Vehicle scrappage incentives to accelerate the replacement decision of heterogeneous consumers

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Abstract: Vehicle scrappage subsidy programs have been widely applied by governments to replace old cars by newer, more fuel-efficient ones. While these programs have been implemented to motivate earlier replacement, they may not be as effective as expected. From a cost-benefit perspective, the consumers who would have replaced anyway, even without the program, need to be addressed when evaluating the net benefits of the program. This requires accounting for variations in consumers’ willingness to replace. Considering consumer heterogeneity in net trade-in valuation, this study investigates a dynamic vehicle-replacement problem based on a life cycle optimization (LCO) approach. We theoretically demonstrate that although increasing the subsidy level motivates low-value consumers to replace earlier, it also induces consumers with a high net trade-in valuation to replace later to become eligible for the subsidy program. We have also developed a simulation program based on real data to show the application of our general model. According to the simulation results, ignoring consumer heterogeneity could result in an overestimation of the net benefits of the scrappage program.

Keywords: Vehicle scrappage program, heterogeneous consumers, dynamic programming, life cycle optimization

Résumé: Les programmes de subvention pour accélérer le remplacement d’une voiture ayant un certain âge ont été très populaires un peu partout dans le monde. Ils avaient essentiellement deux objectifs. Premièrement, stimuler le marché de l’automobile, surtout après la crise financière de 2007–2008, et deuxièmement réduire les émissions de gaz à effet de serre dans la mesure où une nouvelle voiture pollue moins, par kilomètre parcouru, qu’une vieille voiture. Bien que ces programmes aient réussi à stimuler le marché de l’automobile, leur efficacité a été mise en doute par certains auteurs. La raison est qu’en général on ait omis de prendre en compte le fait que certains consommateurs auraient de toute façon procédé au remplacement. Dans ce cas, la subvention est un gâchis de fonds publics. Cette étude étudie le problème dynamique de remplacement de véhicule basé sur une approche d’optimisation du cycle de vie (LCO) et en présence de consommateurs hétérogènes. Nous proposons une analyse théorique, suivie par une étude empirique. Un des résultats est qu’ignorer l’hétérogénéité des consommateurs pourrait entraîner une surestimation des avantages nets de ce type de programmes de subvention.

Mots clés: Programme de mise à la casse des véhicules, consommateurs hétérogènes, programmation dynamique, optimisation du cycle de vie

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1 Introduction

Following the global recession in 2008, many countries introduced car scrappage incentive programs with the dual aim of stimulating the car market and reducing emissions, as new cars typically pollute less than older ones. The CARS (Car Allowance Rebate System) program in the US was introduced in 2009 and provided subsidies of up to $4,500 for individuals trading in their car for a more fuel-efficient one. The program was so successful that its $1 billion budget was used up well ahead of the expected date (Huang et al., 2014). In Canada, the Vehicle Efficiency Incentive (VEI) was introduced in 2007 and offered a $2,000 rebate to consumers purchasing a new fuel-efficient vehicle (Walsh, 2012). Similar scrappage programs were also adopted in European countries to give consumers an incentive to take their cars over a certain age off the road.

Unlike some subsidy programs that provide financial incentive for retailers and recyclers (Liu et al. (2016) and Zhang et al. (2015)), scrappage subsidies are directly offered to consumers. These programs have been reported to be very successful for stimulating car sales, but their effectiveness in terms of environmental impact and cost is debatable (Smit, 2016). For instance, Dill (2004) showed that scrapped vehicles are driven less compared to those of the same vintage that are kept by their owners, and concluded that the reduction in emissions due to a scrappage program is lower than expected. Further, some of the scrapped vehicles might have been retired even without the program, and therefore, the subsidy in such cases is a waste of public funds. Such observations triggered cost-benefit studies of scrappage programs in order to find the optimal subsidy level. Assuming that a consumer will participate in a scrappage program when the subsidy exceeds the price of the old vehicle, Hahn (1995) determined the number of vehicles scrapped for different subsidy levels. With the same assumption as in Hahn (1995), Lavee and Becker (2009) also estimated a supply curve of vehicles for retirement as a function of the subsidy level and used car value. Using a cost-benefit analysis, they next computed the subsidy level that maximizes the net benefit of the scrappage program. Lavee et al. (2014) extended the study by Lavee and Becker (2009) by adding age and maintenance cost in the participation rate function and estimated its parameters using real data.

Unlike the static models in the above-cited papers, Lorentziadis and Vournas (2011) proposed a dynamic model where the demand for new vehicles is equal to the number of scrapped old vehicles and depends on a time-varying subsidy. However, the authors did not account for the impact of vehicle age in the replacement decision. We shall also consider a dynamic model where the replacement decision is endogenous and depends on, among other things, the vehicle’s age, which is intuitively an important driver of such decisions.

Kim et al. (2003) introduced a novel life cycle optimization (LCO) approach and applied it to an automobile replacement problem. As the name suggests, the environmental impact of a vehicle in the LCO approach is determined by taking into account all the stages in the product’s life cycle, that is, material production, manufacturing, use, maintenance, and end-of-life disposal. In addition to single-vehicle analysis, the LCO approach has also been used to find the optimal vehicle fleet conversion (Kim et al., 2004; Figliozi et al., 2013). Kim et al. (2004) studied a vehicle fleet optimization problem from a life cycle perspective where the objective is to minimize the total environmental burden of vehicles. For a comprehensive review of fleet-based life cycle approaches, see Garcia and Freire (2017). Considering such an approach, some studies, e.g., Kim et al. (2004, 2006) and Lenski et al. (2010), have raised doubts about the environmental efficiency of scrappage programs, stating they may increase pollution emissions due to the increase in the production of new cars. Further, while the replacement decision in the above-cited papers is related to the environment, other studies, –see, e.g., Kim et al. (2006), Spitzley et al. (2005) and Kleine et al. (2011)– used an LCO approach with a focus on both the environment and the owner’s cost. Spitzley et al. (2005) showed that when ownership cost is accounted for, the replacement intervals, measured by the number of years a vehicle is kept, are larger than when one considers a minimization problem of emissions damage. Also, when pollution damage cost is combined with ownership cost, the results are identical to those of ownership cost optimization. In our paper, we focus on the usage phase of the LCO approach when evaluating the environmental performance of the subsidy program, with the replacement decision being studied from the consumer’s point of view. That is, the individual’s objective is to minimize her ownership cost, while the environmental consequences are evaluated ex-post.
In all the papers mentioned above, replacement decisions are made by homogeneous consumers. In a study of air conditioners, Kleine et al. (2011) highlighted that consumers differ in their willingness to replace and that there is a need for further research to understand consumer behavior. In particular, they argued that the design and assessment of an incentive program for air conditioner replacement must take into account individuals who would have replaced even without a subsidy. Such consumers are called “free riders” in Blumstein (2010) and Skumatz et al. (2009) where a net saving from the incentive program is calculated by excluding these consumers. Hoekstra et al. (2015) justified the need to account for free riders when concluding that 60% of the Cash for Clunkers program (scrappage program in the US) subsidies went to households that would have purchased anyway while the program was ongoing. Our model accounts for consumer heterogeneity, which seems to be a crucial feature when designing an incentive program.

