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E. K. Kuchesfehani,
E. M. Parilina, G. Zaccour

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Cost-revenue sharing in a dynamic closed-loop supply chain with uncertain parameters

Elnaz Kanani Kuchesfehani [a]
Elena M. Parilina [b,c,d]
Georges Zaccour [a,e]

[a] *GERAD & Department of Decision Sciences, HEC Montréal, Canada, H3T 2A7*

[b] *Saint Petersburg State University, Saint Petersburg 199034, Russia*

[c] *School of Mathematics and Statistics, Qingdao University, Qingdao 266071, PR China*

[d] *Institute of Applied Mathematics of Shandong, Qingdao 266071, PR China*

[e] *Chair in Game Theory and Management, HEC Montréal, Montréal (Québec), Canada, H3T 2A7*

elnaz.kanani@gmail.com
e.parilina@spbu.ru
georges.zaccour@gerad.ca

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Abstract: We represent a closed-loop supply chain formed of one manufacturer and one retailer as a dynamic game played over an event tree, which naturally allows to capture the dynamic nature of product returns for remanufacturing and uncertainties in the parameter values. We characterize and compare the equilibrium results in two different scenarios. In the benchmark scenario, we assume that there is no collaboration between the two players and the manufacturer pays the cost of green activities, which aim at increasing the product return. In the second scenario, namely reverse revenue cost sharing (RRCS) contract, the retailer contributes in the cost of the green activities incurred by the manufacturer and the latter transfers a part of its revenues to the former in return. Numerical experiments are provided.

Keywords: Supply chain management, closed-loop supply chain, cost-revenue sharing, dynamic games, product returns, green activities

Résumé : Nous représentons une chaîne d'approvisionnement en boucle fermée composée d'un fabricant et d'un détaillant comme un jeu dynamique joué sur un arbre d'événements. Ce formalisme permet de tenir compte de la dynamique des produits retournés à la fin de leur vie utile et de l'incertitude dans les valeurs des paramètres. Nous caractérisons et comparons les résultats d'équilibre dans deux scénarios différents. Dans le scénario de référence, nous supposons qu'il n'y a pas de collaboration entre les deux acteurs et que le fabricant paie le coût des activités qui visent à augmenter le taux de retour du produit. Dans le second scénario, à savoir le contrat RRCS (reverse revenue cost sharing), le détaillant participe au coût de ces activités et le fabricant transfère une partie de ses revenus au premier en retour. Des expériences numériques sont fournies.

Mots clés : Gestion de la chaîne d'approvisionnement, chaîne d'approvisionnement en boucle fermée, partage de coûts et de revenus, retour de produits, activités vertes

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1 Introduction

A Closed-Loop Supply Chain (CLSC) combines forward and reverse activities into a unique system to increase economic, environmental, and social performance. Reverse activities include, in particular, collecting back previously sold products when they reach their end of life. The interest in doing so lies in the cost reduction that results from producing by means of used components instead of only with new materials. It is not then surprising to see that the manufacturer is the agent who is most interested in closing the loop and appropriating the returns' residual value, while other members of the supply chain are excluded from the benefits Guide Jr. and Wassenhove (2009). At the same time, as retailers are close to the customers, they can be highly effective in creating awareness about the environmental benefit of recycling, and therefore play a key role in a CLSC De Giovanni (2014), Savaskan et al. (2004). Intuitively, the retailer will participate in closing the loop only if it gets some of the savings that the manufacturer realizes when producing with material extracted from used products Ferguson and Toktay (2006). One straightforward way of sharing this benefit is in reducing the wholesale price. On the other hand, the retailer may find it optimal to pay part of the manufacturer's cost incurred to increase the return of products by consumers at the end of their useful life. This reasoning implies that there is room for a two-way incentive scheme in a CLSC, i.e., sharing both revenues and (some) costs. This is the line of thought pursued in this paper.

In a revenue-sharing contract (RSC), the retailer pays the manufacturer a percentage of the total revenues. The rationale for a RSC is the mitigation of the double-marginalization effect, i.e., a RSC leads to a lower price and higher demand than in a standard wholesale price contract (see, e.g., Tirole (1988) and Cachon and Lariviere (2005)). In a Reverse Revenue Sharing Contract (RRSC), it is typically the manufacturer who transfers part of its revenues to the retailer. For examples of RSC and RRSC, see, e.g., Dana and Spier (2001), Fleischmann et al. (2002), Totty (2002), Geng and Mallik (2007), De Giovanni and Zaccour (2013) and De Giovanni (2014).

In this article, taking into account that the problem's data is stochastic, we address the following research questions:

1. Should the manufacturer rely on its own or financially involve the retailer in the product returns through the implementation of a RRSC?
2. Under what conditions a RRSC equilibrium is Pareto improving with respect to wholesale pricing equilibrium?
3. What is the impact of the main model's parameters on the results?

To answer the above questions, we develop a dynamic game of CLSC played over an uncontrolled event tree, that is, a game where the transition from one node to another is Nature's decision and cannot be influenced by the players' actions, and design a RRSC that adapts to the realization of the stochastic demand at each node of the tree. The dynamic feature of the game stems naturally from the fact that the purchase and return of the product take place at different moments in time. We assume that the manufacturer can influence the return of used products by investing in some "green" activities (GA) such as advertising and communication campaigns about the recycling policies, logistics services, monetary and symbolic incentives, employees-training programs, etc. For a recent survey of game theoretic models in closed-loop supply chains, see De Giovanni and Zaccour (2019b).

We characterize and compare strategies and outcomes in two noncooperative scenarios played à la Stackelberg, where the retailer acts as leader and the manufacturer as follower. In the first scenario, which plays a benchmark role, the retailer does not participate financially in the GA program and the manufacturer does not offer any discount on the wholesale price. In the second scenario, the two members of the supply chain implements a cost-revenue sharing contract. To account for the stochastic process evolving over time, we use the formalism of dynamic games played over event trees (DGPETs); see Haurie et al. (2012).

The rest of the paper is organized as follows. In Section 2, we develop the dynamic game model played over an event tree. In Section 3, we characterize the equilibrium solutions in the benchmark (non-collaborative) and RRSC scenarios. In Section 4, we discuss an illustrative example, and conclude in Section 5.

2 Model

The CLSC is formed of one (re)manufacturer (player M) that sells its product through a retailer (player R). To account for the return by (some) consumers of previously purchased products at the end of their useful life, we naturally retain a dynamic model. Denote by $\mathcal{T} = \{0, 1, \ldots, T\}$ the set of periods, where T is the planning horizon. We suppose that the demand is random and described by a stochastic process defined by an event tree. We denote the root node by n^0 in period 0 and consider a set of nodes $\mathcal{N}^t$ in period $t = 1, \ldots, T$. Let $a(n_l^t) \in \mathcal{N}^{t-1}$ be the unique predecessor of node $n_l^t \in \mathcal{N}^t$ for $t = 1, \ldots, T$, and denote by $S(n_l^t) \in \mathcal{N}^{t+1}$ the set of all possible direct successors of node $n_l^t \in \mathcal{N}^t$ for $t = 0, 1, \ldots, T-1$. In what follows, the dependence on a node $n_l^t \in \mathcal{N}^t$ in period $t = 1, \ldots, T$ is shown as a superscript for parameters and as an argument for variables. We denote by $\pi^{n_l^t}$ the probability of passing through node n_l^t. In particular, we have $\pi^{n_0} = 1$ and $\pi^{n_l^T}$ is equal to the probability of the single scenario that terminates in (leaf) node $n_l^T \in \mathcal{N}^T$. Also, $\sum_{n_l^t \in \mathcal{N}^t} \pi^{n_l^t} = 1, \forall t$.[1]

Denote by $p(n_l^t)$ the price-to-consumer chosen by the retailer in node $n_l^t \in \mathcal{N}^t$, $t = 1, \ldots, T$. At each node, the demand $Q(n_l^t)$ is a decreasing function of price. Following a long tradition in economics, we retain the following linear form:[2]

$$Q\left(n_l^t\right) = \alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t), \tag{1}$$

where $\alpha^{n_l^t} > 0$ is the market potential, and $\beta^{n_l^t} > 0$ represents consumer's sensitivity to price at node $n_l^t \in \mathcal{N}^t$ for $t = 0, 1, \ldots, T$. Note that both parameters' values are node-dependent.

Denote by w the exogenous wholesale price. To have nonnegative demand and positive margin for the retailer, the condition $w < p(n_l^t) \leq \frac{\alpha^{n_l^t}}{\beta^{n_l^t}}$ must be satisfied at all nodes.

The manufacturer can produce with either new materials or old materials extracted from the returned (past-sold) products. Assuming away inventories in production and in consumer's basement, we shall refer to the number of product units that come back as *returns* and denote them by $r\left(n_l^t\right)$, at node $n_l^t \in \mathcal{N}^t$ in period $t = 1, \ldots, T$. In the CLSC literature, the rate (or quantity) of returned products has been modeled following essentially three approaches. A first group of authors assumed that the return rate is exogenous (see, e.g., Geyer et al. (2007), Fleischmann et al. (2001), Ferrer and Swaminathan (2006), Guide Jr. et al. (2006), Atasu et al. (2008)). The second stream also adopted a passive approach, but modelled the return rate as a random variable, e.g., an independent Poisson (see, e.g., van der Laan et al. (1996, 1999), Aras et al. (2010)). The third group of studies considered an active approach, with the return rate being a function of players' strategies (see, e.g., Savaskan et al. (2004), Savaskan and Van Wassenhove (2006), De Giovanni and Zaccour (2019a)). In this paper, we follow this active approach.

We assume that the consumption cycle of a product is one period. Depending on the considered product, a period can be of short duration (soft drink and beer bottles and cans), medium (cartridges, water filters) or long (smart phones, laptops). This does not imply that a consumer either returns the used product after one period or never. Indeed, some consumers may wait longer to return their used product because they missed of an opportunity or forgot, or may take more time to consume what

[1] See Haurie et al. (2012) for an introduction to this class of games and Kanani Kuchesfehani and Zaccour (2015) for the extension to the case where the players face coupling constraints.

[2] To be more rigorous, we should write the demand function as $Q(p(n_l^t))$, but to simplify the notation, we write it as $Q(n_l^t)$.

they bought. To account for these possible lags, we let the returns at any node $n_l^t \in \mathcal{N}^t$ depend on two factors: (i) the investment $G(n_l^t)$ in the GA program at that node; and (ii) the quantities sold at all preceding nodes along the path emanating from the root node and ending in n_l^t. Denote by $\mathcal{P}(n_l^t)$ this path, that is,

$$\mathcal{P}(n_l^t) = (n^0, n_{i_1}^1, \ldots, n_{i_{t-1}}^{t-1}, n_l^t),$$

and by

$$\tilde{Q}\left(\mathcal{P}(n_l^t)\right) = \left(Q\left(n^0\right), \ldots, Q\left(a\left(n_l^t\right)\right), Q\left(n_l^t\right)\right),$$

the vector of sales along the path $\mathcal{P}(n_l^t)$.

We let the returns at any node $n_l^t \in \mathcal{N}^t$ be given by

$$\begin{aligned} r\left(n_l^t\right) =& \theta_1 \times f\left(G\left(n_{i_{t-1}}^{t-1}\right)\right) \times Q\left(n_{i_{t-1}}^{t-1}\right) + \theta_2 \times f\left(G\left(n_{i_{t-2}}^{t-2}\right)\right) \times Q\left(n_{i_{t-1}}^{t-2}\right) + \ldots \\ &+ \theta_t \times f\left(G\left(n^0\right)\right) \times Q\left(n^0\right), \end{aligned} \tag{2}$$

where θ_i are parameters satisfying $1 > \theta_1 > \theta_2 > \cdots > \theta_t > 0$, and $f\left(G\left(\cdot\right)\right)$ is a function defined below satisfying $0 \leq f\left(G\left(\cdot\right)\right) \leq 1$. The initial condition is $r\left(n^0\right) = 0$. The above equation states that the number of items brought back by consumers at a node $n_l^t \in \mathcal{N}^t$ is a weighted sum of past sales along the path from root node n^0 to that node. The weight of each term is given by a scaling parameter θ_i times a function that depends on the green activities effort made at the corresponding node. This specification captures the idea that consumers may wait any given number of periods before returning the product, if they bring it at all.

