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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Subsidies and pricing strategies in a vehicle scrappage program with strategic consumers

Hosain Zaman [a]
Georges Zaccour [a,b]

[a] *GERAD & Department of Decision Sciences, HEC Montréal, Montréal (Québec), Canada, H3T 2A7*

[b] *Chair in Game Theory and Management, HEC Montréal, Montréal (Québec), Canada, H3T 2A7*

hosain.zaman@gerad.ca
georges.zaccour@gerad.ca

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Abstract: We consider the problem of a government that wishes to promote replacing old cars with new ones via a vehicle scrappage program. Since these programs increase consumer's willingness to pay for a new car, manufacturers (or dealers) could respond strategically by raising their prices. In a two-period game between government and a manufacturer, we find equilibrium prices and subsidy levels. Our results demonstrate that price levels are increasing over time and are higher than in the benchmark case where no subsidy is offered. Further, if consumers act strategically, then the equilibrium price levels will be higher than in the scenario where they behave myopically.

Keywords: Vehicle scrappage program, strategic consumers, pricing, government subsidies

Résumé : Nous considérons le problème d'un gouvernement qui souhaite promouvoir le remplacement des vieilles voitures par des neuves via un programme de mise à la casse des véhicules. Ces programmes accroissent la volonté des consommateurs de payer pour une nouvelle voiture, les fabricants (ou concessionnaires) pourraient réagir de manière stratégique en augmentant leurs prix. Nous considérons un jeu à deux périodes entre le gouvernement et un fabricant et nous déterminons les prix d'équilibre et les niveaux de subvention. Nos résultats montrent que les prix sont croissants dans le temps et plus élevés que dans le cas où aucune subvention n'est offerte. En outre, si les consommateurs agissent de manière stratégique, les niveaux de prix d'équilibre seront plus élevés que dans le scénario où ils se comportent de manière myope.

Mots clés : Programme de mise à la casse des véhicules, consommateurs stratégiques, tarification, subventions gouvernementales

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1 Introduction

Vehicle scrappage programs have been introduced by governments in different countries to accelerate the replacement of old cars by less-polluting ones. For instance, the Canadian Vehicle Efficiency Incentive program, introduced in 2007, offered up to \$2,500 for the purchase a new fuel-efficient car (Walsh, 2012). Another example is the German scrappage subsidy program, where every vehicle older than nine years is eligible for a subsidy should it be replaced by a new car (Ewing, 2009). While these scrappage programs are designed with the dual aim of stimulating the car market and taking old cars off the road, they are costly. The German scrappage program, so far the largest one ever, is estimated to cost 2,5 billion euros (Ewing, 2009). Whatever the amount involved, it is important to design these programs in the most cost-efficient way to avoid wasting governmental funds.

A scrappage subsidy aims at increasing the willingness to pay of consumer to replace her car. One concern is that this boost may not fully materialize because manufacturers may increase their prices when a subsidy program is established. Kaul et al. (2009) investigated how much of the €2,500 subsidy in German vehicle scrappage program is actually captured by consumers, and showed that subsidized buyers paid a little more than comparable buyers who did not receive the subsidy. Jiminez et al (2016) obtained that car manufacturers increased vehicle prices by €600 on average after a scrappage program has been announced in Spain. Such observations constitute an invitation to account for the manufacturer role when designing an incentive program.

In this paper, we investigate the problem of designing subsidies over time by a government, when the prices are set by a (representative) car manufacturer (or a car dealer). To capture the strategic interactions between these parties, while keeping the setup parsimonious, we retain a two-period model. The noncooperative game is played à la Stackelberg, with the government, as leader, announces the subsidy program and the manufacturer, as follower, determines the prices. On the demand side, to capture the strategic behavior of consumers, we assume that the decision to replace or not a car is based on current and future subsidies. Moreover, since scrappage programs usually include an eligibility age, it is necessary to group cars based on their age. This enables us to differentiate between consumers whose cars are eligible in the current period with those who can receive the subsidy only in the future.

Governments may apply different strategies when announcing a subsidy program. In particular, they can opt for a constant subsidy over time, or let it varies over time. Further, the government can commit to a fixed subsidy plan, that is, it pre-announces the subsidy levels for all (here two) periods, or adopt a flexible one by avoiding any commitment on the subsidy in later periods. In the parlance of dynamic games, the pre-announcement strategy is referred to as commitment or open-loop strategy, whereas a strategy that depends on the state of the system is referred to as feedback (or Markovian) strategy. For example, the subsidy level in German feed-in-tarif program for solar electricity adoption was adjusted multiple times per year depending on the installed capacity of photovoltaics (International Energy Agency, 2014). In another solar subsidy case, the California Solar Initiative (CSI), there was a planned decrease in the subsidy level that was pre-announced from the outset (Chemama et al., 2019). We will compare different subsidy strategies, i.e., commitment and flexible, and constant versus varying over time subsidy plans. Moreover, we will investigate if the manufacturer is better off implementing dynamic pricing or constant price over time.

1.1 Brief literature background

Subsidy programs designed to accelerate replacement of existing products by greener ones have been studied extensively using both static and dynamic models. Using a static framework, Hahn (1995) performed a cost-benefit analysis to determine the number of scrapped cars for different subsidy levels. Similarly, Lavee and Becker (2009) suggested a function to estimate car replacements based on the subsidy amount and used car values. Unlike in these papers, other researchers retained a dynamic model to investigate subsidy programs over time. For example, De Grrote and Verboven (2019) developed a dynamic model of new technology adoption to study a subsidy program to promote adoption of solar photovoltaic systems. It is shown that consumers significantly discount the future benefits of the new technology. In another study on photovoltaic distributed generation, He et al. (2018) investigated emission reduction benefits of the subsidy program. Taking into account cost learning, they showed that the net benefits of the subsidy program decreases with the reduction of unit cost.

While some researchers studied subsidy programs from the government's viewpoint, others investigated the problem from the manufacturer's perspective. Hirte and Tscharaktschiew (2013) determined subsidy rates for electric vehicles that maximize social welfare. Lobel and Perakis (2011) characterized the optimal trajectory of subsidies for photovoltaic systems that optimizes the government's subsidy cost, instead of social welfare. Whereas in Hirte and Tscharaktschiew (2013) and Lobel and Perakis (2011) the government decides the subsidy rates without considering manufacturers' role, Ding et al. (2015) and Yu et al. (2016) let the manufacturers determine the price and quantity assuming the subsidy given, that is, the government does not play a strategic role. In He et al. (2019), and Janssens and Zaccour (2014) both the government and manufacturers act strategically. In this paper, we too consider the government and manufacturer as players, and add the important feature that consumers also behave strategically.

Strategic behavior of consumers and its effect on subsidy programs has been studied in few papers. Shiraldi (2011) assessed how vehicle choices of heterogeneous consumers with different tastes is affected by a scrappage subsidy program. He et al. (2018) showed that some beneficiaries of the subsidy program would have replaced their car without the subsidy. A similar result is obtained in Zaman and Zaccour (2020), where strategic consumers decide when to replace their cars, assuming that the government plays no strategic role. In Zaman and Zaccour (2021), this assumption is relaxed, that is, the government determines strategically the subsidy levels when designing its program. In this stream of literature, the price of a new car is given, as if car dealers do not react to the existence of a subsidy program. In this paper, the manufacturer is a strategic player in the game and decides upon the price of the subsidized new car.

