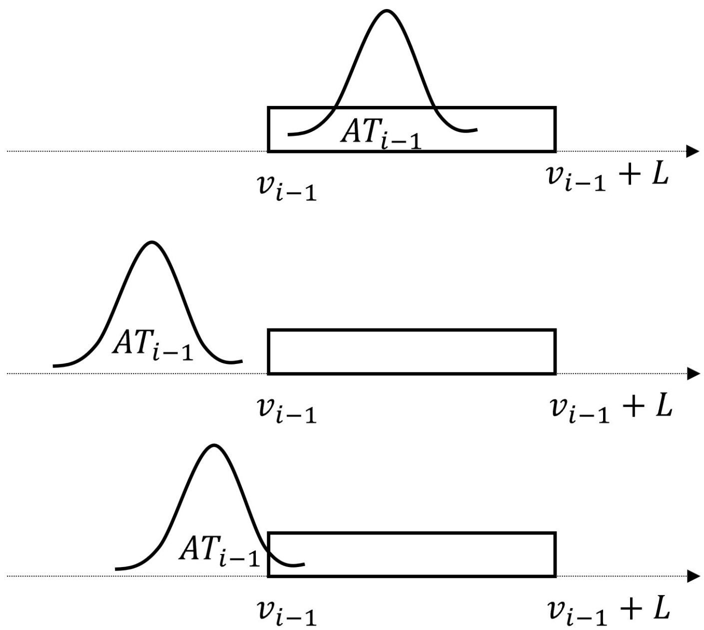

Figure 2: Arrival time distribution possibilities in the presence of visit time window (adapted from Ehmke, Campbell, and Urban (2015))

To demonstrate how the EVT-based approach based on the study of Ehmke, Campbell, and Urban (2015) works, we provide an example as follows. To serve the first patient, the care worker requires $t_{0,1}$ time to travel from their address to this first patient. The start-service time for patient p equals $\max\{t_{0,1}, v_1\}$. This max operator makes the calculations of the mean and variance of patients' start of service time complicated. However, we can find the mean and variance of the maximum of two normally distributed variables $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$, $X = \max\{X_1, X_2\}$ using extreme value theory. In our example, $X_1 = t_{0,1} \sim N(\mu_{0,1}, \sigma_{0,1}^2)$ and $X_2 = v_1$. Once the value of the assignment variable v_1 is determined, the mean and variance of the start-service time for patient 1 can be expressed as:

$$E(x_1) = \mu_{0,1}\Phi[(\mu_{0,1} - v_1)/\sigma_{0,1}] + v_1\Phi[(v_1 - \mu_{0,1})/\sigma_{0,1}] + \sigma_{0,1}\phi[(\mu_{0,1} - v_1)/\sigma_{0,1}] \tag{21}$$

$$V(x_1) = (\mu_{0,1}^2 + \sigma_{0,1}^2)\Phi[(\mu_{0,1} - v_1)/\sigma_{0,1}] + v_1^2\Phi[(v_1 - \mu_{0,1})/\sigma_{0,1}] + (\mu_{0,1} + v_1)\sigma_{0,1}\phi[(\mu_{0,1} - v_1)/\sigma_{0,1}] - [E(x_1)]^2 \tag{22}$$

where $\Phi[...]$ and $\phi[...]$ denote the cumulative distribution function and the probability density function of the standard normal distribution, respectively.

To calculate the arrival time distribution at the subsequent patient, patient 2, we first determine the distribution of the starting time for patient 1, $ss_1 \sim N(E(x_1), V(x_1))$, the service time for patient 1, $s_1 \sim N(\mu_{s_1}, \sigma^2_{s_1})$, and the travel time from patient 1 to patient 2, $t_{1,2} \sim N(\mu_{t_{1,2}}, \sigma^2_{t_{1,2}})$. Although ss_1 is originally not a normally distributed variable because it takes the maximum of two random variables, it is approximated using a normal distribution in order to derive an approximation for arrival time distribution at patient 2; $a_2 = ss_1 + s_1 + t_{1,2}$ (Ehmke, Campbell, and Urban 2015). More specifically, for any patient p, we can find the arrival time distribution based on the start-service time distribution of the previous patient, the normally distributed service time of the previous patient, and the normally distributed travel time from the previous patient to the patient p. Therefore, $\mu_{a_p} = \mu_{ss_{p-1}} + \mu_{s_{p-1}} + \mu_{t_{p-1,p}}$, and $\sigma^2_{a_p} = \sigma^2_{ss_{p-1}} + \sigma^2_{s_{p-1}} + \sigma^2_{t_{p-1,p}}$. The approximate mean of the start-service time distribution for patient p can be determined using the following equation:

$$E(X) = \mu_1\Phi[(\mu_1 - \mu_2)/\theta] + \mu_2\Phi[(\mu_2 - \mu_1)/\theta] + \theta\phi[(\mu_1 - \mu_2)/\theta] \tag{23}$$

where $\mu_1 = \mu_{a_p}$, $\mu_2 = v_p$, and $\theta = \sigma_{a_p}$.