As it stands, $r\left(n_l^t\right)$ involves the entire history of sales till node n_l^t, and requires the estimation of t parameter values, which may be highly demanding in practice. One simplification often made in time series analysis to reduce (drastically) the number of parameters is to assume that the weights (or coefficients) decline geometrically over time.

Proposition 1 *If*

$$\frac{\theta_{i+1}}{\theta_i} = \eta, \quad \textit{for all } i,$$

where $0 < \eta < 1$, *then*

$$r\left(n_l^t\right) = \eta r\left(a\left(n_l^t\right)\right) + \theta_1 \times f\left(G\left(a\left(n_l^t\right)\right)\right) \times Q\left(a\left(n_l^t\right)\right), \quad r\left(n^0\right) = 0. \tag{3}$$

Proof. See Appendix A. □

As we can see, the dynamics in (3) only require the estimation of the two parameters η and θ_1, which is considerably less demanding than before. The interpretation is straightforward: the returns at any node n_l^t are given by a fraction of the returns at antecedent node $a\left(n_l^t\right)$ plus a proportion of sales at $a\left(n_l^t\right)$. We adopt (3) in the rest of the paper.

To interpret $f\left(G\right)$ as a probability or the propensity of consumers to bring back used products, it must satisfy $0 \leq f\left(G\left(\cdot\right)\right) \leq 1$ for $G\left(\cdot\right) \geq 0$. Further, to account for marginal decreasing returns in green activities effort, we suppose that $f\left(G\right)$ is concave increasing, with $\lim_{G\to\infty} f\left(G\right) = 1$. We adopt the following functional form that clearly satisfies these properties:

$$f\left(G\right) = \upsilon_1 - (\upsilon_1 - \upsilon_2)e^{-\kappa G},$$

where κ is a positive parameter, and $\upsilon_1 \in (0, 1]$, $\upsilon_2 \in [0, 1)$, where $\upsilon_1 > \upsilon_2$. Note that $f\left(0\right) = \upsilon_2$ represents the level of returns in the absence of any green activities (passive returns), and $\lim_{G\to\infty} f\left(G\right) = \upsilon_1 \leq 1$ is the maximum level of returns, which may possibly be less than 100%.

The following proposition provides the restriction on parameter's values that guarantees that the returns do not exceed sales.

Proposition 2 *If*

$$\theta_1 \leq \frac{1-\eta}{\upsilon_1\left(1-\eta^{T-1}\right)}, \tag{4}$$

then the returns are always less or equal to past sales.

Proof. See Appendix A. □

We emphasize the upper bound on θ_1 in (4) is determined assuming that the green activities are at their upper limit value. As green activities are costly, it is likely that υ_1 is never reached in practice, and consequently the bound is not tight. Further, as $\upsilon_1 \leq 1$ and $1-\eta^{T-1} < 1$, a sufficient (not necessary) condition for the returns not to exceed sales is $\theta_1 \leq 1-\eta$.

Denote by $d(G(n^t))$ the manufacturer's investment cost in the GA program. We suppose that this cost can be well approximated by the linear function

$$d(G(n_l^t)) = \zeta G(n_l^t), \tag{5}$$

where ζ is a positive constant.

The rationale for returns can be purely environmental. For instance, to avoid having used products ending in the landfill, the government may request the manufacturer to collect back these products and store them in an appropriate site. As mentioned before, here we additionally suppose that there is a cost advantage in recycling, namely, that producing with used parts is cheaper than manufacturing with exclusively new material. Denote by $c(r(n_l^t))$ the unit production cost. As using old material is intuitively subject to marginal decreasing return, we assume $c(r(n_l^t))$ to be convex decreasing, and adopt the following form:

$$c(r(n_l^t)) = c_0 e^{-c_r r(n_l^t)}.$$

where $c_0 > 0$ is the unit production cost when using only new material, and $c_r > 0$ represents the (log-marginal) reduction in cost due to returns.

Remark 1 *While the returns $r(n_l^t)$ measure the environmental performance at node n_l^t, the term $c_r r(n_l^t)$ gives the associated economic gain.*

We shall characterize and compare the equilibrium solutions in two scenarios.

Benchmark scenario: The two members of the supply chain do not share the cost of the GA program, nor the revenues. The game is played noncooperatively, that is, the manufacturer and the retailer maximize their individual profits given by

$$\max_{G(n_l^t)\geq 0} J_M(G,p,r) = \sum_{t=0}^{T} \sum_{n_l^t \in \mathcal{N}^t} \pi^{n_l^t} \delta^t \Big((w - c(r(n_l^t)))Q(n_l^t) - d(G(n_l^t)) \Big), \tag{6}$$

$$\max_{p(n^t)\geq 0} J_R(p) = \sum_{t=0}^{T} \sum_{n_l^t \in \mathcal{N}^t} \pi^{n_l^t} \delta^t \left(p(n_l^t) - w \right) Q(n_l^t), \tag{7}$$

where $\delta \in (0,1]$ is a discount rate, with the returns given by (3).

RRSC scenario: The retailer pays a share $B\left(n_l^t\right)$, $0 \leq B\left(n_l^t\right) \leq 1$, of the manufacturer's GA cost, and the manufacturer discounts the wholesale price by an amount $I(r(n_l^t))$ to compensate the retailer for sharing the cost of the GA program. Therefore, the wholesale price in this scenario is given by $w - I(r(n_l^t))$. Note that the reduction in the wholesale price depends on the returns; it is a way of incentivizing the retailer to contribute at a higher rate in the cost of GA program, whose objective is to increase the returns. The manufacturer's net margin at node $n_l^t \in \mathcal{N}^t$, $t = 0, \ldots, T$ is therefore

$w - c(r(n_l^t)) - I(r(n_l^t))$. As noted before, the noncooperative game is played à la Stackelberg with the retailer as the leader and the manufacturer as the follower, that is, the retailer first decides the support rate and the price to consumer, and next the manufacturer the investment in the GA program.

Following Cachon and Lariviere (2005), Dana and Spier (2001) and Gerchak and Wang (2004), we retain the following form for the incentive function:

$$I(r(n_l^t)) = \phi(w - c(r(n_l^t))) = \phi\left(w - c_0 e^{-c_r r(n_l^t)}\right), \tag{8}$$

where $\phi \in [0,1]$ is the sharing parameter and stands for the percentage of manufacturer's profit margin transferred to the retailer. In the benchmark scenario, $\phi = 0$ and $B\left(n_l^t\right) = 0$ for all $n_l^t \in \mathcal{N}^t,\ t = 0,\ldots,T$.

In this cost-revenue sharing scenario, the players' optimization problems are as follows:

$$\max_{G(n_l^t)\geq 0} J_M(G,p,B,r) = \sum_{t=0}^{T}\sum_{n_l^t\in\mathcal{N}^t}\pi^{n_l^t}\delta^t\Big(\big(w - c(r(n_l^t)) - I(r(n_l^t))\big)Q(n_l^t) - (1-B(n_l^t))d(G(n_l^t))\Big), \tag{9}$$

$$\max_{\substack{p(n_l^t)>0\\ 0\leq B(n_l^t)\leq 1}} J_R\left(G,p,B,r\right) = \sum_{t=0}^{T}\sum_{n_l^t\in\mathcal{N}^t}\pi^{n_l^t}\delta^t\Big(\big(p(n_l^t) - w + I(r(n_l^t))\big)Q(n_l^t) - B(n_l^t)d(G(n_l^t))\Big), \tag{10}$$

subject to the returns dynamics in (3). Notice that in this scenario, the two players are strategically linked, as their payoffs depend on both players' actions and on the returns.

Remark 2 *The reason for having a fixed wholesale price w, which may be counter intuitive, is to have a meaningful comparison between the two scenarios. If w were a decision variable, then the manufacturer can manipulate its level in both scenarios rendering the incentive $I(r(n_l^t))$ meaningless.*

To warp up, we have defined a two-player dynamic game played over an event tree, with one decision variable for the manufacturer ($G(n_l^t) \geq 0$) and two decision variables for the retailer ($p(n_l^t) \geq 0$ and $0 \leq B(n_l^t) \leq 1$).

3 Equilibria

In this section, we characterize equilibrium strategies and outcomes for both scenarios. First, we solve for the benchmark scenario in which the retailer does not participate in the green efforts undertaken by the manufacturer, and the manufacturer does not discount the wholesale price to the retailer. Second, we solve for the cost-revenue sharing (RRSC) scenario in which the retailer, as the leader, sets first her support rate and the price, and next the manufacturer, as the follower, decides about her GA efforts. Next, we compare the two equilibrium results.

3.1 Benchmark scenario

In this scenario, the retailer's optimization problem is independent of the manufacturer's control variable G and of the state variable r. Consequently, the retailer optimizes, at each node, a static problem without any regard to what the manufacturer is doing. The implication of this structure is that it does not matter if the game is played simultaneously à la Nash or sequentially à la Stackelberg, the result would be the same.

Introduce the manufacturer's Hamiltonian function

$$\mathcal{H}_M(n_l^t,\lambda(S(n_l^t)),r(n_l^t),G(n_l^t)) = \left(\alpha^{n_l^t}-\beta^{n_l^t}p(n_l^t)\right)\left(w-c_0e^{-c_r r(n_l^t)}\right)-\zeta G(n_l^t)$$
$$+\,\delta\sum_{\nu\in S(n_l^t)}\frac{\pi^\nu}{\pi^{n_l^t}}\lambda(\nu)\Big\{\eta r\left(n_l^t\right)+\theta_1\left(\upsilon_1-(\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)}\right)\left(\alpha^{n_l^t}-\beta^{n_l^t}p(n_l^t)\right)\Big\},$$

where $\lambda(S(n_l^t)) = (\lambda(\nu):\nu\in S(n_l^t))$ is the costate variable collection over set $S(n_l^t)$ appended by the manufacturer to the state variable r. We set the values $\lambda(\nu)=0$ for any $\nu\in S(n_l^T)$. The value of λ at node n_l^t corresponds to the shadow price or marginal value of the returns at that node. The following proposition characterizes the equilibrium strategies (superscipted with $\sim$) in this scenario.

Proposition 3 *Assuming an interior solution, the equilibrium GA and price values at node* $n_l^t\in\mathcal{N}^t$ *for* $t=0,\ 1,\ldots,T$ *are as follows:*

$$\tilde{G}(n_l^t) = \kappa^{-1}\ln\left[\frac{\theta_1\kappa\delta(\upsilon_1-\upsilon_2)}{2\zeta}\left(\alpha^{n_l^t}-\beta^{n_l^t}w\right)\Phi(n_l^t,\tilde{\lambda})\right],\quad n_l^t\in\mathcal{N}\setminus\mathcal{N}^T \tag{11}$$
$$\tilde{G}(n_l^T)=0,\quad n_l^T\in\mathcal{N}^T \tag{12}$$
$$\tilde{p}(n_l^t)=\frac{w}{2}+\frac{\alpha^{n_l^t}}{2\beta^{n_l^t}},\quad n_l^t\in\mathcal{N}, \tag{13}$$

where $\tilde{\lambda}(\cdot)$ *satisfies the following difference equation:*

$$\tilde{\lambda}(n_l^t)=\frac{1}{2}\left(\alpha^{n_l^t}-\beta^{n_l^t}w\right)c_0c_re^{-c_r r(n_l^t)}+\eta\delta\Phi(n_l^t,\tilde{\lambda}),\quad n_l^t\in\mathcal{N}\setminus\mathcal{N}^T,$$
$$\tilde{\lambda}(n_l^T)=\frac{1}{2}\left(\alpha^{n_l^T}-\beta^{n_l^T}w\right)c_0c_re^{-c_r r(n_l^T)},\quad n_l^T\in\mathcal{N}^T,$$

and $\Phi(n_l^t,\lambda)=\sum_{\nu\in S(n_l^t)}\frac{\pi^\nu}{\pi^{n_l^t}}\lambda(\nu)$.