1.2 Research questions and contributions

To the best of our knowledge, this paper is the first analysis of equilibrium scrappage subsidy and car prices in the presence of strategic consumers. Our aim is to answer the following questions:

1. Does a scrappage subsidy program affect the equilibrium pricing strategy of a car manufacturer?
2. What is the impact of consumer's strategic behavior on equilibrium prices and subsidies?
3. What is the impact of the manufacturer's pricing strategy choice on the equilibrium subsidies, prices and outcomes?
4. Is it in the best interest of the government to pre-announce its subsidy policy or to adopt a flexible one? What is the impact of its choice on the manufacturer and consumers?

Our results are summarized as follows:

1. Price levels in the presence of scrappage program are higher than in the benchmark case where no subsidy is offered. In addition, when subsidy is a given constant, prices are increasing over time.
2. Price and subsidy levels are higher when consumers act strategically than when they behave myopically. Moreover, manufacturer profit and subsidy cost are higher, when consumers are strategic.
3. When the manufacturer applies dynamic pricing, it realizes a higher profit than when adopting a constant price. Dynamic pricing increases the cost of the subsidy program.

4. Comparing the results obtained under pre-announced and flexible subsidies, we obtain that the government is better off, that is, the program cost is lower, and the manufacturer's profit is lower, when flexible subsidies are implemented.

2 Model

Consider a two-period model where the government subsidizes consumers who replace their car older than a specified age, η, with a new one. For example, the scrappage program in Germany subsidizes all cars older than 9 years should they are replaced by a new one. The total number of vehicles targeted by the government program is normalized to 1, with the proportions of cars with age $\eta - 1$ and $\geq \eta$ being $1 - \beta$ and β, respectively. In each period, a (representative) manufacturer chooses the new-car price p_t, $t = 1, 2$, and the government sets subsidy level s, assumed to be the same in both periods. Later on we consider a time-varying subsidy.

In our two-period framework, a vehicle can be eligible either in both periods (i.e., cars aged $\geq \eta$) or only in the second period for cars aged $\eta - 1$. We disregard cars that are not eligible in either periods, as the program has no effect on their replacement. Therefore, we assume that the demand for new cars originates from the replacement of cars older than $\eta - 1$ by new cars. As a result we distinguish between demand functions representing the number of replacements of the two retained categories of cars. For cars aged $\geq \eta$, we assume that the number of replacements in the first period can be approximated by the following linear demand function:

$$d_1^{\geq \eta} = \beta - p_1 + \theta s, \tag{1}$$

where the superscript $\geq \eta$ stands for cars older than η, β represents the market potential, and θ is a positive parameter measuring consumer's sensitivity to the subsidy.

Since cars aged $\eta - 1$ are not eligible for the subsidy in the first period, the demand function in (1) does not apply to these vehicles. One alternative is to assume that $(\eta - 1)$-car owners ignore the subsidy program in the first period as they cannot cash in the subsidy in this period. In this case, the demand in the first period linked to cars aged $\eta - 1$ is obtained by omitting the term θs from Equation (1). However, according to Zaman and Zaccour (2020), expectations for future subsidies could affect the actual demand. In other words, consumers who are not eligible in current period but are in the next one, may consider postponing their car replacement to become eligible for the subsidy program. In our two-period model, some consumers owning a car aged $\eta - 1$ who would have replaced in the first period when there is no subsidy program, could delay their car replacement to the second period when there is a subsidy program. Note that, while these consumers do not receive any subsidy in both periods when there is no subsidy program, they can benefit from a subsidy in the second period when a subsidy program is available. As a result, a part of the demand in the first period is moved forward, to the second period, which affects the first-period demand negatively. For cars aged $\eta - 1$, we assume the demand for replacements in the first period to be given by the following linear function:

$$d_1^{\eta - 1} = 1 - \beta - p_1 - \gamma s, \tag{2}$$

where the superscript $\eta - 1$ stands for cars aged $\eta - 1$ and γ is a nonnegative parameter measuring the consumer sensitivity to the future subsidy. A positive γ means that consumers are strategic (or farsighted), while $\gamma = 0$ implies that consumers are myopic. A series of studies showed a negative impact of consumers' strategic behavior on retailer's profit. In particular, some researchers found that if firms set their prices as if all consumers were myopic, they might end up with a revenue loss estimated to be between 20 and 60% (see, e.g., Aviv and Pazgal (2008), Besanko and Winston (1990)). These results constitute an invitation to sellers to account for strategic consumers when making their pricing and other marketing decisions.

We assume, not unrealistically, that consumers are more sensitive to present subsidy than to a future one, which we translate by setting $\gamma < \theta$. We can refer to θ and γ as the direct and indirect effect of the subsidy on demand, respectively.

Summing (1) and (2), we obtain the following total demand in period 1:

$$d_1^{\geq \eta} + d_1^{\eta-1} = 1 - 2p_1 + (\theta - \gamma)\, s. \tag{3}$$

Unlike in the first period, in the second period all vehicles are eligible for the subsidy and it is not necessary to differentiate between cars with different ages. The second-period demand is given by

$$d_2 = d_2^{\eta} + d_2^{\eta-1} = (1 - d_1^{\eta-1} - d_1^{\geq \eta}) - p_2 + \theta s. \tag{4}$$

In this demand function, the market potential is the number of remaining old cars after the first period. The total demand in period 2 can be rewritten as

$$d_2 = 2p_1 + \gamma s - p_2,$$

which clearly shows that the demand in the second period depends on both prices and the subsidy. Note that, if consumers are myopic, i.e., $\gamma = 0$, then the total demand in period 1 would be higher and the total demand in period 2 lower.

Assuming that the manufacturer maximizes its profit, Π, over the two periods, its optimization problem reads as follows:

$$\max_{p_1, p_2 \geq 0} \Pi \triangleq \Pi_1 + \Pi_2 = p_1(d_1^{\eta-1} + d_1^{\geq \eta}) + p_2 d_2, \tag{5}$$

$$= p_1 \Big(1 - 2p_1 + (\theta - \gamma)\, s\Big) + p_2 \left(2p_1 + \gamma s - p_2\right), \tag{6}$$

where Π_1 and Π_2 are the profit in the first and second period, respectively.

Considering the best response of the manufacturer, the government sets the subsidy level. In order to formulate the government problem, we use the setting in Zaman and Zaccour (2021) where the government minimizes the total cost of subsidy subject to a car replacement target level, denoted by Γ. The government optimization problem is as follows:

$$\min_{s \geq 0} C \triangleq C_1 + C_2 = s.d_1^{\geq \eta} + s.d_2,$$

$$\text{subject to: } d_1^{\geq \eta} + d_1^{\eta-1} + d_2 \geq \Gamma, \tag{7}$$

where C_1 and C_2 are the subsidy costs in the first and second period, respectively. The objective function is composed of the total subsidy paid in the first period for cars aged $\geq \eta$ plus the subsidy cost in the second period spent on all vehicles replaced in that period. The constraint states that the total number of scrapped vehicles must be at least equal to the target level.

We make the following two remarks:

1. Throughout the rest of the paper, we assume that $\Gamma > \dfrac{1}{2}$; otherwise, the government will not offer a subsidy, which is a less interesting case.
2. As the government aims at minimizing the cost of its program, it is clear that it will never exceeds the target. Consequently, the inequality constraint can be written, without any loss of generality, as an equality constraint, i.e.,

$$d_1^{\geq \eta} + d_1^{\eta-1} + d_2 = \Gamma.$$

3 Results

In this section, we determine the Stackelberg equilibrium, with the government acting as leader and the manufacturer as follower. The following proposition characterizes the equilibrium subsidy and prices set by the two players.

