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G–2018–44
June 2018
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June 2018

Les Cahiers du GERAD

G–2018–44

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Abstract: It is well known that strategic consumers can harm firms’ profits by delaying their purchases, to buy at discounted price. A retailer can induce consumers to purchase at the right price and time by acting on the two components of the consumer’s surplus in each period, that is, the willingness to pay (WTP) and the selling price. We develop a multi-period model to investigate pricing and advertising decisions in a supply chain, in the presence of two types of consumers, namely, myopic and strategic. We assume that the retailer’s advertising positively affects the consumer’s WTP at a decreasing rate over time. The manufacturer sets the wholesale price and its share in the retailer’s advertising cost, while the retailer determines the retail prices during the different periods of the selling season, along with the advertising budget. Our approach makes it possible to determine endogenously the number of price drops during the selling season and the depth of each discount. Assuming decentralized decision-making in the supply chain, we determine the conditions under which the retailer prefers a single-pricing policy to a markdown-pricing policy. We show that the manufacturer has a say in the retailer’s choice through its participation rate in the retailer’s advertising cost. Interestingly, we obtain that an integrated supply chain would, roughly speaking, adopt the same pricing policy as in the decentralized case. To start with, we assume that the WTP is distributed uniformly, and later on, we assess the impact of changing the distribution on the results.

Keywords: Pricing Policy, advertising, supply chain, strategic consumers

Résumé: Il est bien connu que les consommateurs stratégiques peuvent nuire aux profits des entreprises en retardant leurs achats pour acheter leurs produits à prix réduit. Un detaillant peut inciter les consommateurs à acheter au bon prix et au bon moment en agissant sur les deux composantes du surplus du consommateur à chaque période, soit le prix de réserve et le prix de vente. Nous développons un modèle multi-périodes pour étudier les décisions de prix et de publicité dans une chaîne d’approvisionnement, en présence de deux types de consommateurs: myope et stratégique. Nous supposons que la publicité du détaillant affecte positivement le prix de réserve du consommateur selon un taux décroissant en fonction du temps. Le manufacturier fixe le prix de gros et sa part dans les coûts publicitaires du détaillant, tandis que le détaillant détermine les prix de détail pendant les différentes périodes de la saison de vente, ainsi que le budget publicitaire. Notre approche permet de déterminer de manière endogène le nombre de baisses de prix pendant la saison de vente et la profondeur de chaque réduction. En supposant une prise de décision décentralisée dans la chaîne d’approvisionnement, nous déterminons les conditions dans lesquelles le détaillant préfère une politique de prix unique à une politique de prix de démarque. Nous montrons que le fabricant a son mot à dire dans le choix du détaillant via son taux de participation aux frais de publicité du détaillant. Fait intéressant, nous obtenons qu’une chaîne d’approvisionnement intégrée adopte, grosso modo, la même politique de prix que dans le cas décentralisé. Pour commencer, nous supposons que le prix de réserve est distribué uniformément, et plus tard, nous évaluons l’impact de la modification de la distribution sur les résultats.

Mots clés: Politique de prix, publicité, chaîne d’approvisionnement, consommateurs stratégiques
1 Introduction

Consumers are smarter and more informed than ever. Thanks to Internet and various mobile apps, they can get substantial information about retailers and their offerings. Shopping engines such as Google and Yahoo and shopping websites such as pricegrabber.com make it superfast and easy for consumers to find out local and online sales listings and maximize their benefits. Also, there are several tools that enable consumers to track the pricing patterns of many products and consequently choose the appropriate purchasing time.

In this paper, we develop a multi-period model to investigate pricing and advertising decisions in a supply chain made up of one manufacturer and one retailer, in the presence of two types of consumers, namely, myopic and strategic (or farsighted). A myopic consumer makes a purchase in the first period in which she gets a positive surplus, whereas a strategic consumer purchases at the period that maximizes her utility. We suppose that the retailer advertises the product at the beginning of the selling season, and that advertising positively affects the consumer’s willingness to pay (WTP), at a decreasing rate over time. The manufacturer sets the product’s wholesale price and its share in the retailer’s advertising cost, while the retailer determines the optimal selling horizon and retail prices during the different periods of the selling season, along with the advertising budget.

Our objective is to answer the following research questions:

1. In a decentralized supply chain, under what conditions would the retailer prefer a single-pricing (SP) policy over a markdown-pricing (MP) policy?
2. Would the manufacturer and retailer choose a different pricing policy if the supply chain coordinated?
3. How can the manufacturer influence the retailer’s pricing policy and the selling horizon?
4. In an MP policy, what is the optimal number of price markdowns and the depth of each one?
5. What is the impact on the results of varying the distribution of the population’s WTP?

Many researchers have reported a negative impact of consumers’ strategic behavior on the retailer’s profit. They have argued that if firms assume that all consumers are myopic and ignore strategic decision making pattern of strategic consumers, they might end up with a revenue loss estimated to be between 20 and 60% (see, e.g., Aviv and Pazgal (2008), Besanko and Winston (1990)). These figures constitute a clear invitation to retailers to take into account strategic consumers when making their pricing and other marketing decisions. This research is meant to provide a guideline for supply chain members on how to mitigate the adverse effects of strategic behavior by consumers.

1.1 Literature background

The idea that some consumers behave strategically when making purchasing decisions is clearly not new. In fact, a number of studies have even considered the case where all consumers are farsighted; see, e.g., Liu and van Ryzin (2008); Yu et al. (2015); Surasvadi et al. (2017). Behavioral research has highlighted the importance of the strategic consumers’ group and the need to consider both strategic and myopic consumers in any study aimed at understanding what is actually happening in the market (Mak et al., 2014; Li et al., 2014; Osadchiy and Bendoly, 2015). According to Li et al. (2014), 5 to 20% of consumers exhibit forward-looking behavior in the air-travel industry. Based on experimental data, Osadchiy and Bendoly (2015) found that 37% of consumers behaved strategically in a full-information setting about future prices. Further, Kremer et al. (2017) showed that the retailer’s pricing strategy depends on the fraction of strategic consumers in the market. The wisdom is that when most consumers are myopic, the retailer could price-discriminate by charging a premium price to myopic consumers with high WTP, and then, drop the price to sell to those with a lower WTP (Coase, 1972). However, the presence of strategic consumers may discourage the retailer to deeply reduce the (regular) price, and instead try to incentivize them to purchase the product earlier to avoid the loss of profit due to delayed purchasing. In the next paragraphs, we briefly discuss how the literature has dealt with strategic consumers.\footnote{We also refer the reader to several recent reviews and book chapters on strategic consumer behavior (Wei and Zhang (2018b), Chen and Chen (2015), Gönsch et al. (2013), Netessine and Tang (2009), and Shen and Su (2007)).}
**Pre-announcing prices.** To prevent consumers from waiting to buy at a discounted price, the seller must eliminate (or drastically reduce) consumers’ expected surplus in the promotional period. One option is to intentionally limit the inventory (or hide information about it) to reduce the likelihood of the product being available in the salvage period (Liu and van Ryzin, 2008; Huang and Liu, 2015; Wei and Zhang, 2018a). Another option, which is directly related to our work, is to announce the future prices up front and commit to a smaller markdown to discourage strategic consumers from waiting (Zhang and Cooper, 2008; Gallego et al., 2008; Mersereau and Zhang, 2012). Aviv and Pazgal (2008) compared a strategy of pre-announcing pricing to a responsive pricing strategy (that is, where the discounted price is decided upon later, based on inventory) and obtained that, in the presence of strategic consumers, a pre-announced policy is more profitable for the seller (up to 8.32% more profits) than is responsive pricing. The reason is that pre-announced prices remove consumers’ rational expectations about the future price. Examples of the implementation of pre-announced prices include Boston store of Filene’s Basement, Wanamaker’s discount department store in Philadelphia, Tuesday Morning discount stores, Land’s End Overstocks, Dress for Less, and TKTS ticket booths in London and New York City (Yin et al., 2009).