Consumer heterogeneity in scrappage policies is taken into account in a few recent papers. Miao et al. (2017) and Li and Xu (2015) considered consumer heterogeneity in trade-in models. Shiraldi (2011) used a dynamic discrete-choice model in which heterogeneous consumers with different tastes decide whether to replace their automobile. He evaluated different incentive policies and checked whether or not the beneficiaries of such policies would have replaced their vehicle without the subsidy. Wei and Li (2016) retained the same discrete-choice setting to analyze vehicle scrappage programs with consumer heterogeneity. They stated that targeting marginal consumers, i.e., those who would not have replaced without a subsidy, is the key factor in designing scrappage programs. Further, they concluded that programs that do not have environmental objective and target marginal consumers are more cost-effective than those with explicit environmental objectives and no targeting of marginal consumers. Unlike these two papers, where the focus is on the effect of the scrappage program on heterogeneous consumers’ vehicle choice, we concentrate in this paper on the impact of the subsidy level on the timing of the replacement decisions. In this regard, the closest paper to ours is Langer and Lemoine (2017) where the reaction of strategic consumers to government subsidies for new products is studied. However, they investigated the effect of subsidies on purchasing decisions rather than replacement decisions as we do here. As far as we know, consumer heterogeneity and strategic behavior have not been considered when analyzing scrappage programs and vehicle replacement decisions.

We consider a dynamic life cycle optimization problem from an ownership-cost perspective, where at each period, heterogeneous consumers decide about replacing or keeping their vehicle. In order to accelerate replacement, the government offers a subsidy to consumers who replace their vehicle with a new one. To our knowledge, this paper is the first work to study the impact of the level of incentive payment on the supply of vehicles retired for scrappage in the presence of consumer heterogeneity. While in the literature on scrappage programs, subsidy has always been considered an incentive to accelerate replacement, we show that it can motivate consumers to postpone replacement to become eligible for the incentive program. Similar consumer behavior has been observed in the dynamic adoption of new products and new technologies, where consumers wait for cheaper options and for the entry of other firms onto the market (Gowrisankaran and Rysman, 2012), and firms postpone investing in new technologies to wait for new inventions when they approach a rapid technological progress period (Feichtinger et al., 2006). In fact, our results imply that, besides “pulling forward” more demand from the future, increasing the subsidy level, can “move the demand forward” too, affecting net benefit calculations of scrappage subsidy programs.

In this work, it is assumed that a new vehicle is less polluting than an older one, as a result of technological progress in the car industry as well as deterioration in vehicle emissions levels due to age (Zachariadis et al., 2001; Pastramas et al., 2014; Trusts, 2010; Zyl et al., 2015). We develop a vintage model in which different generations of cars can be distinguished in terms of their usage emissions. In such a setting, since pollution generated from vehicle use differs for each car model, the total pollution at each period can be calculated only by knowing the age distribution of the current vehicle fleet. Put differently, one needs to have information about the number of vehicles in use for each car model. In terms of modeling, such an information requirement creates a high burden on the dimension of the state space, making the problem of pollution from use much more complicated than that of pollution from production. When pollution is seen as a result of production, the emissions in each period are usually assumed to be a function of the production quantity in the same period (Jorgensen et al., 2010), whereas pollution caused by consumption at each period is the aggregation of emissions from the use of all products (e.g., vehicles) of different ages. Consequently, in
a dynamic setting, many state variables are required to keep track of the generation from all products still in use in each period.

In summary, this paper aims to answer the following questions:

1. Does the subsidy affect all consumers’ replacement decisions in the same way?
2. What is the effect of subsidy on the environment when consumer heterogeneity is taken into account?
3. What is the impact of varying the parameter values on the results?

We believe that answering these questions can help a government designing a more efficient policy and also have better understanding about the real benefits of a scrappage program. In a nutshell, our main results can be summarized as follows:

1. The effect of the subsidy on consumers differs from high-value to low-value consumers. While the subsidy induces low-value consumers to replace their vehicle earlier, it motivates high-value consumers to delay replacements until their cars become eligible for the subsidy. This implies that investing on high-value consumers is somehow a waste of funds. However, direct price discrimination between different consumer groups is not possible for the regulator. Instead, she should assign a subsidy level considering both groups, i.e., high-value and low-value consumers. On the one hand, the subsidy should be high enough to accelerate purchases by low-value consumers, and on the other hand, it should be lower than the threshold above which high-value consumers delay their vehicle replacement to benefit from the subsidy.

2. When consumer heterogeneity is taken into account, the estimated benefits of the scrappage program are lower than when all consumers are assumed homogeneous. Specifically, pollution reduction and new vehicle purchases caused by the program are overestimated when consumer heterogeneity is ignored. This finding provides better understanding about the real benefits of a scrappage program, which can be useful for assigning budgets for future subsidy programs. Also, having better knowledge about the advantage of subsidy programs, policy makers will be able to choose among different options to reach their objective in the most efficient way.