Proposition 1 *Assuming an interior solution, the unique Stackelberg equilibrium strategies of the government and manufacturer are given by*

$$s = \frac{2\Gamma - 1}{\theta - \gamma},$$
$$p_1 = \frac{2\Gamma\theta - \gamma}{2\left(\theta - \gamma\right)},$$
$$p_2 = \frac{(\gamma + \theta)\Gamma - \gamma}{\theta - \gamma},$$

and the manufacturer profit and subsidy cost by

$$\Pi = \frac{(2T^2 - 2\Gamma + 1)\gamma^2 - 2\Gamma\gamma\theta + 2T^2\theta^2}{2\left(\theta - \gamma\right)^2},$$
$$C = \frac{\Big(2\beta\left(\theta - \gamma\right) + \left(2\Gamma - 1\right)\left(2\theta + \gamma\right)\Big)(2\Gamma - 1)}{2\left(\theta - \gamma\right)^2}.$$

Proof. See Appendix. □

First, we note that under the assumptions made earlier on the parameter values, i.e., $\Gamma > 1/2$ and $\theta > \gamma$, the equilibrium is indeed interior, that is, p_1, p_2 and s are strictly positive. Second, it is easy to verify that p_1, p_2 and Π are always increasing in γ for all admissible parameter values. Therefore, the price levels and manufacturer profit are higher than in the case where the strategic behavior of consumers is ignored, i.e., $\gamma = 0$. If all consumers are myopic, then the manufacturer's equilibrium pricing strategy would be to set $p_1 = p_2 = \Gamma$. To avoid that strategic consumers wait to make their purchase, which is how they behave when they expect future discounts, the manufacturer implements a penetration pricing strategy by setting $p_1 < p_2$.

Moreover, we have $\frac{\partial p_2}{\partial \gamma} > \frac{\partial p_1}{\partial \gamma}$, that is, the second-period price is more sensitive towards the propensity of farsightedness of consumers than the first-period price. In other terms, as the strategic behavior of consumers is intensified the second-period price increases with a higher rate than that of the first period. This intuitive result is in line with the fact that when γ is positive, a part of the demand originated from the replacement of cars aged $\eta - 1$ is moved forward from the first to the second period. Therefore, since the manufacturer is a profit maximizer, parameter γ has more influence on p_2 than p_1. In addition, we note that γ has a positive effect on subsidy cost too, i.e., $\frac{\partial C}{\partial \gamma} > 0$. Intuitively, the higher γ is, the more $(\eta - 1)$-car owners go to the second period to benefit from the subsidy, which results in higher subsidy cost.

It is also interesting to look at the equilibrium demands in the two periods. Since p_2 is higher than $p_1\Big(p_2 - p_1 = \dfrac{\gamma(2\Gamma - 1)}{2\left(\theta - \gamma\right)} > 0\Big)$, the replacement of cars aged $\geq \eta$ in the second period is equal to zero. Note that the demand for these cars has the same functional form in both periods. In contrast, the replacement of cars aged $\eta - 1$ is always positive in the second period, as these cars are only eligible for the subsidy in that period. Further, the replacement of cars aged η and $\eta - 1$ in the first period can be either positive or zero.

Corollary 1 describes how the first-period demand depends on the parameter values.

Corollary 1 *The equilibrium number of cars with age $\geq \eta$ and $\eta - 1$ replaced in the first period are as follows:*

$$d_1^{\eta-1} = \begin{cases} \dfrac{(1 + 2\beta - 4\Gamma)\gamma + 2\theta(1 - \beta - \Gamma)}{2\left(\theta - \gamma\right)}, & \textit{if} \ \ \Gamma < \frac{2\theta(1-\beta)+\gamma(2\beta+1)}{2(\theta+2\gamma)}, \\ 0, & \textit{otherwise.} \end{cases} \tag{8}$$

$$d_1^{\geq\eta} = \begin{cases} \dfrac{2\theta(\beta+\Gamma-1)+\gamma(1-2\beta)}{2\,(\theta-\gamma)}, & \textit{if}\ \ \Gamma > \frac{2\theta(1-\beta)+\gamma(2\beta-1)}{2\theta}, \\ 0, & \textit{otherwise.} \end{cases} \tag{9}$$

The total demand in the second period is given by

$$d_2 = d_2^{\eta} + d_2^{\eta-1} = \frac{\Gamma\,(\theta+\gamma)-\gamma}{(\theta-\gamma)}.$$

Proof. It suffices to insert the equilibrium strategies from Proposition 1 in the demand functions to get the results. □

According to Corollary 1, both demands in the first period are positive if

$$\frac{2\theta\,(1-\beta)+\gamma\,(2\beta-1)}{2\theta} < \Gamma < \frac{2\theta\,(1-\beta)+\gamma\,(2\beta+1)}{2\,(\theta+2\gamma)}.$$

Moreover, $d_1^{\eta-1}$ is equal to zero, unless the target level is less than a certain threshold, i.e., $\frac{2\theta(1-\beta)+\gamma(2\beta+1)}{2(\theta+2\gamma)}$. (Recall that these car holders are only eligible in the second period.) In addition, increasing the target leads to higher subsidy levels. As a result, when the target level is higher than a threshold, the subsidy is high enough to push all the demand related to cars aged $\eta-1$ to the second period. On the other hand, the number of replacements of cars aged $\geq\eta$ is positive only if the target level is higher than a threshold, i.e., $\frac{2\theta(1-\beta)+\gamma(2\beta+1)}{2\theta\gamma}$. Besides, as mentioned before, the number of replacements of cars aged $\geq\eta$ in the second period is equal to zero. Consequently, as the target level increases, the first-period demand related to these cars increases too. So, it is intuitive to have a target level threshold after which the demand is positive.

Remark 1 *In this game, feedback (or Markov-perfect) and open-loop Stackelberg equilibria coincide. Indeed, it is easy to verify that if the manufacturer solves simultaneously for p_1 and p_2, the result would be the same. The reason is that the leader (government) sets the same subsidy in both periods.*

4 Extensions

In this section, we extend the basic model in two ways. First, we consider the case where the manufacturer sets the same price in both periods, which will allow us to assess the benefit or loss resulting from dynamic pricing. In an optimization problem, it is obvious that constant pricing cannot be better than dynamic pricing. Indeed, if it were, then $p_1 = p_2$ would be the solution of the dynamic optimization problem. This result does not necessarily carry over to games, because of the strategic interactions between the players. In the second extension, we relax the assumption of constant subsidy over time.

4.1 Constant pricing

We compare two different pricing scenarios for the manufacturer, i.e., dynamic pricing and constant pricing. The constant pricing version is obtained by setting $p_1 = p_2$ in the demand functions and manufacturer's profit introduced in the previous section.