Similarly, to approximate the variance of the start-service time distribution, we have:

$$\begin{aligned} V(X) = (\mu_1^2 + \sigma_1^2)\Phi[(\mu_1 - \mu_2)/\theta] + (\mu_2^2 + \sigma_2^2)\Phi[(\mu_2 - \mu_1)/\theta] \\ +(\mu_1 + \mu_2)\theta\phi[(\mu_1 - \mu_2)/\theta] - [E(X)]^2 \end{aligned} \tag{24}$$

where $\mu_1 = \mu_{a_p}$, $\sigma_1 = \sigma_{a_p}$, $\mu_2 = v_p$, $\sigma_2 = 0$, and $\theta = \sigma_{a_p}$.

Considering that v_p in equations (23) and (24) is a decision variable, these equations are nonlinear. However, CP handles complex problems involving nonlinear constraints, logical statements, or non-convex solution space (Wang, Meskens, and Duvivier 2015). To impose the chance constraint $P(a_p \leq v_p + L) \geq \alpha$, we impose the following constraint for each patient p in the SP:

$$\mu_{a_p} + z_\alpha \sigma_{a_p} \leq v_p + L \tag{25}$$

where z_α is the α-quantile of the standard normal distribution.

3.3 Branch-and-Check algorithm

We describe the B&Ch procedure, which is applied to solve both the scenario-based and EVT-based models. In our B&Ch framework, no-good cuts (9) are added to the master problem during the branch-and-bound (B&B) process. This approach is a variant of LBBD (Hooker and Ottosson 2003) in which the LBBD cuts are added in a branch-and-cut fashion during the B&B procedure of the master problem. It has been shown that B&Ch can be more efficient than standard LBBD when the master problem is harder to solve than the subproblem (Hooker 2000, Heching, Hooker, and Kimura 2019). In this approach, when a feasible solution candidate for the MP is found at a node of the B&B, the SP is called to validate the feasibility of this given solution candidate. If the subproblem finds the solution infeasible, one or more Benders cuts (9) are generated and added to the B&B tree. Otherwise, this solution is considered truly feasible and then the B&B continues. The same process is repeated at all nodes in the B&B tree when a feasible solution candidate for the MP is found until the B&B terminates. By using the B&Ch algorithm to solve our stochastic problem, we can tackle instances of practical size because the SP, which comprises non-linear chance constraints, can be efficiently solved for each care worker using CP. If the SP of the care worker based on assignment decisions determined by the MP is infeasible, a no-good cut of the form $\sum_d \sum_{p \in \bar{P}_{a,d} | p \in \{\text{new patient}\}} (1 - y_{a,p,d}) \geq 1$ is added as a global inequality to eliminate this infeasible assignment for the care worker from the MP. This cut hinders the master problem from assigning the same set of patients, which results in an infeasible schedule for the care worker a in the subsequent iterations in the B&B process.

3.4 Time window inequalities

Inequalities derived from a relaxation of the subproblem could be added to the master problem to enhance computational performance. Heching, Hooker, and Kimura (2019) demonstrates that using time window relaxation can reduce computational time significantly. This type of relaxation is based on forward and backward intervals. A forward interval for patient p begins with the start of their time window and ends with the termination of the care worker's time window. In addition, for patient p, a backward interval begins with the start of the care worker's time window and ends with the termination of the patient's time window. For each patient p assigned to the care worker a on day d, forward and backward augmented durations (p', p'') are calculated using following equations:

$$p'_{a,p,d} = \bar{s}_p + \min\{\min_{q \in P_{a,d}}\{\bar{t}_{p,q}\},\ \bar{t}_{p,l_a}\},$$

$$p''_{a,p,d} = \bar{s}_p + \min\{\bar{t}_{l_a,p},\ \min_{q \in P_{a,d}}\{\bar{t}_{q,p}\}\}$$

where $P_{a,d}$ is the set of patients that are already assigned to care worker a on the day d.