**Increasing willingness to pay.** Whereas pre-announcing prices aims at reducing the surplus from delaying purchasing, here the firm aims at increasing the surplus of buying earlier, that is, at the regular price, with the surplus being equal to the WTP minus the list price. Clearly, the WTP depends on the consumers’ assessments of product performance (or product quality) and on the brand’s image (Kirmani and Rao, 2000; Kirmani and Wright, 1989; Zeithaml, 1988; Koetz et al., 2015; Abbey et al., 2017). In particular, when quality is hard to evaluate before purchasing, then the firm can use advertising to convey information about this quality and to raise the brand’s image (Nelson, 1974; Moorthy and Zhao, 2000; Barone et al., 2005). For instance, Erdem and Keane (1996) and Erdem et al. (2008a) showed that when consumers are uncertain about brand attributes and risk averse with regard to attribute variation, providing a noisy signal of product quality through advertising would have positive effects on WTP and generate greater returns for companies. Wang et al. (2018) studied the efficacy of healthy eating and anti-obesity advertising on the willingness to pay for healthy and unhealthy food, and their results show that both types of advertisements have a significant impact on WTP. Further, Tsui (2012) showed that, regardless of product quality, advertising effectively increases the consumer’s perceived quality and WTP. Also, they noted that the percentage increase in WTP when advertising low-quality products is higher than when advertising high-quality products. Erdem et al. (2008b) used Nielsen scanner panel data in four categories of consumer goods to examine how TV advertising affects demand for a brand. They found that advertising raises individual consumer’s willingness to pay and shifts the whole distribution of WTP upward in the population.

**Cooperative advertising.** If advertising can play an active role in raising the WTP, then a relevant question in a supply chain context is who should be responsible for it. When retailers are single-brand sellers, as in, e.g., a franchising system, and if we forget about (the clearly important) free-riding issues in such a system, then who does the advertising is not that important in terms of influencing consumer behavior. When the retailers carry different brands, it has traditionally been the case that brand advertising is done by manufacturers, and local advertising (in-store displays, flyers, etc.) by retailers. Still, it is not rare nowadays to see retailers, especially large ones, running advertising campaigns that feature manufacturers’ brands (see Pnevmatikos et al. (2018)). Examples include Best Buy’s 2015 campaign for the Samsung UHD TV, Walmart’s 2011 campaign for Coca-Cola, and Fnac’s 2013 campaign for Apple products. These advertisements emphasize the product’s brand name and features, together with the retailer’s own information. Interestingly, these TV campaigns do not focus, as one might expect, on the retail chain itself, on the product categories it sells, or on its special offers, but are co-branding advertising campaigns.

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One way to account for a shared responsibility in advertising is to assume that the supply chain’s members adopt a cooperative advertising program (CAP), that is, a contract in which the retailer advertises the product and shares the cost with the manufacturer. There is a huge literature on CAPs in supply chains, and we refrain from reviewing it, but instead, refer the reader to the comprehensive recent surveys in Jørgensen and Zaccour (2014) and Aust and Buscher (2014). A general result in a bilateral monopoly, both in static and dynamic settings, is that CAPs effectively increase the retailer’s advertising efforts and lead to an increase in demand and total channel profit, which confers unto CAPs a supply chain (partial) coordination flavor.

To recapitulate, in this paper, the retailer (i) endogenously determines the selling season horizon, that is, the number of different prices over time, and pre-announces them; and (ii) advertises the product, with the cost being shared with the manufacturer according to a cooperative advertising contract. Mitigating the adverse effect of strategic consumers by suppressing their expected surplus from waiting, is the most suggested contribution in the literature (see the recent review in Wei and Zhang (2018b)). We adopt the point of view that, by spending on advertising, the retailer raises the willingness to pay, and by the same token, increases the consumer’s surplus in early periods. In the existing literature, the effects of strategic consumer behavior have only been studied from retailers’ perspective. How the upstream firms and the entire supply chain could have been affected by the presence of strategic consumers is a question that has merely been addressed (Lin et al., 2018). The closest paper to ours is Farshbaf-Geranmayeh et al. (2018). Here, the authors also assumed that the WTP increases with advertising and that the supply chain implements a CAP. However, the regular and discounted prices in the two-period model were given parameters, not decision variables. To the best of our knowledge, pricing and advertising decisions in a supply chain in the presence of myopic and strategic consumers have not been addressed in the literature. By acting on the two components of the consumer’s surplus in each period, that is, the WTP and the selling price, the retailer ends up having a decisive say regarding the purchasing time. Further, by assuming infinite capacity (as in, e.g., Besanko and Winston (1990); Mantin and Granot (2010); Liu and Zhang (2013); Du and Chen (2014)) and decreasing the advertising effect over time, we show that strategic consumers will wait until the last period to buy at the lowest possible price. We highlight that our approach makes it possible to determine endogenously the optimal selling horizon for the retailer (and consequently the number and depth of price markdowns), which has not been done in general in the strategic consumer behavior literature.

The remainder of this paper is organized as follows: In Section 2, we set the general model. To gain some qualitative insight into pricing strategies, we retain a two-period model in Section 3 and fully characterize the noncooperative equilibrium and compare it to the vertically integrated channel solution. In Section 4, we derive and discuss our numerical results obtained for more than two periods, a context where the equilibrium results cannot be obtained in closed form. In addition, the impact of changing the distribution of WTP on the results is assessed in this section. Finally, we conclude in Section 5. Proofs are provided in the Appendix.

2 Model

Consider a supply chain made up of one manufacturer and one retailer offering a product during a selling season divided into \( n \) periods. Denote by \( p_i \) the price to the consumer set by the retailer in period \( i \in N = \{1, \ldots, n\} \), and by \( D_i \) the corresponding demand. The total demand during the selling season is denoted by \( D = \sum_{i=1}^{n} D_i \). The retailer also chooses a one-time advertising effort, denoted by \( A \), which is made at the start of the selling season. The total cost of advertising is assumed to be convex increasing and taken quadratic for simplicity, that is, \( C(A) = A^2 \). The manufacturer decides the wholesale price \( w \) and its share \( t \) in the retailer’s advertising cost.

Denote by \( v_i \) the consumer’s willingness to pay in period \( i = 1, 2, \ldots, n \). We assume that the retailer’s advertising affects the WTP as follows:

\[
v_i = v + \theta_i A, \quad \forall i = 1, \ldots, n,
\]

where \( \theta_i \) is the advertising efficiency and \( v > 0 \) is the intrinsic consumer WTP in the absence of advertising (\( A = 0 \)). Put differently, by advertising, the retailer provides information to consumers, which raises their perception of the product’s quality, and consequently increases their WTP from \( v \) to \( v + \theta_i A \). Due
to forgetting, the impact of advertising is decreasing over time (periods), that is, $\theta_{i+1} < \theta_i$, and so is the consumer’s WTP. We assume that $v$ is uniformly distributed over $[0, \alpha]$, with cumulative distribution function $F_v(\cdot)$, which is common knowledge to the retailer and customers. To simplify the exposition, now we set $\alpha = 1$, but will consider other values in the numerical illustrations. Further, we shall later consider a beta distribution and assess the impact on the results.

The consumer’s utility (surplus) of purchasing the product in period $i$ is denoted by $u_i$ and given by

$$u_i = v_i - p_i, \quad \forall i = 1, ..., n, \quad (2)$$

or equivalently,

$$u_i = v + \theta_i A - p_i, \quad \forall i = 1, ..., n.$$  

That is, the consumer’s utility in period $i$ depends on her intrinsic WTP ($v$), advertising level ($A$), and retail price ($p_i$).