Proof. See Appendix A. □

We make the two following comments:

1. As expected, the retail price is increasing in the market potential $\alpha^{n_l^t}$ and decreasing in consumer sensitivity to price given by $\beta^{n_l^t}$. Note that $p(n_l^t)$ is increasing in the wholesale price w, a result that has a strategic complementarity flavor, that is, increasing the value of one strategic variable leads to an increase in the other one. Of course, the interpretation here is a metaphor as w is a parameter and not a decision variable.
2. The equilibrium GA level is determined by the familiar rule of marginal cost equals marginal benefit. To see it, consider the following equilibrium condition (see Appendix):

$$\frac{\partial\mathcal{H}_M}{\partial G(n_l^t)} = -\zeta+\delta\sum_{\nu\in S(n_l^t)}\frac{\pi^\nu}{\pi^{n_l^t}}\lambda(\nu)\Big\{\kappa(\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)}\theta_1\left(\alpha^{n_l^t}-\beta^{n_l^t}p(n_l^t)\right)\Big\}=0.$$

The marginal cost is given by ζ. Note that

$$\kappa(\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)}\theta_1\left(\alpha^{n_l^t}-\beta^{n_l^t}p(n_l^t)\right)=\frac{\partial r\left(n_l^t\right)}{\partial G(n_l^t)},$$

which is the marginal increase in returns at node n_l^t due to the investment in GA. Consequently, the above equilibrium condition can be rewritten as

$$\zeta=\delta\sum_{\nu\in S(n_l^t)}\frac{\pi^\nu}{\pi^{n_l^t}}\lambda(\nu)\frac{\partial r\left(n_l^t\right)}{\partial G(n_l^t)}.$$

Recalling that $\lambda(\cdot)$ is the shadow price of the returns, the term $\sum_{\nu\in S(n_l^t)} \frac{\pi^\nu}{\pi^{n_l^t}}\lambda(\nu)$ is the expected average shadow price taken over all successor nodes of node n_l^t. Therefore, the right-hand side of the above equation is the average future marginal benefit due to investing in GA at node n_l^t, scaled by the discount factor δ.

3.2 Reverse revenue sharing contract scenario

In this scenario, at node $n_l^t \in \mathcal{N}^t$ for $t = 0, 1, \ldots, T$, the manufacturer transfers a part of its revenues $\phi\left(w - c_0 e^{-c_r r(n_l^t)}\right)$, where ϕ is a given parameter, and the retailer pays a percentage $B(n_l^t)$ of manufacturer's GA cost, where $B(n_l^t)$ is a strategic variable to be determined endogenously. In this scenario, the returns influence the retailer's payoff and therefore become a relevant variable.

Denote by μ_r the retailer's costate variable appended to the returns dynamics. Recall that λ_M is the costate variable appended by the manufacturer to the state dynamics, and let μ_λ be the costate variable appended by the retailer to the state equation describing the evolution of λ_M, which becomes an additional state variable in its optimization problem (see Appendix A for details). The following proposition characterizes the equilibrium strategies in the leader-follower dynamic game.

Proposition 4 *Denote by $B(n_l^t)$, $p(n_l^t)$, $n_l^t \in \mathcal{N}^t$, $t = 0, 1 \ldots, T$ a Stackelberg S-adapted equilibrium strategy for the retailer. Then, there exist finite vector sequences $(\mu_\lambda(n_l^t) : n_l^t \in \mathcal{N})$, $(\mu_r(n_l^t) : n_l^t \in \mathcal{N})$, $(\nu_\lambda(n_l^t) : n_l^t \in \mathcal{N})$ that satisfy the following relations:*

$$\begin{aligned}
&\alpha^{n_l^t} - \beta^{n_l^t}\Big\{2p(n_l^t) - (1-\phi)w - \phi c_0 e^{-c_r r(n_l^t)} + \delta\theta_1(\upsilon_1 - (\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)})\mu_r(n_l^t) \\
&\quad + (1-\phi)c_0 c_r e^{-c_r r(n_l^t)}\mu_\lambda(n_l^t) + \theta_1\kappa\delta(\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)}\Phi(n_l^t,\lambda_M)\nu(n_l^t)\Big\} = 0, \qquad (14)\\
&n_l^t \in \mathcal{N}\setminus\mathcal{N}^T,\\
&\alpha^{n_l^T} - \beta^{n_l^T}\Big\{2p(n_l^T) - (1-\phi)w - \phi c_0 e^{-c_r r(n_l^T)} + (1-\phi)c_0 c_r e^{-c_r r(n_l^T)}\mu_\lambda(n_l^T)\Big\} = 0, \qquad (15)\\
&n_l^t \in \mathcal{N}^T,\\
&-\zeta B(n_l^t) + \delta\theta_1\kappa(\upsilon_1-\upsilon_2)(\alpha^{n_l^t} - \beta^{n_l^t}p(n_l^t))e^{-\kappa G(n_l^t)}\Big\{\mu_r(n_l^t) - \kappa\nu(n_l^t)\Phi(n_l^t,\lambda_M)\Big\} = 0, \qquad (16)\\
&n_l^t \in \mathcal{N}\setminus\mathcal{N}^T,\\
&-\zeta(1-B(n_l^t)) + \delta\theta_1\kappa(\upsilon_1-\upsilon_2)(\alpha^{n_l^t} - \beta^{n_l^t}p(n_l^t))e^{-\kappa G(n_l^t)}\Phi(n_l^t,\lambda_M) = 0, \qquad (17)\\
&n_l^t \in \mathcal{N}\setminus\mathcal{N}^T,\\
&G(n_l^t) - \nu(n_l^t) = 0,\ \ n_l^t \in \mathcal{N}\setminus\mathcal{N}^T, \qquad (18)\\
&B(n_l^T) = \nu(n_l^T) = G(n_l^T) = 0,\ n_l^T \in \mathcal{N}^T, \qquad (19)\\
&r(n_l^{t+1}) = \eta r(n_l^t) + \theta_1(\upsilon_1 - (\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)})(\alpha^{n_l^t} - \beta^{n_l^t}p(n_l^t)),\ r(n^0) = 0, \qquad (20)\\
&n_l^t \in \mathcal{N}^t, t = 0, 1, \ldots, T-1,\\
&\lambda_M(n_l^t) = (1-\phi)c_0 c_r e^{-c_r r(n_l^t)}\left(\alpha^{n_l^t} - \beta^{n_l^t}p(n_l^t)\right) + \delta\eta\Phi(n_l^t,\lambda_M), \quad n_l^t \in \mathcal{N}\setminus\mathcal{N}^T, \qquad (21)\\
&\lambda_M(n_l^T) = (1-\phi)c_0 c_r e^{-c_r r(n_l^t)}\left(\alpha^{n_l^t} - \beta^{n_l^t}p(n_l^t)\right), \quad n_l^T \in\ \mathcal{N}^T, \qquad (22)\\
&\mu_r(n_l^t) = \delta\sum_{\nu\in S(n_l^t)}\frac{\pi^\nu}{\pi^{n_l^t}}\Big\{c_0 c_r e^{-c_r r(\nu)}\left(\alpha^\nu - \beta^\nu p(\nu)\right)\left(\phi - (1-\phi)c_r\mu_\lambda(\nu)\right) + \delta\eta\mu_r(\nu)\Big\}, \qquad (23)\\
&n_l^t \in \mathcal{N}\setminus\mathcal{N}^T,\\
&\mu_r(n_l^T) = 0,\ n_l^T \in \mathcal{N}^T, \qquad (24)\\
&\mu_\lambda(n_m^{t+1}) = \eta\mu_\lambda(n_l^t) + \theta_1\kappa(\upsilon_1-\upsilon_2)(\alpha^{n_l^t} - \beta^{n_l^t}p(n_l^t))e^{-\kappa G(n_l^t)}, \qquad (25)\\
&n_m^{t+1} \in \mathcal{N}^{t+1}, t = 0, \ldots, T-1, \quad n_l^t = a(n_m^{t+1}),\\
&\mu_\lambda(n^0) = 0, \qquad (26)
\end{aligned}$$

where $\Phi(n_l^t, \lambda_M) = \sum_{\nu \in S(n_l^t)} \frac{\pi^\nu}{\pi^{n_l^t}} \lambda_M(\nu)$. *Furthermore,* $G(n_l^t)$, $n_l^t \in \mathcal{N}^t$, $t = 0, 1 \ldots, T$ *is the corresponding S-adapted Stackelberg strategy of the manufacturer, and* $r(n_l^t)$, $n_l^t \in \mathcal{N}^t$, $t = 0, 1, \ldots, T$, *is the returns trajectory associated with the Stackelberg solution.*

Proof. See Appendix A. □

The above proposition gives the conditions that determine the Stackelberg equilibrium. As it can be seen in the Appendix, the first step is to obtain the reaction function of the follower (manufacturer) to the leader's (retailer's) announcement. Next, we insert the follower's reaction function in the leader's optimization problem and solve it. Interpreting the relationships in Proposition 4 is harder than interpreting the equations in Proposition 3 because the two players' optimization problems are coupled and an additional state variable and corresponding costate variable must be introduced (see Appendix A for details). To obtain additional insights into the two equilibrium solutions and their implications for the two players, we resort to numerical illustrations.

4 Illustrative examples

Although we can solve for any finite event tree, we retain a binary tree, i.e., any node has two descendants or possible realizations of future parameter values. First, we consider an example with 6 periods ($T = 5$). The event tree is depicted in Figure 1 and Table 1 summarizes some useful information of its structure. In the numerical simulations, the probability of realization of a left-handed (right-handed) successor of a node is constant over time and equals 1/2 (1/2). Next, we extend the planning horizon to $T = 15$. More specifically, in Section 4.1 we will discuss and compare the results for short and long planning horizons.