For patient $p \in P_{a,d}$, the sum of forward augmented durations of patients in $P[r_p, d_a]$, which is the set of patients whose time windows are in the forward interval of p, must observe the forward interval's width of p. It is the same for the patient's p backward interval. The following inequalities can be added to the MP:

$$\sum_{p \in P[r_p, d_a]} p'_{a,p,d} y_{a,p,d} \leq d_a - r_p, \qquad p \in P_{a,d}, \tag{26}$$

$$\sum_{p \in P[r_a, d_p]} p''_{a,p,d} y_{a,p,d} \leq r_a - d_p, \qquad p \in P_{a,d}. \tag{27}$$

4 Experimental results

This section presents our computational results obtained by three models, that is, deterministic, chance-constrained with scenario generation, and chance-constrained with the approximation of start-service and arrival time distributions, on different instances based on real data provided by a home care agency from the US (Heching, Hooker, and Kimura 2019).

4.1 Experimental setting

Each instance contains new and existing patients assigned to the care workers whose visit days have been decided. For existing patients, the assigned care worker and visit days assignment must remain the same to ensure service consistency, but we can reschedule their visit times. In other words, we optimize the decisions to accept and assign new patients in conjunction with modifications of the rolling schedule, which has previously been determined for existing patients. For each care worker, the revisions of the schedule to visit existing patients are done if there is a change (addition or removal) in the list of patients assigned to them. Otherwise, we keep their schedules unchanged. In this instance set, there are eight care workers available to serve patients, and the planning horizon is five working days. To accept any new patients, the decision-maker must ensure that there is a feasible schedule that considers service level and other service requirement constraints as described in Section 3. We run the models for the different numbers of new and existing patients in the ranges of $[8, 18]$ and $[10, 17]$, respectively.

In our instances, some patients ask for multiple visits per week. When they require two visits per week, there must be a minimum gap of two days between consecutive visits, and when there are three weekly visits, there must be a minimum gap of one day between consecutive visits. With this information, we can replace constraints (8) with the following linear inequality constraint (Heching, Hooker, and Kimura 2019):

$$y_{a,p,d} + y_{a,p,d+i} \leq 1 \qquad \forall a, p \text{ with } w_p \in \{2, 3\},$$
$$\forall i, d \text{ with } 1 \leq i \leq 4 - w_p, 1 \leq d \leq 5.$$

We implemented the models in Python 3.8.10. The experiments were performed on Compute Canada Servers with 8 Gb RAM and one core for each instance. To solve the master problem, we used Gurobi 9.5.0, and to solve the subproblems, we used the constraint programming solver of Cplex 22.1.0. The maximum computation time was set to 7200 seconds per instance.

To evaluate the performance of the models, we ran simulations with $10,000$ scenarios for service times and travel times. Service times follow a normal distribution in which the mean is set equal to the deterministic service times, and the coefficient of variation (CoV) of the service time (denoted by $\mathrm{CoV_s}$) was set to 10, 25, or 50 percent of the mean service times in different instances. Travel times follow normal, shifted-gamma, and shifted-exponential distributions. We considered three probability distributions with different skewness (i.e., skewness = 0 in a normal distribution, skewness = 1 in shifted-gamma, and skewness = 2 in shifted-exponential) to study the effect of skewness on the performance of the models. The mean of travel time between each two locations equals the nominal value in the original instance set. The CoV of travel times was set to 10, 25, or 50 percent of the mean travel times to represent different travel conditions (we refer to CoV for travel times as $\mathrm{CoV_t}$). The allowable delay parameter (L parameter) was set to 10, 20, or 30 minutes. This implies that we consider a late arrival a delay only when a care worker arrives later than the assigned visit time plus the allowable delay. Additionally, we considered service levels, α, 95% and 98% ($z_\alpha = 1.64, 2.05$) to investigate the impact of different service levels. Finally, to study the effects of increasing the number of scenarios in the chance-constrained model with scenario generation, we used the following different numbers of scenarios: 10, 50, 100, 300, and 500.

4.2 Performance of the stochastic optimization approach for HHCRSP

We evaluate the performance of the models using several criteria. First, we compare different models in terms of solving times in seconds. In Table 3, the deterministic model adapted from Heching, Hooker, and Kimura (2019) can always solve instances faster than approximation and scenario models, which is to be expected. When we compare approximation and scenario models, we can see that when using 100 or more scenarios, the approximation model outperforms the scenario model in all instances. However, using only 10 scenarios, the scenario model solves instances faster. Both are comparable when using 50 designs.