Let $P = (p_1, \ldots, p_n)$ be the vector of prices during the selling season. We distinguish between two groups of consumers, namely, myopic and farsighted (or strategic) consumers. A myopic consumer makes a purchase in the first period, call it $i_m$, in which she gets a positive surplus. Period $i_m$ is then determined as follows:

$$i_m(v, A, P) = \min_i \{i | v_i \geq p_i\}. \quad (3)$$

A strategic consumer purchases at the period $i_f$ that maximize her utility, i.e.,

$$i_f(A, P) = \min \arg \max_i \{u_i = v + \theta_i A - p_i | u_i \geq 0\}. \quad (4)$$

Observe that the value of $i_f(A, P)$ is the same for all consumers.

Without any loss of qualitative insight, we set the manufacturer’s unit production cost $c$ to zero. Assuming profit-maximization behavior, the manufacturer and retailer optimization problems are as follows:

$$\max_{w,t} \Pi_M = wD - tA^2,$$

subject to : $w \geq 0, \quad 0 \leq t \leq 1,$

$$\max_{p_1, \ldots, p_n, A} \Pi_R = \sum_{i=1}^{n} (p_i - w)D_i - (1 - t) A^2,$$

subject to : $A \geq 0, \quad p_i \geq 0$ for all $i \in N.$

Following the long tradition in supply chains, especially in the presence of cooperative advertising (see the surveys in Aust and Buscher (2014) and Jørgensen and Zaccour (2014)), we assume that the game is played à la Stackelberg, where the manufacturer, as leader, first announces its wholesale price and support rate, and the retailer, as follower, reacts to this announcement and chooses the retail prices in different periods and the advertising budget.

### 2.1 Demand

Denote by $S$ the market size and let $\rho$ be the proportion of myopic consumers, with the rest, $\bar{\rho} = 1 - \rho$, being the proportion of strategic consumers. We need to determine the number of buyers of each type in each period.

We start by strategic consumers. From the definition of $i_f(A, P)$ in (4), it is clear that if they buy, then all strategic consumers will buy in the same period $i_f$, whose value depends on the retailer’s pricing and advertising decisions. Given our assumption that $v$ is uniformly distributed over $[0, 1]$, the demand by strategic (farsighted) consumers is then given by $D_{ij}^f = \bar{\rho}SF_{v_i}(p_{ij})$, that is,

$$D_{ij}^f = \bar{\rho}S \left( \theta_{ij} A + 1 - p_{ij} \right).$$
We now turn to myopic consumers. Recall that a myopic consumer makes a purchase in the first period where she gains a positive utility, meaning that a myopic consumer having a positive utility in period \( i_m \), and negative utility in all the previous periods \( (i < i_m) \), will purchase in period \( i_m \). Contrary to strategic consumers, purchasing time is not the same for all myopic consumers, because their intrinsic WTP \( v \) is not. Therefore, the demand of myopic consumers in each period corresponds to the number of myopic consumers with a positive surplus in that period minus the number of myopic consumers who purchased in the previous period. Note that no myopic consumer will purchase after period \( i_f \), because it is assumed that all consumers achieve their highest utility in \( i_f \). Consequently, it is impossible to have a negative utility in \( i_f \) and a positive one in a subsequent period. Therefore, the demand of myopic consumers is as follows:

\[
D^m_{i_m} = \begin{cases} 
\rho S(F_{v_{i_m}}(p_{i_1})) = \rho S(\theta_1 A + 1 - p_1), & \text{for } i_m = 1, \\
\rho S(F_{v_{i_m}}(p_{i_m}) - F_{v_{i_m}}(p_{i_{m-1}})) = \rho S(p_{i_{m-1}} - p_{i_m} - (\theta_{i_{m-1}} - \theta_{i_m})A), & \text{for } 2 \leq i_m \leq i_f, \\
0, & \text{for } i_m > i_f.
\end{cases}
\]

The total demand in each period is then given by

\[
D_i = \begin{cases} 
D^m_{i_m} & \text{for } i < i_f, \\
D^m_{i_m} + D^f_{i_f} & \text{for } i = i_f, \\
0 & \text{for } i > i_f,
\end{cases}
\]

and the total demand during the whole selling season by

\[
D = D^f_{i_f} + \sum_{i_m=1}^{i_f} D^m_{i_m} = S(F_{v_{i_f}}(p_{i_f})) = S(\theta_{i_f} A + 1 - p_{i_f}).
\]

We make the following remarks on the demand system:

1. Prices are decreasing over time. Indeed, the nonnegativity of demand implies that for period 1 (see (5)), we have

\[
D^m_1 \geq 0 \iff F_{v_{i_1}}(p_1) \geq 0 \iff p_1 \leq \theta_1 A + 1,
\]

and for \( 2 \leq i_m \leq i_f \):

\[
D^m_{i_m} \geq 0 \iff F_{v_{i_m}}(p_{i_m}) \geq F_{v_{i_m}}(p_{i_{m-1}}) \iff p_{i_{m-1}} - p_{i_m} \geq (\theta_{i_{m-1}} - \theta_{i_m})A.
\]

As \( \theta_{i_{m-1}} - \theta_{i_m} \) is positive, then \( p_{i_{m-1}} - p_{i_m} \geq 0 \).

2. The result that after \( i_f \) no sales can take place amounts to considering that the selling season ends at \( i_f \) and this date is endogenously determined. Therefore, in the rest of paper, \( i_f \) can also be used as the length of the retailer’s selling season.

3. The utility is nondecreasing over time. Recall that

\[
\forall i = 1, \ldots, n, \quad u_i = v + \theta_i A - p_i,
\]

and therefore

\[
u_i - u_{i-1} = \begin{cases} 
p_{i-1} - \theta_i A \geq 0, & \text{for } 2 \leq i < i_f, \\
p_{i-1} - (\theta_{i-1} - \theta_i)A > 0, & \text{for } i = i_f.
\end{cases}
\]

The inequalities follow from the nonnegativity of the demand and the definition of \( i_f \). The utility in period \( i_f \) is, by definition, strictly larger than the utility in other periods.

4. Adding the features that (i) the demand is nonnegative in all periods; (ii) the highest utility is achieved in period \( i_f \); and (iii) the total over-the-season sales must not exceed the total market size, that is, \( F_{v_{i_f}}(p_{i_f}) \leq 1 \), we get

\[
0 \leq p_{i_f} - \theta_{i_f} A < p_{i_f-1} - \theta_{i_f-1} A \leq \cdots \leq p_2 - \theta_2 A \leq p_1 - \theta_1 A \leq 1.
\]
5. The manufacturer’s payoff depends on the total demand, which in turn depends on advertising and only the price \( p_i \) (see (6)). Therefore, for each specified \( i_f \), its wholesale pricing strategy should take into account only this price and not the different prices that the retailer implements during the selling season.

In determining its best response, the retailer must find out the optimal value of \( i_f \), which requires solving \( n \) subproblems and comparing their corresponding profit values. Formally, the retailer’s problem is as follows:

\[
\Pi_R(P, A) = \max_{i_f \in \{1, 2, \ldots, n\}} \sum_{i=1}^{i_f} (p_i - w)D_i - (1 - t)A^2,
\]

\[
\text{subject to } A \geq 0 \quad \text{and} \quad (7).
\]

**Remark 1** As the market size \( S \) does not play any significant qualitative role in the paper, we shall from now on normalize it to one to save on notation.

### 3 A two-period model

To gain some qualitative insights into the pricing strategies in the presence of strategic consumers, we shall retain and analytically fully solve a two-period model. To keep it as simple as possible, without altering the pricing structure, we suppose that advertising has no effect on the consumer’s WTP in the second period, that is, we have \( 0 = \theta_2 < \theta_1 \triangleq \theta \). Further, to have an interior solution, we require \( \theta^2 < 2 \). The retailer’s optimization problem can be rewritten as follows:

\[
\Pi_R(P, A) = \max_{i_f \in \{1, 2\}} \Pi_{R}^{ij}(P, A),
\]

where

\[
\Pi_{R}^{1j}(P, A) = \max_{p_1, A} (p_1 - w)(\theta A + 1 - p_1) - (1 - t)A^2,
\]

\[
\text{subject to } : \theta A \leq p_1 \leq \theta A + 1, \quad \text{and } A \geq 0,
\]

and

\[
\Pi_{R}^{2j}(p_1, p_2, A) = \max_{p_1, p_2, A} \rho(p_1 - p_2)(\theta A + 1 - p_1) + (p_2 - w)(1 - p_2) - (1 - t)A^2,
\]

\[
\text{subject to } : \theta A + p_2 < p_1 \leq \theta A + 1, \quad \text{and } A \geq 0, \quad p_2 \geq 0.
\]

The optimization problem in (10) corresponds to the case where the retailer only sells in the first period, and (11), to the case where it sells in both periods.