Proposition 2 *Assuming an interior solution, when the manufacturer implements a constant pricing strategy, the unique equilibrium price and subsidy levels, the manufacturer profit and the subsidy cost are given by*

$$\tilde{p} = \Gamma,$$
$$\tilde{s} = \frac{2\Gamma-1}{\theta},$$

$$\tilde{\Pi} = \Gamma^2,$$
$$\tilde{C} = \frac{\Big(\theta(\beta + 2\Gamma - 1) + \gamma\,(2\Gamma - 1)\Big)(2\Gamma - 1)}{\theta^2}.$$

Proof. See Appendix. □

Proposition 3 *If the manufacturer sets a constant price over time, then it achieves a lower profit than under dynamic pricing, i.e.,* $\Pi > \tilde{\Pi}$*. The government's total program cost is lower under constant pricing, i.e.,* $C > \tilde{C}$*.*

Proof. It suffices to compute the following differences to get the result:

$$\Pi - \tilde{\Pi} = \frac{\gamma\,(2\Gamma - 1)\,(2\Gamma\theta - \gamma)}{2\,(\theta - \gamma)^2} > 0,$$
$$C - \tilde{C} = \frac{\gamma\,(2\Gamma - 1)}{2\theta^2\,(\theta - \gamma)^2}\Big(3\theta^2\,(2\Gamma - 1) + 2\,(\theta - \gamma)\,(\gamma\,(2\Gamma - 1) + \theta\beta)\Big) > 0.$$

□

Proposition 3 shows that the manufacturer is better off with dynamic pricing, while the government is worse off. By setting a higher price in the second period, the manufacturer takes advantage of consumers who can benefit from the subsidy only in this period, and as a result obtains a higher profit with this intertemporal price discrimination strategy. One implication is a higher government's subsidy cost under dynamic pricing. Indeed, comparing the results in Propositions 1 and 2, it is easy to verify that

$$p_2 > p_1 > \tilde{p},$$
$$s - \tilde{s} = \frac{\gamma}{\theta}\frac{2\Gamma - 1}{\theta - \gamma} > 0.$$

What about consumers who are the third party in this game? Comparing the net prices paid by consumers in the two scenarios, we have

$$(p_1 - s) - (\tilde{p} - \tilde{s}) = \frac{\gamma\,(\theta - 2)}{2\theta}\frac{2\Gamma - 1}{\theta - \gamma} < 0,$$
$$(p_2 - s) - (\tilde{p} - \tilde{s}) = \frac{\gamma\,(\theta - 1)}{\theta}\frac{2\Gamma - 1}{\theta - \gamma} < 0,$$

that is, consumers pay a lower net price in each period under dynamic pricing. Consequently, the only party that would prefer constant pricing is the government.

It is also interesting to compare the two pricing strategies taking into account the consumer's type, i.e., strategic or myopic. In the dynamic pricing scenario, the prices, subsidies, manufacturer's profit and subsidy cost increase with γ, the parameter that measures the intensity of strategic behavior of consumers. Under constant pricing, the price and subsidy are independent of γ. In addition, manufacturer's profit is independent of γ because prices are equal in both periods, which makes farsightedness irrelevant. Interestingly, this result does not apply to the subsidy cost. Indeed, while the subsidy levels are equal in both periods, consumers are not treated equally because of the program's eligibility age requirement. This differentiation between consumers is reflected in the government's subsidy cost.

4.2 Dynamic subsidies

In our base model, while the car prices set by the manufacturer vary over time, the government's subsidy remain constant over the two periods. Here, we consider the case where the government sets different subsidy levels in the two periods. In this setup, the demand functions become

$$\begin{aligned} d_1^{\geq\eta} &= s_1\theta + \beta - p_1, \\ d_1^{\eta-1} &= -\gamma s_2 - p_1 + 1 - \beta, \\ d_2 &= 1 - (d_1^{\eta-1} + d_1^{\geq\eta}) + s_2\theta - p_2, \\ &= \gamma s_2 + 2p_1 - s_1\theta + s_2\theta - p_2. \end{aligned}$$

In this context, the government can adopt one of the following two strategic options: (i) a commitment strategy in which it announces (and sticks to) a subsidy plan at the beginning of the first period; and (ii) a flexible strategy where the government sets the subsidy level at the beginning of each period. The main difference between the commitment and flexibility is in the sequence of events. Under commitment, the government announces the subsidy levels for both periods and, then, the manufacturer decides on the prices in the two periods. Under flexibility, the government announces the subsidy in the first period, and next the manufacturer sets the first-period price. Afterwards, the government sets the second period subsidy, just before the manufacturer decides on its second-period price. In order to solve for commitment and flexible policies, we need to find open-loop and feedback equilibrium strategies, respectively. Propositions 4 and 5 characterize the dynamic prices and subsidies equilibria in the two scenarios.

Proposition 4 *Assume $\Gamma > \frac{1}{2}$ and $\beta > \frac{1}{2}$. If $\theta > 2\gamma$, then the open-loop Stackelberg equilibrium strategies are as follows:*

$$\begin{aligned} \check{s}_1 &= -\frac{1}{2}\frac{4\Gamma\theta + 2\beta\gamma - \gamma - 2\theta)}{\theta(2\gamma - \theta)}, \\ \check{s}_2 &= -\frac{1}{2}\frac{2\beta - 5 + 8\Gamma}{2\gamma - \theta}, \\ \check{p}_1 &= \frac{(-2\beta - 8\Gamma + 3)\theta + 4\gamma}{8\gamma - 4\theta}, \\ \check{p}_2 &= \frac{(-2\beta - 6\Gamma + 3)\theta - 4\gamma(\Gamma - 1)}{4\gamma - 2\theta}. \end{aligned}$$

Proof. See Appendix. □

Proposition 5 *Suppose $\Gamma > \frac{2\theta+\gamma}{3\theta+2\gamma}$. The unique Markov-perfect (feedback) Stackelberg equilibrium is given by*

$$\hat{p}_1 = \frac{\theta + \gamma}{2\theta + \gamma}\Gamma, \tag{10}$$

$$\hat{p}_2 = 1 - \Gamma, \tag{11}$$

$$\hat{s}_1 = \frac{6\Gamma\theta + 4\Gamma\gamma - 4\theta - 2\gamma}{\theta\,(2\theta + \gamma)}, \tag{12}$$

$$\hat{s}_2 = 0. \tag{13}$$

Proof. See Appendix. □

The above propositions show that the government and manufacturer apply different strategies in the two scenarios. This result is not surprising as it is well known that in a dynamic game, the equilibrium strategies vary with the information structure, that is, open-loop and feedback (see, e.g., Haurie et al. (2012)). We make the following comments:

1. Whether the conditions stated in Proposition 4 are reasonable or not is ultimately an empirical matter. The condition $\beta > \frac{1}{2}$ means that more than half of the cars aged $\geq \eta - 1$ are of age $\geq \eta$. Moreover, $\theta > 2\gamma$ implies that the present effect of the subsidy is at least twice the future effect of the subsidy.
2. The results in Proposition 4 show that subsidy levels under commitment are increasing in γ for all admissible parameter values. Therefore, both subsidy levels are higher than when the consumer is myopic, i.e., $\gamma = 0$. Moreover, $\frac{\partial \check{s}_2}{\partial \gamma} > \frac{\partial \check{s}_1}{\partial \gamma}$, that is, the second-period subsidy is more sensitive towards the strategic behavior of consumers than the first-period subsidy. In addition, price levels are increasing in γ and $\frac{\partial \check{p}_2}{\partial \gamma} > \frac{\partial \check{p}_1}{\partial \gamma}$.
3. When the government commits, $\check{s}_2 > \check{s}_1$ and $\check{p}_2 > \check{p}_1$, i.e., prices and subsidies are increasing over time. This result shows that the pricing strategy of the manufacturer is in line with the government's subsidy plan. In fact, as the government increases the subsidy level from the first period to second period, the manufacturer responds by raising the price.
4. The results in Proposition 5 show that when the government implements a flexible policy, subsidy and price in the first period are increasing in γ. However, both subsidy and price in the second period are independent of γ. Moreover, while the first-period price is increasing in Γ, the second-period price decreases when the target level increases.
5. When the government adopts a flexible policy, the subsidy in the second period is zero, while it is positive in the first period. The manufacturer responds to this subsidy plan by increasing its prices, i.e., $\hat{p}_1 > \hat{p}_2$. However, the net prices paid by consumers are decreasing over the two periods, that is, $\hat{p}_1 - \hat{s}_1 < \hat{p}_2 - \hat{s}_2$.