Table 3: Solution times (in seconds) until optimality for different models based on simulations

$\alpha = \mathbf{0.98}$, $\mathbf{L = 10}$, $\mathbf{CoV_t = 25\%}$, $\mathbf{CoV_s = 10\%}$, **distribution = normal**								
New Patients	**Total Patients**	**Deterministic**	**Approximation**	**Scenario**				
				10	**50**	**100**	**300**	**500**
8	25	4.7	26.4	12	22.7	92.3	207.4	429
10	26	6.2	26.4	19.3	43.6	167.3	296.4	1011
12	26	30.2	70.9	35.2	67.2	119.9	452.9	1291.8
14	26	91.9	175.5	169.3	370.6	663.5	2375.2	3087.3
16	27	111.5	196.9	202.9	502.4	1026.8	3464.5	5570.1
18	28	150.5	664.7	362.3	771.8	2435.4	⋆	⋆

⋆ Computational time exceeded two hours

The difference between the deterministic and stochastic models concerns service level constraints embedded in the latter. Therefore, we investigate the effects of incorporating these constraints into the model on the actual service levels for patients based on the simulation results. To do so, we compare the results of two criteria: minimum service level (MinS) between all scheduled patients and average service level (AvgS) between different models. According to Table 4, the deterministic approach produces solutions that do not meet the acceptable service level, 98%. However, using as few as 10 scenarios, the service quality could be improved considerably, indicating the effectiveness of entering uncertainties into the model. Furthermore, the approximation model generally outperforms the scenario model, especially when the number of scenarios is smaller than 300. Although the approximation model could solve all the instances within two hours, for many scenarios, the scenario model sometimes exceeded the time limit of two hours.

Table 4: Service level comparisons between different models based on simulations

$\alpha = 0.98$, L = 10, $CoV_t = 25\%$, $CoV_s = 10\%$, distribution = normal															
New Patients	**Deterministic**		**Approximation**		**Scenario (10)**		**Scenario (50)**		**Scenario (100)**		**Scenario (300)**		**Scenario (500)**	
	MinS	**AvgS**	**MinS**	**AvgS**	**MinS**	**AvgS**	**MinS**	**AvgS**	**MinS**	**AvgS**	**MinS**	**AvgS**	**MinS**	**AvgS**
8	0.449	0.762	**0.985**	0.994	0.670	0.903	0.948	0.981	0.974	0.991	0.978	**0.997**	0.977	0.995
10	0.440	0.754	0.981	0.993	0.670	0.906	0.948	0.988	0.976	0.995	0.981	0.997	**0.993**	**0.999**
12	0.440	0.784	0.979	0.994	0.670	0.917	0.930	0.990	0.958	0.995	**0.987**	**0.997**	0.971	0.996
14	0.486	0.786	0.987	0.995	0.670	0.925	0.904	0.985	0.962	0.992	0.977	0.995	**0.993**	**0.998**
16	0.454	0.739	0.980	0.994	0.670	0.928	0.930	0.985	0.974	0.991	**0.989**	**0.998**	0.975	0.996
18	0.484	0.739	**0.980**	0.990	0.670	0.919	0.948	0.989	0.967	**0.991**	⋆	⋆	⋆	⋆

⋆ Computational time exceeded two hours

In addition to service level metrics, we must consider the number of accepted new patients in the objective function. However, studying only the number of accepted new patients without considering the number of those who get service on time, at least α percentage of times, fails to provide us with a comprehensive view of the performance. In this regard, in Table 5, we compare the models in terms of not only the number of accepted patients (NA) but also on-time-served newly accepted patients with the satisfied service levels (ON).

Table 5: Comparison of models in terms of the number of accepted new patients and accepted new patients with satisfied service levels based on simulations

$\alpha = 0.98$, L = 10, $CoV_t = 25\%$, $CoV_s = 10\%$, distribution = normal															
New Patients	**Deterministic**		**Approximation**		**Scenario (10)**		**Scenario (50)**		**Scenario (100)**		**Scenario (300)**		**Scenario (500)**	
	NA	ON	NA	ON	NA	ON	NA	ON	NA	ON	NA	ON	NA	ON
8	8	1	7	7	7	1	7	5	6	5	7	7	5	5
10	9	2	8	8	8	2	8	7	7	7	8	8	6	6
12	11	4	10	10	10	4	10	9	10	9	10	10	9	8
14	13	3	12	12	13	7	12	10	12	10	12	12	11	11
16	16	3	14	14	15	6	14	11	14	12	14	14	13	12
18	18	2	16	16	17	6	17	15	17	16	⋆	⋆	⋆	⋆

⋆ Computational time exceeded two hour

According to Table 5, although the deterministic solutions generally accept more new patients than the solutions from the stochastic approximation model, most of the patients could not be served on time. Furthermore, the maximum difference in the number of accepted patients between the solutions from these two approaches is two patients. In contrast, the stochastic approximation model can achieve a satisfactory service level for all accepted patients. As for the scenario model, the metrics depend on the number of scenarios used in the model. With an increase in the number of scenarios, the number of accepted new patients decreases, such that with 500 scenarios, the scenario model accepts fewer patients than the approximation model.