The following proposition characterizes the best response functions of the retailer to any pair \((w, t)\) announced by the manufacturer.

**Proposition 1** Suppose that the manufacturer announces a wholesale price \( w \) and a support rate \( t \). Assuming an interior solution, the best response of the retailer is given by

\[
\text{Pricing policy} = \begin{cases} 
SP, & \text{if } t \geq 1 - \frac{\theta^2(1+\sqrt{1-\rho^2})}{2\rho}, \\
MP & \text{otherwise},
\end{cases}
\]

\[\text{Remark} \quad \text{The results are available for any } S \text{ upon request.}\]
and for SP,

\[ A(w, t) = \frac{\theta(1 - w)}{4(1 - t) - \theta^2}, \quad (13) \]

\[ p_1(w, t) = \frac{2(1 + w)(1 - t) - \theta^2 w}{4(1 - t) - \theta^2}, \quad (14) \]

\[ \Pi_R(p_1(w, t), A(w, t)) = \frac{(1 - w)^2(1 - t)}{4(1 - t) - \theta^2}. \quad (15) \]

and for MP,

\[ A(w, t) = \frac{\rho\theta(1 - w)}{2(4 - \rho)(1 - t) - 2\rho\theta^2}, \quad (16) \]

\[ p_1(w, t) = \frac{2(w + 3 - \rho)(1 - t) - \rho\theta^2(1 + w)}{2(4 - \rho)(1 - t) - 2\rho\theta^2}, \quad (17) \]

\[ p_2(w, t) = \frac{2(2w + 2 - \rho)(1 - t) - \rho\theta^2(1 + w)}{2(4 - \rho)(1 - t) - 2\rho\theta^2}, \quad (18) \]

\[ \Pi_R(p_1(w, t), p_2(w, t), A(w, t)) = \frac{(1 - w)^2(4(1 - t) - \rho\theta^2)}{4(4 - \rho)(1 - t) - 4\rho\theta^2}. \quad (19) \]

**Proof.** See Appendix.

The results in the above proposition call for the following comments:

1. It can be easily checked that the conditions for having an interior solution are

\[ 0 \leq t \leq 1 - \frac{\theta^2}{2} \quad (20) \]

\[ 0 \leq w < 1 \quad (21) \]

The first condition means that the support rate should not be too high. For the retailer’s revenues to be positive in the absence of advertising, the prices must be between 0 and 1. The second condition is a consequence of the (implicit) bounds on the retail prices.

2. From (12), we see that the retailer’s choice of the pricing policy, the value of \( i_f \), depends on the manufacturer’s participation rate in advertising, and not on the wholesale price. Indeed, looking at the difference of the retailer’s profit in (15) and (19), we see that \( w \) factors out, and hence does not influence, the sign of this difference. More specifically, the retailer implements a single-price policy when the support rate exceeds the threshold defined in (12). It is easy to verify that this threshold is decreasing in the advertising efficiency parameter \( \theta \), and increasing in the percentage of myopic consumers. Figure 1 shows the SP (single-price) and MP (markdown-price) regions in the \((t, \theta, \rho)\)-space. It is clearly seen that the single-price region enlarges with \( \theta \) and \((1 - \rho)\).

3. The retailer’s advertising is negatively related to the manufacturer’s wholesale price, which means that there is strategic substitution between the two decision variables. The short explanation is that a larger \( w \) leaves the retailer with less available revenue for advertising. The retailer’s advertising is increasing in the support rate, i.e., we have strategic complementarity, which is an expected result since the role of the support rate is precisely to stimulate the retailer’s advertising.

4. The retail prices in both the SP and MP scenarios are increasing in the relevant wholesale price, which replicates the classical strategic complementarity due to double marginalization.

5. The regular price, which is the unique price under SP and the first-period price under MP, are both increasing in the manufacturer’s support rate. This result follows the principle that the higher is the consumer’s WTP, the higher the price. Advertising leads to an increase in this WTP, and the manufacturer’s support rate boosts the retailer’s advertising. The discounted price in the second period under MP is decreasing in \( t \), which is a by-product of the fact that advertising has a decreasing impact on utility and WTP over time.
Turning to the manufacturer, we need to solve the following two subproblems:

If the pricing policy of the retailer is $SP$ then solve

\[
\max_{w,t} \Pi_M(w,t) = w(\theta A(w,t) + 1 - p_1(w,t)) - tA^2(w,t),
\]
subject to:

\[
0 \leq w < 1,
\]
\[
1 - \frac{\theta^2 \left(1 + \sqrt{1 - \rho^2}\right)}{2\rho} \leq t \leq 1 - \frac{\theta^2}{2}.
\]

If the pricing policy of the retailer is $MP$ then solve

\[
\max_{w,t} \Pi_M(w,t) = w(1 - p_2(w,t)) - tA^2(w,t),
\]
subject to:

\[
0 \leq w < 1,
\]
\[
0 \leq t \leq 1 - \frac{\theta^2 \left(1 + \sqrt{1 - \rho^2}\right)}{2\rho}.
\]

The following proposition characterizes the unique Stackelberg equilibrium of the game.

**Proposition 2** Given the retailer’s best response, then the manufacturer’s optimal wholesale price and support rate, and the consequent retailer’s prices and advertising, are defined as follows:

**Under $SP$,**

\[
SP_1: \begin{cases} 
\text{if } \theta^2 \leq \frac{4}{3}, \\
\quad t = \frac{1}{3}, \\
\quad w = \frac{16 - 3\theta^2}{32 - 9\theta^2}, \\
\quad A = \frac{6\theta}{32 - 9\theta^2}, \\
\quad p_1 = \frac{24 - 3\theta^2}{32 - 9\theta^2}, \end{cases} \quad SP_2: \begin{cases} 
\text{if } \frac{4}{3} \leq \theta^2 \leq 2, \\
\quad t = 1 - \frac{\theta^2}{4}, \\
\quad w = \frac{2}{2 + \theta^2}, \\
\quad A = \frac{\theta}{2 + \theta^2}, \\
\quad p_1 = 1. \end{cases}
\]
Under MP,

\[
\begin{align*}
MP_1: & \quad \begin{cases}
    \text{if } \theta^2 \leq \frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2}, \\
    t = \frac{1}{2} - \frac{\rho\theta^2}{12w}, \\
    w = \frac{64-16\rho(\theta^2+1)+\rho^2\theta^2}{128-32\rho(\theta^2+1)-\rho^2\theta^2}, \\
    A = \frac{12\rho t}{128-32\rho(\theta^2+1)-\rho^2\theta^2}, \\
    p_1 = \frac{112-2\rho(116^2+16)-\rho^2\theta^2}{128-32\rho(\theta^2+1)-\rho^2\theta^2}, \\
    p_2 = \frac{96-8\rho(3\theta^2+4)-\rho^2\theta^2}{128-32\rho(\theta^2+1)-\rho^2\theta^2}.
\end{cases} \\
MP_2: & \quad \begin{cases}
    \text{if } \frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 \leq \frac{2\rho}{1+\sqrt{1-\rho^2}}, \\
    t = 1 - \frac{\theta^2(1+\sqrt{1-\rho^2})}{2\rho}, \\
    w = \frac{\theta^2[4\sqrt{1-\rho^2}(4-\rho-2\rho^2)+2(8-2\rho-8\rho^2+\rho^3+\rho^4)]}{2\rho^2}, \\
    A = \frac{\rho t(1-w)}{2(4-\rho)(1-t)-2\rho\theta^2}, \\
    p_1 = \frac{2(w+3-\rho)}{2(4-\rho)(1-t)-2\rho\theta^2}(1+w), \\
    p_2 = \frac{4(w+1-\rho)(1-t)-\rho\theta^2(1+w)}{2(4-\rho)(1-t)-2\rho\theta^2}.
\end{cases}
\end{align*}
\]

Proof. See Appendix.