The remainder of the paper is organized as follows: In Section 2, we introduce our general model; and we analytically investigate two particular instances in Section 3. In Section 4, we report and discuss our empirical results. In Section 5, we briefly conclude.

2 The model

Denote by \( t \) the time (year) index, with \( t \in \mathcal{T} = \{0, \ldots, T^*, \ldots, T\} \), where \( T^* \) corresponds to the terminal date of the subsidy program, which is assumed to be shorter than the planning horizon \( T \). Let \( j, j = 0, \ldots, \omega \), be the vehicle’s age, where \( \omega \) is the maximum life span of a vehicle. We assume \( \omega \) to be constant for all vehicles, irrespective of their date of introduction onto the market. At time period \( t \) a vehicle of age \( j \) is of the year model \( t - j \). Consumers are heterogeneous in terms of their valuation of the environment and are indexed by \( i \in I \), where \( I \) represents the set of types. Denote by \( n_t, t \in \mathcal{T} \), the exogenously given number of first-time buyers being of type \( i \) at period \( t \), and by \( N_t \) the number of new cars purchased at \( t \). The difference \( N_t - n_t \) is the replacement demand at year \( t \) and will be determined endogenously.

Denote by \( p_t \) the price of a new car at time \( t \), and let \( \theta_i \) be the “utility” that a consumer of type \( i \) derives from replacing her used car by a new one. We shall refer to \( \theta_i \) as the net trade-in valuation, which is measured by the difference between the environmental valuation of a new car minus the valuation of a retained car. Consequently, we define by \( p_t - \theta_i \) the effective price of a vehicle sold at \( p_t \). This definition of an effective vehicle-replacement price is in line with Huang et al. (2014), where a parameter \( \theta \) is added to the consumer’s utility function. The probability \( P_i \) of a first-time buyers being of type \( i \) is constant over time. We denote the distribution of first-time buyer by \( (P_i)_{i \in I} \). The distribution of all buyers, including those replacing their vehicle at period \( t \), is denoted by \( (\pi_{i,t})_{i \in I, t \in \mathcal{T}} \), and will be determined endogenously.
Deciding whether or not to replace a vehicle involves an investment cost (effective price of the new vehicle) and the usage cost. We assume that the usage cost of a vehicle of age \( j \) at time \( t \) can be well approximated as follows:

\[
C_t(j) = f(t - j).D.O_t + m(j),
\]

where \( D \) is the number of kilometers traveled per year (km/y), \( f(t - j) \) is the per-kilometer fuel consumption, \( O_t \) is the unit fuel cost at time \( t \), and \( m(j) \) is the maintenance cost of a car with age \( j \). In our formulation, fuel consumption varies by car vintage (given by \( t - j \)), while maintenance cost is only affected by age \( j \). Also, consumers are assumed to be homogeneous in terms of their yearly travel demand.

The government (or the regulator) can affect the vehicle replacement decision through a scrappage subsidy. The subsidy, denoted by \( r \), is given to any consumer who scraps a vehicle older than a given age \( \eta \) (measured in years), independently of the type of consumer and of the actual age of the vehicle (assuming eligibility, of course, that is, \( j \geq \eta \)).

Denote by \( q_t \) the environmental quality index of a vintage \( t \) vehicle, defined by

\[
q_t = 1 - \left( \frac{f(t) - f(T)}{f(T)} \right).
\]

Reflecting the empirical observation that the per-kilometer fuel consumption is decreasing over vintage, the quality index increases over time and reaches its maximum value at the terminal date, i.e., \( q_T = 1 \).

Given the usage cost, new vehicle price and government policy, the replacement decision problem of consumer \( i \) having a vehicle of age \( j \) can be modeled by the following dynamic optimization program:

\[
V_{t,i}(j) = \begin{cases} 
C_t(j) + \beta.V_{t+1}(j + 1) & \text{if } 0 \leq j < \omega, \\
\min \{ C_t(j) + \beta.V_{t+1}(j + 1) + \frac{(p_t - q_t.\theta_t)}{f(t)} + C_t(0) + \beta.V_{t+1}(1) \} & \text{if } \omega \leq j < \eta, \\
\min \{ C_t(j) + \beta.V_{t+1}(j + 1) + \frac{(p_t - q_t.\theta_t) - r + C_t(0) + \beta.V_{t+1}(1)}{f(t)} \} & \text{if } \eta \leq j \leq \omega \text{ and } 0 \leq t \leq T^*, \\
\min \{ C_t(j) + \beta.V_{t+1}(j + 1) + \frac{(p_t - q_t.\theta_t) + C_t(0) + \beta.V_{t+1}(1)}{f(t)} \} & \text{if } \eta \leq j \leq \omega \text{ and } t > T^*, 
\end{cases}
\]

where \( V_{t,i} \) is consumer \( i \)'s cumulative cost of keeping or replacing the vehicle from period \( t \) to \( T \), and \( \omega = 1, \ldots, \eta \) is the minimum age under which no individual will consider replacing her vehicle. The above program has four parts. If the car is still brand new, that is, its age is below \( \omega \), then no replacement is considered, and the total cost is the sum of the current cost and the discounted future cost. In the second case (\( \omega \leq j < \eta \)), no subsidy is yet available, and consumer type \( i \) compares the total cost of replacing her vehicle, which is given by the sum of the effective price of a new car (\( p_t - q_t.\theta_t \)) and its usage cost, to the cost of continuing with the same car. The same type of comparison is done for \( \eta \leq j \leq \omega \), with the difference that here, the price includes the subsidy, that is, it is given by \( p_t - q_t.\theta_t \). Recall that the subsidy is only available in periods \( t = 0, \ldots, T^* \). Note that the case \( \omega < \eta \) allows us to assess the effect of the government program on those consumers who would have replaced their vehicle before becoming eligible for a subsidy but postpone their purchase until they are eligible. As alluded to in the introduction, one issue with subsidies offered to all individuals satisfying a given condition is that some of them would have purchased the product anyway, without the subsidy. Our framework will allow us to assess the size of this group.