4.3 Sensitivity analysis

We investigate the effects of parameter changes on the performance of the models. The sensitivity analyses were conducted to analyze the changes in allowable delay (L), CoV_s, CoV_v, types of the probability distribution for travel times, and acceptable service levels (α).

Table 6 shows that, by increasing the value of the allowable delay, the number of accepted new patients can be increased in the stochastic approximation model. Additionally, increasing the allowable delay can improve the deterministic model's service level metrics and the number of on-time-served new patients, but the results are still inferior to the stochastic models.

Table 6: Sensitivity analysis of the allowable delay parameter

$\alpha = \mathbf{0.98}$, $\mathbf{CoV_t = 25\%}$, $\mathbf{CoV_s = 10\%}$, distribution = normal																	
New Patients	**L**	**Deterministic**				**Approximation**				**Scenario (10)**				**Scenario (100)**			
		NA	**ON**	**MinS**	**AvgS**	**NA**	**ON**	**MinS**	**AvgS**	**NA**	**ON**	**MinS**	**AvgS**	**NA**	**ON**	**MinS**	**AvgS**
8	10	8	1	0.449	0.762	7	7	0.985	0.994	7	1	0.670	0.903	6	5	0.974	0.991
8	20	8	2	0.642	0.888	7	7	0.982	0.993	7	2	0.670	0.915	6	6	0.976	0.993
8	30	8	3	0.786	0.950	8	8	0.976	0.993	7	4	0.670	0.955	6	6	0.967	0.996
10	10	9	2	0.440	0.754	8	8	0.981	0.993	8	8	0.670	0.906	7	7	0.976	0.995
10	20	9	2	0.631	0.882	8	8	0.982	0.992	8	3	0.670	0.915	7	7	0.976	0.995
10	30	9	5	0.773	0.946	9	9	0.977	0.993	8	4	0.670	0.941	7	7	0.976	0.997
12	10	11	4	0.440	0.784	10	10	0.979	0.994	10	4	0.670	0.917	10	9	0.958	0.995
12	20	11	6	0.631	0.905	11	11	0.983	0.993	11	6	0.670	0.938	10	8	0.943	0.991
12	30	11	7	0.773	0.957	11	11	0.976	0.994	11	7	0.853	0.971	10	8	0.958	0.993
14	10	13	3	0.486	0.786	12	12	0.987	0.995	13	7	0.670	0.925	12	10	0.962	0.992
14	20	13	4	0.669	0.907	13	13	0.980	0.993	13	10	0.670	0.933	12	12	0.976	0.997
14	30	13	9	0.805	0.961	13	13	0.977	0.993	13	7	0.670	0.954	12	11	0.967	0.996
16	10	16	3	0.454	0.739	14	14	0.980	0.994	15	6	0.670	0.928	14	12	0.974	0.991
16	20	16	4	0.620	0.873	15	15	0.977	0.992	15	5	0.670	0.930	14	13	0.950	0.995
16	30	16	7	0.745	0.941	16	14	0.962	0.990	15	8	0.670	0.942	14	13	0.971	0.997
18	10	18	2	0.484	0.739	16	16	0.980	0.990	17	6	0.670	0.919	17	16	0.967	0.991
18	20	18	3	0.669	0.878	17	17	0.980	0.992	17	7	0.544	0.904	17	17	0.976	0.995
18	30	18	7	0.808	0.984	17	16	0.972	0.991	17	7	0.654	0.923	17	17	0.971	0.997

According to Table 7, when the uncertainty in travel time CoV_t is lower, service levels for customers generally improve. The approximation model can always satisfy the service level requirements for all newly accepted patients. However, when the CoV_t is lower, the approximation model can accept more patients. Moreover, although the CoV_t parameter does not have any effect on the number of accepted patients in the deterministic model, a higher value can cause worse performance. As for the scenario models, we can say that higher CoV_t requires more scenarios to find high-quality solutions in terms of service level satisfaction. This pattern can be observed in Table 8.

To investigate whether a non-normal probability distribution of travel times can affect the solution quality of the approximation model, we performed this sensitivity analysis and reported the results in Table 9. When shifted-gamma or shifted-exponential is the actual probability distribution for travel times, the minimum service level metric is not perfectly satisfied but very close to the acceptable service level α.