In the next proposition, we show which party in the game is better off in each setting, i.e., flexible or commitment.

Proposition 6 *If the government commits to a subsidy plan, then it incurs a higher subsidy cost than under flexible subsidy strategy, i.e.,* $\check{C} > \hat{C}$. *The manufacturer's total profit is higher under a government's commitment strategy, i.e.,* $\check{\Pi} > \hat{\Pi}$.

Proof. See Appendix. □

Proposition 6 shows that the government is better off with flexible setting, while the manufacturer is worse off. Note that in the feedback case, the government foresees the strategic implications of the subsidy on actual and future prices, which leads to lower subsidies than in the commitment case. In summary, it is easy to verify that

$$\check{p}_1 > \hat{p}_1, \ \check{p}_2 > \hat{p}_2,$$
$$\check{s}_1 > \hat{s}_1, \ \check{s}_2 > \hat{s}_2.$$

Another interesting insight is that our result under commitment verifies those provided in paper (Zaman and Zaccour, 2020), where it is shown that scrappage subsidies are increasing over time. Similar to the open-loop equilibrium found in this paper, in (Zaman and Zaccour, 2020), the government commits to its strategy. However, while in (Zaman and Zaccour, 2020) the price is a given constant parameter, in this paper we account for the strategic pricing of the manufacturer. We conclude that in both cases the government applies increasing subsidy strategy.

5 Conclusion

In this paper, we find equilibrium price and subsidy levels offered to strategic consumers using a two-period game model. Various subsidy and pricing strategies are investigated and compared. We obtain that when the subsidy is constant, the manufacturer applies an increasing price strategy, i.e., the second-period price is higher than that of the first period. It is also shown that, when consumers act strategically, price and subsidy levels are higher than in the case where they are myopic.

Our results show that, while the government incurs higher cost in dynamic pricing than under constant pricing, the manufacturer's profit is higher in dynamic pricing. Moreover, we compare two different subsidy strategies applied by the government, i.e., commitment and flexible policies. It is illustrated that when the government commits, manufacturer's profit and subsidy costs are both higher than when it adopts a flexible policy. We believe that these results could help a government designing an efficient scrappage program.

Several extensions of this work can be envisioned. One is to consider more than one manufacturer and assume imperfect competition between them. This market structure would lead to a game with one leader and multiple followers. Another project is to investigate the effect of strategic behavior of consumers in different subsidy programs such as those offered for the adoption of electric vehicles (EVs). Currently, Transport Canada offers consumers who purchase an EV an incentive of $2,500 to $5,000 based on the car type and lease duration. A specific problem to look at is the determination of the subsidy levels taking into account lease type and different EV categories, while accounting for consumer's strategic behavior.

Appendix: Proofs

Proof of Proposition 1

We start by solving for the manufacturer to obtain its reaction to the subsidy announced by the government. Recall that the manufacturer's profit is given by

$$\Pi = \Pi_1 + \Pi_2 = p_1\Big(1 - 2p_1 + (\theta - \gamma)\, s\Big) + p_2\,(2p_1 + \gamma s - p_2)\,. \tag{14}$$

First, we consider the second-period manufacturer's profit. Taking the derivative of Π_2 with respect to p_2, we get

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \Leftrightarrow p_2 = \frac{2p_1 + \gamma s}{2}.$$

Substituting for p_2 in Π yields

$$\Pi = p_1\Big(1 - 2p_1 + (\theta - \gamma)\, s\Big) + \left(\frac{2p_1 + \gamma s}{2}\right)^2.$$

Differentiating with respect to p_1 gives

$$\Pi' = 1 - 2p_1 + \theta s = 0 \Leftrightarrow p_1 = \frac{1 + \theta s}{2}.$$

Substituting in p_2, we obtain the manufacturer's reaction functions, that is,

$$\begin{aligned} p_1 &= \frac{1}{2}\,(\theta s + 1)\,, \\ p_2 &= \frac{1}{2}\,(\gamma s + \theta s + 1)\,. \end{aligned} \tag{15}$$

To solve the government's problem, we write down the Lagrangian

$$\mathcal{L}\,(s, \mu) = s.d_1^{\geq\eta} + s.d_2 + \mu(\Gamma - d_1^{\geq\eta} - d_1^{\eta-1} - d_2),$$

where μ is the Lagrange multiplier appended to the target constraint. Developing the above equation and inserting for p_1 and p_2 from (15), we get

$$\mathcal{L}(s,\mu) = \Gamma\mu - \frac{1}{2}\mu + s\beta + \frac{1}{2}s^2\gamma - \frac{1}{2}s\theta\mu + \frac{1}{2}s\gamma\mu.$$

Assuming an interior solution, the first-order optimality conditions are

$$\frac{\partial\mathcal{L}}{\partial s} = 2\beta + 2s\gamma + \mu(\gamma-\theta) = 0,$$
$$\frac{\partial\mathcal{L}}{\partial\mu} = 2\Gamma - 1 - s(\theta-\gamma) = 0.$$

Solving, we obtain

$$s = \frac{2\Gamma-1}{\theta-\gamma},$$
$$\mu = \frac{2\Big((2\Gamma-1)\gamma + \beta(\theta-\gamma)\Big)}{(\theta-\gamma)^2}.$$

The values of p_1 and p_2 can be found by inserting s into their formulas. Straightforward computations give Π and C.

Proof of Proposition 2

As in Proposition 1, we solve for the manufacturer and get the following reaction function:

$$p = \frac{1}{2}(\theta s + 1).$$

To solve the the government's problem, we introduce the Lagrangian

$$\mathcal{L} = s(s\theta + \beta - p_c) + s(\gamma s + p) + \mu(-s\theta + \Gamma + p - 1),$$

where μ is the Lagrange multiplier. Substituting for p and solving, we obtain the following unique subsidy and value of the Lagrange multiplier:

$$\tilde{s} = \frac{2\Gamma-1}{\theta},$$
$$\mu = \frac{2(4\Gamma\gamma + 4\Gamma\theta + \beta\theta - 2\gamma - 2\theta)}{\theta^2}.$$

Substituting in the price, manufacturer's profit and government's cost, we obtain the $\tilde{p}$, $\tilde{\Pi}$ and $\tilde{C}$ in the statement of the Proposition. We note that the equilibrium is indeed interior.

Proof of Proposition 4

Same as proposition 1, we solve for the manufacturer and we get the following price functions:

$$p_1 = \frac{1}{2}\theta s_2 + \frac{1}{2}$$
$$p_2 = \frac{1}{2}\gamma s_2 + \theta s_2 - \frac{1}{2}\theta s_1 + \frac{1}{2}.$$

The Lagrangian function of the government is given by:

$$\mathcal{L} = s_1(\theta s_1 + \beta - \frac{1}{2}\theta s_2 - \frac{1}{2}) + s_2(\frac{1}{2}\gamma s_2 - \frac{1}{2}\theta s_1 + \theta s_2 + \frac{1}{2}) + \mu(-s_2\theta + \Gamma + p_2 - 1),$$

which results in the following solution:

$$\check{s}_1 = -\frac{1}{2}\frac{4\Gamma\theta + 2\beta\gamma - \gamma - 2\theta)}{\theta(2\gamma - \theta)},$$
$$\check{s}_2 = -\frac{1}{2}\frac{2\beta - 5 + 8\Gamma}{2\gamma - \theta},$$
$$\mu = -\frac{1}{2}\frac{8\Gamma\gamma + 12\Gamma\theta + 4\beta\theta - 6\gamma - 7\theta}{\theta(2\gamma - \theta)}.$$

p_1 and p_2 can be found accordingly.