Let \( d_{i,t}(j) \) be the decision made at time \( t \) by consumer \( i \) with a vehicle of age \( j \), defined as follows:

\[
d_{i,t}(j) = \begin{cases} 
0 & \text{if keep}, \\
1 & \text{if replace}.
\end{cases}
\]

In order to keep track of consumers corresponding to each vehicle vintage at each period, let \( K_t(j - 1) \) be the set of consumers who have a vehicle of age \( j - 1 \) at time \( t \) and \( R_t(j - 1) \) be the set of consumers who replace their vehicles of age \( j - 1 \) at time \( t \). Consequently, we have \( K_{t+1}(j) = K_t(j - 1) \setminus R_t(j - 1) \), that is, the set of individuals who have a vehicle of age \( j \) at period \( t + 1 \) is made up of those who had a vehicle of age \( j - 1 \) at period \( t \), minus those who replaced their vehicles at period \( t \). Further, let \( W_t(i) \) be the set of vehicles that are replaced by consumers of type \( i \) at time \( t \).
Using the above-defined sets, the total number of used cars replaced at time $t$ is then given by

$$\sum_{j=1}^{\omega} \sum_{i \in R_t(j)} \pi_{i,t-j} \cdot N_{t-j},$$

and the total number of new vehicles purchased at $t$ is

$$N_t = n_t + \sum_{j=1}^{\omega} \sum_{i \in R_t(j)} \pi_{i,t-j} \cdot N_{t-j},$$

with $\pi_{i,t}$ being determined by

$$\pi_{i,t} = \frac{P_t \cdot n_t + \sum_{j \in W_t(i)} \pi_{i,m,j} \cdot N_{t-j} \cdot f(0) \cdot D + n_t \cdot e \cdot f(0) \cdot D}{N_t}$$

To determine the pollution dynamics, denote by $e$ the emissions generated by the consumption of one liter of fuel and let $E_t$ be the accumulated stock of pollution (a state variable) by time $t$. To capture the fact that the vehicle emissions rate increases with age (Zyl et al., 2015), we introduce the increasing deterioration factor $df(j)$. The dynamics of pollution accumulation are given by the following difference equation:

$$E_{t+1} = E_t + \sum_{j=1}^{\omega} \sum_{i \in K_t(j) \cap R_t(j)} \pi_{i,t-j} \cdot N_{t-j} \cdot e \cdot f(t - j) \cdot D \cdot df(j) + \sum_{j=1}^{\omega} \sum_{i \in R_t(j)} \pi_{i,m,j} \cdot N_{t-j} \cdot e \cdot f(0) \cdot D + n_t \cdot e \cdot f(0) \cdot D$$

As we are considering a short-term planning horizon, we do not account for emissions absorption by nature. As car sales depend on the subsidy program, so does the stock of pollution. Consequently, the government can easily assess the environmental impact of the subsidy.

Table 1 summarizes the notations used throughout the paper.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Related to Consumers</th>
<th>Related to Vehicles</th>
<th>Related to Government</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$</td>
<td>Net trade-in value of consumer type $i$</td>
<td>$C_j$</td>
<td>Ownership cost of a car of age $j$ at time $t$</td>
</tr>
<tr>
<td>$n_t$</td>
<td>Number of new buyers at time $t$</td>
<td>$D$</td>
<td>Travel demand per year</td>
</tr>
<tr>
<td>$\pi_{i,t}$</td>
<td>Distribution of new buyers</td>
<td>$f(t - j)$</td>
<td>Per-kilometer fuel use for a car of age $j$ at time $t$</td>
</tr>
<tr>
<td>$V_{i,t}(j)$</td>
<td>Value function related to consumer $i$ who has a vehicle of age $j$ at period $t$</td>
<td>$O_t$</td>
<td>Unit fuel price at time $t$</td>
</tr>
<tr>
<td>$d_{i,m,j}$</td>
<td>Decision variable related to consumer $i$ who has a vehicle of age $j$ at period $t$</td>
<td>$m(j)$</td>
<td>Maintenance cost of a car of age $j$</td>
</tr>
<tr>
<td>$K_t(j)$</td>
<td>Set of consumers who have a vehicle of age $j$ at period $t$</td>
<td>$\omega$</td>
<td>Maximum life span of a vehicle</td>
</tr>
<tr>
<td>$R_t(j)$</td>
<td>Set of consumers who replace a vehicle of age $j$ at period $t$</td>
<td>$W_t(i)$</td>
<td>Set of vehicles (ages) replaced at time $t$ by consumer type $i$</td>
</tr>
<tr>
<td>$s_t$</td>
<td>Incremental threshold related to the subsidy’s adverse effect</td>
<td>$p_t$</td>
<td>Price of the vehicle at time $t$</td>
</tr>
<tr>
<td>$t$</td>
<td>Set of all consumers</td>
<td>$df(j)$</td>
<td>Parameter related to emissions deterioration by age</td>
</tr>
<tr>
<td>$n_t$</td>
<td>Number of new buyers at time $t$</td>
<td>$q_t$</td>
<td>Fuel economy coefficient of a new car produced at time $t$</td>
</tr>
<tr>
<td>$N_t$</td>
<td>Total number of consumers who buy a new car at time $t$ including those who replace their car</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A simplified model

The dynamic program defined in (3) cannot be solved analytically in its full generality. We shall solve the model numerically in the next section, using real data. To gain qualitative hints about its solution, we analytically investigate a simplified version of the model. Specifically, instead of having the detailed cost function in (1), we omit specifying $C_i(j)$ and simply assume that it is an increasing concave function in $j$. Further, we only deal with the consumer’s problem, without dealing with the environmental performance of the subsidy program.