The manufacturer's strategies are parameterized in the pricing policy of the retailer (SP or MP), and for each one, we have a unique solution, whose expression depends on some conditions on the parameter values.

From the results in Proposition 1, we clearly see that the manufacturer, through its support rate, can induce the retailer to implement a dynamic pricing, or not. To select between the two pricing possibilities, the manufacturer must compare its profits under the two scenarios, which depend on two parameters, namely, \(\theta\) and \(\rho\). Given the complexities of the expressions of the profit values, it is much better to illustrate the result graphically rather than writing out the very long equations. Figure 2 shows the regions where the manufacturer is better off with one selling period and with two selling periods with a price markdown, respectively. This figure requires the following comments: First, we see, roughly speaking, that each of the pricing strategies is the best option for the manufacturer in half of the \((\rho, \theta)\)-space, with the MP region expanding with the values of both parameter values. When \(\theta\) increases, advertising is more efficient in raising WTP and the retailer could induce consumers to purchase at the high price just by setting the appropriate regular price and advertising level (SP). On the other hand, when the proportion of myopic consumers is larger, it becomes more profitable to implement a price skimming strategy, which is (tautologically) not possible under SP. It is important for the sequel to keep this observation in mind, namely, that SP requires efficient advertising and/or a high fraction of strategic consumers. Second, each region is divided into two subregions, that is, \(MP_1\) and \(MP_2\) for markdown pricing, and \(SP_1\) and \(SP_2\), for single pricing. The values of the equilibrium strategies in these different regions are given in Proposition 2. As a technical remark, we note that \(w, p_1, p_2, A,\) and \(t\) in the markdown pricing policy are continuous with respect to \(\rho\).

![Figure 2: Manufacturer’s Stackelberg Equilibria](image-url)
Looking at how the strategies vary with $\rho$, we can make the following observations:

1. The participation rate $t$ in the $SP$ policy is always higher than in the $MP$ policy, with an upper bound of $1/3$ reached in region $SP_1$. One interpretation of this result is that when the proportion of strategic consumers is sufficiently high, it is worth it for the manufacturer to incentivize the retailer to invest heavily in advertising, to induce these consumers to purchase at full (regular) price. Further, it can be easily verified that the participation rate in $MP$ is first increasing and next decreasing in $\rho$. These results are illustrated graphically in Figure 3 for two values of $\theta$. Note, however, that the larger the value of $\theta$, the smaller the region where an $MP$ policy emerges in equilibrium.

2. Although the expressions of the wholesale prices are surprisingly complex in the $MP$ equilibrium, we can still make two important observations. First, $w^{MP_1}$ and $w^{MP_2}$ are both increasing in $\rho$. Second, the wholesale price under the $SP$ policy is larger than in the $MP$ policy.

3. Irrespective of its efficiency, as measured by $\theta$, advertising is increasing in the proportion of myopic consumers when the equilibrium is $MP$. The reason is that the larger is the value of $\rho$, the more attractive the price skimming strategy encompassed in the $MP$ policy. By raising the WTP, advertising helps in capture more demand during the (higher) regular-price period. In the $SP$ policy, since the retailer needs to spend a considerable amount on advertising to induce the consumers to purchase at full price, the advertising level under the $SP$ policy is always higher than under $MP$.

4. Comparing the prices in the $SP$ and $MP$ scenarios, one would expect the single price to lie somewhere between the regular and the salvaged price under an $MP$ policy. The intuition behind this conjecture is that an overly high single price will exclude too many potential consumers and an overly low one will leave too much money on the table. From Figure 4, which exhibits how these prices behave for different values of $\theta$, we see that this conjecture is verified so long as the efficiency of advertising is not too high, that is, $\theta \leq 0.9$. It is noteworthy that the salvage price is decreasing in the proportion of myopic customers for all values of $\theta$ and is always lower than the single price, but that the regular price under $MP$ is decreasing in $\rho$ for low values of $\theta$ (i.e., $\theta \leq 0.58$) and increasing in $\rho$ for high values of $\theta$ (i.e., $\theta \geq 0.83$).

Looking at the regular-price demand when the equilibrium is $MP$, that is, $\delta \triangleq \frac{D_1}{D_1 + D_2}$, we have

$$\delta^{MP_1} = \frac{\rho(8 + \rho \theta^2)}{4(4 - \rho \theta^2)}, \quad \delta^{MP_2} = \frac{\rho(1 + \sqrt{1 - \rho^2})}{2(1 + \sqrt{1 - \rho^2}) - \rho^2}.$$  

Clearly, $\delta^{MP_1}$ and $\delta^{MP_2}$ are increasing in $\rho$ and non-decreasing in $\theta$ (in fact, $\delta^{MP_2}$ is independent of $\theta$). In the extreme case, where all consumers are myopic ($\rho = 1$), at least half the total demand is realized in the first period. In the other extreme case where all consumers are strategic, $MP$ is not feasible and the equilibrium is $SP$. 

![Figure 3: Participation rate of the manufacturer: (a) $\theta = 0.5$, (b) $\theta = 0.71$](image-url)
3.1 Comparison with an integrated supply chain

As a benchmark to our decentralized Stackelberg equilibrium, we consider the scenario in which the supply chain members coordinate their operations by maximizing their joint profit. The optimal solution is given in the following proposition.

**Proposition 3** Assuming an interior solution, the optimal cooperative solution is $MP$ if $\rho > \frac{4\theta^2}{\theta^4 + 4}$ and $SP$ otherwise. Advertising, prices, and total profits are as follows:

$$SP^{CO} = \begin{cases} A = \frac{\theta}{4-\theta^2}, \\ p_1 = \frac{2}{4-\theta^2}, \\ \Pi_{M+R} = \frac{1}{4-\theta^2}. \end{cases}$$

$$MP^{CO} = \begin{cases} A = \frac{\rho^{\theta^2}}{2(4-\rho^{\theta^2})}, \\ p_1 = \frac{6-2\rho - \rho^2}{2(4-\rho^{\theta^2})}, \\ p_2 = \frac{4-2\rho - \rho^2}{2(4-\rho^{\theta^2})}, \\ \Pi_{M+R} = \frac{4 - \rho^2}{4(4-\rho^{\theta^2})}. \end{cases}$$

**Proof.** See Appendix. \qed

Under our assumption of $\theta^2 < 2$, the solution is indeed interior. As in the Stackelberg equilibrium, an $MP$ policy requires a sufficiently high proportion of consumers to be myopic ($\rho > \frac{4\theta^2}{\theta^4 + 4}$). If advertising is inefficient, that is, $\theta$ is close to zero, than we would need to have $\rho$ close to one for an $MP$ policy to be attractive for the supply chain. Figure 5 shows the regions of $MP$ and $SP$ for both solutions (coordination and Stackelberg equilibrium). With the exception of a very small part of the $(\rho, \theta)$-space, the regions are by and large the same in both solutions. Qualitatively speaking, this means that what matter in defining these regions are the percentage of myopic consumers and the advertising efficiency, and not the mode of play: a decentralized or centralized supply chain. Of course, the result that the total profit is higher under centralization remains valid.

To wrap up, our main takeaways from the two-period model with regard to the two pricing policies are as follows: when most consumers are myopic and/or advertising has a low effect on WTP, an $MP$ policy,
consisting of charging a high regular price to myopic consumers with high WTP and diverting all strategic consumers to the late season, is optimal. However, when sufficiently many consumers are strategic (low $\rho$) and/or advertising has a significant effect on WTP (high $\theta$), the firm charges a single price in the early season in order to sell to both strategic and myopic consumers. In this case, there is no need for sales in the clearance period ($SP$ policy).