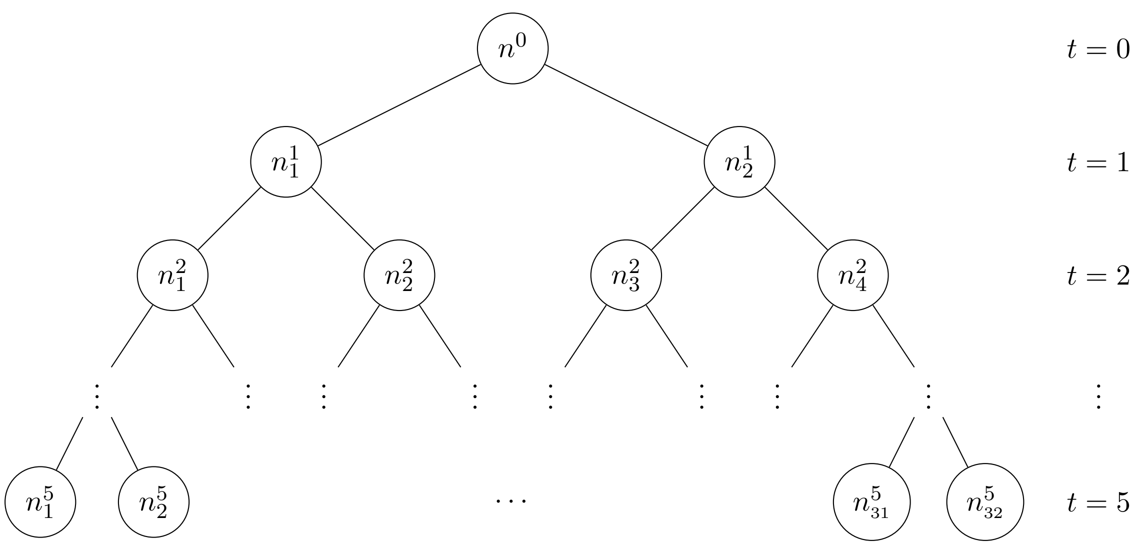


Figure 1: The event tree, $T = 5$

Table 1: Important numbers on the binary event tree

Number of stages	$T + 1$
Number of nodes in the last stage	2^T
Total number of scenarios	2^T
Total number of nodes	$2^{T+1} - 1$

The model has the following parameters:

$$
\begin{aligned}
\text{Stochastic demand parameters} &: \alpha, \beta \\
\text{Cost parameters} &: c_0, c_r, \zeta \\
\text{Return dynamics parameters} &: \eta, \theta_1, \kappa, \upsilon_1, \upsilon_2 \\
\text{Wholesale price} &: w \\
\text{Sharing parameter} &: \phi \\
\text{Discount rate} &: \delta
\end{aligned}
$$

In the rest, we shall fix once for all the values of three parameters, namely, c_0, ζ and δ, whose impact on the results is expected to be purely quantitative, without much qualitative insight. More precisely, we normalize the two cost parameters to one ($c^0 = \zeta = 1$) and set the discount factor $\delta = 0.9$.

Let the parameters characterizing the dynamics of returns be as follows: $r_0 = 0$ (no returns at the root node n^0), $\eta = 0.4$ (rate of geometrical decay of returns), $\theta_1 = 0.6$ (the initial level of returns). The propensity of consumers to bring back used products is characterized by three parameters κ, υ_1 and υ_2. We fix the values as $\kappa = 5$, $\upsilon_1 = 0.9$ and $\upsilon_2 = 0.1$. The graph of function $f(G)$ is depicted on Figure 2.

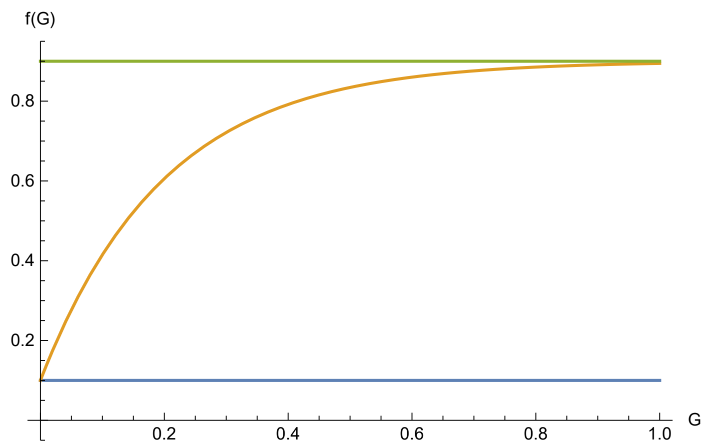


Figure 2: Graph of function $f(G) = 0.9 - 0.8e^{-5G}$ (orange) with limit values $\upsilon_2 = 0.1$ (blue) and $\upsilon_1 = 0.9$ (green)

The stochastic demand parameters evolve as follows along the tree:

$$
\begin{aligned}
&\alpha^{n_l^t} = (1 \pm \rho_\alpha) \cdot \alpha^{a(n_l^t)}, && \alpha^{n^0} = 5, && \rho_\alpha = 0.05, && t = 1, \ldots, T, \\
&\beta^{n_l^t} = (1 \mp \rho_\beta) \cdot \beta^{a(n_l^t)}, && \beta^{n^0} = 0.8, && \rho_\beta = 0.01, && t = 1, \ldots, T.
\end{aligned}
$$

The positive variation rate for α and the negative variation rate for β are applied to left-handed pointing successor nodes, and the negative rate for α and positive variation rate for β to right-handed pointing successor nodes, for $t = 0, \ldots, T$. In the following simulations, we shall consider the event tree made of $T + 1 = 6$ periods and the initial demand parameters are as given in Table 2.

Table 2: Demand parameters over the tree

Parameters	α	β
Value at root	5	0.8
Variation rate	0.05	0.01

In the base case, we fix three parameters $c_r = 0.5$, $w = 4$, and $\phi = 0.3$ and solve for both the benchmark and the RRSC scenarios. The profits of the manufacturer and retailer are given in Table 3.

Table 3: Profits for benchmark and RRSC scenarios in the base case

Scenario	Benchmark	RRSC
Parameters	$c_r = 0.5, w = 4, \phi = 0.3, c_0 = \zeta = 1, \delta = 0.9$	
Manufacturer	13.690	14.580
Retailer	5.035	10.410

The equilibrium strategies of the manufacturer G (for both scenarios), and the retailer p (for both scenarios) and B (for cost revenue sharing scenario) are given in Table 4 for two realizations of the stochastic demand parameters. The first realization of the stochastic demand (nodes n^0, n_1^1, n_1^2, n_1^3, n_1^4, n_1^5) is the most favorable as only left-handed pointing successors are realized. The second realization of the stochastic demand (nodes n^0, n_2^1, n_4^2, n_8^3, n_{16}^4, n_{32}^5) is the least favorable one as only right-handed pointing successors are realized. Figure 3 shows the trajectories of p, G, Q and r for one path along the event tree.

Table 4: Equilibrium strategies, return trajectory and demand trajectory for benchmark and RRSC scenarios in positive (upper table) and negative (lower table) realization of stochastic demand parameters

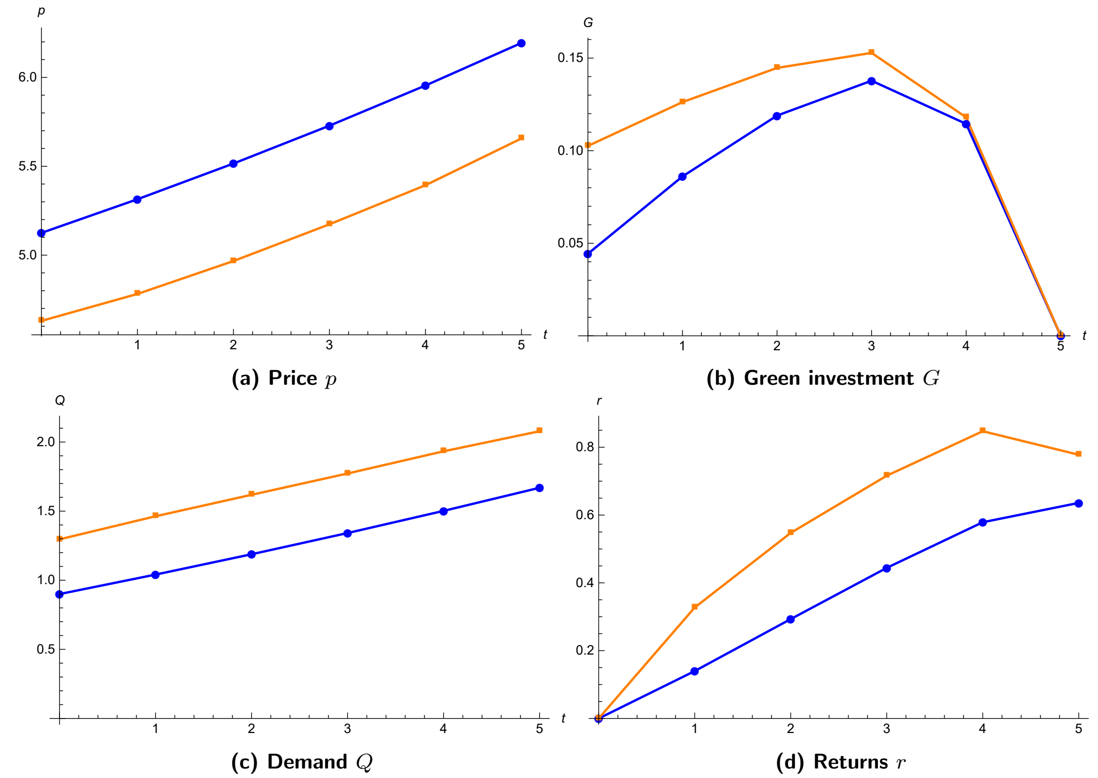
***Positive** realization of stochastic demand*									
Scenario	Benchmark				RRSC				
Parameters	$c_r = 0.5, w = 4, \phi = 0.3, c_0 = \zeta = 1, \delta = 0.9$								
Node	G	p	r	Q	G	p	B	r	Q
n^0	0.044	5.125	0.000	0.900	0.103	4.630	0.000	0.000	1.296
n_1^1	0.086	5.314	0.140	1.041	0.126	4.782	0.000	0.327	1.463
n_1^2	0.119	5.515	0.293	1.188	0.144	4.967	0.000	0.547	1.618
n_1^3	0.138	5.728	0.444	1.342	0.151	5.173	0.000	0.716	1.773
n_1^4	0.114	5.954	0.579	1.502	0.118	5.401	0.000	0.844	1.927
n_1^5	0.000	6.194	0.636	1.669	0.000	5.666	0.000	0.865	2.071
***Negative** realization of stochastic demand*									
Scenario	Benchmark				RRSC				
Parameters	$c_r = 0.5, w = 4, \phi = 0.3, c_0 = \zeta = 1, \delta = 0.9$								
Node	G	p	r	Q	G	p	B	r	Q
n^0	0.044	5.125	0.000	0.900	0.103	4.630	0.000	0.000	1.296
n_2^1	0.000	4.939	0.140	0.759	0.062	4.411	0.000	0.327	1.186
n_4^2	0.000	4.765	0.101	0.624	0.037	4.241	0.112	0.353	1.052
n_8^3	0.000	4.601	0.078	0.495	0.003	4.103	0.252	0.290	0.905
n_{16}^4	0.000	4.446	0.061	0.371	0.000	3.971	0.155	0.176	0.767
n_{32}^5	0.000	4.301	0.047	0.253	0.000	3.838	0.000	0.117	0.642

For the next three sets of simulations, the aim is to investigate the impact of different values of ϕ, w, and c_r on the expected payoffs separately.

More precisely, in the first run we investigate the impact of changing ϕ on the expected payoffs of the manufacturer and the retailer in RRSC scenario. In these simulations, the idea is to vary ϕ around the base case value while the other parameters are kept at their levels in the base case (see Table 5). Thus, we have $\phi \in \{0.05, 0.1, 0.2, 0.3, 0.4, 0.5\}$.

Table 5: Impact of ϕ on the expected payoffs of the players

Parameters	$c_r = 0.5, w = 4, c_0 = \zeta = 1, \delta = 0.9$						
	RRSC						Benchmark
ϕ	**0.05**	**0.10**	**0.20**	**0.30**	**0.40**	0.50	NA
Manufacturer	**14.054**	**14.370**	**14.702**	**14.580**	**13.991**	12.9097	13.690
Retailer	**5.759**	**6.551**	**8.343**	**10.410**	**12.752**	15.3635	5.035



(a) Price p **(b) Green investment G**

(c) Demand Q **(d) Returns r**

Figure 3: Equilibrium values of p, G, Q and r with the benchmark (blue) and RRSC (orange) scenarios while positive realization of stochastic parameters of demand along the event tree with $T = 5$

In the next run, we aim at studying the impact of w on players' expected payoffs. We take two values below the base case and two others above for the parameter $w = 4$ while the other two parameters are kept at their levels in the base case (see Table 6). Consequently, we have $w \in \{3.50, 3.75, 4.00, 4.25\}$.