Proof of Proposition 5

To solve for a Markov-perfect (feedback) Stackelberg equilibrium, we solve the game backward, that is, we start by the second stage.

$$\begin{aligned} d_1^{\geq\eta} &= s_1\theta + \beta - p_1, \\ d_1^{\eta-1} &= -\gamma s_2 - p_1 + 1 - \beta, \\ d_2 &= 1 - (d_1^{\eta-1} + d_1^{\geq\eta}) + s_2\theta - p_2, \\ &= \gamma s_2 + 2p_1 - (s_1 - s_2)\,\theta - p_2, \end{aligned}$$

Second-period equilibrium

For any given s_2 announced by the government, the manufacturer solves the following optimization problem:

$$\max_{p_2 \geq 0} \Pi_2 = p_2\left(\gamma s_2 + 2p_1 - (s_1 - s_2)\,\theta - p_2\right).$$

Introduce the manufacturer's Lagrangian

$$\mathcal{L}_{M_2}(p_2, \lambda_2) = p_2\left(\gamma s_2 + 2p_1 - (s_1 - s_2)\,\theta - p_2\right) + \lambda_2 p_2,$$

where λ_2 is the Lagrange multiplier appended to the constraint $p_2 \geq 0$. The first-order optimality conditions are

$$\frac{\partial \mathcal{L}_{M_2}}{\partial p_2} = \gamma s_2 + 2p_1 - (s_1 - s_2)\,\theta - 2p_2 + \lambda_2 = 0,$$
$$\lambda_2 \geq 0, \quad p_2 \geq 0, \quad \lambda_2 p_2 = 0.$$

Solving the first equation, gives

$$p_2(s_2) = \frac{\gamma s_2 + 2p_1 - (s_1 - s_2)\,\theta + \lambda_2}{2}.$$

Now, we consider the second-period government's optimization problem, which is given by

$$\min_{s_2 \geq 0}\; C_2 = s_2.d_2.$$

Substituting for p_2 in d_2, the above optimization problem becomes

$$\min_{s_2 \geq 0}\; C_2 = s_2.\left(p_1 - \frac{1}{2}\left(\lambda_2 + \theta\,(s_1 - s_2) - \gamma s_2\right)\right).$$

Introduce the second-period government's Lagrangian

$$\mathcal{L}_2(s_2, \mu_2) = s_2\left(p_1 - \frac{1}{2}\left(\lambda_2 + \theta\,(s_1 - s_2) - \gamma s_2\right)\right) + \eta_2 s_2,$$

where η_2 is the Lagrange multiplier appended to the constraint $s_2 \geq 0$. The first-order optimality conditions are

$$\frac{\partial \mathcal{L}_2}{\partial s_2} = p_1 - \frac{1}{2}\left(\lambda_2 + \theta\left(s_1 - 2s_2\right) - 2\gamma s_2\right) + \eta_2 = 0,$$
$$\eta_2 \leq 0, \quad s_2 \geq 0\,, \quad \eta_2 s_2 = 0.$$

Solving the first equation, we obtain

$$s_2\left(s_1, p_1\right) = \frac{1}{\theta+\gamma}\left(\frac{1}{2}\lambda_2 - \eta_2 - p_1 + \frac{1}{2}\theta s_1\right). \tag{16}$$

Substituting in p_2 yields

$$p_2\left(s_1, p_1\right) = \frac{1}{4}\left(2p_1 + 3\lambda_2 - 2\eta_2 - \theta s_1\right). \tag{17}$$

The second-period demand is given by

$$d_2\left(s_1, p_1\right) = \frac{1}{4}\left(2p_1 - 2\eta_2 - \lambda_2 - \theta s_1\right). \tag{18}$$

To wrap up, in (16) and (17), we express the second-period strategies in terms of the first-period decision variables.

First-period equilibrium (or overall equilibrium problem)

The manufacturer overall optimization problem is as follows:

$$\max_{p_1 \geq 0} \Pi = p_1(d_1^{\eta-1} + d_1^{\geq\eta}) + p_2\left(s_1, p_1\right) d_2\left(s_1, p_1\right), \tag{19}$$

where $p_2\left(s_1, p_1\right)$ and $d_2\left(s_1, p_1\right)$ have been determined in the previous step and the product $p_2\big(s_1, p_1\big) d_2\left(s_1, p_1\right)$ plays the role of a salvage value in the current optimization problem.

Substituting for the second-period equilibrium strategies, the above optimization problem becomes:

$$\begin{aligned}\max_{p_1 \geq 0} \Pi =& p_1\left(\frac{2\left(\theta+\gamma\right) - \gamma\lambda_2 + 2\gamma\eta_2 + \left(2\theta+\gamma\right)\left(\theta s_1 - 2p_1\right)}{2\left(\theta+\gamma\right)}\right) \\ &+ \frac{1}{16}\left(2p_1 + 3\lambda_2 - 2\eta_2 - \theta s_1\right)\left(2p_1 - 2\eta_2 - \lambda_2 - \theta s_1\right).\end{aligned} \tag{20}$$

The Lagrangian is given by

$$\begin{aligned}\mathcal{L}_M\left(p_1, \lambda_1\right) =& p_1\left(\frac{2\left(\theta+\gamma\right) - \gamma\lambda_2 + 2\gamma\eta_2 + \left(2\theta+\gamma\right)\left(\theta s_1 - 2p_1\right)}{2\left(\theta+\gamma\right)}\right) \\ &+ \frac{1}{16}\left(2p_1 + 3\lambda_2 - 2\eta_2 - \theta s_1\right)\left(2p_1 - 2\eta_2 - \lambda_2 - \theta s_1\right) + \lambda_1 p_1,\end{aligned}$$

where λ_1 is the Lagrange multiplier appended to the constraint $p_1 \geq 0$.

The first-order optimality conditions are

$$p_1\left(s_1\right) = \frac{4\left(\theta+\gamma\right)\left(1+\lambda_1\right) + \left(\lambda_2 - 2\eta_2\right)\left(\theta-\gamma\right) + \left(3\theta+\gamma\right)\theta s_1}{14\theta + 6\gamma},$$
$$p_1 \geq 0, \quad \lambda_1 \geq 0, \quad \lambda_1 p_1 = 0.$$

Now, we turn to the government's optimization problem. Substituting for $p_1\left(s_1\right)$ in (16)–(18) and in the demands, we get

$$s_2\left(s_1\right) = \frac{-2\left(\theta+\gamma\right)\left(1+\lambda_1\right) + \left(3\theta+2\gamma\right)\left(\lambda_2 - 2\eta_2\right) + \left(2\theta+\gamma\right)\theta s_1}{7\theta^2 + 10\theta\gamma + 3\gamma^2}, \tag{21}$$

$$p_2(s_1) = \frac{2(\theta+\gamma)(1+\lambda_1) + (11\theta+4\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta+6\gamma}, \tag{22}$$

$$d_2(s_1) = \frac{2(\theta+\gamma)(1+\lambda_1) - (3\theta+2\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta+6\gamma}, \tag{23}$$

$$d_1^{\geq\eta}(s_1) = \frac{-4(\theta+\gamma)(1+\lambda_1) + 14\theta\beta + 6\beta\gamma + (2\eta_2-\lambda_2)(\theta-\gamma) + (11\theta+5\gamma)\theta s_1}{14\theta+6\gamma} \tag{24}$$

$$\begin{aligned} d_1^{\eta-1}(s_1) = &- \frac{((10\theta+6\gamma)(\beta-1) + 4\theta(\beta+\lambda_1))(\theta+\gamma) + (\lambda_2-2\eta_2)\left(\theta^2+3\gamma^2+6\theta\gamma\right)}{14\theta^2+20\theta\gamma+6\gamma^2} \\ &- \frac{\left(3\theta^2+3\gamma^2+8\theta\gamma\right)\theta s_1}{14\theta^2+20\theta\gamma+6\gamma^2} \end{aligned} \tag{25}$$