4 Numerical results

As it is not possible to obtain a closed-form solution in the general $n$-period model, we shall numerically determine the players’ strategies and outcomes. More specifically, let us assume that the selling season has 4 periods, which implies that the retailer has 3 opportunities to mark down the initial regular price. We believe that such a planning horizon is realistic in view of what is observed in seasonal products retailing, and is, in any case, long enough to shed light on the impact on the pricing policy of having more than two periods. The retained parameter values are given in Table 1. As the proportion of myopic consumers played an important role in the equilibrium results of the previous section, we shall consider, and present the findings, for the whole range of values of $\rho$.

<table>
<thead>
<tr>
<th>Table 1: Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notation</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>$S$</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>$\theta_1$</td>
</tr>
<tr>
<td>$\theta_2$</td>
</tr>
<tr>
<td>$\theta_3$</td>
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<td>$\theta_4$</td>
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</tbody>
</table>

To determine the equilibrium values, we proceed similarly to what we did in the simpler two-period model. More specifically (i) we solve the parameterized retailer’s optimization problem for $i_f = 1, \ldots, 4$ and obtain its reaction function to the manufacturer’s announcement of $w$ and $t$; (ii) we incorporate the retailer’s reaction function in the manufacturer’s optimization problems and solve them; (iii) we compare the resulting profits and determine the conditions under which $i_f = 1, \ldots, 4$ is optimal for the manufacturer; and (iv) we insert the manufacturer’s equilibrium values in the retailer’s reaction functions to get the prices and advertising effort. With the parameter values in Table 1, the equilibrium results are as follows for $\rho \in \{0, 0.1, \ldots, 1\}$:
Table 2: Equilibrium values

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$i_f$</th>
<th>$w$</th>
<th>$t$</th>
<th>$A$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$\Pi_R$</th>
<th>$\Pi_M$</th>
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<tr>
<td>0</td>
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<td>0.333</td>
<td>15.2</td>
<td>8.17</td>
<td></td>
<td></td>
<td></td>
<td>686.6</td>
<td>1,449.8</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>5.27</td>
<td>0.333</td>
<td>15.2</td>
<td>8.17</td>
<td></td>
<td></td>
<td></td>
<td>686.6</td>
<td>1,449.8</td>
</tr>
<tr>
<td>0.2</td>
<td>1</td>
<td>5.27</td>
<td>0.333</td>
<td>15.2</td>
<td>8.17</td>
<td></td>
<td></td>
<td></td>
<td>686.6</td>
<td>1,449.8</td>
</tr>
<tr>
<td>0.3</td>
<td>1</td>
<td>5.27</td>
<td>0.333</td>
<td>15.2</td>
<td>8.17</td>
<td></td>
<td></td>
<td></td>
<td>686.6</td>
<td>1,449.8</td>
</tr>
<tr>
<td>0.4</td>
<td>2</td>
<td>5.11</td>
<td>0.311</td>
<td>10.1</td>
<td>9.07</td>
<td>7.43</td>
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<tr>
<td>0.5</td>
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<td>5.04</td>
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<tr>
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</tr>
<tr>
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<td>8.12</td>
<td>7.14</td>
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<tr>
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<td>9.8</td>
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<td>11.9</td>
<td>9.33</td>
<td>7.82</td>
<td>6.68</td>
<td>5.77</td>
<td>1,038.2</td>
<td>2,116.1</td>
</tr>
</tbody>
</table>

As in the two-period model, we obtain that the retailer’s pricing best response is independent of the manufacturer’s wholesale price. Figure 6 illustrates this result for three values of $\rho$.

To get a general sense of the results, we show in Figure 7 the retailer’s choice of pricing policy for different values of $\rho$ and $t$. We make two general observations. First, the larger is $\rho$, the larger the number of price drops implemented by the retailer during the selling season. This result, which generalizes what we obtained analytically in a two-period setup, can be explained as follows: when the proportion of strategic consumers is large, the retailer is better off practicing a single-price policy to prevent these consumers from waiting to buy at a discounted price, which would damage its profit. However, when the proportion of myopic consumers is large, the profit loss caused by strategic consumers buying at a discounted price is reduced. Here, the retailer is better off implementing a price skimming strategy, with the number of drops in price over time being an increasing function of the proportion of myopic consumers. This is a standard case of price discrimination when there are different market segments having a different WTP. Second, the larger is the manufacturer’s participation in the advertising cost, the larger is the threshold value of $\rho$ needed to mark down the price. Recalling that a higher participation rate leads to a higher advertising by the retailer and a higher WTP, this result means that (i) advertising can be effective in encouraging consumers to buy earlier, and (ii) the manufacturer can in fact influence the retailer’s pricing policy.
Now, we take a closer look at the numerical results.

**Prices.** Figure 8 shows that a single-pricing policy is applied when strategic consumers constitute more than 70% of the total. Clearly, the actual threshold value will generally depend on the other parameter values. From Figure 9, which gives the wholesale and regular price as a function of $\rho$, we see that the (first-period) regular price in a markdown-pricing policy is higher than in a single-pricing policy. This result recalls the fact that dynamic pricing gives the firm the opportunity to sell at decreasing prices over time to market segments ordered by their decreasing WTP. To complete the picture, we plot the number of price drops in Figure 10a, and the maximum discount, that is, the ratio of the price in the last period to the price in first period, for different values of $\rho$ in Figure 10b. As one might expect, the larger is the value of $\rho$, the larger the number of price drops and the depth of the price promotion. Looking at the wholesale price, we observe an opposite direction from the retail price, that is, it is lower under markdown pricing. We earlier alluded to this result when stating that the manufacturer’s wholesale price only depends on the retailer’s last posted price, which is decreasing in $i_f$.

**Advertising.** The results in Table 2 call for three comments. First, as in the analytical results in the previous section, the highest value of the manufacturer’s support rate is one-third, and it occurs, not surprisingly, when the market is essentially populated by strategic consumers (low $\rho$) and the retailer implements a single-pricing policy. So, it seems that this maximum value of one-third for $t$ is robust with respect to a change in the number of periods during the selling season. Further, the lowest rate occurs when the market is divided half-and-half between the two types of consumers. Second, advertising expenditures
by the retailer are the highest, when $\rho$ is low, for the same reason as for $t$. Third, similarly to the analytical results, for a specified $i_f$, the advertising level is increasing in the proportion of myopic consumers (see Figure 11). Since advertising increases the difference between the WTP of consumers in different periods $v_i - v_{i-1} = (\theta_i - \theta_{i-1})A$, a larger proportion of myopic consumers, induces the retailer to increase its advertising to price discriminate and capture more demand during the early periods.
Profits. As shown in Figure 12, the presence of strategic consumers (low value of $\rho$) results in a reduction in both the channel members’ profit. This confirms what the literature has obtained about the retailer’s revenue in the presence of strategic consumers (see, e.g., Kremer et al. (2017)).

4.1 Varying the distribution of willingness to pay

Up to now, we have assumed that the willingness to pay of consumers is distributed uniformly between zero and $\alpha$. A relevant methodological question is how the supply chain members’ pricing and advertising decisions change when the distribution of WTP is not uniform. We respond to this question in this subsection.

To keep the WTP, before accounting for the impact of advertising, between zero and $\alpha$ and also to prevent negative values of WTP, a beta distribution function (for details, see, e.g., Haab and McConnell (1998)) with different parameter values for the density of $\frac{\rho}{\alpha}$ is used. Figure 13 exhibits the probability density function of beta distribution for different parameter values. Note that Beta(1, 1) is equivalent to a uniform distribution.

As it is not possible to get closed-form solutions with a beta distribution, we solve numerically for the Stackelberg equilibrium, assuming a two-period selling horizon and setting $S = \alpha = 1$. Although our setting is somewhat distinctive, it will still allow us to answer our research question. As previously, the results are mainly presented in terms of $\rho$ and $\theta$.