Table 6: Impact of w on the expected payoffs of the players

Parameters	$c_r = 0.5, \phi = 0.3, c_0 = \zeta = 1, \delta = 0.9$			
w	3.50	3.25	4.00	4.25
	RRSC			
Manufacturer	14.035	14.372	14.580	14.669
Retailer	12.809	11.582	10.410	9.298
	Benchmark			
Manufacturer	14.669	14.268	13.690	12.907
Retailer	7.378	6.148	5.035	4.039

Finally, we examine the impact of varying c_r on the players' payoffs. We consider $c_r \in \{0.3, 0.4, 0.5, 0.6, 0.7\}$ and keep the other parameters at their levels in the base case. The results of the simulations are given in Table 7.

Note that in each simulation, expected payoff of each player should be compared with her associated benchmark payoff. Clearly, the benchmark results are independent of ϕ, which is an irrelevant parameter in this scenario. For other simulations the benchmark results are given in two bottom rows of Tables 6 and 7.

Payoff wise comparisons lead to the following immediate observations based on these simulations results:

Table 7: Impact of c_r on the expected payoffs of the players

Parameters	$w = 4, \phi = 0.3, c_0 = \zeta = 1, \delta = 0.9$				
c_r	0.3	0.4	0.5	0.6	0.7
			RRSC		
Manufacturer	13.518	14.125	14.580	14.923	15.180
Retailer	9.996	10.226	10.410	10.547	10.641
			Benchmark		
Manufacturer	13.109	13.387	13.690	13.929	14.141
Retailer	5.035	5.035	5.035	5.035	5.035

1. Table 4 gives the following observations. The green investments in the benchmark scenario are larger compared to RRSC scenario along the most positive realization of demand parameters, but the returns are larger in RRSC scenario compared to the benchmark scenario for both positive and negative stochastic realization of demand parameters (except in the last stage in positive realization). This result is due to the lower price and larger demand in the RRSC scenario in comparison with the benchmark scenario. For consumers, RRSC scenario is preferable because the price is lower compared to the benchmark scenario. If we compare the positive and negative realizations of stochastic demand parameters, we observe that in positive realization strategy B decreases over time until one stage before the terminal one, and increases at the stage 4. In negative realization, strategy B of the retailer increases until the terminal stage 5. As expected, the green investments decrease over time in both scenarios.
2. In the RRSC scenario, the retailer prefers higher sharing parameter ϕ, while the manufacturer is more interested in lower levels of ϕ, which is fully expected (see Table 5).
3. From Table 6 we observe that the retailer prefers lower w in both scenarios, benchmark and RRSC, while the manufacturer prefers higher w in both scenarios, benchmark and RRSC. Again, this result is intuitive.
4. Table 7 shows that only the manufacturer benefits from a higher c_r in the benchmark scenario. The retailer does not care about c_r in this scenario, because it is not directly affected by the value of this parameter. Both players, the retailer and the manufacturer, prefer higher c_r in the RRSC scenario.

Based on the above last item, we may conclude that whereas both players' perspectives are aligned regarding c_r, they hold opposite views on the preferable levels of w and ϕ. In particular, if w is too low, then the manufacturer loses with a RRSC contract in comparison with benchmark scenario. It is the opposite for the retailer. This shows that a RRSC is not always a Pareto-improving contract. For the higher wholesale price, $w = 4$, it is Pareto improving when the sharing parameter is not too high (see Table 5). The bold entries in Table 5 show the Pareto-improving cases (in particular, for $\phi \leq 0.43$). To warp up, qualitatively speaking, both players prefer a RRSC contract to the benchmark when the wholesale price is sufficiently high and the sharing parameter sufficiently low. When the wholesale price is high enough, the retailer finds it worthwhile to financially support the manufacturer's GA efforts to receive a discount on the wholesale price. Clearly, the higher its support is, the higher the discount will be. On the other hand, the manufacturer prefers a RRSC contract when the discount is not so high, i.e., the manufacturer does not find it rational to condone a huge share of its profit even when it is charging the retailer a high wholesale price and the latter is paying a significant portion of GA efforts' cost.

In Appendix B, we also show the impact of parameters ζ, c_0 and η on players' payoffs.

4.1 Short vs. long planning horizon

In this section, we look at the impact of length of the planning horizon on values of the variables in the model. We let $T = 15$, and assume that for $t = 0, \ldots, 5$, we have the binary event in Figure 1, and for $t = 6, \ldots, 15$, we prolong each branch until the end, that is, the parameter values after period

5 are kept at their levels at $t = 5$. Consequently, we have $2^5 = 32$ scenarios. We suppose that we have equal probabilities for each of the two branches stemmed from node n^t, $t = 0, \ldots, 4$. The event tree is depicted in Figure 4 and Table 8 summarizes some useful information about the tree structure.

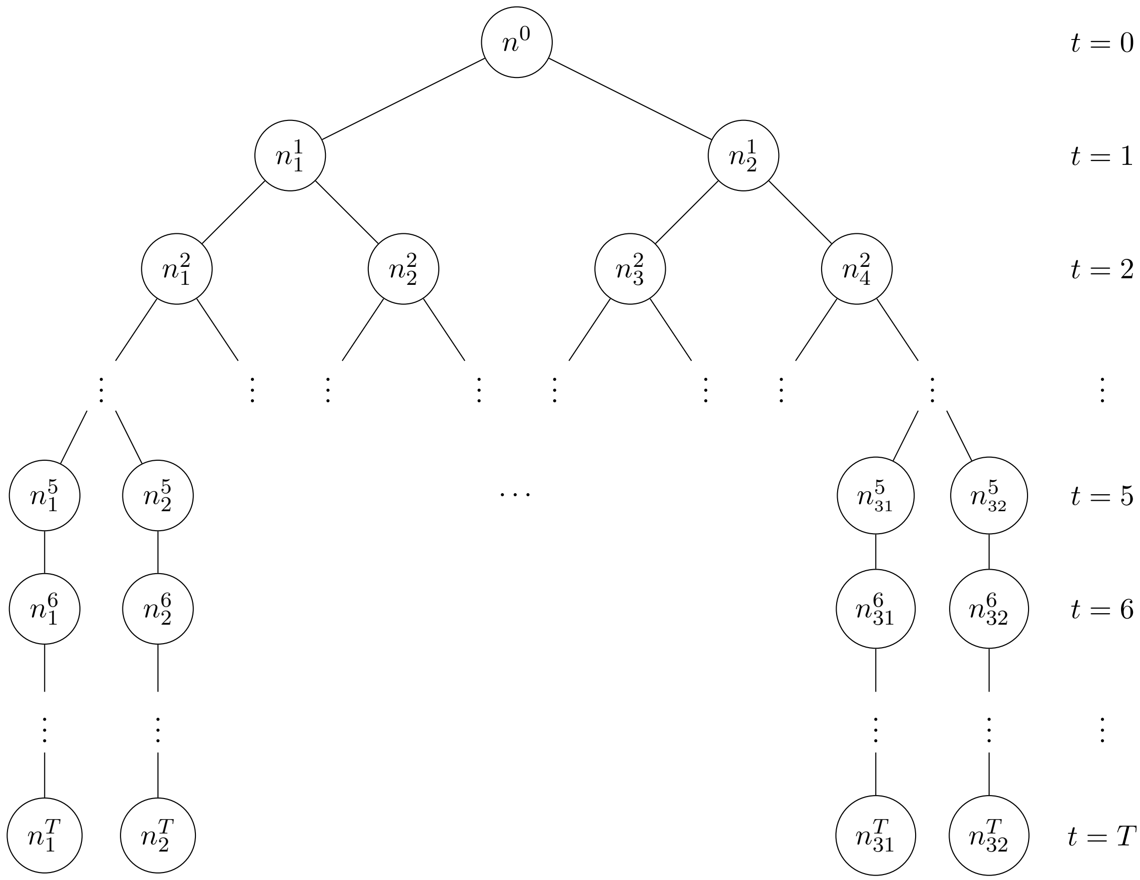


Figure 4: Event tree for long horizon analysis, $T = 15$

Table 8: Important numbers of the extended tree

Number of stochastic nodes	63
Number of nodes in the last stochastic stage = Total number of scenarios	32
Number of deterministic nodes	$32(T-5)$
Total number of nodes	$32(T-3)-1$

The demand parameters are given in Table 2. The positive variation rate for α and the negative variation rate for β are applied to left-handed pointing successor nodes, and the negative rate for α and positive variation rate for β to right-handed pointing successor nodes for $t = 0, \ldots, 5$. For $t = 6, \ldots, T$, the values of α and β remain at their levels at $t = 6$. The stochastic demand parameters evolve as follows along the tree:

$$\alpha^{n^t_l} = (1 \pm \rho_\alpha) \cdot \alpha^{a\left(n^t_l\right)}, \qquad \alpha^{n^0} = 5, \qquad \rho_\alpha = 0.05, \qquad t = 1, \ldots, 5, \tag{27}$$

$$\alpha^{n^t_l} = \alpha^{a\left(n^t_l\right)}, \qquad t = 6, \ldots, T, \tag{28}$$

$$\beta^{n^t_l} = (1 \mp \rho_\beta) \cdot \beta^{a\left(n^t_l\right)}, \qquad \beta^{n^0} = 0.8, \qquad \rho_\beta = 0.01, \qquad t = 1, \ldots, 5. \tag{29}$$

$$\beta^{n^t_l} = \beta^{a\left(n^t_l\right)}, \qquad t = 6, \ldots, T. \tag{30}$$

We fix $c_0 = \zeta = 1, \delta = 0.9, c_r = 0.5, w = 4$, and $\phi = 0.3$ as before. The results of the two scenarios, benchmark and RRSC over long horizon with the most favorable realization of stochastic parameters of demand (the scenario terminating at node n_1^{15}) and most negative (the scenario terminating at node n_{32}^{15}) are introduced in Table 13 (see Appendix C).

At a first glance, the results show that, regardless of the length of the planning horizon, the price is always lower in the RRSC scenario compared to the benchmark scenario, which leads to higher demand in the RRSC scenario. We have higher return rate in the RRSC scenario for both lengths of the game. The reason is in significantly higher demand in RRSC scenario and in larger green investments until termination with short horizon and until $t = 5$ with long horizon. In the $T = 15$ case, the green investments are slightly higher in benchmark scenario than in RRSC scenario after $t = 5$ (see Table 13 in Appendix C).

In order to visualize the difference between the results in the different scenarios, we use the most favorable scenario that starts at root node and terminates at node n_1^{15}, that is, the scenario in which the demand parameter α always increases while β always decreases. As shown in Figure 5, we have lower price in RRSC scenario at each node compared to the price at the same node in benchmark scenario. The demand at each node is higher in RRSC scenario compared to the same node in the benchmark scenario. The results are qualitatively the same for any other complete scenario over the event tree. All the variables follow the same trend over short planning horizon as well.