Inserting for the above values in the following optimization problem

$$\begin{aligned} &\min_{s_1\geq 0} C = s_1.d_1^{\geq\eta} + s_2.d_2, \\ &\text{subject to: } d_1^{\geq\eta} + d_1^{\eta-1} + d_2 = \Gamma, \end{aligned} \tag{26}$$

we get

$$\begin{aligned} \min_{s_1\geq 0} C = &s_1\left(\frac{-4(\theta+\gamma)(1+\lambda_1) + 14\theta\beta + 6\beta\gamma + (2\eta_2-\lambda_2)(\theta-\gamma) + (11\theta+5\gamma)\theta s_1}{14\theta+6\gamma}\right) \\ &+ \left(\frac{-2(\theta+\gamma)(1+\lambda_1) + (3\theta+2\gamma)(\lambda_2-2\eta_2) + (2\theta+\gamma)\theta s_1}{7\theta^2+10\theta\gamma+3\gamma^2}\right) \\ &\times\left(\frac{2(\theta+\gamma)(1+\lambda_1) - (3\theta+2\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta+6\gamma}\right), \end{aligned} \tag{27}$$

subject to:

$$\begin{aligned} &\frac{1}{14\theta^2+20\theta\gamma+6\gamma^2}\ \left(8\theta^2+4\gamma^2+12\theta\gamma - \left(6\theta^2+2\gamma^2+8\theta\gamma\right)\lambda_1\right. \\ &\left. - \left(5\theta^2+4\gamma^2-11\theta\gamma\right)\lambda_2 - 2\left(2\theta^2-\gamma^2-\theta\gamma\right)\eta_2 + \left(6\theta^2+\gamma^2+5\theta\gamma\right)\theta s_1\right) = \Gamma, \end{aligned} \tag{28}$$

The Lagrangian is given by

$$\begin{aligned} \mathcal{L}_1(s_1,\mu) = &s_1\left(\frac{-4(\theta+\gamma)(1+\lambda_1) + 14\theta\beta + 6\beta\gamma + (2\eta_2-\lambda_2)(\theta-\gamma) + (11\theta+5\gamma)\theta s_1}{14\theta+6\gamma}\right) \\ &+ \left(\frac{-2(\theta+\gamma)(1+\lambda_1) + (3\theta+2\gamma)(\lambda_2-2\eta_2) + (2\theta+\gamma)\theta s_1}{7\theta^2+10\theta\gamma+3\gamma^2}\right) \\ &\times\left(\frac{2(\theta+\gamma)(1+\lambda_1) - (3\theta+2\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta+6\gamma}\right) \\ &+ \frac{\mu}{14\theta^2+20\theta\gamma+6\gamma^2}\left(8\theta^2+4\gamma^2+12\theta\gamma - \left(6\theta^2+2\gamma^2+8\theta\gamma\right)\lambda_1 - \left(5\theta^2+4\gamma^2-11\theta\gamma\right)\lambda_2\right. \\ &\left. -2\left(2\theta^2-\gamma^2-\theta\gamma\right)\eta_2 + \left(6\theta^2+\gamma^2+5\theta\gamma\right)\theta s_1 - \left(14\theta^2+20\theta\gamma+6\gamma^2\right)\Gamma\right) + \eta_1 s_1, \end{aligned}$$

where μ and η_1 are the Lagrange multipliers appended to the target constraint and $s_1 \geq 0$, respectively.

The first-order optimality conditions are

$$\begin{aligned} \frac{\partial\mathcal{L}_1}{\partial s_1} = &- \frac{1}{14\theta+6\gamma}\left(4(\theta+\gamma)(1+\lambda_1) - 14\theta\beta - 6\beta\gamma + (\lambda_2-2\eta_2)(\theta-\gamma) - 22\theta^2 s_1 - 10\theta\gamma s_1\right) \\ &+ \frac{\theta(2\theta+\gamma)}{(7\theta+3\gamma)^2(\theta+\gamma)}\left(2(1+\lambda_1)(\theta+\gamma) - 3\theta\lambda_2 - 2\gamma\lambda_2 - \theta\eta_2 + \gamma\eta_2 - 2\theta^2 s_1 - \theta\gamma s_1\right) \\ &- \theta\mu\frac{6\theta^2+5\theta\gamma+\gamma^2}{14\theta^2+20\theta\gamma+6\gamma^2}\left(6\theta^2\lambda_1 - 4\gamma^2 - 12\theta\gamma - 8\theta^2 + 5\theta^2\lambda_2 + 2\gamma^2\lambda_1 + 4\gamma^2\lambda_2 + 8\theta\gamma\lambda_1 - 11\theta\gamma\lambda_2\right) \\ &+ \eta_1 = 0 \end{aligned}$$

$$\frac{\partial \mathcal{L}_1}{\partial \mu} = \left(8\theta^2 + 4\gamma^2 + 12\theta\gamma - \left(6\theta^2 + 2\gamma^2 + 8\theta\gamma\right)\lambda_1 - \left(5\theta^2 + 4\gamma^2 - 11\theta\gamma\right)\lambda_2 \right.$$
$$\left. -2\left(2\theta^2 - \gamma^2 - \theta\gamma\right)\eta_2 + \left(6\theta^2 + \gamma^2 + 5\theta\gamma\right)\theta s_1 - \left(14\theta^2 + 20\theta\gamma + 6\gamma^2\right)\Gamma\right) = 0$$
$$\eta_1 \leq 0, \quad s_1 \geq 0\,, \quad \eta_1 s_1 = 0.$$

From the second condition, we can get s_1 as function of the model's parameters and the Lagrange multipliers, that is,

$$\begin{aligned} s_1\left(\lambda_1, \lambda_2, \eta_2\right) = & \frac{1}{6\theta^3 + \theta\gamma^2 + 5\theta^2\gamma}\left(-8\theta^2 - 4\gamma^2 - 12\theta\gamma + 14\Gamma\theta^2 + 6\Gamma\gamma^2 + 20\Gamma\theta\gamma \right. \\ & \left. +2\left(3\theta^2 + \gamma^2 + 4\theta\gamma\right)\lambda_1 + \left(5\theta^2 + 4\gamma^2 - 11\theta\gamma\right)\lambda_2 + 2\left(2\theta^2 - \gamma^2 - \theta\gamma\right)\eta_2\right). \end{aligned} \tag{29}$$

Inserting for s_1 in p_1 we get

$$p_1\left(\lambda_1, \lambda_2, \eta_2\right) = \frac{\left(14\Gamma\theta^2 + 6\Gamma\gamma^2 + \left(14\theta^2 + 6\gamma^2 + 20\theta\gamma\right)\lambda_1 + \left(7\theta^2 + 3\gamma^2 - 12\theta\gamma\right)\lambda_2 + 20\Gamma\theta\gamma\right)}{28\theta^2 + 26\theta\gamma + 6\gamma^2}. \tag{30}$$