First, we consider a two-period setting, with the subsidy available in both periods. In the first period, vehicles that can either be kept or replaced by a new one are of age 0 to $\omega$. Vehicles of age $j$ that are replaced in the first period are vehicles of age 1 in the second period, and those that are kept will be of age $j+1$. The terminal value of all vehicles at the end of the second period is assumed to be zero. The value functions in the first and second periods are as follows:

$$V_{i,1}(j) = \begin{cases} \min\{C_1(j) + \beta V_{i,2}(j+1) - q_1 \theta_i + C_1(0) + \beta V_{i,2}(1)\} & \text{if } j < \eta, \\ \min\{C_1(j) + \beta V_{i,2}(j+1) - p - q_1 \theta_i + C_1(0) - r + \beta V_{i,2}(1)\} & \text{if } j \geq \eta, \end{cases}$$

$$V_{i,2}(j) = \begin{cases} \min\{C_2(j) - q_2 \theta_i + C_2(0)\} & \text{if } j < \eta, \\ \min\{C_2(j) - q_2 \theta_i + C_2(0) - r\} & \text{if } j \geq \eta. \end{cases}$$

In the first period, strategic consumers make their vehicle replacement decision based not only on the cost in the first period, but also on the cost in the second period. Consequently, the effect of the subsidy on different groups of consumers may not be the same. On the one hand, offering a subsidy to those who replace only after age $\eta$ in the absence of a subsidy induces them to replace their vehicle earlier. This result is quite intuitive since higher subsidies make the purchase of new vehicles cheaper, thereby motivating the consumer to replace her vehicle when it is younger. On the other hand, increasing the government payment level may have the opposite effect on the replacement decision of those who would have replaced before $\eta$ without the subsidy. This adverse effect of the subsidy occurs because forward-looking consumers find it more cost-efficient to delay their purchase and become eligible for the subsidy in the next period rather than replacing it in the current period and receiving no subsidy.

To analyze this adverse effect in the subsidy, consider the consumers who replace their vehicle with age $\eta - 1$ in the first period. Such consumers satisfy the following inequality:

$$C_1(\eta - 1) + \beta V_{i,2}(\eta) \geq p - q_1 \theta_i + C_1(0) + \beta V_{i,2}(1), \tag{4}$$

where the left-hand side (LHS) is the cost of keeping a vehicle of age $\eta - 1$ in the first period plus the discounted cost-to-go of the vehicle of age $\eta$ in the second period, while the right-hand side (RHS) is the cost of replacing the vehicle in the first period plus the discounted cost-to-go of the vehicle of age 1 in the second period. Next, we show that, for some parameter values, increasing the subsidy level incites consumers to postpone the replacement of their vehicles aged $\eta - 1$, while they would have replaced them if the subsidy level had been lower.

**Proposition 1** Let $\tilde{r}$ be a subsidy level satisfying

$$\tilde{r} \geq p - q_2 \theta_i + C_2(0) - C_2(\eta),$$

that is, consumer of type $i$ replaces her vehicle of age $\eta$ in period 2. Suppose also that the inequality in (4) holds true for such $\tilde{r}$.
If the subsidy level is set at \( \tilde{r} + s_i \), where

\[
s_i = \frac{1}{\beta} (C_1(\eta - 1) + \beta(p - q_2.\theta_i + C_2(0) - \tilde{r} - V_{i,2}(1)) - p + q_1.\theta_i - C_1(0)),
\]

then the same consumer would postpone the replacement of her vehicle from period 1 to period 2.

**Proof.** Suppose that for a given subsidy, there are consumers who replace their vehicle of age \( \eta \) in the second period. Also, assume that among these consumers, there is a subgroup who replace their vehicle aged \( \eta - 1 \) in the first period with the same subsidy, that is, the inequality in (4) holds true. We need to show that when the subsidy exceeds a certain level, some consumers in such a group prefer to keep their vehicle aged \( \eta - 1 \) in the first period and replace it at age \( \eta \) in the second period.

Suppose that the subsidy is \( \tilde{r} \). For consumers who replace their vehicle at age \( \eta \) in period 2 at this subsidy level \( \tilde{r} \), it holds that

\[
C_2(\eta) \geq p - q_2.\theta_i + C_2(0) - \tilde{r},
\]

or equivalently,

\[
\tilde{r} \geq p - q_2.\theta_i + C_2(0) - C_2(\eta).
\]

(5)

Note that, if in the second period, the replacement happened for a subsidy level \( r \), then it would also happen at higher levels. Therefore, 5 holds for all \( r \geq \tilde{r} \).

Besides, since vehicle \( \eta \) is replaced at subsidy level \( \tilde{r} \), \( V_{i,2}(\eta) \) reads

\[
V_{i,2}(\eta) = p - q_2.\theta_i + C_2(0) - \tilde{r}.
\]

Therefore, the inequality 4 turns into the following:

\[
C_1(\eta - 1) + \beta(p - q_2.\theta_i + C_2(0) - \tilde{r}) \geq p - q_1.\theta_i + C_1(0) + \beta.V_{i,2}(1)
\]

If the value of \( \tilde{r} \) increases enough, say to \( \tilde{r} + s_i \), the direction of the above inequality changes, that is, the consumer will no longer replace the vehicle aged \( \eta - 1 \) in the first period. \( s_i \) can be simply characterized as following:

\[
s_i = \frac{1}{\beta} (C_1(\eta - 1) + \beta(p - q_2.\theta_i + C_2(0) - \tilde{r} - V_{i,2}(1)) - p + q_1.\theta_i - C_1(0)).
\]

This result illustrates the idea that a subsidy program may have the opposite effect than the intended one, that is, it also benefits those who would have replaced their vehicles without the subsidy.\(^1\)

Now, we extend the model to \( T \) periods, with the subsidy being available till \( T_s < T \). Suppose that in period \( t \), consumer \( i \) has a vehicle of age \( j < \eta \), say \( j = \eta - \tau \), where \( \tau \) is the number of periods she needs to wait to become eligible for the subsidy program, with \( \tau = 1, \ldots, \eta - 1 \). Assume that the consumer’s valuation is so high that she replaces the vehicle in period \( t \), while she is not eligible to cash the subsidy. Also, assume that a subsidy is available in period \( t + \tau \) and that the consumer is forward-looking, that is, that she anticipates her eligibility for the subsidy, \( r = \tilde{r} \), in period \( t + \tau \). We are interested in characterizing a threshold for the subsidy at which the consumer postpones the replacement from period \( t \) to \( t + \tau \) to benefit from the subsidy.