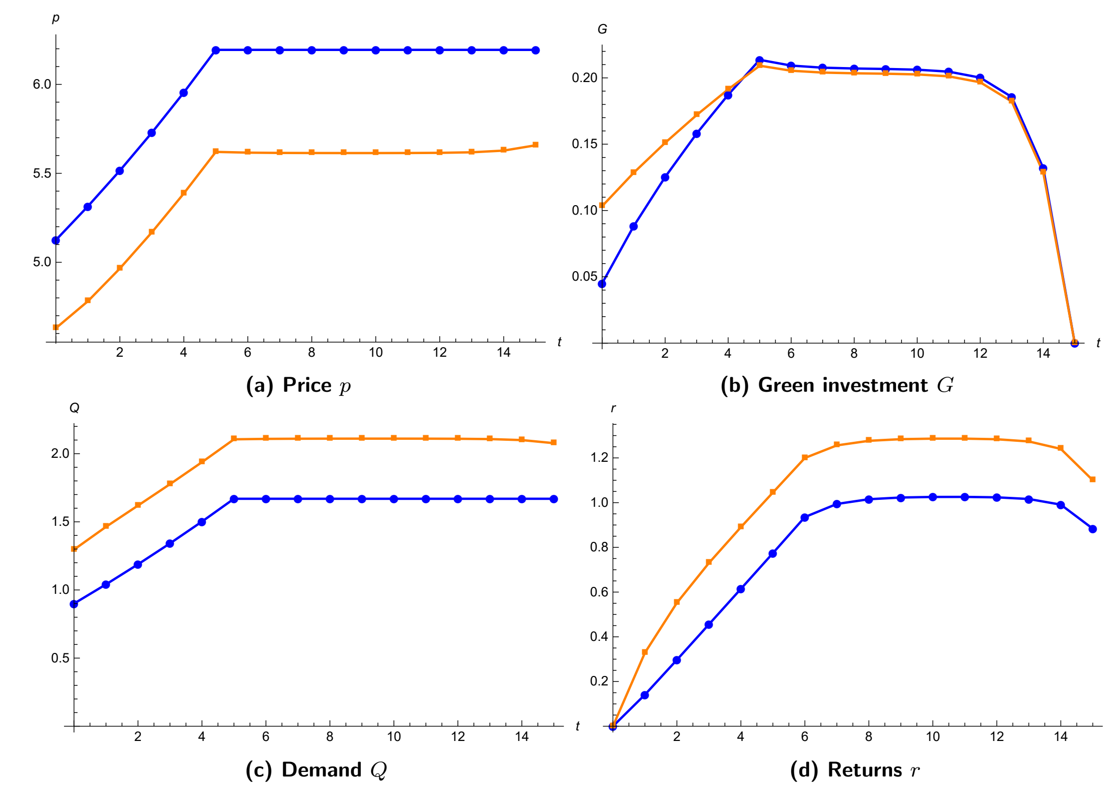
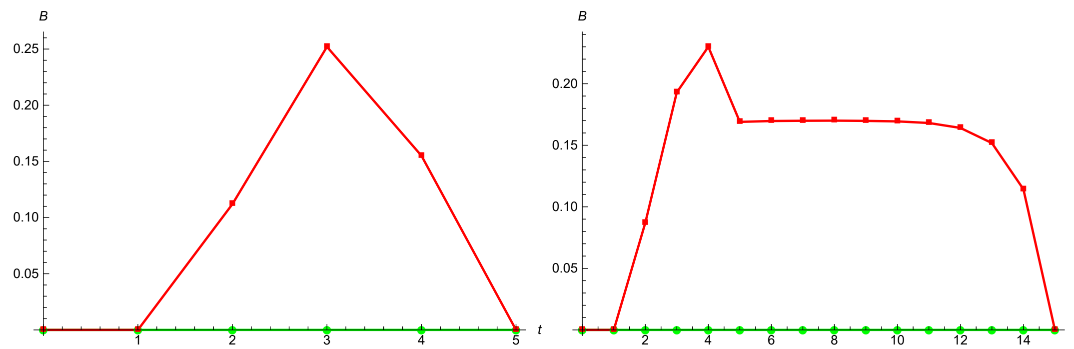


Figure 5: Equilibrium values of p, G, Q and r with the benchmark (blue) and RRSC (orange) scenarios while positive realization of stochastic parameters of demand along the event tree with $T = 15$

Getting closer to the end of planning horizon, in RRSC scenarios, regardless of the length of the planning horizon, we have lower demand in the last stage, which is due to setting higher prices. Another interesting observation about RRSC scenarios is that in short planning horizon we have lower demand (higher price) in stage $t = 5$ (the last stage) compared to the demand (price) in the same stage in long planning horizon (an intermediate stage $t = 5$). Hence, we may conclude that under RRSC contract, regardless of the length of the planning horizon, getting closer to the end of planning horizon, we face a lower demand (higher price) level; see also Table 13 in Appendix C.

Figure 6 demonstrates the evolution of B over time in RRSC scenario. We show the results with the positive and negative realizations, depicted in green and red colors, respectively. We have the same qualitative relations for short and long horizon. With both length planning horizons, in positive realization of the demand stochastic parameters, strategy B equals zero along the trajectory. And in negative realization it increases and then decreases while at the terminal node it equals zero (see the values in Table 13 in Appendix C).



(a) $T = 5$ **(b)** $T = 15$

Figure 6: Equilibrium values of B in RRSC scenarios while positive (green) and negative (red) realizations of stochastic parameters of demand along the event tree

5 Conclusion

This paper is the first attempt to analyze a RRSC contract in a dynamic game with uncertain parameter values. The model included an endogenous return process, with the returns directly affecting the manufacturer's production cost. We characterized and contrasted the equilibrium strategies in two different scenarios, i.e., RRSC and benchmark, where there is no sharing of the revenue nor the cost of the green activities done to incentivize consumers to return the product. Based on numerical experiments, we highlighted the conditions under which a RRSC contract is Pareto improving with respect to the benchmark case.

In any modeling effort, some assumptions are made for mathematical tractability, or simply to focus on the most important issues under consideration. We believe that some extensions in terms of modeling are worth conducting. We assumed that cost saving is the main purpose of the supply chain. It has been shown in the literature that the customers who return a product, usually purchase a new one De Giovanni (2014). One interesting extension would be to let the returns both influence the production cost and the demand. Also, we assumed that the a product can be reused in the manufacturing process at will. It may well be the case that some used parts could only be exploited few times. Another implicit assumption is that the returned products are remanufactured and sold as new product and the customers do not distinguish between the two versions of the product, i.e., there is no secondary market. Investigating the results without these simplifying assumptions would be interesting though challenging.

The other avenue for the research is to investigate the effect of offering some monetary incentives to the consumers to return the product. Differently, a penalty can be added per non-retuned product. Clearly, such modification will have an impact on the modeling as well as on the results.

Finally, competition might be another interesting factor to investigate. One may consider a situation with more than one CLSC whose decisions influence each other. In addition to demand level, they may compete in the collection of end-of-use products.

Appendix A

5.1 Proof of Proposition 1

Substituting $\frac{\theta_{i+1}}{\theta_i} = \eta$ in (2), we obtain

$$\begin{aligned} r\left(n_l^t\right) =&\theta_1 \times f\left(G\left(n_{i_{t-1}}^{t-1}\right)\right) \times Q\left(n_{i_{t-1}}^{t-1}\right) + \theta_1\eta \times f\left(G\left(n_{i_{t-2}}^{t-2}\right)\right) \times Q\left(n_{i_{t-1}}^{t-2}\right) \\ &+ \theta_1\eta^2 \times f\left(G\left(n_{i_{t-3}}^{t-3}\right)\right) \times Q\left(n_{i_{t-2}}^{t-3}\right) + \ldots + \theta_1\eta^{t-1} \times f\left(G\left(n^0\right)\right) \times Q\left(n^0\right), \end{aligned} \tag{31}$$

Consider one-period lag

$$\begin{aligned} r\left(a\left(n_l^t\right)\right) =&\theta_1 \times f\left(G\left(n_{i_{t-2}}^{t-2}\right)\right) \times Q\left(n_{i_{t-1}}^{t-2}\right) + \theta_1\eta \times f\left(G\left(n_{i_{t-3}}^{t-3}\right)\right) \times Q\left(n_{i_{t-2}}^{t-3}\right) \\ &+ \theta_1\eta^2 \times f\left(G\left(n_{i_{t-4}}^{t-4}\right)\right) \times Q\left(n_{i_{t-3}}^{t-4}\right) + \ldots + \theta_1\eta^{t-2} \times f\left(G\left(n^0\right)\right) \times Q\left(n^0\right), \end{aligned} \tag{32}$$

and multiply the above result by η, to get

$$\begin{aligned} \eta r\left(a\left(n_l^t\right)\right) =&\theta_1\eta \times f\left(G\left(n_{i_{t-2}}^{t-2}\right)\right) \times Q\left(n_{i_{t-1}}^{t-2}\right) + \theta_1\eta^2 \times f\left(G\left(n_{i_{t-3}}^{t-3}\right)\right) \times Q\left(n_{i_{t-2}}^{t-3}\right) \\ &+ \theta_1\eta^3 \times f\left(G\left(n_{i_{t-3}}^{t-3}\right)\right) \times Q\left(n_{i_{t-2}}^{t-3}\right) + \ldots + \theta_1\eta^{t-1} \times f\left(G\left(n^0\right)\right) \times Q\left(n^0\right). \end{aligned} \tag{33}$$

Compute the difference between (31) and (33) to obtain

$$r\left(n_l^t\right) - \eta r\left(a\left(n_l^t\right)\right) = \theta_1 \times f\left(G\left(n_{i_{t-1}}^{t-1}\right)\right) \times Q\left(n_{i_{t-1}}^{t-1}\right),$$

or equivalently

$$r\left(n_l^t\right) = \eta r\left(a\left(n_l^t\right)\right) + \theta_1 \times f\left(G\left(a\left(n_l^t\right)\right)\right) \times Q\left(a\left(n_l^t\right)\right), \quad r\left(n^0\right) = 0.$$

5.2 Proof of Proposition 2

Consider the total returns along the path from the root node n^0 to the terminal node $n^T \in \mathcal{N}^T$, that is, $\mathcal{P}(n^T) = (n^0, n^1, \ldots, n^T)$, where $n^t \in S(a(n_{t-1}))$ for any $t = 1, \ldots, T$, which are given by

$$r(\mathcal{P}(n^T)) = \sum_{\nu \in \mathcal{P}(n^T)} r(\nu), \tag{34}$$

when $r(\cdot)$ satisfies the dynamics in (31) with initial condition $r(n^0) = 0$. Substituting r from (31) into (34) and grouping terms, we obtain

$$\begin{aligned} r(\mathcal{P}(n^T)) =&Q(n^0)f(G(n^0))\theta_1\left(1 + \eta + \ldots + \eta^{T-1}\right) \\ &+ Q(n^1)f(G(n^1))\theta_1\left(1 + \eta + \ldots + \eta^{T-2}\right) \\ &+ \ldots \\ &+ Q(n^{T-1})f(G(n^{T-1}))\theta_1. \end{aligned} \tag{35}$$

The coefficient of $Q(n^0)$ can be interpreted as a share of sales at node n^0 that is totally returned along the path. It is natural to constraint this share to be no larger than one. As the upper bound of $f\ (\cdot)$ is υ_1, we need to satisfy the inequality:

$$\theta_1\upsilon_1\left(1 + \eta + \ldots + \eta^{T-1}\right) \leq 1,$$

which is equivalent to

$$\theta_1\upsilon_1 \leq \frac{1-\eta}{1-\eta^{T-1}}.$$

If this condition is satisfied, the returns from sales in any further nodes on the path will be smaller than the sales because $1 + \eta + \ldots + \eta^{T-1} \geq 1 + \eta + \ldots + \eta^t$ for any $0 < t < T - 1$.