In the stochastic approximation model, CoV values of the travel times between nodes are input parameters. To specify a valid value concerning travel CoV of the actual data, we investigate the performance of the approximation model when we use CoV values that are smaller than, equal to, or larger than the actual CoV in the simulation in the problem (Table 10). When we use a smaller value for CoV in the approximation, we underestimate the actual variation in travel times. Therefore, more new patients may be accepted, but not all of them can be served at the acceptable service level. In contrast, an overestimation happens when we consider a larger value for CoV in the approximation model. As a result, we might accept fewer new patients than we should since the solution can be overly conservative. Thus, it is essential to properly estimate the CoV values used in the approximation model.

Table 7: Travel CoV effects on the performance of the models

$\alpha = 0.98$, $CoV_s = 10\%$, $L = 10$, distribution = normal																	
New Patients	CoV_t	Deterministic				Approximation				Scenario (10)				Scenario (100)			
		NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS
8	0.1	8	2	0.569	0.846	8	8	0.983	0.993	8	5	0.871	0.975	7	7	0.992	0.998
8	0.25	8	1	0.449	0.762	7	7	0.985	0.994	7	1	0.670	0.903	6	5	0.974	0.991
8	0.5	8	0	0.316	0.662	5	5	0.976	0.990	7	2	0.623	0.857	5	5	0.962	0.991
10	0.1	9	3	0.576	0.856	9	9	0.982	0.993	9	5	0.799	0.966	8	8	0.992	0.998
10	0.25	9	2	0.440	0.754	8	8	0.981	0.993	8	2	0.670	0.906	7	7	0.976	0.995
10	0.5	9	1	0.329	0.643	7	7	0.979	0.993	8	2	0.673	0.889	6	6	0.958	0.988
12	0.1	11	4	0.576	0.869	11	11	0.982	0.993	11	7	0.893	0.960	11	11	0.983	0.998
12	0.25	11	4	0.440	0.784	10	10	0.979	0.994	10	4	0.670	0.917	10	9	0.958	0.995
12	0.5	11	1	0.329	0.688	9	9	0.982	0.991	11	2	0.773	0.914	8	6	0.958	0.989
14	0.1	13	4	0.602	0.865	13	13	0.984	0.995	13	7	0.799	0.962	13	13	0.932	0.995
14	0.25	13	3	0.486	0.786	12	12	0.987	0.995	13	7	0.670	0.925	12	10	0.962	0.992
14	0.5	13	0	0.358	0.682	10	10	0.981	0.992	13	2	0.622	0.854	10	10	0.962	0.994
16	0.1	16	3	0.569	0.822	15	15	0.981	0.995	15	10	0.799	0.954	15	15	0.991	0.998
16	0.25	16	3	0.454	0.739	14	14	0.980	0.994	15	6	0.670	0.928	14	12	0.974	0.991
16	0.5	16	0	0.316	0.645	13	13	0.980	0.992	14	2	0.637	0.901	12	9	0.962	0.991
18	0.1	18	4	0.591	0.819	17	17	0.980	0.993	17	12	0.771	0.950	17	16	0.975	0.996
18	0.25	18	2	0.484	0.739	16	16	0.980	0.990	17	6	0.670	0.919	17	16	0.967	0.991
18	0.5	18	0	0.363	0.647	14	14	0.983	0.992	16	3	0.673	0.877	14	10	0.930	0.984