When the consumer replaces her vehicle at period \( t \), her current and discounted cost-to-go from period \( t \) to \( t + \tau \) is as follows:

\[
C_t(0) + p - q_t.\theta_t + \sum_{a=1}^{\tau} \beta^a C_{t+a}(a)
\]

\(^1\) This result is familiar in price promotions on frequently purchased products. Indeed, a discount that is offered to attract non-users of the product or buyers of competitive brands also benefits regular users who would have bought the product anyway. Worse, these regular users can also buy during the promotion for later consumption, inflicting a double loss in revenues to the firm.
On the other hand, if she decides to wait until period \( t + \tau \) to be able to cash the subsidy when she replaces at age \( \eta \), then she will incur the following cost:

\[
\beta^{\tau} \left( p - q_{t+\tau,0} \beta_{t+\tau} - \hat{r} + C_{t+\tau}(0) \right) + \sum_{a=0}^{\tau-1} \beta^a C_{t+a}(\eta - \tau + a),
\]

where the first part is the discounted cost of purchasing a new vehicle in period \( t + \tau \) and the second part is the discounted cost of keeping the vehicle for \( \tau - 1 \) periods. Note that at period \( t + a, a = 1, \ldots, \tau - 1, \) the age of the vehicle is \( \eta - \tau + a \). Since we assumed that the consumer replaces at period \( t \) when the subsidy is \( \hat{r} \), the cost in 6 is lower than in 7. However, if the government increases the subsidy from \( \hat{r} \) to \( \hat{r} + s(i, \tau) \) where \( s(i, \tau) \) is given by

\[
s(i, \tau) = \frac{1}{\beta^{\tau}} \left( \beta^{\tau} \left( p - q_{t+\tau,0} \beta_{t+\tau} - \hat{r} + C_{t+\tau}(0) \right) + \sum_{a=0}^{\tau-1} \beta^a C_{t+a}(\eta - \tau + a) \right. \\
\left. - C_t(0) - p + q_t \theta_t - \sum_{a=1}^{\tau} \beta^a C_{t+a}(a) \right),
\]

then, the consumer keeps the vehicle for \( \tau \) periods and replaces at period \( t + \tau \).

### 4 Empirical results

In this section, we illustrate our general model using some available data and run a sensitivity analysis to assess the impact of the main model’s parameter values on the results.

To run our model, we need to estimate some parameter values and to fit some functions. Recall that our model involves the following 13 parameters:

- Duration of the subsidy program : \( T^s \)
- Planning horizon : \( T \)
- Average life duration of a vehicle : \( \omega \)
- Discount factor : \( \beta \)
- Eligibility age for a subsidy : \( \eta \)
- Minimum age for replacement : \( \zeta \)
- Travel demand per year in kilometers : \( D \)
- Vehicle price in thousands of dollars : \( p \)
- Gasoline price (\$/liter) : \( O_t \)
- \( CO_2 \) emissions per liter : \( e \)
- Number of new buyers : \( n_t \)
- Subsidy level : \( r \)
- Trade-in valuation in thousands of dollars : \( \theta \)

To start with, we shall solve the problem for a base-case scenario characterized by the following parameter values:

\[
T^s = 3, \quad T = 10, \quad \omega = 14, \quad \beta = 0.9, \quad \eta = 10, \\
p = 33.5, \quad O_t = 1, \quad e = 2.6, \quad n_t = 0, \quad D = 25000,
\]

where we let the subsidy rate and trade-in valuation have different values to capture the impact of the subsidy program in different market segments, that is,

\[
r \in [0, 4000], \quad \theta \in \{22, 23, \ldots, 38\}.
\]
In order to account for consumer heterogeneity, we follow Huang et al. (2014), where the net trade-in valuation is assumed to be normally distributed, with a mean $E(\theta) = 30,000$ and standard deviation $\sigma = 4,000$. We constrain the values of $\theta$ to $E(\theta) \pm 2\sigma$ and, also as in Huang et al. (2014), we fix $p = 33,500$. Following scrappage programs in some European countries such as France, Spain, and Ireland, we assume the vehicle’s subsidy eligibility age to be 10 years. Note that, while the duration of the subsidy program is 3 years, the (consumer’s and government’s) planning horizon is longer, i.e., 10 years. Further, in order to identify the number of vehicles with different ages at the initial date, we use the density distribution of vehicles aged 1 to 14 provided in Engers et al. (2012).

To run the model, we need to give values to $f(t - j), m(j)$ and $df(j)$. To estimate $f(t - j)$, we follow Spitzley et al. (2005), where the miles per gallon (MPG) for passenger cars is assumed to increase 1% yearly with respect to 22.8 MPG in 1995. Then, we use the MPG data to estimate $f(t - j)$ as follows:

$$f(t - j) = \frac{3.8}{1.6} \left( \frac{1}{\text{MPG}(t - j)} \right),$$

which results in fuel consumption in liters per kilometer. To estimate the maintenance cost $m(j)$, we took data from YourMechanic (2016), where this cost is given per block of 25,000 miles driven (see Table 2).