Proof of Proposition 3

The retailer's optimization problem is given by

$$\max_{p(n_l^t)\geq 0} J_R\left(p\right) = Q(n^0)\Big(p(n^0) - w\Big) + \sum_{t=1}^{T} \sum_{n_l^t \in \mathcal{N}^t} \pi\left(n_l^t\right) \delta^t \Big\{ Q(n_l^t) \Big(p(n_l^t) - w \Big) \Big\},$$

which is independent of the manufacturer's decision variable G and of the state variable r. Assuming an interior solution, the first-order optimality condition at node n_l^t reads

$$\frac{dJ_R}{dp(n_l^t)} = \pi^{n_l^t} \left(\alpha^{n_l^t} - 2\beta^{n_l^t} p(n_l^t) + \beta^{n_l^t} w \right) = 0,$$

which yields

$$p(n_l^t) = \frac{\alpha^{n_l^t} + \beta^{n_l^t} w}{2\beta^{n_l^t}}, \quad n_l^t \in \mathcal{N}^t, t = 0, \ldots, T. \tag{36}$$

Clearly, $p(n_l^t) > 0$ and as $J_R\left(p\right)$ is concave in $p(n_l^t)$, we have an interior maximum. We should mention that $p(n_l^t)$ should be not less than w. Therefore, we need to satisfy condition

$$\frac{\alpha^{n_l^t}}{\beta^{n_l^t}} \geq w$$

for any node $n_l^t \in \mathcal{N}^t, t = 0, \ldots, T$.

Introduce the manufacturer's pre-Hamiltonian

$$\begin{aligned} \mathcal{H}_M(n_l^t, \lambda(S(n_l^t)), r(n_l^t), G(n_l^t)) &= \left(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t) \right) \left(w - c_0 e^{-c_r r(n_l^t)} \right) - \zeta G(n_l^t) \\ &+ \delta \sum_{\nu \in S(n_l^t)} \frac{\pi^\nu}{\pi^{n_l^t}} \lambda(\nu) \delta \Big\{ \eta r\left(n_l^t\right) + \theta_1 \left(\upsilon_1 - (\upsilon_1 - \upsilon_2) e^{-\kappa G(n_l^t)} \right) \left(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t) \right) \Big\}, \end{aligned}$$

defined for any node $n_l^t \in \mathcal{N}^t, t = 0, \ldots, T$, where $\lambda(S(n_l^t))$ is the vector of costate variables.

For any non-terminal node we differentiate $\mathcal{H}_M$ with respect to $G(n_l^t)$ and equating it to zero, we obtain

$$\frac{\partial \mathcal{H}_M}{\partial G(n_l^t)} = -\zeta + \delta \sum_{\nu \in S(n_l^t)} \frac{\pi^\nu}{\pi^{n_l^t}} \lambda(\nu) \Big\{ \theta_1 \kappa (\upsilon_1 - \upsilon_2) e^{-\kappa G(n_l^t)} \left(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t) \right) \Big\} = 0,$$

$$\Leftrightarrow G(n_l^t) = \kappa^{-1} \ln \left[\frac{\theta_1 \kappa \delta (\upsilon_1 - \upsilon_2)}{2\zeta} \left(\alpha^{n_l^t} - \beta^{n_l^t} w \right) \left(\sum_{\nu \in S(n_l^t)} \frac{\pi^\nu}{\pi^{n_l^t}} \lambda(\nu) \right) \right], \quad n_l^t \in \mathcal{N} \setminus \mathcal{N}^T.$$

We introduce the operator

$$\Phi(n_l^t, \lambda) = \sum_{\nu \in S(n_l^t)} \frac{\pi^\nu}{\pi^{n_l^t}} \lambda(\nu), \tag{37}$$

so the strategy $G(n_l^t)$ can be rewritten in the following way:

$$G(n_l^t) = \kappa^{-1} \ln \left[\frac{\theta_1 \kappa \delta (\upsilon_1 - \upsilon_2)}{2\zeta} \left(\alpha^{n_l^t} - \beta^{n_l^t} w \right) \Phi(n_l^t, \lambda) \right], \quad n_l^t \in \mathcal{N} \setminus \mathcal{N}^T. \tag{38}$$

At the terminal node $n_l^T \in \mathcal{N}^T$, the manufacturer's strategy does not influence on the state variable r at any node of period T. As the payoff function of the manufacturer is linear and decreasing in G, then his equilibrium strategy equals zero for any terminal node:

$$G(n_l^T) = 0, \quad n_l^t \in \mathcal{N}^T. \tag{39}$$

The costate variables are derived using the following system of equations,

$$\lambda(n_l^t) = \frac{\partial \mathcal{H}_M}{\partial r(n_l^t)} = \frac{c_0 c_r}{2} e^{-c_r r(n_l^t)} \left(\alpha^{n_l^t} - \beta^{n_l^t} w\right) + \delta\eta\Phi(n_l^t, \lambda), \quad n_l^t \in \mathcal{N} \setminus \mathcal{N}^T, \tag{40}$$

$$\lambda(n_l^T) = \frac{\partial \mathcal{H}_M}{\partial r(n_l^t)} = \frac{c_0 c_r}{2} e^{-c_r r(n_l^t)} \left(\alpha^{n_l^t} - \beta^{n_l^t} w\right), \quad n_l^T \in \mathcal{N}^T,$$

where we substitute $p(n_l^t)$ given by (36).

The transversality condition is $\lambda(\nu) = 0$ for any node $\nu \in S(n^t)$, meaning that returns after period T have no value to the manufacturer.

Substituting (40) in the expression of $G(n_l^t)$ leads to the results in the Proposition.

Proof of Proposition 4

We first determine the optimal control problem of the manufacturer to the announcement of the retailer of a retail price $p(n_l^t)$ and a support rate $B(n_l^t)$ for any $n_l^t \in \mathcal{N}^t$, $t = 1, \ldots, T$. The manufacturer aims to maximize

$$J_M(G, p, B, r) = \sum_{t=0}^{T} \sum_{n_l^t \in \mathcal{N}^t} \pi^{n_l^t} \delta^t \Big(\big(w - c(r(n_l^t)) - I(r(n_l^t))\big) Q(n_l^t) - (1 - B(n_l^t)) d(G(n_l^t)) \Big)$$

with $G(n_l^t) \geq 0$ with respect to returns dynamics

$$r(n_l^{t+1}) = \eta r(n_l^t) + \theta_1(\upsilon_1 - (\upsilon_1 - \upsilon_2) e^{-\kappa G(n_l^t)})(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t)), \; r(n^0) = r_0 \text{ given.} \tag{41}$$

The manufacturer's Hamiltonian function for any node $n_l^t \in \mathcal{N} \setminus \mathcal{N}^T$ is as follows:

$$\begin{aligned}
&\mathcal{H}_M(n_l^t, \lambda_M(S(n_l^t)), r(n_l^t), G(n_l^t), p(n_l^t), B(n_l^t)) = (1 - \phi) \left(w - c_0 e^{-c_r r(n_l^t)}\right) \left(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t)\right) \\
&- \zeta \left(1 - B(n_l^t)\right) G(n_l^t) + \delta\Phi(n_l^t, \lambda_M) \left[\eta r(n_l^t) + \theta_1(\upsilon_1 - (\upsilon_1 - \upsilon_2) e^{-\kappa G(n_l^t)}) \left(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t)\right)\right],
\end{aligned}$$

where $\lambda_M(\cdot)$ is the costate variable appended by the manufacturer to the state dynamics in (41), and operator Φ is defined by (37). For any $n_T \in \mathcal{N}^T$ function $\mathcal{H}_M$ is defined as:

$$\begin{aligned}
\mathcal{H}_M(n_l^T, r(n_l^T), G(n_l^T), B(n_l^T)) =& (1 - \phi) \left(w - c_0 e^{-c_r r(n_l^T)}\right) \left(\alpha^{n_l^T} - \beta^{n_l^T} p(n_l^{Tt})\right) \\
&- \zeta \left(1 - B(n_l^T)\right) G(n_l^T).
\end{aligned}$$

Maximizing $\mathcal{H}_M$ with respect to $G(n_l^t)$, we obtain

$$\frac{\partial \mathcal{H}_M}{\partial G(n_l^t)} = -\zeta \left(1 - B(n_l^t)\right) + \theta_1 \kappa \delta (\upsilon_1 - \upsilon_2) e^{-\kappa G(n_l^t)} \Phi(n_l^t, \lambda_M) \left(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t)\right) = 0, \tag{42}$$

and for the terminal nodes $n_l^T \in \mathcal{N}^T$ we have

$$G(n_l^T) = 0, \tag{43}$$

because function $\mathcal{H}_M(n_l^T, r(n_l^T), G(n_l^T), B(n_l^T))$ is a linear decreasing function of $G(n_l^T)$.

Notice that under the assumptions of an interior solution and positive demand, $\Phi(n_l^t, \lambda_M)$ must satisfy the following condition

$$\Phi(n_l^t, \lambda_M) Q(n_l^t) \geq \frac{\zeta \left(1 - B(n_l^t)\right)}{\theta_1 \kappa \delta (\upsilon_1 - \upsilon_2)}$$

to have a nonnegative $G(n_l^t)$. The expression in the left-hand side of the inequality is the expected demand on the stage which follows the current node n_l^t.

The conditions on $\lambda_M(n_l^t)$ are given by

$$\lambda_M(n_l^t) = \frac{\partial \mathcal{H}_M}{\partial r(n_l^t)} = F^{n_l^t} = (1-\phi)c_0c_r e^{-c_r r(n_l^t)} \left(\alpha^{n_l^t} - \beta^{n_l^t} p(n_l^t)\right) + \delta\eta\Phi(n_l^t, \lambda_M),\ n_l^t \in \mathcal{N}\setminus\mathcal{N}^T, \tag{44}$$

$$\lambda_M(n_l^T) = \frac{\partial \mathcal{H}_M}{\partial r(n_l^T)} = F^{n_l^T} = (1-\phi)c_0c_r e^{-c_r r(n_l^T)} \left(\alpha^{n_l^T} - \beta^{n_l^T} p(n_l^T)\right),\ n_l^T \in \mathcal{N}^T. \tag{45}$$

This costate variable will play the role of an additional state variable in the retailer's (leader's) problem.

Following the proof of Theorem 7.1 in (Basar and Olsder, 1998, p. 368–370), we obtain the Stackelberg strategy of the retailer. We have to maximize $J_R(G,p,B,r)$, in view of the unique optimal response of the follower (manufacturer). Therefore, the retailer is faced with the optimal control problem

$$\max_{\substack{p(n_l^t)>0 \\ 0\le B(n_l^t)\le 1}} J_R(G,p,B,r) = \sum_{t=0}^{T}\sum_{n_l^t\in\mathcal{N}^t} \pi^{n_l^t}\delta^t\Big(\big(p(n_l^t) - w + I(r(n_l^t))\big)Q(n_l^t) - B(n_l^t)d(G(n_l^t))\Big),$$

subject to state dynamics (41), relations on λ_M dynamics given by (44) and (45), Equations (42) and (43).