Figure 14 shows the retailer’s best response function for $\theta^2 = 0.4$ and $\rho = 0.8$. As we can clearly see, the retailer’s choice of pricing policy, that is, between $SP$ and $MP$, depends on both the manufacturer’s
strategies (\(w\) and \(t\)), which was not the case with a uniform distribution. This is sufficient to claim that the choice of a WTP distribution is not neutral and has important methodological and strategic impacts.

Figure 15 exhibits the \(SP\) and \(MP\) regions in the \((\rho, \theta)\)-space for different parameter values of the beta distribution. In a nutshell, the results here are qualitatively similar to what we had with a uniform distribution of the WTP. Essentially, increasing the proportion of strategic consumers and/or advertising efficiency leads to a region where single pricing would be implemented.

In Figures 16 and 17, we plot the advertising effort and participation rate for different values of \(\rho\) and \(\theta^2 = 0.4\). From Figure 16, we see that the same pattern emerges for all considered distributions. More specifically, the advertising level is at maximum in the single-pricing policy and is increasing in the proportion of myopic customers in the markdown-pricing policy. Regarding the participation rate, we observe that independently of the distribution parameters, the manufacturer pays one-third of the retailer’s advertising cost when the pricing policy is a single price throughout the selling season. Further, the participation rate in a markdown-pricing policy is generally non-monotone in the proportion of myopic consumers. In the particular case where most consumers have low WTP (i.e., for \(Beta(1.25, 2)\)), the participation rate is almost independent of the value of \(\rho\).

![Figure 14: Best response of the retailer in different WTP distribution functions for \(\theta^2 = 0.4, \rho = 0.8\)](image1)

![Figure 15: Threshold for markdown and single-price strategies in different WTP distribution functions](image2)
5 Concluding remarks

Let us start by recalling our research questions and then we will wrap up our answers.

1. **Under what conditions would the retailer prefer a single-pricing (SP) policy over a markdown-pricing (MP) policy?** The general answer is that a single-pricing policy will be implemented when advertising is efficient in raising consumer’s WTP and/or the proportion of strategic consumers is high enough in the population. Under single-pricing policy, the manufacturer pays one-third of the retailer’s advertising cost, which leads to high advertising expenditure by the retailer. Therefore, to avoid being played out by sophisticated and farsighted consumers, the retailer could induce them to purchase at the premium price and eliminate the prospect of a price promotion. When most consumers are myopic and/or advertising has a low effect on WTP, an MP policy, consisting of charging a high regular price to myopic consumers with high WTP and diverting all strategic consumers to the late season, is optimal. Under this pricing policy, an increase in the proportion of myopic customers leads the retailer to increase its advertising to price discriminate and capture more demand during the regular period from myopic consumers. What this paper offers, are specific and precise guidelines in terms of when to adopt an SP or MP policy. A simple way of highlighting the difference between our results and the literature is to recall the finding in Gallego et al. (2008), namely, that a single-price policy is optimal only if all consumers are strategic. Here, we show that, under some conditions, the retailer can still prefer an SP over an MP policy even when the market is populated by both myopic and strategic consumers.

2. **Would the manufacturer and retailer choose a different pricing policy if the supply chain was coordinated?** The short answer is no. In fact, the choice between the two pricing options is so heavily driven by the type of consumers that little room is left for any other considerations, including the mode of play in the supply chain.

3. **How can the manufacturer influence the retailer’s pricing policy and the selling horizon?** The manufacturer has a say in the retailer’s choice of a pricing policy, and therefore, on the selling horizon, through the participation rate in the retailer’s advertising cost.

4. **In an MP policy, what is the optimal number of price markdowns and the depth of each discount?** Our approach endogenously determines the number of price drops during the selling season and the depth of each discount. A main driver of the results is (again) the proportion of myopic consumers and efficiency of the advertising in raising consumers’ WTP.

5. **What is the impact on the results of varying the distribution of the population’s WTP?** The main takeaway here is that, under a beta distribution of the WTP, it is no longer the case that the choice of the pricing policy by the retailer solely depends on the support rate as under a uniform distribution.
Indeed, the wholesale price itself becomes a determinant of this choice. Generally speaking, we found that the choice of the WTP distribution is not neutral, but has a qualitative (and of course quantitative) effect on the results.

Finally, we mention a few possible future research directions. First, it would be interesting to assess the impact on the supply chain’s profit and strategies of considering capacity rationing along with cooperative advertising. Second, following the finding that the proportion of strategic consumers plays an important role in the pricing policy, a natural question is how to deal with these consumers when we do not a priori know what proportion they account for. Developing a methodology to determine equilibrium strategies when the proportion of strategic consumers is unknown would be clearly relevant for scholars and practitioners. Finally, we assumed in this paper that advertising is done once. Extending the framework to dynamic advertising is also of interest.

### A Appendix: Proofs of propositions

As mentioned before, in all propositions, we assume that $2(1 - t) - \theta^2 \geq 0$ and $w < 1$ (Equations (20)–(21)).

#### Proof of Proposition 1

First, the optimal solution for each of subproblems (1) and (2) should be obtained separately. Then, by comparing their optimal profits, we obtain the conditions under which MP is preferred to an SP policy.

**Subproblem 1:**

The Hessian matrix of $\Pi_{1R}$ (Equation (10)) and its principal minors are given by

$$H(\Pi_{1R}) = \left( \frac{\partial^2 \Pi_{1R}^2}{\partial p_1^2} \right),$$

$$H(1) = \frac{\partial^2 \Pi_{1R}^2}{\partial A^2} = -2(1 - t) < 0,$$

$$H(2) = \left( \frac{\partial^2 \Pi_{1R}^2}{\partial A^2} \right) - \left( \frac{\partial^2 \Pi_{1R}^2}{\partial A \partial p_1} \right) \left( \frac{\partial^2 \Pi_{1R}^2}{\partial p_1^2} \right) = (4(1 - t) - \theta^2) > 0.$$  

The negative sign of the first principal minor and the positive sign of the second principal minor of $H(\Pi_{1R})$ prove concavity of function $\Pi_{1R}$ relative to $A$ and $p_1$. Hence, their optimal values can be obtained by setting the first-order partial derivatives equal to zero, to get

$$A = \frac{\theta(1 - w)}{4(1 - t) - \theta^2},$$

$$p_1 = \frac{2(1 + w)(1 - t) - \theta^2 w}{4(1 - t) - \theta^2}.$$  

It can be shown that, under assumed conditions, the above solution satisfies constraints of this subproblems.

**Subproblem 2:**

The Hessian matrix of function $\Pi_{2R}$ (Equation (11)) and its principal minors are

$$H(\Pi_{2R}) = \left( \frac{\partial^2 \Pi_{2R}^2}{\partial p_1^2} \right),$$

where

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}.$$
\[ H(1) = -2(1 - t) < 0 \]  
\[ H(2) = \rho(4(1 - t) - \rho t^2) > 0 \]  
\[ H(3) = 2\rho(\rho t^2 - (1 - t)(4 - \rho)) < 0 \]  

The negative sign of the first and third principal minors and the positive sign of the second principal minor prove the concavity of function \( \Pi^2_R \) relative to \( \rho_1, \rho_2, \) and \( A \). Therefore, their optimal value can be obtained by setting the first-order partial derivatives equal to zero, to get

\[ A = \frac{\rho \theta (1 - w)}{2(4 - \rho)(1 - t) - 2\rho \theta^2}, \]  
\[ p_1 = \frac{2(w + 3 - \rho)(1 - t) - \rho \theta^2(1 + w)}{2(4 - \rho)(1 - t) - 2\rho \theta^2}, \]  
\[ p_2 = \frac{4(w + 1 - \frac{\theta^2}{2})(1 - t) - \rho \theta^2(1 + w)}{2(4 - \rho)(1 - t) - 2\rho \theta^2}. \]

It can be shown that the obtained solution satisfies the corresponding constraints of the subproblem.