<table>
<thead>
<tr>
<th>Mileage</th>
<th>Total Maintenance Cost per 25k Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-25,000</td>
<td>$1,400</td>
</tr>
<tr>
<td>25,000-50,000</td>
<td>$2,200</td>
</tr>
<tr>
<td>50,000-75,000</td>
<td>$3,000</td>
</tr>
<tr>
<td>75,000-100,000</td>
<td>$3,900</td>
</tr>
<tr>
<td>100,000-125,000</td>
<td>$4,100</td>
</tr>
<tr>
<td>125,000-150,000</td>
<td>$4,400</td>
</tr>
<tr>
<td>150,000-175,000</td>
<td>$4,800</td>
</tr>
<tr>
<td>175,000-200,000</td>
<td>$5,000</td>
</tr>
</tbody>
</table>

The emissions deterioration factor is assumed to increase by 0.09 from age 1 to 10 and then remain constant for the rest of the vehicle’s life (Zyl et al., 2015):

$$df(j) = \begin{cases} 
1 & \text{for } j = 0; \\
 df(j - 1) + 0.09 & \text{for } 1 \leq j \leq 10 \\
 df(10) & \text{for } j \geq 11. 
\end{cases}$$

Considering the functions and parameters described above, we evaluate the effect of the subsidy on replacement decisions and on pollution from usage. Figures 1 and 2 show how the subsidy affects the replacement decisions of the different groups of consumers. The average keep age demonstrates how long vehicles are kept by consumers on average during the horizon.

One main takeaway is that increasing the subsidy does not necessarily result in earlier replacements for all groups of consumers. This surprising result stems from the subsidy program having an indirect effect on some consumers, who will prefer to postpone their replacement to benefit from the subsidy instead of paying the full price themselves. By now we are able to answer the first question of this research, namely, what is the effect does the subsidy have on heterogeneous consumers’ decisions? The answer differs for consumers whose trade-in valuation is high from those with a low valuation. When consumers have a low valuation, say 25,000 or 27,000, as in Figure 1, increasing the subsidy induces earlier replacements, which results in a decreasing trend of the average keep age. This intuitive result holds true for all “low-value” consumers whose trade-in valuation ranges between 22,000 and 31,000. In contrast, “high-value” consumers with a trade-in valuation higher than $31,000, say 35,000 or 37,000, as in Figure 2, respond to an increase in the subsidy by replacing later. Tables 3 and 4 clarify this behavior in high-value consumers more precisely. Consumers with a valuation of 35,000 replace vehicles aged 8 or 9 at subsidy levels under 1,230 and 680 respectively. However, above thresholds, they keep these vehicles until age 10 to benefit from the subsidy. Similarly, consumers with a valuation 37,000 postpone replacing as shown in Table 4.
So far, we have studied separately the reaction of different groups of consumers to a variation in the subsidy level. The next step is to mix consumer groups to find the subsidy’s environmental effect and market impact in the presence of heterogeneity. These are shown in Figures 3 and 4. According to Figure 3a, the effect of increasing the subsidy on the total pollution by all consumer groups is decreasing, despite some ups and downs. When consumers are divided into low-value and high-value groups, this result does not necessarily hold true. More precisely, when consumers have a low valuation ranging between 22,000 and 31,000, the trend is downward (Figure 3b). However, when we only take into account high-value consumers whose valuation is between 32,000 and 38,000, the subsidy has a negative effect on the environment (Figure 3c).

<table>
<thead>
<tr>
<th>Vehicle age</th>
<th>Subsidy range to replace</th>
<th>Subsidy range to keep</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>$r &lt; 1230$</td>
<td>$r \geq 1230$</td>
</tr>
<tr>
<td>9</td>
<td>$r &lt; 680$</td>
<td>$r \geq 680$</td>
</tr>
</tbody>
</table>
Table 4: Replacement decision of consumers with a valuation of 37,000 for a vehicle aged 8 and 9

<table>
<thead>
<tr>
<th>Vehicle age</th>
<th>Subsidy range to replace</th>
<th>Subsidy range to keep</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>( r &lt; 1830 )</td>
<td>( r \geq 1830 )</td>
</tr>
<tr>
<td>9</td>
<td>( r &lt; 930 )</td>
<td>( r \geq 930 )</td>
</tr>
</tbody>
</table>

Similarly, the effect of the subsidy on the total number of replacements during the planning horizon is not the same for high-value and low-value consumers. While increasing the subsidy increases replacements in low-value consumers (Figure 4a), it has no effect on new vehicle purchases by those with a high valuation (Figure 4b). Indeed, the subsidy only delays new vehicle purchases by high-value consumers and does not have any impact on the total number of cars replaced by them. Consequently, older vehicles remain on the roads longer, which is environmentally harmful.

Now, we turn to the second question of the paper, namely, what is the effect of subsidy on the environment when consumer heterogeneity is taken into account? The rationale behind this question is both methodological and practical. If consumers are indeed heterogeneous, then retaining a homogeneous representation would lead to, at best, a very rough approximation of their behavior, and at worst, to a completely wrong result. Practically speaking, replacement predictions based on a homogeneous market-assumption will be erroneous and misleading to the public decision maker attempting to design a subsidy program. To obtain a hint of how the results differ under the two market compositions, that is, homogeneous and heterogeneous consumers, we run our model under the two scenarios. To do so, suppose that heterogeneous consumers are
uniformly distributed over the set \( \Theta \in \{22,000; 23,000; \ldots; 38,000\} \) of trade-in values, and assume that, in a homogeneous setting, the only consumer group has a net trade-in value of \( \tilde{\theta} = 30,000 \), which is the middle point in the set \( \Theta \). In Figure 5, we plot the values of the stock of pollution (panel (a)) and the number of vehicle replacements (panel (b)) for different subsidy level values. From Figure 5a, we see that giving a $2,000 subsidy leads to a reduction in pollution of \( 2 \times 10^8 \) kilograms (from \( 8.7 \times 10^9 \) to \( 8.50 \times 10^9 \)) when we assume consumer homogeneity and to a much lower decrease, that is, \( 3 \times 10^7 \) kilograms (from \( 8.82 \times 10^9 \) to \( 8.79 \times 10^9 \)) under the heterogeneity assumption. Such a large difference is observed for all other subsidy values. Similarly, we obtain very different estimates for the number of vehicle replacements under the two market compositions (see Figure 5b). In particular, for a subsidy of $2,000, the number of additional new vehicle purchases with respect to a no-subsidy case is estimated at 18,500 and 1,088 when consumers are assumed to be homogeneous and heterogeneous, respectively. By any measure, this difference is huge. The immediate implication of these comparative results is that pollution reduction and new vehicle purchases caused by a scrappage program would be highly exaggerated if the regulator retained a mean trade-in valuation to predict the impact of the program.