The Hamiltonian of the optimal control problem of the retailer for any node $n_l^t \in \mathcal{N}\setminus\mathcal{N}^T$ is

$$\begin{aligned} H_R(n_l^t) = &\big(p(n_l^t) - w + \phi(w - c_0 e^{-c_r r(n_l^t)})\big)Q(n_l^t) - \zeta B(n_l^t)G(n_l^t) \\ &+ \delta\mu_r(n_l^t)\big(\eta r(n_l^t) + \theta_1(\upsilon_1 - (\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)})Q(n_l^t)\big) \\ &+ \delta\mu_\lambda(n_l^t)\big((1-\phi)c_0c_r e^{-c_r r(n_l^t)}Q(n_l^t) + \delta\eta\Phi(n_l^t,\lambda_M)\big) \\ &+ \nu(n_l^t)\big(-\zeta\left(1 - B(n_l^t)\right) + \theta_1\kappa\delta(\upsilon_1-\upsilon_2)e^{-\kappa G(n_l^t)}\Phi(n_l^t,\lambda_M)Q(n_l^t)\big), \end{aligned}$$

where $Q(n_l^t) = \alpha^{n_l^t} - \beta^{n_l^t}p(n_l^t)$. And for terminal nodes $n_l^T \in \mathcal{N}^T$ we have

$$\begin{aligned} H_R(n_l^T) = &\big(p(n_l^T) - w + \phi(w - c_0 e^{-c_r r(n_l^T)})\big)Q(n_l^T) - \zeta B(n_l^T)G(n_l^T) \\ &+ \mu_\lambda(n_l^T)(1-\phi)c_0c_r e^{-c_r r(n_l^T)}Q(n_l^T) \\ &- \nu(n_l^T)\zeta(1 - B(n_l^T)). \end{aligned}$$

Theorem 7.1 in Basar and Olsder (1998) (see p. 368–370) or Stankova and Schutter (2011) gives the following system of relations:

$$\frac{\partial H_R(n_l^t)}{\partial p(n_l^t)} = 0, n_l^t \in \mathcal{N}\setminus\mathcal{N}^T, \tag{46}$$

$$\frac{\partial H_R(n_l^T)}{\partial p(n_l^T)} = 0, n_l^T \in \mathcal{N}^T, \tag{47}$$

$$\frac{\partial H_R(n_l^t)}{\partial B(n_l^t)} = 0, n_l^t \in \mathcal{N}\setminus\mathcal{N}^T, \tag{48}$$

$$\frac{\partial H_R(n_l^T)}{\partial B(n_l^T)} = 0, n_l^T \in \mathcal{N}^T, \tag{49}$$

$$\frac{\partial H_R(n_l^t)}{\partial G(n_l^t)} = 0, n_l^t \in \mathcal{N}\setminus\mathcal{N}^T, \tag{50}$$

$$\frac{\partial H_R(n_l^T)}{\partial G(n_l^T)} = 0, n_l^T \in \mathcal{N}^T, \tag{51}$$

$$\mu_r(n_l^t) = \delta \sum_{\nu \in S(n_l^t)} \frac{\pi^\nu}{\pi^{n_l^t}} \frac{\partial H_R(\nu)}{\partial r(\nu)}, n_l^t \in \mathcal{N} \setminus \mathcal{N}^T, \tag{52}$$

$$\mu_r(n_l^T) = 0, n_l^T \in \mathcal{N}^T, \tag{53}$$

$$\mu_\lambda(n_l^{t+1}) = \frac{1}{\delta} \frac{\pi^{a(n_l^{t+1})}}{\pi^{n_l^{t+1}}} \frac{\partial H_R(a(n_l^{t+1}))}{\partial \lambda_M(n_l^{t+1})}, n_l^{t+1} \in \mathcal{N} \setminus n^0, \tag{54}$$

$$\mu_\lambda(n^0) = 0, \tag{55}$$

$$\frac{\partial H_M(n_l^t)}{\partial G(n_l^t)} = 0, n_l^t \in \mathcal{N} \setminus \mathcal{N}^T, \tag{56}$$

$$\frac{\partial H_M(n_l^T)}{\partial G(n_l^T)} = 0, n_l^T \in \mathcal{N}^T, \tag{57}$$

$$\lambda_M(n_l^t) = F^{n_l^t}, n_l^t \in \mathcal{N} \setminus \mathcal{N}^T, \tag{58}$$

$$\lambda_M(n_l^T) = F^{n_l^T}, n_l^T \in \mathcal{N}^T, \tag{59}$$

with the state dynamics (41). Substituting the expressions of H_M, H_R, $F^{n_l^t}$ and $F^{n_l^T}$ we obtain the relations given in the theorem. In particular, (46) and (47) imply (14) and (15); (48) and (49) imply (18) and $\nu(n_l^T) = 0$ given in (19); (50) and (51) imply (16) and $B(n_l^T) = 0$ given in (19); (56) and (57) imply (17) and $G(n_l^T) = 0$ given in (19); (52) and (53) imply (23) and (24); (54) and (55) imply (25) and (26); (58) and (59) imply (21) and (22).

The solution of this system gives the Stackelberg equilibrium strategies G^*, B^*, p^* and equilibrium state trajectory r^*.

Appendix B

Table 9 presents the expected payoff for each player for the base scenario ($T = 5$) for different values of the sharing parameter, ϕ.

Table 9: Expected payoff for each player for different sharing parameters

Parameters	$c_r = 0.5, w = 4, c_0 = \zeta = 1, \delta = 0.9$						
	RRSC						Benchmark
ϕ	**0.05**	**0.10**	**0.20**	**0.30**	**0.40**	0.50	NA
Manufacturer	**14.054**	**14.370**	**14.702**	**14.580**	**13.991**	12.910	13.690
Retailer	**5.759**	**6.551**	**8.343**	**10.410**	**12.752**	15.3635	5.035

Tables 10–12 show how changing parameter ζ, c_0, η affects the expected payoffs to the players over the short planning horizon ($T = 5$). Comparing the numerical results in these tables with those in Table 9 provides more insight about the model. The parameters that are changed are blue-colored in Tables 10–12.

We can observe the following:

1. Increasing ζ does not change the expected payoff for the retailer in the benchmark scenario as expected while reduces the manufacturer's expected payoff. This change leads to a slightly lower expected payoffs for both players in the RRSC scenario (see Table 10 in comparison with Table 9).
2. Increasing c_0 is not in manufacturer's interest as he faces significantly lower expected payoff in both scenarios. Although the retailer is not affected in the benchmark scenario, he is highly penalized for higher c_0 in the RRSC scenario (see Table 11 in comparison with Table 9).

3. The retailer is not affected in the benchmark scenario by decreasing η, but the manufacturer's payoff decreases slightly. Decreasing η leads to a negligible decrease in both players' expected payoffs in RRSC scenario (see Table 12 in comparison with Table 9).
4. The manufacturer is much more sensitive in increasing c_0 than in increasing ζ and decreasing of η. The reason can be in very low level of green investments which implies low sensitivity in increasing ζ.
5. In Tables 10–12 we can observe Pareto-improving RRSC scenarios comparing with benchmark scenario. They are highlighted in bold.

Table 10: Effect of ζ on the expected payoff for each player for different ϕ

Scenario	RRSC					Benchmark
Parameters	$c_r = 0.5, w = 4, \eta = 0.4, c_0 = 1, \delta = 0.9, \zeta = 2$					
ϕ	**0.10**	**0.20**	**0.30**	0.40	0.50	N/A
Manufacturer	**13.695**	**13.825**	**13.713**	13.218	12.269	13.274
Retailer	**6.467**	**8.127**	**10.060**	12.279	14.819	5.035

Table 11: Effect of c_0 on the expected payoff for each player for different ϕ

Scenario	RRSC					Benchmark
Parameters	$c_r = 0.5,\ w = 4,\ \eta = 0.4,\ c_0 = 2,\ \zeta = 1, \delta = 0.9$					
ϕ	**0.1**	**0.2**	**0.3**	0.4	0.5	N/A
Manufacturer	**11.984**	**12.161**	**12.011**	11.498	10.593	11.535
Retailer	**6.361**	**7.930**	**9.743**	11.795	14.072	5.035

Table 12: Effect of η on the expected payoff for each player for different ϕ

Scenario	RRSC					Benchmark
Parameters	$c_r = 0.5,\ w = 4,\ \eta = 0.2,\ c_0 = \zeta = 1, \delta = 0.9$					
ϕ	**0.1**	**0.2**	**0.3**	**0.4**	0.5	N/A
Manufacturer	**14.036**	**14.319**	**14.196**	**13.633**	12.610	13.421
Retailer	**6.511**	**8.241**	**10.243**	**12.538**	15.117	5.035

Appendix C

Table 13: Equilibrium strategies, return trajectory and demand trajectory for benchmark and RRSC scenarios in positive and negative realization of stochastic demand parameters, $T = 15$ for an event tree graph depicted on Figure 4

***Positive** realization of stochastic demand*									
Scenario	Benchmark				RRSC				
Parameters	$c_r = 0.5, w = 4, \phi = 0.3, c_0 = \zeta = 1, \delta = 0.9$								
Node	G	p	r	Q	G	p	B	r	Q
n^0	0.045	5.125	0.000	0.900	0.103	4.630	0.000	0.000	1.296
n_1^1	0.088	5.314	0.141	1.041	0.128	4.781	0.000	0.329	1.463
n_1^2	0.125	5.515	0.297	1.088	0.151	4.965	0.000	0.552	1.619
n_1^3	0.158	5.728	0.455	1.342	0.172	5.168	0.000	0.730	1.776
n_1^4	0.187	5.954	0.614	1.502	0.191	5.387	0.000	0.890	1.938
n_1^5	0.214	6.194	0.774	1.669	0.209	5.621	0.000	1.045	2.105
n_1^6	0.209	6.194	0.936	1.669	0.205	5.616	0.000	1.200	2.109
n_1^7	0.208	6.194	0. 994	1.669	0.204	5.615	0.000	1.256	2.110
n_1^8	0.207	6.194	1.015	1.669	0.2034	5.6141	0.000	1.277	2.1102
n_1^9	0.2067	6.194	1.023	1.669	0.2031	5.61395	0.000	1.284	2.11036
n_1^{10}	0.2062	6.194	1.0255	1.669	0.202	5.61396	0.000	1.28629	2.11035
n_1^{11}	0.2047	6.194	1.0257	1.669	0.201	5.614	0.000	1.28628	2.1101
n_1^{12}	0.200	6.194	1.024	1.669	0.197	5.615	0.000	1.2835	2.109
n_1^{13}	0.186	6.194	1.016	1.669	0.182	5.618	0.000	1.273	2.107
n_1^{14}	0.132	6.194	0.911	1.669	0.129	5.628	0.000	1.241	2.100
n_1^{15}	0.000	6.194	0.884	1.669	0.000	5.658	0.000	1.100	2.077

***Negative** realization of stochastic demand*									
Scenario	Benchmark				RRSC				
Parameters	$c_r = 0.5, w = 4, \phi = 0.3, c_0 = \zeta = 1, \delta = 0.9$								
Node	G	p	r	Q	G	p	B	r	Q
n^0	0.045	5.125	0.000	0.900	0.103	4.630	0.000	0.000	1.296
n_2^1	0.000	4.939	0.141	0.759	0.065	4.411	0.000	0.329	1.186
n_4^2	0.000	4.765	0.102	0.624	0.041	4.238	0.087	0.360	1.054
n_8^3	0.000	4.601	0.078	0.495	0.017	4.093	0.193	0.302	0.914
n_8^4	0.000	4.446	0.061	0.371	0.000	3.962	0.230	0.212	0.774
n_8^5	0.000	4.301	0.047	0.253	0.000	3.832	0.169	0.131	0.647
n_8^6	0.000	4.301	0.034	0.253	0.000	3.838	0.1697	0.091	0.642
n_8^7	0.000	4.301	0.029	0.253	0.000	3.841	0.1699	0.075	0.640
n_8^8	0.000	4.301	0.027	0.253	0.000	3.842	0.1700	0.068	0.639
n_8^9	0.000	4.301	0.026	0.253	0.000	3.842	0.1698	0.066	0.6382
n_8^{10}	0.000	4.301	0.0255	0.253	0.000	3.84256	0.1694	0.065	0.638
n_8^{11}	0.000	4.301	0.0254	0.253	0.000	3.84265	0.168	0.064	0.6379
n_8^{12}	0.000	4.301	0.0253	0.253	0.000	3.84276	0.164	0.0639	0.6378
n_8^{13}	0.000	4.301	0.02529	0.253	0.000	3.84301	0.152	0.06384	0.6377
n_8^{14}	0.000	4.301	0.025289	0.253	0.000	3.844	0.114	0.06379	0.637
n_8^{15}	0.000	4.301	0.025285	0.253	0.000	3.846	0.000	0.06374	0.635

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