**Optimal pricing policy**

To determine the optimal pricing policy (single-price or markdown price), a comparison must be made between \( \Pi^1_R \) and \( \Pi^2_R \). Eq. The following equation shows that if the manufacturer’s participation rate is less than \( \frac{\theta^2(1 + \sqrt{1 - \rho^2})}{2\rho} \), then the best response of the retailer will be the MP policy; otherwise, it will be the SP policy:

\[ \Pi^1_R(A^*, p_1^*, p_2^*) < \Pi^2_R(A^*, p_1^*, p_2^*) \rightarrow t < 1 - \frac{\frac{\theta^2(1 + \sqrt{1 - \rho^2})}{2\rho}}{2} \]

**Proof of Proposition 2** To obtain the Stackelberg equilibrium, we insert the retailer’s best response function in the manufacturer’s profit function. Since the best response function of the retailer is a piecewise function for a different range of participation rate \( t \) (Proposition 1), the manufacturer’s optimization problem is divided into two subproblems.

**Subproblem 1**

Considering the retailer’s best response function, the manufacturer’s subproblem 1 is as follows:

\[ \max_{w,t} \Pi^1_M(w, t) = w(\theta A + 1 - p_1) - tA^2, \]  
subject to:

\[ A = \frac{\theta(1 - w)}{4(1 - t) - \theta^2}, \]  
\[ p_1 = \frac{2(1 + w)(1 - t) - \theta^2 w}{4(1 - t) - \theta^2}, \]  
\[ 1 - \frac{\theta^2(1 + \sqrt{1 - \rho^2})}{2\rho} \leq t \leq 1 - \frac{\theta^2}{2}, \]  
\[ 0 \leq w < 1. \]
\[
\left( \frac{\partial(-\Pi_M^1)}{\partial w} \right) + \sum_{i=1}^{4} u_i \left( \frac{\partial g_i}{\partial w} \right) = 0
\]

where \( \varepsilon_w \) is a very small positive value. In this subproblem, two combinations of active constraints can be feasible and are shown in Table 3. According to the results of the KKT conditions, optimum solutions of subproblem 1 are as in Table 4.

Table 3: KKT conditions for the manufacturer’s profit function in subproblem 1

<table>
<thead>
<tr>
<th>Row</th>
<th>Active constraint</th>
<th>Description</th>
<th>Feasibility condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No constraint</td>
<td>all ( u_i = 0 )</td>
<td>( \theta^2 \leq \frac{4}{3} )</td>
</tr>
<tr>
<td>2</td>
<td>( g_4 )</td>
<td>( u_1 = u_2 = u_3 = 0, \quad u_4 = \frac{3w^2 - 4}{(2w^2 + 2)^2} )</td>
<td>( \frac{4}{3} &lt; \theta^2 \leq 2 )</td>
</tr>
</tbody>
</table>

Table 4: Optimal solution of subproblem 1

<table>
<thead>
<tr>
<th>If ( \theta^2 \leq \frac{4}{3} )</th>
<th>( \frac{4}{3} &lt; \theta^2 \leq 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular and salvage price</td>
<td>( p_1^{SP_1} = \frac{24 - 39\theta^2}{32 - 99\theta^2} ) ( p_1^{SP_2} = 1 )</td>
</tr>
<tr>
<td>Wholesale price</td>
<td>( w_1^{SP} = \frac{16 - 36\theta^2}{32 - 99\theta^2} ) ( w_2^{SP} = \frac{2}{2 + \theta^2} )</td>
</tr>
<tr>
<td>Advertising level</td>
<td>( A_1^{SP_1} = \frac{6\theta}{32 - 99\theta^2} ) ( A_1^{SP_2} = \frac{\theta}{2 + \theta^2} )</td>
</tr>
<tr>
<td>Participation rate</td>
<td>( t_1^{SP_1} = \frac{1}{3} ) ( t_1^{SP_2} = 1 - \frac{\theta^2}{2} )</td>
</tr>
</tbody>
</table>

Subproblem 2

\[
\max_{w,t} \Pi_M^2(w,t) = w(1 - p_2) - tA^2,
\]

subject to:

\[
A = \frac{\rho(1 - w)}{2(4 - \rho)(1 - t) - 2\rho\theta^2}, \quad p_2 = \frac{4(w + 1 - \frac{\theta}{2})(1 - t) - \rho\theta^2(1 + w)}{2(4 - \rho)(1 - t) - 2\rho\theta^2},
\]

\[
0 \leq t < 1 - \frac{\theta^2(1 + \sqrt{1 - \rho^2})}{2\rho}, \quad 0 \leq w < 1.
\]

The manufacturer’s profit function is bounded and differentiable. So, based on the extreme value theorem, the function attains its maximum value at least once. The KKT conditions for subproblem 2 are

\[
\left( \frac{\partial(-\Pi_M^2)}{\partial w} \right) + \sum_{i=1}^{4} u_i \left( \frac{\partial g_i}{\partial w} \right) = 0
\]

\[
g_1 = -w \leq 0, \quad g_2 = w + \varepsilon_w - 1 \leq 0
\]
Proof Proposition 3

To obtain the cooperative (joint-optimization) solution, it suffices to set

\[
g_3 = -t \leq 0, \quad g_4 = t - 1 + \frac{\theta^2(1 + \sqrt{1 - \rho^2})}{2\rho} + \epsilon_t \leq 0 \quad (45)
\]

where \(\epsilon_t\) is a very small positive value. In this subproblem, two combinations of active constraints can be feasible and are shown in Table 5. According to the results of the KKT conditions, optimum solutions of the subproblem 1 are as in Table 6.

### Table 5: KKT conditions for the manufacturer’s profit function in subproblem 2

<table>
<thead>
<tr>
<th>Row</th>
<th>Active constraint</th>
<th>Description</th>
<th>Feasibility condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No constraint</td>
<td>all (u_i = 0)</td>
<td>(\theta^2 &lt; \frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2})</td>
</tr>
<tr>
<td>2</td>
<td>(g_4)</td>
<td>(u_1 = u_2 = u_3 = 0, u_4 &gt; 0)</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
</tr>
</tbody>
</table>

### Table 6: Optimal solution of subproblem 2

<table>
<thead>
<tr>
<th>(P_1)</th>
<th>(P_1^{MP_1})</th>
<th>(P_2^{MP_1})</th>
<th>(P_2^{MP_2})</th>
<th>(w^{MP_1})</th>
<th>(w^{MP_2})</th>
<th>(A^{MP_1})</th>
<th>(A^{MP_2})</th>
<th>(t^{MP_1})</th>
<th>(t^{MP_2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>If (\theta^2 &lt; \frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
<td>(\frac{8\rho}{6(1+\sqrt{1-\rho^2})-\rho^2} \leq \theta^2 &lt; \frac{2\rho}{1+\sqrt{1-\rho^2}})</td>
</tr>
</tbody>
</table>

Optimal pricing policy

It can be shown that by decreasing \(\rho\) or increasing \(\theta\), the \(SP\) policy will be optimal; therefore, to determine the optimal pricing policy, it would be enough to make a comparison between \(\Pi_2^w(w^{MP_1}, t^{MP_1})\) and \(\Pi_2^w(w^{SP_1}, t^{SP_1})\) (in case of \(\theta^2 \leq \frac{2}{3}\)) and between \(\Pi_2^w(w^{MP_2}, t^{MP_2})\) and \(\Pi_1^w(w^{SP_2}, t^{SP_2})\) (in case of \(\frac{4}{3} \leq \theta^2 \leq 2\)). Figure 2 shows the borderline between the two above-mentioned pricing policies. It can be shown that the border being located in the feasible region of the \(MP\) policy (that is, \(\theta^2 < \frac{2\rho}{1+\sqrt{1-\rho^2}}\)).

Proof Proposition 3 To obtain the cooperative (joint-optimization) solution, it suffices to set \(t = w = 0\) in the retailer’s best response function in Proposition 1.

References