We also compared the cost of the program under both assumptions and the results are exhibited in Figure 6. As discussed before, accounting for heterogeneity makes it possible to assess the magnitude of high-value consumers who replace their vehicle before it reaches the program eligibility age, in the absence of the program. It enables us to measure the money wasted on these consumers who do not need the subsidy as an incentive to replace their vehicle. Figure 6b illustrates that in our case study, up to 40% of the total subsidy spending is invested in consumers with a high valuation. This result is in line with those in Hoekstra et al. (2015) and Li and Linn (2013), where it has been empirically shown that 60% and 45%, respectively, of the subsidies in Cash for Clunkers went to households that would have replaced their vehicle even without the program. As stated in Dill (2004), Kim et al. (2004), and Lenski et al. (2010), although scrappage programs improve both the environment and the car market overall, their efficiency may not be what one might imagine. Now, in these papers it is pollution from car production that was identified as the main cause of inefficiency in the scrappage program. Here, by considering consumer heterogeneity, we are able to better estimate the positive effect of such a program on pollution emissions due to vehicle usage. This leads to a better understanding of the efficiency of scrappage programs by scholars and policy makers.

Finally, we conduct a sensitivity analysis to assess the robustness of our results with respect to changes in some of the parameter values. The retained parameters and their values are listed below:

- Planning horizon: \( T \in \{9, 10, 11\} \),
- Duration of the subsidy program: \( T^* \in \{2, 3, 4\} \),
- Discount factor: \( \beta \in \{0.85, 0.9, 0.95\} \),

![Figure 5: Effect of subsidy on a) stock of pollution and b) total replacements, for homogeneous and heterogeneous consumers](image)
The results can be summarized as follows:

1. Increasing the value of $T$ leads to an increase of a lower magnitude on the stock of pollution. For instance, increasing $T$ from 9 to 10 years, which corresponds to an increment of 11%, leads to an increase in the pollution stock of roughly 8.65%. In any event, we see in Figure 7 that varying $T$ only has a quantitative impact on the stock of pollution, without affecting the qualitative behavior.

2. Varying the duration of the subsidy program seems to have very little effect on the stock of pollution (see Figure 8). This robust result tells a policy maker wishing to design a scrappage program that this parameter is of little importance when it comes to environmental performance.

3. Despite the non-monotonic behavior observed in the stock of pollution when varying the discount factor (only for low subsidy values), it is safe to state that, qualitatively speaking, increasing $\beta$ has little effect on the result (see Figure 9). The quantitative change is, more or less, of the same order of magnitude as the change in $\beta$.

4. From Figure 10 we see that varying the eligibility age $\eta$ has a very low effect on the stock effect, in relative terms, when the subsidy level is low. For higher subsidy values, the impact is of the same order of magnitude as the change in $\eta$.

5. The relative impact of increasing the minimum age of replacement on the pollution stock is rather small. Indeed, changing $\zeta$ from 7 years to 9, which is a large boost, leads to an increase in the stock of pollution of less than 3% (see Figure 11).

6. Increasing the technology progress rate by 50%, that is, from 0.008 to 0.012, leads to an increase in the stock of pollution of roughly 10% (see Figure 12).

7. Finally, for all subsidy levels, a larger travel demand induces an increase in pollution of the same magnitude (see Figure 13).

In summary, the main takeaway of our sensitivity analysis is that varying the parameter values does not affect the qualitative results, that is, we only see parallel shifts in the curves. Quantitatively speaking, the impact on pollution is at most of the same magnitude as the change in the parameter values. The conclusion is that our results are clearly robust.
Figure 7: Effect of subsidy on stock of pollution for different values of $T$

Figure 8: Effect of subsidy on stock of pollution for different values of $T^a$

Figure 9: Effect of subsidy on stock of pollution for different values of $\beta$

Figure 10: Effect of subsidy on stock of pollution for different values of $\eta$

Figure 11: Effect of subsidy on stock of pollution for different values of $\zeta$

Figure 12: Effect of subsidy on stock of pollution for different values of $f(t-j)$
Conclusion

In this paper, we applied a dynamic programming model to investigate the effect of vehicle scrappage programs on strategic consumers’ replacement decisions. Assuming that consumers are heterogeneous in terms of their net trade-in valuation, we examined the impact on different consumer groups of changing the government payment level. We theoretically show that under certain circumstances, increasing the government subsidy may have an adverse effect on some consumers. In particular, when the government payment exceeds a certain threshold, forward-looking consumers with a high net trade-in valuation prefer to delay their replacement decision until their vehicle become eligible for the program.

Our simulation results also demonstrate that while a scrappage program induces low-value consumers to scrap their old vehicle earlier, it may also motivate high-value consumers who would have replaced their vehicle even without the subsidy to replace it latter. Therefore, in addition to wasting the subsidy, delaying the decision to replace in high-value consumers is also harmful to the environment degrades the environment too. These results emphasize the importance of accounting for heterogeneity in consumer valuation when a subsidy program is designed. We find that net benefits of scrappage programs could be overestimated when consumers are taken to be homogeneous. More precisely, ignoring heterogeneity and relying only on the mean group of consumers results in higher estimates of pollution reduction and new vehicle purchases when calculating program benefits. These results are in line with Li and Linn (2013), where 45 percent of the Cash for Clunkers program expenditure is estimated to be spent on those who would have replaced even without the program.

Two challenging extensions to our work are worth considering. First, our approach aimed at providing the regulator with results describing the impact of different subsidy levels on the outcomes (pollution and replacement). It would be interesting to give the regulator a more strategic role by assuming, for instance, that it plays as the leader in a non-cooperative game where consumers are followers reacting to the leader’s subsidy announcement. The main challenge when computing Stackelberg equilibria is in solving the resulting high-dimensional dynamic program. Second, it is worth attempting to relax the assumption that the subsidy is constant over time and to introducing a time-varying one. Where needed, this would give the regulator a tool to smooth replacements over time.
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