Table 8: Service CoV effects on the performance of the models

$\alpha = 0.98$, $CoV_t = 25\%$, $L = 10$, distribution = normal																	
New Patients	CoV_s	Deterministic				Approximation				Scenario (10)				Scenario (100)			
		NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS
8	0.1	8	1	0.449	0.762	7	7	0.985	0.994	7	1	0.670	0.903	6	5	0.974	0.991
8	0.25	8	1	0.407	0.687	4	4	0.978	0.992	6	3	0.784	0.957	4	4	0.976	0.995
8	0.5	8	1	0.370	0.628	3	3	0.984	0.993	4	1	0.759	0.933	3	3	0.980	0.995
10	0.1	9	2	0.440	0.754	8	8	0.981	0.993	8	2	0.670	0.906	7	7	0.976	0.995
10	0.25	9	2	0.4	0.701	5	5	0.983	0.994	7	4	0.837	0.962	6	6	0.981	0.994
10	0.5	9	2	0.370	0.654	4	4	0.982	0.992	5	1	0.853	0.945	4	3	0.971	0.994
12	0.1	11	4	0.440	0.784	10	10	0.979	0.994	10	4	0.670	0.917	10	9	0.958	0.995
12	0.25	11	3	0.4	0.722	6	6	0.984	0.991	9	3	0.784	0.953	7	7	0.977	0.995
12	0.5	11	2	0.372	0.668	4	4	0.982	0.991	6	2	0.759	0.949	4	4	0.962	0.987
14	0.1	13	3	0.486	0.786	12	12	0.987	0.995	13	7	0.670	0.925	12	10	0.962	0.992
14	0.25	13	1	0.434	0.709	7	7	0.978	0.992	11	5	0.784	0.958	8	8	0.976	0.994
14	0.5	13	1	0.377	0.640	5	5	0.984	0.993	7	4	0.759	0.957	5	5	0.962	0.991
16	0.1	16	3	0.454	0.739	14	14	0.980	0.994	15	6	0.670	0.928	14	12	0.974	0.991
16	0.25	16	3	0.403	0.67	10	10	0.987	0.996	13	5	0.787	0.953	10	10	0.976	0.993
16	0.5	16	3	0.362	0.619	⋆	⋆	⋆	⋆	9	3	0.838	0.943	7	7	0.962	0.991
18	0.1	18	2	0.484	0.739	16	16	0.980	0.990	17	6	0.670	0.919	17	16	0.967	0.991
18	0.25	18	2	0.424	0.668	12	12	0.985	0.995	15	5	0.756	0.943	13	13	0.976	0.992
18	0.5	18	2	0.355	0.617	⋆	⋆	⋆	⋆	10	6	0.746	0.930	⋆	⋆	⋆	⋆

⋆ Computational time exceeded two hour

Table 9: Performance of the approximate model based on different types of distributions

$\alpha = 0.98$, L = 10, $CoV_t = 25\%$, $CoV_s = 10\%$												
New Patients	Normal				Shifted-gamma				Shifted-exponential			
	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS
8	7	7	0.985	0.994	7	7	0.974	0.988	7	5	0.968	0.983
10	8	8	0.981	0.993	8	7	0.973	0.987	8	5	0.968	0.983
12	10	10	0.979	0.994	10	10	0.967	0.988	10	8	0.959	0.984
14	12	12	0.987	0.995	12	12	0.974	0.989	12	9	0.968	0.985
16	14	14	0.980	0.994	14	14	0.968	0.989	14	12	0.963	0.985
18	16	16	0.980	0.990	16	13	0.968	0.983	16	10	0.962	0.979

Table 10: Proper travel CoV value for approximation model

$\alpha = 0.98$, L = 10, $CoV_t = 25\%$, $CoV_s = 10\%$, distribution = normal												
New Patients	approx CoV (0.15)				approx CoV (0.25)				approx CoV (0.35)			
	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS
8	7	6	0.943	0.977	7	7	0.985	0.994	7	7	0.985	0.997
10	8	6	0.907	0.978	8	8	0.981	0.993	8	8	0.985	0.997
12	11	7	0.944	0.977	10	10	0.979	0.994	10	10	0.985	0.998
14	13	10	0.944	0.982	12	12	0.987	0.995	11	11	0.985	0.997
16	15	10	0.911	0.977	14	14	0.980	0.994	14	14	0.985	0.997
18	17	9	0.907	0.969	16	16	0.980	0.990	15	15	0.993	0.998

Finally, the effect of acceptable service level α on the performance of models are investigated in Table 11. Indeed, the α does not affect NA, MinS, and AvgS for the deterministic model, but it can increase the ON metric when α has a lower value. In addition, decreasing the assigned service level can sometimes increase the number of accepted new patients in the approximate model.

Table 11: Effects of service level on the performance of the models

L = 10, $CoV_t = 25\%$, $CoV_s = 10\%$, distribution = normal																	
New Patients	α	Deterministic				Approximation				Scenario (10)				Scenario (100)			
		NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS	NA	ON	MinS	AvgS
8	0.98	8	1	0.449	0.762	7	7	0.985	0.994	7	1	0.670	0.903	6	5	0.974	0.991
8	0.95	8	2	0.449	0.762	7	7	0.953	0.983	7	2	0.670	0.903	6	6	0.941	0.986
10	0.98	9	2	0.440	0.754	8	8	0.981	0.993	8	2	0.670	0.906	7	7	0.976	0.995
10	0.95	9	2	0.440	0.754	8	8	0.948	0.979	8	4	0.670	0.906	7	7	0.941	0.989
12	0.98	11	4	0.440	0.784	10	10	0.979	0.994	10	4	0.670	0.917	10	9	0.958	0.995
12	0.95	11	4	0.440	0.784	11	11	0.948	0.982	10	5	0.670	0.917	10	10	0.941	0.993
14	0.98	13	3	0.486	0.786	12	12	0.987	0.995	13	7	0.670	0.925	12	10	0.962	0.992
14	0.95	13	4	0.486	0.786	13	13	0.948	0.981	13	9	0.670	0.925	12	12	0.941	0.994
16	0.98	16	3	0.454	0.739	14	14	0.980	0.994	15	6	0.670	0.928	14	12	0.974	0.991
16	0.95	16	3	0.454	0.739	15	15	0.948	0.982	15	10	0.670	0.928	14	14	0.941	0.987
18	0.98	18	2	0.484	0.739	16	16	0.980	0.990	17	6	0.670	0.919	17	16	0.967	0.991
18	0.95	18	3	0.484	0.739	16	16	0.954	0.982	17	11	0.670	0.919	17	16	0.938	0.984