Substituting for $s_1\left(\lambda_1, \lambda_2, \eta_2\right)$ in $s_2\left(s_1\right)$ and $p_2\left(s_1\right)$, we obtain

$$s_2\left(\lambda_2, \eta_2\right) = -\frac{\left(2\theta\gamma - 14\theta^2 - 6\gamma^2\right)\lambda_2 + \left(14\theta^2 + 6\gamma^2 + 20\theta\gamma\right)\left(1 + \eta_2 - \Gamma\right)}{21\theta^3 + 37\theta^2\gamma + 19\theta\gamma^2 + 3\gamma^3}, \tag{31}$$

$$p_2\left(\lambda_2, \eta_2\right) = \frac{\left(7\theta^2 + 3\gamma^2 + 10\theta\gamma\right)\left(1 - \Gamma\right) + \left(14\theta + 17\gamma\right)\theta\lambda_2 - \left(14\theta + 6\gamma\right)\theta\eta_2}{21\theta^2 + 16\theta\gamma + 3\gamma^2}, \tag{32}$$

We have four cases to consider:

1. $s_2 > 0$ and $p_2 > 0 \Rightarrow \eta_2 = \lambda_2 = 0$. Then,

$$s_2 = -\frac{\left(14\theta^2 + 6\gamma^2 + 20\theta\gamma\right)\left(1 - \Gamma\right)}{21\theta^3 + 37\theta^2\gamma + 19\theta\gamma^2 + 3\gamma^3}, \tag{33}$$

$$p_2 = \frac{\left(7\theta^2 + 3\gamma^2 + 10\theta\gamma\right)\left(1 - \Gamma\right)}{21\theta^2 + 16\theta\gamma + 3\gamma^2}. \tag{34}$$

   Clearly, s_2 is negative and therefore a contradiction.

2. $s_2 = 0$ and $p_2 > 0 \Rightarrow \eta_2 < 0$ and $\lambda_2 = 0$. Then,

$$\eta_2 = \Gamma - 1 < 0, \tag{35}$$

$$p_2\left(\lambda_2, \eta_2\right) = 1 - \Gamma > 0. \tag{36}$$

3. $s_2 = 0$ and $p_2 = 0 \Rightarrow \eta_2 < 0$ and $\lambda_2 > 0$. Then,

$$\eta_2 = \frac{\left(7\theta^2 + 3\gamma^2 + 10\theta\gamma\right)\left(\Gamma - 1\right)}{11\theta\gamma} < 0, \tag{37}$$

$$\lambda_2 = \frac{\left(7\theta^2 + 3\gamma^2 + 10\theta\gamma\right)\left(\Gamma - 1\right)}{11\theta\gamma} > 0. \tag{38}$$

   The fact that λ_2 is strictly negative is a contradiction.

4. $s_2 > 0$ and $p_2 = 0 \Rightarrow \eta_2 = 0$ and $\lambda_2 > 0$. Then,

$$s_2 = \frac{2\left(7\theta + 3\gamma\right)\left(\Gamma - 1\right)}{14\theta^2 + 17\gamma\theta} < 0, \tag{39}$$

$$\lambda_2 = \frac{\left(\Gamma - 1\right)\left(7\theta^2 + 3\gamma^2 + 10\theta\gamma\right)}{\theta\left(14\theta + 17\gamma\right)} < 0, \tag{40}$$

   a contradiction.

Consequently, the only admissible solution is

$$s_2 = 0, \quad p_2 = 1 - \Gamma, \quad \eta_2 = \Gamma - 1, \quad \lambda_2 = 0.$$

Substituting for these values in (29) and (30), we obtain

$$\begin{aligned} s_1(\lambda_1) =& \frac{1}{6\theta^3 + \theta\gamma^2 + 5\theta^2\gamma}\left(-8\theta^2 - 4\gamma^2 - 12\theta\gamma + 14\Gamma\theta^2 + 6\Gamma\gamma^2 + 20\Gamma\theta\gamma\right. \\ & \left. + 2\left(3\theta^2 + \gamma^2 + 4\theta\gamma\right)\lambda_1 + 2\left(2\theta^2 - \gamma^2 - \theta\gamma\right)(\Gamma - 1)\right), \\ p_1(\lambda_1) =& \frac{\left(14\Gamma\theta^2 + 6\Gamma\gamma^2 + \left(14\theta^2 + 6\gamma^2 + 20\theta\gamma\right)\lambda_1 + 20\Gamma\theta\gamma\right)}{28\theta^2 + 26\theta\gamma + 6\gamma^2}. \end{aligned}$$

Clearly, $p_1(\lambda_1)$ is strictly positive and therefore λ_1 must be equal to zero. Therefore, the final value of p_1 is

$$p_1 = \frac{(\theta + \gamma)\Gamma}{2\theta + \gamma}, \tag{41}$$

and

$$s_1 = \frac{6\Gamma\theta + 4\Gamma\gamma - 4\theta - 2\gamma}{\theta(2\theta + \gamma)},$$

which is positive for

$$\Gamma > \frac{2\theta + \gamma}{3\theta + 2\gamma}.$$

To determine the Lagrange multiplier associated with the target constraint, it suffices to substitute for the equilibrium values in $\frac{\partial \mathcal{L}_1}{\partial s_1} = 0$ to obtain

$$\mu = -\frac{2\theta^3(32\Gamma - 23 + 7\beta) + \theta^2(27\beta\gamma - 93\gamma + 137\Gamma\gamma) + \theta\left(91\Gamma\gamma^2 - 57\gamma^2\right) + \gamma^3(19\Gamma + 3\beta - 11) + 16\theta\beta\gamma^2}{2\theta(3\theta + \gamma)(\theta + \gamma)(2\theta + \gamma)^3}.$$

Proof of Proposition 6

The manufacturer profits and subsidy costs in both settings are given by:

$$\check{C} = \frac{(8\beta^2 + (64\Gamma - 36)\beta + 96\Gamma^2 - 112\Gamma + 32)\theta^2 + 4(\beta^2 16\Gamma^2 + \beta^2 - 24\Gamma - 3\beta + \frac{37}{4})\gamma\theta - 8(\beta - \frac{1}{2})^2\gamma^2}{8\theta(2\gamma - \theta)^2}$$

$$\check{\Pi} = \frac{(40\Gamma^2 + (24\beta - 36)\Gamma + 4(\beta - \frac{3}{2})^2)\theta^2 + 16\gamma(2\Gamma^2 + (\beta - \frac{9}{2})\Gamma - \beta + \frac{3}{2})\theta + (32(\Gamma^2 - \Gamma + \frac{1}{2}))\gamma^2}{(2\gamma - \theta)^2}$$

$$\hat{C} = \frac{4((\frac{3}{2}\Gamma - 1)\theta + \gamma(\Gamma - \frac{1}{2}))((2\beta + 5\Gamma - 4)\theta + \gamma(\beta + 3\Gamma - 2))}{(2\theta + \gamma)^2\theta}$$

$$\hat{\Pi} = \frac{(3\gamma + 4\theta)\Gamma^2 + (-3\gamma - 5\theta)\Gamma + 2\theta + \gamma}{2\theta + \gamma}$$

Considering the circumstances in propositions 4 and 5, $\check{C} - \hat{C}$ and $\check{\Pi} - \hat{\Pi}$ are increasing in Γ and positive at $\Gamma = \frac{2\theta+\gamma}{3\theta+2\gamma}$, that is to say, $\check{C} > \hat{C}$ and $\check{\Pi} > \hat{\Pi}$.

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