4.4 Performance for the case with correlated travel times

To evaluate the performance of the models in actual service levels for patients when travel times are correlated to each other, we present Table 12. The existence of correlations between parameters has negative effects on the performance of deterministic and approximation models. However, the scenario-based model can still perform well when the number of scenarios is 100 or more. This clearly demonstrates the benefits of the scenario generation model, which is agnostic to the sampling method and can thus be used to solve the instances in which samples are generated from distributions with dependent parameters.

Table 12: Performance of models with correlated travel times

$\mathbf{L = 10}$, $\mathbf{CoV_t = 25\%}$, $\mathbf{CoV_s = 10\%}$, **distribution = normal**									
New Patients	**Corrolation**	**Deterministic**		**Approximation**		**Scenario (100)**		**Scenario (300)**	
		MinS	**AvgS**	**MinS**	**AvgS**	**MinS**	**AvgS**	**MinS**	**AvgS**
8	no	0.449	0.762	**0.985**	0.994	0.974	0.991	0.978	**0.997**
8	yes	0.148	0.546	0.903	0.961	0.970	0.992	**0.987**	**0.997**
10	no	0.440	0.754	**0.981**	0.993	0.976	0.995	**0.981**	**0.997**
10	yes	0.139	0.507	0.903	0.963	0.970	0.993	**0.983**	**0.997**
12	no	0.440	0.784	0.979	0.994	0.958	0.995	**0.987**	**0.997**
12	yes	0.139	0.581	0.871	0.961	0.965	0.993	**0.984**	**0.996**
14	no	0.486	0.786	0.987	0.995	0.962	0.992	**0.977**	**0.995**
14	yes	0.139	0.572	0.903	0.964	0.968	0.993	**0.974**	**0.995**
16	no	0.454	0.739	0.980	0.994	0.974	0.991	**0.989**	**0.998**
16	yes	0.123	0.518	0.887	0.966	0.962	0.991	**0.972**	**0.993**
18	no	0.484	0.739	**0.980**	0.990	0.967	**0.991**	⋆	⋆
18	yes	0.133	0.528	0.880	0.945	**0.954**	**0.990**	⋆	⋆

⋆ Computational time exceeded two hours

5 Conclusions and future research

In this paper, we provide a stochastic solution framework to solve a home health care routing and scheduling problem with service consistency consideration when travel and service times are uncertain. This problem can be considered a variant of the consistent vehicle routing problem that incorporates personnel and time consistency as hard constraints. A chance-constraint model is proposed to deal with the uncertain parameters for the first time in the HHCRSP with hard service consistency and TST uncertainty. The chance constraint is reformulated using two different methods: a set of discrete scenarios and approximations of start-service and arrival time distributions using EVT analysis. In both stochastic approaches, an exact method called branch-and-check, a variant of LBBD, is implemented. This method decomposes the original problem into a master problem and multiple subproblems, in which the master problem is solved by mixed integer linear programming and subproblems by CP.

Different metrics are evaluated through simulation to compare the efficiency of our proposed stochastic models with the deterministic model proposed by Heching, Hooker, and Kimura (2019) to investigate the value of stochastic solutions. Our computational experiments demonstrated that the deterministic model can not reach an acceptable service level while the stochastic models can easily do so. In addition, the number of newly accepted patients in the stochastic solutions is generally not much lower than the one obtained using the deterministic model while the stochastic model guarantees a much higher service quality. Furthermore, we can achieve more efficient computing performance by adopting the nonlinear stochastic approximation function, which can be handled efficiently using constraint programming.

This paper can be extended in different directions. For example, given that we have studied the problem in a static setting, its dynamic variants could be promising in future studies. Although hard service constraints are more difficult to solve exactly, it would be interesting to develop new models and methods for a soft constraint version of service consistency, where personnel consistency and/or time consistency constraints can be relaxed and penalized in the objective function.

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